

Choice of timescale and its implications for longitudinal cognitive aging research (work in progress)

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Background

- In longitudinal cognitive aging research (for example, cohort study), generalized linear mixed models (GLMM) are widely used to describe the change over time of outcomes and association with risk factors
 - With continuous outcomes, linear mixed models (LMM) are often used
- However, the timescale seems an arbitrary choice in the LMM literature
 - Time on study
 - Age
 - Age + adjust for baseline age

Aims

Use simulations to evaluate how different parameterizations of timescales affect longitudinal cognitive aging research results

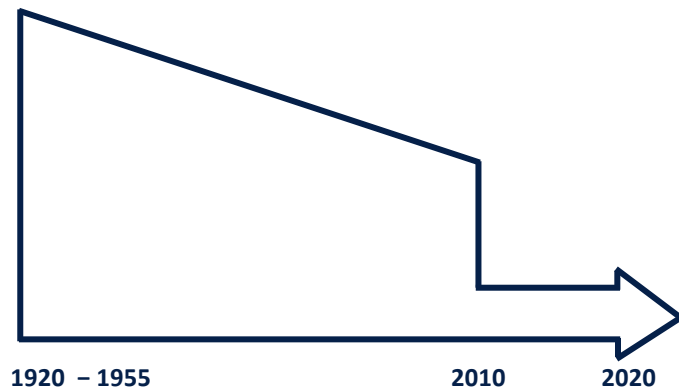
- Estimated effects of education on the rate of cognitive decline
- Estimated within- and between-person age effects

Simulation- Procedure

1. Generate a hypothetical cohort
2. Create an analytical sample
3. Generate data according to data generating rules
4. Estimate parameters of interest
 - Timescale: study time / age / age + adjustment
5. Repeat 1-4 for multiple iterations (B=1000)
6. Quantify magnitude of bias
 - Percentage bias = $\frac{\hat{\beta} - \beta}{\beta} * 100\%$

Simulation- Hypothetical cohort study

- Hypothetical cohort:
 - Study the effect of completion of high school education on late-life cognitive decline
 - $N = 50,000$ subjects born between 1920 and 1955 (uniformly)
 - In 2010, a random sample of $n = 2,000$ subjects who survived to late life (55-90 years old) enrolled in the study
 - $P(survival_i) = \frac{\exp(\gamma_{0t} + \gamma_1 educ_i + \gamma_2 U_i)}{1 + \exp(\gamma_{0t} + \gamma_1 educ_i + \gamma_2 U_i)}$, γ_{0t} selected to match the age distribution based on the 2010 Census
 - Cognitive function was measured on 10 waves over 10 years

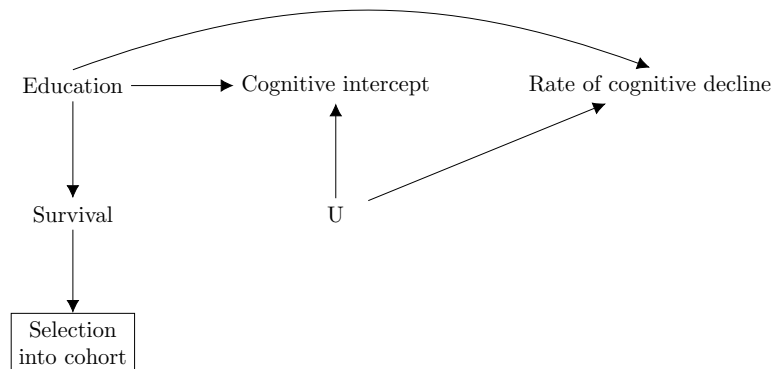


Simulation- Data generating rules

- Causal structures

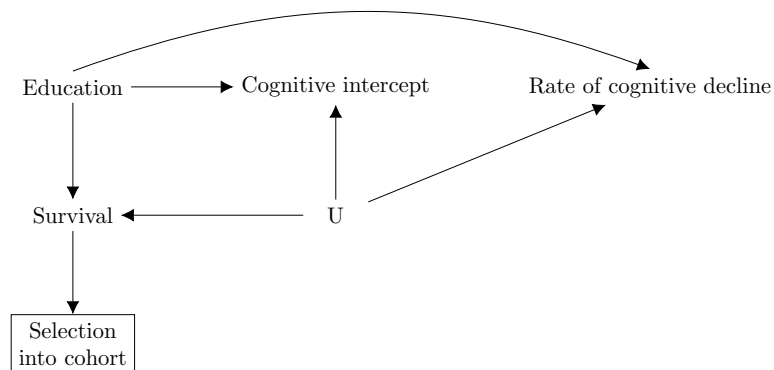
- Causal structure 1 (no bias anticipated)

- U is the unmeasured determinants of cognitive decline



- Causal structure 2 (selection bias)

- U is the unmeasured determinants of cognitive decline



Simulation- Data generating rules

- Unified causal structures
- Generation of repeated measures of cognitive function

- $$Y_{it} = \beta_{00} + b_{0i} + \beta_{01}educ_i + \beta_{02}U_i + \beta_{03}pe_{ij} \\ + (\beta_{10} + b_{1i} + \beta_{11}educ_i + \beta_{12}U_i) \times age_{it} + \epsilon_{ij}$$

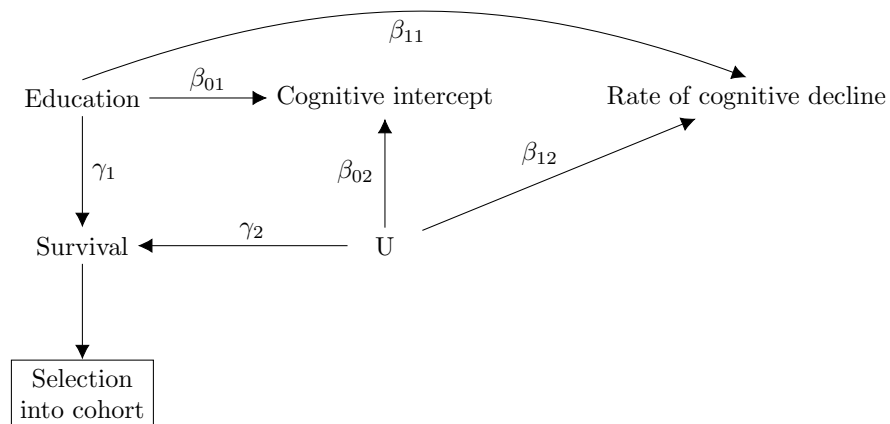
- Age as timescale

- $$\begin{bmatrix} b_{0i} \\ b_{1i} \end{bmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_0^2 & \rho\sigma_0\sigma_1 \\ \rho\sigma_0\sigma_1 & \sigma_1^2 \end{bmatrix}\right), \epsilon_{ij} \sim N(0, \sigma^2)$$

- $educ_i \sim \text{Bernulli}(0.4)$

- $U_i \sim N(0,1)$

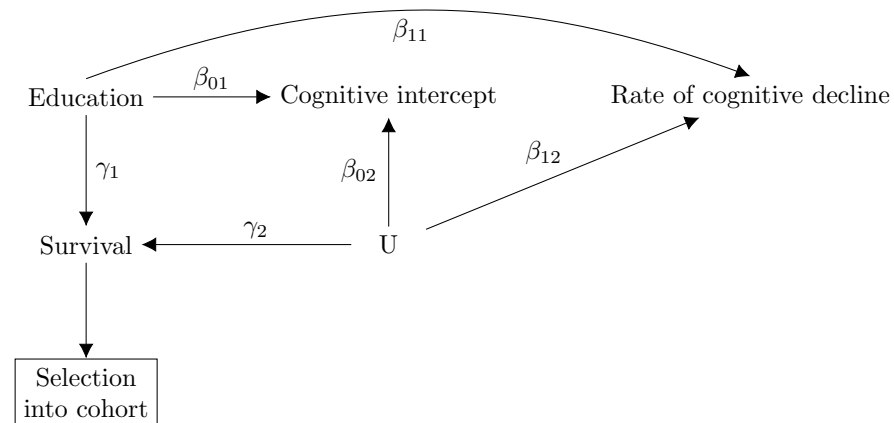
- True within-person age effect = true
between-person age effect



Simulation- Input parameter values

Causal structure	Cognitive intercept			Cognitive decline			Mortality	
	β_{00}	β_{01}	β_{02}	β_{10}	β_{11}	β_{12}	γ_1	γ_2
1: no bias anticipated	0	-0.05	-0.005	-0.05	-0.05	-0.005	$-\log(2)$	0
2: selection bias	0	-0.05	-0.005	-0.05	-0.05	-0.005	$-\log(2)$	$-\log(2)$

Parameter	Definition	Value
β_{03}	Practice effect	0.05
σ_0^2	Variance of random cognitive intercept	0.2
σ_1^2	Variance of random cognitive slope	0.005
ρ	Correlation between random intercepts and random slopes	0.01
σ^2	Variance of measurement error	0.7



Simulation- Model fitting

- Three parameterizations

Timescale	LMM model with a random intercept and slope
Time on study	$E[Y_{ij}] = (\beta_{00} + b_{i0}) + (\beta_{10} + b_{i1})time_{ij} + \beta_2 baseline_age_i + \beta_{02}educ_i + \beta_{03}pe_{ij} + \beta_{11}time_{ij} \times educ_i$
Age	$E[Y_{ij}] = (\alpha_{00} + v_{i0}) + (\alpha_{10} + v_{i1})age_{ij} + \alpha_{02}educ_i + \alpha_{03}pe_{ij} + \alpha_{11}age_{ij} \times educ_i$
Age + adjustment	$E[Y_{ij}] = (\gamma_{00} + u_{i0}) + (\gamma_{10} + u_{i1})age_{ij} + \gamma_2 baseline_age_i + \gamma_{02}educ_i + \gamma_{03}pe_{ij} + \gamma_{11}age_{ij} \times educ_i$

Simulation- Model fitting

- Three parameterizations

Timescale	LMM model with a random intercept and slope
Time on study	$E[Y_{ij}] = (\beta_{00} + b_{i0}) + \boxed{\beta_{10}} + b_{i1})time_{ij} + \boxed{\beta_2}baseline_age_i + \beta_{02}educ_i + \beta_{03}pe_{ij} + \beta_{11}time_{ij} \times educ_i$
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Sanity check 1: under the data generation rules, the coefficient for time on study and baseline age should be approximately the same

Sanity check 2: the coefficient for baseline age should be approximately zero

Simulation- Model fitting

- Three parameterizations

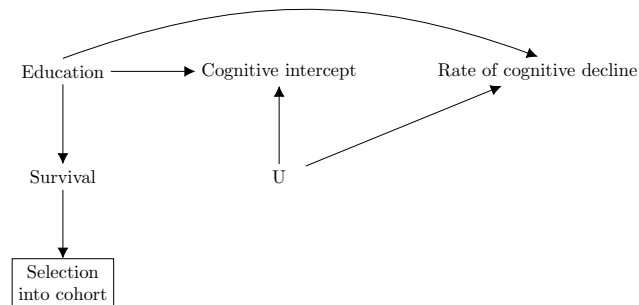
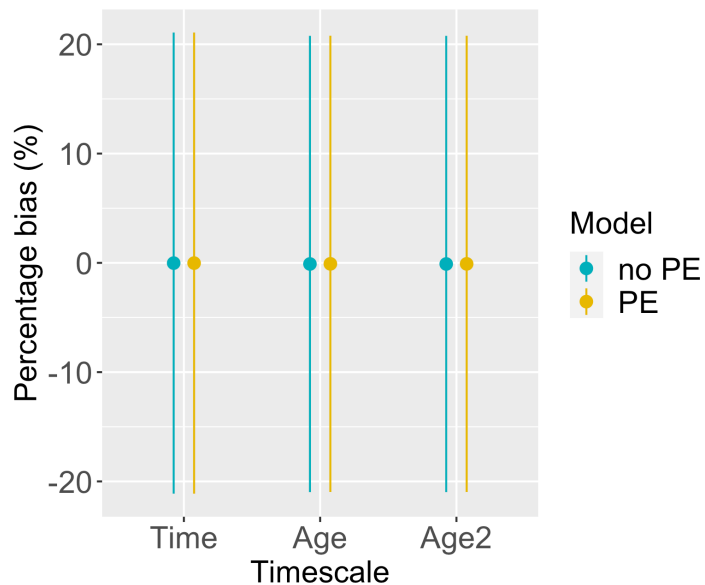
Timescale	LMM model with a random intercept and slope	Within-person age effect	Between-person age effect
Time on study	$E[Y_{ij}] = (\beta_{00} + b_{i0}) + (\beta_{10} + b_{i1})time_{ij} + \beta_2baseline_age_i + \beta_{02}educ_i + \beta_{03}pe_{ij} + \beta_{11}time_{ij} \times educ_i$	β_{10}	β_2
Age	$E[Y_{ij}] = (\alpha_{00} + v_{i0}) + (\alpha_{10} + v_{i1})age_{ij} + \alpha_{02}educ_i + \alpha_{03}pe_{ij} + \alpha_{11}age_{ij} \times educ_i$	α_{10}	α_{10}
Age + adjustment	$E[Y_{ij}] = (\gamma_{00} + u_{i0}) + (\gamma_{10} + u_{i1})age_{ij} + \gamma_2baseline_age_i + \gamma_{02}educ_i + \gamma_{03}pe_{ij} + \gamma_{11}age_{ij} \times educ_i$	γ_{10}	$\gamma_{10} + \gamma_2$

Simulation- Results

- Causal structure 1 (no bias anticipated)

Effect of education on cognitive decline

$\beta_{11} / \alpha_{11} / \gamma_{11}$

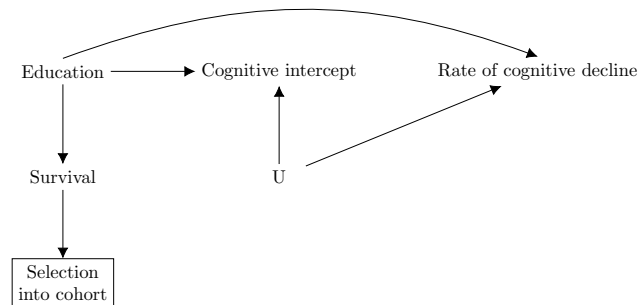


- Takeaway:

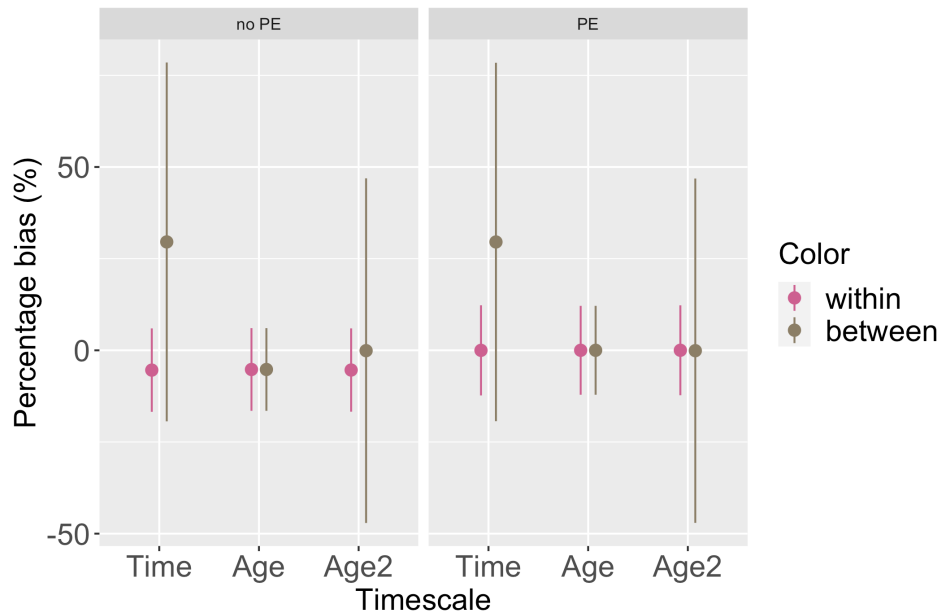
- In causal structure 1, the estimated effect of education on rate of cognitive change was unbiased, independent of timescale

Simulation- Results

- Causal structure 1 (no bias anticipated)



Within- and between-person age effects



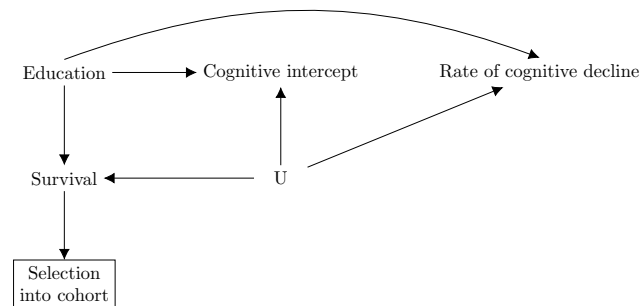
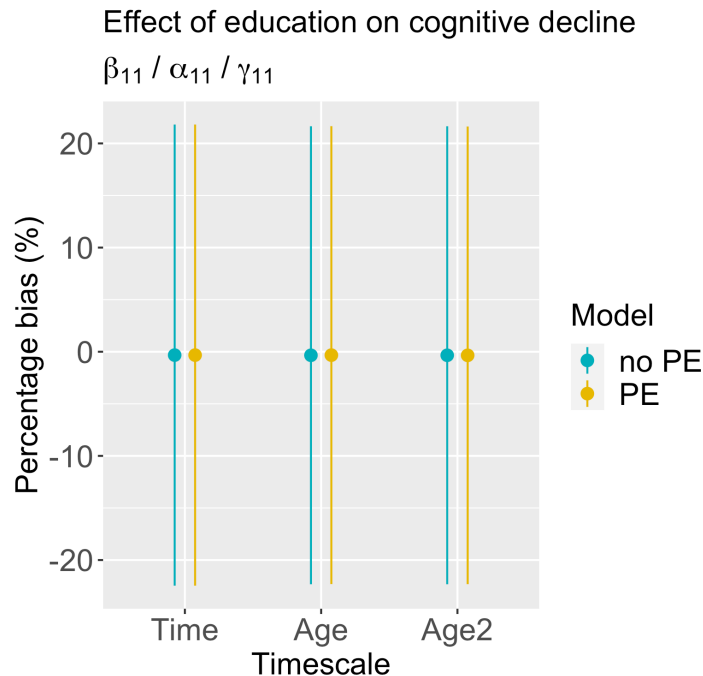
Time scale	LMM model with a random intercept and slope	Within	Between
Time	$E[Y_{ij}] = \beta_{10}time_{ij} + \beta_2baseline_age_i + \dots$	β_{10}	β_2
Age	$E[Y_{ij}] = \alpha_{10}age_{ij} + \dots$	α_{10}	α_{10}
Age2	$E[Y_{ij}] = \gamma_{10}age_{ij} + \gamma_2baseline_age_i + \dots$	γ_{10}	$\gamma_{10} + \gamma_2$

- Takeaway:

- In causal structure 1, all timescales gave unbiased estimates for within-person age effect. Time as timescale overestimated between-person age effect

Simulation- Results

- Causal structure 2 (selection bias)

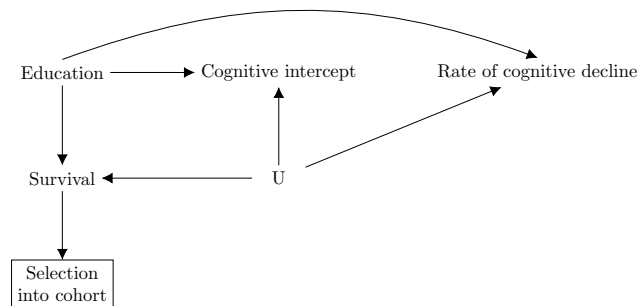


- Takeaway:

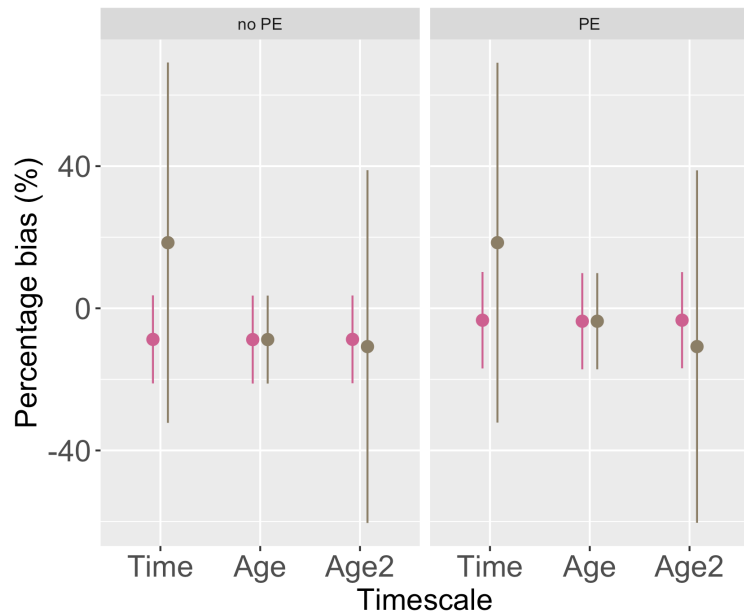
- In causal structure 2, the magnitude of bias with moderate input parameters was relatively small, but independent of timescale

Simulation- Results

■ Causal structure 2 (selection bias)



Within- and between-person age effects



Color
 ● within
 ● between

Time scale	LMM model with a random intercept and slope	Within	Between
Time	$E[Y_{ij}] = \beta_{10}time_{ij} + \beta_2baseline_age_i + \dots$	β_{10}	β_2
Age	$E[Y_{ij}] = \alpha_{10}age_{ij} + \dots$	α_{10}	α_{10}
Age2	$E[Y_{ij}] = \gamma_{10}age_{ij} + \gamma_2baseline_age_i + \dots$	γ_{10}	$\gamma_{10} + \gamma_2$

■ Takeaway:

- In causal structure 2, all timescales underestimated within-person age effect and the magnitude of bias was independent of timescale

Summary

- Under current data generating structures (true within- and between-person age effects are the the same), analytical timescale did not affect the magnitude of bias in within-person age effect
 - “Wrong” analytical timescale may give biased between-person age effect
- Analytical timescale did not affect the magnitude of bias in estimated effect of education on rate of cognitive decline
 - In the absence of bias, time on study and current age resulted in almost identical unbiased results
 - In the presence of selection bias, the magnitudes of bias were similar (and small)

Next steps- simulation

- Nonlinear age effect
- Cohort effect
- Healthy participation effect
 - A 75-year-old person who enrolls in a study may be healthier (higher cognitive level, lower rate of decline) than an otherwise comparable person who turns 75 after several years of follow-up
- Loss to follow-up

Next steps

- Assess and compare the associations between education and cognitive decline in the Health and Retirement Study and the Life After 90 Study under the following three scenarios:
 - Using time as the timescale
 - Using current age as the timescale
 - Using current age as the timescale and adjust for baseline age

Comments or suggestions?

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