Q2. If Y is not one of $X_1, X_2 \ldots X_k$, then $sup(Y) \le \min\{sup(X_1), sup(X_2), \ldots sup(X_k)\}$,
since $X_1, X_2, \ldots X_k$ are the k most frequent items in T

If Y is one of $X_1, X_2, \ldots X_k$,

~~There are~~ $\because$ Y is length-k itemset and $sup(Y)$ is less than or equal to its subset Z_{k-1}

$\therefore sup(Y) \le sup(Z_{k-1}) \le sup(Z_{k-2}) \ldots \le sup(Z_1)$ (Z_{k-1} is lenght $k-1$ itemset)

There must be $k-1$ itemsets whose supports are larger or equal than Y

Therefore, Y can only be X_k, which means $sup(Y) \le \min\{sup(X_1), sup(X_2), \ldots sup(X_k)\}$

Q3.

t_1	abc
t_2	abc
t_3	abc

$X = abc$, $sup(X) = 1$

$Y = abc$, $sup(Y) = 1$

$X \cap Y = abc$, $sup(X \cap Y) = 1$

$X \cup Y = abc$, $sup(X \cup Y) = 1$

The relationship should be $sup(X \cap Y) = sup(X \cup Y)$

Proof: $\because$ ~~$sup(X \cup Y) = sup(X) + sup(Y)$~~

$\because sup(X) = sup(Y) = sup(X \cap Y)$

$\therefore$ transactions in X are also in $X \cap Y$ and Y, $X = Y = X \cap Y$

$\therefore X \cup Y = X = Y = X \cap Y$

$\therefore sup(X \cup Y) = sup(X \cap Y)$