Particle Swarm Optimization for Sustainable Development in Zambia

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Problem Statement

With the United Nations (UN) predicting that the world's population will be over 9 billion by 2050, the strain on the Earth's resources will be significant. Concerns regarding the balance of human needs with ecosystem health have drawn attention to the concept of sustainable development. The International Conglomerate of Money (ICM) has asked us to help them understand how positive economic development can be achieved while still ensuring sustainable consumption of resources so that the environment will not be compromised for future generations. The solution that we propose offers insight into these problems.

Plan of Attack

Our objective is to develop a metric to determine the sustainability of a country and then to quantify the trade-off between maintaining ecosystem health and economic development.

We create a two-part metric to assess the country's ecological responsibility and socioeconomic development (the two main pillars of sustainability). Then, we design a Zambia-specific 20-year sustainable development plan by optimizing microeconomic and macroeconomic models of sustainability indicators.

Lastly, we project the plan for 20 years and use our two-part metric to evaluate its effect.

The UMAP Journal 36 (2–3) (2015) 151–170.

Assumptions

- We assume only air pollution to be important in quantifying net pollution of a developing country [Tucker 1995]. Because there are no readily-available data for water / ground pollution in most countries, this restriction allows us to quantify the relationship between pollution and our Ecological Impact Index.
- There are numerous trends in worldwide development of countries based on a variety of indicators (GDP, population, industry, etc.). Because these relationships generally hold true, we assume Zambia will follow these trends. This assumption allows for relationship quantification, between a number of indicators based on world-wide data, to predict Zambian growth from current values for each indicator.
- We assume that ICM funding over 20 years follows a discrete model rather than a continuous one (i.e., funding for Zambia is paid incrementally over the course of 20 years). This assumption allows us to simplify our optimization model by optimizing only a finite number of resource allocations over the 20-year period.
- While effects of money may be semi-erratic, we assume a deterministic instead of probabilistic effect of ICM spending. This assumption allows us to create a solvable and reproducible optimization function that gives a distribution of resource allocation based on an input spending cost by the ICM and desire for ecological over developmental growth.
- Because industry growth and decay in Zambia is small [World Bank 2015], we assume that without ICM funding, Zambia's net change in industry size would be 0 (that is, only redistribution of wealth between industries would occur). This assumption allows us to quantify the effect of ICM spending in Zambia and help determine what measures would give the ICM the most bang for their buck.
- Zambia is politically stable with little history of political corruption when receiving external funding. Thus, we assume that ICM funding would not be squandered but used in full for the programs for which ICM allots the money. This assumption allows us to quantify the effect of ICM spending on sustainable development.

Environmental Resource Model

Quantifying Pollution

A desirable measure of the ecological aspect of sustainability would be a weighted pollution index. Since not all pollutants damage the environment and people's health to the same degree, each would be individually weighted.

Global warming potential (GWP) is a measure of how much heat—which damages the environment through global warming—is trapped in the environment due to an atmospheric pollutant [Harvey 2009]. The GWP of various greenhouse gases is measured in comparison to the amount of heat trapped by the same mass of CO_2 gas (whose GWP is normalized to 1) over a specified time frame [Albritton et al. 1996]. The Intergovernmental Panel on Climate Change (IPCC) and Kyoto Protocol both use GWP measures as the de facto standard to measure emission damage when creating environmental policy [Smith and Wigley 2000]. We used the standard 100-year GWP values, shown in**Table 1** for the air pollutants for which we could get country emission data.

Table 1.

Global Warming Potentials (IPCC 2013) [Myhre et al. 2013].

Pollutant	100-year GWP
Carbon Dioxide (CO_2)	1
Methane (CH_4)	28
Nitrous Oxide (N_2O)	265
Perfluorocompounds	11,100
Hydrofluorocarbons	12,400
Sulfur Hexafluoride	23,500

For each country, we calculate pollution as

$$\text{Pollution} = \sum_{p}^{\text{Pollutants}} A_p W_p,$$

where A is amount of pollutant p emitted per capita and W is the damage-based weighting of that pollutant based on normalized GWP data.

We combine the pollution data with normalized nonrenewable energy use per capita data, with both weighted equally. We transform the sum onto a $[0, 1]$ scale, and subtract the value from 1 to find an Ecological Impact Index E for country i. So low E represents high pollution and high nonrenewable energy consumption, and vice versa for a high E. We run our Ecological Impact Index on all countries with available pollution and energy-use data and display the results in **Figure 1**.

Strengths and Weaknesses

Our metric E for ecological impact of a country is both easily calculable (when data are available) and encompasses the primary environmental impacts on a developing country.

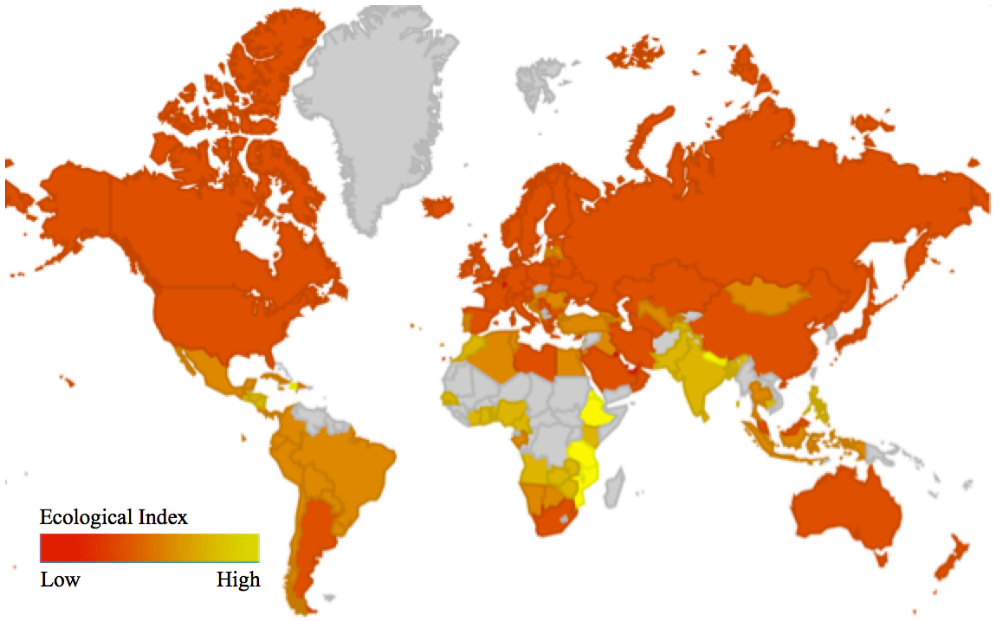


Figure 1. Worldwide view of country Ecological Impact Index (E) values. Countries in light grey (such as those in central Africa) have missing data.

This metric overlooks ground and water pollution in favor of air pollution. Because some underdeveloped countries suffer from a lack of clean water, this model should be readjusted (if sufficient data exists) to incorporate water pollution.

Development for Better Life Quality

Human Development Index

The Human Development Index, or HDI, is a composite measurement used to measure the quality of life in countries around the world. The HDI consists of real GDP per capita, life expectancy, adult literacy, and years of schooling, which are combined to give a single value between 0 and 1.

However, a major weakness of the HDI is that it uses GDP per capita while taking no account of income distribution [Stanton 2007]. If income is very unevenly distributed, then GDP per capita is a misleading and inaccurate measure of the financial well-being of the people.

Socioeconomic Development Index

To make up for the flaws of the HDI, we use an inequality-adjusted measurement of monetary well-being. We define $I(x)$ to be purchasing power (income in terms of local commodities) for the income distribution percentile x. We account for income inequality by weighting $I(x)$ with weight

$1 - x$. The product $A(x) = (1 - x)I(x)$ is an adjusted income distribution; we graph it in **Figure 2**.

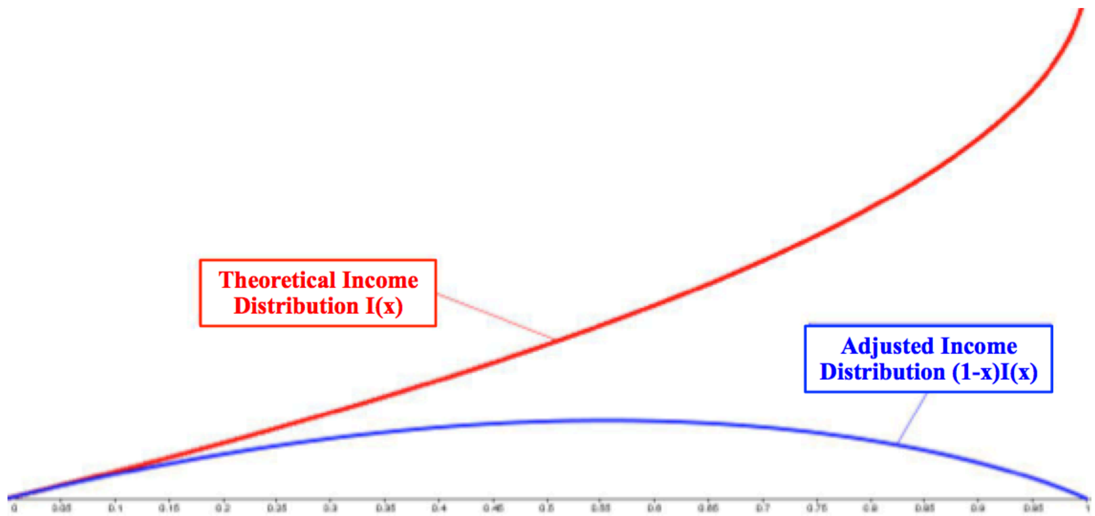


Figure 2. Graph of the theoretical income distribution $I(x)$ and adjusted income distribution $A(x)$

Let

- E be the the average years of education, and
- L be the average life expectancy.

We formulate a country's Socioeconomic Development Index D as

$$D = \frac{2}{5} \log \mathrm{Norm} \left(\int_0^1 I(x)(1 - x) \right) + \frac{2}{5} \mathrm{Norm}(L) + \frac{1}{5} \mathrm{Norm}(E),$$

where Norm indicates data normalization with respect to worldwide maxima and minima, a procedure utilized by the Human Development Index (HDI) [Kovacevic 2010]. We weight education less than the other two factors because it has less influence on socioeconomic development and because it already shares a strong correlation with those two values.

We calculate this Socioeconomic Development Index on all countries with available data and display the results in **Figure 3**.

Strengths and Weaknesses

Our model, by accounting for income inequality and share of income by percentile, gives more weight to a uniform distribution of wealth throughout the country, valuing equality over mean income. Additionally, because we integrate the product of economic share and inequality, we provide a more inclusive metric for measuring wealth than by separately calculating GDP and inequality. Thus, when this measure is combined with education and longevity, we have a more effective measure of the effect of wealth on

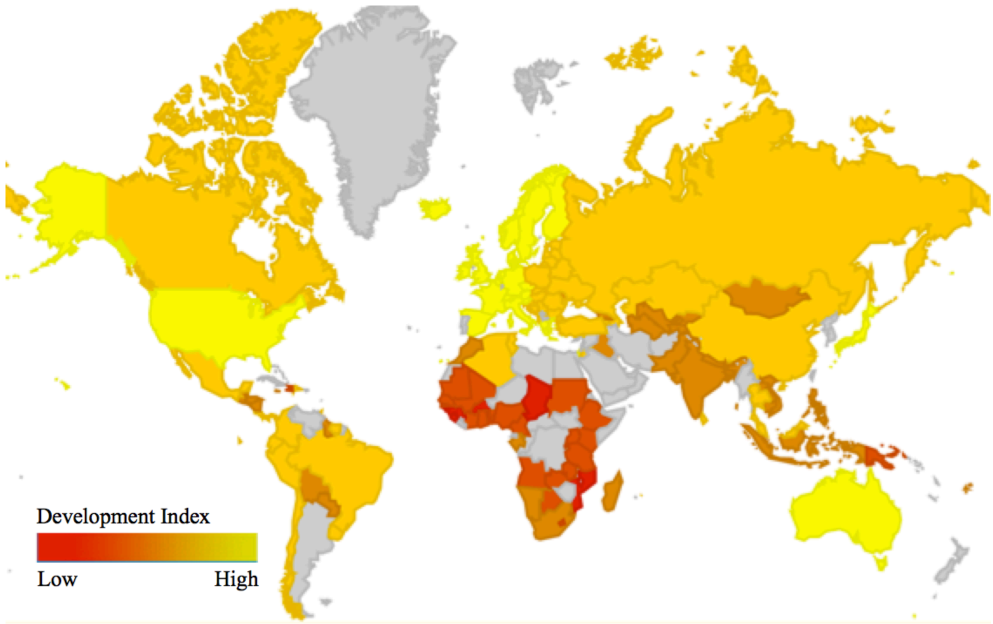


Figure 3. Worldwide view of our Socioeconomic Development Index (D) values. Countries in very light gray (such as Argentina and in central Africa) have missing data.

the daily socioeconomic well-being of the populace. Increasing this metric increases not just wealth but overall health and well-being.

Because this model combines economy and well-being into one value, it can be seen as weak compared to using two values. However, it is easier to optimize one value than two simultaneously. Because this metric combines only three sources of information, we omit other factors that have an unmeasured effect on health and well being.

It is very difficult to optimize a country's sustainability because there is often a trade-off between environmental protection and industrial development. This dilemma is highlighted in **Figure 4**, where nations from the UN's list of the 48 Least Developed Countries (LDC) [UNCTAD 2013] are clustered in the top left corner of low environmental damage but low human well-being.

Ecological-Economic Trade off

More-developed countries seem to have better life quality but much worse impact on the ecosystem. Therefore, we can hypothesize that as countries develop, they follow a relatively negative trend between D and E. An ideal configuration—perfect sustainability—would be at the point $(1, 1)$ in **Figure 4**, where a country is both economically sustainable and environmentally sustainable.

We rank the top 10 countries on our index and compare the rankings

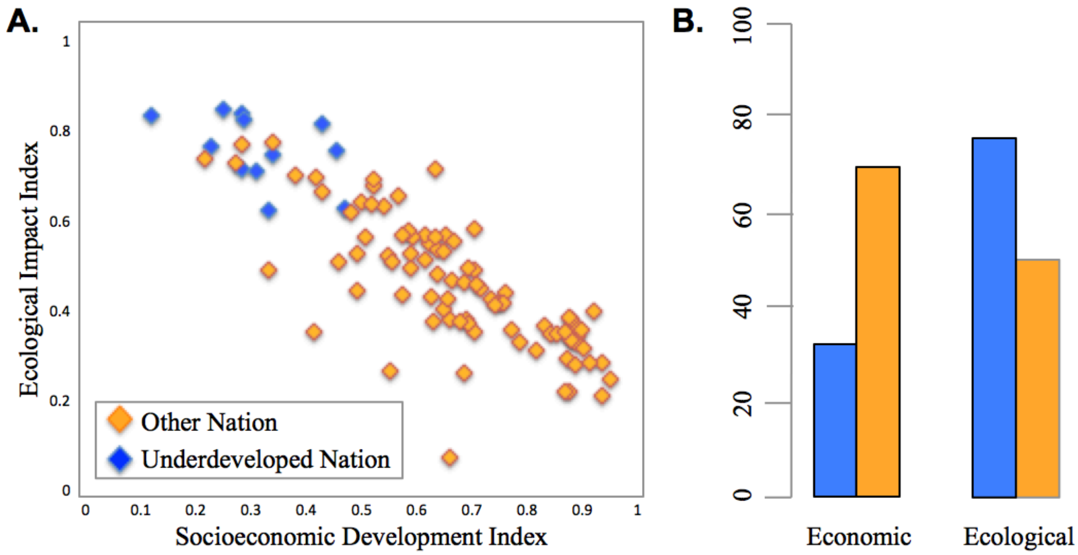


Figure 4. Tradeoff dynamics.
A: Socioeconomic Development Index (D) vs. Ecological Impact Index E.
B: Mean Index levels for underdeveloped nations (left) vs. other nations (right).

to the most recent Society Sustainability Index (SSI) rankings by the Sustainable Society Foundation (SSF) [van de Kerk and Manuel 2008]. The SSI includes three Well-being dimensions: Human, Environmental, and Economic. Similarly, our measure of sustainability includes an Ecological Impact (similar to the Environmental Well-being) and a Socioeconomic Development Index, which is comprised of elements in the SSI's Human Well-being Index (education, life expectancy). The SSF has stated that development towards sustainability requires an integrated approach that simultaneously focuses on Human Well-being and Environmental Well-being, and that from an anthropocentric point of view, Economic Well-being (GDP, etc.) should be just a means to achieve these goals [van de Kerk and Manuel 2012].

We rank only countries for which both we and the SSF have data. As seen in **Table 2**, 7 out of the top 10 countries on our Socioeconomic Development Index are among the top 10 on the SSF Human Well-being Index; and 7 out of the top 10 countries on our Ecological Impact Index are also found among the top 10 on the SST Environmental Well-being Index. So, the metrics that we have created affirm the results of UN-accepted rankings. Our sustainability model has a clear, easy-to-understand basis in trends historically followed by countries as they change from underdeveloped countries to developed countries.

Table 2.
Top 10 most sustainable countries according to various indexes.

Our D	SSF Human Well-being	Our E	SSF Enviro. Well-being
Norway	Finland	Haiti	Nepal
Iceland	Iceland	Ethiopia	Mozambique
Australia	Germany	Mozambique	Zambia
Switzerland	Japan	Tanzania	Tanzania
Netherlands	Sweden	Nepal	Kenya
Ireland	Denmark	Kenya	Cameroon
Sweden	Norway	Nigeria	Ethiopia
Denmark	Austria	Togo	Tajikistan
Germany	Hungary	Bangladesh	Benin
Finland	Ireland	Zambia	Haiti

20-year Plan for Zambia

Zambia, an underdeveloped, landlocked country in sub-Saharan Africa, is one of the 48 Least Developed Countries, and currently has $D = .34$ and $E = .74$. It occurs in both the third (E) and fourth (SSF Enviro. Well-being) columns of **Table 4**.

We propose a set of programs, policies, and aid that ICM could fund to promote sustainable development in Zambia over 20 years. Key features that we must consider in developing Zambia include [World Bank 2015]:

- High fertility and birth rates
- Poor health care and high mortality rates
- Significant understaffing of doctors
- Rampant levels of HIV/AIDS (over 15% of the population)
- High income disparity (the top 10% control 47% of the income, with Gini coefficient = 0.6)
- Agriculture, mining, and tourism are a significant portion of Zambian industry.

Our plan is built on the two indices that we created, D and E.

Population Growth and Economic Development

Knowing where to focus money and aid in a developing country is essential to the efficiency of country development. To optimize the ICM's "bang per buck" ratio, we create a cellular automaton model to determine optimal aid location with respect to both population density and wealth.

Wealth distribution and population dynamics are intertwined. To model the growth of population and wealth over time, we create a paired system of iterated microeconomic and agent-based behavioral functions to define the rules of of a two-dimensional cellular automaton, looking at the effect of various starting paradigms. We model the diffusion of both wealth ($W(n)$ in year n) and population ($P(n)$) as cofactors of each other, based on an adjusted version of Epstein and Axtell's agent-based sugarscape model for social simulation [Epstein and Axtell 1996].

We write recursive (difference) equations for W and P with time step 1 year. We let B be the birth rate and L be the life expectancy. Both B and L are functions of $W(n)$ and hence change over time. Thus, we can write iterative equations as follows:

$$W_{n+1} = (1 - 8\epsilon)W_n + \epsilon \sum_{i=0}^{\text{neighbors}} W_{n,i},$$

$$P_{n+1} = P_n + P_n(B(W_n) - \frac{1}{L(W_n)}) + \Delta\text{Migration},$$

where

- W_n is the wealth in Zambia in year n;
- $W_{n,i}$ is the wealth in neighbor i of Zambia in year n;
- P_n is the population in Zambia in year n;
- $B(W)$ is the birth rate as a function of wealth, calculated from a best-fit logarithmic curve;
- $L(W)$ is life expectancy as a function of wealth, using the Preston curve for life expectancy as a function of national GDP (see the **Appendix**) [Preston 1975];
- ϵ is the coefficient of employment (the percentage of wealth of a cell's neighbors that is distributed in Zambia via employment);
- β is the percentage of the population that migrates each iteration;

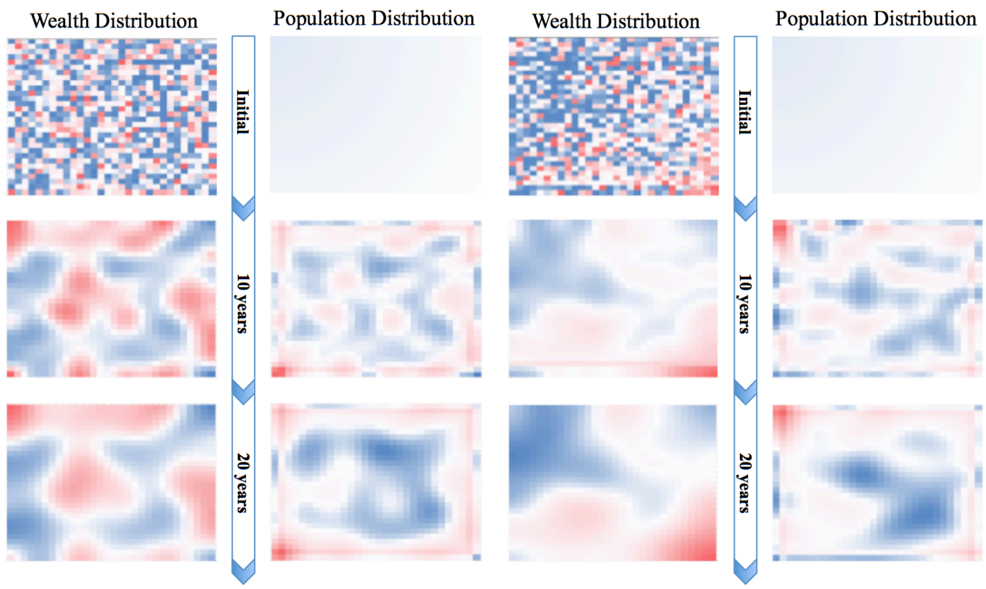
and

$$\Delta\text{Migration} = \beta P_n \left(1 - \frac{W_n}{\sum W_{n,i}}\right) - \beta \sum^{\text{neighbors}} \frac{W_n}{\sum W_{n,j}} P_{n,i}. \qquad \textbf{(1)}$$

Wealth is given to each neighbor as a fraction of current wealth, and population grows as both a function of itself and a function of migration. Net migration is based on the migration percentage β of the population that exit the cell and enter the neighboring cells, in proportion with which neighbors have the highest wealth per person (GDP/capita).

Figure 5a shows the results of our cellular automata model for a random initial wealth distribution and **Figure 5b** shows results for a biased initial wealth distribution (concentrated in the bottom right corner) after 10 and

20 years, allowing us to see the spatial-economic equilibrium of wealth as a function of time. We see the natural segregation of wealth, and the corresponding segregation of population density, over the course of time, indicating natural increase in wealth inequality given no external factors.



(a) Random initial wealth distribution. **(b)** Trended initial wealth distribution.

Figure 5. Comparison of two initial wealth distributions: (a) random and (b) biased, for a uniform initial population distribution. Red indicates higher values of wealth / population while blue indicates lower values.

We see that wealth segregates itself to areas of lower population, thus increasing income disparity. To counter this with external aid, we need to focus efforts on areas of increased poverty, which will adjust the seed distribution from biased (**Figure 5(b)**) to more random (**Figure 5(a)**), allowing for greater impact of funding throughout Zambia.

A further suggestion is to introduce a progressive tax, which could be beneficial for Zambia due to its uneven wealth distribution [Diamond and Saez 2011]. This income inequality is reflected in Zambia's low D value. By implementing a progressive tax, income disparity could be reduced and national GDP could be increased.

Iterated Function Map

Through the cellular automata model, we can geographically locate where the ICM should focus its efforts. However, we also need to know in exactly which sectors of Zambia's economy and society the ICM can invest . To accurately help plan Zambia's 20-year plan for development, we create a map of the main factors that influence the Socioeconomic (D) and Ecological (E) Indices of Zambia (**Figure 6**).

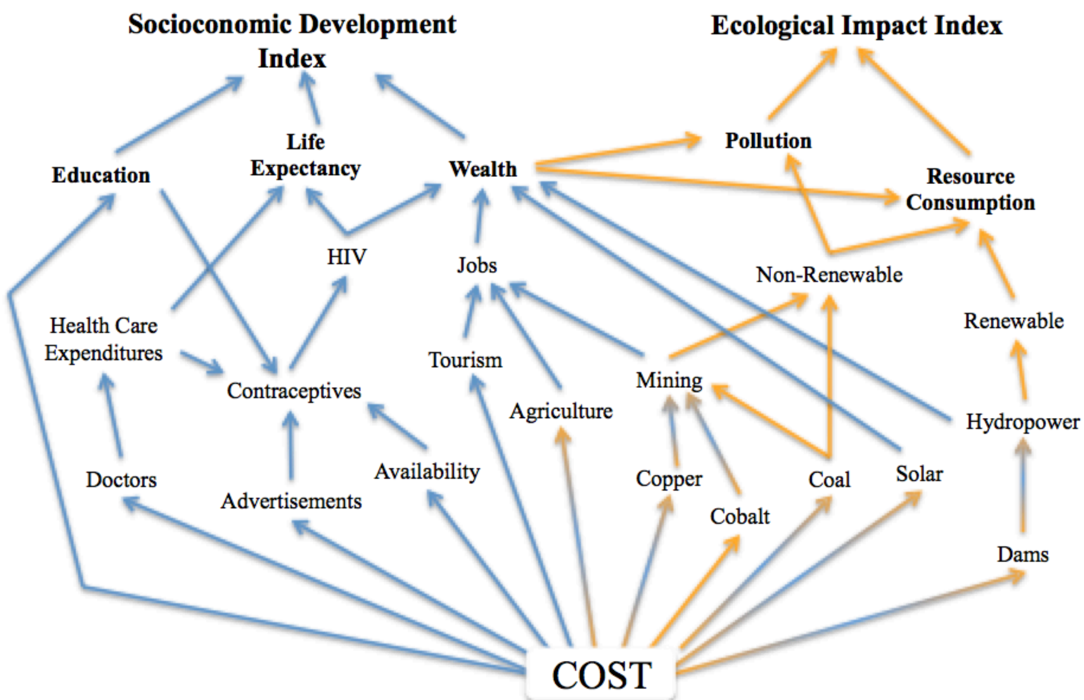


Figure 6. D and E with regards to features of Zambia.

The factors in the map are interdependent. Given a budget over 20 years, we must find the optimal distribution of funding to the resources shown in the map. We investigate four techniques (Genetic Algorithm, Nelder-Mead Search, Powell Search, and Particle Swarm Optimization) to find the best investment distribution.

Function Map Justification

Disease and Health

Zambia remains one of the top 10 HIV / AIDS killing zones in the world, with an average of 45,000 HIV-related deaths each year and a prevalence of over 13.5% of the current population (almost a million people) [AfricaW 2015]. HIV-infected people often have weakened immune systems and thus can easily fall prey to infectious diseases such as tuberculosis. As a result, Zambia has a life expectancy of only 48 years, far below the world average, even when adjusted for Zambia's poverty and lack of resources. More significantly, the average citizen in Zambia at birth has a healthy life expectancy (HALE) of only 40 years [WHO 2010, 40].

Our goal is to implement programs in Zambia that can increase average life expectancy and decrease the prevalence of HIV. Using data from the WHO [2010] and World Bank [2015], we quantify life expectancy as a function of each variable linked to it, directly or indirectly, in the map

(HIV prevalence, Health Care Expenditures, Education, etc.). These equations are derived through best fits of linearized relationship trends (see the **Appendix**).

Wealth

Zambia is a struggling economy with great income disparity [Zambia Institute for Policy Analysis and Research 2013]. With the goal to increase the sustainable development of Zambia, we model wealth's effect on both E and D indices in the sustainability-cost map (**Figure 6**) through a number of relationships. Intuitively, wealth increases with employment (jobs) and decreases with HIV prevalence [UN Department of Economic and Social Affairs 2004].

Wealth also increases with renewable resource use, because of the wealth saved from renewable energy that would otherwise be spent on fossil-fuel imports. Also, E decreases with increased wealth, because increased wealth (when funneled into nonrenewables) causes increased resource consumption and pollution. The D index, intuitively, increases with wealth and income equality. The equations for each relationship are found in the **Appendix**.

Objective Function

We maximize sustainability in terms of the measure S of an investment distribution, using a weighted average with weighting factor α:

$$S = \frac{D + \alpha E}{1 + \alpha},$$

where α measures the country's desire for environmental responsibility, with higher values of α corresponding to greater desire.

Nelder-Mead, Powell, and Genetic Optimization

Both the Nelder-Mead [1965] and Powell [1964] optimization methods start with a random distribution of investment money into various resources in the map, and proceed to optimize investment by iteratively generating better distributions that are hoped to allow for more successful sustainable development.

Optimization using either of these algorithms for our iterative function map has the same two major flaws: Both methods quickly find and lock to local optima; and neither of these methods is meant to be bounded, which reduces their ability to converge and therefore further decreases their likely effectiveness.

To avoid those weaknesses, we implement a genetic algorithm. The genetic algorithm, because it is based on crossover and mutation of seed chromosomes, is bounded and is more effective than both Nelder-Mead's and Powell's methods at finding global optima; however, it is far too slow to be useful. Therefore, we seek yet another alternative.

Particle Swarm Optimization

We implement particle swarm optimization. Particle swarm optimization works by initializing a set of particles to random positions in the investment distribution search space. Each particle is attracted to both the best investment distribution it has so far found and also to the best investment distribution any particle has found. For various applications, the complex search and swarm behavior of the particles that results has been shown to be more efficient and successful than genetic algorithms at finding global optima [Hassan et al. 2005].

Indeed, we find that by setting the size of the swarm to 128 particles, we converge to optima faster than the genetic algorithm, and these optima are more likely to be the global optimum.

Using the particle swarm optimization, we find the optimal investment distributions and sustainability for varying values of investment and α. Due to processing power and time constraints, we split the 20-year plan into two 10-year plans instead of the year-by-year annual plans that we had hoped to create. **Figure 7** shows the relationship between investment and the resulting sustainability.

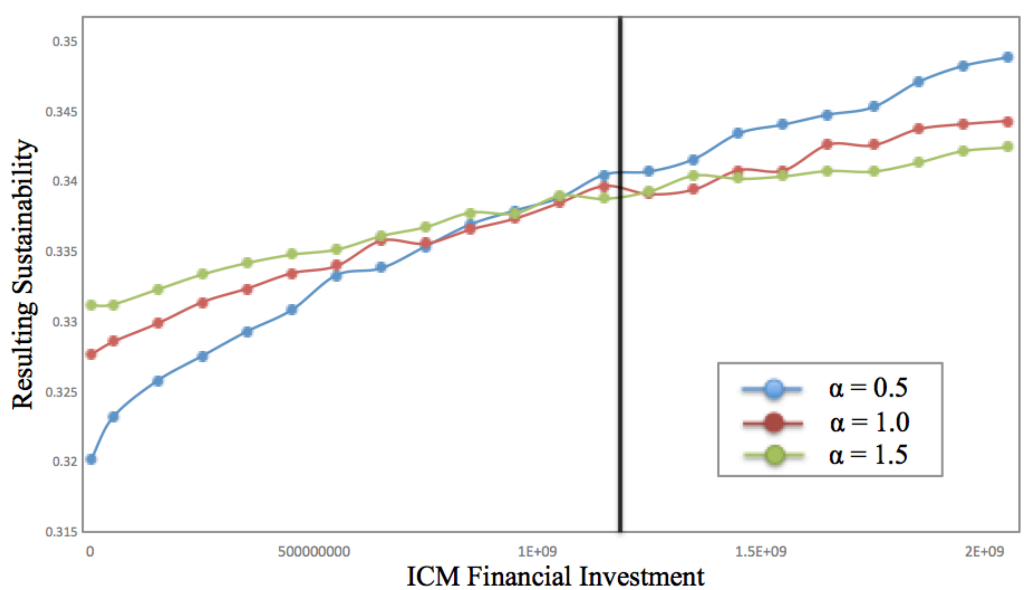


Figure 7. Effect of alpha on resulting sustainability, for varying values of investment. The vertical black line represents a sample budget for investment of $1.2 billion over the 20 years.

For differing values of investment, there are corresponding sustainability level outcomes, depending on the α value. At any level of investment, it would be most beneficial in terms of ecosystem preservation to choose the plan with the highest resulting sustainability level; but to do so would depend on the government of Zambia's desire for environmental responsibility and their preference for the corresponding investment distribution. For example, given an investment of \$1.2 billion (see the vertical line in **Figure 7**), **Figure 8** gives optimal investment distributions, for the first 10 years and for the second 10 years, for three values of α.

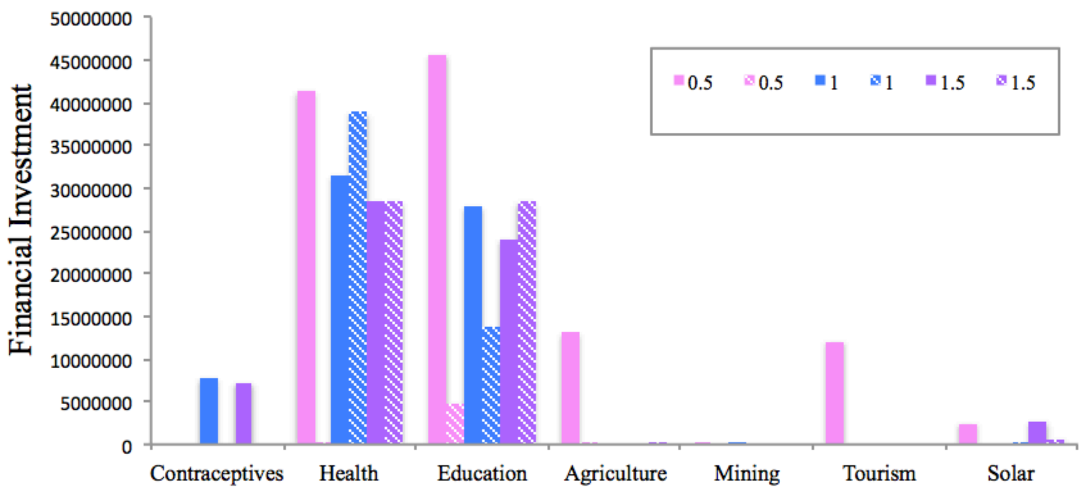


Figure 8. Optimal amount of investment in each sector for a budget of \$1.2 billion, for three different levels of α (from left to right: 0.5, 1.0, 1.5, hence in order of increasing desire for environmental responsibility). Solid rectangles are for the first 10 years of the plan and adjacent striped rectangles are for the second 10 years.

Evaluating the 20-year Plan

We evaluate our plan over the course of 20 years for the optimal resource allocation plan as determined by particle swarm optimization. **Figure 9** demonstrates the trends in D, E, and S.

The trends in **Figure 9** give insight on what to expect for an optimized financial investment distribution of \$ 1.2 billion for scenarios of different levels of α. All three scenarios cause a rapid increase in socioeconomic development, and as a result there is an increase in sustainability as well. However, there is a cost: a decrease in the ecological index E. After about 10 to 12 years, socioeconomic growth tapers off and sustainability stabilizes to a value higher than it was before. The ecological index begins to increase.

These trends show that for an underdeveloped country to undergo efficient and sustainable development, factors of the socioeconomic index D such as wealth, life expectancy, and education must first increase positively at the expense of the environment. Then, once overall sustainability reaches a plateau higher than its initial value, ecosystem health can slowly be improved. Overall, we recommend following the $\alpha = 0.5$ trajectory for

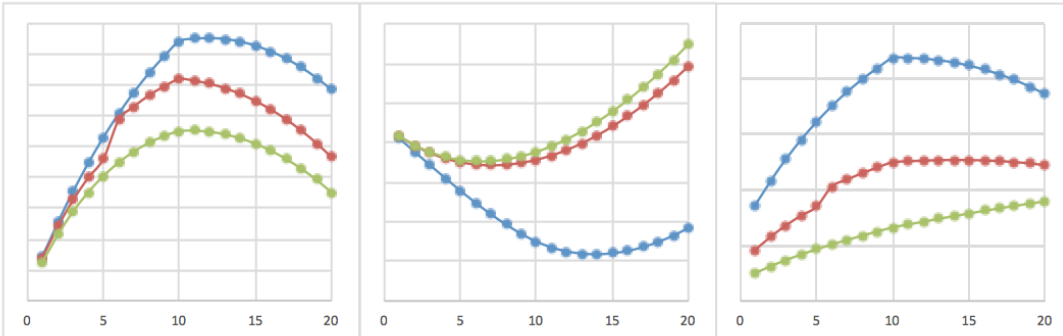


Figure 9. D, E, and S over time, for an investment of \$1.2 billion.
The blue curve (top curve in left and right graphs, bottom curve in middle graph) is for $\alpha = 0.5$;
the red curve (middle curve in all three graphs) is for $\alpha = 1$; and
the green curve (bottom curve in left and right graphs, top curve in middle graph) is for $\alpha = 1.5$.

the first 10 years, and thereafter placing more emphasis on environmental impact by following the $\alpha = 1.5$ trajectory to ensure more growth in overall sustainability.

Strengths and Weaknesses

The major strength of our 20-year plan is that it successfully models a complex iterative network of relationships among a variety of factors, all of which are quantified in terms of cost. This allows choice of an optimal distribution of resources and allows the ICM to maximize return on investment in terms of how sustainable they (and the Zambian government) desire development to be. By combining otherwise uncoupled socioeconomic and environmental factors, our model demonstrates versatility and maximizes use of a variety of input factors uniquely tailored to Zambia.

On the other hand, a weakness of the model is that it is calibrated on two iterations of 10-year-long periods. This is due to constraints in computer processing power and time limitations. Given greater computing power, our model could be used to create yearly optimal ICM investment distribution plans.

Another weakness is that since we do not include import or exports in our model, total wealth does not increase as much as it would had we accounted for international trade.

Sensitivity Analysis

We want our model to be tolerant of a small amount of error in its inputs. Such errors could result from incorrect population caps for job growth, Zambia not following fitted world trends, slight over- or under-budgeting by the ICM, etc. So, to test our model's sensitivity, we intentionally create small sources of error in our data and compare the results to the results using unmodified data.

One error we introduce is raising the population cap in the agricultural sector by 10%. Doing so should increase total agriculture jobs but have little impact on the total economic and developmental indices, because our algorithms should compensate for this cost elsewhere in the Zambian sustainability-cost web. We find that agricultural jobs increase by 7.6% but the overall E and D indices are affected by only -0.2% and 0.1% percent. We find similar results for a 10% increase in the mining industry.

To further test the robustness of our model, we introduce a second error: not accounting for solar energy in the calculation for wealth. In doing so, we believe that our algorithm should instead account for renewable energy with hydroelectric power, maintaining near constant values of E and D. The results match our expectations: Our program adds dams instead of solar panels, accounting for a -0.2% in D and no change in E. These values make sense, for wealth should decrease with the higher expense of energy with dams than with solar panels.

Overall, we find that our model successfully deals with errors in input and demonstrates solid robustness, matching expected behavior with only small deviations. Such robustness is understandable, because small input errors are only a fraction of a percent of the total sustainability metric.

Conclusion

We model sustainability and devise a 20-year sustainable development plan for Zambia based on our findings.

We find, as expected, that our Socioeconomic Development Index D and Ecological Impact Index E have a negative correlation: Better human-economic life quality results in lower ecosystem health.

We offer a choice of 20-year plans that maximize economic expansion but also maintain a high level of environmental preservation, and we create a sustainability map for Zambia that includes factors specific to the country. We thus indicate strategies and programs to implement for a successful 20 years of sustainable development in Zambia.

References

AfricaW: Africa and the World. 2015. Major problems facing Zambia today. `http://www.africaw.com/major-problems-facing-zambia-today.`

Albritton, D., R. Derwent, I. Isaksen, M. Lal, and D. Wuebbles. 1996. Trace gas radiative forcing indexes. Chapter 2.5 in *Climate Change 1995, The*

Science of Climate Change, edited by John T. Houghton et al., 118–131. New York: Cambridge University Press.

Brundtland Commission. 1987. *World Commission on Environment and Development: Our Common Future*. New York: Oxford University Press.

Diamond, Peter A., and Emmanuel Saez. 2011. The case for a progressive tax: From basic research to policy recommendations. *Ecological Economics* 25 (4): 165–190. `http://tinyurl.com/ovomryx` = `http://papers.ssrn.com/sol3/Delivery.cfm/SSRN_ID1915957_code459177.pdf?abstractid=1915957&mirid=2`.

Epstein, J.M., and R. Axtell. 1996. *Growing Artificial Societies: Social Science from the Bottom*. Cambridge, MA: MIT Press.

Harvey, L.D. Danny. 2009. A guide to global warming potentials (GWPs). *Energy Policy* 106 (28): 11812–11817. `http://tinyurl.com/pqeuvr8` = `http://faculty.geog.utoronto.ca/Harvey/Harvey/papers/Harvey%20(1993a_Energy_Policy_GWPs).pdf`.

Hassan, Rania, Babak Cohanim, Olivier de Weck, and Gerhard Venter. 2005. A comparison of particle swarm optimization and the genetic algorithm. *Proceedings of the 1st AIAA (American Institute of Aeronautics and Astronautics) Multidisciplinary Design Optimization Specialist Conference*. `http://tinyurl.com/d2g4s9f` = `http://web.mit.edu/deweck/www/PDF_archive/3%20Refereed%20Conference/3_50_AIAA-2005-1897.pdf`.

van de Kerk, Geurt, and Arthur Manuel. 2008. A comprehensive index for a sustainable society: The SSI—The Sustainable Society Index. *Ecological Economics* 66 (2–3): 228–242.

________. 2012. Measuring wellbeing and progress towards sustainability. `http://ec.europa.eu/environment/beyond_gdp/download/factsheets/bgdp-ve-ssi.pdf`.

Kovacevic, Milorad. 2010. Review of HDI critiques and potential improvements. `http://tinyurl.com/naqw7zd` = `http://www.researchgate.net/profile/Milorad_Kovacevic/publication/235945302_Human_Development_Research_Paper_201033_Review_of_HDI_Critiques_and_Potential_Improvements/links/54412e550cf2e6f0c0f602b7.pdf`.

Myhre, G., D. Shindell, F.M. Bréon, W. Collins, J. Fuglestvedt, J. Huang, D. Koch, J.F. Lamarque, D. Lee, B. Mendoza, T. Nakajima, A. Robock, G. Stephens, T. Takemura, and H. Zhang. 2013. *2013: Anthropogenic and Natural Radiative Forcing in Report of the Intergovernmental Panel on Climate Change*. Cambridge, UK: Cambridge University Press.

Nelder, J.A., and R. Mead. 1965. A simplex method for function minimization. *The Computer Journal* 7 (4): 308–313. doi:10.1093/comjnl/7.4.308 . http://www.ii.uib.no/~lennart/drgrad/Nelder1965.pdf .

Powell, M.J.D. 1964. An efficient method for finding the minimum of a function of several variables without calculating derivatives. *The Computer Journal* 7 (2): 155–162. doi:10.1093/comjnl/7.2.155 . http://folk.uib.no/ssu029/Pdf_file/Powell64.pdf .

Preston, Samuel H. 1975. The changing relation between mortality and level of economic development. *Population Studies* 29 (2): 231–248.

Smith, Stephen J., and T.M.L. Wigley. 2000. Global warming potentials: 1. Climatic implications of emissions reductions. *Climate Change* 44 (4): 445–457.

Stanton, Elizabeth A. 2007. The Human Development Index: A history. http://tinyurl.com/n9uohbf = http://scholarworks.umass.edu/cgi/viewcontent.cgi?article=1101&context=peri_workingpapers&sei-redir=1&referer=http%3A%2F%2Fscholar.google.com%2Fscholar%3Fhl%3Den%26q%3DStanton%252C%2BElizabeth%2BA.%2B2007%2Bhuman%2Bdevelopment%2Bindex%253A%2BA%2B%2Bhistory.%26btnG%3D%26as_sdt%3D1%252C50%26as_sdtp%3D#search=%22Stanton%2C%20Elizabeth%20A.%202007%20human%20development%20index%3A%20history.%22.

Tucker, M. 1995. Carbon dioxide emissions and global GDP. *Ecological Economics* 15 (3): 215–223. http://tinyurl.com/pndqesw = http://www.researchgate.net/profile/Michael_Tucker8/publication/4838985_Carbon_dioxide_emissions_and_global_GDP/links/5509cfba0cf20f127f90ad98.pdf .

UN Conference on Trade and Development (UNCTAD). 2013. Map of the LDCs. http://unctad.org/en/Pages/ALDC/Least%20Developed%20Countries/LDC-Map.aspx .

UN Department of Economic and Social Affairs Population Division. 2004. HIV Impact on economic growth. Chapter VIII in *The Impact of AIDS*. New York: United Nations. http://www.un.org/esa/population/publications/AIDSimpact/91_CHAP_VIII.pdf .

World Bank. 2015. Indicators. http://data.worldbank.org/indicator .

World Health Organization. 2010. *World Health Statistics 2010*. http://www.who.int/gho/publications/world_health_statistics/EN_WHS10_Full.pdf .

Zambia Institute for Policy Analysis and Research. 2013. The distribution of household income and the middle class in Zambia.
http://tinyurl.com/q4nbur8 = http://www.zipar.org.zm/

`publications/working-papers/22-the-distribution-of-household-income-and-the-middle-class-in-zambia.`

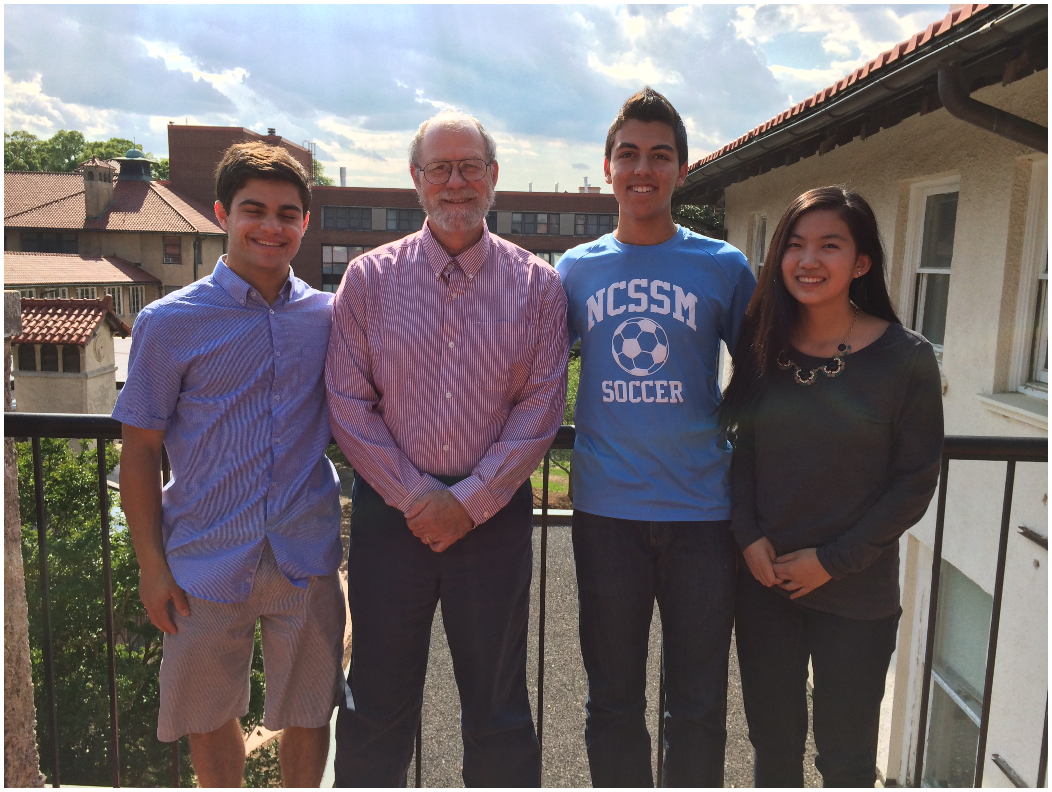
Appendix

Table 3.

Inputs and equations for the recurrences.

Feature	Inputs	Equation
Life expectancy	x = doctors per 1000	$5.058 \ln x + 65.02$
% Use of contraception	x = years of education	$2.07x + 27.54$
Years of education	x = government HIV health spending per capita	$1.4856 \ln x + 9.9122$
Life expectancy	x = health care spending per capita	$5.8066 \ln x + 33.7923$
Birth rate	x = GDP per capita	$.070428 - 5.62 \ln x$
Life expectancy	x = GDP per capita	$6.6354 \ln x + 10.754$
Energy consumption	x = wealth per capita, P = population	$1.41812x^{0.7587} \times (1.628 \times 10^{-6})P$
HIV_{n+1}	HIV_n = total # HIV in year n, U = % Use of contraception	$\mathrm{HIV}_n + \mathrm{HIV}_n[5.51 \times 10^{-9}(P - \mathrm{HIV}_n)(1 - 0.9U)] - 0.02455\mathrm{HIV}_n$
Nonrenewable energy use	total energy, renewable energy	Total energy $-$ renewable energy
Hydropower	x = total wealth	$(3.1536 \times 10^{-10})x$
P_{n+1}	P_n = population in year n, x = wealth per capita	$P_n + (-1.316 \ln x + 14.576)(.01)P_n$
W_{n+1}	W_n = wealth in year n, x = hydropower, J = jobs	$W_n(1 - 0.06\mathrm{HIV}_n) + (1.58 \times 10^8)x + 7000J$
Jobs	M = mining jobs, T = tourism jobs, A = agriculture jobs	$\dfrac{150000}{1 + 2e^{-(2.9\times10^{-6})M}} + \dfrac{150000}{1 + 4.7692e^{-DT(7.5\times10^{-6})}} + \dfrac{125000}{1 + \frac{2}{3}e^{-(2.4\times10^{-6})A}}$

The equations in **Table 3** were derived through linearized plots of data extracted from World Bank and WHO data, giving net relationship trends between each variable connected by an arrow in the **Figure 6** map.



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