Announcements

- H1 posted. Due Next Tuesday

Nondeterministic Finite Automata

CS 536

Previous Lecture

Scanner: converts a sequence of characters to a sequence of tokens

Scanner and parser: master-slave relationship

Scanner implemented using FSMs

FSM: DFA or NFA

This Lecture

NFAs from a formal perspective

Theorem: NFAs and DFAs are equivalent

Regular languages and Regular expressions

NFAs, formally

$$M \equiv (Q, \Sigma, \delta, q, F)$$

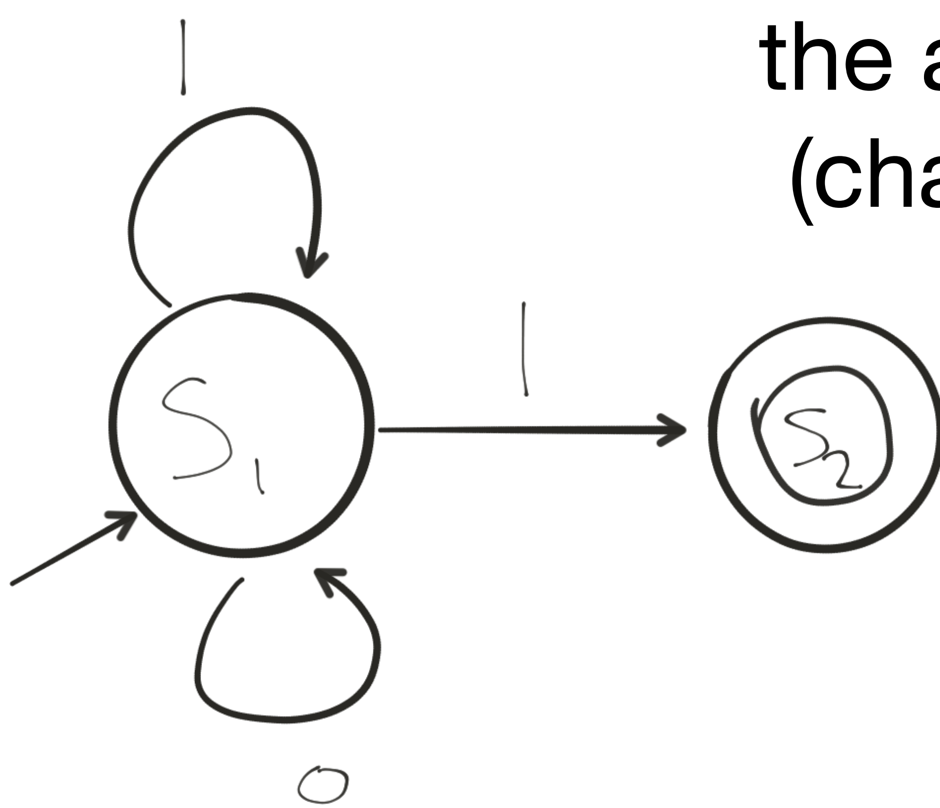
finite set of states

the alphabet
(characters)

transition function
$\delta : Q \times \Sigma \to 2^Q$

start state
$q \in Q$

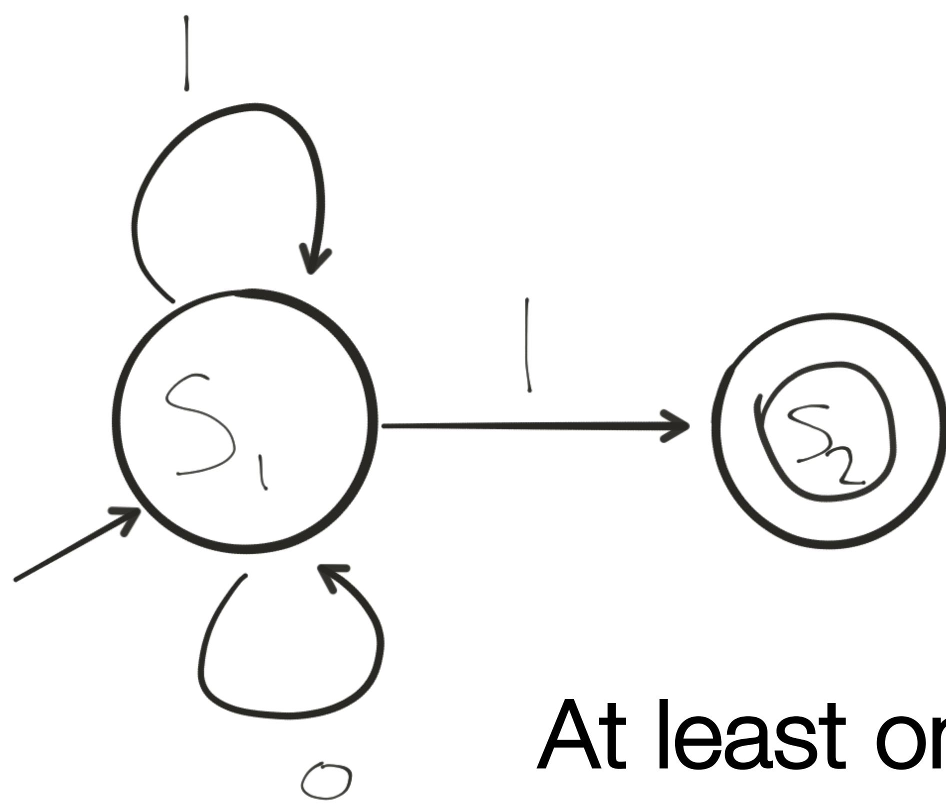
final states
$F \subseteq Q$



	0	1
s1	{s1}	{s1, s2}
s2		

NFA

To check if string is in *L*(*M*) *of* NFA *M*, simulate **set** of **choices** it could make



	1	1	1
s1	s2	st	st
s1	s1	s2	st
s1	s1	s1	**s2**
s1	s1	s1	s1

At least one sequence of transitions that:

Consumes all input (without getting stuck)

Ends in one of the final states

NFA and DFA are Equivalent

Two automata M and M’ are equivalent iff L(M) = L(M’)

Lemmas to be proven

Lemma 1: Given a DFA M, one can construct an NFA M’ that recognizes the same language as M, i.e., L(M’) = L(M)

Lemma 2: Given an NFA M, one can construct a DFA M’ that recognizes the same language as M, i.e., L(M’) = L(M)

Proving lemma 2

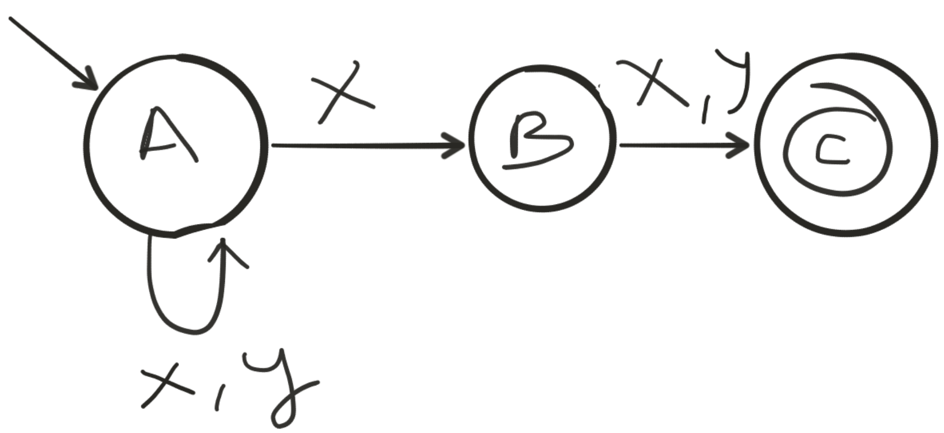
Lemma 2: Given an NFA M, one can construct a DFA M' that recognizes the same language as M, i.e., L(M') = L(M)

Idea: we can only be in finitely many subsets of states at any one time

$2^{|Q|}$ possible combinations of states

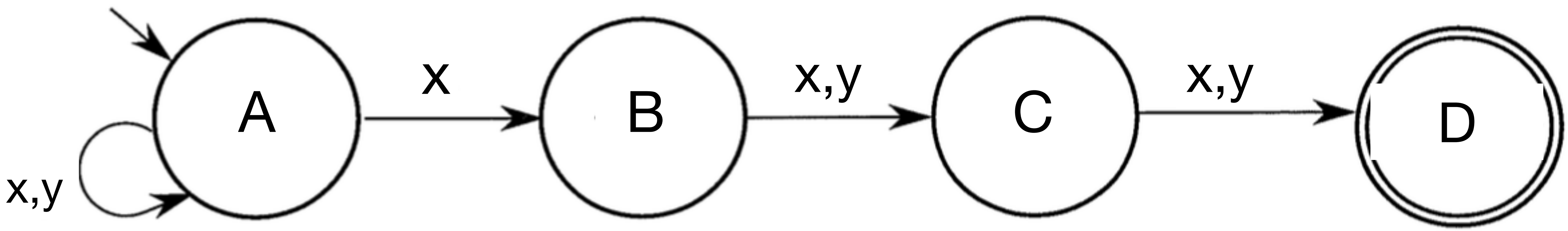
Why?

Why 2^|Q| states?



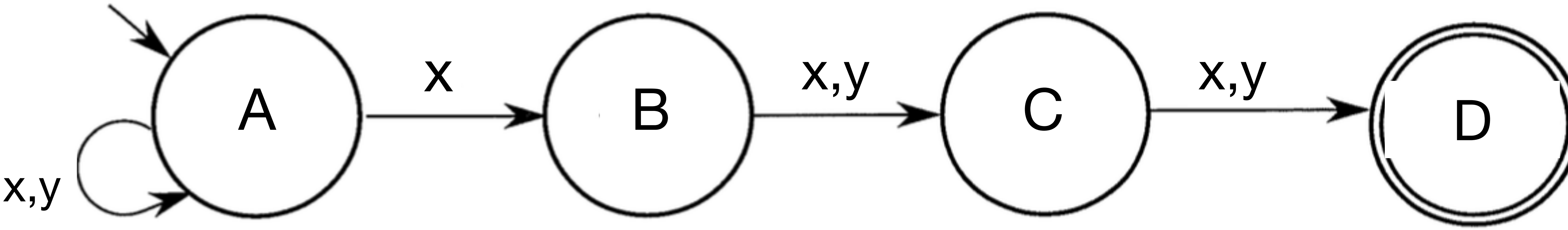
Build DFA that tracks set of states the NFA is in!

A	**B**	**C**		
0	0	0	=	{}
0	0	1	=	{C}
0	1	0	=	{B}
0	1	1	=	{B,C}
1	0	0	=	{A}
1	0	1	=	{A,C}
1	1	0	=	{A,B}
1	1	1	=	{A,B,C}

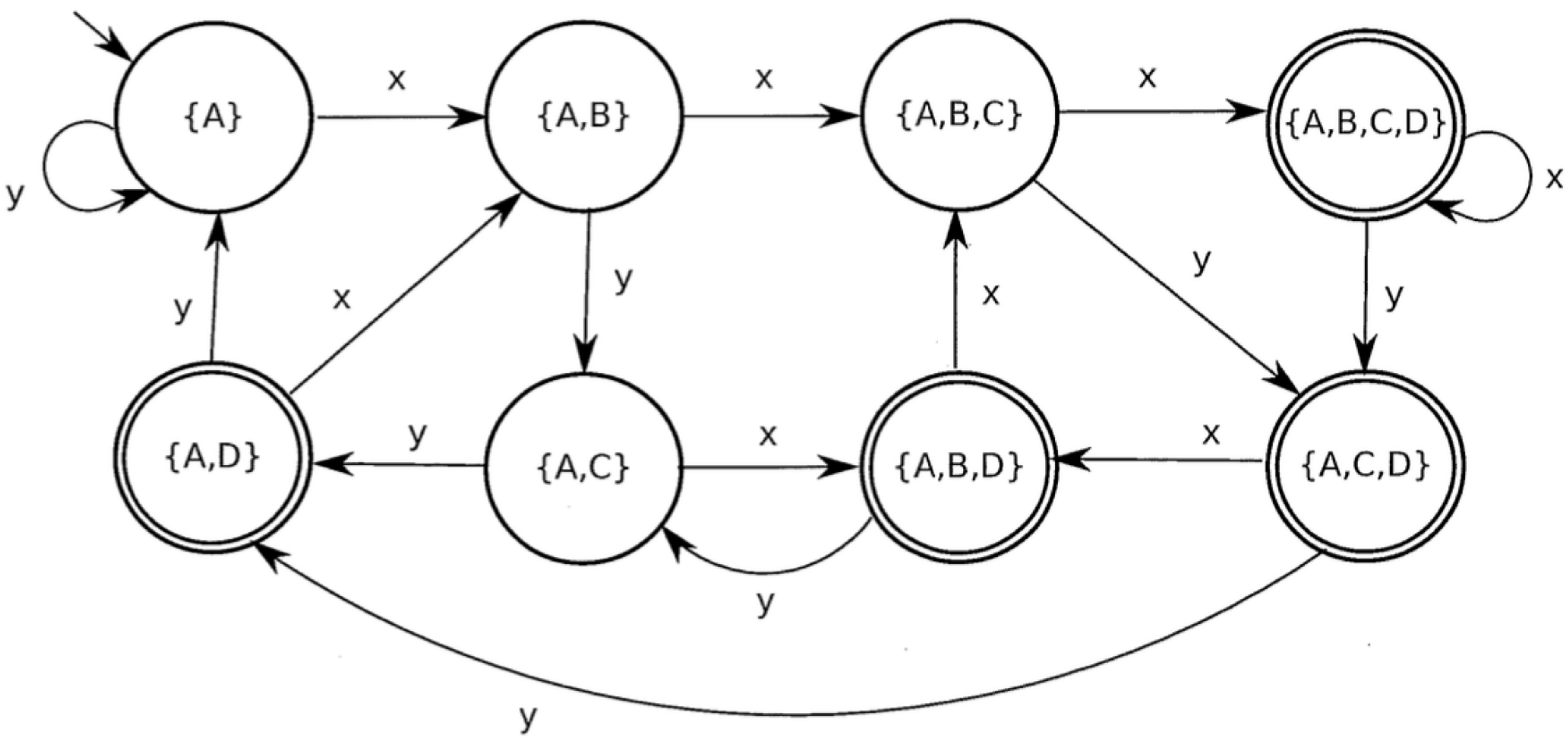


Defn: let succ(s,c) be the set of choices the NFA could make in state s with character c

succ(A,x) = {A,B}
succ(A,y) = {A}
succ(B,x) = {C}
succ(B,y) = {C}
succ(C,x) = {D}
succ(C,y) = {D}



Build new DFA M’ where Q’ = 2^Q

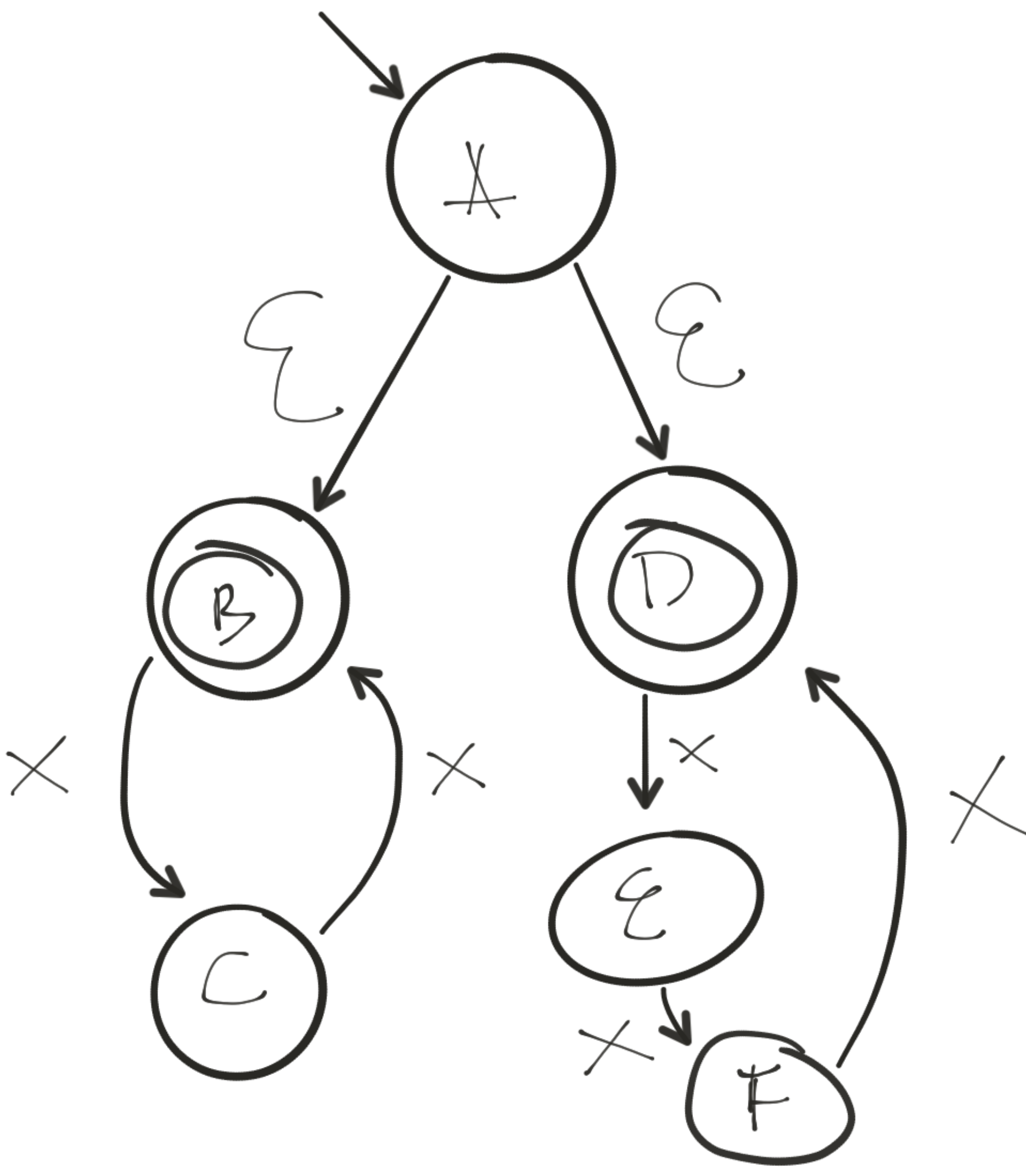


succ(A,x) = {A,B}
succ(A,y) = {A}
succ(B,x) = {C}
succ(B,y) = {C}
succ(C,x) = {D}
succ(C,y) = {D}

To build DFA: Add an edge from state S on character c to state S’ if S’ represents the union of states that all states in S could possibly transition to on input c

ε-transitions

Eg: x^n, where n is even **or** divisible by 3



Useful for taking union of two FSMs

In example, left side accepts even n; right side accepts n divisible by 3

	x		x	
AB		C		B
AD		E		F
A				

Eliminating ε-transitions

We want to construct **ε**-free FSM M’ that is equivalent to M

Definition:

eclose(s) = set of all states reachable from s in
zero or more epsilon transitions

M’ components

s is an accepting state of M’ iff eclose(s) contains an accepting state

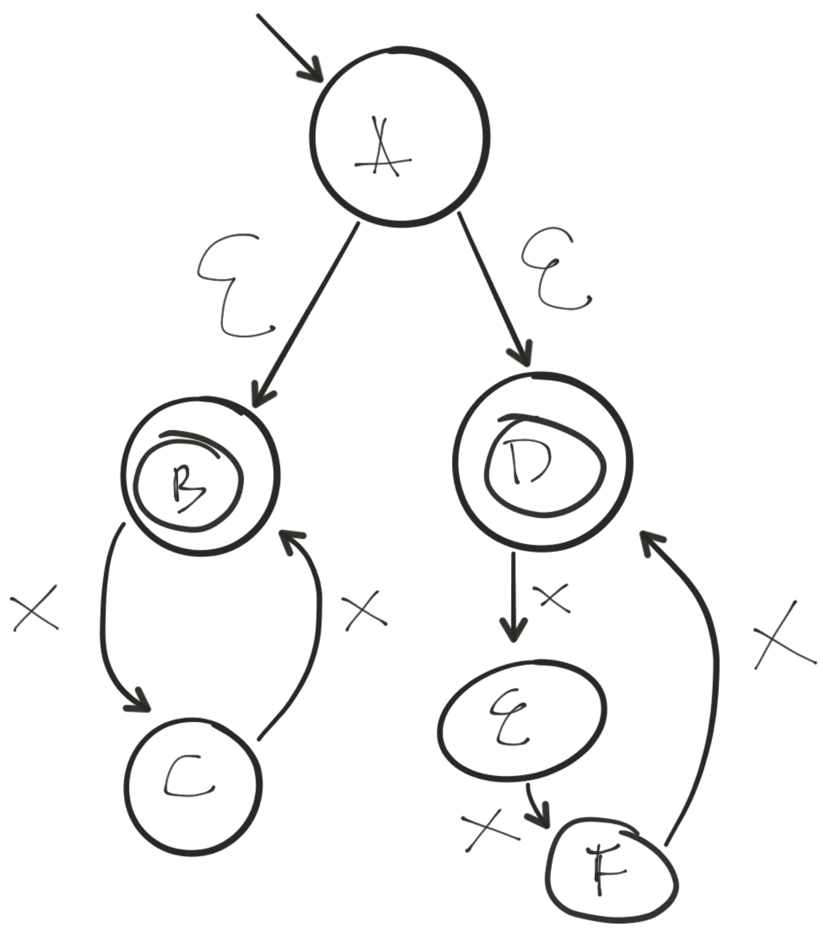
s —c—> t is a transition in M’ iff
q —c—> t for some q in eclose(s)

Eliminating ε-transitions

We want to construct ε-free NFA M’ that is equivalent to M

Definition: Epsilon Closure

eclose(s) = set of all states reachable from s using zero or more epsilon transitions

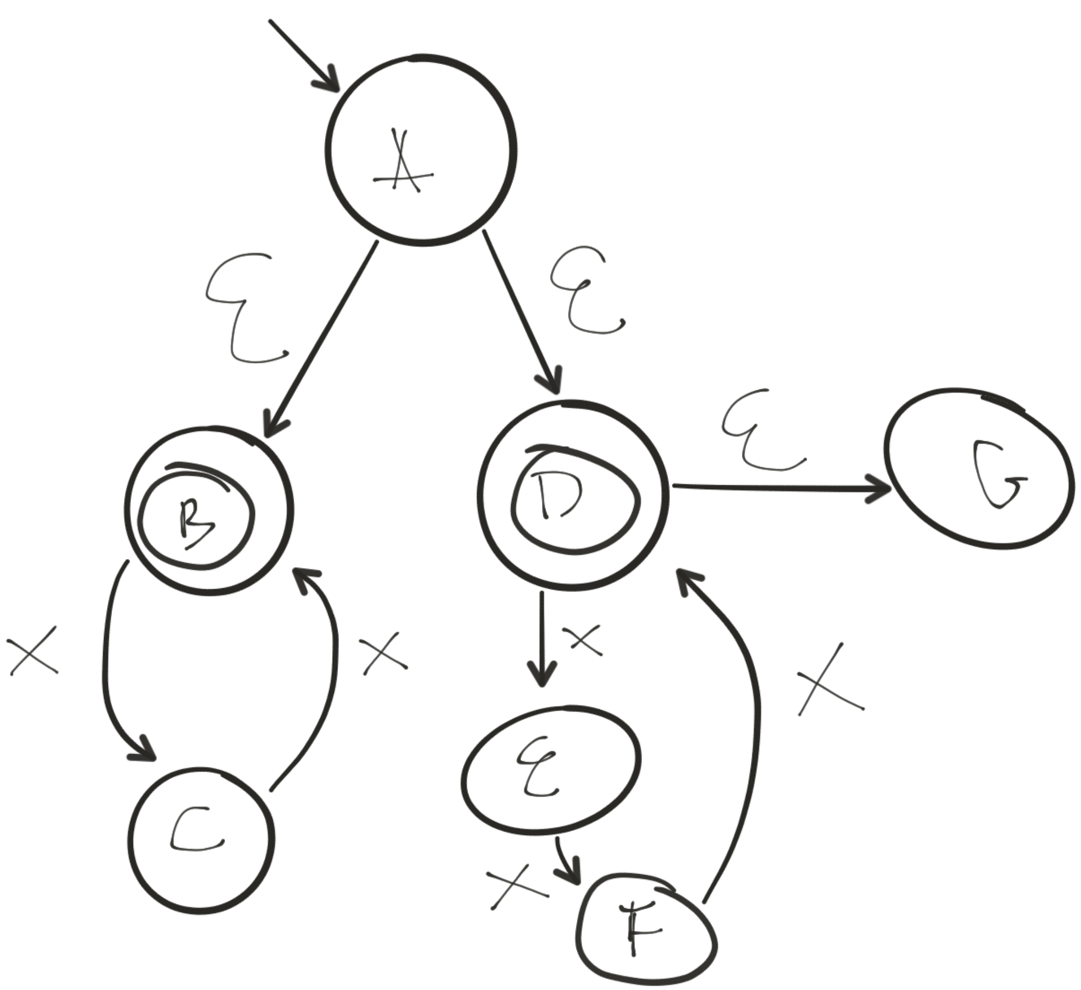


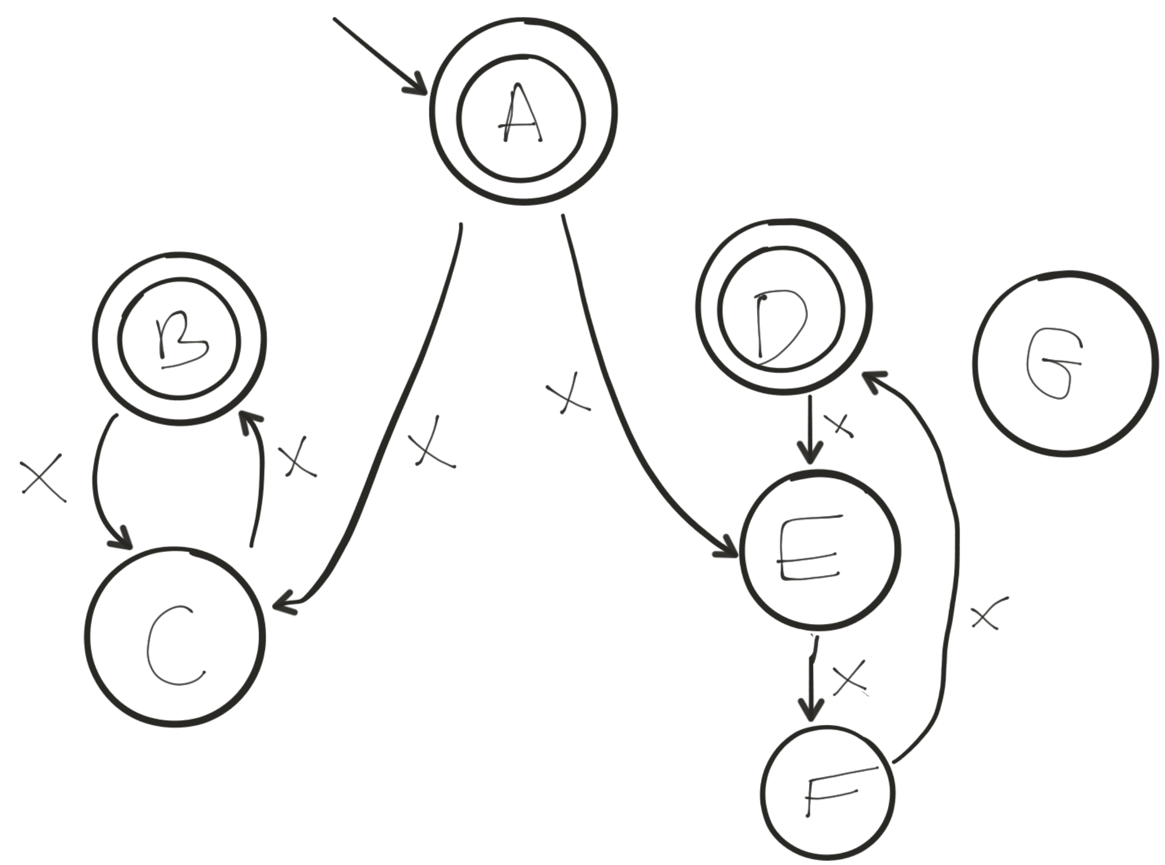
	eclose
A	{A, B, D}
B	{B}
C	{C}
D	{D}
E	{E}
F	{F}

Def: eclose(s) = set of all states reachable from s in zero or more epsilon transitions

s is an accepting state of M’ iff eclose(s) contains an accepting state

s —c—> t is a transition in M’ iff
q —c—> t for some q in eclose(s)





Recap

NFAs and DFAs are equally powerful

any language definable as an NFA is definable as a DFA

ε-transitions do not add expressiveness to NFAs

we showed a simple algorithm to remove epsilons

Regular Languages and Regular Expressions

Regular Language

Any language recognized by an FSM is a regular language

Examples:

- Single-line comments beginning with //
- Integer literals
- {ε, ab, abab, ababab, abababab, …. }
- C/C++ identifiers

Regular expressions

Pattern describing a language

operands: single characters, epsilon

operators: from low to high precedence

alternation “or”: a | b

catenation: a.b, ab, a^3 (which is aaa)

iteration: a* (0 or more a’s) aka Kleene star

Why do we need them?

Each token in a programming language can be defined by a regular language

Scanner-generator input: one regular expression for each token to be recognized by scanner

Regular expressions are inputs to a scanner generator

Regexp, cont'd

Conventions:

- a+ is aa*
- letter is a|b|c|d|…|y|z|A|B|…|Z
- digit is 0|1|2|…|9
- not(x) all characters except x
- . is any character
- parentheses for grouping, e.g., (ab)*
  - ε*, ab, abab, ababab*

Regexp, example

Hex strings

- start with 0x or 0X
- followed by one or more hexadecimal digits
- optionally end with l or L

0(x|X)hexdigit+(L|l|ε)

- where hexdigit = digit|a|b|c|d|e|f|A|…|F

Regexp, example

Single-line comments in Java/C/C++

// this is a comment

//(not('\n'))*'\n'

Regexp, example

C/C++ identifiers: sequence of letters/digits/underscores; cannot begin with a digit; cannot end with an underscore

Example: a, _bbb7, cs_536

Regular expression

letter | (letter|_)(letter|digit|_)*(letter|digit)

Recap

Regular Languages

- Languages recognized/defined by FSMs

Regular Expressions

- Single-pattern representations of regular languages
- Used for defining tokens in a scanner generator

Creating a Scanner