Announcements

HW4 due today

HW5 assigned today

Building a Predictive Parser

I.e., How to build the parse table for a recursive-descent parser

Last Time: Intro LL(1) Predictive Parser

Predict the parse tree top-down

Parser structure

– 1 token of lookahead
– A stack tracking parse tree frontier
– Selector/parse table

Necessary conditions

– Left-factored
– Free of left-recursion

Today: Building the Parse Table

Review Grammar transformations

- Why they are necessary
- How they work

Build the selector table

- FIRST(X): Set of terminals that can begin at a subtree rooted at X
- FOLLOW(X): Set of terminals that can appear after X

Review of LL(1) Grammar Transformations

Necessary (but not sufficient conditions) for LL(1) parsing:

- Free of left recursion
  - “No left-recursive rules”
  - Why? Need to look past the list to know when to cap it
- Left-factored
  - “No rules with a common prefix, for any nonterminal”
  - Why? We would need to look past the prefix to pick the production

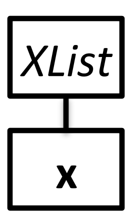
Why Left Recursion is a Problem (Blackbox View)

CFG snippet: $XList \longrightarrow XList\ \mathbf{x}\ \mid\ \mathbf{x}$

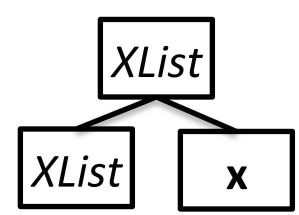
Current parse tree: $\boxed{XList}$

Current token: **x**

How should we grow the tree top-down?



(OR)



Correct if there are no more **x**s

Correct if there <u>are</u> more **x**s

We don't know which without more lookahead

Why Left Recursion is a Problem (Whitebox View)

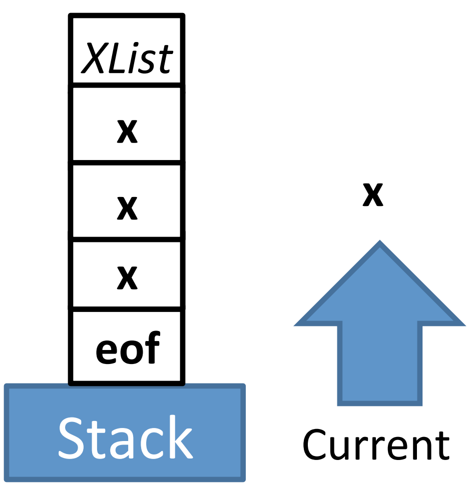
CFG snippet: *XList* ⟶ *XList* **x** | **x**

Current parse tree: $\boxed{XList}$

Current token: **x**

Parse table:

	x	**eof**
XList	*XList* **x**	ε



(Stack overflow)

Left Recursion Elimination: Review

Replace $A \longrightarrow A\ \alpha \mid \beta$

Head of the list

With $A \longrightarrow \beta\ A'$

$A' \longrightarrow \alpha\ A' \mid \varepsilon$

Where β does not start with *A or* may not be present

Preserve order (a list of α starting with β) but use right recursion

Left Recursion Elimination: Ex1

$$A \longrightarrow A\,\alpha \mid \beta \quad \Rightarrow \quad \begin{array}{l} A \longrightarrow \beta\, A' \\ A' \longrightarrow \alpha\, A' \mid \varepsilon \end{array}$$

$$E \longrightarrow E\,\underbrace{\textbf{cross id}}_{\alpha} \mid \underbrace{\textbf{id}}_{\beta} \quad \Rightarrow \quad \begin{array}{l} E \longrightarrow \overbrace{\textbf{id}}^{\beta}\, E' \\ E' \longrightarrow \underbrace{\textbf{cross id}}_{\alpha}\, \mathrm{E}' \mid \varepsilon \end{array}$$

Left Recursion Elimination: Ex2

$$A \longrightarrow A\,\alpha \mid \beta \quad \Rightarrow \quad \begin{array}{l} A \longrightarrow \beta\, A' \\ A' \longrightarrow \alpha\, A' \mid \varepsilon \end{array}$$

$$\begin{array}{l} E \longrightarrow E \; \mathbf{+} \; T \mid T \\ T \longrightarrow T \; \mathbf{*} \; F \mid F \\ F \longrightarrow \mathbf{(}\ \mathrm{E}\ \mathbf{)} \mid \mathbf{id} \end{array} \quad \Rightarrow \quad \begin{array}{l} E \longrightarrow TE' \\ E' \longrightarrow \mathbf{+}\ \mathrm{TE'} \mid \varepsilon \\ T \longrightarrow FT' \\ T' \longrightarrow \mathbf{*}\mathrm{FT'} \mid \varepsilon \\ F \longrightarrow \mathbf{(}\ \mathrm{E}\ \mathbf{)} \mid \mathbf{id} \end{array}$$

Left Recursion Elimination: Ex3

$A \longrightarrow A\ \alpha \mid \beta \quad \Rightarrow \quad A \longrightarrow \beta\ A' \qquad A' \longrightarrow \alpha\ A' \mid \varepsilon$

$SList \longrightarrow SList\ D \mid \varepsilon$

$D \longrightarrow Type$ **id semi**

$Type \longrightarrow$ **bool** | **int**

$\Rightarrow$

$SList \longrightarrow \varepsilon\ SList'$

$SList' \longrightarrow D\ SList' \mid \varepsilon$

$D \longrightarrow Type$ **id semi**

$Type \longrightarrow$ **bool** | **int**

$\Rightarrow$

$SList \longrightarrow D\ SList \mid \varepsilon$

$D \longrightarrow Type$ **id semi**

$Type \longrightarrow$ **bool** | **int**

Left Factoring: Review

Removing common prefix from grammar

Replace $\quad A \longrightarrow \alpha\,\beta_1 \mid \ldots \mid \alpha\,\beta_m \mid y_1 \mid \ldots \mid y_n$

With $\quad A \longrightarrow \alpha\,A' \mid y_1 \mid \ldots \mid y_n$

$\quad\quad\quad A' \longrightarrow \beta_1 \mid \ldots \mid \beta_m$

Where β_i and y_i are sequence of symbols with no common prefix
y_i May not be present, one of the β may be ε

Squash all "problem" rules starting with α together into one rule $\alpha\,A'$
Now A' represents the suffix of the "problem" rules

Left Factoring: Example 1

$$A \longrightarrow \alpha\,\beta_1 \mid \dots \mid \alpha\,\beta_m \mid \gamma_1 \mid \dots \mid \gamma_n \quad \Rightarrow \quad \begin{array}{l} A \longrightarrow \alpha\,A' \mid \gamma_1 \mid \dots \mid \gamma_n \\ A' \longrightarrow \beta_1 \mid \dots \mid \beta_m \end{array}$$

$$X \longrightarrow \overbrace{\mathbf{<}}^{\alpha}\ \overbrace{\mathbf{a\ >}}^{\beta_1} \mid \overbrace{\mathbf{<}}^{\alpha}\ \overbrace{\mathbf{b\ >}}^{\beta_2} \mid \overbrace{\mathbf{<}}^{\alpha}\ \overbrace{\mathbf{c\ >}}^{\beta_3} \mid \overbrace{\mathbf{d}}^{\gamma_1}$$

$$X \longrightarrow \overbrace{\mathbf{<}}^{\alpha}\ X' \mid \overbrace{\mathbf{d}}^{\gamma_1}$$

$$X' \longrightarrow \underbrace{\mathbf{a\ >}}_{\beta_1} \mid \underbrace{\mathbf{b\ >}}_{\beta_2} \mid \underbrace{\mathbf{c\ >}}_{\beta_3}$$

Left Factoring: Example 2

$$A \longrightarrow \alpha\, \beta_1 \mid \ldots \mid \alpha\, \beta_m \mid y_1 \mid \ldots \mid y_n \quad \Rightarrow \quad \begin{array}{l} A \longrightarrow \alpha\, A' \mid y_1 \mid \ldots \mid y_n \\ A' \longrightarrow \beta_1 \mid \ldots \mid \beta_m \end{array}$$

$$\mathit{Stmt} \longrightarrow \overbrace{\textbf{id assign } E}^{\beta_1} \mid \overbrace{\textbf{id (} \mathit{EList} \textbf{)}}^{\beta_2} \mid \textbf{return}$$

$$E \longrightarrow \textbf{intlit} \mid \textbf{id}$$

$$\mathit{Elist} \longrightarrow E \mid E \textbf{ comma } \mathit{EList}$$

$$\mathrm{Stmt} \longrightarrow \textbf{id } \mathit{Stmt'} \mid \textbf{return}$$

$$\mathrm{Stmt'} \longrightarrow \textbf{assign } E \mid \textbf{(} \mathit{EList} \textbf{)}$$

$$E \longrightarrow \textbf{intlit} \mid \textbf{id}$$

$$\mathit{Elist} \longrightarrow E \mid E \textbf{ comma } \mathit{EList}$$

Left Factoring: Example 3

$$A \longrightarrow \alpha\, \beta_1 \mid \ldots \mid \alpha\, \beta_m \mid \gamma_1 \mid \ldots \mid \gamma_n \quad \Rightarrow \quad \begin{array}{l} A \longrightarrow \alpha\, A' \mid \gamma_1 \mid \ldots \mid \gamma_n \\ A' \longrightarrow \beta_1 \mid \ldots \mid \beta_m \end{array}$$

$$S \longrightarrow \overbrace{\textbf{if } E \textbf{ then } S}^{\alpha}\ \overbrace{\vphantom{S}}^{\beta_1 = \varepsilon} \mid \overbrace{\textbf{if } \mathrm{E} \textbf{ then } S}^{\alpha}\ \overbrace{\textbf{else } S}^{\beta_2} \mid \textbf{semi}$$

$$E \longrightarrow \textbf{boollit}$$

$$\mathrm{S} \longrightarrow \textbf{if } \mathrm{E} \textbf{ then } \mathrm{S}\ \mathrm{S'} \mid \textbf{semi}$$

$$\mathrm{S'} \longrightarrow \textbf{else } \mathrm{S} \mid \varepsilon$$

$$\mathrm{E} \longrightarrow \textbf{boollit}$$

Left Factoring: Not Always Immediate

$A \longrightarrow \alpha\, \beta_1 \mid \ldots \mid \alpha\, \beta_m \mid y_1 \mid \ldots \mid y_n$ ⇒ $A \longrightarrow \alpha\, A' \mid y_1 \mid \ldots \mid y_n$

$A' \longrightarrow \beta_1 \mid \ldots \mid \beta_m$

This snippet yearns for left-factoring

S ⟶ A | C | **return**
A ⟶ **id assign** *E*
C ⟶ **id (** *EList* **)**

but we cannot! At least without *inlining*

S ⟶ **id assign** *E* | **id (** *Elist* **)** | **return**

Let’s be more constructive

So far, we’ve only talked about what <u>precludes</u> us from building a predictive parser

It’s time to actually build the parse table

Building the Parse Table

What do we actually need to <u>ensure</u> arbitrary production $A \longrightarrow \alpha$ is the correct one to apply?

Assume α is an arbitrary sequence of symbols.

1. What terminals could α possibly <u>start</u> with
   → we call this the FIRST set
2. What terminal could possibly come <u>after</u> *A*
   → we call this the FOLLOW set

Why is FIRST Important?

Assume the top-of-stack symbol is *A* and current token is **a**

- Production 1: $A \longrightarrow \alpha$
- Production 2: $A \longrightarrow \beta$

FIRST lets us disambiguate:

- If **a is in FIRST(α)**, we know Production 1 is a viable choice
- If **a is in FIRST(β)**, we know Production 2 is a viable choice
- If **a** is in only in one of FIRST(α) and FIRST(β), we can predict the production we need

FIRST Sets

FIRST(α) is the set of terminals that begin the strings derivable from α, and also, if α can derive ε, then ε is in FIRST(α).

Formally, let's write it together

FIRST(α) =

FIRST Sets

FIRST(α) is the set of terminals that begin the strings derivable from α, and also, if α can derive ε, then ε is in FIRST(α).

Formally, let's write it together

FIRST(α) = $\{t \mid (t \in \Sigma \wedge \alpha \Rightarrow\uparrow * \ t\beta) \vee (t = \epsilon \wedge \alpha \Rightarrow\uparrow * \ \epsilon)\}$

FIRST Construction: Single Symbol

We begin by doing FIRST sets for a single, arbitrary symbol X

- If X is a terminal: FIRST(X) = { X }
- If X is ε: FIRST(ε) = { ε }
- If X is a nonterminal, for each $X \longrightarrow Y_1\ Y_2 \dots Y_k$
  - Put FIRST(Y_1) - {ε} into FIRST(X)
  - If ε is in FIRST(Y_1) , put FIRST(Y_2) - {ε} into FIRST(X)
  - If ε is also in FIRST(Y_2), put FIRST(Y_3) - {ε} into FIRST(X)
  - …
  - If ε is in FIRST of all Y_i symbols, put ε into FIRST(X)

Repeat this step until there are no changes to any nonterminal's FIRST set

FIRST(X) Example

<u>Building FIRST(X) for nonterm X</u>

for each $X \longrightarrow Y_1\ Y_2\ \dots\ Y_k$

- Add FIRST(Y_1) - {ε}
- If ε is in FIRST($Y_{1 \text{ to } i-1}$): add FIRST(Y_i) - {ε}
- If ε is in all RHS symbols, add ε

Exp	⟶	*Term Exp'*
Exp'	⟶	**minus** *Term Exp'* | ε
Term	⟶	*Factor Term'*
Term'	⟶	**divide** *Factor Term'* | ε
Factor	⟶	**intlit** | **lparens** *Exp* **rparens**

FIRST(*Factor*) = { **intlit, lparens** }

FIRST(*Term'*) = { **divide,** ε }

FIRST(*Term*) = { **intlit, lparens** }

FIRST(*Exp'*) = { **minus,** ε}

FIRST(*Exp*) = { **intlit, lparens**}

FIRST(α)

We now extend FIRST to strings of symbols α

- We want to define FIRST for all RHS

Looks very similar to the procedure for single symbols

Let $\alpha = Y_1\ Y_2\ \ldots\ Y_k$

- Put FIRST(Y_1) - {ε} in FIRST(α)
  - If ε is in FIRST(Y_1): add FIRST(Y_2) – {ε} to FIRST(α)
  - If ε is in FIRST(Y_2): add FIRST(Y_3) – {ε} to FIRST(α)
  - …
  - If ε is in FIRST of all Y_i symbols, put ε into FIRST(α)

Building FIRST(α) from FIRST(X)

<u>Building FIRST(X) for nonterm X</u>

for each $X \longrightarrow Y_1\ Y_2\ \dots\ Y_k$

- Add FIRST(Y_1) - {ε}
- If ε is in FIRST($Y_{1\text{ to }i-1}$): add FIRST(Y_i) - {ε}
- If ε is in all RHS symbols, add ε

<u>Building FIRST(α)</u>

Let $\alpha = Y_1\ Y_2\ \dots\ Y_k$

- Add FIRST(Y_1) - {ε}
- If ε is in FIRST($Y_{1\text{ to }i-1}$): add FIRST(Y_i) – {ε}
- If ε is in all RHS symbols, add ε

FIRST(α) Example

<u>Building FIRST(α)</u>

Let $\alpha = Y_1\ Y_2\ \ldots\ Y_k$

- Add FIRST(Y_1) - {ε}
- If ε is in FIRST($Y_{1 \text{ to } i-1}$): add FIRST(Y_i) – {ε}
- If ε is in all RHS symbols, add ε

E → TX
X → +TX | ε
T → FY
Y → *FY | ε
F → (E) | id

FIRST(*E*) = {(, id}
FIRST(*T*) = {(, id}
FIRST(*F*) = {(, id}
FIRST(*X*) = {+, ε}
FIRST(*Y*) = {*, ε}

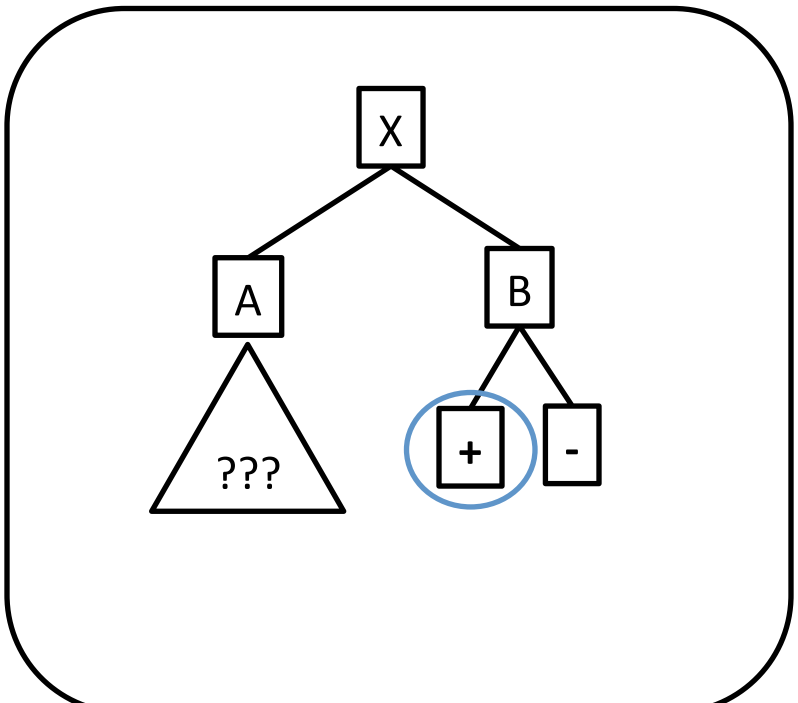
FIRST(*TX*) = {**(**, **id**}
FIRST(**+***TX*) = { **+** }
FIRST(*FY*) = { **(**, **id** }
FIRST (**FY*) = { ***** }
FIRST(**(** *E* **)**) = { **(** }
FIRST(**id**) = { **id** }

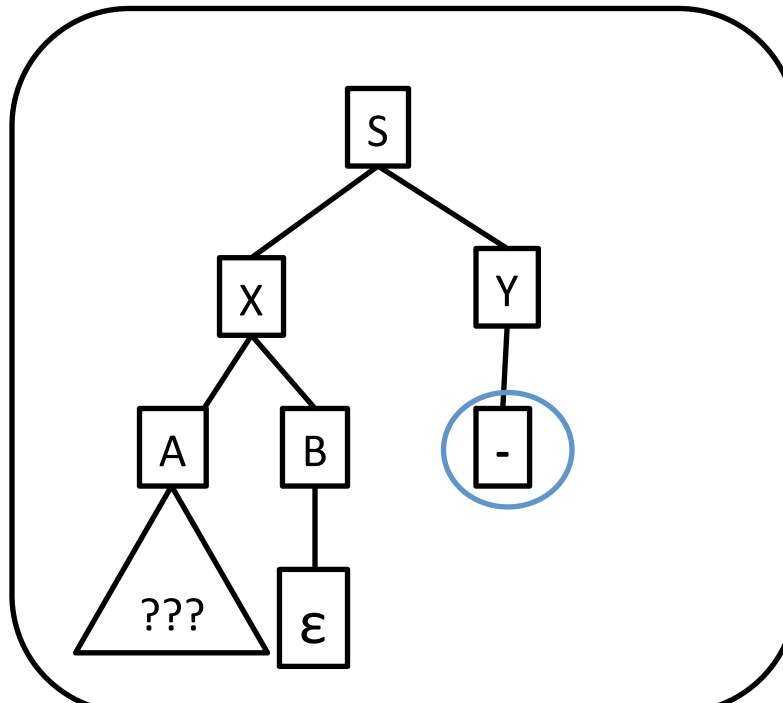
FIRST sets alone do not provide enough information to construct a parse table

If a rule R can derive ε, we need to know what terminals can come just <u>after</u> R

FOLLOW Sets: Pictorially

For nonterminal A, FOLLOW(A) is the set of terminals that can appear immediately to the right of A





FOLLOW Sets: Pictorially

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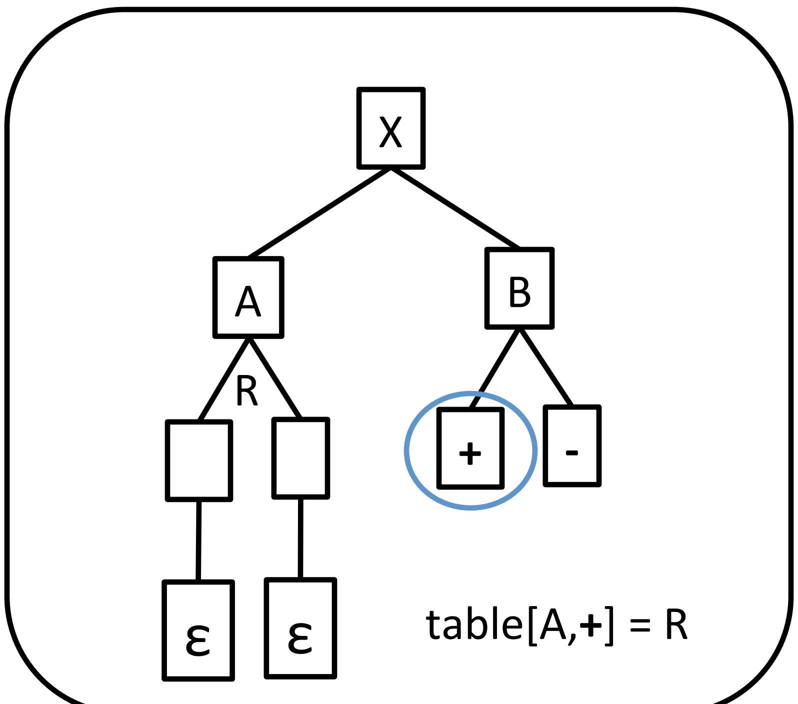
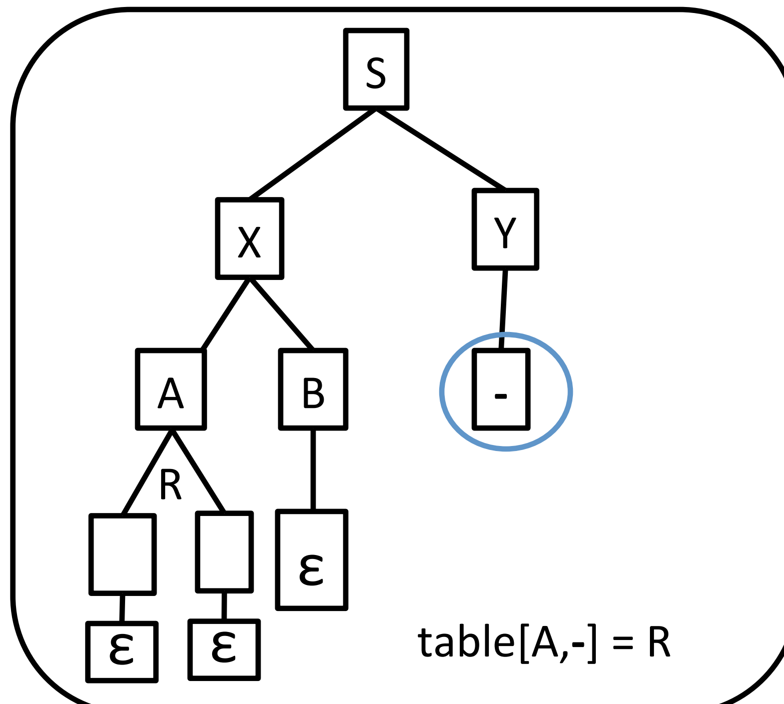




FOLLOW Sets

For <u>nonterminal</u> A, FOLLOW(A) is the set of <u>terminals</u> that can appear immediately to the right of A

Let’s write it together,

FOLLOW(A) =

FOLLOW Sets

For nonterminal A, FOLLOW(A) is the set of terminals that can appear immediately to the right of A

Let's write it together,

FOLLOW(A) =

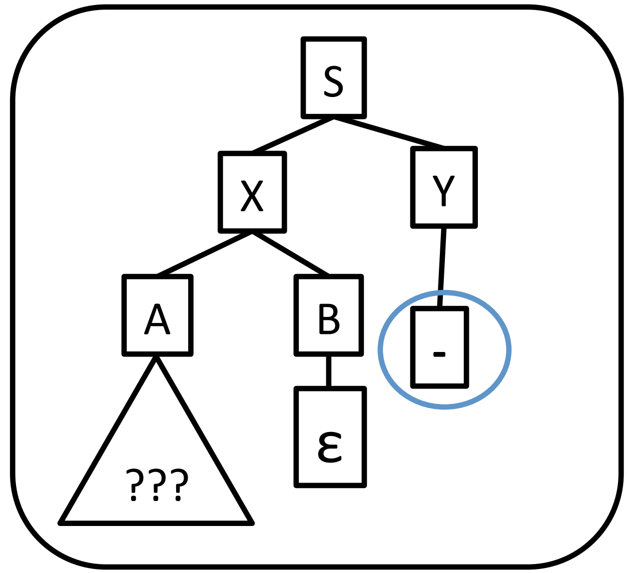
$$\{t \mid (t \in \Sigma \wedge S \Rightarrow\upharpoonleft+\ \alpha A t \beta) \vee (t = EOF \wedge S \Rightarrow\upharpoonleft*\ \alpha A)\}$$

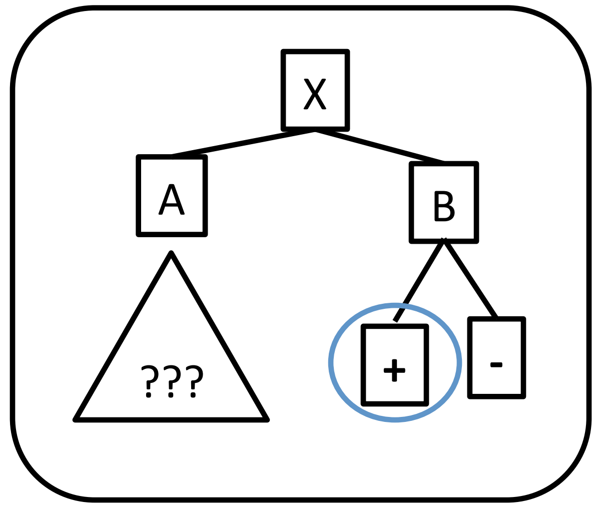
FOLLOW Sets: Construction

To build FOLLOW(A)

- If A is the start nonterminal, add **eof**
- For rules $X \longrightarrow \alpha\ A\ \beta$ **Where α, β may be empty**
  - Add FIRST(β) – {ε}
  - If ε is in FIRST(β) or β is empty, add FOLLOW(X)

Continue building FOLLOW sets until reach a fixed point (i.e., no more symbols can be added)





FOLLOW Sets Example

<u>FOLLOW(A) for $X \longrightarrow \alpha\ A\ \beta$</u>

If A is the start, add **eof**

Add FIRST(β) – {ε}

Add FOLLOW(X) if ε in FIRST(β) or β is empty

S ⟶ B **c** | D B

B ⟶ **a b** | **c** S

D ⟶ **d** | ε

FIRST (S) = { **a**, **c**, **d** }

FIRST (B) = { **a**, **c** }

FIRST (D) = { **d**, ε }

FIRST (B **c**) = { **a**, **c** }

FIRST (D B) = { **d**, **a**, **c** }

FIRST (**a b**) = { **a** }

FIRST (**c** *S*) = { **c** }

FOLLOW (S) = { **eof** }

FOLLOW (B) = { **c**, **eof** }

FOLLOW (D) = { **a, c** }

FOLLOW (S) = { **eof, c** }

FOLLOW (B) = { **c**, **eof** }

FOLLOW (D) = { **a, c** }

FOLLOW (S) = { **eof, c** }

FOLLOW (B) = { **c**, **eof** }

FOLLOW (D) = { **a, c** }

Building the Parse Table

```
for each production X ⟶ α {
  for each terminal t in FIRST(α){
    put α in Table[X][t]
  }
  if ε is in FIRST(α){
    for each terminal t in FOLLOW(X){
      put α in Table[X][t]
    }
  }
}
```

Table collision ⇔ Grammar is not in LL(1)

Putting it all together

Build FIRST sets for each nonterminal

Build FIRST sets for each production's RHS

Build FOLLOW sets for each nonterminal

Use FIRST and FOLLOW to fill parse table for each production

Tips n' Tricks

FIRST sets

- Only contain alphabet terminals and ε
- Defined for arbitrary RHS and nonterminals
- Constructed by starting at the beginning of a production

FOLLOW sets

- Only contain alphabet terminals and **eof**
- Defined for nonterminals only
- Constructed by jumping into production

FIRST(α) for $\alpha = Y_1\, Y_2 \ldots Y_k$

Add FIRST(Y_1) - {ε}

If ε is in FIRST($Y_{1 \text{ to } i-1}$): add FIRST(Y_i) – {ε}

If ε is in all RHS symbols, add ε

FOLLOW(A) for $X \longrightarrow \alpha\ A\ \beta$

If A is the start, add **eof**

Add FIRST(β) – {ε}

Add FOLLOW(X) if ε in FIRST(β) or β empty

Table[X][t]

```
for each production X ⟶ α
 for each terminal t in FIRST(α)
   put α in Table[X][t]
 if ε is in FIRST(α){
   for each terminal t in FOLLOW(X){
     put α in Table[X][t]
```

CFG

S ⟶ B **c** | D B

B ⟶ **a b** | **c** S

D ⟶ **d** | ε

FIRST (S) = { **a**, **c**, **d** }

FIRST (B) = { **a**, **c** }

FIRST (D) = { **d**, ε }

FIRST (B **c**) = { **a**, **c** }

FIRST (D B) = { **d**, **a**, **c** }

FIRST (**a b**) = { **a** }

FIRST (**c** *S*) = { **c** }

FIRST (**d**) = { **d** }

FIRST (ε) = { ε }

FOLLOW (S) = { **eof**, **c** }

FOLLOW (B) = { **c**, **eof** }

FOLLOW (D) = { **a**, **c** }

Not LL(1)

	a	**b**	**c**	**d**	**eof**
S	B **c** D B		B **c** D B	D B	
B	**a b**		**c** *S*		
D	ε		ε	**d**	

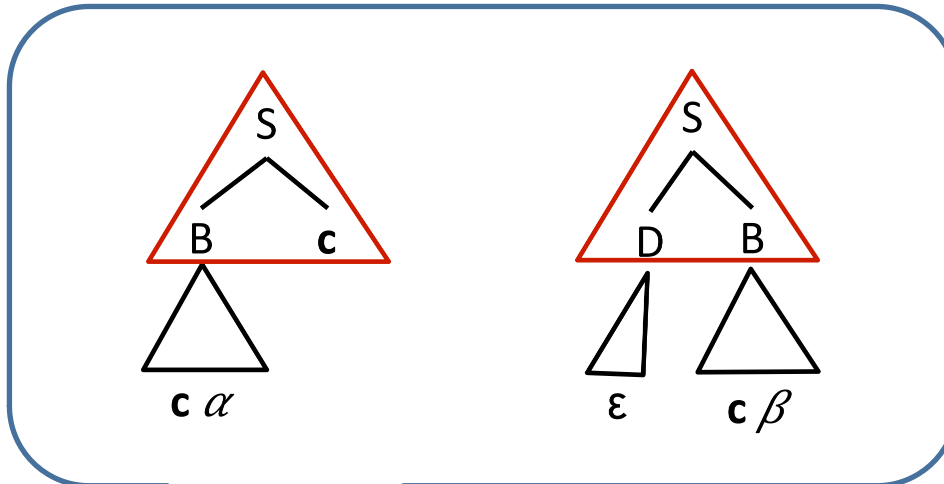
Why is a Table Collision a Problem?

<u>CFG</u>

S → B **c** | D B

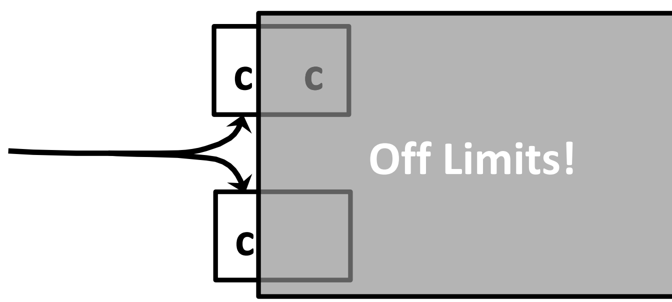
B → **a b** | **c** S

D → **d** | ε



	a	**b**	**c**	**d**	
S	B **c** D B		B **c** D B	D B	
B	**a b**		**c** *S*		
D	ε		ε	**d**	

current token



Recap

FIRST and FOLLOW sets define the parse table

If the grammar is LL(1), the table is unambiguous

- i.e., each cell has at most one entry

If the grammar is not LL(1) we can attempt a transformation sequence:

1. Remove left recursion
2. Left-factoring

Next time: Grammar transformations affect the structure of the parse tree. How does this affect syntax-directed translation (in particular, parse tree AST)?