Physica A 586 (2022) 126438

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

PageRank centrality and algorithms for weighted, directed networks

Panpan Zhang [a,*], Tiandong Wang [b], Jun Yan [c]

[a] Department of Biostatistics, Epidemiology and Informatics, University of Pennsylvania, Philadelphia, 19104, PA, USA
[b] Department of Statistics, Texas A&M University, College Station, 77843, TX, USA
[c] Department of Statistics, University of Connecticut, Storrs, 06269, CT, USA

ARTICLE INFO

Article history:
Received 6 April 2021
Received in revised form 12 August 2021
Available online 22 September 2021

MSC:
91D03
05C82

Keywords:
Node centrality
Weighted directed networks
Weighted PageRank
World Input-Output Tables

ABSTRACT

PageRank (PR) is a fundamental tool for assessing the relative importance of the nodes in a network. In this paper, we propose a measure, weighted PageRank (WPR), extended from the classical PR for weighted, directed networks with possible non-uniform node-specific information that is dependent or independent of network structure. A tuning parameter leveraging node degree and strength is introduced. An efficient algorithm based on R program has been developed for computing WPR in large-scale networks. We have tested the proposed WPR on widely used simulated network models, and found it outperformed the classical PR. Additionally, we apply the proposed WPR to the real network data generated from World Input–Output Tables as an example, and have seen the results that are consistent with the global economic trends, which renders it a preferred measure in the analysis.

1. Introduction

Centrality measures are widely accepted tools for assessing the relative importance of the entities in networks. A variety of centrality measures have been developed in the literature, including position/degree centrality [1], closeness centrality [2], betweenness centrality [2], eigenvector centrality [3], Katz centrality [4], and PageRank [5], among others. Centrality measures have been applied to different types of real networks; for instance, ranking the cities with at least one operating airport in the air transport network of China [6] and evaluating the impact of research papers in a citation network [7]. See Das et al. [8] for a concise review and Newman [9, Chapter 7] for a text-style elaboration.

The classical PageRank [PR,10] was designed to precisely rank web pages in Google search via hyper-textual information (primarily link structure). Today, PR and its extensions are popular tools for the analyses of all kinds of networks, such as co-authorship networks [11], citation networks [12], and biological networks [13]. One limitation of the classical PR is that it does not account for edge weight in definition. Although ignoring edge weight may sometimes help with a quick exploration of the fundamental structure of a network, the discarded edge weight can lead to incorrect inference [14]. Only a limited number of works considered weight for PR. Xing and Ghorbani [15] asserted that the popularity of a web page should be based on the numbers of its in-links and out-links. Ding [12] suggested replacing the random restart of the new process with a probability distribution based on the weights assigned to the nodes. No PR centrality measures have put edge weight and node weight in a unified framework.

* Corresponding author.
E-mail address: panpan.zhang@pennmedicine.upenn.edu (P. Zhang).

https://doi.org/10.1016/j.physa.2021.126438

As most real networks are weighted and directed, possibly with node-specific auxiliary information, here we consider a weighted PageRank (WPR) measure. The WPR uses edge direction and weight as well as auxiliary information at the node level to better characterize the centrality of a network. The computation of WPR boils down to finding the principal eigenvector of a big matrix, which can be efficiently done for large networks. We assess the performance of the proposed WPR in comparison with a few extended PR measures in the literature though numerical studies with synthetic networks. In the applications to the World Input-Output networks (WIONs) constructed from World Input-Output Tables [WIOTs,16], the proposed WPR measure gives more intuitive results than the existing PR measures. The implementation of the proposed and competing measures is publicly available in an open-source R package wdnet [17].

The rest of the manuscript is organized as follows. In Section 2, we propose a WPR measure and demonstrate the computation strategy. We carry out some synthetic data analyses in Section 3, where two classes of widely used network models, namely scale-free networks and stochastic block models, are adopted. We apply the proposed WRP measure to the WIONs in Section 4, followed by some concluding remarks and discussions in Section 5.

2. Weighted PageRank

We begin with some basic network notations. Let $G(V, E)$ denote a weighted and directed network, where V and E are respectively its node and edge sets. The structure of G is characterized by its weighted adjacency matrix $\boldsymbol{W} := (w_{ij})$, where w_{ij} is the weight of the directed edge from $i \in V$ to $j \in V$. If no edge exists from i to j, then $w_{ij} = 0$. When edge weight is ignored, $\boldsymbol{W}$ is reduced to the standard adjacency matrix $\boldsymbol{A} := (a_{ij})$, where a_{ij} takes value 0 or 1. For any $i \in V$, let $d_i^{(\text{out})} := \sum_{j\in V} a_{ij}$ and $d_i^{(\text{in})} := \sum_{j\in V} a_{ji}$ respectively denote the out-degree and in-degree of i, referring to the numbers of edges emanating out from and pointing into i. Analogously, we have $s_i^{(\text{out})} := \sum_{j\in V} w_{ij}$ and $s_i^{(\text{in})} := \sum_{j\in V} w_{ji}$, respectively called the out-strength and in-strength of i when weight is accounted. For undirected networks, there is no need to distinguish out-strength and in-strength. We call $s_i := \sum_{j\in V} w_{ij} = \sum_{j\in V} w_{ji}$ the strength of i.

2.1. Formulation

Brin and Page [10] defined PR recursively as

$$\text{PR}(i) = \gamma \sum_{j\in V} \frac{a_{ji}}{d_j^{(\text{out})}} \text{PR}(j) + \frac{1-\gamma}{n}, \tag{1}$$

where $\text{PR}(i)$ is the PR of node i, $n = |V|$ counts the number of nodes in $G(V, E)$, and $\gamma \in [0, 1)$ is a damping factor ensuring the algorithm never gets stuck in a "sinking node". This definition is based on a random surfer model. Suppose that an Internet surfer keeps clicking on links bringing her to different web pages. With probability $(1-\gamma)$ she restarts the process by randomly selecting a web page as the new initial state, where γ is the probability that she continues in the current process. The inclusion of damping factor in the model ensures that the process will not be forced to terminate when the surfer arrives at a web page with no outbound link, called sinking node. Eq. (1) suggests that a node would get a high PR score if: (1) it receives a large number of incoming edges; (2) senders of those incoming edges have small out-degrees; or (3) the PR scores of the senders are high.

In practice, lots of real networks are directed, weighted, and affiliated with important node-specific information. For instance, in Section 4, we consider the WIONs whose nodes correspond to different region-sectors, and edge weights are determined by transaction volumes. In addition, the total value added for each region-sector is considered as node-specific information. Here we extend the classical PR to a weighted version by simultaneously considering edge weights and auxiliary information contained in nodes. Let $\phi(i)$ denote the weighted PR of i, and β_i be some node-specific quantifiable information attached to i. We assume that β_i is independent of w_{ij} for all $i, j \in V$.

Analogous to Eq. (1), we define the weighted PR recursively by

$$\phi(i) = \gamma \sum_{j\in V} \left(\theta \frac{w_{ji}}{s_j^{(\text{out})}} + (1-\theta) \frac{a_{ji}}{d_j^{(\text{out})}} \right) \phi(j) + \frac{(1-\gamma)\beta_i}{\sum_{i\in V} \beta_i}, \tag{2}$$

where $\theta \in [0, 1]$ is a tuning parameter adjusting the relative importance of weights in the definition. The value of θ can be chosen according to practical needs and actual interpretations. For instance, the value of a business project may be heavily reflected in the investment amount it has received rather than the number of investors, and the popularity of a product mainly depends on the sales volume. In many situations, a balance between the two factors is needed. For example, the strength of a researcher is related to the number of publications as well as the prestige of the journals (measured by a unified metric such as impact factor) of the publications simultaneously. The tuning parameter θ controls the balance between weight and degree. For the special case of $\theta = 0$, the proposed WPR is equivalent to the weighted PR introduced by Ding [12]. The vector $\boldsymbol{\beta} := (\beta_1, \beta_2, \dots, \beta_n)^\top$ usually takes the non-uniform relative importance of the nodes into account. When no such information is available, we let $\beta_i = 1$, $i = 1, \dots, n$, so that the second term on the right-hand-side (RHS) of Eq. (2) coincides with what has been defined in Eq. (1).

2.2. Computation

We propose an efficient algorithm for the computation of the proposed WPR in large networks. The standard method to compute classical PR is the power iteration. It is well known that the power iteration converges slowly, especially for massive and dense networks. This leads to the development of accelerated algorithms, many of which have been surveyed in Berkhin [18]. When $\gamma \neq 1$, the underlying process of the random surfer model for the classical PR is a irreducible Markov chain [18], where every state in the chain can be accessed with positive probability from other states. Here we regard nodes in a network as states in a Markov chain.

A similar argument can be applied to the proposed WPR. Let $\boldsymbol{M} := (m_{ij})$ be the transition matrix of the associated Markov chain for WPR, where

$$m_{ij} = \begin{cases} \theta w_{ji}/s_j^{(\text{out})} + (1-\theta)a_{ji}/d_j^{(\text{out})}, & \text{if } d_j^{(\text{out})} \neq 0; \\ \beta_i / \sum_{i\in V} \beta_i, & \text{if } d_j^{(\text{out})} = 0. \end{cases}$$

Notice that

$$\sum_{i\in V} \left(\theta \frac{w_{ji}}{s_j^{(\text{out})}} + (1-\theta)\frac{a_{ji}}{d_j^{(\text{out})}} \right) = \theta + (1-\theta) = 1.$$

That is, matrix $\boldsymbol{M}$ is non-negative and column stochastic. Let $\boldsymbol{P} := \big(\phi(1), \phi(2), \ldots, \phi(n)\big)^\top$ be a column vector collecting the WPR for each node. Eq. (2) is equivalent to

$$\boldsymbol{P} = \gamma \boldsymbol{M}\boldsymbol{P} + (1-\gamma)\boldsymbol{\beta}^*, \tag{3}$$

where $\boldsymbol{\beta}^* = \boldsymbol{\beta}/\|\boldsymbol{\beta}\|_1$ is the normalization of $\boldsymbol{\beta}$.

Since $\boldsymbol{M}$ is column stochastic, we can rewrite Eq. (3) as

$$\boldsymbol{P} = (\gamma \boldsymbol{M} + (1-\gamma)\boldsymbol{B})\, \boldsymbol{P} =: \boldsymbol{M}^* \boldsymbol{P}, \tag{4}$$

where $\boldsymbol{B}$ is an $(n \times n)$ matrix such that the ith column is given by $\beta_i^* \cdot \mathbf{1}$ for $i = 1, 2, \ldots, n$. It is obvious that $\boldsymbol{B}$ is also column stochastic, rendering that $\boldsymbol{M}^*$ is strictly positive and column stochastic provided that $\boldsymbol{\beta} \neq \mathbf{0}$. By the Perron–Frobenius theorem [19], the largest eigenvalue of $\boldsymbol{M}^*$ is equal to 1, and the solution to Eq. (4) is the corresponding eigenvector. In the context of stochastic process, we regard the normalized solution of $\boldsymbol{P}$ as a stationary distribution of the Markov chain associated with the probability transition matrix $\boldsymbol{M}^*$. Based on the above representation, the computation of $\boldsymbol{P}$ in a massive network is converted to finding the principal eigenvector of a large-scale matrix. One of the most efficient approaches is the ARPACK software [20], with a recent interface for R through package **rARPACK** [21]. It is worthy of mentioning that we choose $\gamma = 0.85$ throughout the synthetic and real network analyses in Sections 3 and 4 as recommended in [10]. It is evident that the optimal selection of γ is related to network type, but this is our main focus in the present analysis. We refer the interested readers to Yan and Ding [11],Fu et al. [22],Bressan and Peserico [23] for more discussions.

3. Synthetic data examples

We assess the performance of the proposed WPR measure with scale-free networks and stochastic block networks. Comparison is done with respect to classical PR [5] and two existing weighted PR measures respectively introduced by Xing and Ghorbani [15] and Ding [12]. The definition of the former is relegated A, where the latter is a special case of the proposed WPR as mentioned. Under each network model, we introduced a parameter $\rho \in [0, 1]$ to control the strength of the dependency between the node-specific prior information and the node strength. Specifically, let S_i be the strength of node i, $i \in V$. Then, the prior information β_i of this node was generated such that the correlation between S_i and β_i is $\rho \in [0, 1]$. This can be done by setting $\beta_i = \alpha S_i + (1-\alpha)X_i$, where X_i is a positive random variable independent of S_i and

$$\alpha = \left(1 + \sqrt{\frac{(1-\rho^2)\text{Var}(S_i)}{\rho^2 \text{Var}(X_i)}} \right)^{-1}. \tag{5}$$

As $\rho \to 0$, we have $\beta_i \to X_i$, which coincides with the case of Ding [12]. When $\rho = 1$, $\beta_i = S_i$. For any $\rho \in (0, 1)$, β_i's generated with Eq. (5) were used as prior information for WPR computations.

3.1. Scale-free network

In the literature, the preferential attachment (PA) rule [24] is one way to generate scale-free networks. We used the algorithm of Yuan et al. [25] to generate weighted directed PA networks via R package `wdnet` [17]. Specifically, the simulated PA network initiates with a directed edge from node 1 to 2, where the weight is drawn from Bin(100, 0.75). At each subsequent step, an edge is added according to one of the following three scenarios: With probability $\alpha = 0.05$,

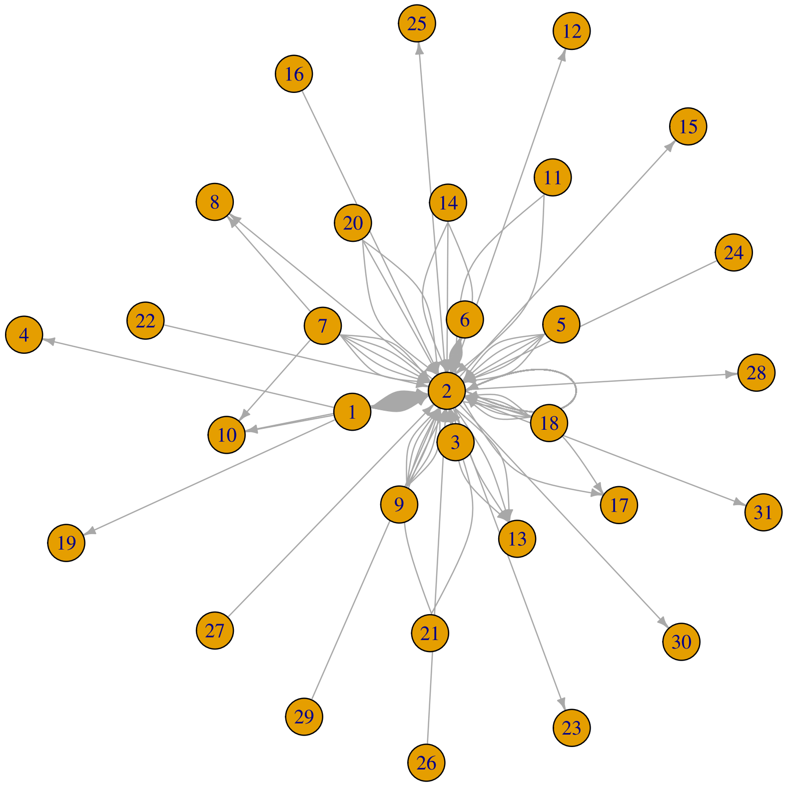

Fig. 1. A simulated (weighted and directed) PA network mimicking a Facebook wall post data.

Table 1
A comparison of the proposed WPR measure with those respectively proposed in Ding [12] (equivalent to the case of $\theta = 0$) and Xing and Ghorbani [15] for the simulated PA network; the damping factor for the proposed WPR is fixed $\gamma = 0.85$.

ϕ ($\theta = 0$)		ϕ ($\theta = 0.5$)		ϕ ($\theta = 1$)		X-G's measure	
Node	WPR (%)	Node	WPR (%)	Node	WPR (%)	Node	WPR (%)
2	26.216	2	38.032	2	66.030	3	1.710
10	4.415	10	3.404	13	2.177	5	1.710
8	4.051	8	3.225	17	1.643	6	1.710
13	3.567	13	3.185	10	1.429	9	1.710
31	3.567	17	3.031	8	1.377	11	1.710
17	3.567	28	2.931	28	1.294	14	1.710
25	3.567	23	2.928	23	1.286	16	1.710
28	3.567	30	2.928	30	1.286	18	1.710
30	3.567	25	2.927	25	1.282	20	1.710
12	3.567	15	2.925	15	1.273	21	1.710

the edge is added from a new node to an existing one; with probability $\beta = 0.9$, the edge is added between two existing nodes; with probability $\eta = 0.05$, the edge is added from an existing node to a new one. The source and target nodes of the added edge are selected proportional to their current out- and in-strengths, respectively. Upon the edge being added, its weight is independently drawn from Bin(100, 0.75). The leveraging parameters δ_{in} and δ_{out} that respectively control the growth rates of in-strengths and out-strengths are fixed, both taking value 1. We refer the readers to Wang and Zhang [26] for the statistical properties and detailed interpretations of these parameters. The evolution proceeds in this fashion for 300 steps, where the resulting PA network is depicted in Fig. 1.

In the first experiment, we did not consider any kind of node-specific prior information, that is, $\beta_i = 1$ for all $i \in V$. Table 1 summarizes the top 10 nodes based on Xing and Ghorbanis' PR measure and the proposed WPR measures (with $\gamma = 0.85$ and $\theta = \{0, 0.5, 1\}$) With Xing and Ghorbanis' PR measure, all of the top 10 nodes have the same score, so they are simply ordered by their appearance timing. This does not provide much practical guidance. In particular, node 2, which emerges at the central position in Fig. 1, does not appear in the top 10 list. Hence, Xing and Ghorbanis' PR measure will not be considered in the sequel.

Table 2
Nodes of top 10 WPR scores with/without (independent) prior information in the simulated PA network for $\theta \in \{0, 0.5, 1\}$ and $\gamma = 0.85$.

$\theta = 0$		$\theta = 0.5$		$\theta = 1$	
no prior	with prior	no prior	with prior	no prior	with prior
2	2	2	2	2	2
10	19	10	19	13	19
8	10	8	10	17	7
13	8	13	7	10	10
31	7	17	17	8	4
17	17	28	4	28	17
25	4	23	8	23	29
28	23	30	23	30	23
30	25	25	25	25	25
12	29	15	29	15	8

Node 2 ranks first in all three lists in Table 1, which is consistent with the observation from Fig. 1. The nodes of rank 2 and 3 are the same for $\theta = 0$ and $\theta = 0.5$, but not for $\theta = 1$. Ding's PR measure ($\theta = 0$) could not distinguish the nodes from rank 4 to 10, as they have exactly the same score. This is expected; a measure not accounting for edge weight is not suitable for weighted networks. From the list of $\theta = 0.5$, nodes of lower ranks (in top 10) have become identifiable, albeit with tiny gaps. When edge weights are fully accounted, node 2 is extensively dominant in the network with a much higher WPR score than the rest. Meanwhile, the normalized WPR score of node 2 in the list of $\theta = 1.0$ is higher than the counterparts in the lists of $\theta = 0$ and $\theta = 0.5$ as well. The ranks of nodes 13 and 17 both rise in the list of $\theta = 1.0$, while those of nodes 10 and 8 drop. Further investigation reveals that node 13 and 17 both have links of high weight 295 and 165, respectively, from node 2. Except for edges pointing towards node 2, these two edges are the most weighted in the network. Though node 10 receives three links respectively with weight 72 from node 1, weight 80 from node 2, and weight 72 from node 7, and node 8 receives two links respectively with weight 71 from node 2 and weight 76 from node 7, the incoming edges from node 2 are relatively small and the WPR scores of all of the other nodes are much smaller than that of node 2. As a result, nodes 10 and 8 are ranked lower than nodes 13 and 17.

In the second experiment, we incorporated an independent node-specific prior ranking weight ($\rho = 0$) into the WPR computation. The priors were generated independently from the network with an exponential distribution with mean 5; see Fig. 2. Table 2 summarizes the top 10 nodes based on proposed WPRs obtained under $\theta \in \{0, 0.5, 1\}$ and $\gamma = 0.85$ in comparison to those without considering the prior information. Drastic changes are observed. Take $\theta = 1$ as an example. Four nodes in the top 10 list are new. Two of them, nodes 19 and 7 rank 2 and 3, respectively, compared to 13 and 31 without the prior information. Indeed, these two nodes indeed have the top two prior scores as shown in Fig. 2. Although node 2 remains at the top, inclusion of the prior information has brought several low rank nodes to much higher ranks. This suggest that a non-negligible impact of the prior information on WPR and the ultimate ranking.

Lastly, we carried out a sensitivity analysis for $\theta \in \{0, 0.5, 1\}$ and $\rho \in \{0.25, 0.5, 0.75\}$, where ρ is the correlation between the prior information and the in-strength. Table 3 summarizes the results of the top 10 nodes. Especially for $\theta = 1$, the ranks are almost identical for different choices of ρ, suggesting that the proposed WPR is robust when edge weight is fully accounted in the computation. For $\theta = 0.5$, we do not observe significant difference in node labels across the three lists. The new participant for the list of $\rho = 0.75$, node 13, is ranked 12 and 11 respectively in the lists of $\rho = 0.25$ and $\rho = 0.5$, and node 29 has dropped to rank 11 in the list of $\rho = 0.75$. There have been some changes in the rank orders as expected, as the quantities of resulting priors change with the value of ρ. For instance, the prior of node 7 is much higher than that of node 17 for small ρ, but the deviation gets smaller as ρ gets larger. As node 17 receives a moderately weighted link from node 2 (the one with the largest WPR score), it takes the fourth place in the lists of $\rho = 0.5$ and $\rho = 0.75$. Furthermore, node 8 has surpassed node 7 in the list of $\rho = 0.75$, too, as it also gets a link from node 2, despite its small prior. When weight is not considered, we again only see difference in the order of ranks in the presented lists. For $\rho = 0.75$, node 10 has replaced node 19 taking the second place. Node 19 is always ranked high since it has the largest prior of all, but there is only one link pointing to it. The difference between the priors of node 10 and node 19 is not big for $\rho = 0.75$, but node 10 receives more links from the others in the network, including one from node 2, rendering it to take a higher rank ultimately.

No significant change in WPR scores is observed for different selections of θ and ρ. This is due to the characteristic of PA rule that nodes of high in-degree (in-strength) are likely to attract more incoming connections. When there is a subset of nodes that have received the majority of incoming edges at an early stage, the newly generated edges will be connected towards these nodes with high probabilities. While the in-strengths of these nodes keep growing, their in-degrees increase as well, which results in high scores of classical PR measure. Accordingly, the effect of edge weight on the final ranking results, especially the top 5, has become limited for PA networks. Rankings sensitive to θ are illustrated in the next example.

3.2. Stochastic block model

Stochastic block models (SBMs) are a class of network models for characterizing community structure [27–29]. In essence, an SBM is comprised of a certain number of within-block Erdös–Renyi [ER,30] models, where the cross-block

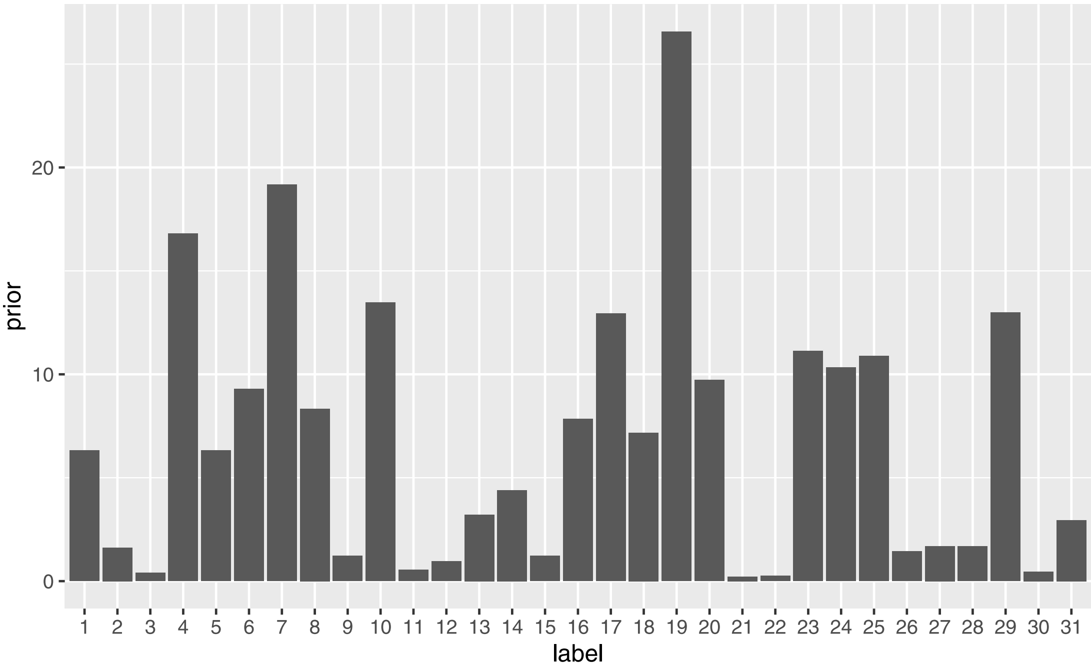

Fig. 2. Generated scores (as prior information) for the nodes in the simulated PA network.

Table 3
Nodes of top 10 WPR scores with a variety of correlated prior information, $\rho \in \{0.25, 0.5, 0.75\}$, in the simulated PA network for $\theta \in \{0, 0.5, 1\}$ and $\gamma = 0.85$.

$\theta = 0$			$\theta = 0.5$			$\theta = 1$		
$\rho = 0.25$	$\rho = 0.5$	$\rho = 0.75$	$\rho = 0.25$	$\rho = 0.5$	$\rho = 0.75$	$\rho = 0.25$	$\rho = 0.5$	$\rho = 0.75$
2	2	2	2	2	2	2	2	2
19	19	10	19	19	19	19	19	19
10	10	19	10	10	10	7	7	7
8	8	8	7	17	17	10	10	10
7	17	17	17	7	8	17	17	17
17	7	23	8	8	7	4	4	4
4	4	25	4	23	23	23	23	13
23	23	7	23	4	25	29	25	23
25	25	4	25	25	4	25	29	25
29	29	29	29	29	13	8	8	8

structure is specified by Bernoulli models. We generated a weighted and directed SBM network consisting of two communities, C_1 and C_2, each containing 50 members. The link densities within C_1 and C_2 were respectively 0.2 and 0.3, whereas the link density between C_1 and C_2 was 0.02. The weights for edges in C_1 and C_2 were independently drawn from Bin(500, 0.75) and Bin(20, 0.5), respectively. The weights for edges between C_1 and C_2 (either from C_1 to C_2 or from C_2 to C_1) were independently drawn from Bin(5, 0.5). Community C_1 is sparser than community C_2, but their within-community links are both much denser than the between-community links. In addition, edge weights in C_1 are much larger than those in C_2, and edge weights between the communities are the smallest. The generated SBM network is presented in Fig. 3, where the node sizes are proportional to the logarithm of their strengths. The nodes in C_1 and C_2 are colored with blue and red, respectively.

Table 4 summarizes the top 10 nodes based on the WPR scores with $\theta \in \{0, 0.5, 1\}$ and $\gamma = 0.85$. When edge weight is not accounted for (i.e., $\theta = 0$), nine of the top 10 nodes come from C_2 since the nodes therein are more densely connected. A close inspection shows that the top 3 nodes 59, 63 and 93 have the largest in-degrees. For $\theta = 1$, all top 10 nodes belong to C_1. Node 39 has in-strength 2,230, which is less than that of node 50 (2,415), but still ranks higher than node 50. This is desired as node 50 has a large amount of inputs from insignificant nodes, like nodes 22 (rank 87), 23 (rank 100), 28 (rank 85) and 29 (rank 90), whereas node 39 has inputs mostly from high rank nodes including itself. The top 3 nodes in the list of $\theta = 0$ rank only, respectively, 56, 97 and 54, as they have low in-strengths, and the WPR scores of the nodes linking towards them are relatively low. For the hybrid case of $\theta = 0.5$, we observe a mixture of the nodes from the top 10 lists of $\theta = 0$ and $\theta = 1$ with five each. The top 1 is node 59 (top 1 from $\theta = 0$), whose WPR score is slightly

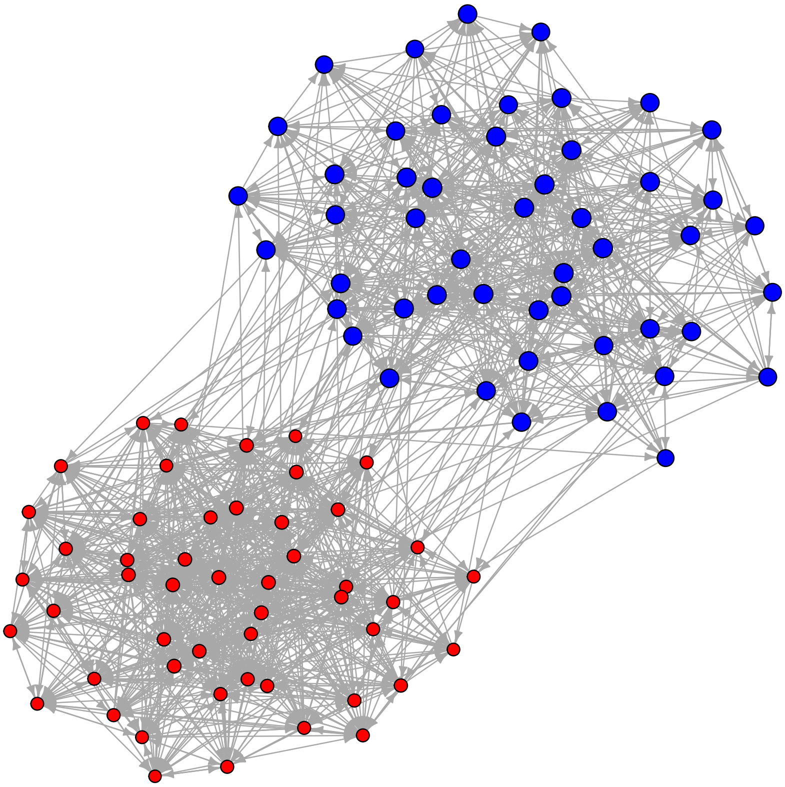

Fig. 3. A simulated (weighted and directed) SBM network consisting of two communities; the blue nodes are from C_1 and the red ones are from C_2; self-loops exist but are not presented.

Table 4
The nodes with top 10 proposed WPR scores in the simulated SBM network for $\theta \in \{0, 0.5, 1\}$; the damping factor is fixed $\gamma = 0.85$.

ϕ ($\theta = 0$)		ϕ ($\theta = 0.5$)		ϕ ($\theta = 1$)	
Node	WPR (%)	Node	WPR (%)	Node	WPR (%)
59	1.775	59	1.536	39	1.750
63	1.500	39	1.492	8	1.605
93	1.494	8	1.466	50	1.583
98	1.444	63	1.424	32	1.576
78	1.429	32	1.367	36	1.525
57	1.417	93	1.364	5	1.421
86	1.403	36	1.352	25	1.420
71	1.390	98	1.343	38	1.384
8	1.390	78	1.328	46	1.353
72	1.372	50	1.328	30	1.327

higher than that of the second largest, node 39 (top 1 form $\theta = 1$). The node with third highest WPR score is node 8, which also comes from the $\theta = 1$ list. We see that the difference between the WPR scores between nodes 39 and 8 is smaller than that in the list of $\theta = 1$ since the edge weight is not yet fully accounted.

The SBM example provides strong evidence for the necessity of accounting for edge weight in the WPR computation. Unlike the example of PA network, the top 10 nodes here are almost completely different between $\theta = 0$ and $\theta = 1$. Noticeable changes have been observed in the WPR scores as well. Such drastic changes in node ranks are primarily due to the structure of the network. There is no preferential attachment feature in the generation of SBMs, so the impact of edge weight on the WPR scores remains compelling.

4. World Input-Output networks

In economics, World Input–Output Table (WIOT) is a multi-regional input–output table, which records the intermediate transaction volumes among the sectors from different countries/regions. It has great research value in analyzing the inter-dependency across multi-regional sectors in the global economy. In the literature, there are a great deal of research

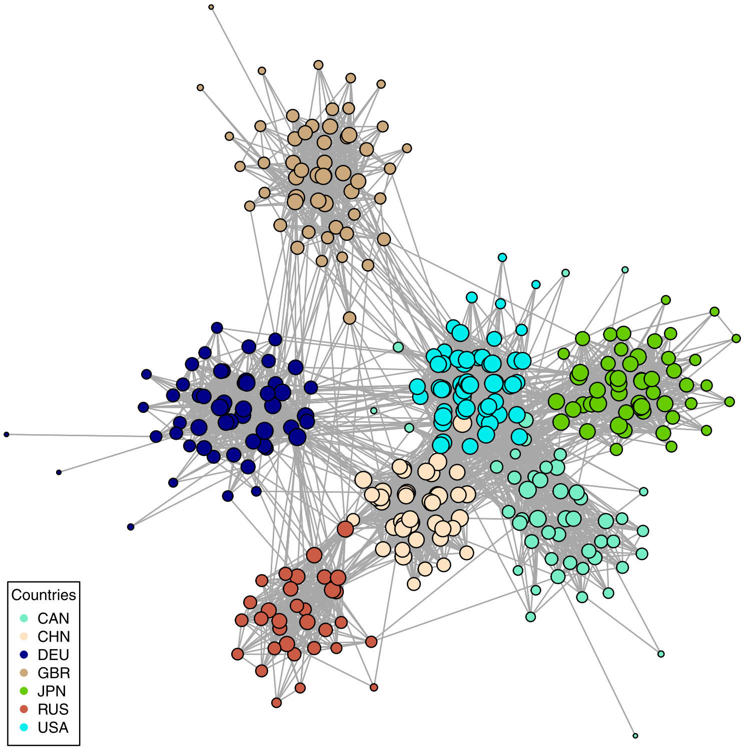

Fig. 4. An example of sub-network of the WION consisting of seven major economies in 2014; self-loops and isolated nodes have been removed; node sizes are proportional to the natural logarithm of their strengths; the edges of weight greater than or equal to 500 (with unit 1 million USD) are presented.

Table 5
The region-sectors with top 10 WPR scores (no prior information) in the WIONs from 2000, 2007 and 2014, with $\gamma = 0.85$ and $\theta \in \{0, 0.5, 1\}$.

Rank	2000			2007			2014		
	$\theta = 0$	$\theta = 0.5$	$\theta = 1$	$\theta = 0$	$\theta = 0.5$	$\theta = 1$	$\theta = 0$	$\theta = 0.5$	$\theta = 1$
1	IND39	ROW27	USA51	IND39	ROW27	ROW27	IND39	ROW27	ROW27
2	IND31	DEU20	ROW27	IND31	DEU20	GBR53	IND31	ROW4	USA51
3	IND40	USA51	GBR53	MEX5	ROW4	USA51	IND40	DEU20	GBR53
4	IND27	ROW24	USA53	MEX50	USA51	USA53	MEX50	USA51	CHN27
5	IND32	ROW6	DEU20	MEX27	ROW24	ROW24	MEX5	ROW24	ROW4
6	IND1	ROW29	USA20	MEX28	ESP27	DEU20	MEX27	ROW5	ROW24
7	IND42	USA27	ROW6	MEX29	FRA27	ESP27	MEX30	ROW51	USA53
8	MEX5	DEU27	USA27	MEX30	ROW6	ROW4	IND27	GBR53	ROW5
9	MEX50	USA20	USA44	IND40	GBR53	ROW6	MEX29	CHN27	DEU20
10	MEX27	ROW31	ROW24	MEX45	ROW17	USA44	MEX45	ROW29	ROW6

assessing the importance of sectors or industries from a variety of economic networks [31–34]. We applied the proposed WPR to the WIONs constructed from the annual WIOTs [16] from 2000 to 2014 using the 2016 release of the WorldInput-OutputDatabase. The 2016 release covers 56 sectors from 44 countries/regions, including a region called "the rest of the world" (ROW). The dictionary for the sector codes are given in Table C.9 in C. The 2,464 region-sectors are the nodes of the WIONs. A transaction from one region-sector to another forms a weighted, directed edge, where the edge weight is represented by the transaction volume (in the unit of 1 million USD). The link densities of the WIONs are high with average 83%. A sub-network consisting of seven major economies for 2014 is depicted in Fig. 4. Only edges of weight $\geq$ 500 are presented, while the self-loops and isolated nodes have been removed. The node size is proportional to the natural logarithm of its total strength. A few studies have investigated centrality measures (not limited to PR and its variants) of the WIONs [35–37].

Table 5 presents the top 10 region-sectors ranked by the proposed WPR with $\theta \in \{0, 0.5, 1\}$ and $\gamma = 0.85$ for the WIONs from 2000, 2007 and 2014. When edge weights are not taken into account (i.e. $\theta = 0$), all top 10 nodes are sectors from India (IND) and Mexico (MEX) for all three years. Since neither of the two countries was regarded as the most influential in the world economy during the study period, we think $\theta = 0$ as inadequate for ranking region-sectors using WPR. When weight is partially accounted ($\theta = 0.5$), results are more reasonable, but completely different from those

Table 6
The region-sectors with top 10 WPR scores (with TVA prior information) in the WIONs from 2000, 2007 and 2014, with $\gamma = 0.85$ and $\theta = 1$.

Rank	2000		2007		2014	
	no prior	TVA prior	no prior	TVA prior	no prior	TVA prior
1	USA51	USA51	ROW27	USA51	ROW27	CHN27
2	ROW27	USA53	GBR53	USA53	USA51	USA51
3	GBR53	USA44	USA51	USA44	GBR53	ROW27
4	USA53	USA27	USA53	ROW27	CHN27	USA53
5	DEU20	USA20	ROW24	USA27	ROW4	USA44
6	USA20	USA5	DEU20	GBR53	ROW24	CHN20
7	ROW6	USA30	ESP27	ROW24	USA53	CHN17
8	USA27	USA36	ROW4	USA5	ROW5	ROW4
9	USA44	USA29	ROW6	CHN27	DEU20	ROW24
10	ROW24	JPN27	USA44	USA29	ROW6	CHN15

with $\theta = 0$. Construction (27) from ROW took the first place in all three years. Manufacture of motor vehicles, trailers and semi-trailers (20) from Germany (DEU) took the second place in 2000 and 2007, but the third place in 2014, and the second place in 2014 was taken by mining and quarrying (4) from ROW. Several other traditional leading sectors from influential economies are also included in the top 10 list, such as public administration and defense and compulsory social security (51) from the United States of America (USA) and human health and social work activities (53) from the United Kingdom (GBR). Quite a few sectors from ROW besides construction (27) are in the top 10 lists, which is understandable since ROW aggregates over those outside of the 43 countries/regions.

When edge weight is fully accounted ($\theta = 1$), no significantly different results have been observed from those with $\theta = 0.5$. The reason that we see much more significant changes when increasing θ from 0 to 0.5 than when further increasing θ from 0.5 to 1 could be that the variation in edges weights is much greater than the variation in node strengths. Over all three years, the top 3 sectors were public administration and defense and compulsory social security (51) from USA, human health and social work activities (53) from GBR, and construction (27) from ROW, except in different orders. Fewer sectors are from ROW in the top 10 lists of $\theta = 1$ compared to those of $\theta = 0.5$. As ROW contains many countries/regions, its sectors have input and output connections with all of the other region-sectors in the networks. Those edge weights are not necessarily large even though they are aggregated counts. Therefore, more leading sectors from the strong economies appear in the top 10 lists. In 2000, four sectors were from the world's largest economy USA, but the number was reduced to three in 2007. Human health and social work activities (53) and real estate activities (44) from USA were indeed world-leading region-sectors. After the subprime mortgage crisis in 2008, real estate activities (44) did not appear in the top 10 list in 2014 like in 2000 or 2007. Construction (27) from China (CHN) joined the top 10 list in 2014. As China became the world's second largest economic power in 2010, construction as a sector with the largest pulling effect has been expected to have a high rank [38]. The inclusion of a non European Union (EU) country or USA in the top 10 list suggests a changed landscape and increased diversity of the global economy.

We next used the total value-added (TVA) in the WIOTs as prior information in the computation of WPR. TVA was adopted as it is one of the most commonly used measures indicating the contribution by each region-sector to the global economy [38], so it appears to be appropriate for identifying the most influential regional-sectors in the global economy. The results with edge weights fully accounted (i.e., $\theta = 1$) are summarized in Table 6, whereas the additional comparisons for $\theta = 0.5$ and $\theta = 0$ are given in B. Significant changes have been observed over time. In 2000, the top 9 region-sectors were from USA. The top 5 are public administration and defense and compulsory social security (51), human health and social work activities (53), real estate activities (44), construction (27), and manufacture of motor vehicles, trailers and semi-trailers (20). The only non-USA region-sector was construction (27) from Japan, which has been a large component of the Japanese economy in terms of output and employment, and a robust force for the economic recovery and expansion in Japan in the post-war years till today [39,40]. In 2007, USA was not as dominant as in 2000 but still with six in the top 10. The top 3 remain unchanged. Construction (27) from China ranked the 9th. This result seems to be more reasonable, as the Chinese government provided unlimited support to the construction industry in 2007 in preparation for the 2008 Olympic Game. As the most influential sector in China, construction (27) has driven the development of a large number of domestic sectors as well as international cooperation over that period [41,42]. In 2014, the top 10 nodes had 4 from China, 3 from USA, and 3 from ROW. Construction (27) of China ranked the first. The other three Chinese sectors were manufacture of motor vehicles, trailers and semi-trailers (20), manufacture of computer, electronic and optical products (17) and manufacture of basic metals (15), which, respectively, ranked 6, 7 and 10. The top 3 USA sectors in 2007 now ranked 2, 4, and 5. No EU sectors showed up in the top 10. This result is consistent with the fact that USA and China are the two largest economies in the world, and that the growth of Chinese economy has been significant in the last two decades. Compared to results without TVA prior, fewer region-sectors from ROW ranked in the top 10 as their TVA amounts were small in general.

The synthetic data analyses and the WION example have substantiated the importance of accounting for edge weight and utilizing possible prior information when applying WPR to weighted, directed network data. Which has more effect on the computation of WPR then? To answer this question, we carry out a sensitivity analysis as follows. We select

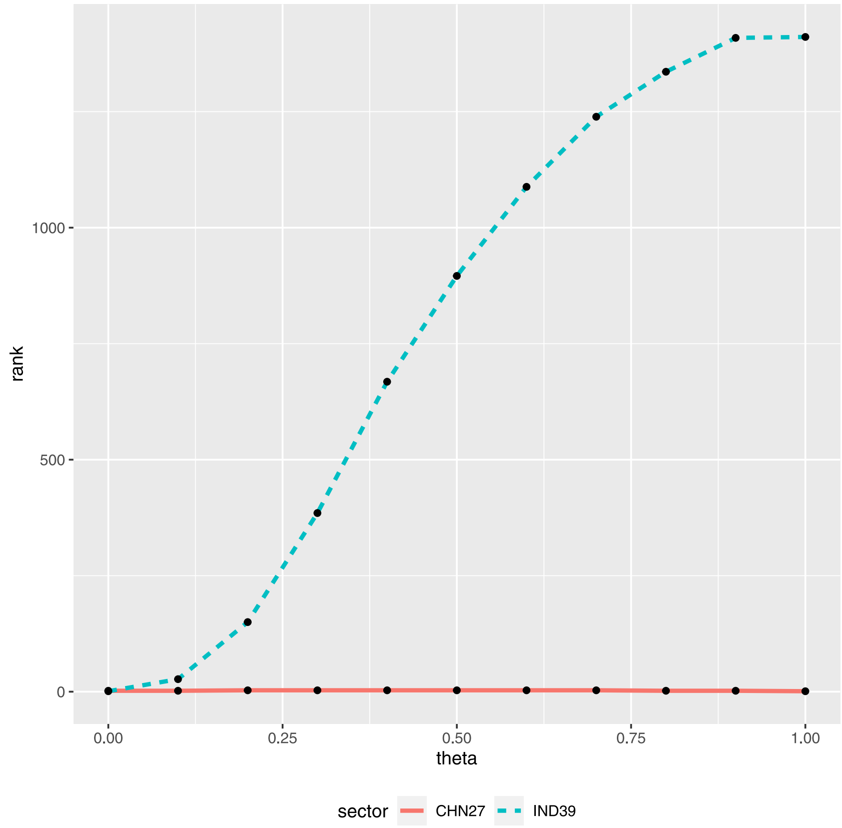

Fig. 5. Rank change in $\theta = \{0, 0.1, \dots, 1\}$ for Construction (27) from China in 2014 and Telecommunications (39) from India in 2000.

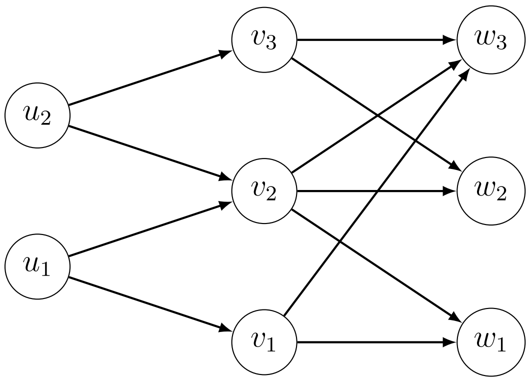

Fig. A.6. A toy example for illustrating the edge weighting process in Xing and Ghorbani [15]; all the edges have a unit weight.

two region-sectors; namely they are Construction (27) from China (Rank 1 in WPR score from 2004 when TVA prior are accounted) and Telecommunications (39) from India (Rank 1 in WPR score when no prior is accounted). For each region-sector, we compute the WPR scores, and determine the corresponding ranks with the change of $\theta = \{0, 0.1, \dots, 1\}$. The graphical results for ranks are shown in Fig. 5.

We find that, when TVA prior is used, the rank of Construction (from China), is stable with the change in θ. Its rank stays in top 3 regardless of the selected value of θ, which suggests the utilization of proper prior information helps ameliorate the misleading conclusions caused by the lack of considering edge weight in the computation. However, when prior information is not accounted, the rank of Telecommunications (39) from India is extensively sensitive to the increase of θ, dropping from rank 1 (for $\theta = 0$) to rank 896 (for $\theta = 0.5$), where the lowest is rank 1411 (for $\theta = 1$).

Table B.7
The region-sectors with top 10 WPR scores (with TVA prior information) in the WIONs from 2000, 2007 and 2014, with $\gamma = 0.85$ and $\theta = 0.5$.

Rank	2000		2007		2014	
	no prior	TVA prior	no prior	TVA prior	no prior	TVA prior
1	ROW27	USA51	ROW27	USA51	ROW27	ROW27
2	DEU20	USA44	DEU20	ROW27	ROW4	USA51
3	USA51	USA53	ROW4	USA44	DEU20	CHN27
4	ROW24	USA27	USA51	USA53	USA51	ROW4
5	ROW6	ROW27	ROW24	ROW24	ROW24	USA53
6	ROW29	JPN27	ESP27	ROW4	ROW5	USA44
7	USA27	USA29	FRA27	USA27	ROW51	ROW24
8	DEU27	USA20	ROW6	DEU20	GBR53	CHN17
9	USA20	USA30	GBR53	USA29	CHN27	CHN15
10	ROW31	USA5	ROW17	CHN27	ROW29	CHN5

Table B.8
The region-sectors with top 10 WPR scores (with TVA prior information) in the WIONs from 2000, 2007 and 2014, with $\gamma = 0.85$ and $\theta = 0$.

Rank	2000		2007		2014	
	no prior	TVA prior	no prior	TVA prior	no prior	TVA prior
1	IND39	USA51	IND39	USA51	IND39	USA51
2	IND31	USA44	IND31	USA44	IND31	CHN27
3	IND40	USA53	MEX5	USA53	IND40	USA44
4	IND27	USA27	MEX50	ROW24	MEX50	ROW4
5	IND32	USA29	MEX27	ROW4	MEX5	ROW27
6	IND1	USA30	MEX28	USA27	MEX27	USA53
7	IND42	JPN27	MEX29	USA29	MEX30	ROW24
8	MEX5	USA50	MEX30	ROW27	IND27	CHN15
9	MEX50	JPN44	IND40	USA30	MEX29	CHN5
10	MEX27	JPN29	MEX45	USA45	MEX45	CHN17

5. Discussions

We propose a weighted PageRank measure that simultaneously accounts for edge weights and prior information on the relative importance of nodes in weighted directed networks. The relative importance of node strengths and edge weights is controlled by a tuning parameter for flexibility. In the present article, we consider θ as a subjective selection based on the research questions to answer. Nonetheless, given a specific type of network class, there may exist an optimal selection of θ, which leads to some further investigations in our future studies. Efficient algorithms are implemented and made publicly available in R package wdnet [17]. Through two simulated network examples and one application to the WIONs, we have observed significant differences between the results from the proposed WPR and other classical measures, where the proposed WPR is preferred. Especially for the WIONs, the proposed WPR has led to conclusions that are more consistent with intuition, providing new insights into the global input–output system. Incorporating edge weight makes large transactions between region-sectors get recognized, and incorporating relevant prior information further improves the results by imposing non-uniform distributions to the region-sectors based on their status in the economic system. Both synthetic and real data studies suggest the need of considering edge weight and prior information in the node centrality measure for weighted directed networks.

We would like to point out that the WIONs are used as examples to illustrate the application of the proposed WPR. Given the complexity of global economy, there exist multiple ways to define the importance of region-sectors therein, depending on the research question of interest. Thus, different centrality measures should be used in corresponding to different economic aspects. Nonetheless, he proposed WPR is expected to be widely applied to different types of weighted, directed networks, but not particularly to economic networks like WION only.

There are several limitations in the present research that merit further studies. So far the proposed measure has been adapted to static networks only. It is of substantial interest to investigate the proposed measure in random network models. Such extension would provide theoretical foundations for statistical inference such as confidence interval and hypothesis testing. Recently, Avrachenkov et al. [43] and Banerjee and Olvera-Cravioto [44] have looked into the asymptotic properties of the classical PR in undirected, unweighted SBMs and directed, unweighted PA networks, respectively. In addition to only being a centrality measure, the classical PR has been used to identify community structure in unweighted networks [45]. Applying the proposed WPR in community detection to weighted networks may lead to fruitful results. Lastly, owing to the increasing complexity in modern networks, we may consider extending the proposed WPR to a multilayer version, which is capable of capturing the characteristics of high dimensional networks potentially.

Table C.9
Description of the codes in the WIOTs.

Code	Sector
1	Crop and animal production, hunting and related service activities
2	Forestry and logging
3	Fishing and aquaculture
4	Mining and quarrying
5	Manufacture of food products, beverages and tobacco products
6	Manufacture of textiles, wearing apparel and leather products
7	Manufacture of wood and of products of wood and cork, except furniture; manufacture of articles of straw and plaiting materials
8	Manufacture of paper and paper products
9	Printing and reproduction of recorded media
10	Manufacture of coke and refined petroleum products
11	Manufacture of chemicals and chemical products
12	Manufacture of basic pharmaceutical products and pharmaceutical preparations
13	Manufacture of rubber and plastic products
14	Manufacture of other non-metallic mineral products
15	Manufacture of basic metals
16	Manufacture of fabricated metal products, except machinery and equipment
17	Manufacture of computer, electronic and optical products
18	Manufacture of electrical equipment
19	Manufacture of machinery and equipment n.e.c.
20	Manufacture of motor vehicles, trailers and semi-trailers
21	Manufacture of other transport equipment
22	Manufacture of furniture; other manufacturing
23	Repair and installation of machinery and equipment
24	Electricity, gas, steam and air conditioning supply
25	Water collection, treatment and supply
26	Sewerage; waste collection, treatment and disposal activities; materials recovery; remediation activities and other waste management services
27	Construction
28	Wholesale and retail trade and repair of motor vehicles and motorcycles
29	Wholesale trade, except of motor vehicles and motorcycles
30	Retail trade, except of motor vehicles and motorcycles
31	Land transport and transport via pipelines
32	Water transport
33	Air transport
34	Warehousing and support activities for transportation
35	Postal and courier activities
36	Accommodation and food service activities
37	Publishing activities
38	Motion picture, video and television program production, sound recording and music publishing activities; programming and broadcasting activities
39	Telecommunications
40	Computer programming, consultancy and related activities; information service activities
41	Financial service activities, except insurance and pension funding
42	Insurance, reinsurance and pension funding, except compulsory social security
43	Activities auxiliary to financial services and insurance activities
44	Real estate activities
45	Legal and accounting activities; activities of head offices; management consultancy activities
46	Architectural and engineering activities; technical testing and analysis
47	Scientific research and development
48	Advertising and market research
49	Other professional, scientific and technical activities; veterinary activities
50	Administrative and support service activities
51	Public administration and defense; compulsory social security
52	Education
53	Human health and social work activities
54	Other service activities
55	Activities of households as employers; undifferentiated goods- and services-producing activities of households for own use
56	Activities of extraterritorial organizations and bodies

CRediT authorship contribution statement

Panpan Zhang: Conceptualization, Methodology, Software, Formal Analysis, Writing – original draft, Writing – review & editing, Visualization. **Tiandong Wang:** Conceptualization, Methodology, Writing – review & editing. **Jun Yan:** Conceptualization, Methodology, Writing – review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Weighted PR by Xing and Ghorbani [15]

Xing and Ghorbani [15] proposed a weighted PR to rank the web pages based on their popularity, where the popularity of a web page is reflected in two aspects: a large number of web pages have links to it and a large number of web pages it is linked to. Xing and Ghorbanis' weighted PR measure, after normalization, is

$$\phi^{\mathrm{XG}}(i) = \gamma \sum_{j\in V} a_{ji} \left(\frac{d_i^{(\mathrm{in})}}{\sum_{k\in V} a_{jk} d_k^{(\mathrm{in})}} \right) \left(\frac{d_i^{(\mathrm{out})}}{\sum_{k\in V} a_{jk} d_k^{(\mathrm{out})}} \right) \phi^{\mathrm{XG}}(j) + \frac{1-\gamma}{n}. \tag{A.1}$$

One of the distinctive features of this weighted PR measure is that the edges of an unweighted network are actually "weighted". The "weight" of an edge is calculated by accounting for not only the in-degree and out-degree of the target node of the edge, but also the in-degrees and out-degrees of all the nodes that are linked by the source node of the edge. To clarify, let us take the edge from u_1 to v_2 in Fig. A.6 as an example. Its in-degree and out-degree generated weight are respectively given by

$$\frac{d_{v_2}^{(\mathrm{in})}}{d_{v_1}^{(\mathrm{in})} + d_{v_2}^{(\mathrm{in})}} = \frac{2}{1+2} = \frac{2}{3} \quad \text{and} \quad \frac{d_{v_2}^{(\mathrm{out})}}{d_{v_1}^{(\mathrm{out})} + d_{v_2}^{(\mathrm{out})}} = \frac{3}{2+3} = \frac{3}{5}.$$

Xing and Ghorbanis' weighted PR does not actually use the information quantified by edge weight; instead, the "weight" is converted from node in- and out-degrees. Thus, the so-called "weight" is always integer-valued, causing the loss of generality. In addition, this PR measure does not seem to be applicable to some social networks. A celebrity may follow only a few users in a social media platform, so is likely to get a low score for Xing and Ghorbanis' PR measure.

Appendix B. Top 10 WPR scores in the WIONs for $\theta = 0.5$ and $\theta = 0$

We present the region-sectors with top 10 WPR scores in the WIONs from 2000, 2007 and 2014 for $\theta = 0.5$ and $\theta = 0$ in this section.

From Table B.7 (for $\theta = 0.5$), we observe notable differences between the results with or without using TVA as prior information across three years. For instance, when no prior information is considered, most of the top 10 region-sectors are from ROW in 2000, whereas the top 10 list is primarily occupied by USA regions-sectors when accounting for TVA prior information. However, comparing the results with TVA prior for $\theta = 0.5$ with the counterparts for $\theta = 1$ (c.f. Table 6), we do not see significant difference in finalized region-sectors but their ranking orders.

Additionally, we list the region-sectors with top 10 WPR scores in the WIONs from 2000, 2007 and 2014 for $\theta = 0$ in Table B.8. By investigating the results, we have arrived at similar conclusions, suggesting that the rational use of relevant prior information is as critical as accounting for edge weight in PR-based centrality computations.

Appendix C. Code dictionary of the WIOTs

Table C.9 summarizes the code and definition of the 56 sectors in the 2016 release of the WIOD.

References

[1] L.C. Freeman, Centrality in social networks conceptual clarification, Social Networks 1 (1978) 215–239.
[2] M.E.J. Newman, Scientific collaboration networks. II. Shortest paths weighted networks, and centrality, Phys. Rev. E 64 (2001) 016132.
[3] P. Bonacich, Factoring and weighting approaches to status scores and clique identification, J. Math. Sociol. 2 (1972) 113–120.
[4] L. Katz, A new status index derived from sociometric analysis, Psychometrika 18 (1953) 39–43.
[5] L. Page, S. Brin, R. Motwani, T. Winograd, The PageRank Citation Ranking: Bringing Order to the Web, Technical Report 1999-66, Stanford University, 1999.
[6] J. Wang, H. Mo, F. Wang, F. Jin, Exploring the network structure and nodal centrality of China's air transport network: A complex network approach, J. Transp. Geogr. 19 (4) (2011) 712–721.
[7] J.E. Hirsch, An index to quantify an individual's scientific research output, Proc. Natl. Acad. Sci. USA 102 (46) (2005) 16569–16572.
[8] K. Das, S. Smanta, M. Pal, Study on centrality measures in social networks: A survey, Soc. Netw. Anal. Min. 8 (2018) 8.
[9] M.E.J. Newman, Networks, Oxford University Press, Oxford, UK, 2018.
[10] S. Brin, L. Page, The anatomy of a large-scale hypertextual web search engine, Comput. Netw. ISDN Syst. 30 (1998) 107–117.
[11] E. Yan, Y. Ding, Discovering author impact: A PageRank perspective, Inf. Process. Manage. 47 (2011) 125–134.
[12] Y. Ding, Applying weighted PageRank to author citation network, J. Am. Soc. Inf. Sci. Technol. 62 (2011) 236–245.
[13] K. Kalecky, Y.-R. Cho, PrimAlign: PageRank-inspired Markovian alignment for large biological networks, Bioinformatics 34 (2018) 537–546.
[14] M.E.J. Newman, Analysis of weighted networks, Phys. Rev. E 70 (2004) 056131.
[15] W. Xing, A.A. Ghorbani, Weighted PageRank algorithm, in: A.A. Ghorbani (Ed.), Proceedings of the 2nd Annual Conference on Communication Networks and Services Research, IEEE, Piscataway, NJ, USA, 2004, pp. 305–314.

[16] M.P. Timmer, E. Dietzenbacher, B. Los, R. Stehrer, G.J. de Vries, An illustrated user guide to the world input-output database: The case of global automotive production, Rev. Int. Econ. 23 (2015) 575–605.
[17] J. Yan, Y. Yuan, P. Zhang, wdnet: Weighted Directed Network, University of Connecticut, 2020, R package version 0.0-3, https://gitlab.com/wdnetwork/wdnet.
[18] P. Berkhin, A survey on PageRank computing, Internet Math. 2 (2005) 73–120.
[19] O. Perron, Zur theorie der matrices, Math. Ann. 64 (1907) 248–263.
[20] R.B. Lehoucq, D.C. Sorensen, C. Yang, ARPACK Users' Guide: Solution of Large-Scale Eigenvalue Problems with Implicitly Restarted Arnoldi Methods, SIAM, Philadelphia, PA, USAc, 1998.
[21] Y. Qiu, J. Mei, rARPACK: Solvers for Large Scale Eigenvalue and SVD Problems, 2016, R package version 0.11-0, https://CRAN.R-project.org/package=rARPACK.
[22] H.-H. Fu, D.K.J. Lin, H.-T. Tsai, Damping factor in Google page ranking, Appl. Stoch. Models Bus. Ind. 22 (2006) 431–444.
[23] M. Bressan, E. Peserico, Choose the damping, choose the ranking? J. Discrete Algorithms 8 (2010) 199–213.
[24] A.-L. Barabási, R. Albert, Emergence of scaling in random networks, Science 286 (1999) 509–512.
[25] Y. Yuan, J. Yan, P. Zhang, Assortativity measures for weighted and directed networks, J. Complex Netw. 9 (2021) cnab017.
[26] T. Wang, P. Zhang, Directed hybrid random networks mixing preferential attachment with uniform attachment mechanisms, 2021, arXiv:2101.04611.
[27] P.W. Holland, K.B. Laskey, S. Leinhadrt, Stochastic blockmodels: First steps, Social Networks 5 (1983) 109–137.
[28] T.A.B. Snijders, K. Nowicki, Estimation and prediction for stochastic blockmodels for graphs with latent block structure, J. Classification 14 (1997) 75–100.
[29] K. Nowicki, T.A.B. Snijders, Estimation and prediction for stochastic blockstructures, J. Amer. Statist. Assoc. 96 (2001) 1077–1087.
[30] P. Erdös, A. Rényi, On random graphs I, Publ. Math. Debrecen 6 (1959) 290–297.
[31] S. Hallegatte, An adaptive regional input-output model and its application to the assessment of the economic cost of Katrina, Risk Anal. 28 (3) (2008) 779–799.
[32] S. Battiston, M. Puliga, R. Kaushik, P. Tasca, G. Caldarelli, DebtRank: Too central to fail? Financial networks, the FED and systemic risk, Sci. Rep. 2 (2012) 541.
[33] H. Inoue, Y. Todo, Propagation of negative shocks across nation-wide firm networks, PLoS One 14 (2019) e0213648.
[34] P. Barucca, M. Bardoscia, F. Caccioli, M. D'Errico, G. Visentin, G. Caldarelli, S. Battiston, Network evaluation in financial systems, Math. Finance 30 (2020) 1181–1204.
[35] F. Cerina, Z. Zhu, A. Chessa, M. Riccaboni, World input-output network, PLoS One 10 (2015) e0134025.
[36] R.M. del Río-Chanona, J. Grujić, H.J. Jensen, Trends of the world input and output network of global trade, PLoS One 12 (2017) e0170817.
[37] M. Xu, L. Sai, Input-output networks offer new insights of economic structure, Physica A 527 (2019) 121178.
[38] T. Wang, S. Xiao, J. Yan, P. Zhang, Regional and sectoral structures of the Chinese economy: A network perspective from multi-regional input-output tabels, Physica A 581 (2021) 126196.
[39] R. Bon, R. Pietroforte, Historical comparison of construction sectors in the United States, Japan, Italy and Finland using input-output tables, Constr. Manag. Econ. 8 (1990) 233–247.
[40] J. Raftery, B. Pasadilla, Y.H. Chiang, E.C. Hui, B.-S. Tang, Globalization and construction industry development: Implications of recent developments in the construction sector in Asia, Constr. Manag. Econ. 16 (1998) 729–737.
[41] A.-M. Broudehoux, Spectacular Beijing: The conspicuous construction of an Olympic Metropolis, J. Urban Aff. 29 (2007) 383–399.
[42] Y. Zhang, K. Zhao, Impact of Beijing Olympic-related investments on regional economic growth of China: Interregional input-output approach, Asian Econ. J. 21 (3) (2007) 261–282.
[43] K. Avrachenkov, A. Kadavankandy, L.O. Prokhorenkov, A. Raigorodskii, PageRank in undirected random graphs, in: D. Gleich, J. Komjáthy, N. Litvak (Eds.), Proceedings of the International Workshop on Algorithms and Models for the Web-Graph, WAW 2015, Springer, Cham, Switzerland, 2015, pp. 151–163.
[44] S. Banerjee, M. Olvera-Cravioto, PageRank asymptotics on directed preferential attachment networks, 2021, arXiv:2102.08894.
[45] I.M. Kloumann, J. Ugander, J. Kleinberg, Block models and personalized PageRank, Proc. Natl. Acad. Sci. USA 114 (1) (2017) 33–38.