The rho calculus 20 years on

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Abstract. We discuss developments and advances in the rho calculus 20 years after its inception.

1 Introduction

This December will find my submission of the first paper about the rho calculus to be 20 years in the rear view mirror. A lot has happened since then. Apart from production implementations of scalable decentralized asset management systems based on the calculus, a lot of work has been done on the calculus itself. Indeed, just this May (2024) i discovered a conservative extension of the calculus that fills a hole in the account of reflection that has long bothered me. And, last April (2023) i found a fuzzy version of the rho calculus that is just waiting for someone to take further. In 2018 i discovered a variant of the calculus (which i dubbed the *space calculus*) that makes it a computational analog (in a certain precise sense) of solutions of the Einstein field equations.

Thankfully, i am not the only one who has found the rho calculus interesting enough to think about. Stay and Williams studied a variant they called the π-rho calculus as an example of a system that could be typed in their native types construction. And Stian Lybech showed that rho calculus is more expressive than the the π-calculus. The list goes on.

The aim of this note is to collect, and summarize in one place, developments over the last 20 years in the study of the rho calculus. i suspect that there are more researchers than i know about who are interested in the questions raised by the calculus. Hopefully, this note can serve that community, and possibly spark wider interest due to the discoveries to date.

1.1 Summary of contributions and outline of paper

More specifically, this note will develop the following.

- the modern, programming-language-friendly syntax for the calculus;
- a handful of useful syntactic sugar coatings that make the calculus a suitable semantic framework for a realistic protocol-oriented programming language, including, but not limited to:
  - adding first class values;
  - adding unforgeable names (the π-rho calculus);
  - adding syntax for joins and sequences and choice;
- a procedurally reflective rho calculus;

– the fuzzy variant of rho;
– multi-level agency mechanisms in rho;
– the space calculus;
– probabilistic and quantum variations of rho;
– revisiting the encodings of π into rho and vice versa.

But, we will also touch briefly on the techniques used to represent reflective computation in the rho calculus and demonstrate that they are not bound to this specific setting. In particular, they can be applied the the λ-calculus and to set theory, yielding variations of these theories that have very useful features.

Additionally, we will look at relating the rho calculus to physical intuition. There has been a great deal of work on using mobile process calculi to describe network protocols and concurrent algorithms. It is not too much of an abstraction to see how these techniques apply to physical processes such as chemical reactions, signaling regimes within and between cells. But the rho calculus can also be used to model ordinary classical physics problems. We illustrate some of these.

Finally, we will conclude with some thoughts about the foundations of mathematics and its relationship to agency. As a preview, i believe that set theory (and even to some extent category theory) are theories of *data structures*. Implicitly, the community has believed that (only) mathematicians have agency and their formalisms merely carry information content between these agents. But the advances in artificial intelligence (AI) tell a different story. To some extent AI has brought their formalisms to life and is on the verge of giving them agency. As a result, i want to argue that a proper foundation of mathematics needs to include a notion of agency and that the ingredients of a foundational account of agency are present in the mobile process calculi, generally, and a pragmatically foundational account is present in the rho calculus, specifically.

2 Concurrent process calculi and spatial logics

In the last thirty years the *process calculi* have matured into a remarkably powerful analytic tool for reasoning about concurrent and distributed systems, such as Internet communication, security, or application protocols ([1], [16], [17]), but also chemical reactions, and biological protocols such as cell signaling regimes ([30], [29].). Process-calculus-based algebraic specification of computational processes began with Milner's Calculus for Communicating Systems (CCS) [22] and Hoare's Communicating Sequential Processes (CSP) [15]. Being largely influenced by the hardware models of the time, these formalisms were based on a fixed communication topology, where computational processes are considered to be something like components soldered to a motherboard.

But as radio and cellular technologies, and protocols like TCP/IP became popularized, this influenced the evolution of these formalisms. In particular, the trend toward physical mobility of computational devices gave rise to the mobile process calculi, called so because they envisioned a network of computational processes connected by a dynamic communication topology that evolves as the processes compute. In a sense the computational processes contemplated by these formalisms are mobile in much the same way

as participants at a conference who randomly bump into one another and trade mobile numbers and email addresses, or molecules that randomly bump into one another and exchange electrons. It was this innovation of mobility that greatly expanded the scope of applicability of these formalisms.

Perhaps the most paradigmatic version of these calculi is Milner, Parrow, and Walker's π-calculus [26], [27], which Milner refined with [23] and [25]. In the π-calculus the dominant metaphor is very much processes as mobile agents (e.g., conference participants, or molecules) bumping into one another exchanging channels over which they can communicate. Yet, another equally compelling metaphor is that processes are mobile agents swimming in various nested compartments or membranes. These membranes are called *ambients* because they both surround and are surrounded by other ambients, but also because they *move*. That is, they can enter or exit one another are. In this sense the membranes themselves are the computational agents. These intuitions are formalized in Cardelli and Gordon's ambient calculus [9], and considerably refined by Cardelli's brane calculi [8].

Note that it is common practice in the mobile process calculi literature to refer to channels as *names* and in this note we will use these terms interchangeably. In the more abstract setting of that includes calculi like the ambient calculus where names are really tags associated with mobile membranes this more generic moniker makes sense. In both the π and ambient calculi names are the means of synchronization and rendezvous for mobile processes.

It is notable that to this day there is no fully abstract encoding of the ambient calculus into the π-calculus, suggesting that the ambient calculus is somehow more expressive than the π-calculus. That said, expressivity is not always a benefit. It is quite difficult to map the ambient style of computation to trustless distributed Internet-based protocols. The expressivity requires a great deal more coordination which, in turn, requires a great deal more trust. Meanwhile, the π-calculus has a ready mapping onto a wide range of Internet protocols, which i exploited vigorously in building Microsoft's BizTalk Process Orchestration. And the rho calculus has an even better mapping to Internet protocols, which i exploited even more vigorously in building RChain and F1R3FLY.io.

2.1 Hennessy-Milner and spatial logics, and session types

Companion to these calculi are logics and type systems which enable practitioners not only to reason about the correctness of individual agents, or specific ensembles of agents, but also about entire classes of agents, identified as the witnesses of logical formulae. These began with the Hennessy-Milner logics which build on Kripke's interpretation of modal logic [25]. More recently, these logics have been given new superpowers in the form of being able to detect structural properties. Notably, we find Cardelli and Caires's groundbreaking spatial logic [4] [6] [7]. The interested reader might also look at *session types* [11].

This topic is worthy of a much richer and deeper discussion. However, we relegate that to a separate paper in which we unify them all with a single algorithm, the OSLF algorithm.

This takes as input a model of computation formatted as a graph structured lambda theory and outputs a spatial-behavioral type system for that model of computation. The type system enjoys the Hennessy-Milner property, namely that two computations inhabit the same bisimulation equivalence class iff they satisfy exactly the same set of types. We leave the details to the paper describing this algorithm.

2.2 Compositionality and bisimulation

Among the many reasons for the continued success of the mobile process calculi's approach to organizing and analyzing computation are two central points. First, the process algebras provide a compositional approach to the specification, analysis and execution of concurrent and distributed systems. Owing to Milner's original insights into computation as interaction [24], the process calculi are so organized that the behavior —the semantics— of a system may be composed from the behavior of its components. This means that specifications can be constructed in terms of components —without a global view of the system— and assembled into increasingly complete descriptions.

The second central point is that process algebras have a potent proof principle, yielding a wide range of effective and novel proof techniques [33] [31] [32]. In particular, *bisimulation* encapsulates an effective notion of process equivalence that has been used in applications as far-ranging as algorithmic games semantics [3] and the construction of model-checkers [5]. The essential notion can be stated in an intuitively recursive formulation: a *bisimulation* between two processes P and Q is an equivalence relation E relating P and Q such that: whatever action of P can be observed, taking it to a new state P', can be observed of Q, taking it to a new state Q', such that P' is related to Q' by E and vice versa. P and Q are *bisimilar* if there is some bisimulation relating them. Part of what makes this notion so robust and widely applicable is that it is parameterized in the actions observable of processes P and Q, thus providing a framework for a broad range of equivalences and up-to techniques [33] all governed by the same core principle [32].

3 The reflective higher order calculus

It is in the context of the evolution of the mobile process calculi that Meredith and Radestock's *r*eflective *h*igher-*o*rder, or rho calculus arises [1] [20]. While it is possible to give a faithful encoding of the ambient calculus into the rho calculus, the rho calculus can be viewed as the natural successor to the π-calculus. One way to look at the relationship between the two is that the π-calculus is not actually a single calculus. Instead, it is a recipe or *function* for constructing a calculus of agents that communicate over *some* collection of channels. That is, it is parametric in a *theory* of channels[2]. This intuition

[1] The π-calculus gets its name because it was the first Greek letter available after the λ in Church's calculus by that name, and just happens to be the Greek letter corresponding to the English 'p' for process. Likewise, the rho calculus gets its name from its delivery of higher-order features through the use of reflection, and it just happens that the acronym spells out *rho*, the English spelling of the Greek letter that comes after π.

[2] Note that here we are using the term 'theory' in a technical sense, such as Peano arithmetic, or a Lawvere theory. We don't mean theoretical in the usual sense of that word, but rather a collection of entities (such as the Natural Numbers) that can be generated from a small set of rules.

can be made very precise and corresponds to a particular strategy of implementation called two-level type decomposition. But, if the π-calculus is a function taking a theory of channels to a theory of agents communicating over those channels, then the rho calculus is the *least fixed point* of that function. The rho calculus says we need look no farther than the *communicating agent themselves* for our theory of channels.

More specifically, the calculus envisions a new kind of distinction, one between the *code* of a process, and a *running instance* of a process. It will not have escaped the reader's attention that this is a commonplace notion in the practice of building concurrent and distributed systems. Indeed, the development of the rho calculus stole a page from the previous evolution of process algebras. The step introducing mobility was the theory coming into better alignment with practice. Computing devices became mobile, to keep up the formalisms had to follow suit and devise a notion of mobile processes. Likewise, the practice of building the software that comprises concurrent and distributed systems makes a distinction between code and running instances of the code. We argue, however, that providing an account of this distinction in our formalism is much more fundamental than just keeping up with practice. We argue that this reason this is such a ubiquitous practice is because it is a fundamental aspect of the underlying order and organization of computation.

3.1 Coding as a fundamental aspect of computation

Indeed, Gödel's incompleteness results fundamentally depend on a notion of *coding*. To achieve the self-reference needed to construct a logical statement of the form "This sentence is not provable." Gödel encoded logical formulae into the natural numbers and then used logical statements about the natural numbers to construct self-referential statements like that one. In his construction, the coding trick appears to be exogenous to logic and arithmetic. Indeed a careful study of these formalisms shows that it is, in fact, *intrinsic*. That is, both the natural numbers, and logic naturally reflect on themselves, if you look more carefully at those mathematical objects. More to the point, however, this intrinsic, loopy self-reference that arises from the notion of code is also at the heart of Turing's solution to the Entscheidungsproblem and Cantor's separation of the countably infinite from the uncountably infinite.

Moreover, the scope of this phenomenon is not restricted to some of the most profound results in mathematics and logic over the last couple of centuries. It's also at the heart of the origin of life. Some four billion years ago the Earth's chemistry stumbled on a way to code for the production of proteins involved in certain self-replicating chemical reactions. Arguments about the validity of the Weissman barrier to one side [39], it is a matter of experimentally established fact that the codons at the heart of the genetic apparatus provide the mechanism for both the self-replication of the chemical processes underlying life, but also the transmission of inherited information about fitness to environment. As such, we believe that phenomenon of coding is a much more profound phenomenon than has been appreciated and deserves to be given a first-class representation in our computational formalisms.

Once reflection is introduced into the mobile process calculus setting, higher-order capabilities and recursion are child's play. Perhaps more surprisingly, so is the mysterious new operator of the π-calculus tasked with generating fresh channels. But, we are getting ahead of ourselves. Suffice it to say that using reflection as a first-class representation of the trick of coding – a trick originally discovered by life and then independently rediscovered by Cantor, Gödel, and Turing – is a real banger.

Intriguingly, not only does introducing coding explicitly into the formalism solve a number of practical problems associated with the π-calculus, it simultaneously solves a number of problems associated with corresponding logics and type systems, as well. The interested reader should see namespace logic [19].

4 An idiosyncratic view of related work

Here we direct the reader to a small sample of the considerable work on the mobile process calculi. This sample is far from representative of the overall body of work, but instead constitutes a snapshot of what your author felt was salient at some point during the last twenty years. Indeed, the reader is encouraged to investigate the literature on her own to get an independent perspective on this rich field of research.

- telecommunications, networking, security and application level protocols [1] [2] [16] [17];
- programming language semantics and design [16] [14] [13] [35];
- webservices [16] [17] [18];
- blockchain [21]
- and biological systems [8] [10] [30] [29].

5 The syntax and semantics of the core rho calculus

We now summarize a technical presentation of the core calculus. The typical presentation of such a calculus generalizes a generators and relations presentation of an algebra [28]. The grammar below, describing term constructors in the language of process expressions (which we abbreviate to processes in the sequel), freely generates the set of processes. We denote this set Proc, and the set of channels (aka names) over which these processes communicate by @Proc. The set Proc (and by extension @Proc) is then quotiented by a relation known as structural congruence, denoted $\equiv$, and it is over this set that the notion of computation is expressed.

In particular, computation is expressed as a small handful of rewrite rules which should be viewed as defining a state transition relation. Thus, process expressions capture *states* and the form of the expression determines how one process (state) *might* evolve to another (state). The word 'might' is doing some heavy lifting in that previous sentence, as the calculus is *not* deterministic, in fact it is not even confluent. As such, the calculus can be used to represent all manner of common human (and natural) resource distribution

protocols, such as first come first served, which sits at the heart of everything from airline reservation systems to concert ticket sales. By comparison, Church's λ-calulus (the core of programming languages such as Haskell or Scala) does not – without considerable modification – natively support the expression of such protocols.

5.1 The rho calculus in less than a page

PROCESS
$$P, Q ::= 0 \mid \mathsf{for}(y \leftarrow x)P \mid x!(Q) \mid *x \mid P|Q$$

NAME
$$x, y ::= @P$$

EQUIV
$$P|0 \equiv P \quad P|Q \equiv Q|P \quad P|(Q|R) \equiv (P|Q)\mathsf{R}$$

ALPHA
$$\mathsf{for}(y \leftarrow x)P \equiv \mathsf{for}(z \leftarrow x)(P\{z/y\}) \; \mathsf{if} z \notin \mathsf{FN}(P)$$

COMM
$$\mathsf{for}(y \leftarrow x)P \mid x!(\overrightarrow{Q}) \rightarrow P\{@Q/y\}$$

PAR
$$\frac{P \rightarrow P'}{P|Q \rightarrow P'|Q}$$

EQUIV
$$\frac{P \equiv P' \qquad P' \rightarrow Q' \qquad Q' \equiv Q}{P \rightarrow Q}$$

This presentation is essentially that of [20] recast with a programmer-friendly syntax that extends smoothly to fork-join patterns commonly found in human decision-making processes. The reader should note that, even with a programmer-friendly syntax, this calculus is considerably smaller than a corresponding presentation of the π-calculus. Indeed is as concise and compact as the λ-calculus, yet considerably more expressive. What follows below is an exegesis of the technical details hidden in this more compact presentation. Notably, we provide definitions of free and bound names, and other housekeeping matters such as name equality. The reader more familiar with these notions should feel free to skim this section, however the section on bisimulation is of some importance and worthy of more focused attention.

5.2 The fine print

Notational interlude when it is clear that some expression t is a sequence (such as a list or a vector), and a is an object that might be meaningfully and safely prefixed to that sequence then we write $a : t$ for the sequence with a prefixed (aka "consed") to t. We write $t(i)$ for the ith element of t. In the sequel we use $\overrightarrow{x}$ (resp. $\overrightarrow{P}$) denotes a vector of *names* (resp. *processes*) of length $|\overrightarrow{x}|$ (resp. $|\overrightarrow{P}|$).

Process grammar

PROCESS	NAME
$P, Q ::= \mathsf{0} \mid \mathsf{for}(y \leftarrow x)P \mid x!(Q) \mid *x \mid P\|Q$	$x, y ::= @P$

As mentioned above we use Proc (resp. @Proc) to denote the language of processes (resp. names) freely generated by this grammar. But how are we to think about expressions in this language, intuitively?

- $\mathsf{0}$ is the ground of this tiny little language. It represents the *stopped* process, or the process that does nothing; in some real sense this is the quintessential neutral element in this computational dynamics, as we shall see;
- $\mathsf{for}(y \leftarrow x)P$ is the process that is waiting at channel x for some data or message that it will bind to y and then proceed to do P; if $\mathsf{0}$ is neutral, then for-comprehension is receptive, it waits for action at x before it can do anything;
- $x!(Q)$ is the process that is outputting the datum or message Q (which is also a process) along the channel, x; if $\mathsf{0}$ is neutral, and $\mathsf{for}(y \leftarrow x)P$ is reception, then $x!(Q)$ is active; it represents actively sending Q floating down the channel x to be picked up by some $\mathsf{for}(y \leftarrow x)P$;
- $*x$ is the process that takes the code represented by x and begins running it;
- $P|Q$ is the process that is the parallel or concurrent composition of the process P with the process Q; it introduces the principle that individual computations can actually be collectives of autonomous, yet coordinating computation.

As foreshadowed by the syntactic category of expressions of the form $*x$, a channel or name is merely the code of some process, $@P$. In this sense we may think of these two operators $*$ and $@$ as dual to one another. The latter delivers the code of some process, while the former delivers a running process from its code.

Next, we quotient this set by the smallest equivalence relation containing α-equivalence that makes $(\mathsf{Proc}, |, 0)$ into a commutative monoid. In order to define this relation we require a definition of free and bound names to define α-equivalence.

Definition 1. Free and bound names *The calculation of the free names of a process, P, denoted $\mathsf{FN}(P)$ is given recursively by*

$$\mathsf{FN}(\mathsf{0}) = \emptyset \qquad \mathsf{FN}(\mathsf{for}(y \leftarrow x)(P)) = \{x\} \cup \mathsf{FN}(P) \setminus \{y\} \qquad \mathsf{FN}(x!(P)) = \{x\} \cup \mathsf{FN}(P)$$

$$\mathsf{FN}(P|Q) = \mathsf{FN}(P) \cup \mathsf{FN}(Q) \qquad \mathsf{FN}(x) = \{x\}$$

An occurrence of x in a process P is bound if it is not free. The set of names occurring in a process (bound or free) is denoted by $\mathsf{N}(P)$.

5.3 Substitution

We use the notation $\{\overrightarrow{y}/\overrightarrow{x}\}$ to denote partial maps, $s : @\mathsf{Proc} \to @\mathsf{Proc}$. A map, s lifts, uniquely, to a map on process terms, $\widehat{s} : \mathsf{Proc} \to \mathsf{Proc}$. Historically, it is convention to use σ to range over lifted subsitutions, $\widehat{s}$, to write the application of a substitution, σ to a process, P, using postfix notation, with the substitution on the right, $P\sigma$, and the application of a substitution, s, to a name, x, using standard function application notation, $s(x)$. In this instance we choose not to swim against the tides of history. Thus,

Definition 2. *given $x = @P'$, $u = @Q'$, $s = \{u/x\}$ we define the lifting of s to $\widehat{s}$ (written below as σ) recursively by the following equations.*

$$0\sigma := 0$$

$$(P|Q)\sigma := P\sigma|Q\sigma$$

$$(\mathsf{for}(y \leftarrow v)P)\sigma := \mathsf{for}(z \leftarrow \sigma(v))((P\widehat{\{z/y\}})\sigma)$$

$$(x!(Q))\sigma := \sigma(x)!(Q\sigma)$$

$$(*y)\sigma := \begin{cases} Q' & y \equiv_{\mathsf{N}} x \\ *y & otherwise \end{cases}$$

where

$$\{\widehat{@Q/@P}\}(x) = \{@Q/@P\}(x) = \begin{cases} @Q & x \equiv_{\mathsf{N}} @P \\ x & otherwise \end{cases}$$

and z is chosen distinct from $@P$, $@Q$, the free names in Q, and all the names in R. Our α-equivalence will be built in the standard way from this substitution.

Definition 3. *Then two processes, P, Q, are alpha-equivalent if $P = Q\{\overrightarrow{y}/\overrightarrow{x}\}$ for some $\overrightarrow{x} \in \mathsf{BN}(Q), \overrightarrow{y} \in \mathsf{BN}(P)$, where $Q\{\overrightarrow{y}/\overrightarrow{x}\}$ denotes the capture-avoiding substitution of $\overrightarrow{y}$ for $\overrightarrow{x}$ in Q.*

Definition 4. *The* structural congruence $\equiv$ *between processes [34] is the least congruence containing alpha-equivalence and satisfying the commutative monoid laws (associativity, commutativity and $\mathsf{0}$ as identity) for parallel composition $|$.*

Definition 5. *The* name equivalence $\equiv_{\mathsf{N}}$ *is the least congruence satisfying these equations*

$$\textsc{Quote-drop}\quad @*x \equiv_{\mathsf{N}} x \qquad\qquad \textsc{Struct-equiv}\quad \frac{P \equiv Q}{@P \equiv_{\mathsf{N}} @Q}$$

The astute reader will have noticed that the mutual recursion of names and processes imposes a mutual recursion on alpha-equivalence and structural equivalence via name-equivalence. Fortunately, all of this works out pleasantly and we may calculate in the natural way, free of concern. The reader interested in the details is referred to the original paper on the rho calculus [20].

Remark 1. One particularly useful consequence of these definitions is that $\forall P.@P \notin \mathsf{FN}(P)$. It gives us a succinct way to construct a name that is distinct from all the names in P and hence fresh in the context of P. For those readers familiar with the work of Pitts and Gabbay, this consequence allows the system to completely obviate the need for a fresh operator, and likewise provides a canonical approach to the semantics of freshness.

Equipped with the structural features of the term language we can present the dynamics of the calculus.

5.4 Operational semantics

Finally, we introduce the computational dynamics. What marks these algebras as distinct from other more traditionally studied algebraic structures, e.g. vector spaces or polynomial rings, is the manner in which dynamics is captured. In traditional structures, dynamics is typically expressed through morphisms between such structures, as in linear maps between vector spaces or morphisms between rings. In algebras associated with the semantics of computation, the dynamics is expressed as part of the algebraic structure itself, through a reduction reduction relation typically denoted by $\rightarrow$. Below, we give a recursive presentation of this relation for the calculus used in the encoding.

$$\text{COMM}\ \frac{x_t \equiv_{\mathsf{N}} x_s}{\mathsf{for}(y \leftarrow x_t)P \mid x_s!(\overrightarrow{Q}) \rightarrow P\{@Q/y\}} \qquad \text{PAR}\ \frac{P \rightarrow P'}{P|Q \rightarrow P'|Q}$$

$$\text{EQUIV}\ \frac{P \equiv P' \qquad P' \rightarrow Q' \qquad Q' \equiv Q}{P \rightarrow Q}$$

We write $P \rightarrow$ if $\exists Q$ such that $P \rightarrow Q$ and $P \not\rightarrow$, otherwise.

5.5 Dynamic quote: an example

Anticipating something of what's to come, let $z = @P$, $u = @Q$, and $x = @y!(*z)$. Now consider applying the substitution, $\widehat{\{u/z\}}$, to the following pair of processes, $w!(y!(*z))$ and $w!(*x) = w!(*@y!(*z))$.

$$w!(y!(*z))\widehat{\{u/z\}} = w!(y!(Q))$$
$$w!(*x)\widehat{\{u/z\}} = w!(*x)$$

The body of the quoted process, $@y!(*z)$, is impervious to substitution, thus we get radically different answers. In fact, by examining the first process in an input context, e.g. $\mathsf{for}(z \leftarrow x)w!(y!(*z))$, we see that the process under the output operator may be shaped by prefixed inputs binding a name inside it. In this sense, the combination of input prefix binding and output operators will be seen as a way to dynamically construct processes before reifying them as names.

6 Replication

As mentioned before, it is known that replication (and hence recursion) can be implemented in a higher-order process algebra [34]. As our first example of calculation with the machinery thus far presented we give the construction explicitly in the rho calculus.

$$\begin{aligned} D_x &:= \mathsf{for}(y \leftarrow x)(x!(y)|*y) \\ !_x P &:= x!(D_x|P)|D_x \end{aligned}$$

$$\begin{array}{ll} !_x P & \\ = & x!((\mathsf{for}(y \leftarrow x)(x!(y)|*y))|P)|\mathsf{for}(y \leftarrow x)(x!(y)|*y) \\ \rightarrow & (x!(y)|*y)\{@(\mathsf{for}(y \leftarrow x)(*y|x!(y)))|P/y\} \\ = & x!(@(\mathsf{for}(y \leftarrow x)(x!(y)|*y))|P)|(\mathsf{for}(y \leftarrow x)(x!(y)|*y))|P \\ \rightarrow & \ldots \\ \rightarrow^* & P|P|\ldots \end{array}$$

Of course, this encoding, as an implementation, runs away, unfolding $!P$ eagerly. A lazier and more implementable replication operator, restricted to input-guarded processes, may be obtained as follows.

$$!\mathsf{for}(v \leftarrow u)P := x!(\mathsf{for}(v \leftarrow u)(D(x)|P))|D(x)$$

Remark 2. Note that the lazier definition still does not deal with summation or mixed summation (i.e. sums over input and output). The reader is invited to construct definitions of replication that deal with these features.

Further, the definitions are parameterized in a name, x. Can you, gentle reader, make a definition that eliminates this parameter and guarantees no accidental interaction between the replication machinery and the process being replicated – i.e. no accidental sharing of names used by the process to get its work done and the name(s) used by the replication to effect copying. This latter revision of the definition of replication is crucial to obtaining the expected identity $!!P \sim !P$.

Remark 3. The reader familiar with the lambda calculus will have noticed the similarity between D and the paradoxical combinator.

[Ed. note: the existence of this seems to suggest we have to be more restrictive on the set of processes and names we admit if we are to support no-cloning.]

Bisimulation The computational dynamics gives rise to another kind of equivalence, the equivalence of computational behavior. As previously mentioned this is typically captured *via* some form of bisimulation.

The notion we use in this paper is derived from weak barbed bisimulation [25].

Definition 6. *An* observation relation, $\downarrow_{\mathcal{N}}$, *over a set of names,* $\mathcal{N}$, *is the smallest relation satisfying the rules below.*

$$\frac{y \in \mathcal{N},\ x \equiv_{\mathsf{N}} y}{x!(v) \downarrow_{\mathcal{N}} x} \qquad (\textsc{Out-barb})$$

$$\frac{P \downarrow_{\mathcal{N}} x \ \text{ or } \ Q \downarrow_{\mathcal{N}} x}{P|Q \downarrow_{\mathcal{N}} x} \qquad (\textsc{Par-barb})$$

We write $P \Downarrow_{\mathcal{N}} x$ *if there is* Q *such that* $P \Rightarrow Q$ *and* $Q \downarrow_{\mathcal{N}} x$.

Definition 7. *An* $\mathcal{N}$-barbed bisimulation *over a set of names,* $\mathcal{N}$, *is a symmetric binary relation* $\mathcal{S}_{\mathcal{N}}$ *between agents such that* $P \ \mathcal{S}_{\mathcal{N}} Q$ *implies:*

1. *If* $P \to P'$ *then* $Q \Rightarrow Q'$ *and* $P' \ \mathcal{S}_{\mathcal{N}} Q'$.
2. *If* $P \downarrow_{\mathcal{N}} x$, *then* $Q \Downarrow_{\mathcal{N}} x$.

P *is* $\mathcal{N}$*-barbed bisimilar to* Q, *written* $P \dot{\approx}_{\mathcal{N}} Q$, *if* $P \ \mathcal{S}_{\mathcal{N}} Q$ *for some* $\mathcal{N}$*-barbed bisimulation* $\mathcal{S}_{\mathcal{N}}$.

Contexts One of the principle advantages of computational calculi from the λ-calculus to the π-calculus is a well-defined notion of context, contextual-equivalence and a correlation between contextual-equivalence and notions of bisimulation. The notion of context allows the decomposition of a process into (sub-)process and its syntactic environment, its context. Thus, a context may be thought of as a process with a "hole" (written $\square$) in it. The application of a context K to a process P, written $K[P]$, is tantamount to filling the hole in K with P. In this paper we do not need the full weight of this theory, but do make use of the notion of context in the proof the main theorem.

$$\overset{\text{CONTEXT}}{K ::= \square \mid \mathsf{for}(\overrightarrow{y} \leftarrow x)K \mid x!(\overrightarrow{P}, K, \overrightarrow{Q}) \mid K|P}$$

Definition 8 (contextual application). *Given a context* K, *and process* P, *we define the* contextual application, $K[P] := K\{P/\square\}$. *That is, the contextual application of* K *to* P *is the substitution of* P *for* $\square$ *in* K.

Remark 4. Note that we can extend the definition of free and bound names to contexts.

7 Useful syntactic sugar

7.1 First class values

While it is standard in calculi such as the λ-calculus to define a variety of common values such as the natural numbers and booleans in terms of Church-numeral style encodings, it is equally common to simply embed values directly into the calculus. Not being higher-order, this presents some challenges for the π-calculus, but for the rho-calculus, everything works out very nicely if we treat values, e.g. the naturals, the booleans, the reals, etc as processes. This choice means we can meaninfully write expressions like $x!(5)$ or $u!(\mathsf{true})$, and in the context $\mathsf{for}(y \leftarrow x)P[*y]|x!(5)$ the value 5 will be substituted into P. Indeed, since operations like addition, multiplication, etc. can also be defined in terms of processes, it is meaningful to write expressions like $5|+|1$, and be confident that this expression will reduce to a process representing 6. Thus, we can also use standard mathematical expressions, such as $5 + 1$, as processes, and know that these will evaluate to their expected values. Further, when combined with for-comprehensions, we can write algebraic expressions, such as $\mathsf{for}(y \leftarrow x)5+*y$, and in contexts like $(\mathsf{for}(y \leftarrow x)5+*y)|x!(1)$ this will evaluate as expected, producing the process (aka value) 6.

VALUES

$$P ::= \cdots \mid \mathsf{Bool} \mid \mathsf{Long} \mid \mathsf{String} \mid \mathsf{Uri} \mid \mathsf{Collection} \mid \ldots$$

COLLECTION

$$\mathsf{Collection} ::= \texttt{[}[P]\texttt{]} \mid \texttt{(}P\texttt{, }[P]\texttt{)} \mid \texttt{Set(}[P]\texttt{)} \mid \{[P\texttt{:}P]\}$$

7.2 Pattern matching

Remember that rho allows processes to be sent: $x!(Q)$. Since we can send values, it will be useful to be able to pattern match on values. The following pattern-matching syntax captures very commonly used idioms.

PATTERN

$$\mathsf{ProcPattern} ::= \_ \mid \mathsf{Var} \mid \texttt{@}\mathsf{ProcPattern} \mid \mathsf{VarRefKind\ Var} \mid \mathsf{LogicalPattern} \mid \mathsf{ValuePattern}$$

LOGICAL-PATTERN

$$\mathsf{LogicalPattern} ::= \mathsf{ProcPattern} \ \vee \ \mathsf{ProcPattern} \mid \mathsf{ProcPattern} \ \wedge \ \mathsf{ProcPattern} \mid \texttt{~}\mathsf{ProcPattern}$$

VALUE-PATTERN

$$\mathsf{ValuePattern} ::= \mathsf{Ground} \mid \mathsf{Collection} \mid \texttt{Nil} \mid \mathsf{SimpleType}$$

VAR-REF-KIND

$$\mathsf{VarRefKind} ::= \texttt{=} \mid \texttt{=*}$$

7.3 Summation

SUMMATION

P ::= . . . `select` { [Branch] }

Branch ::= ReceiptLinearImpl=>Proc3

[Branch] ::= Branch | Branch[Branch]

7.4 Let syntax

We achieve even greater compression and a more familiar notation if we also adopt the notation

$$\text{let } x = v \text{ in } P := (\text{new } u)(\text{for}(@x \leftarrow u)P)|u!(v)$$

7.5 Joins

TBD

7.6 Unguessable versus unforgeable names

Since all the names of the rho calculus are generated from the codes of processes we know all of them up front. Security on channels, therefore, amounts to unguessability. There are an infinite number of names and we have to arrange our protocols to make it very, very hard to guess which channels are in use at any given time. On the one hand, we could delegate that to some black box which delivers us the next unguessable name when we ask for it. In this sense, it is therefore useful to reintroduce the π-calculus' new operator as a standin for that black box.

However, in many settings, such as in the RChain and F1R3FLY.io implementations execution is arranged so that names are not merely unguessable, but *unforgeable*, meaning that the execution environment guarantees not to allow outside agents to generate certain names. Thus, even if some malefactor guesses a particular name, it cannot eavesdrop on the channel associated with it because it can't execute code that uses that name because it didn't have the right to generate it. It could only do so had its code been in a scope where the generated name was sent to it by a (presumably) willing party.

Thus, the new operator not only provides useful abstraction over a black boxed unguessability algorithm, but can also be used to support unforgeability. Hence, we reintroduce to rho as a conservative extension.

UNFORGEABLE

$$P ::= \ldots \mid \texttt{new}\ [\mathsf{NameDecl}]\ \texttt{in}\ P$$

$$\mathsf{NameDecl} ::= \mathsf{Var} \mid \mathsf{Var}(\mathsf{Uri})$$

$$[\mathsf{NameDecl}] ::= \mathsf{NameDecl} \mid \mathsf{NameDecl}, [\mathsf{NameDecl}]$$

8 Fuzzy rho

TBD

9 Multilevel Agency

The rho calculus provides a number of important perspectives on multilevel agency. In this section we focus on three of them that can be reduced to calculi that can be coded up and executed on modern computer hardware.

9.1 Annihilation

Another important variation has to do with the rewrite rules. There is a nested recursion version of the COMM-rule that aligns with intuitions about the recursive nature of behavior in compositionally defined agents. While for your author the maths make it clearer than the English it is worth setting out some of the motivations for this variation of the dynamics of the rho calculus.

The reductionist view of science is that phenomena like medicinal cures rest on the phenomena of biology. That is, we find explanation of the mechanism underlying the medical procedure in the biological phenomena the procedure interacts with in some fashion. Meanwhile, the phenomena of biology rests, in turn, on the phenomena of chemistry; and chemistry rests on (nuclear) physics. By way of an example, in the reductionist narrative the interaction of two blood cells in the veins of the body of a person is ultimately explained in terms of the interactions of various chemical compounds, which are in turn explained in terms of the interaction of various atoms and their subatomic components, such as electrons, etc.

While this is accepted philosophy and pedagogy, it is quite difficult to reduce this account to actual computations, except in very limited or schematic and abstract fashion. For example, getting differential equations to work effectively across such scales (from cells to quarks) is simply not practical. The reductionist picture is comforting and to some extent supported by evidence, but not reducible to practice except in extremely limited cases.

The variation of the rho calculus dynamics presented here was developed to explore the power of compositionality to capture this kind of tower of reduction. Essentially, we

develop of notion of process interaction that in turn relies on the interaction of "lower level" processes, and so on all the way down to a real bottom. The notion gives a precise definition of what it means to be "lower level" and how that relates to computational dynamics. In some sense, this is the crudest of pictures of multilevel agency, and yet given the efficacy of the process calculi to faithfully represent chemical, biochemical, and biological phenomena, there is some hope that it might not just be a pleasant abstraction.

First, we define what it means for two processes to annihilate each other.

Definition 9. *Annihilation: Processes P and Q are said to annihilate one another, written $P \perp Q$, just when $\forall R.P|Q \to^* R \Rightarrow R \to^* \mathsf{0}$.*

Thus, when $P \perp Q$, all rewrites out of $P|Q$ eventually lead to $\mathsf{0}$. Evidently, $P \perp Q \iff Q \perp P$, and $\mathsf{0} \perp \mathsf{0}$. Naturally, we can extend annihilation to names: $x \perp y \iff \dot{x} \perp \dot{y}$.

Annihilation affords a new version of the COMM-rule:

$$\text{COMM}\quad \frac{x_t \perp x_s, \quad |\overrightarrow{y}| = |\overrightarrow{Q}|}{(R_1 + \mathsf{for}(\overrightarrow{y} \leftarrow x_t)P) \mid (x_s!(\overrightarrow{Q}).P' + R_2) \to P\{\overrightarrow{@Q}/\overrightarrow{y}\}|P'}$$

All annihilation-based reduction happens in terms of reductions that happen at a lower degree of quotation, and grounds out in the fact that $\mathsf{0} \perp \mathsf{0}$ and thus $@\mathsf{0} \perp @\mathsf{0}$.

Example 1. For example, let $P_1 := \mathsf{for}(@\mathsf{0} \leftarrow @\mathsf{0})\mathsf{0}|@\mathsf{0}!(\mathsf{0})$. Then $P_1 \to \mathsf{0}$ because $\mathsf{0} \perp \mathsf{0}$. Suppose now that we set $x_0^-, x_0^+ := @\mathsf{0}$. Then, we can write P_1 as $\mathsf{for}(x_0^- \leftarrow x_0^-)\mathsf{0}|x_0^+!(\mathsf{0})$. Now, set

$$x_1^- := @(\mathsf{for}(x_0^- \leftarrow x_0^-)\mathsf{0}) \qquad\qquad x_1^+ := @(x_0^+!(\mathsf{0}))$$

Then define $P_2 := \mathsf{for}(x_1^- \leftarrow x_1^-)\mathsf{0}|x_1^+!(\mathsf{0})$. Then $P_2 \to \mathsf{0}$ because $x_1^- \perp x_1^+$, and hence $\mathsf{for}(x_1^- \leftarrow x_1^-)\mathsf{0} \perp x_1^+!(\mathsf{0})$.

More generally, set

$$x_i^- := @(\mathsf{for}(x_{i-1}^- \leftarrow x_{i-1}^-)\mathsf{0}) \qquad\qquad x_i^+ := @x_{i-1}^+!(\mathsf{0})$$

$$P_i := \mathsf{for}(x_{i-1}^- \leftarrow @x_{i-1}^-)\mathsf{0}|@x_{i-1}^+!(\mathsf{0})$$

Then $P_i \to \mathsf{0}$ and hence $x_i^- \perp x_i^+$.

This allows for a measure of reduction complexity.

9.2 Procedural reflection

The perspective on multilevel agency embodied in this conservative extension to the calculus is focused on a process being able to "see into the future of another process". That is, for a given process P to probe how another process, say Q, might evolve. This ability to "imagine" the behavior of another becomes introspection when a process turns this imaginative capacity onto itself. This introspection creates a kind of reflective tower, ala $3-\mathsf{Lisp}$ [36] or Brown [37]. Different floors or levels in this tower correspond to different levels of introspection. That is, level n is one level of introspection deeper than level $n-1$. It is in this sense that we view procedural reflection as providing a kind of multilevel agency.

Note that the fact that the π-calculus can be encoded into the rho calculus means that rho is Turing complete, or a model of universal computation. The reason we say that the procedurally reflective rho calculus is a conservative extension is because it doesn't add any fundamentally new expressive power in the way that adding an oracle that answers certain halting questions would. As such, we can write a meta-circular interpreter for the rho calculus in the rho calculus. Having done so, we could make reflection on the evolution of a process available to the process undergoing evolution. In some sense, this is the rho calculus equivalent of functional programming phenomena like call/cc, or more generally, delimited continuations [12].

To achieve this ability for a process, say P to look into its future we introduce an additional syntactic category, $x?P$. Then, if P evolves in a single step to P', then $x?P$ evolves to $x!(P')$. That is, a next step in P's evolution is made available at x. Rendering this idea in symbols, is simply adding the new syntactic category, extending the definition of free names in the obvious way, and one additional rewrite rule.

PROCESS
$$P, Q ::= 0 \mid \mathsf{for}(y \leftarrow x)P \mid x!(Q) \mid *x \mid x?P \mid P|Q$$

NAME
$$x, y ::= @P$$

EQUIV
$$P|0 \equiv P \quad P|Q \equiv Q|P \quad P|(Q|R) \equiv (P|Q)R$$

ALPHA
$$\mathsf{for}(y \leftarrow x)P \equiv \mathsf{for}(z \leftarrow x)(P\{z/y\} \text{ if } z \notin \mathsf{FN}(P)$$

$$\text{COMM}\quad \mathsf{for}(y \leftarrow x)P \mid x!(\overrightarrow{Q}) \rightarrow P\{@Q/y\}$$

$$\text{PAR}\quad \frac{P \rightarrow P'}{P|Q \rightarrow P'|Q} \qquad \text{EQUIV}\quad \frac{P \equiv P' \qquad P' \rightarrow Q' \qquad Q' \equiv Q}{P \rightarrow Q}$$

$$\text{REFL}\quad \frac{P \rightarrow P'}{x?P \rightarrow x!(P')}$$

What makes this version of procedural reflection distinct and interesting is that it fits well within a concurrency setting. It is common folklore that call/cc-like continutations and concurrency are not the best bedfellows. Essentially, call/cc provides the ability to revert *global* state. But, in some very real sense there is no global state, or the state shared amongst a collection of agents is partitioned in such a way that not every agent has access and there is no shared way to roll back the clock for all agents. Once one agent has fired a missile, there is no time machine that rolls everyone's state back and puts the missile back in the silo. This fact about the partitioning of state is closely related to the famous arrow of time problem in physics, but we won't delve into it here [38]. Instead, we note that the form procedural reflection takes in the rho calculus is perfectly amenable to multi-agent settings in which state is partitioned and time cannot be globally rolled back.

9.3 Programmable contexts

Process grammar

$$\text{PROCESS}\quad P, Q ::= 0 \mid \mathsf{U}(x) \mid \mathsf{for}(y \leftarrow x)P \mid x!(Q) \mid P|Q \mid *x \mid \mathsf{COMM}(K) \qquad \text{NAME}\quad x, y ::= @\langle K, P\rangle$$

$$\text{CONTEXT}\quad K ::= \square \mid \mathsf{for}(y \leftarrow x)K \mid x!(K) \mid P|K$$

Definition 10. Free and bound names *The calculation of the free names of a process, P, denoted* $\mathsf{FN}(P)$ *is given recursively by*

$$\mathsf{FN}(0) = \emptyset \qquad \mathsf{FN}(\mathsf{U}(x)) = \{x\} \qquad \mathsf{FN}(\mathsf{for}(y \leftarrow x)P) = \{x\} \cup \mathsf{FN}(P) \setminus \{y\}$$

$$\mathsf{FN}(x!(P)) = \{x\} \cup \mathsf{FN}(P) \qquad \mathsf{FN}(P|Q) = \mathsf{FN}(P) \cup \mathsf{FN}(Q) \qquad \mathsf{FN}(*x) = \{x\}$$

$$\mathsf{FN}(\mathsf{COMM}(K)) = \mathsf{FN}(K) \qquad \mathsf{FN}(\square) = \emptyset \qquad \mathsf{FN}(\mathsf{for}(y \leftarrow x)K) = \{x\} \cup \mathsf{FN}(K) \setminus \{y\}$$

$$\mathsf{FN}(x!(K)) = \{x\} \cup \mathsf{FN}(K) \qquad \mathsf{FN}(P|K) = \mathsf{FN}(P) \cup \mathsf{FN}(K)$$

An occurrence of x in a process P is bound if it is not free. The set of names occurring in a process (bound or free) is denoted by $\mathsf{N}(P)$.

9.4 Operational semantics

Finally, we introduce the computational dynamics. What marks these algebras as distinct from other more traditionally studied algebraic structures, e.g. vector spaces or polynomial rings, is the manner in which dynamics is captured. In traditional structures, dynamics is typically expressed through morphisms between such structures, as in linear maps between vector spaces or morphisms between rings. In algebras associated with the semantics of computation, the dynamics is expressed as part of the algebraic structure itself, through a reduction reduction relation typically denoted by $\rightarrow$. Below, we give a recursive presentation of this relation for the calculus used in the encoding.

$$\textsc{Catalyze}\quad \mathsf{U}(x) \mid *@\langle K, Q\rangle \rightarrow \mathsf{COMM}(K) \mid x!(Q)$$

$$\textsc{Comm}\quad \frac{x_t \equiv_{\mathsf{N}} x_s}{\mathsf{COMM}(K) \mid \mathsf{for}(y \leftarrow x_t)P \mid x_s!(Q) \rightarrow P\{@\langle K, Q\rangle/y\}}$$

$$\textsc{Par}\quad \frac{P \rightarrow P'}{P|Q \rightarrow P'|Q}$$

$$\textsc{Equiv}\quad \frac{P \equiv P' \qquad P' \rightarrow Q' \qquad Q' \equiv Q}{P \rightarrow Q}$$

We write $P \rightarrow$ if $\exists Q$ such that $P \rightarrow Q$ and $P \not\rightarrow$, otherwise.

9.5 Movement in space: an example

In the following example let $x = @\langle \square, 0\rangle$ and $y = @\langle K, Q\rangle$ for some K and Q.

$$\begin{aligned} & \mathsf{for}(y \leftarrow x) *y | x!(P) | \mathsf{COMM}(K_1) \\ \rightarrow \quad & \\ & *y\{@\langle K_1, P\rangle/y\} \\ = \quad & \\ & K_1[P] \end{aligned}$$

Now, if K_1 is also of the form $\mathsf{for}(y' \leftarrow x') *y' | x'!(\square) | \mathsf{COMM}(K_2) | R$, then P will move to the location $K_2[P]|R$. That is, $K_1[P] \rightarrow K_2[P]|R$. And if K_2 is likewise of the form $\mathsf{for}(y'' \leftarrow x'') *y'' | x''!(\square) | \mathsf{COMM}(K_3) | R'$, then P will move to the location $K_3[P]|R|R'$.

Thus, we have a means to describe movement of a process from location to location.

10 Stochastic and quantum rho

TBD

11 From formalism to physical intuition and back

TBD

12 Conclusions and future work

One way to understand the rho-calculus is a study in the seams of the π-calculus. There are several holes in Milner's calculus:

- *the zero process*, $\mathsf{0}$, is an input to the theory; adding other elements here corresponds to supplying "builtin" processes;
- *names* are an input to the theory, which we discussed at length;
- *the source of non-determinism* is an input to the theory.

The results here can really be seen as an initial exploration of the a general theory of this third dependency.

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13 Appendix: rholang specification

13.1 Rholang grammar

PROCESS

Proc ::= Proc1 | Proc|Proc1

CONDITIONAL

Proc1 ::= Proc2 | `if` (Proc) Proc2 | `if` (Proc) Proc2 `else` Proc1
| `new` [NameDecl] `in` Proc1 | Name `!?`([Proc]) SynchSendCont

IO

Proc2 ::= Proc3 | `contract` Name ([Name] NameRemainder) = { Proc }
| `for` ([Receipt]){ Proc } | `select` { [Branch] } | `match` Proc4 { [Case] }
| Bundle { Proc } | `let` Decl Decls `in` { Proc }

OUTPUT

Proc3 ::= Proc4 | Name Send ([Proc])

DISJUNCTION

Proc4 ::= Proc5 | Proc4 `or` Proc5

CONJUNCTION

Proc5 ::= Proc6 | Proc5 `and` Proc6

MATCHING

Proc6 ::= Proc7 | Proc7 `matches` Proc7 | Proc6 == Proc7 | Proc6 `!=` Proc7

COMPARISON

Proc7 ::= Proc8 | Proc7 < Proc8 | Proc7 <= Proc8 | Proc7 > Proc8
| Proc7 >= Proc8

ADDITION

Proc8 ::= Proc9 | Proc8 + Proc9 | Proc8 - Proc9 | Proc8 ++ Proc9
| Proc8 -- Proc9

MULTIPLICATION

Proc9 ::= Proc10 | Proc9 * Proc10 | Proc9 / Proc10 | Proc9 % Proc10
| Proc9 %% Proc10

NEGATION

Proc10 ::= Proc11 | `not` Proc10 | -Proc10

NESTING

Proc11 ::= Proc12 | Proc11 . Var([Proc]) | (Proc4)

DEREFERENCE

Proc12 ::= Proc13 | *Name

MATCH-DISJUCTION

Proc13 ::= Proc14 | VarRefKind Var | Proc13 \/ Proc14

MATCH-CONJUNCTION

Proc14 ::= Proc15 | Proc14 /\ Proc15

MATCH-NEGATION

Proc15 ::= Proc16 | ~Proc15

???

Proc16 ::= {Proc} | Ground | Collection | ProcVar | `Nil`
| SimpleType

???

[Proc] ::= **eps** | Proc | Proc , [Proc]

Decl ::= [Name] NameRemainder ← [Proc]

LinearDecl ::= Decl

[LinearDecl] ::= LinearDecl | LinearDecl ; [LinearDecl]

ConcDecl ::= Decl

[ConcDecl] ::= ConcDecl | ConcDecl & [ConcDecl]

Decls ::= **eps** | ; [LinearDecl] | & [ConcDecl]

SynchSendCont ::= . | ; Proc1

ProcVar ::= _ | Var

Name ::= _ | Var | @Proc12

[Name] ::= **eps** | Name | Name , [Name]

Bundle ::= `bundle+` | `bundle-` | `bundle0` | `bundle`

Receipt ::= ReceiptLinearImpl | ReceiptRepeatedImpl | ReceiptPeekImpl

[Receipt] ::= Receipt | Receipt ; [Receipt]

ReceiptLinearImpl ::= [LinearBind]

LinearBind ::= [Name]NameRemainder ← NameSource

[LinearBind] ::= LinearBind | LinearBind&[LinearBind]

NameSource ::= Name | Name?! | Name!?([Proc])

ReceiptRepeatedImpl ::= [RepeatedBind]

RepeatedBind ::= [Name]NameRemainder<=Name

[RepeatedBind] ::= RepeatedBind | RepeatedBind&[RepeatedBind]

ReceiptPeekImpl ::= [PeekBind]

PeekBind ::= [Name]NameRemainder<<-Name

[PeekBind] ::= PeekBind | PeekBind&[PeekBind]

Send ::= ! | !!

Branch ::= ReceiptLinearImpl=>Proc3

[Branch] ::= Branch | Branch[Branch]

Case ::= Proc13=>Proc3

[Case] ::= Case | Case[Case]

NameDecl ::= Var | Var(UriLiteral)

[NameDecl] ::= NameDecl | NameDecl , [NameDecl]

BoolLiteral ::= `true` | `false`

Ground ::= BoolLiteral | LongLiteral | StringLiteral | UriLiteral

Collection ::= [[Proc]ProcRemainder] | Tuple | `Set`([Proc]ProcRemainder)
| {[KeyValuePair]ProcRemainder}

KeyValuePair ::= Proc : Proc

[KeyValuePair] ::= **eps** | KeyValuePair | KeyValuePair , [KeyValuePair]

Tuple ::= (Proc ,) | (Proc , [Proc])

ProcRemainder ::= . . . ProcVar | **eps**

NameRemainder ::= . . . @ ProcVar | **eps**

VarRefKind ::= = | =*

SimpleType ::= `Bool` | `Int` | `String` | `Uri` | `ByteArray`

13.2 Rholang operational semantics

TBD