Lecture Note (Part 3)

CSCI 4470/6470 Algorithms, Fall 2023

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October 3, 2023

Part 3. Algorithms on graphs (Chapters 3 and 4)

Topics to be discussed:

- Basics and representations of graphs
- Depth-first search and applications
- Shortest path algorithms
- priority queue

1. Fundamentals of graphs

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vertex, edge, graph, degree, neighbor, weight, directed edge, di-graph, subgraph, tree, path, cycle, connected component, strongly connected component, complete graph, planar graph, non-planar graph, bi-partite graph

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But why would non-biological trees need a root?

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Proper definitions of graphs may incur some structural views on graphs and help solve various computational problems on graphs.

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Graph traversal by exploiting the recursive definition of graphs.

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  visit vertex v and then recursively **visit** u, for all $(v, u) \in E$.
- two different traversal methods: DFS and BFS, depending on which vertex is to visit next

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Assume graph G that was created with x being any vertex

Explore all vertices reachable from vertex x:

```
function explore(G: graph; x: vertex)
 1. visited(x) = true;
 2. for each edge (x, y) in G
 3.   if not visited(y) explore(G, y);
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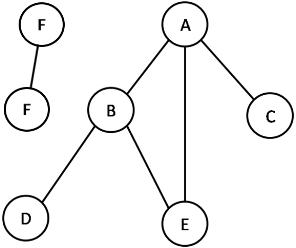
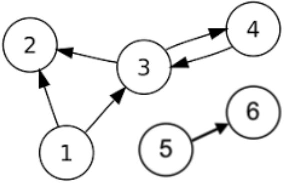
Adding time stamps:

```
function explore(G: graph; x: vertex)
visited(x) = true;
pre(x) = time_stamp;         // pre-visit work
time_stamp = time_stamp + 1;
for each edge (x,y) in G
  if not visited(y)
     parent(y)=x;            // record tree edge
     explore(G, y);
post(x) = time_stamp;          // post-visit work
time_stamp = time_stamp + 1;
```

Examples for DFS





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  - tree edges
  - back edges

2. Depth-first search

DFS on directed graphs

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- DFS on directed acyclic graphs (DAGs)

  Theorem If there is a path $x \rightsquigarrow y$, then post$(x) >$ post(y)

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All have time complexity: $O(|E| + |V|)$

Topological Sort problem

Input: directed acyclic graph $G(V, E)$,
Output: vertices of V in order: $v_1, v_2, \ldots, v_n$ such that
$\forall i < j, (v_j, v_i) \notin E$.

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- DFS on G^T from vertex `v` with the highest `post(v)` value;

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3. Shortest path problems

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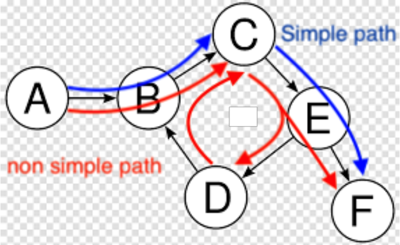

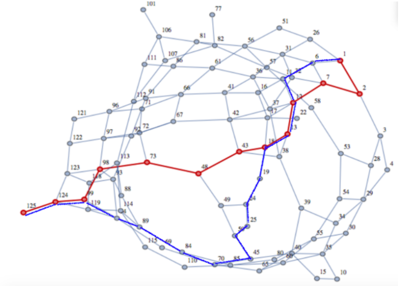

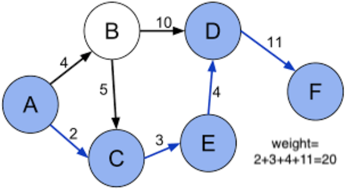

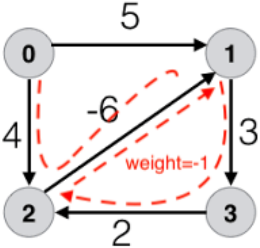

3. Shortest path problems

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Input: di-graph $G = (V, E)$, weights $w : E \to R$; source $s \in V$
Output: $dist(v)$, shortest distance s to every vertex $v \in V$.

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Shortest Distance problem on DAG

- what does DFS tree on DAG look like?
- are there back, forward, and crossing edges?
- Solving the shortest distance problem
  with DFS, i.e, Topological Sort
- So vertices can be arranged linearly, beginning from source s
  how to calculate all the shortest distances from s?

3. Shortest path problems

Input: DAG $G = (V, E)$, edge lengths $l : E \to R_{\geq 0}$, and $\mathtt{s} \in V$,
Output: $\forall\ \mathtt{u} \in V$, the smallest distance `dist(u)` from `s` to `u`.

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Input: DAG $G = (V, E)$, edge lengths $l : E \to R_{\geq 0}$, and s $\in V$,
Output: $\forall$ u $\in V$, the smallest distance dist(u) from s to u.

```
function dag-shortest-path(G, l, s)
for all u in V
  dist(u) = infinity;
  prev(u) = nil;  // predecessor of u in the path
dist(s) = 0;
topological sort V;
for all u in V in the sorted order
  for all edge (u, v) in E
     if dist(v) > dist(u) + l(u, v);
        dist(v) = dist(u) + l(u, v);
        prev(v) = u;
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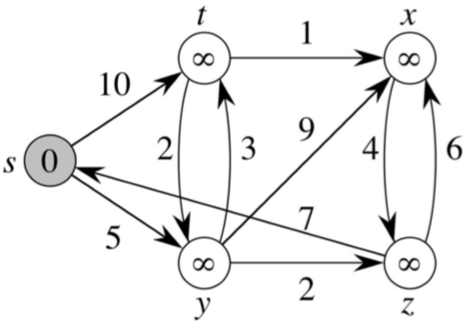
Shortest Distance problem: `Dijkstra's algorithm`
on general directed graphs, without negative weights

Input: $G = (V, E)$, edge lengths $l : E \to R_{\geq 0}$, and s $\in V$,
Output: $\forall$ u $\in V$, the smallest distance dist(u) from s to u.

```
function Dijkstra(G, l, s)
for all u in V
  dist(u) = infinity;
  prev(u) = nil;
dist(s) = 0;
H = makequeue(V);
While H is not empty
u = dequeue(H);
for all edges (u, v) in E
    if dist(v) > dist(u) + l(u, v);
       dist(v) = dist(u) + l(u, v);
       prev(v) = u;
return (prev)
```

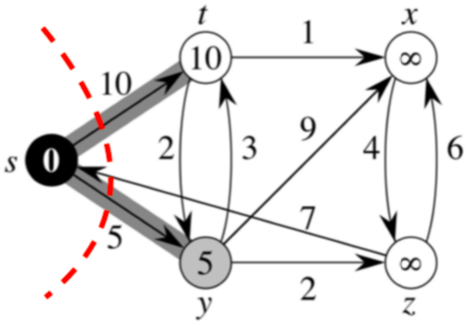
3. Shortest path problems



(a)

Note: the gray-colored vertex is to be dequeued
while the black-colored vertices are in set S.

3. Shortest path problems



(b)

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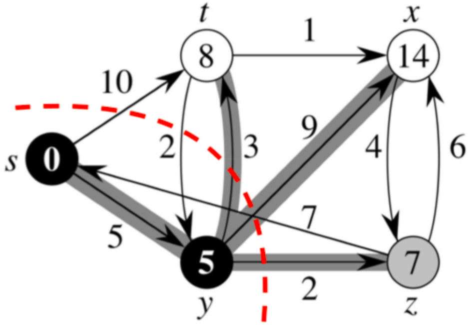

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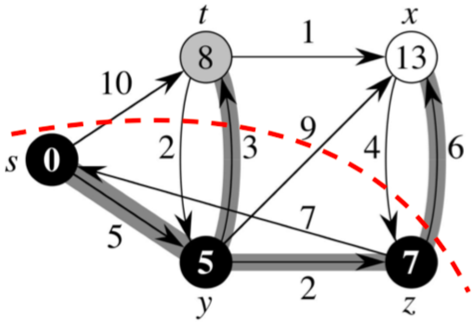

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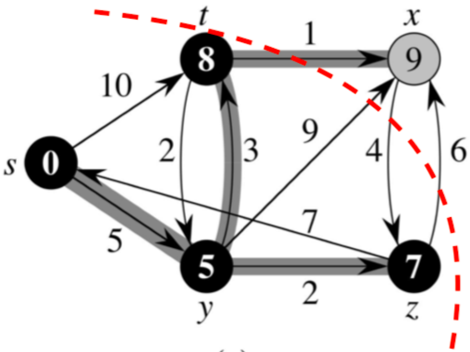

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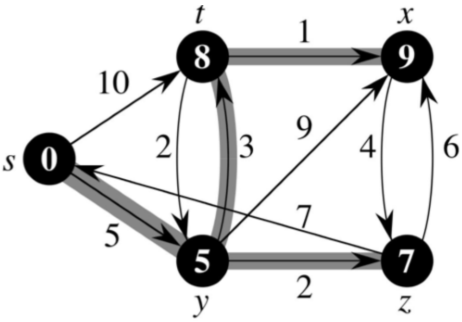

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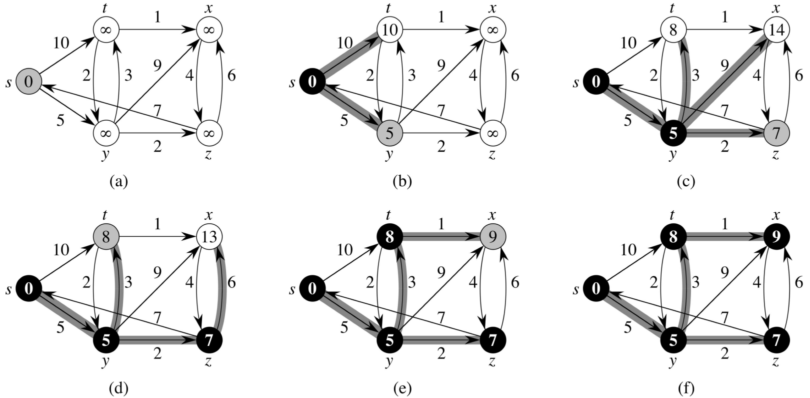

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- what would happen if negative edges are present?

Dijkstra's algorithm

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- what would happen if negative edges are present?
- edge relaxation on edge (u, v):

```
if dist(v) > dist(u) + l(u, v)
    dist(v) = dist(u) + l(u, v)
    prev(v) = u
```

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Shortest path (distance) problem is solved by repeatedly relaxing edges coming out from selected vertices.

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- on graphs without negative edges (`Dijkstra's`), what order?
- on arbitrary graphs, what order? (`Bellman-Ford`)

3. Shortest path problems

For graphs that may contain negative edges, edge relaxations cannot be done in specific order.

But every shortest path consists of at most $n-1$ edges;
$n-1$ rounds of edge relaxations suffices

```
function Bellman-Ford(G=(V,E): graph; s: vertex)
for all u in V
  dist(u) = infinite;
  prev(u) = nil;
dist(s) = 0;
for k=1 to n-1
  for all edge (u,v) in E
    if dist(v) >  dist(u) + l(u,v)       // l(u,v) is the
      then dist(v) = dist(u) + l(u,v);  // weight of edge
      prev(v) = u;
if there is an edge (u, v):  dist(u) + l(u,v) < dist(v)
  then return "G contains a negative cycle"
  else return (dist, prev)
```

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- the path cannot be shorter than $s \rightsquigarrow y \rightarrow x \rightarrow z \rightsquigarrow v$ **unless** cycle $x \rightsquigarrow x$ is negative.

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Priority queue implementation using heap

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- how to use a heap to realize a priority queue?

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Time complexities of different shortest path algorithms

- `dag-shortest-path`: $O(|V| + |E|)$
- `Dijkstra's`: $O(|V| \log_2 |V| + E)$
- `Bellman-Ford`: $O(|V||E|)$.