# CSCI 6470 Quiz #7 Questions Answers

**November 13, 2023 (12:40pm-1:10pm EST)**

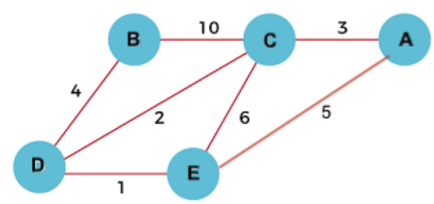
**Student Name** ______________________________ **Student ID**____________________________

**Rules**. Violation will result in **zero credit** for the exam and possibly the final grade.
1. Closed book/note/electronics/neighborhood.
2. Surrender your cell phone to the podium before using the restroom.

**There are 4 questions and 60 points in total. Good luck!**

1. (**15 points**) Run Prim's algorithm on the following graph to find a minimum spanning tree. Show in the table changes of `cost` values for vertices after every vertex is dequeued. Assume the algorithm starts from vertex `C`. $\boxed{\text{for the table total: } 10\,points}$

| Vertex | A | B | C | D | E | vertex dequeued |
|---|---|---|---|---|---|---|
| `cost` | $\infty$ | $\infty$ | 0 | $\infty$ | $\infty$ | |
| `cost` | 3 | 10 | 0 | 2 | 6 | C |
| `cost` | 3 | 4 | 0 | 2 | 1 | D |
| `cost` | 3 | 4 | 0 | 2 | 1 | E |
| `cost` | 3 | 4 | 0 | 2 | 1 | A |
| `cost` | 3 | 4 | 0 | 2 | 1 | B |




In addition,

(1) Show the cut for which edge (D, E) is a light edge: $(\{C, D\}, V - \{C, D\})$ $\boxed{3\,points}$

(2) Show the cut for which edge (C, E) is a light edge: There is NO such cut $\boxed{2\,points}$

2. (**15 points**) Prim's algorithm does NOT work well on disconnected non-directed graphs. However, it can be fixed to suit the need. Let the input graph $G$ consist of two disconnected components $H_1$ and $H_2$. Assume the algorithm starts from some vertex in $H_1$. Answer the following questions:

(1) Will the algorithm output correctly an m.s.t. for component $H_1$? "Yes" $\boxed{3\,points}$

(2) After the algorithm is done with all vertices in $H_1$, what vertices and of what `cost` values are still in the priority queue? All vertices in $H_2$ and of the cost values $\infty$ $\boxed{4\,points}$

(3) Why the algorithm may not find an m.s.t. for component $H_2$? vertices with $\infty$ cost will be dequeued but costs and `prev` will not be updated. $\boxed{4\,points}$

(4) Suggest a simple modification to Prim's so it can find m.s.t. for all disconnected components in the input graph.

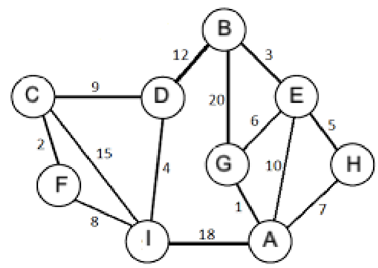
when there is vertex with cost $\infty$ is dequeued, its cost should be set to 0. $\boxed{4\,points}$

3. (**15 points**) Run Kruskal's algorithm on the following graph to find an m.s.t. Fill in the table changes of disjoint sets after each edge is selected.

| Edge selected | Disjoint sets | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | [A] | [B] | [C] | [D] | [E] | [F] | [G] | [H] | [I] |
| $(A, G)$ | [A,G] | [B] | [C] | [D] | [E] | [F] | | [H] | [I] |
| $(C, F)$ | [A,G] | [B] | [C,F] | [D] | [E] | | | [H] | [I] |
| $(B, E)$ | [A,G] | [B,E] | [C,F] | [D] | | | | [H] | [I] |
| $(D, I)$ | [A,G] | [B,E] | [C,F] | [D,I] | | | | [H] | |
| $(E, H)$ | [A,G] | [B,E,H] | [C,F] | [D,I] | | | | | |
| $(E, G)$ | [A,G, B,E,H] | [C,F] | [D,I] | | | | | | |
| $(F, I)$ | [A,G, B,E,H] | [C,F, D,I] | | | | | | | |
| $(B, D)$ | [A,G, B,E,H, C,F, D,I] | | | | | | | | |

$\boxed{\text{Every edge selected: } 1\, points}$

$\boxed{\text{every corresponding set joining: } 1\, points}$




4. (**15 points**) This question concerns greedy-choice property. Answer the following questions.

(1) State a greedy-choice property for the Fractional Knapsack problem: $\boxed{\text{total: } 7\, points}$

Greedy-choice property: The item with the maximum value density $d_k = \frac{v_k}{s_k}$

(1) belongs to some optimal solution, $\boxed{3\, points}$
(2) where $f_k = 1$ if knapsack size $W \geq s_k$, $\boxed{2\, points}$
(3) or $f_k = W/s_k$ if $W < s_k$. $\boxed{2\, points}$

(2) A greedy-choice property for the Activities Scheduling problem states that the activity $a$ with the earliest finish time belongs to some optimal solution. If a given optimal solution $S$ does not contains $a$, show that the greedy-choice property still holds for $a$. $\boxed{\text{total: } 8\, points}$

Let $S = \{e, x_1, \ldots, x_m\}$, where $e$ is the activity of the earliest finish time in $S$; $\boxed{3\, points}$

Define $S' = S \cup \{a\} - \{e\}$ $\boxed{2\, points}$ ;

$x_1, \ldots, x_m$ do not conflict with $a$; so $S'$ is an optimal solution that contains $a$ $\boxed{3\, points}$

[The following space will not be graded.]