CSCI 6470 Quiz #6 Questions Answers

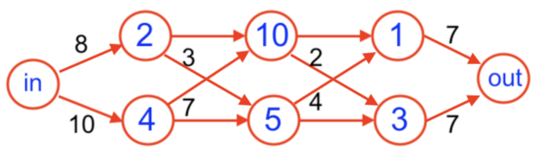
November 2, 2023 (12:45pm-1:15pm EST)

Student Name ______________________________ **Student ID**____________________________

> **Rules**. Violation will result in **zero credit** for the exam and possibly the final grade.
> 1. Closed book/note/electronics/neighborhood.
> 2. Surrender your cell phone to the podium before using the restroom.

There are 4 questions and 60 points in total. Good luck!

1. (**15 points**) Fill out the DP table for the following given FASTEST ASSEMBLY PATH problem. **Make sure to include `prev` information in the table too.**



	1	2	3	out
1	10	← 20	← 21	28
2	14	↖ 18	← 21	

every time value in the table has 1 *points* ,

every prev info, regardless format has 1 *points*

Also the fastest assembly path is (in) $\to (1,1) \to (2,2) \to (2,3) \to$ (out) or
(in) $\to (1,1) \to (1,2) \to (1,3) \to$ (out) 3 *points*

2. (**20 points**) For the following given 0-1 KNAPSACK problem, fill out the DP table **up to row** $k = 2$. No need to include `prev` information.

knapsack W = 5

item	1	2	3
size	1	3	2
value	2	4	5

k= \ X=	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	2	2	2	2	2
2	0	2	2	4	6	6
3						

first row has 1 *points*

first column has 1 *points*

second row has 3 *points*

everyone of other cells has 1 *points*

The following questions assume n items and knapsack size W. **Give answers in the allowed spaces only**. Every answer has 2 *points*

(1) Where does the traceback start to find selected items? (n, w) where to end? (0, X) for some X, or (k, 0) for some k.

(2) During the trackback process, how do we know which items are selected or abandoned? if prev points to up cell, it is abandoned; otherwise (to up-left) selected

(3) The time complexity on n items and knapsack of size W is: $O(nW)$

(4) While the time complexity looks in a polynomial form, it may not be a polynomial time in the **input size**. This is because W can be as large as an exponential of the input size

3. (**10 points**) The Coin Change problem has the following recursive formulation for its objective function $c(m)$, with base case $c(0) = 0$. **Give answers in the allowed spaces only**.

$$c(m) = \min \begin{cases} c(m-25)+1 & m \geq 25 \\ c(m-10)+1 & m \geq 10 \\ c(m-5)+1 & m \geq 5 \\ c(m-1)+1 & m \geq 1 \end{cases}$$

(1) Give two cases of c that share an overlapping subproblem: $c(2)$ and $c(3)$ share $c(1)$, 2 *points*

(2) Where in objective function formulation shows that the problem has optimal substructure $c(m)$ is optimized over 4 optimal solutions, 2 *points*

Assume that table cells `prev`$[k]$ are used to remember the corresponding type of coin chosen in computing $c(k)$, for all $k = 0, 1, \dots$.

(3) When a trackback process encounters cell `prev`[14] that has the information *dime* (10 cents) in it, what cell should the trackback go to next? `prev`(4) 3 *points*

(4) How was the *dime* put in cell `prev`[14]? when $c(4) \leq$ both $c(9)$ and $c(13)$ 3 *points*

4. (**15 points**) Problem Edit Distance has recursive objective function $E(i, j)$ as follows,

$$E(i,j) = \min \begin{cases} E(i-1, j-1) & \text{when } x_i = y_j \\ E(i-1, j-1)+2 & \text{when } x_i \neq y_j \\ E(i, j-1)+1 \\ E(i-1, j)+1 \end{cases}$$

where base case $E(0, 0) = 0$ and the penalty scores are: 0 point for a match, 2 for a mismatch, and 1 for an insertion/deletion.

Let the two input sequences be $x = x_1 \dots x_n$ and $y = y_1 \dots y_m$. Matrix table `Ed` is used to compute function E via dynamic programming. **Give answers in the allowed spaces only**.

(1) What is the dimensions of the matrix table `Ed`? $(n+1) \times (1+m)$ 2 *points*

(2) What does cell `Ed`$[4, 5]$ represent? edit distance between $x_1 \dots x_4$ and $y_1 \dots y_5$ 5 *points*

(3) Give the formula to compute cell $\texttt{Ed}[4,5]$ from other cells: $\boxed{2\,points}$

$$\texttt{Ed}[4,5] = \min \begin{cases} \texttt{Ed}[3,4]+2 & x_4 \neq y_5 \\ \texttt{Ed}[3,4] & x_4 = y_5 \\ \texttt{Ed}[4,4]+1 & \\ \texttt{Ed}[3,5]+1 & \end{cases}$$

(4) If cells $\texttt{Ed}[4,5] = 20$ and $\texttt{Ed}[3,4] = 20$, what do they tell you? $x_4 = y_5$ $\boxed{2\,points}$

(5) If cells $\texttt{Ed}[4,5] = 22$ and $\texttt{Ed}[3,4] = 20$, what do they tell you? $x_4 \neq y_5$ $\boxed{2\,points}$

(6) If cells $\texttt{Ed}[3,4] = 22$, but $\texttt{Ed}[4,5] = 21$ and $\texttt{Ed}[4,4] = 21$, what do they tell you?
$\texttt{Ed}[4,5]$ is computed from $\texttt{Ed}[3,5] = 20$ OR x_4 is aligned with gap "_" $\boxed{2\,points}$

[The following space will not be graded.]