

# *Enumeration*

## **Pseudo-code**

There are a lot of websites on the internet that talk about the enumeration in the form of codes and the logic in the enumeration is almost the same for all. Basically, the pseudo code that I do here is based on python, but it should work with any other language since the idea is the same.

Here is the Enumeration pseudo-code :

```
Enum(A[1,...,N])
  if N == 0
    return 0, A
  else
    max_sum = -Infinity
    for i from 1 to N
      for j from i to N
        curr_sum = 0
        for k from i to j
          curr_sum = curr_sum + A[k]
          if curr_sum > max_sum
            max_sum = curr_sum
            start_index = i
            end_index = j
    return max_sum, A[begIndex, ..., endIndex]
```

## **Theoretical Run-time Analysis**

Here we would like to know from our pseudo-code and estimate the theoretical run-time for the enumeration. From the iterative pseudo-code that we have, we can try to count the theoretical run-time.

First, we can see from the outer  $i$  loop. So the first outer loop for  $i$  runs from 1 to  $N$ , and then we can see there are two inner loops inside that loop, which is  $j$  that runs from  $i$  to  $N$  and then  $k$  that runs from  $i$  to  $j$ . Here from the iterative work, we can know that all the for loops are connected each other and work to result the  $\text{curr\_sum}$ , which means that based on that, our  $f(n)$  is  $\Theta(N^3)$ . Then we can compute the number of iterations as :

$\sum_{i=1}^N \sum_{j=i}^N \sum_{k=i}^j \theta(1)$ . Inputting that calculation of iteration to calculator, and we will get :  $\sum_{i=1}^N \frac{1}{2}(i^2 - 2iN - 3i) + \sum_{i=1}^N \frac{1}{2}(N^2 + 3N + 2)\theta(1)$  with i will remain as a constant . By keep doing the calculation and i remain as a constant, We will get an equation in the end with  $N^3$ ,  $N^2$ , and  $N$ , which is :  $\frac{1}{6}N^3 + \frac{1}{2}N^2 + \frac{1}{3}N \cdot \theta(1) = \theta(N^3)$ . Finally, based on our calculation, theoretically, the run-time for this algorithm is Cubical.

## *Divide and Conquer*

Pseudo-code

The "Divide and Conquer" maximum sub-array algorithm is described by the following pseudo-code:

```

Div_and_Conq(A[1,...,N]){
    if N == 0 {
        return 0, A
    } else if N == 1 {
        return A[0], A
    }

    tmp_max = 0
    mid_max = 0
    mid_start = 0
    mid_end = 0
    left_max = 0
    right_max = 0

    midpoint = N / 2

    mid_start = midpoint
    mid_end = midpoint

    for i from A[N,...,midpoint] {
        tmp_max = tmp_max + A[i]
        if tmp_max > left_max {
            left_max = tmp_max
            mid_start = i
        }
    }

```

```

    }
}
tmp_max = 0

for i from A[midpoint,...,N] {
    tmp_max = tmp_max + A[i]
    if tmp_max > right_max {
        right_max = tmp_max
        mid_end = i + 1
    }
}
mid_max = left_max + right_max

left_max, left_subarray = DIVIDE_AND_CONQUER(A[0,...,midpoint])
right_max, right_subarray = DIVIDE_AND_CONQUER(A[midpoint,...,N])

if mid_max >= left_max and mid_max >= right_max {
    return mid_max, A[mid_start,...,mid_end]
} else if left_max >= right_max and left_max > mid_max {
    return left_max, left_subarray
} else if right_max > left_max and right_max > mid_max {
    return right_max, right_subarray
}
}

```