Midterm Exam 2 *(Wednesday, February 19, 8:30 p.m.)*

GENERAL DIRECTIONS: Fill out the cover sheet completely, as indicated.

Please follow the specific directions on each page of the answer form. Your test form letter is E. **For all items**, unless directed otherwise: In all expressions and symbolic solutions, reduce them to simplest form; in all final numerical answers, use standard SI units and proper significant digits. ***No item will be given credit if it does not include valid reasoning/work to justify the solution or answer.***

Physical constants and other possibly useful information:

$g_{earth.surface} = 9.80\ \mathrm{m/s^2}$		$V_{cube} = s^3$	$V_{sphere} = (4/3)\pi r^3$
1 atm = 1.01×10^5 Pa	$n_{air} \approx 1.00$	$\rho_{water} = 1000\ \mathrm{kg/m^3}$	$\rho_{aluminum} = 2700\ \mathrm{kg/m^3}$

DIRECTIONS for T/F/N items 1-3: Evaluate each statement as being demonstrably True (**T**), demonstrably False (**F**), or as having Not enough information (**N**) to declare it either True or False; AND briefly but fully explain your reasoning. **Note: No credit will be given for any T/F/N answer without a valid explanation to accompany it.**

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1. *Evaluate (T/F/N) each statement. Justify your answers fully with any valid mix of words, drawings and calculations.*

(A) The buoyant force exerted by a fluid on an object is equal to the volume of the fluid displaced by that object.

(B) $\rho g(\Delta h)$ has units of $\mathrm{J/m^3}$.

(C) If all else remains unchanged, when a barometer's (accurate) reading falls, you can sip water through a straw slightly more easily than before.

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2. *Evaluate (T/F/N) each statement. Justify your answers fully with any valid mix of words, drawings and calculations.*

(A) The force exerted on a square meter of floor by the earth's atmosphere (at sea level) is greater than the rest weight of a 1.5-meter cube of pure aluminum.

(B) The image created by a diverging lens becomes taller as the object is moved closer to the lens.

(C) A single object and a set of two identical converging lenses can be arranged so that the total absolute magnification of the system is in this range: $0 < m < 1$

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3. *Evaluate (T/F/N) each statement. Justify your answers fully with any valid mix of words, drawings and calculations.*

(A) A far-sighted person has a near point less than 25 cm.

(B) A near-sighted person would use lenses with negative focal lengths to correct his vision.

(C) A standard two-lens telescope uses two converging lenses, both with very short focal lengths.

DIRECTIONS for free-answer items 4-5: Fill in or provide the answers or solutions indicated. For these items, you do not need to do the full seven-step ODAVEST problem-solving procedure, but **you must still justify every answer/solution. No credit will be given for any answer or solution without a valid explanation and/or valid work to accompany it.**

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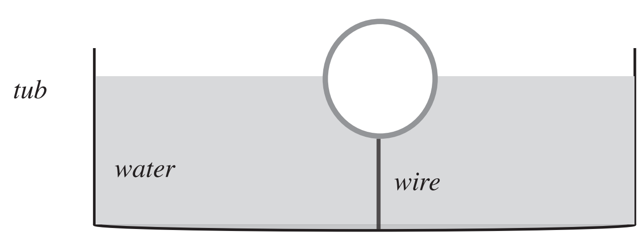
4. A certain hollow, sealed ball with an outer radius of 12.0 cm will float freely in a tub of pure water, with just 10% of its volume immersed.

But now a thin anchor wire, connected to the bottom of the tub, is holding that ball at rest, so that the ball is 50% immersed in water, as shown here.



What is the magnitude of the tension force exerted by the wire on the ball?

5. Suppose you have eyeglasses with identical +2.5 Diopter lenses. And suppose you're also carrying a 5x magnifying glass, a 2.5x magnifying glass, and a 1.25-mm-long splinter of wood.

a. What's the very best angular view (in radians) that you can get of the splinter, using just one of the four lenses (assuming that you can bring the splinter as close as you wish)?

b. What's the best comfortable angular view (in radians) that you can get of the splinter, using just two of the four lenses (again, assuming that you can bring the splinter as close as you wish), if you hold the lenses no more than 50 cm apart?

You may use the small-angle approximation here for all angles (if they are measured in radians): θ (rad) $\approx \tan\theta$

DIRECTIONS for item 6: For this item, the full seven-step ODAVEST problem-solving procedure is required. Note that each part will be awarded with points, so even if you don't get all the way through the problem, there are many ways to earn partial credit for parts that are valid). *Keep in mind that **you're not being asked to actually solve for the final expression**. In fact, you're not being asked to do any math at all—**not even any algebra**. Rather, for the **S**olve step, you are to write a series of succinct instructions on how to solve this problem. Pretend that your instructions will be given to someone who knows math but not physics. And for the **T**est step, you should consider the situation and predict how the solution would change if the data were different—changing one data value at a time.*

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6. Refer to the diagram below (*which is not necessarily to scale*). A solid, clear-plastic brick of known width W, height H, and index of refraction, n_p, is floating freely, at rest, in a tub of water. The cube floats with its top and bottom faces level (parallel to the water's surface). The water also has a known index of refraction, n_w; and it is known that $n_w < n_p$.

Nearby (in the open air), a powerful point source of monochromatic (single-color) light has been placed at the focal point, F, of a converging lens, whose central optical axis is directed as shown, at some unknown angle.

At least three light rays (A, B and C) coming from that point source all pass through the lens, then through the brick, then meet again at point X. Point X's depth is the same as that of the brick's bottom, but X is located a horizontal distance d_X from the brick's right face.

At all times, the three rays travel in the same plane, which is parallel to the page and to the brick's front face.

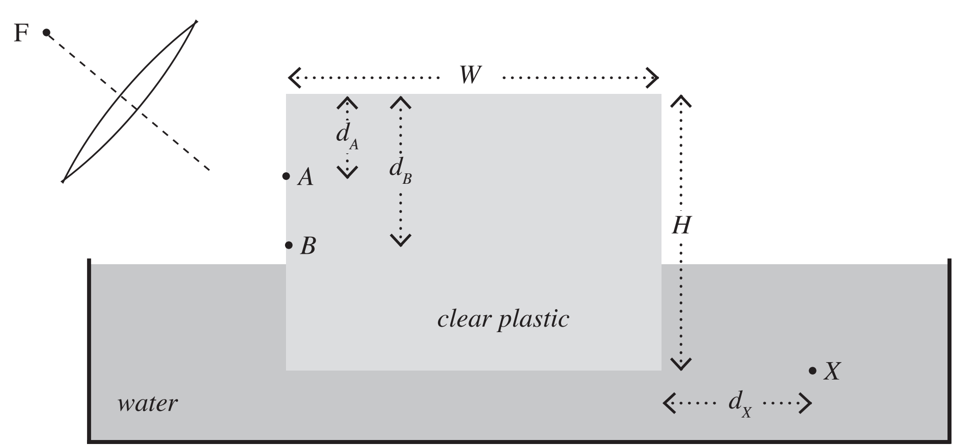
Ray A enters the side of the brick at point A, which is a known vertical distance d_A from the top of the brick.
Ray A exits the brick back into air before entering the water.

Ray B enters the side of the brick at point B, which is a known vertical distance d_B from the top of the brick.
Ray B exits the brick directly into the water.

Ray C enters water before entering the brick.
Ray C is traveling horizontally when it passes through point X.

Find the distance d_X.

Here is a summary of all known values: W, H, n_p, n_w, n_{air} (≈ 1.00), d_A, d_B



Scoring: ***Each part (A-B-C) of each problem on this page is worth 8 points (so each of the 3 problems is worth 24 points.)***
For each part (A-B-C), the answer (T/F/N) is worth 4 points, and the reasoning is worth 4 points.
If the answer is correct, but the physics reasoning is missing or incorrect: 0 points for the problem.
If the physics reasoning is correct, but the answer is missing or incorrect: 4 points for the problem.

1

(A) The buoyant force is equal to the ***weight*** of the fluid displaced, not the volume.

A force (N, or kg·m/s^2) can never be equated to a volume (m^3); they are inherently different physical measures.

A **F**

(B) In the SI system, it is true that $\rho g(\Delta h)$ could be expressed as J/m^3, as follows:

$\rho g(\Delta h) = (\text{kg/m}^3)(\text{m/s}^2)(\text{m}) = \text{kg/(m·s}^2)$ $\quad$ $\text{J/m}^3 = (\text{N·m/m}^3) = [(\text{kg·m/s}^2)\text{·m}]/\text{m}^3 = \text{kg/(m·s}^2)$

But note that (still within the SI system) $\rho g(\Delta h)$ could be expressed instead as Pa or N/m^2.
Moreover, there are other (*non-SI*) units that $\rho g(\Delta h)$ could be expressed in (e.g. lbs/in^2, etc.).

B **T/N**

(C) Liquid is pushed up a drinking straw by the difference in pressure between the atmosphere and the chamber formed by your mouth and throat; that chamber must be lower in pressure than the atmosphere. So when the barometer reading drops, indicating lower atmospheric pressure, in order to push the liquid just as far up the straw as before, you must create an even lower pressure in your mouth and throat, which is more difficult.

C **F**

2

(A) $P = F/A$, so $F = P·A$. So the force on the square meter of floor by the atmosphere is given by:

$F_{atm.floor} = (P_{atm})(A_{floor}) = (1.01 \times 10^5 \text{ Pa})(1 \text{ m}^2) = \mathbf{1.01 \times 10^5 \text{ N}}$

The rest weight of a cube of aluminum is simply the force of gravity exerted on it by the earth:

$F_{G.al} = \rho_{al} V_{al} g = (2700)(1.5^3)(9.80) = \mathbf{8.93 \times 10^4 \text{ N}}$

A **T**

(B) The value of m for a single diverging lens (imaging from a single object) is always positive and less than 1; the image is always smaller than the object. The question is, how does m change as d_o approaches zero (i.e. as you move the object toward the lens)? Solve the thin-lens equation for d_i: $d_i = f·d_o/(d_o - f)$ $\quad$ Thus: $m = -(d_i/d_o) = f/(f - d_o)$, where f is negative and d_o is positive. As d_o gets larger, so does the denominator; m becomes a smaller fraction. But as d_o approaches zero, m ***increases***—approaches 1.
Other acceptable forms of reasoning: Choose two different values of d_o and numerically solve the thin-lens equation for d_i, then calculate m, showing that its value increases (approaches closer to 1) for the smaller d_o. Or, construct two reasonably good ray diagrams that compare the image sizes/distances for the same object placed nearer or farther from the lens.

B **T**

(C) There are many possibilities. Example: Place the object at a distance greater than 2f (to the left) from the first lens, which will form a reduced but upside-down image. Let that image land in a similar position relative to the second lens (again more than twice the focal distance to the left of that lens). The final image will be further reduced, but re-inverted, so that it's now right-side up. Voila: $\mathbf{0 < m < 1}$
Other acceptable avenues of reasoning: The above summary as a clear diagram, or a numerical calculation of same using the thin-lens equation and the magnification equation.

C **T**

3

(A) By definition, a far-sighted person has a near point greater than 25 cm.

A **F**

(B) A near-sighted person has a **far point that is less than standard** (i.e. less than ∞), so he/she needs a lens that will accept a very large object distance and produce a smaller image distance—and yet not invert the image with respect to the object. This is exactly what a single diverging lens does (and the one task a single converging lens cannot do).

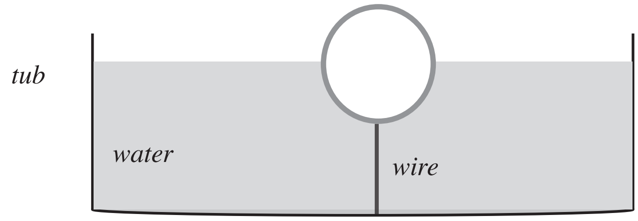
B **T**

(C) The objective lens on a telescope should be of a very long focal length; only the eyepiece should be very short f (and the angular magnification equation for a standard two-lens telescope shows this clearly: $M \approx -f_o/f_e$). The long distance from the objective lens to its focal point allows it to better grow its image size before "serving" it to the eyepiece for further magnification.

C **F**

4. A certain hollow, sealed ball with an outer radius of 12.0 cm will float freely in a tub of pure water, with just 10% of its volume immersed.

But now a thin anchor wire, connected to the bottom of the tub, is holding that ball at rest, so that the ball is 50% immersed in water, as shown here.

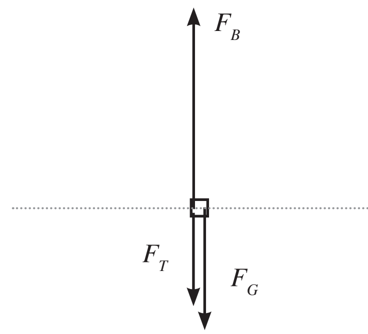


What is the magnitude of the tension force exerted by the wire on the ball?

Sum the y-forces on the ball:

$$\Sigma \boldsymbol{F} = m\boldsymbol{a}_y$$
$$F_B - F_T - F_G = m\boldsymbol{a}_y$$
$$(\rho_{water})(V_{water.displ.})(g) - F_T - (\rho_{ball})(V_{ball})(g) = 0$$

12 pts.



6 pts.

But: $V_{water.displ.} = V_{ball}/2$ *2 pts.*

And: $\rho_{ball} = 0.10\rho_{water}$ *4 pts.*

So: $(\rho_{water})(V_{ball}/2)(g) - F_T - (0.10\rho_{water})(V_{ball})(g) = 0$ *4 pts.*

Solve for F_T: $F_T = (V_{ball})(g)[(4/10)\rho_{water}]$ *4 pts.*

$= (4/3)\pi(0.12)^3(9.80)[(4/10)(1000)] =$ **28.4 N** *value = 2 pts., units = 1 pt., sig. figs. = 1 pt.*

5. Suppose you have eyeglasses with identical +2.5 Diopter lenses. And suppose that you're also carrying a 5x magnifying glass, a 2.5x magnifying glass, and a 1.25-mm-long splinter of wood.

a. What's the <u>very best</u> angular view (in radians) that you can get of the splinter, using just <u>one</u> of the four lenses (assuming that you can bring the splinter as close as you wish)?

b. What's the best <u>comfortable</u> angular view (in radians) that you can get of the splinter, using just <u>two</u> of the four lenses (again, assuming that you can bring the splinter as close as you wish), if you hold the lenses no more than 50 cm apart?

You may use the small-angle approximation here for all angles (if they are measured in radians): θ (rad) $\approx \tan\theta$

First, find the focal length of your eyeglass lens(es): $2.5 = 1/f$, where f is in meters

Or: $2.5/100 = 1/f$, where f is in cm. <u>Thus</u>: $f = 40$ cm *3 pts.*

Next, find the near point (NP) of your eye. A thin eyeglass lens that corrects for far-sightedness must accept an object held at the <u>standard</u> distance (25 cm) *from the eye* and form an image located at the eye's near point (with a negative image distance); but the lens is located 2 cm *closer* to its object and its image than the eye is.

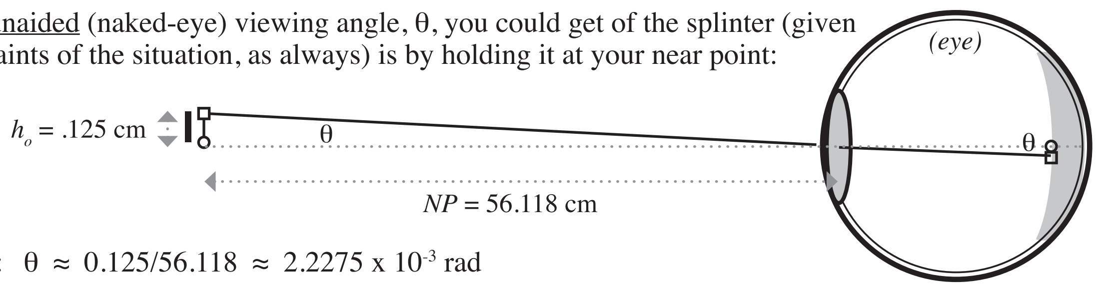
<u>Thus</u> for that lens: $1/f = 1/d_o + 1/d_i$ becomes $1/40 = 1/23 + 1/[-(NP - 2)]$

<u>Solving now for NP</u>: $NP = 56.118$ cm *3 pts.*

Next, find the focal lengths of each of the magnifying glasses. A magnifier glass's "power" rating is the ***angular magnification*** (M) value that would be achieved by a "normal" eye (with a standard NP, 25 cm) when holding the object right at the focal point of the magnifier lens, thus allowing <u>comfortable</u> viewing of the image formed out on the distant horizon (∞). And we know this $M_{standard.comfort} = NP/f = 25/f$.

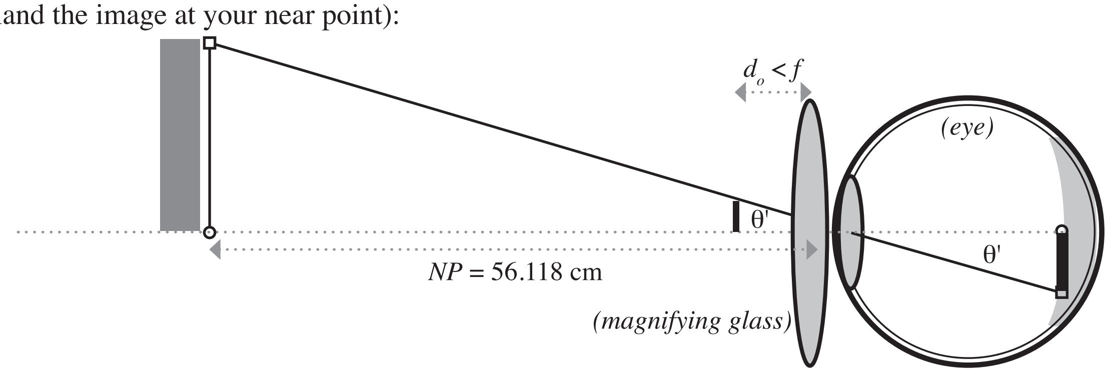
So for these two magnifiers: $5 = 25/f_1$ so $f_1 = 5$ cm *and* $2.5 = 25/f_2$ so $f_2 = 10$ cm *3 pts.*

a. The best <u>unaided</u> (naked-eye) viewing angle, θ, you could get of the splinter (given the constraints of the situation, as always) is by holding it at your near point:



<u>Therefore</u>: $\theta \approx 0.125/56.118 \approx 2.2275 \times 10^{-3}$ rad *3 pts.*

The <u>very best angular magnification</u> with one lens would be when using that lens as a magnifying glass in maximum configuration (i.e. holding the splinter a little distance inside the focal point of the lens in order to land the image at your near point):



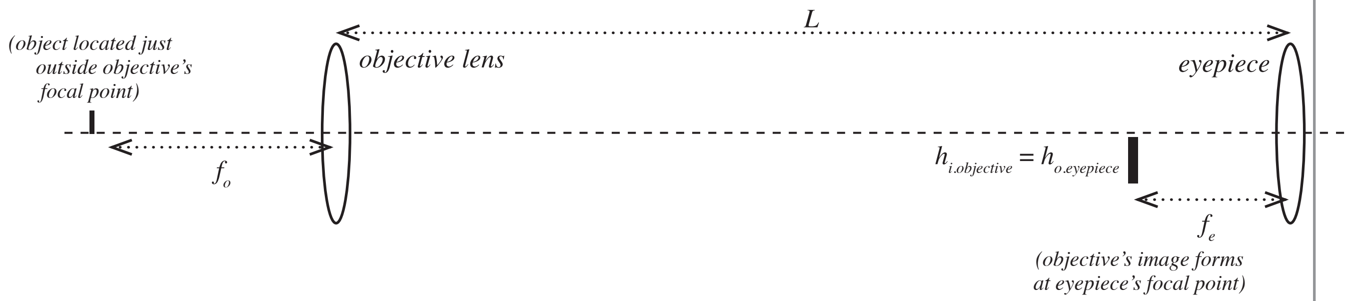
In this maximized viewing configuration, we know that $M_{max} = NP/f + 1$. *3 pts.*

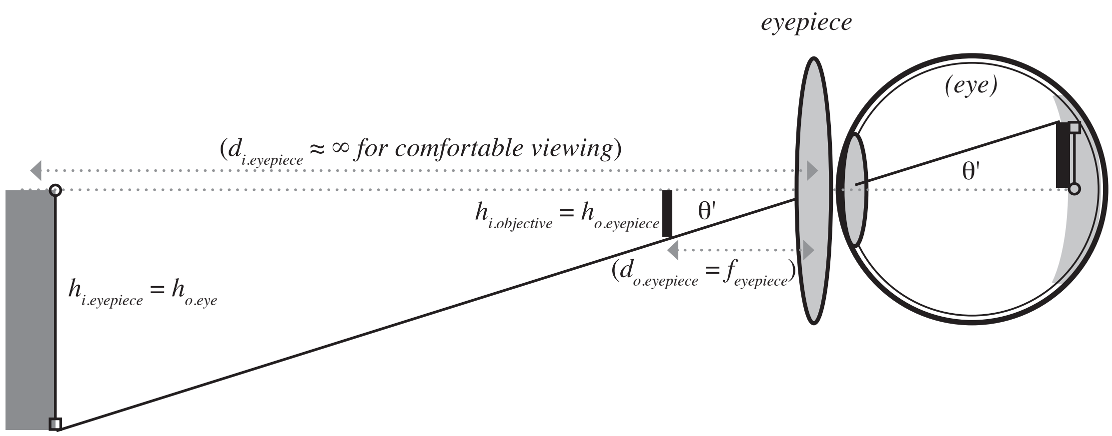
This is largest when using the lens with the <u>shortest</u> focal length, *so choose the magnifier with f_1 (5 cm).* Thus the <u>best</u> M_{max} is given by $M_{max} = NP/f + 1 = 56.118/5 + 1 = 12.224$ *2 pts.*

<u>Therefore</u>: Because $M = \theta'/\theta$, we have $\theta' = M\theta$ *3 pts.*

$= 12.224(2.2275 \times 10^{-3}) =$ **0.0272 rad** *value = 1 pt., units = 1/2 pt., sig. figs. = 1/2 pt.*

b. A standard microscope configuration will allow good magnification and comfortable viewing:





3 pts.

In this configuration, we know that $M \approx -NP(L - f_e)/(f_e \cdot f_o)$

<u>Looking at the equation</u>, we see that to get the best value (greatest magnitude) of M here, we must use…

- The longest barrel length, L, allowed.
  So in this case: $L = 50$ cm
- The two lenses with the <u>shortest</u> focal lengths (and of the two, f_e must be shorter).
  So in this case: $f_e = 5$ cm and $f_e = 10$ cm

4 pts.

<u>Thus</u> your makeshift microscope offers $M \approx -(56.12)(50 - 5)/(5 \cdot 10) \approx -50.51$ *2 pts.*

And (see diagram and calculations from part **a**) we already know that the best naked-eye view of the splinter is about 0.0022275 rad.

<u>Therefore</u>: Again, because $M = \theta'/\theta$, we have $\theta' = M\theta$

$= -50.51(2.2275 \times 10^{-3}) = \mathbf{-0.113\ rad}$

3 pts.

value = 1 pt., units = 1/2 pt., sig. figs. = 1/2 pt.

6. **Refer to the diagram below (*which is not necessarily to scale*). A solid, clear-plastic brick of known width W, height H, and index of refraction, n_p, is floating freely, at rest, in a tub of water. The cube floats with its top and bottom faces level (parallel to the water's surface). The water also has a known index of refraction, n_w; and it is known that $n_w < n_p$.**

Nearby (in the open air), a powerful point source of monochromatic (single-color) light has been placed at the focal point, F, of a converging lens, whose central optical axis is directed as shown, at some unknown angle.

At least three light rays (A, B and C) coming from that point source all pass through the lens, then through the brick, then <u>meet again at point X</u>. Point X's depth is the same as that of the brick's bottom, but X is located a horizontal distance d_X from the brick's right face.

At all times, the three rays travel in the same plane, which is parallel to the page and to the brick's front face.

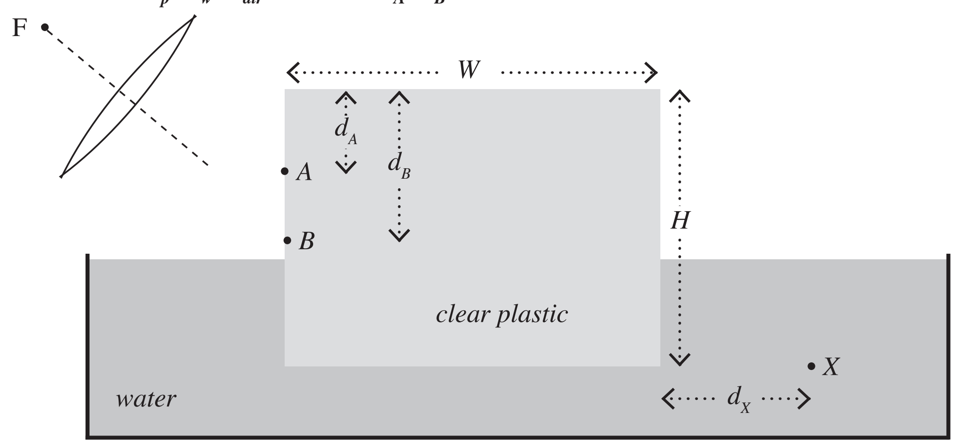
Ray A enters the side of the brick at point A, which is a known vertical distance d_A from the top of the brick. Ray A exits the brick back into air before entering the water.

Ray B enters the side of the brick at point B, which is a known vertical distance d_B from the top of the brick. Ray B exits the brick directly into the water.

Ray C enters water before entering the brick.
Ray C is traveling horizontally when it passes through point X.

Find the distance d_X.

<u>Here is a summary of all known values</u>: W, H, n_p, n_w, n_{air} (≈ 1.00), d_A, d_B



For this item, the full seven-step ODAVEST problem-solving procedure is required. Note that <u>each part</u> will be awarded with points, so even if you don't get all the way through the problem, there are many ways to earn partial credit for parts that are valid).
Keep in mind that you're not being asked to actually solve for the final expression. In fact, you're not being asked to do any math at all—not even any algebra. Rather, for the <u>S</u>olve step, you are to write a series of succinct instructions on <u>how</u> to solve this problem. Pretend that your instructions will be given to someone who knows math but not physics. And for the <u>T</u>est step, you should consider the situation and predict how the solution would change if the data were different—changing one data value at a time.

*For a complete ODAVEST (seven-step) problem-solving procedure, each of the seven steps is <u>equally weighted</u> (in this case, **8 points each**, for a total of 56 points for the problem).*

Objective: A solid plastic brick of known height and width is floating at rest in a tub of pure water.

The brick's top and bottom faces are horizontal;
the brick's front and back faces are parallel to the page.

A point source of monochromatic light is positioned at
the focal point of a motionless converging lens.

The lens' optical axis is angled downward toward a point
above the water line on the left side of the brick.

At least three rays (A, B and C) that emerge from the source and
pass through the lens then eventually meet again at a certain point X.

Point X is located in the water an unknown distance d_x directly
(horizontally) to the right of the bottom-front-right corner of the brick.

At all times, rays A, B and C remain co-planar with one another
and with the front/rear faces of the brick.

Ray A strikes the left side of the brick at point A, which is a known
vertical distance below the top of the brick.

Ray A enters the plastic from the air, then exits back into the air
before entering the water.

Ray B strikes the left side of the brick at point B, which is a known vertical
distance below the top of the brick.

Ray B enters the plastic from the air, then exits directly into the water.

Ray C enters the water before entering the plastic.

Ray C is travelling horizontally when it passes through point X.

The refraction indexes of the air, water and plastic are all known;

$n_{air} \approx 1 < n_{water} < n_{plastic}$

We want to calculate the distance d_x.

1/2 pt. each.

A total of 8 points possible here.

Data:

W	The width of the brick.
H	The height of the brick.
n_{air} ($\approx$ 1.00)	The index of refraction of the air (for this particular light frequency).
n_w	The index of refraction of the water (for this particular light frequency).
n_p	The index of refraction of brick's plastic (for this particular light frequency).
d_A	The vertical distance from the top of the brick to point A.
d_B	The vertical distance from the top of the brick to point B.

2 pts for the first correct item, then 1 pt. ea. for more.

8 points possible

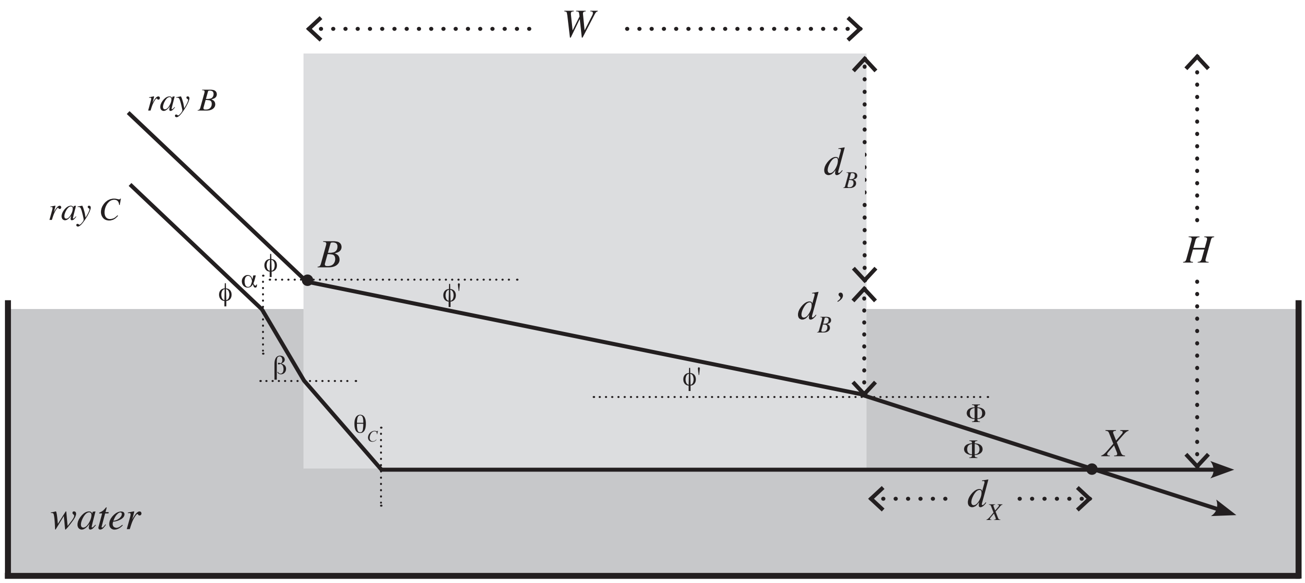
Assumptions:

Air	We assume that the air is uniform—free of impurities or anomalies.
Water	We assume that the water is uniform—free of impurities or anomalies.
Plastic	We assume that the plastic is uniform—free of impurities or anomalies.

4 pts for the first correct item, then 2 pts. ea. for more.

8 points possible

Visual Rep(s):



8 pts.

Note: You can solve for d_X using the information for ray C and for either ray A or B; as it turns out, there is *more than enough* information provided in the problem. The following solution uses the above diagram involving rays B and C. (The solution that uses C and A would be similar, but using B requires fewer steps.)

Equations:

I. $n_p \sin\theta_C = n_w \sin 90$

II. $n_p \sin(90 - \theta_C) = n_w \sin\beta$

III. $n_W \sin(90 - \beta) = n_{air} \sin\alpha$

IV. $\phi = 90 - \alpha$

V. $n_{air} \sin\phi = n_p \sin\phi'$

VI. $n_p \sin\phi' = n_w \sin\Phi$

VII. $\tan\phi' = d_B'/W$

VIII. $\tan\Phi = [H - (d_B + d_B')]/d_X$

1 pt. each.

A total of 8 points possible here.

Solving:

Solve **I** for θ_C. Substitute that result into **II**.

Solve **II** for β. Substitute that result into **III**.

Solve **III** for α. Substitute that result into **IV**.

Solve **IV** for ϕ. Substitute that result into **V**.

Solve **V** for ϕ'. Substitute that result into **VI** and **VII**.

Solve **VI** for Φ. Substitute that result into **VIII**.

Solve **VII** for d_B'. Substitute that result into **VIII**.

Solve **VIII** for d_X.

1 pt. each.

A total of 8 points possible here.

Testing: Dimensions: d_X should have dimensions of length.

Dependencies:

If *W* were greater, then with all other variables the same, this would imply a greater value for d_B', thus leaving less vertical distance available for ray B to travel to reach X. Since the refraction angles have not changed, this also implies less horizontal distance: $\boldsymbol{d_X}$ **would be shorter.**

If *H* were greater, then with all other variables the same, this would imply a greater depth at point X, so, since the refraction angles have not changed, this also implies greater horizontal distance: $\boldsymbol{d_X}$ **would be greater.**

If n_{air} **were greater**, then with all other variables the same, this would result in less refractive bending as ray B enters the plastic, so the ray's overall trajectory would be more downward (nearer to the original angle Φ, *which would also be steeper*), so this would cause $\boldsymbol{d_X}$ **to be shorter.**

If n_w **were greater**, then with all other variables the same, this would imply a lesser value for Φ, which would cause all rays to be more horizontal (and diminish the bending as ray B enters water), so this would cause $\boldsymbol{d_X}$ **to be longer.**

If n_p **were greater**, then with all other variables the same, this would imply a greater value for Φ, which would cause all rays to be more vertical (and increase the bending as ray B enters water), so this would cause $\boldsymbol{d_X}$ **to be shorter.**

If d_A **were greater**, then with all other variables the same, this would have **no effect** on the above solution for $\boldsymbol{d_X}$, which does not depend on any information about ray A.

If d_B **were greater**, then with all other variables the same, this would imply a shorter vertical distance from the point of ray B's entrance into the water to the bottom of the brick; Φ being the same, $\boldsymbol{d_X}$ **would be shorter.**

1 pt. each.

A total of 8 points possible here.