

HW 2

Recommended finish date: *Monday, April 14, 2014*

These HW problems were taken/adapted from past exam problems.
To get an idea of how best to approach various problem types (there are three basic types), refer to these [example HW problems](#).

1. Evaluate the following statements (T/F/N). *As always, you must explain your reasoning with a valid mix of words, diagrams and/or calculations.*
- a. The center of mass of an object is always located somewhere within the object (i.e. at a point occupied by part of the object).
 - b. If you double your distance from the rotation axis of the carousel you're riding on, you double your contribution to its total moment of inertia about that axis.
 - c. The units of moment of inertia could be expressed as $\text{N}\cdot\text{m}\cdot\text{s}^2$.
 - d. The moment of inertia of a solid sphere about its center is the same on the moon as on the earth.
 - e. Moment of inertia is a scalar quantity.
 - f. The moment of inertia, I , of a rigid object depends on its angular acceleration as it rotates.
 - g. The moment of inertia, I , of a rigid object varies according to the location of the rotation axis relative to the particles that form the object.
 - h. The moment of inertia, I , of a rigid object varies according to the orientation of the rotation axis relative to the particles that form the object.

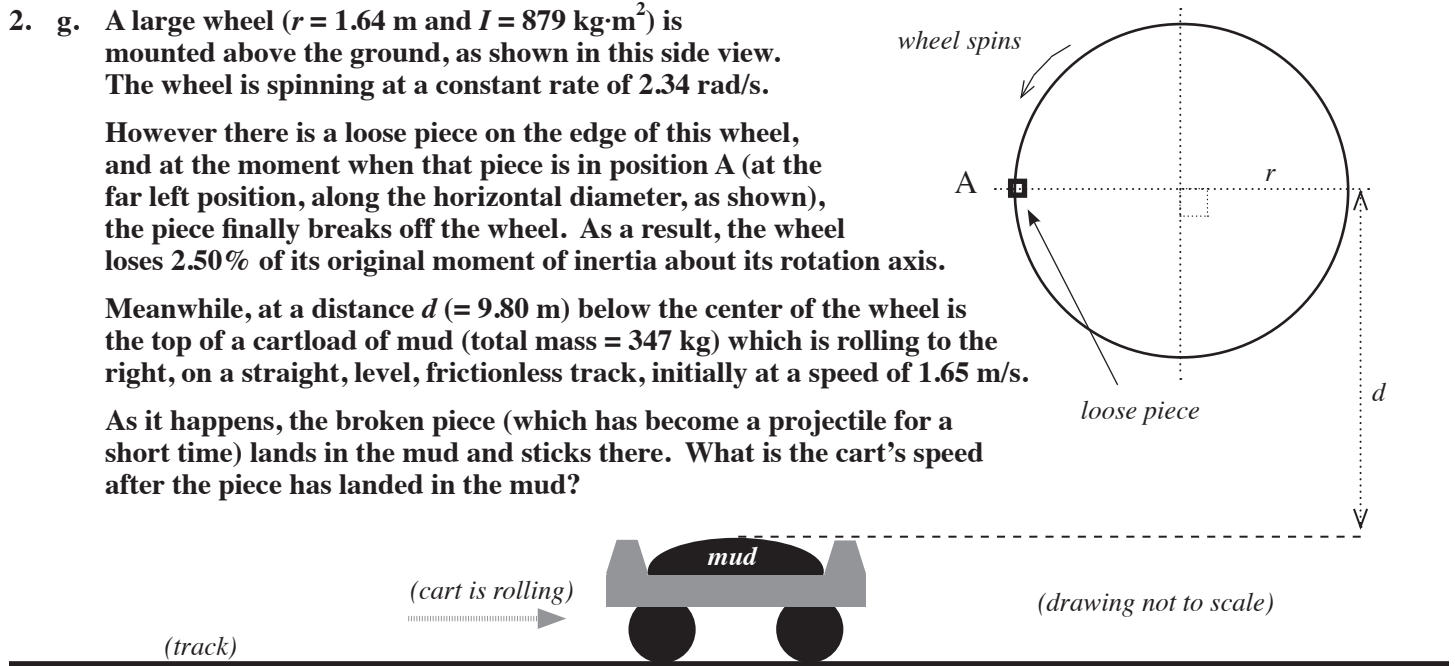
2. a. Moment of inertia is a measure of an object's _____.
- b. Exercise 16, page 348: A 25-kg, solid door is 220 cm tall and 91 cm wide. Find the moment of inertia for its rotation along its hinges (i.e. how a door normally swings); then for a vertical axis within the door 15 cm from one edge.
- c. A disk rotating about its center has a 1.00 kg mass attached 1.50 m from the axis. Where should you place a 2.50 kg mass so that you can remove the 1 kg mass without changing the total moment of inertia of the system?
- d. You're standing on a carousel 1.34 m from its axis of rotation. What minimum distance would you need to walk to arrive at a position where your contribution to the total moment of inertia has doubled?
- e. Suppose you have a thin, uniform rod with a total mass of 3.00 kg and a total length of 2.00 m. By what percentage does the moment of inertia decrease if you rotate it about a point located 20% (0.40 m) in from one end—rather than about the very end of the rod?
- f. A certain thin, uniform rod has a mass of 2 kg and length 3 m. To what shorter length must you trim it (discarding the smaller part) so that the remaining length has half the original moment of inertia about its end?

2. g. A large wheel ($r = 1.64 \text{ m}$ and $I = 879 \text{ kg}\cdot\text{m}^2$) is mounted above the ground, as shown in this side view. The wheel is spinning at a constant rate of 2.34 rad/s .

However there is a loose piece on the edge of this wheel, and at the moment when that piece is in position A (at the far left position, along the horizontal diameter, as shown), the piece finally breaks off the wheel. As a result, the wheel loses 2.50% of its original moment of inertia about its rotation axis.

Meanwhile, at a distance $d (= 9.80 \text{ m})$ below the center of the wheel is the top of a cartload of mud (total mass = 347 kg) which is rolling to the right, on a straight, level, frictionless track, initially at a speed of 1.65 m/s .

As it happens, the broken piece (which has become a projectile for a short time) lands in the mud and sticks there. What is the cart's speed after the piece has landed in the mud?



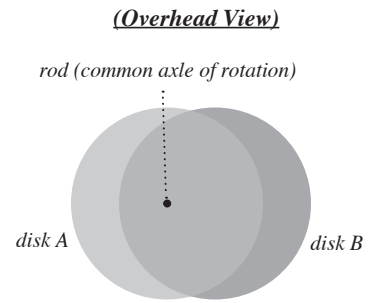
3. Evaluate the following statements (T/F/N). *As always, you must explain your reasoning with a valid mix of words, diagrams and/or calculations.*
- a. ΔL is a vector quantity.
 - b. If a spinning skater draws in her arms, she increases her angular momentum.
 - c. An isolated rotating object (constant m) can change its own angular velocity.
 - d. If one object's angular momentum is changing, there must be another object whose angular momentum is changing.
 - e. An object of constant mass can change its own moment of inertia.
 - f. An object of constant moment of inertia about an axis can change its own angular velocity about that axis.
 - g. The angular momentum of a collection of objects is always conserved.
 - h. An object's angular momentum may be changing even as its angular velocity remains constant.

4. a. Write the SI unit of angular momentum, using only fundamental (“base”) SI units:
- b. An ice skater is spinning in place on frictionless ice. Initially her angular velocity is 10.0 rad/s , but then she extends her arms, increasing her moment of inertia by 12.5% . What is her resulting angular velocity?
- c. A playground carousel (“merry-go-round”) is free to rotate frictionlessly in the horizontal plane (and air resistance is negligible). Without riders, the carousel has a moment of inertia of $152 \text{ kg}\cdot\text{m}^2$. But there is a single rider, initially standing 1.85 m from the axis of rotation, as the carousel turns at an angular speed of 0.640 rad/s . Then the person moves to another location, 0.75 m from the axis, and the angular speed is then 0.973 rad/s . Find the person’s mass.

5. a. You're sitting in a chair that can rotate freely about a vertical axis. The moment of inertia of you and the chair (together as one object) about that axis is I_C . Initially, you and the chair are at rest. But you're holding a spinning disk whose axis of rotation aligns with the axis of the chair. The disk's moment of inertia about its axis is I_D . The disk's initial angular velocity is ω_D . You then use your free hand to stop the disk from spinning axis. As a result, you, the chair and the disk all rotate together with angular velocity ω_f . If $\omega_D/\omega_f = 10.0$, what is I_C/I_D ?
- b. You're sitting in a chair that can rotate freely about a vertical axis. The moment of inertia of you and the chair (together as one object) about that axis is I_C . Initially, you and the chair are at rest. But you're holding a disk whose axis of rotation aligns with the axis of the chair. The disk's moment of inertia about its axis is I_D . The disk is initially at rest. You then use your free hand to start the disk spinning on its axis at a velocity of ω_D . As a result, you and the chair also begin to rotate together, with an angular velocity ω_C . If $\omega_D/\omega_C = x$, what is I_D/I_C ?
- c. You're sitting in a chair that is free to rotate horizontally, but it's initially at rest. With one hand, you're holding the axle of a wheel so that the axle is aligned with the axis of your chair. Initially, the wheel is spinning with an angular velocity of 19.7 rad/s. Then you use your other hand to slow the wheel to 10.2 rad/s. As a result, you and the chair now have an angular velocity of 0.321 rad/s. (All velocities are measured with respect to the floor.)
- Assuming the wheel is a solid disk of mass 8.90 kg and an outer radius of 0.345 m, what is the moment of inertia of you and chair (together, as one object)?
 - If the same wheel as in part (i) were instead just spinning initially on a fixed axis set on the floor (without you and the chair), what mass would you then place on its outer rim to achieve the same slowing (19.7 rad/s to 10.2 rad/s)?

6. a. A flat, uniform, circular disk ($R = 2.25$ m, $m = 84.0$ kg) is initially at rest, but it is free to rotate in the horizontal plane about a vertical, frictionless axis through its center. A 45.0-kg person, standing 1.5 m from the axis, begins to run on the disk in a clockwise circular path, with a tangential speed of 2.00 m/s (measured with respect to the stationary ground—not the disk). Find the resulting angular velocity of the disk.
- b. Two uniform, solid disks are rotating freely about the same axis. Then they are linked together without the aid of any external influences. Before being connected, disk 1 had an angular velocity of 4.60 rad/s; disk 2 had an angular velocity of -8.50 rad/s. Both disks have the same mass (2.00 kg), but disk 1 has a radius of 12.0 cm, and disk 2 has a radius of 8.00 cm. At what angular velocity does the set of disks rotate after being connected?

6. c. Two identical solid disks are rotating independently about the same thin, frictionless rod. That rod goes through the center of disk A, but in disk B, the rod goes through a point midway between that disk's center and its outer edge (see overhead view here). Initially, the disks are rotating about the rod with the same angular speed but in opposite directions. Then they collide (without outside interference or change in height) and they stick together as a result. *Evaluate (T/F/N) each statement. Justify your answers fully with any valid mix of words, drawings and calculations.*



(i) Disk B reverses its direction of rotation as a result of the collision.

(ii) As a result of the collision, $\Delta L_A = -\Delta L_B$.

6. d. An old firework shell from last year's July 4th show was launched but failed to explode. Instead, it fell and came to rest, hanging by a thread from a tree branch above a school playground. Its mass is 7.98 kg. Now (many months later), it finally explodes—into three pieces.

Immediately after the explosion:

Piece A ($m_A = 1.30$ kg) is moving horizontally eastward at 32.4 m/s.

Piece B ($m_B = 0.65$ kg) is moving horizontally westward at 64.8 m/s.

All three pieces land eventually somewhere on the playground. As it happens, Piece C lands on (and sticks to) the surface of the merry-go-round ($I = 120 \text{ kg}\cdot\text{m}^2$), at a distance $r = 1.30$ m from its center hub, as shown here.

The surface of the merry-go-round is 7.19 m lower in elevation than the position of the shell when it was hanging from the tree.

Just before piece C lands on it, the merry-go-round is rotating at a rate of 2.50 rad/s. At what rate is it rotating just after piece C lands on it?

