# HW 3

*Due: Monday, January 27, 2014, 5:00 p.m.*

***Print*** *your full* ***LAST*** *name:* ______________________________________________

***Print*** *your full* ***first*** *name:* ______________________________________________

***Print*** *your HW Grader's name:* ________________________________________

*What is your HW Grader's box # (located outside of Wngr 234)?* _________

"I affirm and attest that this HW assignment is my own work. While I may have had help from (and/or worked with) others, all the reasoning, solutions and results presented in final form here are my own doing—and expressed in my own words."

*Sign your name (full signature):* ________________________________________

***Print*** *today's date:* ____________________________________________________

# General Instructions for HW 3

Requirements: The PH 212 HW graders will select **one** out of the six problems. They will then score that problem according to the points rubric posted with the HW 3 solutions (for a total possible 10 points for the entire HW assignment).

Format: The formats (type, length, scope) of the HW problems have been purposely created to closely parallel those of a typical exam (indeed, most HW problems have been taken from past exams).

***You are required to include the completed, signed cover sheet—that's the first page of this file—as the front page of your HW submission when you turn it in.*** *(*Please staple together all pages, in order—cover sheet first—with one staple at upper left.)

Scoring: The rubrics (scoring guides—the points breakdowns) for HW problem solutions will closely match those for exam solutions. And in those rubrics, *when any item asks for an explanation (and most do), it means exactly what it says*. To get full credit, you must include a short but informative verbal explanation (in your own words) of your reasoning.

***To get an idea of how best to approach various problem types (there are three basic types), refer to these example HW problems.***

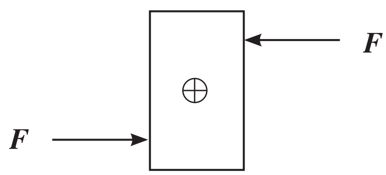
1. a. **Evaluate the following statements (T/F/N).** ***As always, you must explain your reasoning with a valid mix of words, diagrams and/or calculations.***

(i) **If a steady net torque $\tau$ (a known value) is applied to an object (initially at rest) about some fixed axis of rotation, for a known time interval, $\Delta t$, and its resulting angular speed is measured as $\omega$ at the end of that time interval, then this is enough information to calculate the object's moment of inertia about that axis of rotation.**

True. We can calculate $\alpha$ via $\omega_f = \omega_i + \alpha\Delta t$ (where $\omega_i = 0$); then we get $I$ via $\Sigma\tau = I\alpha$.

(ii) **For a moving object, it is possible that $\Sigma F = 0$ and $\Sigma\tau \neq 0$.**

True. For example:




(iii) **For a moving object, it is possible that $\Sigma F = 0$ and $\Sigma\tau = 0$.**

True. An object could be rotating and traveling (translating) through the universe with absolutely no net forces or torques on it. *Equilibrium does not necessarily mean static equilibrium.*

(iv) **It is possible that an object moving (translating) in a circle might have no forces acting on it.**

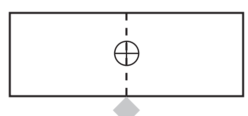
False. Only stright-line translation is possible without a net force to cause *a change of direction*.

(v) **A rotating object can be in total mechanical equilibrium.**

True. See item **(iii)** above.

(vi) **If the gravitational force on an object produces zero torque about a certain axis, the object's center of gravity is located on the same vertical line as that axis.**

True. $\tau = |\boldsymbol{F}_G|\cdot|\boldsymbol{l}_G|\cdot\sin\theta$, where $\theta$ is the angle between the direction of $\boldsymbol{F}_G$ and the direction of $\boldsymbol{l}_G$. So if $\tau = 0$, but if neither $\boldsymbol{F}_G$ nor $\boldsymbol{l}_G$ is zero in magnitude, *then* $\sin\theta$ *must be zero*; that is, $\boldsymbol{F}_G$ and $\boldsymbol{l}$ are collinear:



(vii) **Torque is a vector quantity.**

True. It has both a magnitude ($|\boldsymbol{F}|\cdot|\boldsymbol{l}|\cdot\sin\theta$) and direction (right-hand cross-product: ccw = +, conventionally).

(viii) **A radial force is a constant vector value.**

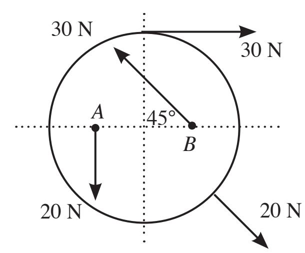
False. A radial force has a constantly changing direction, as the object turns on its circular path.

b. **Express torque in fundamental (base) SI units.** $kg\cdot m^2/s^2$

c. **You are facing a wheel initially at rest. A force is applied to the right edge of the wheel, accelerating the wheel in a clockwise direction. The direction of the torque vector here is** away from you—along the axis of rotation of the wheel—indicated by your right thumb when you curl the fingers of your right hand in the direction of angular acceleration.

d. **Exercise 20, page 349.**

**The disk shown here is 20 cm in diameter and is free to rotate on a frictionless axle at its center (perpendicular to the page). Points A and B are each 5.0 cm from the axle.**




**Find the net torque on the disk.**

$\Delta\tau$ = (20)(0.05)sin90° + (30)(0.05)sin135° + (20)(0.10)sin0° – (30(0.10)sin90°

= **–0.939 N·m**

**2. a. The second hand on a certain standard clock has a length of 12.0 cm, measured from the central hub to its tip. Normally, that second hand sweeps smoothly and steadily around one full revolution every minute (60 seconds). Suppose that it is indeed operating normally—until a certain moment, when it begins to speed up.…**

**(i) What angular acceleration would it need to have—at that first moment of speeding up—so that the tip of the second hand then has equal magnitudes of tangential and radial acceleration?**

At that first speeding up moment, the second hand is still moving at its normal angular velocity.
Normal $\omega = -2\pi/60 = -0.10472$ rad/s (negative because it's clockwise, of course)
$|a_R| = r|\omega|^2$ and $|a_T| = r|\alpha|$, so if these two magnitudes are to be equal, then: $r|\omega|^2 = r|\alpha|$
Thus: $|\alpha| = |\omega|^2 = 0.10472^2 = 1.0966 \times 10^{-2}$ rad/s$^2$
But $\alpha$ is speeding up $\omega$, so it's in the *same direction* as $\omega$. Therefore: $\alpha =$ **$-1.10 \times 10^{-2}$ rad/s$^2$**

**(ii) Assuming the behavior described in part (i), find the total acceleration magnitude of the tip at that first moment of speeding up.**

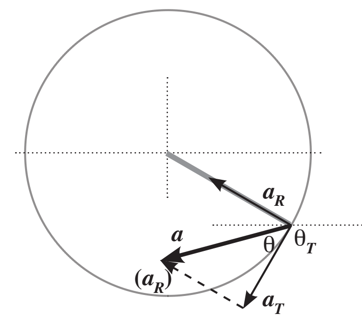
$|a| = \sqrt{[|a_R|^2 + |a_T|^2]} = \sqrt{[(r|\omega|^2)^2 + (r|\alpha|)^2]}$
$= \sqrt{\{[(0.120)(0.10472)^2]^2 + [(0.120)(1.0966 \times 10^{-2})]^2\}} =$ **$1.86 \times 10^{-3}$ m/s$^2$**

**(iii) Assuming the behavior described in part (i), if the tip of the second hand is located at the "4 o'clock" position at that first moment of speeding up, find the direction of its total acceleration, *a*, at that moment.**

The "4 o'clock" position is –30°. So the direction of $a_R$ is given by $\theta_R = -30 + 180 = 150°$

And $a_T$ is perpendicular to the position and in the direction of $\alpha$ (clockwise), so that's 90° more negative than the position.
So the direction of $a_T$ is given by $\theta_T = -30 - 90 = -120°$

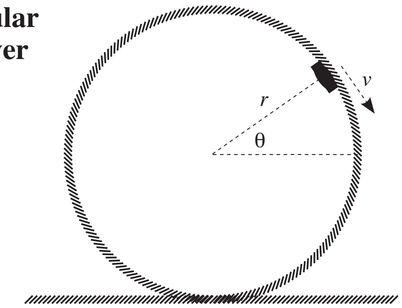
The acceleration vector triangle is a right triangle with equal legs (since $|a_R| = |a_T|$ ).
So the angle $\theta = 45°$, and $\theta_a = \theta_T - 45° = -120 - 45 =$ **–165°**

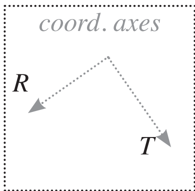


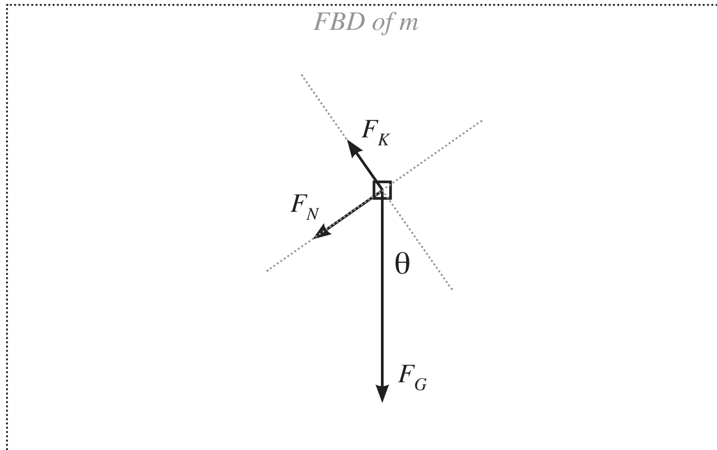
**2. b. At the moment depicted here, this roller coaster car of mass $m$ is at the angular position $\theta$, sliding clockwise at speed $v$, in a vertical "loop-the-loop" maneuver on this circular track of radius $r$, which has a kinetic friction coefficient $\mu_K$ with the car.**

**Find an expression for the net force (both magnitude and direction) acting on the car.**

**You may consider these values as <u>known</u>: $m, \theta, v, r, \mu_K, g$.**




*T-analysis*

$\Sigma F_T = ma_T$

$F_{G.T} - F_K = ma_T$

$mg\cos\theta - \mu_K F_N = ma_T$

*C-analysis*

$\Sigma F_R = ma_R$

$F_N + F_{G.R} = ma_R$

$F_N + mg\sin\theta = mv^2/r$

**I.** Find the magnitude of the net radial force, $F_{R.net}$, acting on the car: $F_{R.net} = mv^2/r$

**II.** Solve for the normal force, $F_N$, acting on the car: $F_N = (mv^2/r) - mg\sin\theta$

**III.** Find the net tangential force, $F_{T.net}$, acting on the car: $F_{T.net} = mg\cos\theta - \mu_K[(mv^2/r) - mg\sin\theta]$

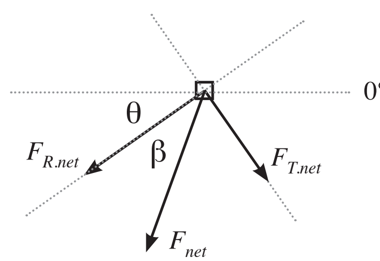
**IV.** Find $|F_{net}|$, the magnitude of the total net force acting on the car: $|F_{net}| = \sqrt{(|F_{R.net}|^2 + |F_{T.net}|^2)}$

$= \sqrt{\{(mv^2/r)^2 + \{mg\cos\theta - \mu_K[(mv^2/r) - mg\sin\theta]\}^2\}}$

**V.** Find the direction, $\theta_R$, of $\boldsymbol{F_{R.net}}$: $\theta_R = \theta + 180$

**VI.** Find the angle, $\beta$, between $\boldsymbol{F_{R.net}}$ and $\boldsymbol{F_{net}}$: $\tan\beta = |F_{T.net}|/|F_{R.net}|$

Or: $\beta = \tan^{-1}(\{[mg\cos\theta - \mu_K[(mv^2/r) - mg\sin\theta]\}/\{mv^2/r\})$




**VII.** Find the direction angle, $\theta_{net}$, of $\boldsymbol{F_{net}}$: $\theta_{net} = \theta_R + \beta$

$= \theta + 180 + \tan^{-1}(\{[mg\cos\theta - \mu_K[(mv^2/r) - mg\sin\theta]\}/\{mv^2/r\})$

**2. c. A record with a 10 cm radius and 200 g mass is dropped vertically (and without rotation) onto a turntable that is rotating but not being driven by a motor; rather, it is gradually slowing, due to a constant net torque of 0.1 N·m exerted by the friction in its hub. When the record lands on the turntable, the two objects stick together and rotate around the turntable's hub. The radius of the turntable is 10 cm; its mass is 2 kg; its angular velocity just before the record lands on it is 5 rad/s.**

**(i) What is earliest <u>common</u> angular speed achieved by the record and the turntable?**

Model both objects as solid, uniform disks:

Data: $\omega_{i.t} = 5$ $\quad I_r = (1/2)M_rR_r^2 = (1/2)(0.200)(0.10)^2 = 0.001$ $\quad I_t = (1/2)M_tR_t^2 = (1/2)(2)(0.10)^2 = 0.01$

This is a collision between two objects that can rotate around a common axis.

Use conservation of angular momentum: $\quad \boldsymbol{L}_{i.TOTAL} = \boldsymbol{L}_{f.TOTAL}$

In this case, two objects become one: $\quad I_r\omega_{i.r} + I_t\omega_{i.t} = (I_r + I_t)\omega_f$

And $\omega_{i.r} = 0$: $\quad I_t\omega_{i.t} = (I_r + I_t)\omega_f$

Solve for $\omega_f$: $\quad \omega_f = I_t\omega_{i.t}/(I_r + I_t)$

$\quad = (0.01)(5)/[.001 + .01] =$ **4.55 rad/s** $\quad$ *(4.54545…)*

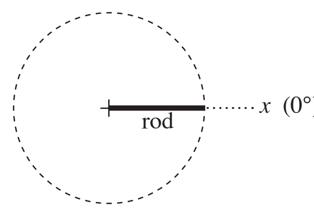
**(ii) How long will it take for the record and turntable to stop rotating altogether?**

Newton's 2nd Law: $\quad \alpha = \tau_{net}/I_{total}$

Kinematics: $\quad \Delta t = (\omega_f - \omega_i)/\alpha = I_{total}(\omega_f - \omega_i)/\tau_{net} = (.011)(0 - 4.54545)/(-0.1) =$ **0.500 s**

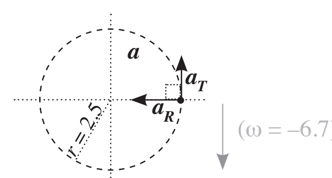
*[Note here: I supplied an unrealistic value for the torque—slipped by one decimal point, basically. The intended value was 0.01, which would have produced a 5-second stopping time for the record and turntable.]*

**d. A uniform rod (2.50 m long, 41.3 kg) is rotating in a horizontal circle about one end, as shown in this overhead view. When the rod is in the position shown, it is rotating at –6.7 rad/s, and a torque of 89.0 N·m is being applied to it. What is the direction of the net force being exerted on the outer tip of the rod at that instant?**




The net force is in the same direction as the total (net) acceleration, $\boldsymbol{a}$. And the total (net) acceleration is the <u>vector</u> sum of the tangential and centripetal acceleration components: $\quad \boldsymbol{a} = \boldsymbol{a}_T + \boldsymbol{a}_C$

The mass is <u>slowing</u>, so that means the tangential acceleration ($\boldsymbol{a}_T$) is in the opposite direction to the tangential velocity:




$\Sigma\tau = I\alpha$

$\Sigma\tau = 89$

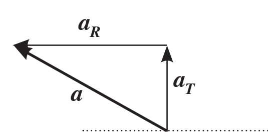
$I = (1/3)ML^2 = (1/3)(41.3)(2.5^2) = 86.042 \text{ kg·m}^2$

$\alpha = \Sigma\tau/I = 89/86.042 = 1.0344 \text{ rad/s}^2$

$|\boldsymbol{a}_T| = r|\alpha| = 2.5(1.0344) = 2.586 \text{ m/s}^2$

$|\boldsymbol{a}_R| = r|\omega|^2 = 2.5(6.7^2) = 112.225 \text{ m/s}^2$

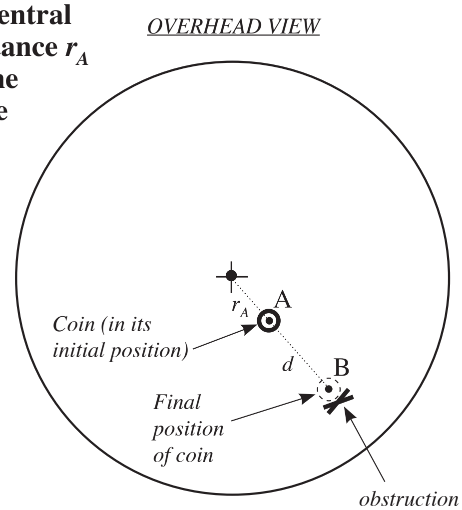
$\theta_a = \tan^{-1}(|\boldsymbol{a}_R|/|\boldsymbol{a}_T|) + 90° = 179°$



**2. e** **A horizontal turntable is a disk free to rotate frictionlessly around its central axis. Initially it is at rest, and a coin is resting on it at position A (a distance $r_A$ from the axis), as shown here. Then a constant torque $\tau$ is applied to the turntable (around its central axis) for a certain time interval, $\Delta t$. At the end of $\Delta t$, the torque ceases.**

**At that very same moment (the end of $\Delta t$), the coin slips and begins to slide across the surface of the turntable. It stops slipping at position B (a radial distance *d* from A), as shown, because it encounters an obstruction in the turntable surface.**

**The mass of the coin is *m*. The moment of inertia of the turntable (without the coin), around its central axis, is *I*. After the coin has arrived at position B (after it has stopped slipping), the friction force on the coin is half the maximum possible. Find the steady force magnitude exerted by the obstruction on the coin after it has arrived at position B (after it has stopped slipping).**




**You may consider these values as <u>known</u>: $r_A$, $\tau$, $\Delta t$, $d$, $m$, $I$, $g$.**

***This is an ODAVEST item—use the full seven-step problem-solving protocol—but keep in mind that*** *you're not being asked to actually solve for the final expression*. ***In fact, you're not being asked to do any math at all—*** *not even any algebra*. ***Rather, for the <u>S</u>olve step, you are to write a series of succinct instructions on <u>how</u> to solve this problem. Pretend that your instructions will be given to someone who knows math but not physics. And for the <u>T</u>est step, you should consider the situation and predict how the solution would change if the data were different—changing one data value at a time.***

<u>**O**bjective</u>: A horizontal turntable, with a known moment of inertia, is free to rotate on a frictionless central axle.

A coin of known mass is sitting at rest at a known location on the turntable (also initially at rest).

A steady known torque is applied to the turntable (about its central axle) for a known time interval.

At the end of that time interval, two events occur simultaneously: The torque ceases, and the coin slips from its initial position and begins to slide across the surface of the turntable.

The coin stops against an obstruction after sliding a known distance—measured with respect to the turntable—radially outward from the axle.

At that final position, the static friction force exerted on the coin by the turntable is half the maximum possible.

We need to find the magnitude of the force exerted on the coin by the obstruction.

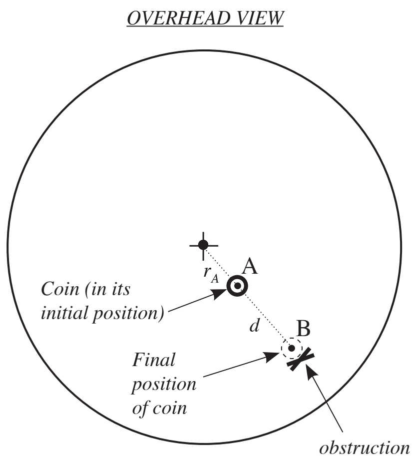
<u>**D**ata</u>:

| | |
|---|---|
| $r_A$ | The radial distance from the axle of the coin's initial position. |
| $\tau$ | The magnitude of the steady torque (about the axle) exerted on the turntable. |
| $\Delta t$ | The time interval during which the torque, $\tau$, is applied to the turntable. |
| $d$ | The distance across the surface of the carousel that the coin slides. |
| $m$ | The mass of the coin. |
| $I$ | The moment of inertia of the turntable (without the coin). |
| $g$ | The local value of free-fall acceleration. |

<u>**A**ssumptions</u>:

| | |
|---|---|
| *Surfaces* | We assume the surfaces of both the turntable and coin are parallel and uniform (except for the obstruction stipulated on the surface of the turntable), so that the coefficients of static and kinetic friction between these surfaces remains constant throughout this scenario. |
| *Alignment* | We assume that the coin remains motionless with respect to the turntable all during the time interval when the torque is being applied. |
| *Coin* | We model the coin as point mass, for the purposes of computing its contribution to the overall moment of inertia. |
| *Air* | We disregard any effects of wind or air drag. |
| *Gravity* | We assume the local value of *g* is constant throughout this scenario. |

Visual Rep(s):




Equations:

**I.** $I_A = I + mr_A^2$

**II.** $\tau = I_A\alpha$

**III.** $\omega_A = \omega_i + \alpha\Delta t$

**IV.** $F_s^{max} = m(r_A\omega_A^2)$

**V.** $I_B = I + m(r_A + d)^2$

**VI.** $I_A\omega_A = I_B\omega_B$

**VII.** $F_{obstr.} + F_s^{max}/2 = m[(r_A + d)\omega_B^2]$

Solving:

Solve **I** for $I_A$. Substitute that result into **II**.

Solve **II** for $\alpha$. Substitute that result into **III**.

Solve **III** for $\omega_A$ (noting that $\omega_i = 0$). Substitute that result into **VI**.

Solve **IV** for $F_s^{max}$. Substitute that result into **VII**.

Solve **V** for $I_B$. Substitute that result into **VI**.

Solve **VI** for $\omega_B$. Substitute that result into **VII**.

Solve **VII** for $F_{obstr.}$.

Testing:

Dimensions: $F_{obstr.}$ should have units of force (dimensions of mass·length/time$^2$).

Dependencies: If the coin starts farther from the axle (a greater $r_A$), then for the given torque, $\omega_A$ will be less, due to the larger moment of inertia. That implies a lesser value for $F_S^{max}$, but since $\omega_B$ would also be less, it's unclear without an explicit solution how all this would affect $F_{obstr.}$.

If the torque magnitude is greater, then $\omega_A$ will be greater, which would imply a stronger $F_S^{max}$. But since $\omega_B$ would also be greater, it's unclear without an explicit solution how all this would affect $F_{obstr.}$.

If the torque interval, $\Delta t$, is greater, then $\omega_A$ will be greater, which would imply a stronger $F_S^{max}$. But since $\omega_B$ would also be greater, it's unclear without an explicit solution how all this would affect $F_{obstr.}$.

If the coin's sliding distance, $d$ is greater, this will reduce $\omega_B$ and therefore the net centripetal force acting on the coin at point B. That would imply a <u>lesser</u> role necessary for $F_{obstr.}$.

If the coin's mass, $m$, were greater, then for the given torque, $\omega_A$ will be less, due to the larger moment of inertia. That implies a lesser value for $F_S^{max}$, but since $\omega_B$ would also be less, it's unclear without an explicit solution how all this would affect $F_{obstr.}$.

If the carousel has a greater moment of inertia, $I$, then for the given torque, $\omega_A$ will be less. That implies a lesser value for $F_S^{max}$, but since $\omega_B$ would also be less, it's unclear without an explicit solution how all this would affect $F_{obstr.}$.

If the local g value were greater, this would increase the value of $F_S^{max}$, so the obstruction force, $F_{obstr.}$, would play a lesser role at point B.

**3. a. Early peoples exploited the effects of Newton's laws when they discovered a fulcrum could be used for mechanical advantage. To lift a large 200-kg-boulder, a 3-m-long board is setup with a fulcrum. If a person weighing 750 N stands on the opposite end, where along the board (d) must the fulcrum be placed to lift the boulder. Assume the mass of the board is negligible and the maximum force the person can apply is simply his (resting) weight.**


$\Sigma\tau = I\alpha = 0$

$[(200)(9.80)(d)\sin 90°] - [(750)(3 - d)\sin 90°] = 0$

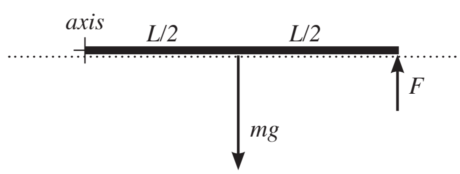
$1960d - (2250 - 750d) = 0$

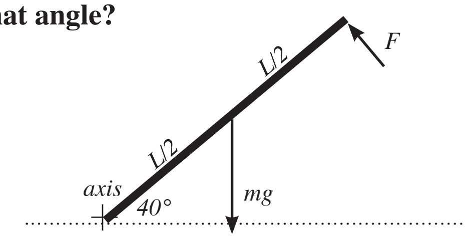
$2710d - 2250 = 0$

$d = 2250/2710 = \underline{\mathbf{0.830\ m}}$

(That's 83.0 cm from the end holding the boulder.)

**b. A long, narrow, heavy but uniform board rests on the ground. To just lift one end off the ground with a vertically-directed force while the other end stays on the ground, you effectively have to lift half the board's weight. You continue to lift the one end of the board until it makes a 40° angle with the ground. If your force at this point is perpendicular to the board itself, how much force (expressed as a fraction of the board's weight) must you now supply to hold the board at that angle?**




$\Sigma\tau = I\alpha = 0$

$(F)(L)\sin 90° - (mg)(L/2)\sin 90° = 0$

$FL - mgL/2 = 0$

$F = mg/2$

$\Sigma\tau = I\alpha = 0$

$(F)(L)\sin 90° - (mg)(L/2)\sin 50° = 0$

$FL - mg\cdot\sin 50°\cdot L/2 = 0$

$\boldsymbol{F = (mg\sin 50°)/2}$

(or: $\boldsymbol{F = (mg\cos 40°)/2}$)

**c. Exercise 30, page 349.**
**The 1.00-kg board is 2.00 m long. The 4.00-kg block is 1.00 m long. Find the distance *d* locating the fulcrum to balance the situation shown.**


The center of mass of the board is located 1.00 m from the left end; that's ($d$ – 1.00) m from the fulcrum.
The center of mass of the block is located 1.50 m from the left end; that's (1.50 – $d$) m from the fulcrum.

$\Sigma\tau = I\alpha = 0$

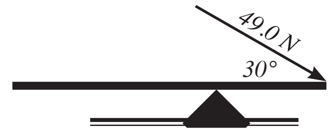
$[(1.00)(9.80)(d - 1.00)\sin 90°] - [(4.00)(9.80)(1.50 - d)\sin 90°] = 0$

$(9.80d - 9.80) - (58.8 - 39.2d) = 0$

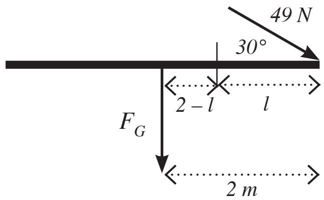
$49d - 68.6 = 0$

$d = 68.6/49 = \underline{\mathbf{1.40\ m}}$

**d. This uniform, 10-kg board is 4.00 m long and is at rest. Find the distance from its fulcrum to its right end.**



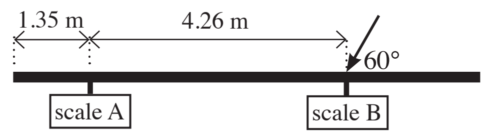

The board is uniform, so its center of gravity is the geometric center of the board (midway between ends). The fulcrum is located at an unknown distance $l$ from the board's right end. And the board is in (static) mechanical equilibrium: $\Sigma\tau = 0$
Sum the torques about the fulcrum axis:

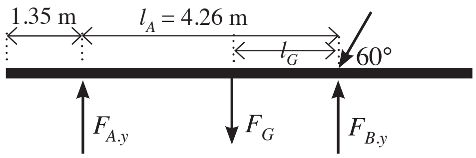



Solve for $l$:

$\Sigma\tau = I\alpha$
$F_G(2-l) - 49\sin 30°(l) = I\alpha$
$mg(2-l) - 49\sin 30°(l) = 0$
$2mg - lmg - 49\sin 30°(l) = 0$
$2mg = lmg + 49\sin 30°(l)$
$2mg = l(mg + 49\sin 30°)$
$2mg/(mg + 49\sin 30°) = l =$ **<u>1.60 m</u>**

**e. A uniform board has a mass of 35 kg and sits horizontally at rest, supported as shown, by two scales that <u>measure</u> vertical force only (but the scales can exert both vertical and horizontal forces). The force shown is 107 N and is applied directly over scale B. The reading on scale B is 294 N. How long is the entire board?**



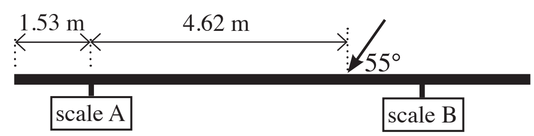



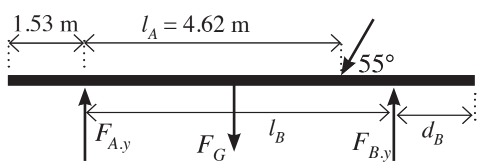

$\Sigma F_y = ma_y$
$F_{A.y} + F_{B.y} - F_y - F_G = ma_y$
$F_{A.y} + 294 - 107\sin\theta - mg = 0$
$F_{A.y} = 141.66$ N

Find center of board (i.e. cg): sum torques about the point above scale B.

$\Sigma\tau_B = I_B\alpha_B$
$F_G(\sin 90°)l_G - F_{A.y}(\sin 90°)l_A = I_B\alpha_B$
$mg(l_G) - 141.66(4.26) = 0$
$l_G = 1.759$ m
Board length: $L = 2(4.26 + 1.35 - l_G) =$ **<u>7.70 m</u>**

**f. A uniform beam, sitting at rest, has a mass of 53 kg and length of 8.6 m. It is supported as shown, by two scales that <u>measure</u> vertical force only (but the scales can exert both vertical and horizontal forces). The force shown is 170 N and is applied at the point shown. The reading on scale A is 294 N. How far is scale B from the right end?**



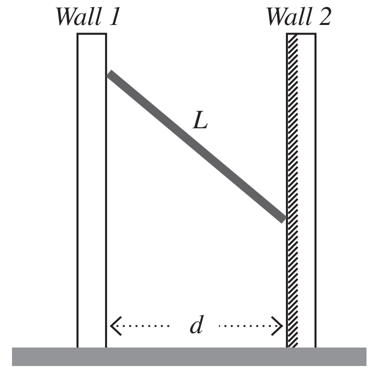




$\Sigma F_y = ma_y$
$F_{A.y} + F_{B.y} - F_y - F_G = ma_y$
$F_{B.y} + 294 - 170\sin\theta - mg = 0$
$F_{B.y} = 364.66$ N

Sum torques about $A$:

$\Sigma\tau_A = I_A\alpha_A$
$-F_G[(8.6/2) - 1.53] - 170(\sin 55°)(4.62) + F_{B.y}\cdot l_B = 0$
$-mg(2.77) - 170(\sin 55°)(4.62) + 364.66\cdot l_B = 0$
$l_B = 5.710$ m
$d_B = 8.6 - (l_B + 1.53) =$ **<u>1.36 m</u>**

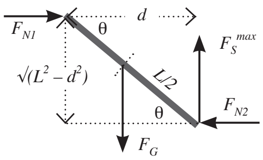
**g. A uniform board of mass *m* is propped between two parallel, vertical walls that are separated by a distance *d*, as shown here in this "edge-on" view. Wall 1 is frictionless, but Wall 2 has static friction (coefficient $\mu_S$) with the board. The board length, *L*, is the maximum length possible so that the board does not slip. Let $F_{N1}$ and $F_{N2}$ denote the normal forces exerted by each wall, respectively, on the board.**




***Evaluate (T/F/N) each statement. <u>Justify your answers fully</u> with any valid mix of words, drawings and calculations.***

**(i)** $\boldsymbol{F_{N1} = F_{N2}}$

<u>True</u>. FBD analysis of the board… <u>*x*-forces</u>:

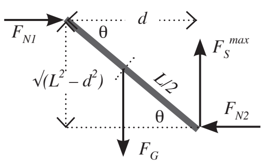



$$\Sigma F_x = ma_x$$
$$F_{N1} - F_{N2} = ma_x$$
$$F_{N1} - F_{N2} = 0$$

<u>Thus</u>: $F_{N1} = F_{N2}$

**(ii)** $\boldsymbol{\mu_S F_{N1} = mg}$

<u>True</u>. FBD analysis of the board… <u>*y*-forces</u>:

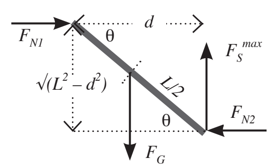



$$\Sigma F_y = ma_y$$
$$F_S^{max} - F_G = ma_y$$
$$\mu_S F_{N2} - mg = 0$$

<u>But</u>: $F_{N1} = F_{N2}$ *(from above)*
<u>So</u>: $\mu_S F_{N1} - mg = 0$
<u>So</u>: $\mu_S F_{N1} = mg$

**(iii)** $\boldsymbol{mgd/2 = (F_{N1})\sqrt{(L^2 - d^2)}}$

<u>True</u>. Sum the torques around the lower end of the board:




$$\Sigma\tau_B = I\alpha_B$$
$$(F_G)(L/2)\sin(90\text{-}\theta) - (F_{N1})(L)\sin\theta = I\alpha_B$$
$$(mg)(L/2)\cos\theta - (F_{N1})(L)\sin\theta = 0$$

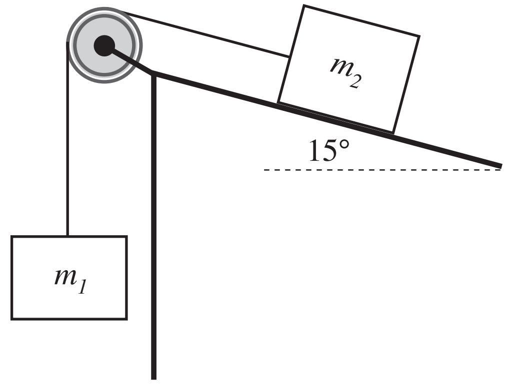
<u>But</u>: $(L/2)\cos\theta = d/2$
<u>And</u>: $(L)\sin\theta = \sqrt{(L^2 - d^2)}$
<u>So</u>: $mgd/2 - (F_{N1})\sqrt{(L^2 - d^2)} = 0$

**4. a. Blocks with masses $m_1 = 30.0$ kg and $m_2 = 40.0$ kg are connected by a string (with negligible mass) that passes over a pulley. The pulley is a solid disk that has a radius of 6.00 cm and a mass of 10.0 kg.**

**The string is taut at all times, and it does not slip as it passes over the pulley. The inclined ramp is frictionless.**

**Find the magnitude of the acceleration of $m_1$.**

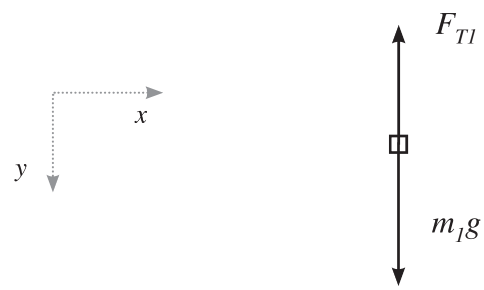
***Be sure to do a FBD and use Newton's 2nd Law for each of the three masses!***




Before starting the problem, notice: Since the string links all three objects, you should *choose the positive direction for each object's acceleration to correspond to the assumed acceleration of the string*. For example, if you assume that $m_1$ will be accelerating downward, choose that direction to be positive for $m_1$; and so the direction of positive acceleration for $m_2$ should be up the slope; and the direction of positive angular acceleration for the pulley ($M$) should be counter-clockwise.

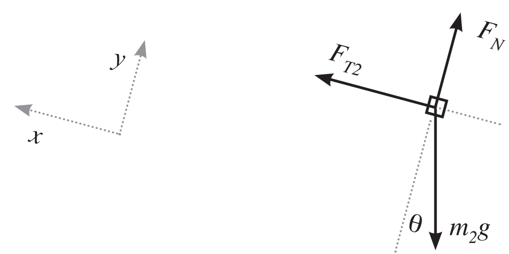
For $m_1$: $\Sigma F_y = m_1 a_1$

$m_1 g - F_{T1} = m_1 a_1$




For $m_2$: $\Sigma F_x = m_2 a_2$

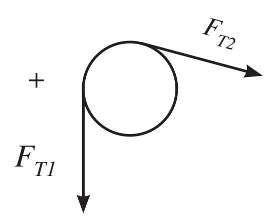
$F_{T2} - m_2 g\sin(15°) = m_2 a_2$




For $M$: $\Sigma\tau = I\alpha$ (but $\alpha = a_T/R$)

$F_{T1}(R) - F_{T2}(R) = (MR^2/2)(a_T/R) = (MRa_T/2)$

*Or:* $F_{T1} - F_{T2} = Ma_T/2$




Now, because you chose the axes carefully, $a_1 = a_2 = a_T = a$.
If you substitute this fact into the above three equations, you'll have three equations (shown at right), ready to solve for $a$:

$m_1 g - F_{T1} = m_1 a$
$F_{T2} - m_2 g\sin(15°) = m_2 a$
$F_{T1} - F_{T2} = Ma/2$

Solve the first two equations for $F_{T1}$ and $F_{T2}$, respectively, (shown at right), and substitute those results into the third equation.

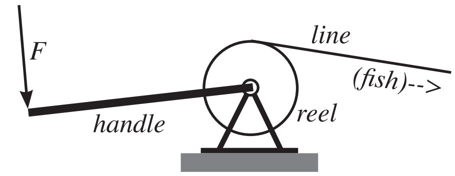
$F_{T1} = m_1 g - m_1 a$
$F_{T2} = m_2 a + m_2 g\sin(15°)$

$m_1 g - m_1 a - [m_2 a + m_2 g\sin(15°)] = Ma/2$

Solve for $a$: $a = g[m_2\sin(15°) - m_1]/(-m_1 - m_2 - M/2)$

$= (9.8)[(40)\sin(15°) - (30)]/(-30 - 40 - 10/2) =$ **2.57 m/s$^2$**

**4. b. A fish is hooked on a fishing line and is pulling on it with a constant force, trying to escape. The line is wound around a reel, which is a solid disk (r = 0.213 m; m = 654 kg). At first, the reel is held motionless by the fisherman, who exerts a perpendicular force, *F*, of 98.7 N on the 1.02 m (massless) handle attached to the hub of the reel, as shown. Suddenly, the entire handle breaks off of the reel, allowing it to spin freely in response to the fish's pull.**
**Assuming the fish maintains the same steady tension in the line as it swims away, how far will it have swum (directly away from the reel) when the reel's angular speed is 54.3 rad/sec?**




This problem has two parts: before and after the handle breaks. In each part, you analyze the reel as the free body. Notice that at all times, the reel is in translational equilibrium—it's not accelerating up/down or sideways. But only in the first part is the reel is in rotational equilibrium; after the handle breaks, the tension in the fishing line (which is then the only force causing any torque about the reel's axis—all other forces go through that axis). This causes an angular acceleration of the reel.

Strategy: Use the first part to determine the tension, $F_T$, caused by the fish. Then use that tension (maintained constantly by the fish after the handle breaks) to find the resulting angular acceleration, $\alpha$. Finally, use that con stant $\alpha$ value in a kinematics calculation to find out how much line has been paid out as the fish swims away.

Part 1: $\Sigma\tau = 0$

$F \cdot d_{handle}\sin(90°) - F_T R\sin(90°) = 0$ (where R is the radius of the reel). Solve this for $F_T$

$F_T = (F \cdot d_{handle})/R$

Part 2: $\Sigma\tau = I\alpha$

$-F_T R = (MR^2/2)\alpha$ Solve this for $\alpha$....

$\alpha = -2F_T/(MR)$ (Keep just the magnitude—let the outbound direction be positive.)

Substitute from Part 1: $\alpha = 2(F \cdot d_{handle})/(MR^2)$

Now do kinematics. You know 3 values: $\omega_0$ (= 0), $\omega$ (= 54.3) and $\alpha$ (just solved for).

Use $\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$, rearranged to solve for $\Delta\theta$: $\Delta\theta = (\omega^2 - \omega_0^2)/2\alpha$

And the amount of line paid out is just $\Delta s = R\Delta\theta = R(\omega^2 - \omega_0^2)/2\alpha$.

Substitute for $\alpha$:

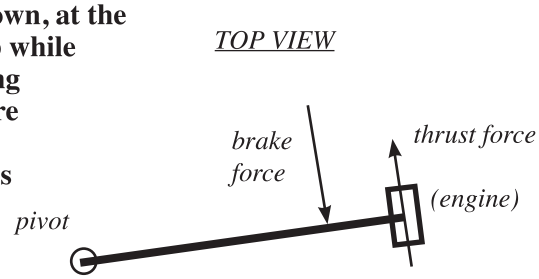
$$\Delta s = R(\omega^2 - \omega_0^2)/\{2[2(F \cdot d_{handle})/(MR^2)]\}$$
$$= MR^3(\omega^2 - \omega_0^2)/[4(F \cdot d_{handle})]$$
$$= (654)(.213)^3[(54.3)^2 - (0)^2]/[4(98.7)(1.02)] = \mathbf{46.3\ m}$$

**c.** ***This is an ODAVEST item—use the full seven-step problem-solving protocol—but keep in mind that you're not being asked to actually solve for the final expression. In fact, you're not being asked to do any math at all—not even any algebra. Rather, for the S̲olve step, you are to write a series of succinct instructions on how to solve this problem. Pretend that your instructions will be given to someone who knows math but not physics. And for the T̲est step, you should consider the situation and predict how the solution would change if the data were different—changing one data value at a time.***

**A jet engine (mass $m$) that produces a constant thrust force is mounted for testing on the end of a pivoting arm (length = $L$; moment of inertia = $I_{arm}$), so that the engine pushes itself around in a horizontal circle with its thrust force, which is perpendicular to the pivot arm, as shown here.**

**At first, there is a perpendicular braking force applied as shown, at the 3/4 position on the pivot arm, to allow the engine to warm up while still at rest. But when the engine has reached its full operating thrust, the brake is released, and the pivot arm and engine are allowed to rotate freely (with no friction or air resistance). If the pivot and engine turn $n$ revolutions in the first $t$ seconds of motion, what was the force applied by the brake?**

***Assume these are known values: $m$, $L$, $I_{arm}$, $n$, $t$***




Objective: A jet engine of known mass is being tested by attaching it to a horizontal pivot arm of known length and moment of inertia.

The arm is held stationary by a brake, set 3/4 of the distance from the pivot to the engine, until the engine reaches its constant full thrust.

The brake is then released so that the engine can rotate the arm freely for a known time interval.

During that time interval, the arm rotates a known number of revolutions.

We must find the force exerted by the brake just before it was released.

Data:

- $m$ The mass of the jet engine.
- $L$ The length of the pivot arm.
- $I_{arm}$ The moment of inertia of the pivot arm, measured at its end (i.e. at the pivot point).
- $t$ The time interval the engine was freely pivoting the arm while at full thrust.
- $n$ The number of revolutions the arm and engine made during time interval $t$.

Assumptions:

*Surfaces* We assume the pivot bearing is frictionless.

*Alignment* We assume that the pivot arm is exactly horizontal at all times—no bouncing or vibration.

We also assume that the brake force is exactly horizontal and exactly perpendicular to the pivot arm.

We also assume that the engine thrust force is exactly horizontal and exactly perpendicular to the pivot arm.

*Arm* We assume the pivot arm is uniform and that it pivots exactly at one end.

*Engine* We model the engine as point mass, mounted at the exact (other) end of the arm.

*Air* We disregard any effects of wind or air drag.

Visual Rep(s):

Equations:

**I.** $\Delta\theta = 2\pi n$

**II.** $\Delta\theta = \omega_0(t) + (1/2)\alpha(t)^2$

**III.** $I_T = I_{arm} + mL^2$

**IV.** $\tau_{engine} = I_T\alpha$

**V.** $\tau_{engine} - (F_{brake}\cdot\sin 90°)(0.75L) = 0$

Solving: Solve **I** for $\Delta\theta$. Substitute that result into **II**.

Solve **II** for $\alpha$ (noting that $\omega_0 = 0$). Substitute that result into **IV**.

Solve **III** for $I_T$. Substitute that result into **IV**.

Solve **IV** for $\tau_{engine}$. Substitute that result into **V**.

Solve **V** for $F_{brake}$.

Testing: Dimensions: $F_{brake}$, should have dimensions of force (mass·length/time$^2$).

Dependencies: If the mass, $m$, of the engine were greater, that would imply a greater thrust capability overcoming more total moment of inertia in order to achieve the same number of revolutions in the same time. Thus, the braking force, $F_{brake}$, would be need to be greater.

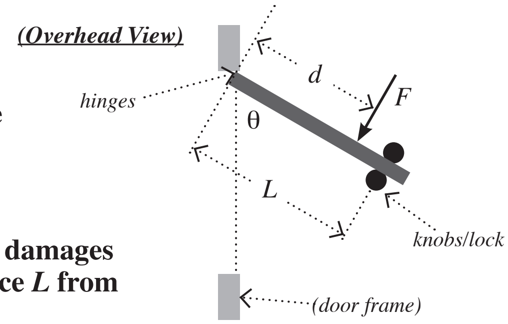
If the length, $L$, of the arm were greater, that would imply a greater thrust capability overcoming more total moment of inertia in order to achieve the same number of revolutions in the same time. Thus, the braking force, $F_{brake}$, would be need to be greater.

If the moment of inertia, $I_{arm}$, were greater, that would imply a greater thrust capability overcoming more total moment of inertia in order to achieve the same number of revolutions in the same time. Thus, the braking force, $F_{brake}$, would be need to be greater.

If the time, $t$, spent rotating under full thrust were greater, that would imply a smaller thrust capability used to accomplish the same $n$ revolutions. Thus, the necessary braking force, $F_{brake}$, would also be smaller.

If the number, $n$, of revolutions accomplished while rotating under full thrust were greater, that would imply a greater thrust capability in order to achieve this in the same time. Thus, the braking force, $F_{brake}$, would be need to be greater.

**d. A small child amuses himself by repeatedly slamming his bedroom door, which has a total moment of inertia (about its hinged edge) of *I*. Each time, he starts with the door at rest—open at the same angle θ, as shown—then exerts the same steady push *F* at the same point, a distance *d* from the door's hinged edge (and he follows the swinging door, so his push is always at right angles to the door) all the way until it slams shut.**




**Of course, this delights him but annoys everyone else—and it soon damages the door. So now the knobs/lock set (of mass *m*, located at a distance *L* from from the hinged edge), must be removed from the door.**

**(i) *How far from the hinged edge* must the child now push on the door (again, always perpendicularly) with the same force *F* as before (again, starting with the door at rest at angle θ) to get the same impact speed)?**

**(ii) <u>Comparing the two scenarios</u> (whole door vs. door without the knobs/lock): What is the difference in the speed of the child's hand (which is still pushing on the door) at the moment of impact?**

**Ignore air drag and hinge friction.**

**Here are the data: $F = 20.0$ N; $I = 5.00$ kg·m$^2$; $\theta = 60°$; $d = 65.0$ cm; $m = 2.00$ kg; $L = 80.0$ cm.**

**(i)** If the kid wants the same impact speed ($\omega_f$) in either scenario (call them 1 and 2), and in both cases he starts the door at rest ($\omega_i = 0$) and swings it through the same angular displacement ($\Delta\theta$), then by simple kinematics ($\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$), he needs to accomplish the same angular acceleration: $\alpha_1 = \alpha_2$

But for the door as a rotating object, $\tau_{net} = I\alpha$. Therefore: $\tau_{net.1}/I_1 = \tau_{net.2}/I_2$

Since the child's pushing force, $F$, is the same in either case: $(F)(d)\sin 90°/I_1 = (F)(x)\sin 90°/I_2$

Simplifying: $x = d(I_2/I_1)$

But: $I_2 = I_1 - mL^2$

Therefore: $x = d\cdot[(I_1 - mL^2)/I_1] = (0.65)[5 - (2)(0.80)^2)/5] =$ **0.484 m**

**(ii)** At impact, the speed of the child's hand is given by $v_{T.f} = r\omega_f$.

So now calculate $\omega_f$, using the data from the starting scenario (and the resulting $\omega_f$ must be the same in either case, as specified above): $\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$, where $\alpha = Fd/I_1$

Therefore: $\omega_f = \sqrt{[2Fd\Delta\theta/I_1]} = \sqrt{[(2)(20)(0.65)(\pi/3)/5]} = 2.334$ rad/s

And now find the difference between the two impact speeds:

$\Delta v_T = v_{T.f.2} - v_{T.f.1} = x\omega_f - d\omega_f = (0.4836)(2.334) - (0.650)(2.334) =$ **–0.388 m/s**

**When slamming the door without the knobs/lock assembly, the child's hands are moving <u>0.388 m/s more slowly</u> at impact.**

**5. a. *Evaluate (T/F/N) this statement (and <u>justify your answers fully</u> with any valid mix of words, drawings and calculations):* An object rotating at speed ω about its center of mass has less rotational kinetic energy than if it's rotating at that same ω about any other parallel axis.**

<u>True</u>. $K_R$ is calculated as $(1/2)I\omega^2$, and $I_{c.m.} < I_{axis}$ for any other parallel axis (evident by inspection of the Parallel Axis Theorem).

**b. A rotating skater draws her arms in, changing her moment of inertia by –8.50% (i.e. <u>reducing</u> it by 8.50%).**

**(i) By what percentage does she change her rotational kinetic energy, $K_R$?**

**(ii) Where does this extra energy come from?**

**(i)** First, use conservation of angular momentum ($\boldsymbol{L_i} = \boldsymbol{L_f}$): $I_i\omega_i = I_f\omega_f$ where $I_f/I_i = 0.915$

Thus: $\omega_i = 0.915(\omega_f)$ or: $(\omega_f/\omega_i) = 1/0.915$

Then note: $\Delta\%(K_R) = [(K_{R.f} - K_{R.i})/K_{R.i}]\cdot 100 = [(K_{R.f}/K_{R.i}) - 1]\cdot 100$

In this case: $K_{R.f}/K_{R.i} = [(1/2)I_f\omega_f^2]/[(1/2)I_i\omega_i^2]$

$= (I_f/I_i)(\omega_f/\omega_i)^2 \quad = (0.915)(1/0.915)^2 = 1/0.915$

Therefore: $\Delta\%(K_R) = [(1/0.915) - 1]\cdot 100$ **<u>Answer</u>: +9.29%**

**(ii) The skater uses chemical energy from her muscles to do work by drawing her arms in.**

**c. In a test laboratory, an electric engine is used to accelerate a wheel from rest. The center of mass of the wheel remains stationary at all times. The wheel's angular acceleration is in the counter-clockwise direction. It rotates through an angular distance Δθ, achieves a final angular speed ω and has a constant moment of inertia *I*. If you assume constant angular acceleration and ignore friction and air resistance, what is the average power output of the engine, expressed in terms of Δθ, ω and/or *I*?**

$$P_{mech.avg} = W/\Delta t$$

There is no elastic source here, nor any change in altitude by the center of mass—nor any translational kinetic energy (since the c.m. is entirely stationary). So the work-energy equation for the wheel simplifies to this:

$K_{R.i} + W_{ext} = K_{R.f}$ And since the wheel starts from rest: $W_{ext} = K_{R.f}$

So our calculation of the average power is: $P_{mech.avg} = K_{R.f}/\Delta t = (1/2)I\omega^2/\Delta t = I\omega^2/(2\cdot\Delta t)$

A bit of kinematics reveals $\Delta t$: $\Delta\theta = (1/2)(\omega_i + \omega_f)\cdot\Delta t$

In this case, that reduces to: $\Delta\theta = (1/2)(\omega)\cdot\Delta t$

Solving for $\Delta t$: $\Delta t = 2\Delta\theta/\omega$

Substituting into the above: $P_{mech.avg} = I\omega^2/(2\cdot\Delta t) = I\omega^3/(4\cdot\Delta\theta)$

**d. One end of a thin, uniform rod, 1.40 m in length, is attached to a pivot. The rod is free to rotate about the pivot without friction or air resistance. Initially, it is hanging straight down (i.e. in the "6 o'clock" position). Then the lower tip of the rod is given an initial horizontal speed $v_{T.i}$, so that the rod rotates upward, about its pivot. Find the value of $v_{T.i}$ so that the rod just reaches the "12 o'clock" position (i.e. straight up), coming to a momentary halt there.**

Modeling this as a rod that is freely rotating about the fixed, frictionless axis at its end, only gravity does work on it, so $W_{ext} = 0$. Therefore (letting the initial position of the center of mass be $h_i = 0$, so that $h_f = L$, we have:

$U_{G.f} = K_{R.end.i}$ or: $MgL = (1/2)I_{pin}\omega^2$

$MgL = (1/6)ML^2\omega^2$ or: $MgL = (1/6)ML^2(v_i/L)^2$

Thus: $6gL = v_i^2$ And: $v_i = \sqrt{[6gL]} = \sqrt{[6(9.80)(1.40)]} =$ **9.07 m/s**

**e. A dead tree is initially stationary and there is a 30.0-degree angle between the tree and the vertical. The roots finally give way and the tree topples to the ground (rotating on an axis that is parallel to the level ground and intersecting the base of the tree perpendicularly through the base of the tree. The length of the tree is 30.0 m. How fast is the top of the tree moving (in m/s) just before it hits the ground? Model the tree as straight, rigid rod, and ignore friction and air resistance.**

Modeling this as a rod that is free-falling (rotating) from rest about a frictionless axis (the root ball), only gravity does work on the tree, so $W_{ext} = 0$. Therefore (letting $h_f = 0$ and noting that the center of mass of the tree is half-way along its leaning length), we have:

$U_{G.i} = K_{R.end.f}$ or: $Mg(L/2)\sin 60° = (1/2)I_{pin}\omega^2$

$Mg(L/2)\sin 60° = (1/6)ML^2\omega^2$ or: $Mg(L/2)\sin 60° = (1/6)ML^2(v_{top}/L)^2$

Thus: $3gL\sin 60° = v_{top}^2$ And: $v_{top} = \sqrt{[3gL\sin 60°]} = \sqrt{[3(9.80)(30)(\sin 60°)]} =$ **27.6 m/s**

**5. f. A merry-go-round (carousel) with a moment of inertia of 594 kg·m$^2$, is initially rotating horizontally at 2 rad/s about a frictionless axis through its center. Then a metal plate located at that center loosens and gradually slides outward along the floor of the merry-go-round. When the plate is 1.4 m from the center, the merry-go-round has lost 1% of its original rotational speed. Assuming that all mechanical energy losses are due to kinetic friction between the plate and floor, find that coefficient of kinetic friction. (Ignore the slow speed of the plate's sliding.)**

Find the reduced angular speed: $\omega_f = 0.99\omega_i = 0.99(2) = 1.98$ rad/s

Angular momentum is conserved: $I_i\omega_i = I_f\omega_f$

Or: $I_i/I_f = \omega_f/\omega_i = 0.99$

Or: $I_i/0.99 = I_f = 594/0.99 = 600$ kg·m$^2$

So the plate has contributed an extra 6 kg·m$^2$ in its new position (modeled as a point mass): $mr^2 = 6$

The mass of the plate is thus given by: $mr^2 = 6/r^2 = 6/(1.40)^2 = 3.061$ kg

The carousel floor is level and no other vertical forces act on the plate.

Therefore: $F_N = F_G = mg$

Now start with the basic equation: $E_{mech.f} = E_{mech.i} + W_{ext}$

There is no spring present, and gravity is not an energy factor.

The external work is by the floor on the plate: $W_{ext} = |\mathbf{F_k}|\cos\theta|\mathbf{d}| = -F_k(d) = -\mu_k F_N(d) = -\mu_k mgd$

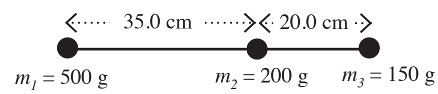
Include non-zero energy terms only: $K_{R.f} = K_{R.i} + W_{ext}$

The details: $(1/2)I_f\omega_f^2 = (1/2)I_i\omega_i^2 - \mu_k mgd$

Solve for $\mu_k$: $\mu_k mgd = (1/2)I_i\omega_i^2 - (1/2)I_f\omega_f^2$

$\mu_k = [(1/2)I_i\omega_i^2 - (1/2)I_f\omega_f^2]/(mgd) =$ **<u>0.283</u>**

**g. A rigid, massless rod connects three point masses as shown here. Doing a total of 192 J of work on this object (which is initially at rest in your hand), you throw it across a level field, launching it so that its center of mass leaves your hand at an angle of 34.0° above the horizontal. At the peak of its arc as a projectile, the object's center of mass has gained 7.00 m in altitude (i.e. above its altitude when released).**




**(i) Assuming that this projectile maintained its rotational speed throughout the upward portion of its arc, what was that ω value?**

**Now suppose that, during the downward portion of the object's arc, $m_2$ slides 10.0 cm toward $m_3$.**

**(ii) What is the rotational speed now—with $m_2$ in its new position?**

**(iii) And now what is the speed of $m_1$ as measured from the object's axis of rotation (i.e. what is $v_{T.1}$ around the axis of rotation)?**

**Ignore any effects of air drag and wind, and assume a constant local $g$ value of 9.80 m/s$^2$.**

**(i)** Unconstrained, a rigid object will rotate around its center of mass. So, set an $x$-axis origin at the left end of the object (i.e. at $m_1$), and calculate $x_{c.m.}$ for this object:

$x_{c.m.} = [(.500)(0) + (.200)(.350) + (.150)(.550)]/(.500+.200+.150) = 0.17941$ m

Next, calculate $I_{c.m.}$, the moment of inertia for rotation about the center of mass:

$I_{c.m.} = [(.500)(.17941)^2 + (.200)(.350-.17941)^2 + (.150)(.550-.17941)^2 = 0.042515$ kg·m$^2$

Now analyze the energy of the <u>projectile</u> (i.e. after the work has been done) from launch to peak:

Only gravity acts on it, so its mechanical energy is unchanged. Setting $U_{G.i} = 0$ at launch level:

$(1/2)mv_i^2 + (1/2)I\omega_i^2 = (1/2)mv_f^2 + (1/2)I\omega_f^2 + mgh_f$

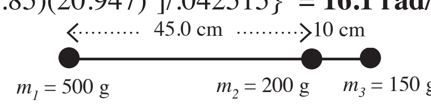
But: $\omega_i = \omega_f$ And: $v_f = v_i\cos 34°$. So: $(1/2)mv_i^2 = (1/2)m(v_i\cos 34°)^2 + mgh_f$

Simplify: $v_i^2 = v_i^2\cos^2(34°) + 2gh_f$ So: $v_i = \sqrt{\{2gh_f/[1-\cos^2(34°)]\}} = 20.947$ m/s

Now analyze the energy "loading" of the system—the object (at $U_G = 0$) as launched:

$W_{ext} = (1/2)mv_i^2 + (1/2)I\omega_i^2$ $\omega_i = \sqrt{[(2W_{ext} - mv_i^2)/I]} = \sqrt{\{[2(192) - (0.85)(20.947)^2]/.042515\}}$ = **16.1 rad/s**

**(ii)** Re-calculate for after $m_2$ shifts, when the object looks like this:




$x_{c.m.} = [(.200)(.450) + (.150)(.550)]/(.500+.200+.150) = 0.20294$ m

$I_{c.m.} = [(.500)(.20294)^2 + (.200)(.450-.20294)^2 + (.150)(.550-.20294)^2 = 0.050868$ kg·m$^2$

Conserve angular momentum: $I_i\omega_i = I_f\omega_f$ $\omega_f = \omega_i(I_i/I_f) = (16.1)(.042515/.050868)$ = **13.5 rad/s**

**(iii)** $v_{T1} = r_1\omega_f = (.20294)(13.5)$ = **2.73 m/s**

6. a. **A solid, uniform sphere of mass 2.0 kg and radius 1.7 m starts from rest and rolls without slipping down an inclined plane of height 5 m. What is the angular velocity of the sphere at the bottom of the inclined plane?**

The fundamental equation: $E_{mech.f} = E_{mech.i} + W_{ext}$

In detail: $K_{T.f} + K_{R.f} + U_{G.f} + U_{S.f} = K_{T.i} + K_{R.i} + U_{G.i} + U_{S.i} + W_{ext}$

Since all rolling is without slipping (static friction) no work is done here except by gravity, so $W_{ext} = 0$. $v_i = \omega_i = 0$, and we will let $h_f = 0$. Also, there's no elastic source in the problem. *1/2 pt.*

With those simplifications: $(1/2)Mv_f^2 + (1/2)I\omega_f^2 = Mgh_i$

Substitute for $I_{sphere}$ and $v_f$: $(1/2)M(R\omega_f)^2 + (1/2)(2/5)MR^2\omega_f^2 = Mgh_i$

Simplify: $(1/2)R^2\omega_f^2 + (1/5)R^2\omega_f^2 = gh_i$

Collect terms: $(7/10)R^2\omega_f^2 = gh_i$

Solve for $\omega_f$: $\omega_f = [(10/7)gh_i/R^2]^{1/2} = [(10/7)(9.80)(5)/1.7^2]^{1/2} =$ **<u>4.92 rad/s</u>** *1/2 pt.*

b. **A bowling ball rolls without slipping, first along a level track, then up a ramp onto another level section of the track, gaining 0.340 m in altitude. If its translational speed along the lower track level was 3.15 m/s, find its translational speed at the top.**

The fundamental equation: $E_{mech.f} = E_{mech.i} + W_{ext}$

In detail: $K_{T.f} + K_{R.f} + U_{G.f} + U_{S.f} = K_{T.i} + K_{R.i} + U_{G.i} + U_{S.i} + W_{ext}$

Since all rolling is without slipping (static friction) no work is done here except by gravity, so $W_{ext} = 0$. Also, there's no elastic source in the problem, and we will let $h_i = 0$. *1/2 pt.*

With those simplifications: $(1/2)Mv_f^2 + (1/2)I\omega_f^2 + Mgh_f = (1/2)Mv_i^2 + (1/2)I\omega_i^2$

Substitute for $I_{sphere}$ and $\omega$: $(1/2)Mv_f^2 + (1/2)(2/5)MR^2(v_f/R)^2 + Mgh_f = (1/2)Mv_i^2 + (1/2)(2/5)MR^2(v_i/R)^2$

Simplify: $(1/2)v_f^2 + (1/5)v_f^2 + gh_f = (1/2)v_i^2 + (1/5)v_i^2$

Collect terms: $(7/10)v_f^2 + gh_f = (7/10)v_i^2$

Solve for $v_f$: $v_f = [v_i^2 - (10/7)gh_f]^{1/2} = [(3.15)^2 - (10/7)(9.80)(0.340)]^{1/2} =$ **<u>2.27 m/s</u>** *1/2 pt.*

c. **A basketball of mass *m* and radius *r* rolls without slipping up a hill. The angle between the hill and the horizontal is θ. The initial speed of the basketball is *v*. The magnitude of the acceleration due to gravity is *g*. Expressed in terms of *m*, *r*, θ, *v* and/or *g*, what vertical height, *h*, does the ball gain before it comes to rest (momentarily)? Assume that the ball is a rigid body, and that friction and air resistance can be ignored.**

The fundamental equation: $E_{mech.f} = E_{mech.i} + W_{ext}$

In detail: $K_{T.f} + K_{R.f} + U_{G.f} + U_{S.f} = K_{T.i} + K_{R.i} + U_{G.i} + U_{S.i} + W_{ext}$

Since all rolling is without slipping (static friction) no work is done here except by gravity, so $W_{ext} = 0$. Also, there's no elastic source in the problem, and we will let $h_i = 0$. And $v_f = \omega_f = 0$. *1/2 pt.*

With those simplifications: $Mgh_f = (1/2)Mv_i^2 + (1/2)I\omega_i^2$

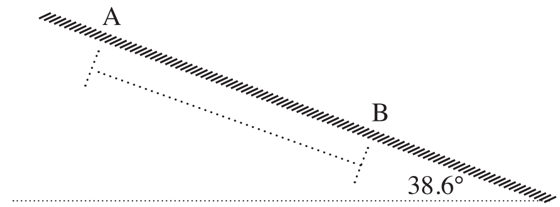
Substitute for $I_{hollow.sphere}$ and $\omega$: $Mgh_f = (1/2)Mv_i^2 + (1/2)(2/3)MR^2(v_i/R)^2$

Simplify: $gh_f = (1/2)v_i^2 + (1/3)v_i^2$

Collect terms: $gh_f = (5/6)v_i^2$

Solve for $h_f$: $h_f = (5/6)v_i^2/g$ $\quad$ $\boldsymbol{h = (5/6)v^2/g}$ *1/2 pt.*

**6. d. A solid sphere and a solid cube have equal masses. You test each, one at a time, on the slope shown here. In each test, you release the object from rest at point A. The sphere rolls without slipping, but the cube slides. In this particular case, it so happens that each object reaches point B with the same translational speed. Find the coefficient of kinetic friction between the cube and the slope.**




**For each case**: $E_{mech.f} = E_{mech.i} + W_{ext}$

The **initial** point is A; the **final** point is B.
The distance traveled (A to B) is $d$.
Let $h_f = 0$
Then $h_i = d\sin\theta \quad (\theta = 38.6°)$
$U_S = 0$ (no spring involved)
$v_i = 0$
$\omega_i = 0$

*1/2 pt.*

**The cube**: The slope <u>does (negative) work</u> via friction:

$W_{ext} = -F_k(d) = -\mu_k F_N(d) = -\mu_k(mg\cos\theta)d$
Thus: $K_{T.f} = U_{G.i} + W_{ext}$
Or: $(1/2)mv_f^2 = mgh_i + W_{ext}$
$(1/2)mv_f^2 = mgd\sin\theta - \mu_k(mg\cos\theta)d$
$(1/2)v_f^2 = gd\sin\theta - \mu_k(g\cos\theta)d$
$v_f^2 = 2gd\sin\theta - 2\mu_k(g\cos\theta)d$

*1/2 pt.*

**The sphere**: During rolling without slipping, <u>no work</u> is done by the surface: $W_{ext} = 0$

Therefore: $K_{T.f} + K_{R.f} = U_{G.i}$
Or: $(1/2)mv_f^2 + (1/2)I_f\omega_f^2 = mgh_i$
But: $\omega_f = v_f/R$ ($R$ is the sphere's radius)
And: $I = (2/5)mR^2$
So: $(1/2)mv_f^2 + (1/2)[(2/5)mR^2](v_f/R)^2 = mgd\sin\theta$
Simplify: $(1/2)v_f^2 + (1/5)v_f^2 = gd\sin\theta$
$v_f^{\mathbf{2}} = (10/7)gd\sin\theta$

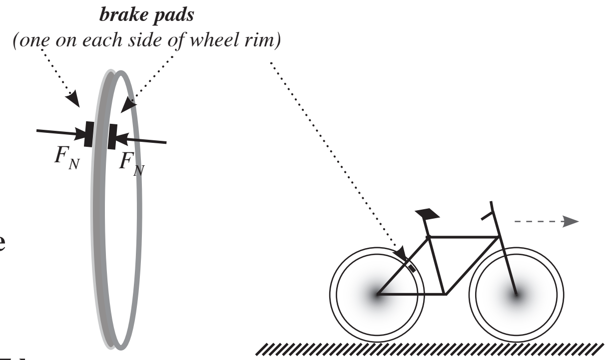
*1/2 pt.*

But the $v_f$ in each of the two cases above is the same. <u>Therefore</u>:

$(10/7)gd\sin\theta = 2gd\sin\theta - 2\mu_k(g\cos\theta)d$
$(10/7)\sin\theta = 2\sin\theta - 2\mu_k(\cos\theta)$
$2\mu_k(\cos\theta) = 2\sin\theta - (10/7)\sin\theta$
$\mu_k(\cos\theta) = \sin\theta - (5/7)\sin\theta$
$\mu_k = (2/7)\tan\theta = \mathbf{0.228}$

*1/2 pt.*

**6. e. The rear brakes on a bicycle consist of a pair of rubber pads that clamp down and rub ($\mu_K = 0.863$) on the rim of the rear wheel in order to slow it. Suppose the bike is rolling (without slipping) along a straight, level road at 9.25 m/s when the rider applies the rear brakes. If the normal force applied by each brake pad on the wheel rim is 74.1 N, find the speed of bike and rider when the wheels have revolved exactly six times after braking began. Ignore air and rolling resistance, and assume that the bike has two identical wheels; and that the brake pads apply their forces at the wheel's outer radius.**




$I_{wheel} = 0.268\ \text{kg·m}^2$ $\quad r_{wheel} = 0.345\ \text{m}$ $\quad m_{bike\&rider} = 107\ \text{kg}$

Use energy to analyze the system (bike and rider) from the moment the braking begins (initial) to the moment when the wheels have turned six full revolutions (final).

The system has $K_T$ (from the speed, $v$, of all parts moving in translational motion) and $K_R$ (from the rotational speed, $\omega$, of the 2 wheels moving in rotational motion). But there's no height change (so no $U_G$); and no spring or other ideal elastic force (so no $U_S$). Thus: $K_{T.i} + K_{R.i} + W_{ext} = K_{T.f} + K_{R.f}$ *1/2 pt.*

The wheels roll without slipping, so the road's static friction does no work on them. But each of the two brake pads does (negative) work on the rear wheel, via a kinetic friction <u>torque</u>, $|\tau_K| = |\boldsymbol{F_K}|r_{wheel}$, acting over an angular displacement (6 revolutions = 12π rad) of that wheel: $W_{ext} = 2|\tau_K|(-1)|\Delta\theta_K| = -2\mu_K F_N r_{wheel}(12\pi)$ *1/2 pt.*

<u>So</u>: $(1/2)mv_i^2 + 2(1/2)I\omega_i^2 + -2\mu_K F_N r_{wheel}(12\pi) = (1/2)mv_f^2 + 2(1/2)I\omega_f^2$

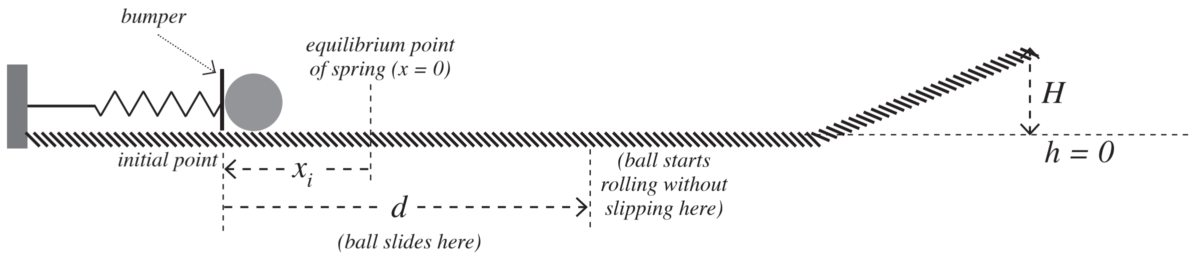
For rolling w/o slipping, $\omega = v/r_{wheel}$: $\quad (1/2)mv_i^2 + (I/r_{wheel}^2)v_i^2 - 2\mu_K F_N r_{wheel}(12\pi) = (1/2)mv_f^2 + (I/r_{wheel}^2)v_f^2$ *1/2 pt.*

Solve for $v_f$: $\quad v_f = \sqrt{\{[(m/2 + I/r_{wheel}^2)v_i^2 - 2\mu_K F_N r_{wheel}(12\pi)]/(m/2 + I/r_{wheel}^2)\}}$

$= \sqrt{\{[(107/2 + 0.268/0.345^2)(9.25)^2 - 2(0.863)(74.1)(0.345)(12\pi)]/(107/2 + 0.268/0.345^2)\}}$ *1/2 pt.*

$=$ <u>**7.46 m/s**</u>

**6. f. An ideal spring (stiffness = 910 N/m) with a frictionless "bumper," as shown, is attached along a level surface to a wall and compressed by a distance of 0.320 from its relaxed equilibrium position. A ball (a solid, uniform sphere, mass = 3.68 kg, radius = 0.465 m) is placed in front of the bumper. When the spring is released, the ball slips ("skids"—i.e. it doesn't roll) horizontally for a distance of 0.791 m. Then it rolls <u>without slipping</u> thereafter, up the ramp and off the end, which is 0.879 m above the level surface. The coefficient of kinetic friction for the entire surface and ramp is 0.245. Find the rotational speed of the ball as it leaves the ramp. Neglect air resistance.**




*1/2 pt.*

*Inventory of known and needed facts:*

$v_i = 0$ $\quad\omega_i = 0$ $\quad h_i = 0$ $\quad x_i = -0.320$ $\quad I = (2/5)mR^2$

$v_f = ?$ $\quad\omega_f = ?$ ***<---the <u>question</u>*** $\quad h_f = H = 0.879$ $\quad x_f = 0$

*1/2 pt.*

*The fundamental Work-Energy equation:*

$$E_{mech.f} = E_{mech.i} + W_{ext}$$

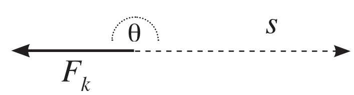
$$K_{T.f} + K_{R.f} + U_{G.f} + U_{S.f} = K_{T.i} + K_{R.i} + U_{G.i} + U_{S.i} + W_{ext}$$

*1/2 pt.*

*Analysis of the external work being done:*

$W_{Fk} = F_k s\cdot\cos\theta = \mu_k F_N d\cdot\cos\theta = -\mu_k F_N d$

$F_N = mg$ therefore $W_{ext} = -\mu_k mgd$




*1/2 pt.*

*Detailed expansion of the Work-Energy equation for this situation:*

$$(1/2)mv_i^2 + (1/2)I\omega_i^2 + mgh_i + (1/2)kx_i^2 - \mu_k mgd = (1/2)mv_f^2 + (1/2)I\omega_f^2 + mgh_f + (1/2)kx_f^2$$

*Simplify:* $\quad (1/2)kx_i^2 - \mu_k mgd = (1/2)mv_f^2 + (1/2)(2/5)mR^2\omega_f^2 + mgH$

Since the ball is rolling without slipping at the final point, its translational speed is equal to the tangential speed of its edge: $v = v_T = R\omega$ And therefore: $v_f^2 = R^2\omega_f^2$

*Substitute for $v_f^2$ and I:* $\quad (1/2)kx_i^2 - \mu_k mgd = (1/2)mR^2\omega_f^2 + (1/2)(2/5)mR^2\omega_f^2 + mgH$

*Simplify:* $\quad (1/2)kx_i^2 - \mu_k mgd = (1/2)mR^2\omega_f^2 + (1/5)mR^2\omega_f^2 + mgH$

*Collect terms:* $\quad (1/2)kx_i^2 - \mu_k mgd = (7/10)mR^2\omega_f^2 + mgH$

*1/2 pt.*

*Solve for ω:* $\quad (1/2)kx_i^2 - \mu_k mgd - mgH = (7/10)mR^2\omega_f^2$

$$\omega_f^2 = [(1/2)kx_i^2 - \mu_k mgd - mgH]/[(7/10)mR^2]$$

$$\omega_f = \{[(1/2)kx_i^2 - \mu_k mgd - mgH]/[(7/10)mR^2]\}^{1/2}$$

*Plug in the numbers:*

$$\omega = \{[(1/2)(910)(.320)^2 - (.245)(3.68)(9.80)(.791) - (3.68)(9.80)(.879)]/[(7/10)(3.68)(.465)^2]\}^{1/2}$$

$= \underline{\textbf{3.77 rad/s}}$

*1/2 pt.*