HW 7

Due: Wednesday, February 26, 2014, 5:00 p.m.

Print *your full* **LAST** *name:* ______________________________

Print *your full* **first** *name:* ______________________________

Print *your HW Grader's name:* ______________________________

What is your HW Grader's box # (located outside of Wngr 234)? ________

"I affirm and attest that this HW assignment is my own work. While I may have had help from (and/or worked with) others, all the reasoning, solutions and results presented in final form here are my own doing—and expressed in my own words."

Sign your name (full signature): ______________________________

Print *today's date:* ______________________________

General Instructions for HW 7

Requirements: The PH 212 HW graders will select **one** out of the six problems. They will then score that problem according to the points rubric posted with the HW 7 solutions (for a total possible 10 points for the entire HW assignment).

Format: ***You are required to include the completed, signed cover sheet—that's the first page of this file—as the front page of your HW submission when you turn it in.*** *(*Please staple together all pages, in order—cover sheet first—with one staple at upper left.)

Scoring: The rubrics (scoring guides—the points breakdowns) for HW problem solutions will closely match those for exam solutions. And in those rubrics, *when any item asks for an explanation (and most do), it means exactly what it says*. To get full credit, you must include a short but informative verbal explanation (in your own words) of your reasoning.

To get an idea of how best to approach various problem types (there are three basic types), refer to these example HW problems.

1. a. ***T/F/N? (And explain, as always.)* Simple harmonic motion can be described by either the *x*- or *y*-motion of an object in uniform circular motion.**

__True__. The "marker" motion of the object going in a circle could be used for either its vertical component (thus using a sine function) or its horizontal component (thus using a cosine function), as we did in class.

b. **A loudspeaker diaphram is producing a sound for 2.50 s by moving back and forth in simple harmonic motion. The angular frequency of the motion is 7.54×10^4 rad/s. How many complete cycles does the diaphragm make in this time interval?**

The cyclical frequency (f) is related to the angular frequency: $\omega = 2\pi f$

So $f = \omega/(2\pi) = (7.54 \times 10^4)/(2\pi) = 1.20 \times 10^4$ Hz. This is the number of cycles per second.

So, in 2.5 seconds, the diaphram completes a total of $(1.20 \times 10^4)(2.5) =$ __3.00×10^4 cycles__.

c. **In the Hinsdale Wave Research Laboratory on the OSU campus, a wavemaker creates water waves for tsunami research. The period of its motion can be set to anywhere from 0.5 s to 10 s *[i.e. the machine can be set to any fixed value in this range for the purposes of the study or experiment]*, and its maximum displacement from equilibrium is 2.1 m. Assuming the wavemaker moves with simple harmonic motion, find its maximum speed.**

The maximum speed of any oscillator undergoing SHM is $v_{max} = A\omega$

And $\omega = (2\pi)/T$, so $v_{max} = (2\pi)A/T$ This will be the greatest for the *least* value of the period, T:

So $v_{max} = (2\pi)(2.1)/(0.5) =$ __26.4 m/s__. (That's about 59 mph)

d. **The acceleration of a simple harmonic oscillator is given by $a(t) = -(15.8 \text{ m/s}^2)\cdot\cos[(2.51 \text{ rad/s})t]$, with t given in seconds. What is the amplitude of the simple harmonic motion?**

$\boldsymbol{a}(t) = -(A\cdot\omega^2)\cdot\cos[(\omega)t]$, so $\omega = 2.51$ rad/s, and therefore: $15.8 = A(2.51)^2$

So: $A = 15.8/[(2.51)^2] =$ __2.51 m__.

e. **The horizontal position of a simple harmonic oscillator is given by $x(t) = (12.0 \text{ cm})\cdot\cos[(12.0 \text{ rad/s})t]$, with t given in seconds. What is this oscillator's maximum acceleration magnitude?**

$\boldsymbol{x}(t) = A\cdot\cos[(\omega)t]$. Therefore: $A = 12.0$ cm ($= 0.12$ m) And: $\omega = 12.0$ rad/s

$a_{max} = A\cdot\omega^2 = (0.12)(12.0)^2 =$ __17.3 m/s^2__.

2. a. **Two equal masses are attached to separate but identical springs. Mass A is displaced 5.00 cm from its equilibrium position. Mass B is displaced 10.0 cm from its equilibrium position. They are released at the same time. (The springs behave "ideally:" Friction and drag are negligible, and neither spring is stretched beyond its elastic limit.) *T/F/N? (Explain.)* Mass B will reach equilibrium position before mass A will.**

__False—they will tie__. Released from its maximum displacement (A), each mass will require time $T/4$ (a quarter of its period) to reach its equilibrium position ($\boldsymbol{x} = 0$). But two identical masses on identical springs have the same angular frequency, $\omega = \sqrt{(k/m)}$, and therefore the same period, $T = (2\pi)/\omega$. *1 pt.*

b. **A block of mass M, at rest on a horizontal frictionless table, is attached to a rigid support by a spring of stiffness k. A bullet of mass m and velocity v strikes the block and is embedded into it.
In terms of M, k, m and v, find:**

(i) **the velocity of the bullet-block system immediately after the collision;**

Use momentum to calculate the starting speed at which the combined mass will begin to oscillate (and since this motion starts at $\boldsymbol{x} = 0$, the equilibrium position, this will be v_{max} of the oscillation): *1 pt.*

$\boldsymbol{P}_{x.total.i} = \boldsymbol{P}_{x.total.f}$: $m\boldsymbol{v} + M(0) = (m + M)v_{max}$ __Thus__: $v_{max} = m\boldsymbol{v}/(m + M)$

(ii) **the amplitude of the resulting simple harmonic motion.**

$v_{max} = A\omega$ and $\omega = \sqrt{(k/m_{total})}$ So: $A = v_{max}/\omega = [m\boldsymbol{v}/(m + M)]/\sqrt{[k/(m + M)]}$ *2 pts.*

Simplifying: $A = v_{max}/\omega = \boldsymbol{mv\sqrt{[(m + M)/k]}}$

2. c. An ideal spring is lying horizontally on a frictionless surface. One end of the spring is attached to a wall. The other end is attached to a moveable block that has a mass of 5 kg. The block is pulled so that the spring stretches from its equilibrium position by 0.65 m. Then the block is released (from rest), and as a result the system oscillates with a frequency of 0.40 Hz (that's 0.40 rev/sec). Find:

(i) the acceleration of the block when the spring is stretched by 0.28 m.
(ii) the maximum force magnitude exerted by the spring on the block.
(iii) the oscillation frequency of a 2.5 kg block using the same spring and initial displacement.

(i) We know that $2\pi f = \omega = \sqrt{(k/m)}$. So: $k = 4\pi^2 m f^2$

And when the spring is stretched by $\boldsymbol{x} = 0.28$ m, we know that $\boldsymbol{F} = -k\boldsymbol{x}$, and so $\boldsymbol{a} = \boldsymbol{F}/m = -k\boldsymbol{x}/m$. *2 pts.*

Thus: $\boldsymbol{a} = -4\pi^2 f^2(\boldsymbol{x}) = -4\pi^2(0.40)^2(0.28) = \underline{\mathbf{-1.77\ m/s^2}}$

At the moment when the spring is stretched by 0.28, the acceleration of the block is –1.77 m/s².

And this negative vector direction makes sense: When the block's displacement is a +0.28 (i.e. to the right of equilibrium), the spring is stretched; the force it exerts on the block is negative—to the left.

(ii) The maximum force magnitude is exerted at the maximum displacement—at the amplitude, *A*. *1 pt.*

So: $|F|_{max} = kA = 4\pi^2 m f^2 A = 4\pi^2(5)(0.40)^2(0.65) = \underline{\mathbf{20.5\ N}}$

The maximum force magnitude exerted by the spring on the block is 20.5 N.

(iii) ω (and therefore f) is proportional to $1/\sqrt{m}$, so multiplying m by a factor of 1/2 would multiply f by a factor of $1/\sqrt{(1/2)}$, which is $\sqrt{2}$. Therefore: $f_2 = (\sqrt{2})f_1 = (\sqrt{2})(0.40) = \underline{\mathbf{0.566\ Hz}}$ *1 pt.*

The frequency of a 2.5-kg block oscillating on the same spring would be 0.566 Hz.

d. A 5-kg block is sent sliding across a frictionless table at a speed of 0.75 m/s. It strikes the free end of an ideal spring whose other end is attached to a wall. After the block has compressed the spring by 4.0 cm, its speed is half of what it was when it first encountered the spring. How much time elapses between the block's first and last contact with the spring?

The question is asking about the duration of a half-period ($T/2$). The block encounters the spring when the spring is at equilibrium—neither stretched nor compressed. The block stays in contact as it compresses the spring to a maximum (i.e. the amplitude), then back to the equilibrium point again—<u>but no farther</u>. At that point, since the block is not attached to the spring, it cannot pull the spring into tension; it slides away freely, losing contact with the spring at that point. Thus the block stays in contact with the spring only for the <u>compression</u> half of the spring's oscillation—half of the spring's total period.

The other point to note is that this system (spring and block) <u>conserves mechanical energy</u> because there is no work done by anything except the spring—not even gravity. (Yes, there is a normal force exerted on the block by the surface, but that does no work, since it's perpendicular to the motion.) So, as the spring goes from no compression (when the block first contacts the spring) to 4.0 cm of compression: $E_{mech.i} = E_{mech.f}$ *2 pts.*

With no rotation or altitude change: $(^1/_2)mv_i^2 + (^1/_2)kx_i^2 = (^1/_2)mv_f^2 + (^1/_2)kx_f^2$ Solve this for k:

$$k = m(v_i^2 - v_f^2)/(x_f^2 - x_i^2) = (5)(0.75^2 - 0.375^2)/(0.04^2 - 0^2) = 1318\ \text{N/m}$$

Now calculate $(1/2)T$: $(1/2)T = (1/2)[(2\pi)/\omega] = \pi/\omega = \pi\sqrt{(m/k)} = \pi\sqrt{(5/1318)} = \underline{\mathbf{0.193\ s}}$

The block is in contact with the spring for 0.193 s.

3. a. ***T/F/N? (And explain, as always.)*** **If a 1-kg mass were oscillating horizontally on a spring of stiffness 4 N/m, a pendulum of arm length $l \approx 2.45$ m would have about the same frequency.**

True. $\omega_{spring\text{-}mass} = \sqrt{(k/m)} = \sqrt{(4/1)} = 2.00$ rad/s $\quad \omega_{pendulum} = \sqrt{(g/L)} = \sqrt{(9.80/2.45)} = 2.00$ rad/s.

b. If a pendulum clock is running too slowly, should you shorten or lengthen the pendulum arm to get it to keep correct time? *Explain.*

Shorten the arm. Since $\omega_{pendulum} = \sqrt{(g/L)}$, if you want to increase $\omega_{pendulum}$, you must decrease L.

c. In principle, both the motion of a simple pendulum and the motion of a mass on a horizontal ideal spring can be used to measure time (via their periods of oscillation). But neither type may be consistently accurate if used outdoors, and one type must be recalibrated if flown to very high altitudes. Explain all this.

Temperature variations will affect both clocks. Thermal expansion or contraction will likely affect the pendulum arm length (L) and the spring stiffness (k). Each of these affects the frequency of its oscillating system.

High altitude will affect the pendulum clock. Any change in the local g value would affect the accuracy of the pendulum's timing, since $\omega_{pendulum} = \sqrt{(g/L)}$. Higher altitude would decrease g (slow the pendulum).

d. A simple pendulum is made from a 0.650-m-long string and a small ball attached to its free end. The ball is pulled to one side through a small angle and then released from rest. After the ball is released, how much time elapses before it attains its greatest speed?

v_{max} is at $\boldsymbol{x} = 0$, when $t = (1/4)T = (1/4)[(2\pi)/\omega] = (1/2)\pi\sqrt{(L/g)} = (1/2)\pi\sqrt{(0.650/9.80)} =$ **0.406 s**

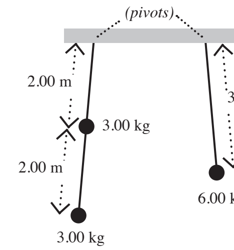
e. An OSU engineering student constructs a simple clock by tying the top of a piece of string to an immovable *[overhead]* post and attaching her cell phone to the bottom end of the string. She then displaces the cell phone to the right so that the string makes an angle of 10.0 degrees with the vertical. When she releases the phone, it oscillates in simple harmonic motion with a period of 2.00 s. The phone's mass is 125 grams. What is the distance from the post to the center of mass of the cell phone?

$T = (2\pi)/\omega = 2\pi\sqrt{(L/g)}$. $\quad$ So: $L = [T/(2\pi)]^2 g = [2/(2\pi)]^2(9.80) =$ **0.993 m**

f. A simple pendulum is used to test the value of g on the surface of a different planet. If the frequency of the pendulum's swing is 1/3 Hz, and the length of the pendulum is 1m, what is the local value of g?

$f = \omega/(2\pi) = \sqrt{(g/L)}/(2\pi)$. $\quad$ So: $\quad g = (2\pi f)^2 L = [(2\pi)(1/3)]^2(1) =$ **4.39 m/s²**

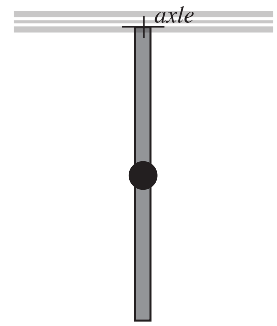
4. a. The two pendulums shown at right swing freely from pivots that are anchored to the ceiling at their upper ends. *T/F/N? (And explain, as always.)* If each pendulum is constructed from point masses attached to rigid massless arms, then for small-angle oscillations, their frequencies are equal.



False. The simple pendulum: $\omega = \sqrt{(g/L)} = \sqrt{(9.80/3)} = \mathbf{1.81\ rad/s}$

The physical pendulum: $\omega = \sqrt{(Mgl/I)} = \sqrt{[(6)(9.80)(3)/(3\cdot 2^2 + 3\cdot 4^2]}$
$= \mathbf{1.71\ rad/s}$

b. A uniform thin rod of length L = 90.0 cm and mass M = 200 g is suspended from a horizontal axle that is fixed at one end; the rod is free to rotate around the axle. A ball-shaped blob of clay, of mass m = 0.0500 kg, is attached to the rod's midpoint.



(i) Treating the ball of clay a a point particle, find the period of (small) oscillations of this physical pendulum.

For small oscillations: $T = (2\pi)/\omega = 2\pi\sqrt{[I_{total.pivot}/(M_{total}gl)]}$

The ball is attached right at the c.m. of the bare rod, so the location of the c.m remains unchanged; its distance, l, from the pivot is still 0.450 m.

$I_{total.pivot} = I_{rod.pivot} + ml^2 = (1/3)ML^2 + ml^2 = (1/3)(0.200)(0.90)^2 + (0.0500)(0.450)^2$
$= 0.064125\ \text{kg}\cdot\text{m}^2$

So: $T = 2\pi\sqrt{\{(0.064125)/[(0.200 + 0.0500)(9.80)(0.450)]\}}$ = **<u>1.52 s</u>**

(ii) What would be the length of a simple pendulum (i.e., a bob on a mass-less string) that would have the same oscillation period as this physical pendulum?

We need $\sqrt{(g/L)} = \sqrt{(Mgl/I)}$ Or: $L = I/(Ml) = 0.064125/[(0.250)(0.45)]$ = **<u>0.570 m</u>**

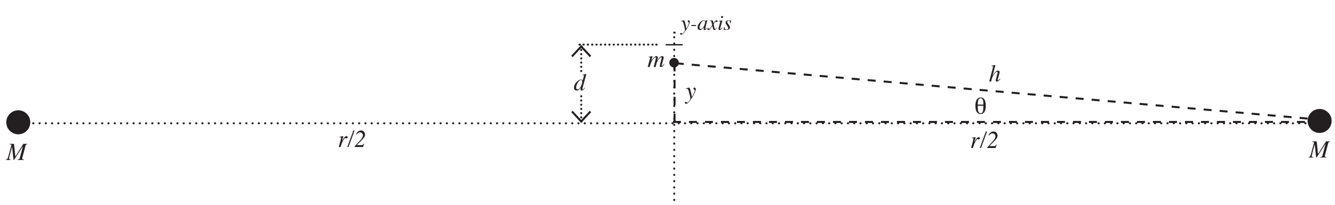
c. Three springs hang separately from the ceiling, with a block attached to the lower end of each spring. When all are at rest, the blocks ($m_1 > m_2 > m_3$) stretch their respective springs by equal distances. Each block-spring system is then set into vertical simple harmonic motion. Rank the three systems according to the period of oscillation, greatest first. *Explain.*

When a mass hangs at rest from a vertical ideal spring, the spring is stretched by x such that $mg = kx$.

So if each spring stretches the same amount, then: $m_1g/k_1 = m_2g/k_2 = m_3g/k_3$

Thus: $k_1/m_1 = k_2/m_2 = k_3/m_3$ That is: $\omega_1 = \omega_2 = \omega_3$ <u>The systems all oscillate with the same period</u>.

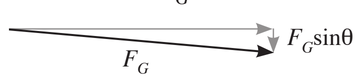
d. Two equally massive stars (mass M) are separated from each other by a distance r. A small observation instrument (of mass m, where $m << M$) is positioned, initially at rest, at a point that is equidistant from the two stars' centers of mass but not directly between them; rather, a small distance d from that center point. The instrument is then released from rest. <u>Describe completely</u> its motion, mathematically and with words and a diagram. You may assume that $d << r$, so that small-angle approximations are appropriate.



$\Sigma F_y = ma_y$ Sum the forces acting on m along the y-axis.

$2(-F_{G.y}) = ma_y$ 2 forces, F_G, act on m, one from each star. By symmetry, they are the same.

$-2(F_G\sin\theta) = ma_y$



(and $F_G = GMm/h^2$)

$-2(GMm/h^2)\cdot\sin\theta = ma_y$

$-2[GMm/(r/2)^2]\cdot[y/(r/2)] = ma_y$ Small-angle approximations: $h \approx r/2$ and so $\sin\theta = y/h \approx y/(r/2)$

$-[16GM/r^3]\cdot y = a_y$ (Simplifying)

So: $d^2y/dt^2 = -[16GM/r^3]\cdot y(t)$ This is the second-order differential equation form that describes SHM:

$\mathbf{y(t) = d\cos(\omega t + \phi_0)}$ **where:** $\mathbf{\omega = \sqrt{[16GM/r^3]}}$ **and (assuming $y(0) = d$):** $\mathbf{\phi_0 = 0}$

5. a. A simple harmonic oscillator with a period of 2.0 s is subject to a damping such that it loses one percent of its amplitude per cycle.

(i) What percent of energy does the oscillator lose per cycle?

For a linearly damped oscillator ($F_{Drag} = bv$, where b is a constant), the position of the oscillator is given by:

$\boldsymbol{x}(t) = A(t)\cdot\cos(\omega t + \phi_0) = A_0[e^{-t/(2\tau)}]\cos(\omega t + \phi_0)$ where τ is the time constant: $\tau = m/b$

And the mechanical energy of the oscillator is given by: $E_{mech}(t) = E_{mech.0}[e^{-t/\tau}]$

In this case: $A(2s) = .99A_0$ That is: $0.99A_0 = A_0[e^{-2/(2\tau)}]$ So: $\tau = -1/\ln(0.99) = 99.499$ s

Then: $E_{mech}(2s) = E_{mech.0}[e^{-2/99.499}]$ That is: $E_{mech}(2s)/E_{mech.0} = e^{-2/99.499} = 0.9801$

So the mechanical energy loss after 2 seconds is **<u>1.99%</u>**.

(ii) How much time will pass before the amplitude has decreased to half the initial value?

We want $A(t) = .50A_0$ That is $0.50A_0 = A_0[e^{-t/(2\cdot 99.499)}]$ That is: $0.50 = e^{-t/(2\cdot 99.499)}$

Thus: $t = -(2\cdot 99.499)[\ln(0.50)] =$ **<u>138 s</u>**

b. A mass (0.500 kg) on a frictionless, horizontal track is attached to a horizontal spring (k = 1000 N/m). The mass is pushed so that the spring is compressed from its equilibrium position by a distance A, then allowed to oscillate.

(i) If the clock isn't started (t = 0) until the mass first returns to the equilibrium position, what is the complete equation of motion (position) for the mass?

$\boldsymbol{x}(t) = A\cdot\cos(\omega t + \phi_0)$ where $\omega = \sqrt{(k/m)} = \sqrt{(1000/.50)} = 44.72$ rad/s and $\phi_0 = \pi/2$ rad

<u>So</u>: $\boldsymbol{x}(t) = \boldsymbol{A\cdot\cos(44.7t + \pi/2)}$

(ii) What is the period of the oscillations? $T = (2\pi)/\omega = 2\pi/44.7 =$ **<u>0.140 s</u>**

(iii) What amplitude, A, was necessary, in order for the speed of the mass at $x = A/2$ to be 2 m/s?

For the frictionless system, $E_{mech} = (1/2)kA^2 = (1/2)k(A/2)^2 + (1/2)mv^2$

That is: $kA^2 = kA^2/4 + mv^2$ Or: $(3/4)kA^2 = mv^2$

Thus: $A = \sqrt{[(4/3)mv^2/k]} = \sqrt{[(4/3)(0.500)(2)^2/1000]} =$ **<u>0.0516 m</u>**

(iv) If there were actually a little friction, so that, at t = 10 seconds, the amplitude is 3/4 of what it initially was, find the damping coefficient (b) for this system.

$A(10s) = 0.75A_0$ That is: $0.75A_0 = A_0[e^{-10/(2\tau)}]$ So: $\tau = -5/\ln(0.75) = 17.38$ s

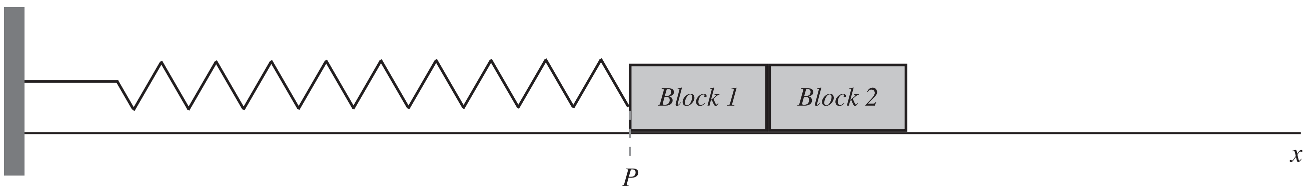
And: $\tau = m/b$ Therefore: $b = m/\tau = 0.500/17.38 =$ **<u>0.0288 kg/s</u>**

6. b. An ideal spring (stiffness k) is attached to a stationary wall on one end and to Block 1 (mass m, length L) on the other end. The system is initially at rest on a level, frictionless surface, with Block 1's left edge at point P. Block 2 (identical to A) is then placed against (to the right of, but not attached to) Block 1, as shown below. Then sufficient force is applied to Block 2 (pushing on it horizontally to the left), so that the blocks move together, compressing the spring until the right edge of Block 2 reaches point P. At that point, the force is removed, and the system moves freely.

Find Block 1's position (relative to point P) 30.0 seconds after it loses contact with Block 2.

<u>The data</u>: k = 100 N/m m = 3.00 kg L = 0.200 m

Let the positive x-direction be to the <u>right</u>. Also, for the 30-second interval in question, assume that air offers a drag force proportional to the speed of Block 1, such that when the block is moving at 1.00 m/s, the drag force is 0.150 N.



The mechanical energy of the system at the moment of release is all $U_{elastic}$: $E_{mech} = (1/2)k(2L)^2 = 2kL^2$

And when the system returns to point P (equilibrium), the energy is all K_T: $E_{mech} = (1/2)(2m)v_{max}^2 = mv_{max}^2$

Thus: $2kL^2 = mv_{max}^2$ So: $v_{max} = (L)\sqrt{(2k/m)}$

Then, at that moment, half the mass departs (and so half the E_{mech}, which is all K_T at that moment, goes with it).

So now the system's total energy is simply $(1/2)mv_{max}^2$, or kL^2.

And the system now has an initial amplitude such that $(1/2)kA^2 = kL^2$. <u>Thus</u>: $A = (\sqrt{2})\,L$

And, if $\boldsymbol{x}$ is positive to the right, then at the moment when Block 2 departs (when you start the clock for the 30-second analysis), the system is already 3/4 of the way through its cycle (the start being at $\boldsymbol{x} = +A$).

<u>Therefore</u>: $\phi_0 = 3\pi/2$ (or: $\phi_0 = -\pi/2$)

So the <u>undamped</u> equation of motion would be: $\boldsymbol{x}(t) = (\sqrt{2})L\cdot\cos(\omega t + 3\pi/2)$, where $\omega = \sqrt{(k/m)}$.

But there is damping, and that model is <u>linear</u>: $F_D = bv$, so $b = F_D/v = (0.150\text{ N})/(1.00\text{ m/s}) = 0.150$ kg/s

So the <u>damped</u> equation of motion would be: $\boldsymbol{x}(t) = (\sqrt{2})L\cdot[e^{-bt/(2m)}]\cos(\omega t + 3\pi/2)$

where $\omega = \sqrt{[(k/m) - b^2/(4m^2)]} = \sqrt{[(100/3) - 0.150^2/(4\cdot3^2)]} = 5.7734$ rad/s

<u>At t = 30 s</u>: $\boldsymbol{x}(t) = (\sqrt{2})(.200)\cdot[e^{-0.150\cdot30/6}]\cdot\cos[5.7734\cdot(30) + 3\pi/2] =$ <u>**–0.0540 m**</u>