Fuentes Name__

ST-314 Homework 2
Due Friday, April 25th at 4:00 PM

Please download this document. Then, complete your solutions and upload as PDF or Word Document. No other formats will be accepted.

Typing or entering answers by hand is accepted as long as solutions are presented neatly and the document is uploaded as PDF. Give the solutions in the space provided.

Instructions

- The homework is due on Friday, Apr. 25th at 4:00 PM. The homework MUST be uploaded to Blackboard as PDF or Word document. No other formats will be accepted.
- The homework is worth 25 points.
- Late homeworks will not be accepted under ANY circumstances.
- You will have 3 attempts to successfully upload your homework. Only your last successful attempt will be graded.
- You may work in groups of 2-3 people, but must submit individual solutions.
- You must provide complete answers in order to receive full credit.
- Please, use the space assigned to provide solutions to the problems.
- Failing to follow any of these instructions may result in a deduction of points from your total score.

Scanners

If you need to scan your homework, eScanners are located in the 2nd floor Copy Center, 1st, 3rd and 5th floors. These scanners allow you to scan documents (color, gray, b/w) in searchable PDF or quick PDF and send them to an email address or store on a flash drive. It is recommended that you scan no more than 17 pages at a time to prevent the eScanners from freezing. Your mailbox will also need to have sufficient storage space for your documents or they will be lost when sent. There is no charge for the service at this time.

Problem 1. (5 Points)
Use your textbook to fill in the following tables:

Discrete Distribution	Binomial	Poisson
pmf	$\binom{n}{k} p^k (1-p)^{n-k}$	$\frac{\lambda^y}{y!} e^{-\lambda}$
E(X)	$\mu = np$	$\mu = \lambda$
V(X)	$np\cancel{q}(1-p)$	λ

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Continuous Distributions	Uniform	Exponential	Gamma
pdf	$f(x) = \begin{cases} \frac{1}{b-a}, & a \le x \le b \\ \emptyset, & \text{otherwise} \end{cases}$	$\lambda e^{-\lambda x}$	$\frac{\lambda^{\alpha}}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x}$
E(X)	$\frac{a+b}{2}$	$\frac{1}{\lambda}$	$\frac{\alpha}{\lambda}$
V(X)	$\frac{(b-a)^2}{12}$	$\frac{1}{\lambda^2}$	$\frac{\alpha}{\lambda^2}$

Problem 2. (5 Points)
A certain sports car comes equipped with either an automatic or a manual transmission, and the car is available in one of four colors. Relevant probabilities for various combinations of transmission type and color are given in the accompanying table.

	Color			
Transmission Type	White	Blue	Black	Red
A	0.13	0.10	0.11	0.11
M	0.15	0.07	0.15	0.18

Let the event A = {the car has an automatic transmission} and the event R = {the car is red}.

a. Calculate P(A), P(R) and $P(A \cap R)$.
b. Given a car has an automatic transmission, what is the probability that it is red?
c. Given a car is red, what is the probability that it has an automatic transmission?
d. Are transmission type and car color independent? Explain.

a. $P(A) = 0.13 + 0.10 + 0.11 + 0.11 = 0.45$

$P(R) = 0.11 + 0.18 = 0.29$

$P(A \cap R) = 0.11$

b) $P(R|A) = \frac{P(R \cap A)}{P(A)} = \frac{0.11}{0.45} = 0.2\overline{4}$

c) $P(A|R) = \frac{P(A \cap R)}{P(R)} = \frac{0.11}{0.29} \cong 0.38$

d) no $P(A|R) = 0.38 \neq P(A)$
$P(R|A) = 0.2\overline{4} \neq P(R)$

Problem 3. (5 Points)

A manufacturer of silicon wafers has encountered an operating problem where too many of the chips made are unacceptable. An inspector selects a sample of five wafers from each shift. The following table describes the distribution of the number of unacceptable wafers in the sample.

y_i	0	1	2	3	4	5
$p(y_i)$	0.3106	0.4313	0.2098	0.0442	0.0040	0.0001

a. Find the probability that at most two wafers in the sample are unacceptable.
b. Find the probability that less than 4 are unacceptable.
c. Find the expected number of unacceptable wafers.
d. Find the variance and standard deviation for the number of unacceptable wafers.

a) $0,3106 + 0,4313 + 0,2098 = 0,9517$

b) $0.3106 + 0,4313 + 0.2098 + 0.0442 = 0,9959$

c) $\mu = \sum x\, p(x) = 0(0.3106) + 1(0.4313) + 2(0.2098) + 3(0.0442) + 4(0.0040) + 5(0.0001)$

$= 1$

d) Variance : $\sigma^2 = \sum (x-\mu)^2 \cdot p(x)$

$(0-1)^2(0.3106) + (1-1)^2 \overset{0.4313}{(\cancel{0.3106})} + (2-1)^2(0.2098) + \cancel{0.84}(3-1)^2(0.0442) + (4-1)^2(0.0040) + (5-1)^2(0.0001)$

$= 0.3106 + 0 + 0.2098 + 4(0.0442) + 9(0.0040) + 16(0.0001) = \boxed{0.7348}$

Std. dev. $= \sigma = \sqrt{0.7348} = 0.8572$

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Problem 4. (5 Points)
A manufacturer of water filters for refrigerators monitors the process for defective filters. Historically, this process averages 5% defective filters. Suppose ten filters are randomly selected for testing.

a. Find the probability that all ten filters are not defective.
b. Find the probability that at least three filters are defective.
c. What is the expected number of defective filters?

a) $(0.95)^{10} = 0.599$

b)

Success

$P(\text{at least } 3 \text{ fail}) = 1 - P(\text{at most } 2 \text{ } \cancel{\text{fail}})$

$= 1 - \left[(0.95)^2 \right] = 0.0975$

c) $E(x) = \mu = (10)(1-p) = (10)(0.05) = 0.5$

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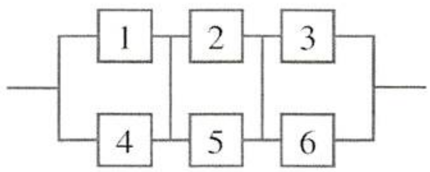
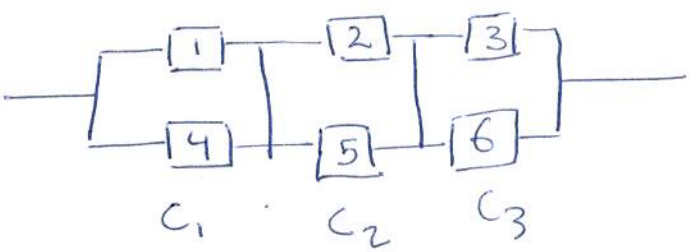
Problem 5. (5 Points)

Consider the total-cross-tied system shown below obtained from the series-parallel array of cells by connecting ties across each column of junctions. A column fails if both cells in that column fail and the system fails as soon as an entire column fails. Therefore, the system lifetime exceeds *100 hours* only if the lifetime of every column does so.



Let A_i denote the event that the lifetime of cell i exceeds 100 hours ($i = 1, 2, \ldots, 6$). Assume that the cells are independent form each other and that $P(A_i) = 0.9$ for every i since the cells are identical. What is the probability that the system lifetime exceeds 100 hours?

$P(Fail) = 1 - P($



$\cancel{P(C_1) = P(C_2) = P(C_3) = (0.9)^2 = 0.81}$

$\cancel{P(\text{all 2,3 exceeds 100 hrs}) = }$

$\cancel{P(x) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}}$

$P(C_1) = P(C_2) = P(C_3) = P(X \geq 1) = 1 - P(X = 0)$

$P(X=0) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$ | $x = 0$, $n = 2$, $p = 0.9$

$= \frac{2!}{0!\,2!} (0.9)^0 (0.1)^2$

$= 0.01$

$P(X \geq 1) = 1 - 0.01 = 0.99$ 2

$$P(\text{System exceeds } 100 \text{ hrs}) = P(C_1) \cdot P(C_2) \cdot P(C_3)$$
$$= 0.99^3 = 0.970$$