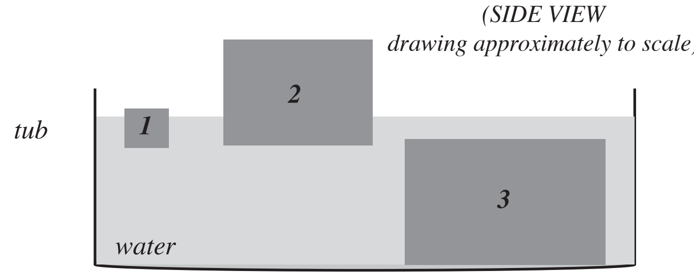
HW 4

Recommended finish date: Monday, April 28, 2014

These HW problems were taken/adapted from past exam problems.
To get an idea of how best to approach various problem types (there are three basic types), refer to these example HW problems.

1. a. **Three blocks are placed in a tub with water, as shown. The blocks have volumes V_1, V_2 and V_3. Only blocks 1 and 2 float. And $V_1 < V_2 < V_3$. Evaluate the following statements (T/F/N).**
As always, explain your reasoning.



(i) **Block 2 has the largest buoyant force acting on it.**

(ii) **Block 1 must be more dense than block 2.**

(iii) **Block 1 must have more mass than block 2.**

b. **Evaluate the following statements (T/F/N). As always, explain your reasoning.**

(i) **Archimedes' Principle states that the buoyant force is the fluid pressure exerted on the underside of any part of any object immersed in that fluid.**

(ii) **For any 1 kg of incompressible fluid, its volume decreases when the pressure on it increases.**

(iii) **If you have two wooden blocks, A and B, with identical outer dimensions (L x W x H), but different densities ($\rho_A > \rho_B$), and both float in the same pool, the buoyant force on each block is the same.**

(iv) **If a bucket of bricks is sitting (fully immersed) on a scale at the bottom of a pool of water, that scale reading is less if the bottom of the pool is 5 m deep than if it is 3 m deep.**

(v) **Any object that is 50% immersed in a fluid has a density that is half of the fluid's density.**

(vi) **If an object's volume increases by 10.0%, then its density decreases by less than 10.0%.**

(vii) **Any object that is 100% immersed in a fluid must have a density greater than the fluid's density.**

(viii) **As an ice cube melts in a glass of water, the uppermost tip of its exposed portion remains at the same height in the glass.**

(ix) **The buoyant force exerted by a fluid on an object is equal to the volume of the fluid displaced by that object.**

1. c. (i) A 579-N lump of steel floats in mercury (Hg). What volume of Hg does it displace? (ρ_{Hg} = 13,600 kg/m^3).

(ii) A 42.2-kg gold statue hangs by a thread, completely immersed in water. The thread tension is 390 N. The density of gold is 19,300 kg/m^3. ***Is the statue pure gold?*** **Show all work and calculations to support your answer.**

2. a. **As you dive toward the bottom of a swimming pool, the pressure increases noticeably. Does the buoyant force also increase? (Assume that water is incompressible.) Explain your answer fully.**

b. **Would you float more easily (i.e. with less of your body immersed) in water on the moon than on earth?**

c. **A glass beaker, filled to the brim with water, is resting on a scale. A block is placed in the water, causing some of it to spill over. The water that spills is wiped away; the beaker is still filled to the brim. Explain fully how and why the initial and final readings on the scale compare if the block is made of...**

(i) wood (ii) iron

d. **You have two identical drinking glasses. One contains free-floating ice cubes, and the other contains no ice. You fill each glass completely to the brim with water. Which glass now weighs more? Explain your answer.**

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e. **Two pencils have the same average density (500 kg/m^3). The mass of the mechanical pencil is twice the mass of the conventional pencil. They are both held completely submerged under the surface of water (1000 kg/m^3) and released. Which accelerates at the greater rate? Explain your answer fully.**

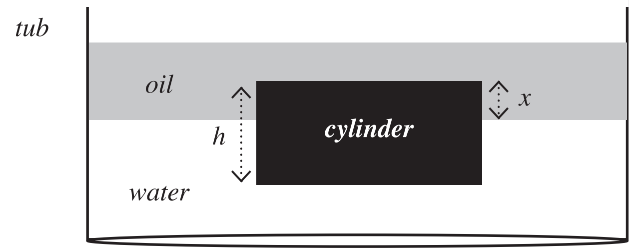
2. f. **A large aluminum salad bowl ($\rho_{aluminum}$ = 2700 kg/m^3), in the shape of a hemisphere of radius 18.0 cm, has a mass of 0.25 kg. You place this bowl, empty and upright (like a boat) into a vat containing pure water. Then you place pieces of wood (ρ_{wood} = 680 kg/m^3) into the bowl until the bowl sinks. What volume of water is now displaced?**

g. **A raft is made of 14 identical logs lashed together. Each log is 35.0 cm in diameter, 9.75 m long, and has a density of 760 kg/m^3. How many 75-kg persons can the raft hold (floating in pure water) while still keeping everyone's feet dry?**

2. h. A solid cylinder (radius = 0.254 m; height = 0.180 m) has a mass of 32.0 kg.

At first, the cylinder is floating freely (with its flat top and bottom horizontal) in a tub of pure water.

Then oil (ρ_{oil} = 690 kg/m^3) is poured into the tub, and the cylinder comes to rest, now fully submerged—but partly in the water, partly in the oil—as shown here. It is still floating (i.e. not touching the bottom of the tub).



(SIDE VIEW – drawing not to scale)

How much of the cylinder (i.e. what part of its height, measured in meters) is in the oil?

(See notes on the next page for more discussion of buoyancy and Archimedes' Principle.)

Why Archimedes' Principle Works

The buoyant force is the <u>net</u> of all fluid forces acting on any part of an object that's in contact with the fluid. Because fluid pressure increases with depth, this net force is always upward, toward the surface of the fluid.

The old reliable way to <u>calculate</u> this net force, F_B, is as the weight of the fluid displaced (Archimedes' Principle), but the <u>physics</u> is always that the buoyant force is the net of the "underneath" fluid pressure forces (acting upward) vs. the "overhead" fluid pressure forces (acting downward); the lateral fluid forces all oppose and cancel one another.

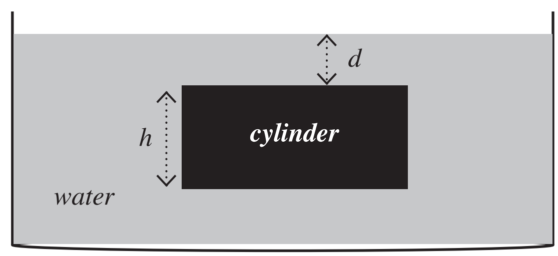
That physics principle is fairly easy to visualize if there's just one fluid and the object is fully immersed, so that we can calculate and compare the upward and downward fluid pressure forces—and the math for Archimedes Principle emerges fairly readily. See <u>Case A</u> below.

But it's not nearly so obvious why Archimedes' Principle still works mathematically when there's no fluid pressure on top at all (the object is only partly immersed). See <u>Case B</u> below.

And it's even less evident in <u>Case C</u> (this HW problem), when there's a combination of two fluids. (Indeed: How can the oil offer any upward forces on the cylinder when no oil contacts the under side? Answer: It doesn't. But the presence of the oil layer increases the <u>water</u> pressure on the underside sufficiently to behave as if it's supporting the weight of the oil that's been displaced.) Below is the solution "from scratch," proving why using Archimedes' Principle is valid even here.

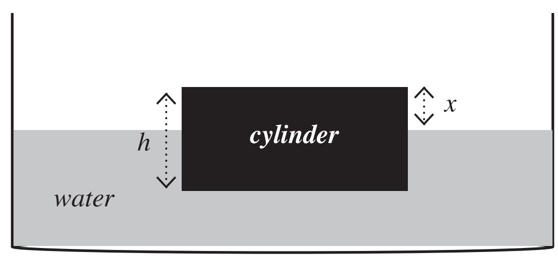
<u>Case A</u>:

$$\begin{aligned}F_B &= F_{bottom} - F_{top}\\ &= P_{bottom}(A) - P_{top}(A)\\ &= [P_{atm} + \rho_{water}g(h+d)](A) - [P_{atm} + \rho_{water}gd](A)\\ &= [\rho_{water}gh](A)\\ &= (\rho_{water}g)Ah\\ &= \rho_{water}gV_{water.displ}\end{aligned}$$



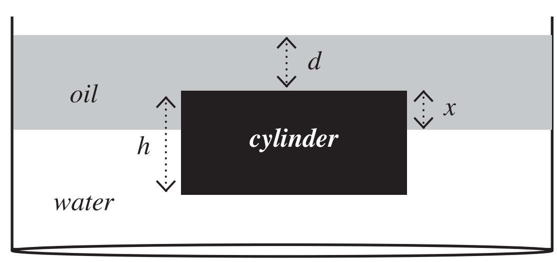
<u>Case B</u>:

$$\begin{aligned}F_B &= F_{bottom} - F_{top}\\ &= P_{bottom}(A) - P_{top}(A)\\ &= [P_{atm} + \rho_{water}g(h-x)](A) - P_{atm}(A)\\ &= [\rho_{water}g(h-x)](A)\\ &= (\rho_{water}g)[A(h-x)]\\ &= \rho_{water}gV_{water.displ}\end{aligned}$$



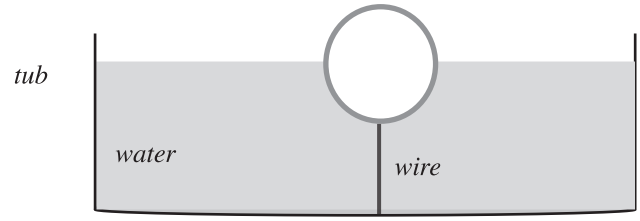
<u>Case C</u>:

$$\begin{aligned}F_B &= F_{bottom} - F_{top}\\ &= P_{bottom}(A) - P_{top}(A)\\ &= [P_{atm} + \rho_{oil}g(d+x) + \rho_{water}g(h-x)](A) - [P_{atm} + \rho_{oil}gd](A)\\ &= [\rho_{oil}gx + \rho_{water}g(h-x)](A)\\ &= \rho_{oil}g[Ax] + \rho_{water}g[(A)(h-x)]\\ &= \rho_{oil}gV_{oil.displ} + \rho_{water}gV_{water.displ}\end{aligned}$$



2. i. **A certain hollow, sealed ball with an outer radius of 12.0 cm will float freely in a tub of pure water, with just 10% of its volume immersed.**

But now a thin anchor wire, connected to the bottom of the tub, is holding that ball at rest, so that the ball is 50% immersed in water, as shown here.



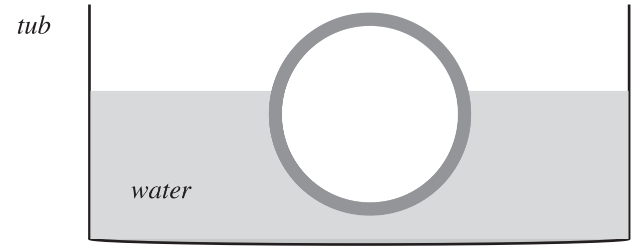
What is the magnitude of the tension force exerted by the wire on the ball?

2. j. A hollow aluminum sphere (outer radius = 0.128 m) floats at rest in a tub of pure water.

The sphere is 66.9% immersed.

The hollow chamber within the aluminum is also a sphere, but it is entirely empty (evacuated).

Find the radius of this hollow chamber.



j. In 1968, engineers retrieved a jettisoned nuclear missile in shallow water in Baffin Bay by attaching two cables to the tip of the missile and raising it slowly ***[slowly; hence, we ignore drag here—and focus on the initial*** a_y ***value]*****.**
Find the magnitude of the missile's ***[initial]*** **upward acceleration with this data:**

- **The missile was completely submerged in seawater of density 1030 kg/m^3.**
- **The missile's mass was 425 kg and its volume was 0.200 m^3.**
- **The tension in each cable was 1750 N.**
- **Each cable made an angle of 30.0° with the vertical.**

3. a. Evaluate the following statements (T/F/N). As always, explain your reasoning.

(i) ρgh is a calculation of gravitational potential energy per unit volume.

(ii) At a certain depth in the Pacific Ocean, the total pressure is P. If you go to twice that depth (treating the seawater as static and incompressible), the total pressure is less than $2P$.

(iii) A manometer can indicate negative gauge pressure.

(iv) If a barometer is 2 m tall, its fluid must have a density less than water.

(v) Gauge pressure is never more than absolute pressure.

(vi) When you sip liquid through a drinking straw, you are creating a space in your mouth that has negative gauge pressure.

(vii) As you place an ice cube in a half-full glass of water (and no water spills out as a result), the water pres sure on the bottom of the glass will increase; and then, as that ice cube melts, the water pressure on the bottom of the glass decreases.

(viii) If water were a <u>compressible</u> fluid, its static pressure at 1 m depth would be greater than it is as an incompressible fluid.

(ix) Neither a soda straw nor a vacuum cleaner would operate effectively on the moon.

(x) "Suction" is a tension force.

b. Evaluate the following statements (T/F/N). As always, explain your reasoning.

(i) Bernoulli's equation equates the mechanical energy density of any two points in a steady flow of incompressible fluid.

(ii) In any steady flow of incompressible fluid, a lower point in the flow always has higher pressure.

(iii) In a steady, vertical flow of incompressible fluid, the fluid pressure in a narrower section of pipe can be greater than in a wider section of pipe.

(iv) In any steady horizontal flow of incompressible fluid, higher pressure is associated with smaller pipe diameter.

3. b. (v) In any steady flow of incompressible fluid, the fluid pressure in a narrower section of pipe is always less than in a wider section of pipe.

(vi) If a strong wind were to blow across the top of a drinking straw sitting in a glass of water, then (assuming the water in the glass is sheltered from this wind) the water level in the straw would rise slightly.

(vii) The mass of an object can also be expressed $\rho_{obj}V_{obj}g$.

(viii) The density of an incompressible fluid may be significantly changed, but not by pressure.

c. What is the gauge pressure at the surface of an open-air swimming pool?

d. What is the gauge pressure in the chamber at the top of a barometer?

e. Express energy density in standard (not base) SI units.

f. Express 3.25 atm in fundamental ("base") SI units.

g. Calculate the fluid pressure potential energy (U_{FL}) contained in 1 cm^3 of pure water at a *[static]* depth of 1.00 m.

4. a. Match the expression on the left, with the one best type of measure that it represents, from the list on the right.

(i) $(1/2)\rho v^2$	*Pressure*
(ii) P/A	*(Nothing meaningful)*
(iii) ρAv	*Mass flow rate*
(iv) ρgh	*Pressure*
(v) PA	*Force*
(vi) Av	*Volume flow rate*
(vii) ρVg	*Weight force*

b. If you were to build a manometer, using pure water as the fluid, how tall would its column need to be to indicate a total pressure of 1.56 atm inside the pressurized container?

c. If you were to build a barometer using a fluid with a density of 6,700 kg/m^3, how tall would its column need to be to measure an air pressure that is 10% greater than standard atmospheric pressure?

d. Evaluate the following statements (T/F/N). As always, explain your reasoning.

(i) The units of $\rho g(\Delta h)$ could be expressed as J/m^3.

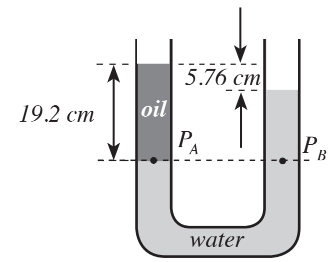
(ii) If all else remains unchanged, when a barometer's (accurate) reading falls, you can sip water through a straw slightly more easily than before.

(iii) The force exerted on a square meter of floor by the earth's atmosphere (at sea level) is greater than the rest weight of a 1.5-meter cube of pure aluminum.

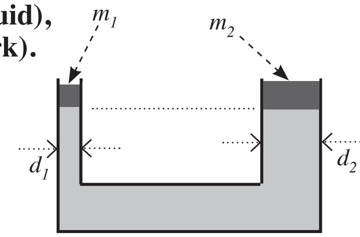
e. A storm gathers over the local neighborhood swimming pool (still filled with pure water). Previously the barometer read exactly 1 atm, but then it drops by 2.75%. By what percentage does the pressure drop at the bottom of the pool, where the depth is 3.00 m?

4. f. **An open tank holds pure water. The pressure at point X in the water is 2.00 atm when the pressure at the surface is 1.00 atm. Find the pressure at X (in atm) when the surface pressure has increased by 7.5%.**

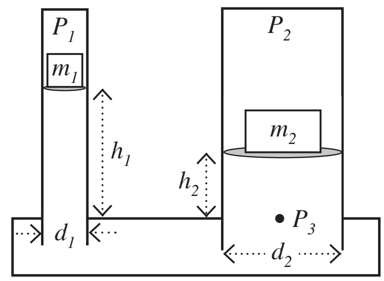
g. **Pure water and then oil are poured into this U-shaped tube, open at both ends, and, as shown here, they come to equilibrium. What is the oil's density?**



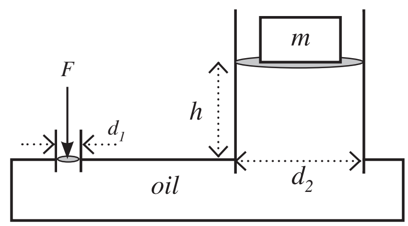
5. a. In the situation shown (where two masses press on equal-height cylinders of fluid), if $d_2 = 3d_1$, and $m_2 = 8m_1$, then is m_2 rising or sinking? Explain (show your work).



b. Evaluate the following statement (T/F/N). *Explain your reasoning.* In the hydraulic lift arrangement shown at right, the two masses sit at rest on massless cylinder heads above a reservoir of incompressible hydraulic fluid. The chambers above the masses are sealed, with gas pressures P_1 and P_2. If $d_2 = 3d_1$, $h_1 = 2h_2$, and $m_2 = 8m_1$, then $P_2 > P_1$.



c. In the diagram shown at right (not to scale), everything is at rest.
$F = 36.0$ N $m = 5{,}000$ kg $d_1 = 4.00$ cm $d_2 = 2.20$ m $h = 2.00$ m
Find the density of the oil.



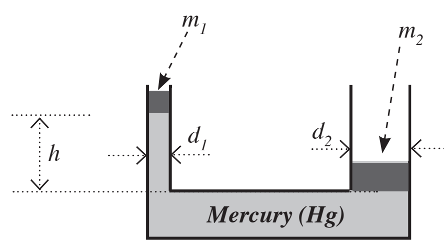
d. Refer again to the diagram above, at right. Again, everything is at rest. Here is the data this time:
$F = 36$ N $m = 5{,}000$ kg $d_1 = 4.00$ cm $d_2 = 2.20$ m $\rho_{oil} = 750$ kg/m^3
By what factor would you need to increase (multiply) F in order to lift m (increase h) by 0.81 m (where everything would again be at rest)? Assume F is still applied in the manner shown.

5. e. A dry wooden log (a cylinder: length = 4.26 m; diameter = 27.8 cm) is placed into a vat of preservative oil. Initially the log floats in the oil with 41.3% of its volume exposed above the level of the liquid. Then gradually, the floating log absorbs 39.1 kg of oil (with no change to the log's volume). As a result, 3/4 of the log's volume is now immersed. Find: (i) The initial mass of the (dry) log. (ii) The density of the oil.

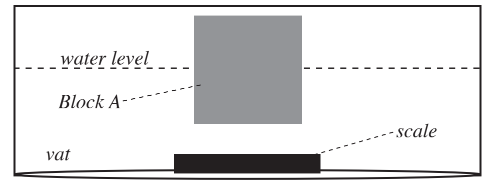
f. Everything is at rest in the initial situation shown. h = 1.42 m; d_1 = .300 m; d_2 = .600 m. Then m_1 is <u>increased</u> by 500 kg, and the system is allowed to come to rest again.

(i) Which column of mercury (the column top) is now higher—and what is the height <u>difference</u> now between the two column tops?

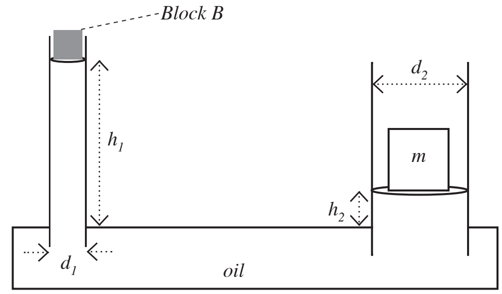
(ii) How much (and in what direction) has each column top moved?



5. g. You have two solid uniform blocks, A and B. As shown in the first diagram, block A (a cube, 0.870 m on each edge) is initially floating, 59.1% immersed in a shallow vat of pure water. Naturally, at the bottom of the vat is a kitchen scale. When block B is set atop block A, the stack of two blocks sinks to the bottom of the vat, coming to rest on the scale. At that point, block A is 87.2% immersed, while block B is still dry. The scale reads 850 N.



If you were to take block B and set it <u>instead</u> on the hydraulic lift mechanism in the position shown here (in the second diagram), what mass, *m*, could it support?



The tank is filled with hydraulic oil of density 892 kg/m^3.
The two cylinders have diameters d_1 = 0.185 m and d_2 = 0.952 m.
The (massless) platforms are at known heights above the top of the tank: h_1 = 5.90 m; h_2 = 1.82 m.

6. a. **If you're pumping water out of a 64 m^3 swimming pool using a hose 15.0 cm in diameter, and the water is flowing in the hose at a rate of 5.72 m/s, how long will it take you to empty the pool?**

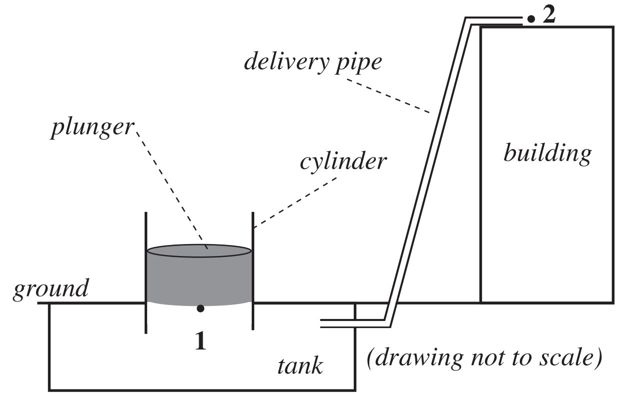
b. **The aorta has a radius of about 1.1 cm, and it carries blood away from the heart at a speed of about 40 cm/s. It branches eventually into a very large number of capillaries that distribute the blood to all cells throughout the body. A capillary typically has a radius of about 6×10^{-4} cm, and blood travels through it at about 0.07 cm/s. Calculate the approximate number of capillaries in the human body.**

c. **In a factory, a pipe that is 2 cm in diameter at ground level delivers water at a rate of 1.5 kg/s to a larger pipe located several meters above. If the gauge pressure at ground level is 2 atm (assign $h = 0$ at ground level), what is the total mechanical energy of one liter (1 L) of water flowing in the larger pipe above?**

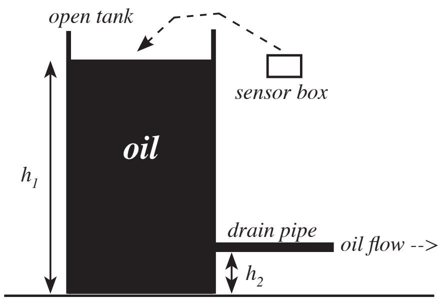
d. **Water flows horizontally through a large, uniform pipe of radius 0.68 m. The water fills the pipe completely, and the flow rate is 1.30 m^3/s. Assuming the same water speed throughout the flow, find the difference in energy contained in 1 L of water flowing at the center vs. 1 L of water flowing along the lowermost surface (the "floor") of the pipe.**

e. **During the 2010 tornado in Aumsville, OR, the pressure outside was 99.0 kPa while the pressure inside the building was 101 kPa. What was the magnitude of the net force caused by this pressure difference on the building's door (which was 1.00 m wide and 2.50 m tall)?**

6. f. Pure water is lifted from a (full) underground tank to the flat roof of a building by means of a cylinder-plunger system, as shown. The plunger has a diameter of 24.0 cm and a mass of 793 kg. Its top is open to the air. The bottom of the plunger is pressing on the water in the tank at ground level and is moving downward at 2.68 cm/s. At the roof level, the delivery pipe has a diameter of 5.10 cm, and it is emptying the water into an open-air basin. How tall is the building?



6. g.



A large tank holds incompressible oil to a depth of h_1. The tank is being drained via a horizontal pipe (radius r), attached at a height h_2 above the tank bottom. Oil is flowing out through the pipe at a flow rate of Q. The descent speed of the oil level at the open-air tank top is negligible (≈ 0). The gauge pressure in the drain pipe is P_2. A sealed box of sensor instruments, of density ρ_s, is to be placed into the oil. What percentage of that box's volume (if any) will remain <u>above</u> the oil surface?

<u>Assume these are known values</u>: $h_1, r, h_2, Q, P_2, \rho_s$

6. i. **Refer to the diagram below. A large, open tank of fluid has a drain pipe leading from it, and fluid is draining through it at a volumetric flow rate Q. The pipe's outlet diameter, d, is known. Also known is the diameter, d_X, of the pipe at point X. The drain rate is such that the motion of the fluid level at the top of the tank is essentially zero. The surface of the fluid in the tank is at a height H (all heights measured above the tank bottom); the outlet pipe is connected to the tank at a height h, and point X is at height h_X.**

Elsewhere in the tank (far away from the drain pipe; the fluid there is essentially motionless), a uniform block (a cube of side s and mass m) is pressed against the bottom of the tank by a force, F, applied vertically downward. As a result, the total pressure on the floor of the tank under the block is P. Find the gauge pressure of the fluid at point X.

<u>Here is a summary of the known values</u>: $Q, d, d_X, H, h, h_X, s, m, F, P, P_{atm}, g$