HW 8

Due: Tuesday, March 4, 2014, 5:00 p.m.

Print *your full* **LAST** *name:* ______________________________

Print *your full* **first** *name:* ______________________________

Print *your HW Grader's name:* ______________________________

What is your HW Grader's box # (located outside of Wngr 234)? ________

"I affirm and attest that this HW assignment is my own work. While I may have had help from (and/or worked with) others, all the reasoning, solutions and results presented in final form here are my own doing—and expressed in my own words."

Sign your name (full signature): ______________________________

Print *today's date:* ______________________________

General Instructions for HW 8

Requirements: The PH 212 HW graders will select **one** out of the six problems. They will then score that problem according to the points rubric posted with the HW 8 solutions (for a total possible 10 points for the entire HW assignment).

Format: ***You are required to include the completed, signed cover sheet—that's the first page of this file—as the front page of your HW submission when you turn it in.*** (Please staple together all pages, in order—cover sheet first—with one staple at upper left.)

Scoring: The rubrics (scoring guides—the points breakdowns) for HW problem solutions will closely match those for exam solutions. And in those rubrics, *when any item asks for an explanation (and most do), it means exactly what it says*. To get full credit, you must include a short but informative verbal explanation (in your own words) of your reasoning.

To get an idea of how best to approach various problem types (there are three basic types), refer to these example HW problems.

1. a. **A traveling wave is described by $D(x,t)$ = (5.00 cm)·sin[(7.85 rad/m)x + (236 rad/s)t)]. Find the wave velocity.**

   $v = f\lambda = \omega/k = 236/7.85 =$ **30.1 m/s**

   b. **A guitar string is plucked and set into vibration. The vibrating string disturbs the surrounding air, resulting in a sound wave. Evaluate the following statements (T/F/N). As always, explain your reasoning.**

   (i) The speeds of the two waves are equal.
   Not enough information. It's possible, that by sheer chance, the speed of the wave on the string could be equal to the speed of sound in air, but these two speeds are not generally related. The string's wave is a transverse displacement wave whose speed depends on the string's tension and mass per meter; the sound in air is a longitudinal displacement wave whose speed depends on several factors, including the temperature of the air (and, by the way, the mass of the air molecules).

   (ii) The wavelengths of the two waves are equal.
   Not enough information. It's not likely, but it's possible, ***if*** by sheer chance, the speeds of the two waves are equal—see item **(i)** above. Then their wavelengths will be equal, too, since their frequencies do match—see item **(iii)** below.

   (iii) The frequencies of the two waves are equal.
   True. The periodic disturbance of the wave passing along the guitar string maximizes once per period; thus so does that string's resulting disturbance of the air.

   (iv) The string wave is transverse. True—see item (i) above.

   (v) The sound wave is transverse. False—see item (i) above.

   (vi) The string wave speed increases if the temperature rises.
   Not enough information. It depends on the thermal expansion properties of whatever the string ends are fixed to. If that material expands less with temperature than the string, then the string will "slack;" its tension (and therefore its wave speed) will decrease. However, if the string's support expands more with temperature than the string, this will indeed increase the string's tension (and therefore its wave speed).

   c. **Evaluate the following statements (T/F/N). As always, explain your reasoning.**

   (i) If you send the the same frequency of sound through 10°C air and through 30°C air, the wavelengths will be longer in the warmer air.
   True. A sound wave travels faster in warmer air, so if v is greater for the same f, then λ must be greater (because $v = f\lambda$).

   (ii) As a sound wave travels westward through air, at any given moment in time, it may include individual particles that are east of their rest positions.
   True. Sound is a longitudinal wave. So air particles are oscillating back and forth sinusoidally (i.e. first west of their equilibrium positions, then east).

2. a. **A wave on a string is described by $y = 0.085\cos[31.4(x) - 47{,}909(t)]$, where position is measured in meters, time is measured in seconds, and the amplitude is measured in mm.**

(i) **If east is the positive x-direction, which direction (north/south/east/west) is this wave traveling?** **East**

(ii) **What property of the medium is being described here?** ***y*-displacement**

(iii) **How much time does the wave require to travel 3 wavelengths?**
It takes a wave 3 periods to travel 3 wavelengths.
In this case, $47909 = 2\pi/T$, so $T = 2\pi/47909$, so $3T = 6\pi/47909 = \mathbf{3.93 \times 10^{-4}\ s}$

b. **A sound wave is described by $P = 8.50\cos[31.4(x) + 47{,}909(t)]$, where position is measured in meters, time is measured in seconds, and the amplitude is measured in Pa.**

(i) **Is this a transverse or longitudinal wave?** Sound is a **longitudinal** wave.

(ii) **If east is the positive x-direction, which direction (north/south/east/west) is this wave traveling?** **West**

(iii) **What property of the medium is being described here?** **Pressure**

(iv) **Traveling in air (under normal atmospheric conditions), a conversation-level sound is produced when this property of the medium is varying by $\pm 3 \times 10^{-5}\%$. If the above sound were indeed traveling in air under normal atmospheric conditions, is it faint, or normal (conversational) level, or really loud? Explain your answer.**
The given pressure wave is varying by 8.50 Pa above or below 1.01×10^5 Pa.
That's $\pm 8.50/101{,}000$, or $8.42 \times 10^{-3}\%$, which is some 280 times the amplitude of conversational sound. This is much **louder** sound.
(Note: Since energy is proportional to the square of the amplitude, this would be about 49 dB louder.)

(v) **Is this sound traveling in air (under standard atmospheric conditions)? Explain your answer.**
The speed of this sound wave is given by $v = \omega/k = 47909/31.4 = 1526$ m/s
That's way too fast for sound traveling in air (≈ 340-350 m/s) under normal atmospheric conditions.
(Note: This would be within typical ranges, however, for the speed of sound in *water*.)

c. **A wave on a string is described by $D(x,t) = (2.00\text{ cm})\cdot\sin[(12.6\text{ rad/m})x - (638\text{ rad/s})t]$, where x is in meters and t is in seconds. The linear mass density of the string is 5.00 g/m.**

(i) **Find the string tension.** $F_T = v^2\mu = (\omega/k)^2\mu = (638/12.6)^2(.005) = \mathbf{12.8\ N}$

(ii) **Find the maximum displacement of a point on the string.** $D_{max} = A = \mathbf{2.00\ cm}$

(iii) **Find the maximum speed of a point on the string.** $v_{max} = A\omega = (2.00\text{ cm/s})(638\text{ rad/s}) = \mathbf{12.8\ m/s}$

3. a. Loudspeaker A is located at $x = 0$, and loudspeaker B is at $x = 5.0$ m. If the speakers emit the same tone (in phase) of wavelength 2 m, find all points along the x-axis between the two speakers which are points of completely Destructive Interference (D.I.).

For complete D.I., $\Delta x = (m + 1/2)\lambda$, where $m = 0, 1, 2, 3...$ Here, $\lambda = 2$, so the condition we're looking for is $\Delta x = (m + 1/2)(2)$, or $\Delta x = 2m + 1$. And $\Delta x = |x_A - x_B| = |x - (5 - x)| = |2x - 5|$ can be no larger than the distance between the speakers, which is 5. So, the only m values that work are: *1 pt.*

$\Delta x = 1$ (when $m = 0$): This would occur at the points **$\underline{x = 2}$** and **$\underline{x = 3}$** (when $|x_A - x_B| = 1$).
$\Delta x = 3$ (when $m = 1$): This would occur at the points **$\underline{x = 1}$** and **$\underline{x = 4}$** (when $|x_A - x_B| = 3$).

(Also—optional, depending on your interpretation of the problem:
$\Delta x = 5$ (when $m = 2$): This would occur at the points **$\underline{x = 0}$** and **$\underline{x = 5}$** (when $|x_A - x_B| = 3$).

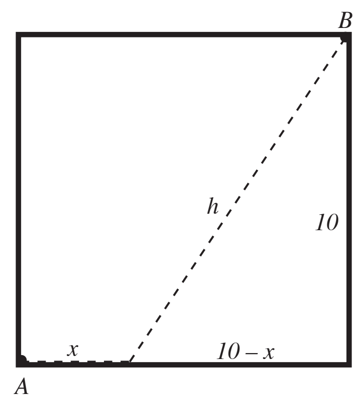
b. Two loudspeakers on a concert stage are vibrating in phase. You are sitting at a distance of 5.50 m from the left speaker and 3.86 m from the right speaker. The speakers are playing music that varies in frequency (i.e. many different frequencies are included in the sound that arrives at your ears), but certain frequencies consistently sound louder to you. If the speed of sound is 343 m/s, what are the 2 lowest such frequencies?

Waves of any given frequency are arriving at your ears via these 2 different paths, and they will sound especially loud to you if they are ***interfering constructively*** (i.e. adding their amplitudes to form a double amplitude). That will occur if the ***path length difference*** is a whole number of wavelengths for that particular frequency: $\Delta r = m\lambda$, where $m = 0, 1, 2, 3...$ (In this case, we already know that $\Delta r \neq 0$, so that means $m \neq 0$ here.) *2 pts.*

Of course, $\lambda = v/f$, so the condition for constructive interference can also be written as $\Delta r = mv/f$. Rearranging, then, to solve for f: $f = mv/\Delta r$ Clearly, the two lowest possible frequencies are for m-values of 1 and 2:

$f = (1)v/\Delta r = 343/(5.50 - 3.86) =$ **<u>209 Hz</u>** $f = (2)v/\Delta r = (2)343/(5.50 - 3.86) =$ **<u>418 Hz</u>**

c. As viewed from above, a square room has speakers in the lower left and upper right corners. Set the lower left corner as the origin $(x, y) = (0, 0)$ and the upper right corner as $(x, y) = (10.0\text{ m}, 10.0\text{ m})$. The sound waves emitted by both speakers are initially in-phase and have a wavelength of 4.00 m. At what location(s) along the wall on the x-axis does destructive interference occur?



For complete D.I., $\Delta r = (m + 1/2)\lambda$, where $m = 0, 1, 2, 3...$
In this situation, $\lambda = 4$, so the condition we're looking for is $\Delta r = (m + 1/2)(4)$, or $\Delta r = 4m + 2$. From the diagram, we see that $\Delta r = h - x = \sqrt{[10^2 + (10 - x)^2]} - x = \sqrt{(200 - 20x + x^2)} - x$, and it can be no larger than the distance between the speakers, which is $10\sqrt{2} = 14.142$ m. So, the only m values that work are: *2 pts.*

$\Delta r = 2$ (when $m = 0$): $\sqrt{(200 - 20x + x^2)} - x = 2$. Solve* for x: **<u>$x = 8.17$</u>**
$\Delta r = 6$ (when $m = 1$): $\sqrt{(200 - 20x + x^2)} - x = 6$. Solve* for x: **<u>$x = 5.13$</u>**
$\Delta r = 10$ (when $m = 2$): $\sqrt{(200 - 20x + x^2)} - x = 10$. Solve* for x: **<u>$x = 2.50$</u>**
$\Delta r = 14$ (when $m = 3$): $\sqrt{(200 - 20x + x^2)} - x = 14$. Solve* for x: **<u>$x = 0.0833$ m</u>**

*(<u>*Note*</u>: The math here calls for a numeric solution on a calculator—this isn't the sort of algebra you would be expected to do on an exam.)

d. You are standing midway between two identical loudspeakers emitting (in phase) a tone of 1300 Hz. The speed of sound is 343 m/s. What minimum (non-zero) distance must you move toward either speaker to reach a point of…

(i) destructive interference (D.I.)? (ii) constructive interference (C.I.)?

The wavelength, λ, here is $343/1300 = 0.26385$ m. Suppose the distance between the speakers is L. When you're midway between them, each sound travels a distance of $L/2$ to reach you. Therefore their path length difference is zero: $\Delta x = L/2 - L/2 = 0$. (And for in-phase speakers, this produces C.I..)
But when you move a distance d from the center (toward one of the speakers), their path length difference is not zero: $\Delta x = (L/2 + d) - (L/2 - d) = 2d$ So, for any desired Δx, you move only <u>half</u> that far: $d = \Delta x/2$
And the smallest non-zero distance d corresponds to the smallest <u>non-zero</u> Δx. *2 pts.*

(i) For destructive interference (D.I.): $\Delta x = (m + 1/2)\lambda$ where m is any integer (including 0)
The smallest non-zero Δx is: $\Delta x = (0 + 1/2)\lambda = \lambda/2$
So the smallest non-zero d is: $d = \Delta x/2 = \lambda/4 = 0.26385/4 =$ **<u>0.0660 m</u>**

(ii) For constructive interference (C.I.): $\Delta x = m\lambda$ where m is any integer (including 0)
The smallest non-zero Δx is: $\Delta x = (1)\lambda = \lambda$
So the smallest non-zero d is: $d = \Delta x/2 = \lambda/2 = 0.26385/2 =$ **<u>0.132 m</u>**

e. Two identical loudspeakers are emitting (in phase) the same sound frequency, f. On x-y coordinate axes, speaker A is located at the point $(-d_A, 0)$; and speaker B is located at the point $(d_B, 0)$. Sound from speaker A arrives at the origin exactly two full cycles sooner than sound from speaker B. Sound from speaker B requires a time Δt to travel all the way to speaker A. <u>Starting from the origin</u>…

(i) What non-zero distance along the <u>positive *y*-axis</u> must you move to arrive at the nearest point of C.I.?

(ii) What non-zero distance along the <u>negative *y*-axis</u> must you move to arrive at the nearest point of D.I.?

It takes two extra cycles for sound B to arrive at the origin (and the time for each cycle is the period, T). So sound B has to travel two extra wavelengths of distance: $d_B - d_A = 2\lambda$

From this result, we see that the origin is a point of Constructive Interference (C.I.), since the path length difference is a whole multiple of the wavelength: $\Delta r = m\lambda$. In this case, $m = 2$.

Now, as we move away from the origin along the y-axis, the path length difference, Δx, will **<u>decrease</u>**. (To see this, use some appropriate example values for d_B, d_A and λ, then choose a new location on the +y-axis and calculate Δr there. It will be <u>less</u> than 2λ.)

So the nearest point of C.I. on the positive y-axis will be where $m = 1$:

$\Delta r = \sqrt{(d_B^2 + y_{CI}^2)} - \sqrt{(d_A^2 + y_{CI}^2)} = (1)\lambda = (d_B - d_A)/2$ You could solve* this for y_{CI}.

Likewise, the nearest point of D.I. on the negative y-axis will be where $m = 1.5$:

$\Delta r = \sqrt{(d_B^2 + y_{DI}^2)} - \sqrt{(d_A^2 + y_{DI}^2)} = (0.75)(d_B - d_A)$ You could solve* this for y_{DI}.

*(<u>Note</u>: This isn't the sort of solving you would be expected to do on an exam. If you had numeric values for d_A and d_B, you'd probably just use a calculator to find a solution.)

2 pts.

f. Loudspeaker A is located on the *x*-axis at (126, 0). Loudspeaker B is located on the *y*-axis at (0, 161). (All coordinates are in cm.) Each speaker emits the same single sound frequency, but when both speakers emit together, they are exactly out of phase with each other. The sound heard at the origin when both speakers are emitting is very faint—much less than either speaker sounding by itself. Could the sound wavelength be 14.0 cm? *Explain your reasoning fully.*

<u>No, the wavelength could not be 14.0 cm</u>. The total phase angle difference between the two waves is given by $\Delta\phi = [\Delta\phi_0 + 2\pi(\Delta r/\lambda)]$. And since they are already out of phase at their starts, we know that $\Delta\phi_0 = \pi$ rad. So if their path length difference *doesn't change* that initial phase difference, they will arrive at the origin *still out of phase* (thus Destructive Interference—the faint sound described). *So the path length difference must be a whole multiple of* λ. Is it? Check: $\Delta r = 161 - 126 = 35$ cm. This is <u>not</u> a whole multiple of 14 cm. (In fact, this is an extra half-λ: $\Delta r/\lambda = 35/14 = 2.5$. So that phase difference ($2.5 \cdot 2\pi = 5\pi$), combined with the out-of-phase start ($\Delta\phi_0 = \pi$ rad), would result in $\Delta\phi = 6\pi$, which would result in C.I.— extra <u>loud</u> sound.)

1 pt.

4. a. Evaluate the following statements (T/F/N). As always, explain your reasoning.

(i) If 2 loudspeakers are emitting one pitch but vibrating exactly out of phase, the point midway between them will have constructive interference (C.I.).

<u>**False**</u>**.** Your ear can detect very slow variations in volume, which is what is varying in a beat frequency.

(ii) Beat frequency is the frequency at which the sound <u>volume</u> varies when two close frequencies are sounded together.

<u>**True**</u>—this is its definition. A beat frequency results from the algebraic addition of two close-but-not-identical frequencies. That addition produces a *varying* ***amplitude*** (as the addition sometimes produces extra positive, extra negative, or near-zero amplitude) which we hear as varying volume (loudness) in our ears.

(iii) You cannot hear a beat frequency less than about 20 Hz.

<u>**False**</u>**.** Your ear can detect very slow variations in volume, which is what is varying in a beat frequency.

(iv) The path length difference that results in simple Constructive Interference is unitless.

<u>**False**</u>**.** Path length difference is indeed a *difference of lengths*, so <u>**it has units of length**</u> (m, for example).

(v) When one sound wave's frequency nearly matches another's, you hear C.I., then D.I., alternating over time. The frequency of this time-varying amplitude is called the beat frequency.

<u>**True**</u>—this is its definition. See item **(ii)** above.

b. Two identical wires are stretched by the same 100 N tension. Each emits a fundamental frequency of 200 Hz when plucked. Then the tension in one wire is increased by 1 N. Calculate the resulting beat frequency heard when both wires are then plucked.

$f_{1.f}/f_{1.i} = [(1)v_f/(2L)]/[(1)v_i/(2L)] = v_f/v_i = \sqrt{[F_{T.f}/\mu]}/\sqrt{[F_{T.i}/\mu]} = \sqrt{[F_{T.f}/F_{T.i}]} = \sqrt{(101/100)}$

Thus: $f_{1.f} = f_{1.i}\sqrt{(101/100)}$ Then $f_{beat} = f_{1.f} - f_{1.i} = f_{1.i}[\sqrt{(101/100)} - 1] = 200[\sqrt{(101/100)} - 1] =$ <u>**0.998 Hz**</u>

c. Two guitar strings (each fixed at both ends) with identical lengths and identical linear mass densities are vibrating at their fundamental frequencies. String A is vibrating at 180 Hz, but because both strings vibrating, you hear a beat frequency of 5 hz. If you were to tighten (increase the tension of) string B, that would increase the beat frequency. But you're allowed to adjust only string A. By what percentage must you tighten (increase the tension of) string A to equalize the frequencies of the two strings?

Tightening string B increases its frequency. If this increases the beat frequency (the difference between the two frequencies), this tells us that $f_B > f_A$ *already*. So f_B = 185 Hz, and you must increase f_A from 180 Hz to 185 Hz.

Thus: $f_{A.f}/f_{A.i} = 185/180 = \sqrt{[F_{T.A.f}/\mu_A]}/\sqrt{[F_{T.A.i}/\mu_A]}$

So: $[F_{T.A.f}/\mu_A]/[F_{T.A.i}/\mu_A] = (185/180)^2$ So: $F_{T.A.f}/F_{T.A.i} = (185/180)^2 =$ <u>**1.0563**</u>

You must increase the tension of string A by 5.63%.

d. Two lasers can be used to generate a beat frequency of 98.5 MHz. The laser with the higher frequency has a wavelength of 780.54510 nm.

(i) What is the period the beats?

$T = 1/f = 1/(98.5 \times 10^6) = \mathbf{1.02 \times 10^{-8}\ s}$

(ii) Find the wavelength of the other laser.

The known frequency is $f_A = c/\lambda_A = (3.00 \times 10^8)/(780.54510 \times 10^{-9}) = \mathbf{3.8434679 \times 10^{14}\ Hz}$

So the other frequency is 98.6 MHz less: $f_B = 3.8434679 \times 10^{14} - 98.5 \times 10^6 = 3.8434669 \times 10^{14}$ Hz

So the other wavelength is: $\lambda_B = c/f_B = (3.00 \times 10^8)/(3.8434669 \times 10^{14}) =$ **780.54530 nm**

(iii) Explain briefly what is observed at the detector.

The detector is receiving light of a frequency that is the <u>average</u> of the two lasers: $f = 3.8434674 \times 10^{14}$ Hz, but this light is fluctuating in <u>**intensity**</u> (brightness) with a frequency of 98.5 MHz.

5. **a.** **A light beam shines through a thin slit and illuminates a distant screen. The central bright fringe on the screen is 1.00 cm wide, as measured between the dark fringes that border it on either side.**
As always, explain your reasoning: **T/F/N?**

(i) Decreasing the wavelength of the light would decrease the width of the central bright fringe.
True: The width of the central bright fringe is $w \approx 2\lambda L/a$, where L is the distance from the slit to the screen and a is the slit width.

(ii) Decreasing the width of the slit would decrease the width of the central bright fringe.
False: The width of the central bright fringe is $w \approx 2\lambda L/a$, where L is the distance from the slit to the screen and a is the slit width.

b. **A diffraction grating with 10,000 openings/cm is used to determine the wavelength of light from a laser. If the 1st-order bright fringe is 50.0 cm from the center of the viewing screen, and the viewing screen is 86.6 cm from the diffraction grating, then what is the wavelength? Assume the diffraction grating and screen are parallel.**

In a diffraction grating, the angle θ_m to the "*m*th" bright fringe, measured from the central axis, is given by $\sin\theta = m\lambda/d$, where d is the slit separation. Here $d = .01/10{,}000 = 10^{-6}$ m, $\theta = \tan^{-1}(50.0/86.6) = 30.00°$ and $m = 1$.
So: $\lambda = (10^{-6})\sin 30°/1 = 5.00 \times 10^{-7}$ m $=$ **<u>500 nm</u>**

c. **Light of wavelength 415 nm passes through the diffraction grating described in the previous problem and makes an interference pattern on the viewing screen described in the previous problem. How many bright fringes appear on the viewing screen? Assume the diffraction grating and screen are again 86.6 cm apart and parallel.**

In a diffraction grating, the angle θ_m to the "*m*th" bright fringe, measured from the central axis, is given by $\sin\theta = m\lambda/d$, where d is the slit separation. But θ can get no greater than 90°, so $m_{max}\lambda/d \le \sin 90° = 1$.
So $m_{max} \le d/\lambda = (10^{-6})/(415 \times 10^{-9}) = 2.4$. So $m_{max} = 2$. So, *counting the central fringe and the other bright fringes on either side*, there are a total of **<u>5 bright fringes</u>**.

d. **In a double-slit experiment, the slits are 2.00 mm apart. A mixture of two wavelengths of light shines on the slits and this light is diffracted though the slits. The wavelengths are λ_1 = 750.0 nm and λ_2 = 900.0 nm. The two interference patterns share a common central maximum. On a screen positioned 2.00 m from the slits, at what minimum distance from the central maximum will a bright fringe of one wavelength light coincide (overlap exactly) with a bright fringe of the other wavelength light?**

In double-slit diffraction, the angle θ_m to the "*m*th" bright fringe, measured from the central axis, is given by $\sin\theta = m\lambda/d$, where d is the slit separation. So we want $\theta_1 = \theta_2$, which means $m_1\lambda_1/d = m_2\lambda_2/d$
That is: $m_1/m_2 = \lambda_2/\lambda_1 = 900/750 = 6/5$ Thus: $\sin\theta_1 = 6\lambda_1/d = 6(750 \times 10^{-9})/.002 = .00225$
Thus: $\theta_1 = 0.128916°$ and $\tan\theta_1 = y/2$. So $y = 2\tan(0.128916°) = 0.00450$ m $=$ **<u>4.50 mm</u>**

e. **Two sources of light illuminate a double slit simultaneously. One has wavelength 630 nm and the second has an unknown wavelength. The *m* = 5 bright fringe of the unknown wavelength overlaps the *m* = 4 bright fringe of the light of 630 nm wavelength. What is the unknown wavelength?**

In double-slit diffraction, the angle θ_m to the "*m*th" bright fringe, measured from the central axis, is given by $\sin\theta = m\lambda/d$, where d is the slit separation. So we want $\theta_1 = \theta_2$, which means $m_1\lambda_1/d = m_2\lambda_2/d$
That is: $m_1/m_2 = \lambda_2/\lambda_1$ But: $m_1/m_2 = 4/5$
Thus: $4/5 = \lambda_2/\lambda_1$ Or: $\lambda_2 = (4/5)\lambda_1 = (4/5)630 =$ **<u>504 nm</u>**

6. a. **When white sunlight is incident perpendicularly on soap bubbles, a certain thickness of soap film ($n \approx 1.33$) brightly reflects OSU orange light ($\lambda_{vac.}$ = 590.0 nm), but it doesn't reflect much UO green-yellow light ($\lambda_{vac.}$ = 540.83 nm). Evaluate the following statements (T/F/N). As always, explain your reasoning.**

(i) The orange light travels a whole number of wavelengths in the film.

False. Of the two rays reflecting (off the top and bottom of the film), only the top ray undergoes a phase shift. So if the other ray (the one traveling in the film) were to traverse exactly a whole number of wavelengths, that would leave the two rays' phase angles differing by π radians when they arrive at your eye—*that would be D.I.*, not the C.I. described above.

(ii) The soap film thickness could be about 450 nm.

False. By the logic in **(i)**, the condition for D.I. is $2t = m\lambda_{film}$, or $2t = m\lambda_{vacuum}/n_{film}$.

That is, there will be D.I. if $2tn_{film}/\lambda_{vacuum} = m$, where m is some integer value.

This is *not true* for the green-yellow light, which is stipulated as undergoing D.I.:

$(2)(450 \times 10^{-9})(1.33)/(540.83 \times 10^{-9}) = 2.21$

(iii) The soap film thickness could be about 1.22 μm.

True. By the logic in **(i)**, the condition for D.I. is $2t = m\lambda_{film}$, or $2t = m\lambda_{vacuum}/n_{film}$.

That is, there will be D.I. if $2tn_{film}/\lambda_{vacuum} = m$, where m is some integer value.

This is *true* for the green-yellow light, which is stipulated as undergoing D.I.:

$(2)(1.22 \times 10^{-6})(1.33)/(540.83 \times 10^{-9}) = 6.00$

Likewise, there will be C.I. if $2tn_{film}/\lambda_{vacuum} = m + 1/2$, where m is some integer value.

This is *true* for the orange light, which is stipulated as undergoing C.I.:

$(2)(1.22 \times 10^{-6})(1.33)/(590.0 \times 10^{-9}) = 5.50$

b. **A disabled tanker leaks kerosene (n = 1.20) into the ocean, creating a layer on top of the water (n = 1.30). You are in an airplane looking straight down while the sun is overhead. The thickness of the layer of kerosene is 460 nm. What wavelengths of visible (400-700 nm) light will be brightly reflected?**

We're looking for C.I. from a double-reflection that involves 2 phase shifts (one for each reflection—off the top and off the bottom of the layer of kerosene), because each reflection is off the boundary to a medium slower (higher n value) than the current.

Thus, for a film thickness t: $2t = m\lambda_{film}$, where $m = 1, 2, 3, \ldots$ and $\lambda_{film} = \lambda_{vacuum}/n_{film}$

Thus: $2t = m\lambda_{vacuum}/n_{film}$

Or: $\lambda_{vacuum} = 2tn_{film}/m = (2)(460 \times 10^{-9})(1.20)/m = (1104 \text{ nm})/m$

Note: m must be an integer, so only $m = 2$ will give a λ_{vacuum} in the visible range:

$\lambda_{vacuum} = (1104 \text{ nm})/2 =$ **552 nm**

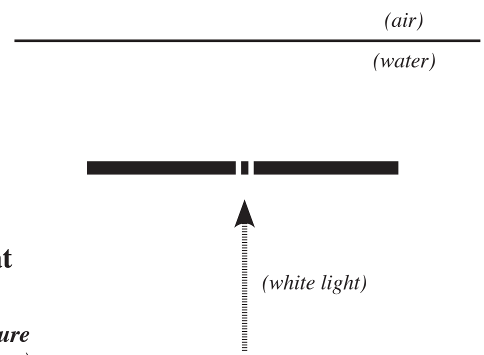
6. c. A white light source is submerged in water and sent vertically upward through two submerged parallel slits, as shown (<u>not</u> to scale). As a result, incident on the water surface is a central maximum (white light), plus various bright and dark fringes in a range of visible colors.

The following question relates to the <u>first set of colored fringes</u> to the left of (<u>adjacent to</u>) the white central maximum.

When the distance between the slits is 1.000 μm, the incidence angles on the surface of the water by blue light (f = 6.379 x 10^{14} Hz) and red light (f = 4.542 x 10^{14} Hz) <u>differ</u> by 9.47°.

If the index of refraction of the blue light in the water is 1.341, at what speed is the red light transmitted through the water?

(For this problem, please include four significant digits in your final answer. Also, be sure to set your calculator back to degrees mode if you were using it for radians trig elsewhere.)

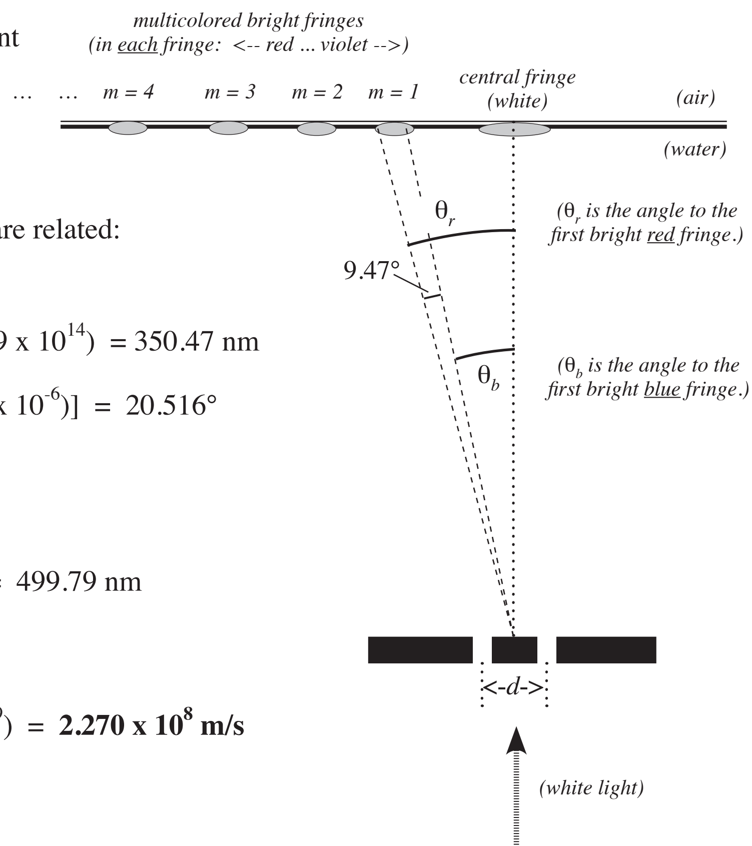


Refer to the more detailed drawing here.

At the first bright multi-colored fringe adjacent to the central white fringe, m = 1, as shown:

$$\sin(\theta_b) = m\lambda_b/d = \lambda_b/d$$

and $$\sin(\theta_r) = m\lambda_r/d = \lambda_r/d$$



Frequency, wavelength and (indexed) speed are related:

$$\lambda_b = v_b/f_b = (c/n_b)/f_b$$

$$= (2.998 \text{ x } 10^8)/1.341]/(6.379 \text{ x } 10^{14}) = 350.47 \text{ nm}$$

Therefore: $\theta_b = \sin^{-1}[(350.47 \text{ x } 10^{-9})/(1.00 \text{ x } 10^{-6})] = 20.516°$

And so: $\theta_r = 20.516° + 9.47° = 29.986°$

And so: $\lambda_r = d\sin(\theta_r)$

$$= (1.00 \text{ x } 10^{-6})\sin(29.986°) = 499.79 \text{ nm}$$

Then: $v_r = f_r\lambda_r$

$$= (4.542 \text{ x } 10^{14})(499.79 \text{ x } 10^{-9}) = \mathbf{2.270 \text{ x } 10^8 \text{ m/s}}$$