

## HW 1

Due: Monday, January 13, 2014, 5:00 p.m.

**Print** your full **LAST** name: \_\_\_\_\_

**Print** your full **first** name: \_\_\_\_\_

**Print** your HW Grader's name: \_\_\_\_\_

What is your HW Grader's box # (located outside of Wngr 234)? \_\_\_\_\_

"I affirm and attest that this HW assignment is my own work. While I may have had help from (and/or worked with) others, all the reasoning, solutions and results presented in final form here are my own doing—and expressed in my own words."

Sign your name (full signature): \_\_\_\_\_

**Print** today's date: \_\_\_\_\_

## General Instructions for HW 1

### Requirements:

The PH 212 HW graders will select **one** out of the six problems. They will then score that problem according to the points rubric posted with the HW 1 solutions (for a total possible 10 points for the entire HW assignment). The optional review items offered on pages 7-8 are just for practice; they will not be scored (but solutions will be posted along with all the other HW 1 solutions).

### Format:

The formats (type, length, scope) of the HW problems have been purposely created to closely parallel those of a typical exam (indeed, most HW problems have been taken from past exams).

***You are required to include the completed, signed cover sheet—that's the first page of this file—as the front page of your HW submission when you turn it in.*** (Please staple together all pages, in order—cover sheet first—with one staple at upper left.)

### Scoring:

The rubrics (scoring guides—the points breakdowns) for HW problem solutions will closely match those for exam solutions. And in those rubrics, *when any item asks for an explanation (and most do), it means exactly what it says.* To get full credit, you must include a short but informative verbal explanation (in your own words) of your reasoning.

***To get an idea of how best to approach various problem types (there are three basic types), refer to these [example HW problems](#).***

1. a. (i) Person A and Person B are both on Earth in the northern hemisphere. Person A is further north (i.e., at a higher latitude) than Person B. Considering only the rotational motion of the Earth on its axis, how do their angular speeds compare? How do their translational speeds compare?

$$\omega_A = \omega_B = \omega_{\text{earth}} \quad (\text{All points on a rotating body have the same angular speed.})$$

$$v_B > v_A \quad (\text{Person A is nearer to the axis of rotation than is Person B.})$$

1 pt.

- (ii) 1 day = 24 hrs. Find the translational speed (around the earth's center) in mph, of a point on the equator ( $r_{\text{earth}} \approx 6.38 \times 10^6 \text{ m}$ ). Find your own translational speed around the earth's center (again, in mph), assuming that you're sitting at rest with respect to the earth, in Corvallis at  $44.56^\circ \text{ N}$  latitude.

$$|\omega_{\text{earth}}| = |\Delta\theta/\Delta t = (2\pi \text{ rad}/24 \text{ hr})(1 \text{ hr}/60 \text{ min})(1 \text{ min}/60 \text{ s}) = 7.2722 \times 10^{-5} \text{ rad/s}$$

$$|v_{\text{Tequator}}| = r_{\text{equator}}|\omega_{\text{earth}}| = (6.38 \times 10^6)(7.2722 \times 10^{-5}) = 464 \text{ m/s}$$

$$464 \text{ m/s}(3600 \text{ s/hr})(1 \text{ mile}/1609 \text{ m}) = 1040 \text{ mph}$$

$$r_{\text{Corvallis}} = r_{\text{equator}}(\cos 44.56^\circ), \text{ so } v_{\text{TCorvallis}} = v_{\text{Tequator}}(\cos 44.56^\circ) = 1040(\cos 44.56^\circ) = 740 \text{ mph}$$

1 pt.

- (iii) A wheel centered at the origin turns through a displacement of  $-8.00$  radians. A bug riding on the wheel rim starts this trip on the  $+x$ -axis. Find its final angular position, in degrees above or below the  $+x$ -axis.

$$-8.00 \text{ rad} = -8.00[360/(2\pi)]^\circ = -458.4^\circ = -(360 + 98.4)^\circ$$

So:  $98.4^\circ$  below the positive  $x$ -axis (or  $262^\circ$  above the positive  $x$ -axis)

1 pt.

- (iv) Two bugs are riding on a meter stick rotating about its  $50.0\text{-cm}$  mark. Bug A is riding on the  $40.0\text{-cm}$  mark. Bug B is riding on the  $75.0\text{-cm}$  mark. How many times faster is bug B traveling (translating) than bug A?

$$\omega_A = \omega_B = \omega_{\text{stick}} \quad (\text{All points on a rotating body have the same angular speed.})$$

$$\text{But: } v_{TA} = r_A\omega \quad \text{and} \quad v_{TB} = r_B\omega \quad \text{Thus: } v_{TB}/v_{TA} = r_B\omega/r_A\omega = r_B/r_A = 25/10 = 2.50$$

Bug B is traveling 2.50 times faster than bug A.

1 pt.

- (v) An auto race is held on a circular track. A car completes one lap in a time of  $19.3 \text{ s}$ , with an average translational speed of  $45.2 \text{ m/s}$ . What is the radius of the track, and what was the car's average angular speed for that lap? (In this problem, it turns out that the second part is actually easier to do first.)

One lap is  $2\pi$  radians. So the race car is sweeping out  $2\pi$  radians every  $19.3$  seconds.

$$\omega_{\text{avg}} \text{ is computed simply as } \omega_{\text{avg}} = \Delta\theta/\Delta t = 2\pi/19.3 = 0.326 \text{ rad/s.}$$

$$v_{T\text{avg}} = r\omega_{\text{avg}}. \quad \text{Therefore, } r = v_{T\text{avg}}/\omega_{\text{avg}} = 45.2/(2\pi/19.3) = 139 \text{ m.}$$

1 pt.

- b. (i) Two persons start at the same place but walk around a circular lake in opposite directions. Their angular speeds are  $1.90 \times 10^{-3} \text{ rad/s}$  and  $3.10 \times 10^{-3} \text{ rad/s}$ . How much time elapses before they meet?

The time interval is the same for both walkers; it's the same value of  $\Delta t$  for both motions. But neither walker travels the entire  $2\pi$  radians; their combined travel sums to  $2\pi$  radians. And we're not interested in directions; they have opposite directions, but it's just the number of radians each sweeps out.

So we write this with absolute values:  $|\Delta\theta_A| + |\Delta\theta_B| = 2\pi$

$$\text{We know that } \Delta\theta_A = \omega_{A,i}(\Delta t) + (1/2)\alpha_A(\Delta t)^2 \quad \text{and} \quad \Delta\theta_B = \omega_{B,i}(\Delta t) + (1/2)\alpha_B(\Delta t)^2$$

$$\text{Since both walking speeds are steady: } \omega_{A,i} = \omega_{A,f} = \omega_A \quad \text{and} \quad \omega_{B,i} = \omega_{B,f} = \omega_B$$

$$\text{And } \alpha_A = \alpha_B = 0, \text{ so the equations reduce to: } \Delta\theta_A = \omega_A(\Delta t) \quad \text{and} \quad \Delta\theta_B = \omega_B(\Delta t)$$

$$\text{Now just substitute: } |\omega_A(\Delta t)| + |\omega_B(\Delta t)| = 2\pi$$

$$\text{Solve for } \Delta t: \Delta t = 2\pi/(|\omega_A| + |\omega_B|) = 2\pi/(1.90 \times 10^{-3} + 3.10 \times 10^{-3}) = 1.25 \times 10^3 \text{ s } (= 20.9 \text{ min})$$

1 pt.

- (ii) A wheel accelerates at a constant rate from  $30 \text{ rpm}$  to  $90 \text{ rpm}$ , turning through  $8.00$  radians in the process. Find its angular speed  $1.00 \text{ s}$  after this process begins.

$$\text{First, convert the disk's initial and final angular velocities to rad/s: } \omega_{\text{rpm}}(2\pi \text{ rad}/1 \text{ rev})(1 \text{ min}/60 \text{ s}) = \omega_{\text{run}}$$

$$\text{Thus: } \omega_i = 30(2\pi/60) = \pi \text{ rad/s} \quad \omega_f = 90(2\pi/60) = 3\pi \text{ rad/s}$$

What acceleration would speed up the disk like that, while it rotates just 8 radians? Find out....

$$\omega_f^2 = \omega_i^2 + 2\alpha(\Delta\theta) \quad \text{Solve for } \alpha: \alpha = (\omega_f^2 - \omega_i^2)/(2\Delta\theta) = [(3\pi)^2 - \pi^2]/(2 \cdot 8) = \pi^2/2 \text{ rad/s}^2$$

What angular velocity would that acceleration produce after just  $1.0 \text{ s}$ ?

$$\text{Find out: } \omega_f = \omega_i + \alpha(\Delta t) = \pi + (\pi^2/2)(1.00) = 8.08 \text{ rad/s}$$

1 pt.

- (iii) **Exercise 4, page 348:** An 18-cm-long bicycle crank arm, with a pedal at one end, is attached (at its hub) to a 20-cm-diameter sprocket (the toothed disk around which the chain moves). A cyclist riding this bike increases her pedaling rate from 60 rpm to 90 rpm in 10s.

**What is the tangential acceleration of the pedal?**

Convert the pedal's angular velocities to rad/s:  $\omega_{rpm}(2\pi \text{ rad/1 rev})(1 \text{ min/60 s}) = \omega_{run}$

Thus:  $\omega_i = 60(2\pi/60) = 2\pi \text{ rad/s}$   $\omega_f = 90(2\pi/60) = 3\pi \text{ rad/s}$

What angular acceleration would speed up the pedal like that, in 10 s? Find out....

$\omega_f = \omega_i + \alpha(\Delta t)$  Solve for  $\alpha$ :  $\alpha = (\omega_f - \omega_i)/(\Delta t) = (3\pi - 2\pi)/(10) = \pi/10 \text{ rad/s}^2$

And then for the pedal:  $|a_{T,pedal}| = r_{pedal}|\alpha| = 0.18(\pi/10) = \mathbf{0.0565 \text{ m/s}^2}$

1 pt.

**What length of chain passes over the top of the sprocket during this interval?**

The travel of the chain is the same as the arc-length travel of the toothed rim of the sprocket, which is given by  $|s_{rim}| = r_{rim}|\Delta\theta|$ , and we can use kinematics to find  $\Delta\theta$ :

$\Delta\theta = \omega_i(\Delta t) + (1/2)\alpha(\Delta t)^2 = (2\pi) \cdot 10 + (1/2)(\pi/10)(10)^2 = 25\pi \text{ rad}$

Thus:  $|s_{rim}| = r_{rim}|\Delta\theta| = 0.100(25\pi) = \mathbf{7.85 \text{ m}}$

- (iv) **When an electric turntable is first switched on, it accelerates at a constant rate through (+)1.25 revolutions in order to attain its full operating speed of 78.0 rpm. At the moment the turntable is switched on, a speck of dust is located at an angular position of (+)36.0° with respect to the +x-axis. Find the speck's angular position, in radians, with respect to that same axis 4.90 seconds after the turntable was switched on. Express your answer between 0 and  $2\pi$  radians.**

**I.** Find the full-speed angular velocity,  $\omega_f$ :  $\omega_f = (2\pi/60)(f_{rpm}) = (2\pi/60)(78.0) = 8.1681 \text{ rad/s}$

**II.** Find the angular acceleration,  $\alpha$ , of the turntable (from rest to full speed):

Use this equation:  $\omega_f^2 = \omega_i^2 + 2\alpha(\Delta\theta_{accel})$  Solve for:  $\alpha$   $\alpha = 4.2474 \text{ rad/s}^2$

Use these knowns:  $\omega_i = 0$  (given);  $\omega_f = 8.1681$  (from **part I**)

$\Delta\theta_{accel} = 1.25(2\pi) = 7.8540$  (given)

**III.** Find the acceleration time,  $\Delta t_{accel}$ , of the turntable (from rest to full speed):

Use this equation:  $\omega_f = \omega_i + \alpha(\Delta t_{accel})$  Solve for:  $\Delta t_{accel}$   $\Delta t_{accel} = 1.9231 \text{ s}$

Use these knowns:  $\omega_i = 0$  (given);  $\omega_f = 8.1681$  (from **part I**)

$\alpha = 4.2474$  (from **part II**)

2 pts.

**IV.** Find the full-speed run time,  $\Delta t_{run}$ :  $\Delta t_{run} = \Delta t_{total} - \Delta t_{accel} = 4.90 - 1.9231 = 2.9769 \text{ s}$

**V.** Find the full-speed displacement,  $\Delta\theta_{run}$ :  $\Delta\theta_{run} = (\omega_f)\Delta t_{run} = (8.1681)(2.9769) = 24.32 \text{ rad}$

**VI.** Find the total displacement,  $\Delta\theta_{total}$ :  $\Delta\theta_{total} = \Delta\theta_{accel} + \Delta\theta_{run} = 7.8540 + 24.32 = 32.17 \text{ rad}$

**VII.** Add the speck's starting angle:  $\theta_f = \theta_i + \Delta\theta_{total} = (36/360)(2\pi) + 32.17 = 32.798 \text{ rad}$

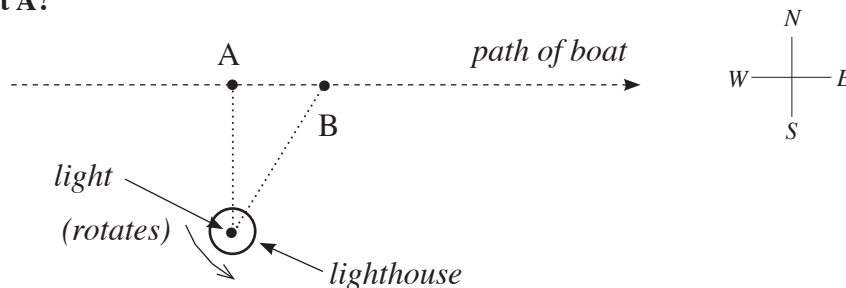
**VIII.** Divide by  $2\pi$  to discard whole revolutions:  $32.798/2\pi = 5.220 \text{ rev.}$

So the dust speck's final position is at (+)0.220 rev.

**IX.** Convert this partial revolution back to radians:  $(0.220)2\pi = \mathbf{1.38 \text{ rad}}$  (or  $\mathbf{0.440\pi \text{ rad}}$ )

2. a. A lighthouse sits on a shoreline, rotating its light at a constant rate of 12.0 rpm in the direction shown in the overhead view below (drawing is not to scale). A speedboat is moving eastward, parallel to the shore, along the path shown, with a constant acceleration of  $3.78 \text{ m/s}^2$  (it's speeding up). At a certain moment, the boat is at point A, directly north of the lighthouse—and the light is shining on it. The next time the light shines on the boat is 4.56 seconds later, and the boat is at point B. Point A is 109 m from the light.

How fast was the boat moving at point A?



- I. Find the angular velocity,  $\omega$ , of the light:

Use this equation:  $\omega = (2\pi/60)(f_{rpm})$

Use these knowns:  $f_{rpm} = 12.0$  (given)

Solve for:  $\omega$  (1.2566 rad/s)

- II. Find the angle swept out by the light between the successive illuminations of the boat:

Use this equation:  $\Delta\theta = \omega_i(\Delta t) + (1/2)\alpha(\Delta t)^2$

Use these knowns:  $\omega_i = 1.2566$  (from part I)

Solve for:  $\Delta\theta$  (5.7303 rad)

$\Delta t = 4.56$  (given)

$\alpha = 0$  (rotation rate,  $\omega$ , is constant)

- III. Find the angle,  $\phi$  (in degrees) shown in the diagram:

Use this equation:  $\phi = (2\pi - \Delta\theta)[360/(2\pi)]$

Use these knowns:  $\Delta\theta = 5.7303$  (from part II)

Solve for:  $\phi$  (31.68°)

- IV. Find the distance from point A to point B:

Use this equation:  $d_{AB}/d_{light.A} = \tan\phi$

Use these knowns:  $d_{light.A} = 109$  (given)

Solve for:  $d_{AB}$  (67.27 m)

$\phi = 31.68$  (from part III)

- V. Find the speed of the boat at point A:

Use this equation:  $\Delta x = v_i(\Delta t) + (1/2)a(\Delta t)^2$

Use these knowns:  $\Delta x = 67.27$  (from part IV)

Solve for:  $v_i$  (6.13 m/s)

$\Delta t = 4.56$  (given)

$a = 3.78$  (given)

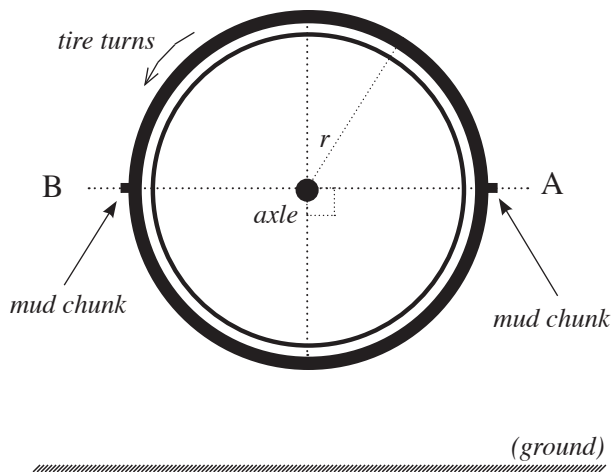
As it passed point A, the boat was moving at a speed of 6.13 m/s.

2. b. A muddy bicycle tire with a radius of 34.0 cm has been mounted above the ground for cleaning. The vertical distance from the tire axle to the ground is 57.2 cm.

The tire has been given a good spin in the direction shown, with a starting rate of 180 rpm, but thereafter it has been gradually slowing at a rate of  $.0692 \text{ rad/s}^2$ .

The tire has been spinning for exactly 1 minute when two chunks of mud, located at positions A and B, simultaneously fly off the tire. Each chunk lands on the ground, (touching no other object meanwhile).

How much time elapses between their moments of impact with the ground?



- I. Find the initial angular speed of the tire in rad/s:

Use this equation:  $\omega_{rad} = \omega_{rpm}(2\pi/60)$

Use these knowns:  $\omega_{rpm} = 180$  (given)

Solve for:  $\omega_{rad}$  ( $6\pi \text{ rad/s}$ )

- II. Find the speed of the tire after it's been spinning (and gradually slowing) for 1 minute (60 s):

Use this equation:  $\omega_f = \omega_i + \alpha(\Delta t)$

Use these knowns:  $\omega_i = 6\pi$  (from **part I**)

Solve for:  $\omega_f$  ( $14.698 \text{ rad/s}$ )

$\alpha = -0.0692$  (given; for positive  $\omega_i$ ,  $\alpha$  is negative to slow)

$\Delta t = 60$  (given)

- III. Find the initial speed at which each mud chunk then flies off.

Use this equation:  $|v_T| = r|\omega|$

Use these knowns:  $r = 0.340$  (given)

Solve for:  $|v_T|$  ( $4.9972 \text{ m/s}$ )

$\omega = 14.698$  (from **part II**)

- IV. Find the impact velocity of each chunk as it hits the ground thereafter:

Use this equation:  $v_f^2 = v_i^2 + 2a(\Delta y)$  (and choose the negative root; impact velocities are downward)

Use these knowns:  $v_{i,A} = 4.9972$  (from **part III**; A goes up)

Solve for:  $v_f$  ( $v_{f,A}$ :  $-6.015 \text{ m/s}$ )

$v_{i,B} = -4.9972$  (from **part III**; B goes down)

( $v_{f,B}$ :  $-6.015 \text{ m/s}$ )

$a = -9.80$  (gravity is known)

$\Delta y = -0.572$  (given)

- V. Find the time from (simultaneous) release to impact for each chunk:

Use this equation:  $v_f = v_i + a(\Delta t)$

Use these knowns:  $v_{f,A} = v_{f,B} = -6.015$  (from **part IV**)

Solve for:  $\Delta t$  ( $\Delta t_A$ :  $1.1237 \text{ s}$ )

$v_{i,A} = 4.9972$  (from **part III**; A goes upward)

( $\Delta t_B$ :  $0.1039 \text{ s}$ )

$v_{i,B} = -4.9972$  (from **part III**; B goes downward)

$a = -9.80$  (gravity is known)

- VI. Find the difference (time delay) between those two impact times:

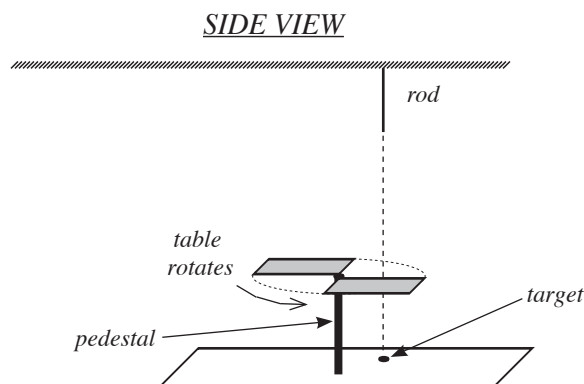
Use this equation:  $\Delta t_{delay} = \Delta t_A - \Delta t_B$

Use these knowns:  $\Delta t_A = 1.1237$  (from **part V**)

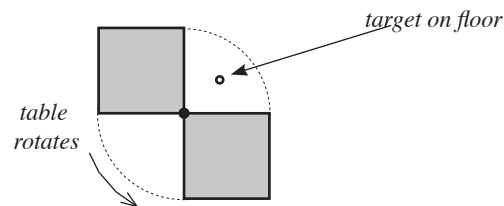
Solve for:  $\Delta t_{delay}$  ( $1.02 \text{ s}$ )

$\Delta t_B = 0.1039$  (from **part V**)

3. a. A thin steel rod of length 1.75 m, initially hanging (vertically) is dropped from rest, and it falls (still in a vertical orientation) down toward a target on the floor. That target, however, is positioned beneath a table whose top is shaped like two squares joined at the corners (see drawings). The table top is 7.98 cm thick, and that top is rotating horizontally about its center pedestal at a constant rate of 50.0 rpm. Measuring from the table's top surface, from what minimum height must the lower end of the rod start when dropped so that the rod has a chance of bypassing the rotating table top and hitting the target?



*TOP VIEW (looking downward along the path of the falling rod)*



- I. Convert the table's rotation rate from rpm to rad/s.

Use this equation:  $\omega_{rpm} (2\pi \text{ rad/1 rev})(1 \text{ min/60 s}) = \omega_{rad/s}$

Use these knowns:  $\omega_{rpm} = 50$  (given)

Solve for:  $\omega_{rad/s}$  **(5.2360 rad/s)**

- II. Find the interval of time for which the table is not obstructing the target. The “window of opportunity” is a quarter-circle of arc, so the time interval is the time necessary for 1/4 revolution:  $\pi/2$  radians.

Use this equation:  $\Delta\theta_{open} = \omega_{i,table}(\Delta t_{open}) + (1/2)\alpha_{table}(\Delta t_{open})^2$

Use these knowns:  $\Delta\theta_{open} = \pi/2$  (given)

Solve for:  $\Delta t_{open}$  **(0.3000 s)**

$\omega_{i,table} = \omega_{rad/s}$  from **part I**

$\alpha_{table} = 0$  (rotation rate  $\omega_{table}$  is constant)

- III. Find the minimum displacement the rod must move through to just evade the table.

Use this equation:  $\Delta y_{min} = -(l_{rod} + h_{tabletop})$

Use these knowns:  $l_{rod} = 1.75$  (given)

Solve for:  $\Delta y_{min}$  **(-1.8298 m)**

$h_{tabletop} = 0.0798$  (given)

- IV. Find the minimum velocity at which the rod must reach the plane of the table top.

Use this equation:  $\Delta y_{min} = v_{i,min}(\Delta t_{open}) + (1/2)a_{fall}(\Delta t_{open})^2$

Use these knowns:  $\Delta y_{min}$  (from **part III**)

Solve for:  $v_{i,min}$  **(-4.6293 m/s)**

$\Delta t_{open}$  (from **part II**)

$a_{fall} = -9.80$  (assumed/known)

- V. Find the minimum height from which the rod must fall to reach that minimum velocity.

Use this equation:  $v_{f,fall}^2 = v_{i,fall}^2 + 2a_{fall}(\Delta y_{fall,min})$

Use these knowns:  $v_{i,fall} = 0$  (rod drops from rest)

Solve for:  $\Delta y_{fall,min}$  **(-1.09 m)**

$v_{f,fall} = v_{i,min}$  from **part IV**

$a_{fall} = -9.80$  (assumed/known)

To slip past the rotating table-top, the rod must drop from a height of at least 1.09 m.

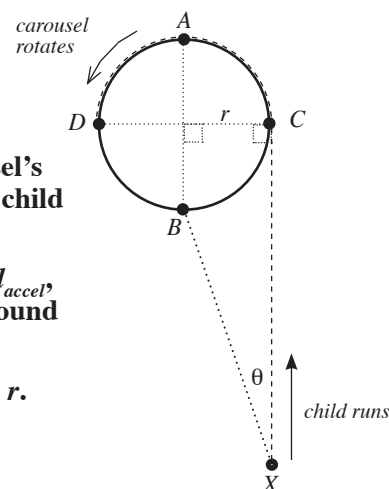
3. b. In an amusement park, a child standing at point X watches a carousel (“merry-go-round”) as it starts from rest and begins to turn (counter-clockwise in the top-view diagram shown), accelerating at a constant rate.

The child’s favorite horse is at point A when the carousel begins to turn. She doesn’t see the horse until it reaches point B (midway around the carousel’s first full turn). At that moment, the horse is moving at speed  $v_{\text{horse},B}$ , and the child begins to run along the path shown (from point X toward point C).

In her run, the child first accelerates at a constant rate  $a_{\text{child}}$  over a distance  $d_{\text{accel}}$ , then continues to run at a steady speed, chasing the horse part of the way around the carousel, finally catching up to it at point D, as shown.

The carousel (which has been accelerating steadily all this time) has a radius  $r$ . Find the angle  $\theta$  shown in the diagram.

You may consider these values as **known**:  $v_{\text{horse},B}$ ,  $a_{\text{child}}$ ,  $d_{\text{accel}}$ ,  $r$ .



*This is an ODAVEST item—use the full seven-step problem-solving protocol—but keep in mind that you’re not being asked to actually solve for the final expression. In fact, you’re not being asked to do any math at all—not even any algebra. Rather, for the **Solve** step, you are to write a series of succinct instructions on **how** to solve this problem. Pretend that your instructions will be given to someone who knows math but not physics. And for the **Test** step, you should consider the situation and predict how the solution would change if the data were different—changing one data value at a time.*

**Objective:** A carnival carousel of known radius accelerates from rest at a constant rate, rotating through 1.25 full revolutions, as marked by the motion of a particular horse (seat) positioned on its outer rim.

The horse has turned through a half-revolution—and is then moving at a known speed—when a child begins to run along a straight-line path tangent to the horse’s path at its 3/4-turn mark.

Upon reaching that point, the child continues to run, but now in a circular path right beside the rim of the carousel, arriving at the carousel’s 1.25-turn mark just as the horse does.

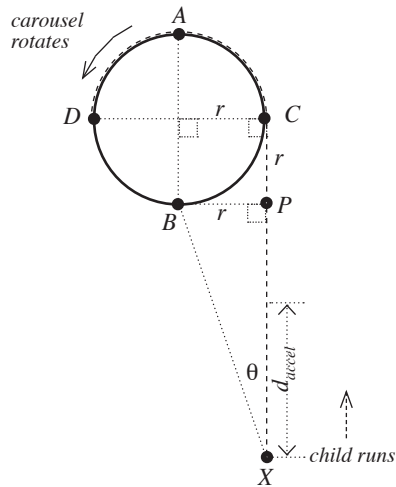
In her run, the child starts from rest and accelerates at a known and constant rate—and for a known distance—up to her full sprint speed, which she then maintains until reaching the 1.25-turn mark of the carousel along the path described.

We must find the angle formed by the horse’s positions at its 1/2-turn mark and its 3/4 -turn mark, as viewed from the child’s starting point.

<b>Data:</b>	$v_{\text{horse},B}$	The speed of the horse at point B (its 1/2-turn mark).
	$a_{\text{child}}$	The acceleration rate of the child during that phase of her run.
	$d_{\text{accel}}$	The distance run by the child during her acceleration phase.
	$r$	The radius of the carousel.

<b>Assumptions:</b>	<i>Horse</i>	We model the horse as a point mass; so its position is on the extreme outer edge of the carousel, and its position at any point (1/2-turn, 1.25-turn, etc.) is well defined.
	<i>Child</i>	We model the child as point mass, so that the circular portion of her run is exactly along the rim of the carousel (same as the horse’s path, therefore); also so that her position at the 1.25-turn, where she catches up to the horse, is well-defined.
	<i>Air</i>	We disregard any effects of wind or air drag.



**Visual Rep(s):****Equations:**

- I.  $v_{horse.B} = r\omega_{horse.B}$
- II.  $\omega_{horse.B}^2 = \omega_{horse.A}^2 + 2\alpha(\Delta\theta_{AB})$
- III.  $\Delta\theta_{AB} = \omega_{horse.A}(\Delta t_{AB}) + (1/2)\alpha(\Delta t_{AB})^2$
- IV.  $\Delta\theta_{AD} = \omega_{horse.A}(\Delta t_{AD}) + (1/2)\alpha(\Delta t_{AD})^2$
- V.  $d_{accel} = v_i(\Delta t_{accel}) + (1/2)a_{child}(\Delta t_{accel})^2$
- VI.  $\Delta t_{accel} + \Delta t_{steady} = \Delta t_{AD} - \Delta t_{AB}$
- VII.  $v_{steady}^2 = v_i^2 + 2a_{child}(\Delta x_{accel})$
- VIII.  $d_{steady} = v_{steady}(\Delta t_{steady})$
- IX.  $d_{accel} + d_{steady} = d_{XP} + r + \pi r$
- X.  $\tan \theta_{BC} = r/d_{XP}$

**Solving:**

- Solve I for  $\omega_{horse.B}$ . Substitute that result into II.
- Solve II for  $\alpha$  (noting that  $\Delta\theta_{AB} = \pi$  rad). Substitute that result into III.
- Solve III for  $\Delta t_{AB}$ . Substitute that result into VI.
- Solve IV for  $\Delta t_{AD}$ . Substitute that result into VI.
- Solve V for  $\Delta t_{accel}$  (noting that  $v_i = 0$ ). Substitute that result into VI.
- Solve VI for  $\Delta t_{steady}$ . Substitute that result into VIII.
- Solve VII for  $v_{steady}$ . Substitute that result into VIII.
- Solve VIII for  $d_{steady}$ . Substitute that result into IX.
- Solve IX for  $d_{XP}$ . Substitute that result into X.
- Solve X for  $\theta_{BC}$ .

**Testing:**

**Dimensions:**  $\theta_{BC}$  should be dimensionless (units of angular measure).

**Dependencies:** If the speed of the horse at point B,  $v_{horse.B}$ , were greater, then in order for the child—with the same running abilities—to catch the horse at point D, she must have started her run closer to the carousel (i.e. distance XP would have to be less). Thus, the angle  $\theta_{BC}$  would be greater.

If the child's acceleration rate,  $a_{child}$ , were greater, then for the same  $d_{accel}$  she accomplishes, this would have taken her less time; so more of her total running time would be spent at full sprint—so she's running farther. Thus the distance XP is longer, so the angle,  $\theta_{BC}$ , would be smaller.

If the child's  $d_{accel}$  were greater for the same acceleration rate,  $a_{child}$ , then this acceleration phase must have taken more time; so less of her total running time would be spent at full sprint—but she's also running faster at that sprint speed. Thus the effect on her total run distance—and thus on XP and  $\theta_{BC}$ —is difficult to assess without seeing the explicit formula for the solution.

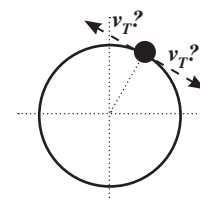
If the carousel's radius,  $r$ , were greater, that would imply a lesser angular acceleration,  $\alpha$ , by the carousel (so a slower horse to catch), but also a longer total path to run. Thus the effect on her starting position—and thus on XP and  $\theta_{BC}$ —is difficult to assess without seeing the explicit formula for the solution.

4. a. Evaluate (T/F/N) the following statements regarding an object traveling along a circular path. *As always, you must fully justify your answer with any valid mix of words, diagrams and equations.*

- (i) **The tangential and radial components of an object's acceleration are never in the same direction.**  
True. By definition, the radial and tangential directions on a circle are always perpendicular to each other. (If the tangential acceleration is zero, it has no direction at all.)
- (ii) **The tangential and radial components of an object's acceleration are always perpendicular to each other.**  
Not enough information. By definition, the radial and tangential directions on a circle are always perpendicular to each other. But if the tangential acceleration is zero, it has no direction at all.
- (iii) **If an object's tangential acceleration vector is in the same direction as its tangential velocity vector, then that object's total acceleration vector is increasing in magnitude.**  
True. If  $a_T$  is parallel to  $v$ , then  $\omega$  is increasing (the object is rotating faster and faster). That means  $|a_R|$  is increasing, because  $|a_R| = r\omega^2$ . That means  $|a|$  is increasing, because  $|a| = \sqrt{|a_T|^2 + |a_R|^2}$ .

- b. (i) **A ball is whirled on a string in a horizontal circle. The string breaks when the ball is at the  $60^\circ$  position on the circle (where  $0^\circ$  is the positive  $x$ -axis). In what direction does the ball move right after the string breaks?**

The ball will move in the tangential direction when the string breaks—which is always at right angles ( $90^\circ$  different) from its position angle. Since you're not told whether the ball has been going clockwise or counterclockwise, there are two options for the tangential direction:  $60^\circ + 90^\circ = \underline{150^\circ}$  or  $60^\circ - 90^\circ = \underline{-30.0^\circ}$



- (ii) **A ceiling fan with a radius of 66.0 cm is rotating steadily at 44.0 rpm. A fly, clinging to the outside edge of one of the fan blades experiences what radial acceleration [magnitude]?**

$|a_R| = r\omega^2$ , where  $r = 0.660$  m, and  $\omega = (44.0 \text{ rev/min})(2\pi \text{ rad/rev})(1 \text{ min}/60\text{s}) = 4.6077 \text{ rad/s}$   
 Thus:  $|a_R| = (0.660)(4.6077)^2 = \underline{14.0 \text{ m/s}^2}$

- (iii) **You're riding on a carousel ("merry-go-round") that is rotating at a constant 6.00 rpm. Initially, you are standing at point A, which is a distance of 1.60 m from the axis of rotation. Then you move to point B, which is a distance of 2.40 m from the axis. By what factor (multiplier) have you changed the magnitude of your total acceleration?**

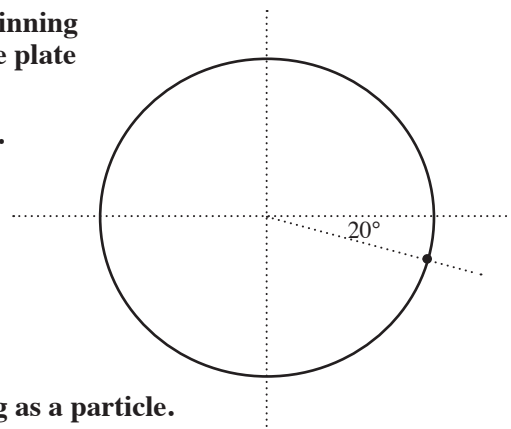
Because  $\omega$  is constant,  $a_T = 0$ . Therefore:  $|a| = |a_R| = r\omega^2$   
 Therefore:  $|a_R|_2/|a_R|_1 = (r_2\omega^2)/(r_1\omega^2) = r_2/r_1 = 2.40/1.60 = \underline{1.50}$

- c. **A car is driving around a horizontal curve whose arc is part of a circle of radius  $r$ . At the moment in question here, its speed is  $v$ . Evaluate (T/F/N) the following statements.** *As always, you must fully justify your answer with a valid mix of words, diagrams and equations.*

- (i) **If the car is moving directly east at this moment, then its radial acceleration direction could be west.**  
False. The car's velocity direction is always tangential, and radial acceleration is, by definition, perpendicular to the tangential direction. So if the car's velocity is eastward, then its radial acceleration could be only northward or southward.
- (ii) **If the car is slowing down at this moment, its total acceleration magnitude is greater than it would be if  $v$  were its steady speed.**  
True.  $|a| = \sqrt{|a_T|^2 + |a_R|^2}$ . If  $v$  is constant, then  $|a_T| = 0$ , whereas if  $v$  is not constant (regardless whether it is increasing or decreasing), then  $|a_T| > 0$ .
- (iii) **If the car is speeding up at this moment, its total acceleration magnitude is greater than it would be if  $v$  were its steady speed.**  
True. See the previous item.

4. d. A ladybug is riding on the edge of a circular dinner plate that is spinning “in place” (its center is not changing position) on a level table. The plate is spinning counterclockwise and its diameter is 30.0 cm.

At a certain moment, the bug is located at the position shown here. At this moment, she is slowing at a rate of  $1.20 \text{ m/s}^2$ , but her total acceleration magnitude is  $2.98 \text{ m/s}^2$



- (i) Find the rotational speed of the plate.  
Express this in revolutions per minute (rpm).
- (ii) Find the direction of the bug’s velocity at this moment.  
Express this as an angle measured from the axes shown.

Ignore any effects of air drag and wind, and model the ladybug as a particle.

- (i) The bug’s acceleration vector triangle is as shown here:

Clearly:  $|a_{total}|^2 = |a_R|^2 + |a_T|^2$

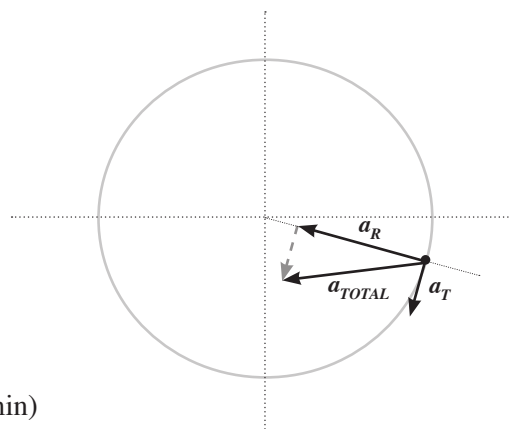
That is:  $|a_R| = \sqrt{(|a_{total}|^2 - |a_T|^2)}$

But:  $|a_R| = v^2/r = r\omega^2$

Therefore:  $r\omega^2 = \sqrt{(|a_{total}|^2 - |a_T|^2)}$

Solve for  $\omega$ :  $\omega = [\sqrt{(|a_{total}|^2 - |a_T|^2)/r}]^{1/2}$   
 $= [\sqrt{(2.98^2 - 1.20^2)/0.15}]^{1/2}$   
 $= 4.264 \text{ rad/s}$

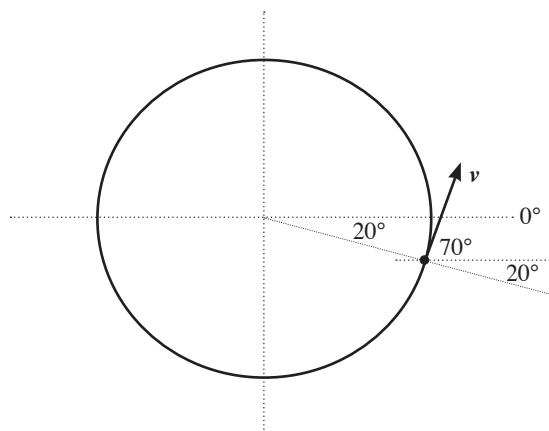
In rpms:  $\omega = (4.264 \text{ rad/s})(1 \text{ rev}/2\pi \text{ rad})(60 \text{ s/min})$   
 $= \underline{40.7 \text{ rpm}}$



- (ii) The bug’s velocity is tangent to her path (but in the counter-clockwise sense)—thus, as shown here:

Because a tangent is always perpendicular to a radius on a circle,  $v$  is  $90^\circ$  (in the positive angular direction) from her position angle ( $-20^\circ$ ).

Thus:  $\theta_v = -20^\circ + 90^\circ = \underline{70^\circ}$



5. a. You are sitting in a seat on a Ferris wheel that is turning at a constant angular speed. Its speed is rapid enough that you can feel the effect of rotation, especially at the topmost and bottommost points. Evaluate these statements (T/F/N), which all refer to the moment when you pass through the highest point in your circular path. As always, fully justify your answer with a valid mix of words, diagrams and equations.

- (i) Your weight is reduced as the radial force cancels out (some of) the gravitational force acting on you.

Not clear enough language to really declare one way or another (so: not enough information, but any answer here could be defended and will be accepted if justified appropriately). *Physically*, it's clearly false: No other force can "cancel out" or reduce the gravitational force; the earth doesn't suddenly reduce its pull on you just because there's some other force acting on you. But speaking *mathematically*, you might be justified in claiming this statement is true. Here's the issue: First of all, the term "the radial force" is misleading. It implies that there's some new (third) force acting on you here—in *addition to* the force by the earth on you ( $F_G$ ) and the seat/harness on you ( $F_N$ ). *No*. Those are the only 2 forces acting on you in the radial direction. Rather, the term "**the radial force**" is intended to mean "**the net** [or total] radial force."

With this in mind, look at what Newton's 2nd Law says mathematically (positive  $R$ -direction is down here):

$$\Sigma F_R = ma_R$$

$$F_G - F_N = ma_R$$

Put that into words:

The gravitational force – your weight = "the radial force"

Now re-arrange it and you get:

Your weight = The gravitational force – "the radial force"

*Mathematically* speaking, therefore, you could argue that your weight is the value obtained when the gravitational force is reduced or "canceled out" (*subtracted from*) by "the radial force."

- (ii) The normal force on you decreases as the centrifugal force adding to your normal force reaches a maximum.

False. Yes, the normal force decreases, but not because of anything called a centrifugal force. There is no such thing as centrifugal force in Newtonian physics, by definition. The action that compels an object away from a circular path is simply the inertia of its mass, which will cause the object to follow a straight line if allowed to.

- (iii) You accelerate upward out of your seat a little bit, thus feeling lighter.

False. Your acceleration is toward the center of your circular path—that's downward here, not upward. Think about it: You get the same feeling here as if you were in an elevator that's accelerating downward.

- (iv) The gravitational force acting on you is greater than the normal force (between the seat and you).

True. The net force at this point must be toward the center of your circular path—that's downward—so the downward radial force of gravity on you must be greater than the upward radial force by the seat on you.

- (v) The radial force acting on you is at its smallest, thus you feel less force acting on you.

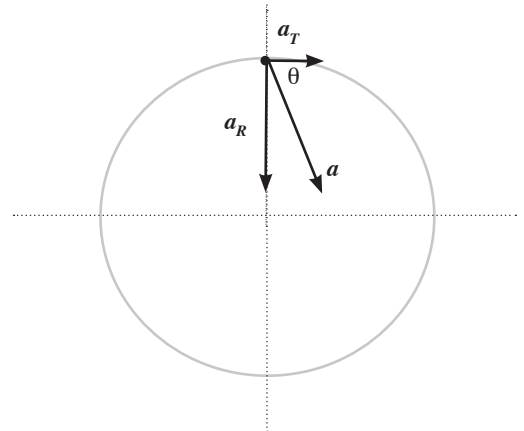
Not enough information (or false—if defended well). Again a language problem. If by "the radial force" we mean "the net radial force," then this is false: In uniform circular motion such as this, the net radial force (magnitude),  $r\omega^2$ , is always the same because both  $r$  and  $\omega$  are always the same. However, if we are referring to, say, the radial force component exerted by either the seat/harness ( $F_{N,R}$ ) or gravity ( $F_{G,R}$ ), that's a different issue. For example,  $F_{G,R}$  minimizes not at the top or bottom but rather, at the "3 o'clock" or 9 o'clock positions—when  $F_G$  is perpendicular to the radius (so  $F_{G,R} = 0$ ). But  $F_{N,R}$  does minimize at the top.

- b. A helicopter propellor blade (4.0 m long) is rotating in a horizontal circle about its hub in a counter-clockwise direction (as viewed from above). It is slowing at a rate of  $10 \text{ rad/s}^2$ . When the tip of the blade is pointing directly north and moving at 20 m/s, what is the direction of the net force being exerted on the tip of the blade?

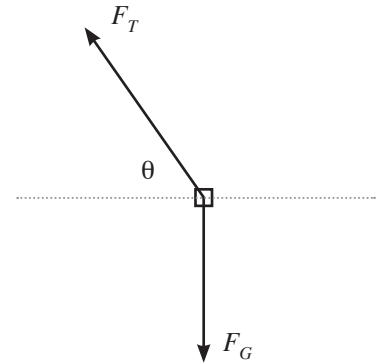
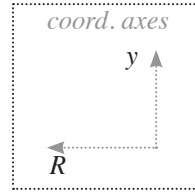
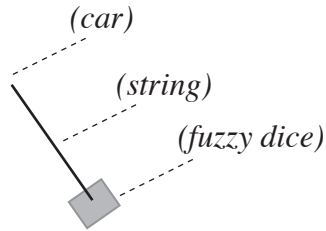
The direction of the net force is the direction of the total acceleration,

$$\begin{aligned} \text{shown here as } \angle \theta &= \tan^{-1}(-|a_R|/|a_T|) \\ &= \tan^{-1}[-(v^2/r)/(r\alpha)] \\ &= \tan^{-1}[-(20^2/4)/(4 \cdot 10)] \\ &= \tan^{-1}(-2.5) \end{aligned}$$

Thus, the net force direction is  $\angle 68.2^\circ$  (or  $68.2^\circ$  south of east).



6. a. As you round a corner on flat ground you notice the dice hanging from your rearview mirror swings outward, making an angle of  $68^\circ$  with respect to the horizontal. The corner can be assumed to be part of a circle that has a diameter of 40 m. What must the speed of your car be to achieve this angle?



$$\Sigma F_y = ma_y$$

$$F_{Ty} - F_G = ma_y$$

$$F_T \sin \theta - mg = 0$$

$$\Sigma F_R = ma_R$$

$$F_{TR} = ma_R$$

$$F_T \cos \theta = mv^2/r$$

$$\text{Thus: } F_T = mg/\sin \theta$$

$$\text{Thus: } (mg/\sin \theta) \cos \theta = mv^2/r$$

$$\text{Thus: } g \cos \theta / \sin \theta = v^2/r$$

$$\text{Thus: } v = \sqrt{(rg \cos \theta / \sin \theta)} = \sqrt{[(20)(9.80) \cos(68)/\sin(68)]} = \mathbf{8.90 \text{ m/s}}$$

- b. The races you are designing tires for will have cars reaching nearly 80 m/s, on level circular tracks as short as 2000 m around. What is the ideal coefficient of friction you will design to?

$$\Sigma F_y = ma_y$$

$$F_N - F_G = ma_y$$

$$F_N - mg = 0$$

$$\Sigma F_R = ma_R$$

$$F_S^{\max} = ma_R$$

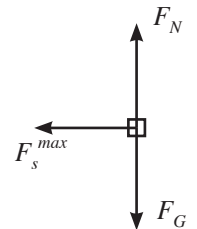
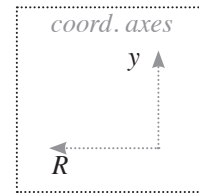
$$\mu_s F_N = mv^2/r$$

$$\text{Thus: } F_N = mg$$

And thus:

$$\mu_s mg = mv^2/r$$

$$\text{Thus: } \mu_s = v^2/(rg) = 2\pi v^2/(Cg) = (2\pi)80^2/[(2000)(9.80)] = \mathbf{2.05}$$



- c. A 30.0-kg child is riding on carousel at a distance of 2.50 m from the center of rotation. For how long would the carousel need to accelerate (starting from rest) at  $0.750 \text{ rad/s}^2$  until the net horizontal force magnitude on the child is 200 N?

$$F_{net}/m = a \quad \text{And: } a = \sqrt{(a_T^2 + a_R^2)}$$

$$\text{Thus: } a_R = \sqrt{(a^2 - a_T^2)} = \sqrt{[(F_{net}/m)^2 - (r\alpha)^2]}$$

$$\text{But also: } a_R = r\omega^2$$

$$\text{Thus: } r\omega^2 = \sqrt{[(F_{net}/m)^2 - (r\alpha)^2]}$$

$$\text{Then note: } \omega = \alpha \Delta t$$

$$\text{Thus: } r(\alpha \Delta t)^2 = \sqrt{[(F_{net}/m)^2 - (r\alpha)^2]}$$

$$\text{Solve for: } \Delta t = [\{\sqrt{[(F_{net}/m)^2 - (r\alpha)^2]}/(r\alpha^2)\}]^{1/2} = \mathbf{2.13 \text{ s}}$$

## OPTIONAL REVIEW ITEMS – Measuring Circular Motion: Angular Sweep vs. Travel Distance

### Exercises:

- (1) Two runners are running along different circular paths.  
Runner A sweeps out an angle of  $400^\circ$ .  
Runner B sweeps out an angle of  $150^\circ$ .  
Which runner travels a greater distance?  
(Circle the one best answer, then *explain your answer/show all work.*)
- A. Runner A
  - B. Runner B**
  - C. They each travel the same distance.
  - D. There is not information to answer.

To know any circular distance (the actual number of meters or feet, etc.), you need to know the radius of the circular path traveled. You don't know the radius of either path here.

- (2) Two runners are running along different circular paths.  
Runner A sweeps out an angle of  $400^\circ$ .  
Runner B sweeps out an angle of  $150^\circ$ .  
The radius of runner B's path is three times greater than that of A.  
Which runner travels a greater distance?  
(Circle the one best answer, then *explain your answer/show all work.*)
- A. Runner A
  - B. Runner B**
  - C. They each travel the same distance.
  - D. There is not information to answer.

$$\text{Runner A: } \theta_A = (400^\circ) \frac{(2\pi \text{ rad})}{360^\circ} = 2.222\pi \text{ radians}$$

$$\text{Runner B: } \theta_B = (150^\circ) \frac{(2\pi \text{ rad})}{360^\circ} = 0.8333\pi \text{ radians}$$

Let the runners' circular path radii be  $r_A$  and  $r_B$ , respectively. So  $r_B = 3r_A$ .

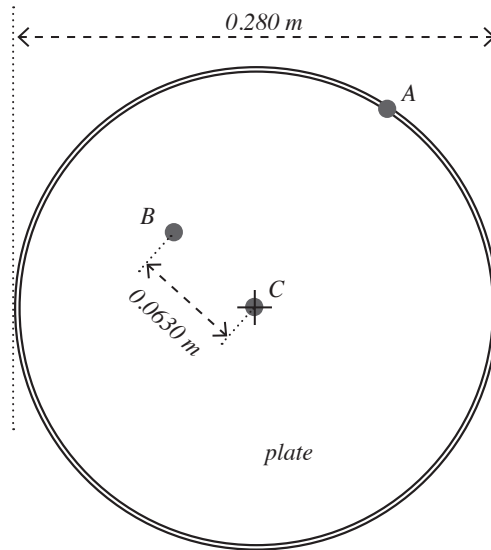
$$\text{Therefore: } s_A = r_A(\theta_A) = 2.222\pi r_A = 6.98r_A$$

$$\text{and: } s_B = r_B(\theta_B) = 3r_A(\theta_B) = 3(0.8333\pi)r_A = 7.85r_A$$

You could also select appropriate values for  $r_A$  and  $r_B$  (e.g. 10 m and 30 m), to show that  $s_B > s_A$ .

- (3)
- a. Convert  $800^\circ$  to radians.  $(800^\circ) \frac{(2\pi \text{ rad})}{360^\circ} = 4.44\pi \text{ radians}$
  - b. Convert  $10\pi$  radians to revolutions.  $(10\pi \text{ rad}) \left( \frac{1 \text{ rev}}{2\pi \text{ rad}} \right) = 5.00 \text{ rev}$
  - c. Convert  $45.0^\circ$  to revolutions.  $(45^\circ) \frac{(1 \text{ rev})}{360^\circ} = 0.125 \text{ rev}$
  - d. Convert 2.00 radians to degrees.  $(2.00 \text{ rad}) \left( \frac{360^\circ}{2\pi \text{ rad}} \right) = 115^\circ$

- (4) Three ladybugs are sitting on a flat, circular dinner plate of diameter 0.280 m. The bugs are not all sitting together (nor are they even sitting in straight row). As shown (see below), bug A is sitting on the rim of the plate; bug B is sitting at a radial distance of 0.0630 m from the plate's center; and bug C is sitting at the exact center of the plate.



Now you rotate the plate around its center, turning it 2.34 full revolutions. How far—what distance (in m)—does each bug travel?

**All bugs turn through the same angle  $\theta$  (2.34 rev). Convert it to radians:**

$$\theta = \frac{(2.34 \text{ rev})(2\pi \text{ rad})}{1 \text{ rev}} = 14.70 \text{ rad}$$

**Therefore:**  $s_A = r_A(\theta) = (0.140)(14.70) = \underline{2.06 \text{ m}}$

**and:**  $s_B = r_B(\theta) = (0.0630)(14.70) = \underline{0.926 \text{ m}}$

**and:**  $s_C = r_C(\theta) = (0)(14.70) = \underline{0 \text{ m}}$