Oregon State University **PH 212** *Winter Term 2014*

Midterm Exam 1 *(Wednesday, January 29, 8:30 p.m.)*

<u>GENERAL DIRECTIONS</u>: Fill out the cover sheet completely, as indicated. Your test form letter is A.

Follow the specific directions on each page of the answer form. **For all items**, unless directed otherwise: In all expressions and symbolic solutions, reduce them to simplest form; in all final numerical answers, use standard SI units and proper significant digits. ***No item will be given credit if it does not include valid reasoning/work to justify the solution or answer.***

Physical constants and other possibly useful information:

$g_{earth.surface} = 9.80\ \text{m/s}^2$	$I_{solid.disk.center} = (1/2)MR^2$
$I_{solid.sphere.center} = (2/5)MR^2$	$I_{spherical.shell.center} = (2/3)MR^2$

<u>DIRECTIONS for T/F/N items 1-3</u>: Evaluate each statement as being demonstrably True (**T**), demonstrably False (**F**), or as having Not enough information (**N**) to declare it either True or False; AND briefly but fully explain your reasoning. **<u>Note</u>: No credit will be given for any T/F/N answer without a valid explanation to accompany it.**

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1. A restless child is wandering randomly on a rotating carousel that is powered by a motor. The carousel is turning counterclockwise, as viewed from above. At a certain moment, the child is standing (at rest relative to the carousel) 1.25 m directly north of the axis (i.e. located at $\angle 90°$ or $\angle \pi/2$ rad); and her speed relative to the ground at that moment is 6.25 m/s. 4.00 seconds later, the child again happens to be standing momentarily at rest relative to the carousel, but she's then 2.00 m from the axis, and her ground speed is then 8.00 m/s.
Evaluate (T/F/N) each statement. <u>Justify your answers fully</u> with any valid mix of words, drawings and calculations.

(A) The carousel's rotational speed decreased during the 4-s time interval.

(B) Assuming a constant angular acceleration by the carousel over the entire 4-s interval, it must have completed at least 3 full rotations in that time.

(C) Assuming a constant angular acceleration by the carousel over the entire 4-s interval, the child's angular position relative to the axis at the end of that interval was about $\angle 41.3°$, or about $\angle 0.721$ rad (defining east as $\angle 0$).

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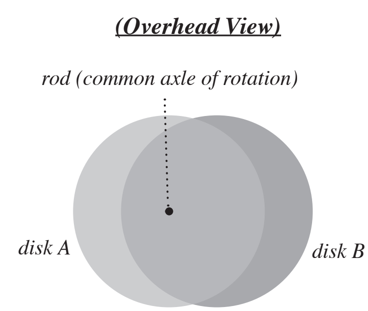
2. Two identical solid disks are rotating independently about the same thin, frictionless rod. That rod goes through the center of disk A, but in disk B, the rod goes through a point midway between that disk's center and its outer edge (see overhead view here). Initially, the disks are rotating about the rod with the <u>same angular speed but in opposite directions</u>. Then they collide (without outside interference or any change in altitude) and they stick together as a result.
Evaluate (T/F/N) each statement. <u>Justify your answers fully</u> with any valid mix of words, drawings and calculations.



(A) Disk B reverses its direction of rotation as a result of the collision.

(B) As a result of the collision, $\Delta \boldsymbol{L}_A = -\Delta \boldsymbol{L}_B$.

(C) About 96.0% of the disks' total initial mechanical energy is lost as a result of the collision.

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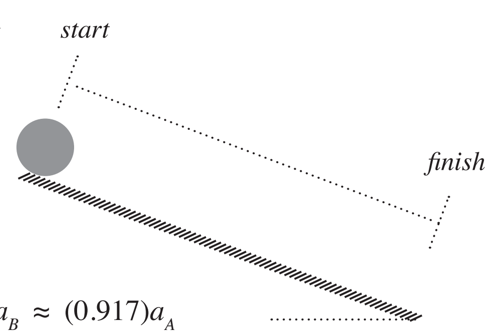
3. In two separate trials, a solid sphere (A) and a hollow spherical shell (B) are each released from rest and allowed to roll for the same distance down the same slope (shown here). *Evaluate (T/F/N) each statement. <u>Justify your answers fully</u> with any valid mix of words, drawings and calculations.*



(A) Their final translational speeds compare like this: $v_{f.B} \approx (1.09)v_{f.A}$

(B) At any given moment during its motion along the slope, the solid sphere has about 28.6% of its total kinetic energy in the form of K_R.

(C) Their average translational acceleration magnitudes compare like this: $a_B \approx (0.917)a_A$

DIRECTIONS for free-answer items 4-5: Fill in or provide the answers or solutions indicated. For these items, you do not need to do the full seven-step ODAVEST problem-solving procedure, but **you must still justify every answer/solution. No credit will be given for any answer or solution without a valid explanation and/or valid work to accompany it.**

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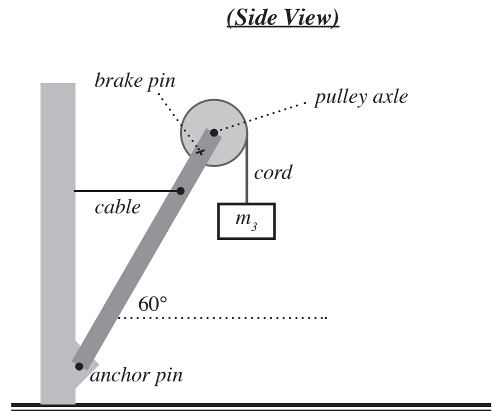
4. A rigid beam (mass = 400 kg; length = 12.0 m) has a pulley (mass = 30.0 kg; outer radius = 1.50 m) mounted at its upper end; and the beam's lower end is anchored with a pin to a wall. The beam is also supported by a horizontal tension cable connected to the wall, as shown here. The distance along the beam from the pulley axle to the cable attach point is 3.50 m.



A strong, thin cord has been wrapped several times around the pulley and then connected to a block (m_3 = 100 kg)—which hangs at rest *so long as the pulley is secured to the beam with a removable "brake" pin* to prevent it from rotating.

a. Fnd the tension in the cable when everything is at rest.

b. Now suppose the brake pin is suddenly removed. When the block has lowered by a vertical distance of 2.50 m, how fast is it moving?

Ignore air drag and any friction in the anchor pin, cable attachment, or pulley axle. Assume that the beam is a solid, uniform rod; the pulley is a solid, uniform disk; and the cord is massless and never slips with respect to the pulley rim.

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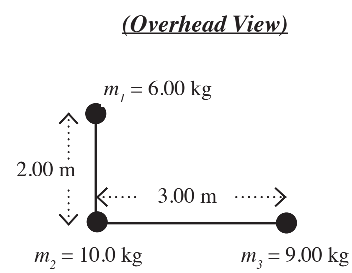
5. Rigid, massless rods connect three point masses as shown in this overhead view. This object is set at rest on level, frictionless ice, then pinned at m_2 to that ice so that it can rotate freely (without friction, in a horizontal plane) around m_2. Then a torque of 87.0 N·m is applied to the object for just 4.00 s (at which time the torque ceases). Ignore wind/air drag.



a. At what translational speed does the object's center of mass now move?

b. If the pin could somehow pull in (shorten) the 3-m rod to 1.50 m, how much work would need to be done by the pin to accomplish this?

DIRECTIONS for item 6: For this item, the full seven-step ODAVEST problem-solving procedure is required. Note that each part will be awarded with points, so even if you don't get all the way through the problem, there are many ways to earn partial credit for parts that are valid). *Keep in mind that **you're not being asked to actually solve for the final expression**. In fact, you're not being asked to do any math at all—**not even any algebra**. Rather, for the **S**olve step, you are to write a series of succinct instructions on how to solve this problem. Pretend that your instructions will be given to someone who knows math but not physics. And for the **T**est step, you should consider the situation and predict how the solution would change if the data were different—changing one data value at a time.*

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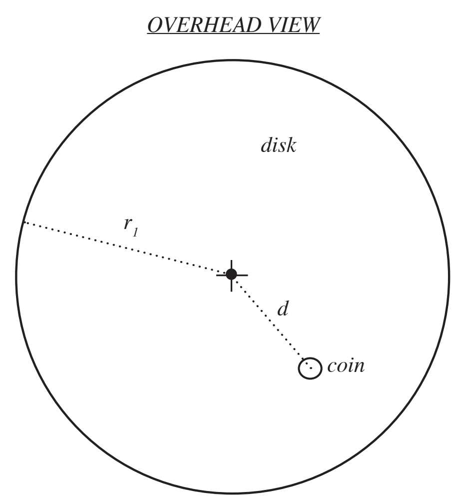
6. Refer to the overhead view here. A large, uniform disk (mass = m_1, outer radius = r_1) is rotating freely on a frictionless central axis. A small coin (mass = m_2) is "riding" on the disk at a distance d from the axis. The coin's speed (relative to the ground) is v, and it does not slip across the disk's surface. The static friction coefficient between the coin and disk surfaces is μ_S. Ignore any effects of wind/air drag.



a. What is the maximum steady torque magnitude you could apply to the disk (without touching the coin) in order to bring the disk (and coin) to rest without the coin ever slipping across the disk's surface?

b. How much time would this slowing-to-a-stop process require?

Here is a summary of all known values (and you may also assume that you know all of the information from the data box on the front page):
$m_1, r_1, m_2, d, \mu_S, v, g$

Scoring: ***Each part (A-B-C) of each problem on this page is worth 10 points (so each of the 3 problems is worth 30 points.)***
For each part (A-B-C), the answer (T/F/N) is worth 5 points, and the reasoning is worth 5 points.
If the answer is correct, but the physics reasoning is missing or incorrect: 0 points for the problem.
If the physics reasoning is correct, but the answer is missing or incorrect: 5 points for the problem.

1

(A) $\omega_i = v_i/r_i = 6.25/1.25 = 5.00$ rad/sec

$\omega_f = v_f/r_f = 8.00/2.00 = 4.00$ rad/sec

A **T**

(B) $\Delta\theta = (1/2)(w_i + w_f)(\Delta t) = (1/2)(5 + 4)(4) = 18$ rad

or: $\alpha = (w_f - w_i)/(\Delta t) = (4 - 5)/4 = -0.250$ rad/s
$\Delta\theta = w_i(\Delta t) + (1/2)\alpha(\Delta t)^2 = 5(4) + (1/2)(-0.250)(4)^2 = 18$ rad

18 rad < $3(2\pi)$, so it's **not sufficient for 3 full revolutions**.

B **F**

(C) Since the child is wandering randomly, there's no telling where she ends up direction-wise relative to the axis—all you know is her distance from that axis. It's possible, of course, that she could end up at the angle stated, but it's not necessarily true.

[Half-credit for a True answer, if showing the math: $(18 + \pi/2) - 3(2\pi) = 0.721$ rad $= 41.3°$]

C **N**

2

(A) The two disks are an isolated system, so their total angular momentum is conserved. This is a completely inelastic collision, where the two objects—always rotating on the same axis—become one.

Let $\boldsymbol{w}_{i.A} = \boldsymbol{w}_i$ (ccw) and $\boldsymbol{w}_{i.B} = -\boldsymbol{w}_i$ (cw)

Thus: $I_A\boldsymbol{w}_{i.A} + I_B\boldsymbol{w}_{i.B} = (I_A + I_B)\boldsymbol{w}_f$
Then: $(I_A - I_B)\boldsymbol{w}_i = (I_A + I_B)\boldsymbol{w}_f$ Or: $\boldsymbol{w}_f = \boldsymbol{w}_i(I_A - I_B)/(I_A + I_B)$

But $I_B > I_A$, since I_A is the $I_{c.m.}$ (the minimum I).
So $\boldsymbol{w}_f$ is negative—that's in the **same rotational direction** as $\boldsymbol{w}_{i.B}$.

A **F**

(B) The two disks are an isolated system, so their total angular momentum is conserved.
Thus the sum of their two individual momentum changes must be zero: $\Delta\boldsymbol{L}_A + \Delta\boldsymbol{L}_B = 0$

B **T**

(C) There was only K_R in the system (no other E_{mech}), and $\Delta\%K_R = 100(K_{R.f} - K_{R.i})/K_{R.i} = 100(K_{R.f}/K_{R.i} - 1)$
From part 2A: $\boldsymbol{w}_f = \boldsymbol{w}_i(I_A - I_B)/(I_A + I_B)$
Then note: $I_{c.m.} = I_A = (1/2)MR^2$ $\quad I_B = I_{c.m.} + M(R/2)^2 = (3/4)MR^2$ (Parallel Axis Theorem)
Therefore: $\boldsymbol{w}_f = \boldsymbol{w}_i[-(1/4)MR^2]/[(5/4)MR^2] = -(1/5)\boldsymbol{w}_i$
$K_{R.f} = (1/2)(I_A + I_B)w_f^2 = (1/2)[(5/4)MR^2][-(1/5)w_i]^2 = 0.025MR^2w_i^2$
$K_{R.i} = (1/2)I_Aw_i^2 + (1/2)I_Bw_i^2 = (1/2)(5/4)MR^2w_i^2 = 0.625MR^2w_i^2$ $\quad \Delta\%K_R = 100(0.025/0.625 - 1) = \mathbf{-96.0\%}$

C **T**

3

(A) In each trial: $U_{G.i} = K_{T.cm.f} + K_{R.cm.f}$ (No K initially and no U_G finally; and rolling w/o slipping assumes $W_{ext} = 0$)
Thus: $mgh_i = (1/2)mv_{cm.f}^2 + (1/2)[I_{cm}]\omega_{cm.f}^2 = (1/2)mv_{cm.f}^2 + (1/2)[I_{cm}](v_{cm.f}/R)^2$
Trial A: $mgh_i = (1/2)mv_{cm.f}^2 + (1/2)[(2/5)mR^2](v_{cm.f}/R)^2 = (7/10)mv_{cm.f}^2$ Thus: $v_{f.A}^2 = (10/7)gh_i$
Trial B: $mgh_i = (1/2)mv_{cm.f}^2 + (1/2)[(2/3)mR^2](v_{cm.f}/R)^2 = (5/6)mv_{cm.f}^2$ Thus: $v_{f.B}^2 = (6/5)gh_i$
Therefore: $v_{f.B}/v_{f.A} = \sqrt{[(6/5)/(10/7)]} = \mathbf{0.917}$

A **F**

(B) Trial A (for any value of h_i—thus for any point along the slope):

$$mgh_i = K_{T.cm.f} + K_{R.cm.f} = (1/2)mv_{cm.f}^2 + (1/2)[(2/5)mR^2](v_{cm.f}/R)^2 = (1/2)mv_{cm.f}^2 + (1/5)mv_{cm.f}^2$$

Thus: $K_{R.A}/(K_{T.A} + K_{R.A}) = (1/5)/[(1/2) + (1/5)] = \mathbf{0.286}$

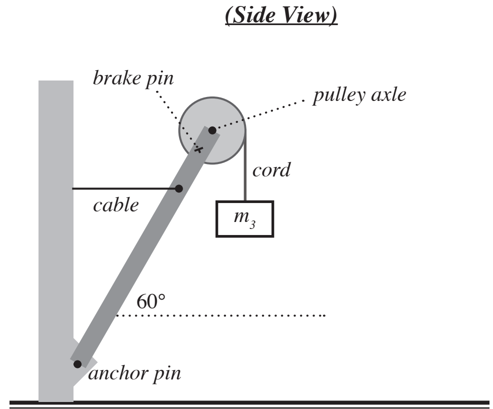
B **T**

(C) Kinematics: $v_f^2 = v_i^2 + 2a\Delta x$
Rearranging (with $v_i = 0$ here): $a = v_f^2/(2\Delta x)$
From 3A: $v_{f.A}^2 = (10/7)gh_i$ and $v_{f.B}^2 = (6/5)gh_i$
Thus: $a_B/a_A = [v_{f.B}^2/(2\Delta x)]/[v_{f.A}^2/(2\Delta x)] = v_{f.B}^2/v_{f.A}^2 = (6/5)/(10/7) = \mathbf{0.840}$

C **F**

4. A rigid beam (mass = 400 kg; length = 12.0 m) has a pulley (mass = 30.0 kg; outer radius = 1.50 m) mounted at its upper end; and the beam's lower end is anchored with a pin to a wall. The beam is also supported by a horizontal tension cable connected to the wall, as shown here. The distance along the beam from the pulley axle to the cable attach point is 3.50 m.

A strong, thin cord has been wrapped several times around the pulley and then connected to a block (m_3 = 100 kg)—which hangs at rest *so long as the pulley is secured to the beam with a removable "brake" pin* to prevent it from rotating.

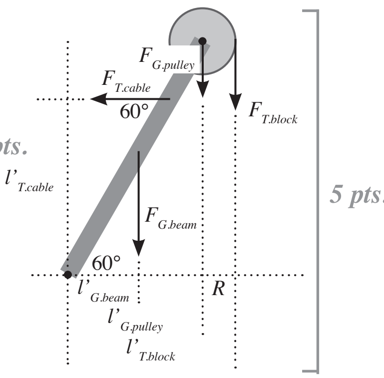


a. Fnd the tension in the cable when everything is at rest.

b. Now suppose the brake pin is suddenly removed. When the block has lowered by a vertical distance of 2.50 m, how fast is it moving?

Ignore air drag and any friction in the anchor pin, cable attachment, or pulley axle. Assume that the beam is a solid, uniform rod; the pulley is a solid, uniform disk; and the cord is massless and never slips with respect to the pulley rim.

a. When all is at rest, it's static equilibrium: $\Sigma\tau = I\alpha = 0$ *2 pts.*
Summing the torques <u>about the anchor pin</u>:



$$\Sigma\tau = (F_{T.cable})(l'_{T.cable}) - (F_{G.beam})(l'_{G.beam}) - (F_{G.pulley})(l'_{G.pulley}) - (F_{T.block})(l'_{T.block}) = 0$$

Thus: $(F_{T.cable}) = [(F_{G.beam})(l'_{G.beam}) + (F_{G.pulley})(l'_{G.pulley}) + (F_{T.block})(l'_{T.block})]/(l'_{T.cable})$ *4 pts.*

From the diagram:
$l'_{T.cable} = (12 - 3.50)\sin 60° = 7.361$ m
$l'_{G.beam} = (12/2)\cos 60° = 3.00$ m
$l'_{G.pulley} = (12)\cos 60° = 6.00$ m
$l'_{T.block} = (6 + 1.50) = 7.50$ m

And:
$F_{G.beam} = 400(9.80) = 3920$ N
$F_{G.pulley} = 30(9.80) = 294$ N
$F_{T.block} = 100(9.80) = 980$ N

1 pt. ea. (7 pts. total)

So: $(F_{T.cable}) = [(3920)(3.00) + (294)(6.00) + (980)(7.50)]/(7.361)$ *2 pts.*

$= \mathbf{2840\ N\ \ (2.84 \times 10^3\ N)}$ *value = 1 pt., units = 1/2 pt., sig. figs. = 1/2 pt.*

b. After the brake pin is removed, the block *and pulley rim* will start to accelerate together. One approach is to analyze the two objects ($\Sigma \boldsymbol{F}_y$ for the block and $\Sigma\tau$ for the pulley) to find that common acceleration value, a, then use kinematics for block (also knowing that $\boldsymbol{v}_i = 0$ and $\Delta\boldsymbol{y} = 2.50$), to calculate its $\boldsymbol{v}_f$.

Or, we can analyze the mechanical energy of the block and pulley system together: $E_{mech.f} = E_{mech.i} + W_{ext}$ *4 pts.*

No work is done except by gravity; and the block loses some U_G, exchanging it for some K_T for itself and some K_R for the pulley:

So: $K_{T.f.block} + K_{R.f.pulley} = U_{G.i.block}$ Or: $(1/2)m_3v_f^2 + (1/2)I_{pulley}\omega_f^2 = m_3gh_i$ *4 pts.*

But: $v_f = r_{pulley}\omega_f$ *3 pts.*

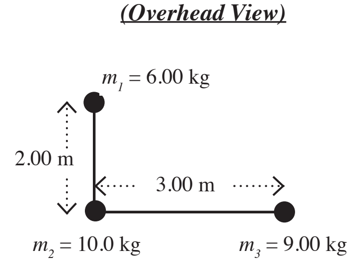
And: $I_{pulley} = (1/2)m_{pulley}r_{pulley}^2$ *3 pts.*

So: $(1/2)m_3v_f^2 + (1/4)m_{pulley}v_f^2 = m_3gh_i$ *3 pts.*

Or: $v_f^2[(m_3/2) + (m_{pulley}/4)] = m_3gh_i$ So: $v_f = \sqrt{\{m_3gh_i/[(m_3/2) + (m_{pulley}/4)]\}}$ *4 pts.*

$= \sqrt{\{[(100)(9.80)(2.50)]/[100/2 + 30/4]\}} = \mathbf{6.53\ m/s}$ *value = 1 pt., units = 1/2 pt., sig. figs. = 1/2 pt.*

5. Rigid, massless rods connect three point masses as shown in this overhead view. This object is set at rest on level, frictionless ice, then pinned at m_2 to that ice so that it can rotate freely (without friction, in a horizontal plane) around m_2. Then a torque of 87.0 N·m is applied to the object for just 4.00 s (at which time the torque ceases). Ignore wind/air drag.



a. At what translational speed does the object's center of mass now move?

b. If the pin could somehow pull in (shorten) the 3-m rod to 1.50 m, how much work would need to be done by the pin to accomplish this?

a. First, find the moment of inertia around the fixed pin: $I_{pin} = \Sigma m_i r_i^2 = (6)2^2 + (9)3^2 = 105 \text{ kg·m}^2$ *4 pts.*

Now find the rotational speed after the torque is finished—using <u>either</u> $\Sigma\tau_{pin} = I_{pin}\alpha_{pin}$ <u>or</u>…

$\Delta \boldsymbol{L}_{pin} = \tau_{pin.net}(\Delta t)$ <u>That is</u>: $I_{pin}(\omega_f - \omega_i) = \tau_{pin.net}(\Delta t)$

In this case, $\omega_i = 0$, so we have: $\omega_f = \tau_{pin.net}(\Delta t)/I_{pin} = (87)(4)/105 = 3.314$ rad/s *5 pts.*

Now find the center of mass (using the pin as the origin):

$x_{c.m.} = (\Sigma m_i x_i)/M = [(6)(0) + (10)(0) + (9)(3)]/(6 + 10 + 9) = 1.08$ *3 pts.*

$y_{c.m.} = (\Sigma m_i y_i)/M = [(6)(2) + (10)(0) + (9)(0)]/(6 + 10 + 9) = 0.48$ *3 pts.*

<u>So</u>: The distance $r_{c.m.}$ from the pin to the c.m. is: $r_{c.m.} = \sqrt{(1.08^2 + 0.48^2)} = 1.182$ m *3 pts.*

<u>Therefore</u>: $v_{c.m.} = r_{c.m.}\omega_f$ *3 pts.*

$= (1.182)(3.314) =$ **3.92 m/s** *value = 1 pt., units = 1/2 pt., sig. figs. = 1/2 pt.*

b. Find the new moment of inertia around the fixed pin:

$I_{pin.new} = (\Sigma m_i r_i^2)_{new} = (6)2^2 + (9)(1.50)^2 = 44.25 \text{ kg·m}^2$ *4 pts.*

Now find the new rotational speed, using conservation of angular momentum as the pin reels in the mass:

$I_{pin.new}\omega_{new} = I_{pin.old}\omega_{old}$ *5 pts.*

<u>That is</u>: $\omega_{new} = (I_{pin.old}\omega_{old})/I_{pin.new} = (105)(3.314)/(44.25) = 7.864$ rad/s *5 pts.*

The change in K_R here is the work done in pulling the mass inward:

$W_{ext} = K_{R.f} - K_{R.i} = (1/2)I_{pin.new}\omega_{new}^2 - (1/2)I_{pin.old}\omega_{old}^2$ *6 pts.*

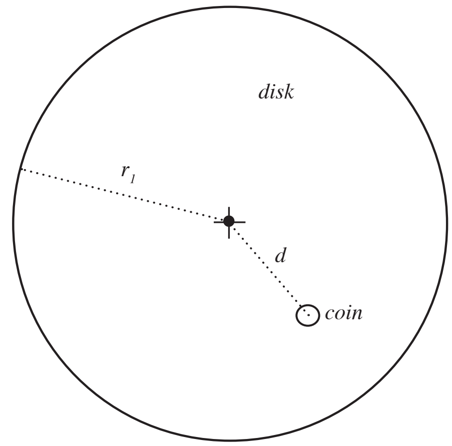
$= (1/2)(44.25)(7.864)^2 - (1/2)(105)(3.314)^2 =$ **792 J** *value = 1 pt., units = 1/2 pt., sig. figs. = 1/2 pt.*

6. Refer to the overhead view here. A large, uniform disk (mass = m_1, outer radius = r_1) is rotating freely on a frictionless central axis. A small coin (mass = m_2) is "riding" on the disk at a distance d from the axis. The coin's speed (relative to the ground) is v, and it does not slip across the disk's surface. The static friction coefficient between the coin and disk surfaces is μ_S. Ignore any effects of wind/air drag.

a. What is the maximum steady torque magnitude you could apply to the disk (without touching the coin) in order to bring the disk (and coin) to rest without the coin ever slipping across the disk's surface?

b. How much time would this slowing-to-a-stop process require?

OVERHEAD VIEW



Here is a summary of all known values (and you may also assume that you know all of the information from the data box on the front page): $m_1, r_1, m_2, d, \mu_S, v, g$

For this item, the full seven-step ODAVEST problem-solving procedure is required. Note that each part will be awarded with points, so even if you don't get all the way through the problem, there are many ways to earn partial credit for parts that are valid).
Keep in mind that you're not being asked to actually solve for the final expression. In fact, you're not being asked to do any math at all—not even any algebra. Rather, for the Solve step, you are to write a series of succinct instructions on how to solve this problem. Pretend that your instructions will be given to someone who knows math but not physics. And for the Test step, you should consider the situation and predict how the solution would change if the data were different—changing one data value at a time.

*For a complete ODAVEST (seven-step) problem-solving procedure, each of the seven steps is equally weighted (in this case, **10 points each**, for a total of 70 points for the problem).*

Objective:

A large, uniform disk of known mass is rotating about its center.

Its axis of rotation is vertical and the disk's large surface is horizontal.

The rotation occurs without any friction in the axle (axis of rotation).

A small coin of known mass is lying on the disk surface, at rest with respect to the disk.

The coin's center is a known distance from the axis of the disk's rotation.

The speed of the coin with respect to the ground is known.

The coefficient of static friction between the coin and disk is known.

Effects of air drag and/or wind are negligible.

We want to estimate the maximum steadytorque that could be applied to the disk (without touching the coin), so that the coin does not move relative to the disk.

We also want to calculate the time that torque would require to slow the disk and coin to a complete stop.

1 pt. each. A total of 10 points possible here.

Data:

m_1 The mass of the large, uniform disk. *1 pt.*

r_1 The outer radius of the large uniform disk. *1 pt.*

m_2 The mass of the coin. *1 pt.*

d The radial distance from the coin center to the axis of rotation. *2 pts.*

μ_S The coefficient of static friction between the surfaces of the disk and the coin. *2 pts.*

v The (translational) speed of the coin relative to the ground, before any torque is applied. *2 pts.*

g The local gravitational free-fall acceleration magnitude. *1 pt.*

Assumptions:

Disk — We assume that the disk is perfectly balanced on its axle, so that there is no "wobble" or vibration as it rotates.

Coin — We assume that the coin is small enough to model as a point mass on the surface of the large disk.

Surfaces — We assume that the given coefficient of static friction applies equally in all horizontal directions for the disk—that its surface has no "grain" or bias.

We assume that the given coefficient of static friction applies equally in all horizontal directions for the coin—that its surface has no "grain" or bias.

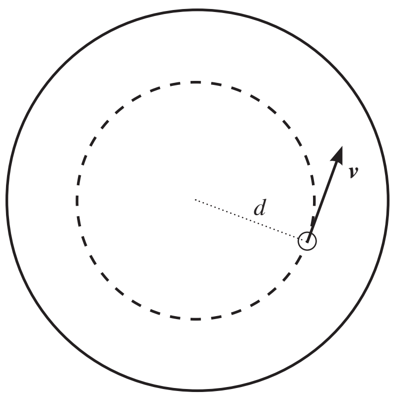
Torque — We assume that the sudden application of the steady torque causes no wobble or vibration.

Speed — We assume that the given speed v is <u>less than</u> the maximum steady speed at which the coin could ride without slipping.

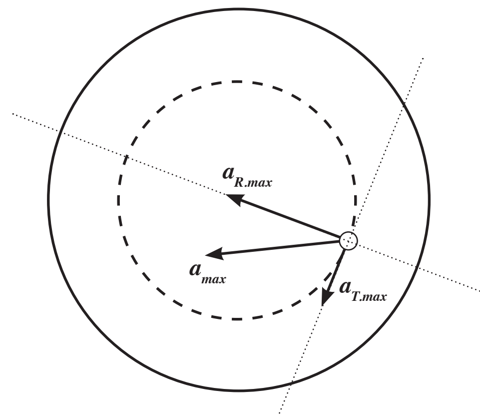
Any five = 2 pts. each (and there may be others, too).

10 points possible here.

Visual Rep(s):

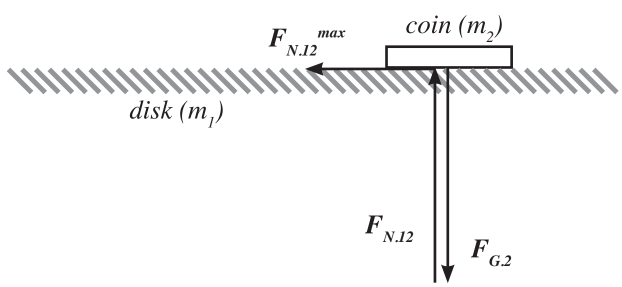


2 pts.



4 pts.

(side view)



4 pts.

Equations:

I. $\tau_{max} = I_{total}\alpha_{max}$

II. $I_{total} = (1/2)m_1 r_1^{\,2} + m_2 d^2$

III. $v = d\omega$

IV. $|\boldsymbol{a_R}|_{max} = d\omega^2$

V. $|\boldsymbol{a_T}|_{max} = d\alpha_{max}$

VI. $a_{max} = \sqrt{[|\boldsymbol{a_R}|_{max}^{\,2} + |\boldsymbol{a_T}|_{max}^{\,2}]}$

VII. $F_{S.12}^{\ max} = m_2 a_{max}$

VIII. $F_{S.12}^{\ max} = \mu_S F_{N.12}$

IX. $F_{N.12} = m_2 g$

X. $\omega = (0) + \alpha_{max}(\Delta t_{stop.min})$

1 pt. each.

A total of 10 points possible here.

Solving:

Solve **II** for I_{total}. Substitute that result into **I**.

Solve **III** for ω. Substitute that result into **IV** and **X**.

Solve **IV** for $|\boldsymbol{a_R}|_{max}$. Substitute that result into **VI**.

Solve **IX** for $F_{N.12}$. Substitute that result into **VIII**.

Solve **VIII** for F_S^{max}. Substitute that result into **VII**.

Solve **VII** for a_{max}. Substitute that result into **VI**.

Solve **VI** for $|\boldsymbol{a_T}|_{max}$. Substitute that result into **V**.

Solve **V** for α_{max}. Substitute that result into **I** and **X**.

Solve **I** for τ_{max}.

Solve **X** for $\Delta t_{stop.min}$.

1 pt. each.

A total of 10 points possible here.

Testing:

Dimensions:

τ_{max} should have dimensions of force·length.

$\Delta t_{stop.min}$ should have dimensions of time.

Dependencies:

If m_1 were greater, then with all other variables the same, this *would not change a_{max}*, thus not affect α_{max}. But it would produce a larger I_{total}. So: **τ_{max} would be greater, but $\Delta t_{stop.min}$ would not change.**

If r_1 were greater, then with all other variables the same, this *would not change a_{max}*, thus not affect α_{max}. But it would produce a larger I_{total}. So: **τ_{max} would be greater, but $\Delta t_{stop.min}$ would not change.**

If m_2 were greater, then with all other variables the same, this *would not change a_{max}*, thus not affect α_{max}. But it would produce a larger I_{total}. So: **τ_{max} would be greater, but $\Delta t_{stop.min}$ would not change.**

If d were greater, then with all other variables the same, this would produce smaller values for ω and for $|\boldsymbol{a_T}|_{max}$, which would allow a greater value for $|\boldsymbol{a_T}|_{max}$ (since F_S^{max} is unchanged), thus a greater α_{max}. So:
τ_{max} would be greater, and $\Delta t_{stop.min}$ would be smaller.

If μ_S were greater, then with all other variables the same, this would mean F_S^{max} would be greater, allowing a greater a_{max}. This would allow a greater $|\boldsymbol{a_T}|_{max}$, thus a greater α_{max}. So:
τ_{max} would be greater, and $\Delta t_{stop.min}$ would be smaller.

If v were greater, then with all other variables the same, this would produce greater values for ω and for $|\boldsymbol{a_T}|_{max}$, which would require a smaller "allowance" for $|\boldsymbol{a_T}|_{max}$ (since F_S^{max} is unchanged), thus a lesser α_{max}. So:
τ_{max} would be smaller, and $\Delta t_{stop.min}$ would be greater.

If g were greater, then with all other variables the same, this would mean F_S^{max} would be greater, allowing a greater a_{max}. This would allow a greater $|\boldsymbol{a_T}|_{max}$, thus a greater α_{max}. So:
τ_{max} would be greater, and $\Delta t_{stop.min}$ would be smaller.

Any item: 1 pt. each., + 1 pt. extra for getting at least one item correct.

A total of 10 points possible here.