

HW 4

Due: Wednesday, February 5, 2014, 5:00 p.m.

All solutions posted now except 6h.

Print your full **LAST** name: _____

Print your full **first** name: _____

Print your HW Grader's name: _____

What is your HW Grader's box # (located outside of Wngr 234)? _____

“I affirm and attest that this HW assignment is my own work. While I may have had help from (and/or worked with) others, all the reasoning, solutions and results presented in final form here are my own doing—and expressed in my own words.”

Sign your name (full signature): _____

Print today's date: _____

General Instructions for HW 4

Requirements:

The PH 212 HW graders will select **one** out of the six problems. They will then score that problem according to the points rubric posted with the HW 4 solutions (for a total possible 10 points for the entire HW assignment).

Format:

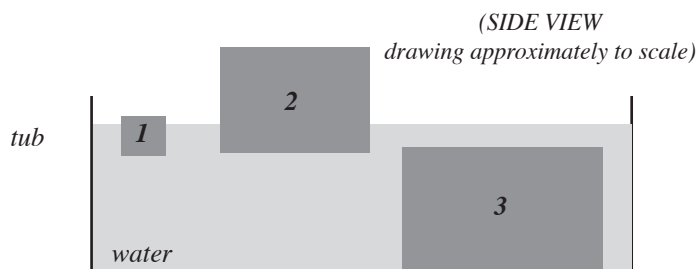
You ARE required to include the completed, signed cover sheet—that's the first page of this file—as the front page of your HW submission when you turn it in. (Please staple together all pages, in order—cover sheet first—with one staple at upper left.)

Scoring:

The rubrics (scoring guides—the points breakdowns) for HW problem solutions will closely match those for exam solutions. And in those rubrics, *when any item asks for an explanation (and most do), it means exactly what it says.* To get full credit, you must include a short but informative explanation (any valid mix of your own words, equations or diagrams) of your reasoning.

To get an idea of how best to approach various problem types (there are three basic types), refer to these [example HW problems](#).

1. a. Three blocks are placed in a tub with water, as shown. The blocks have volumes V_1 , V_2 and V_3 . Only blocks 1 and 2 float. And $V_1 < V_2 < V_3$. Evaluate the following statements (T/F/N). As always, explain your reasoning.



- (i) **Block 2 has the largest buoyant force acting on it.**

False. The buoyant force on any object is equal to the weight of the fluid it is displacing. Block 3 is clearly displacing the most water here.

- (ii) **Block 1 must be more dense than block 2.**

True. For any **floating** object, the percent of its volume that is immersed in the fluid indicates the ratio of the object's density to that of the fluid. Blocks 1 and 2 are both floating, but block 1 is immersed to a greater percentage of its volume, so its density is nearer to that of water than is block 2's density.

- (iii) **Block 1 must have more mass than block 2.**

False. Judging from the diagram, block 2 is displacing more water, so the buoyant force supporting it is greater than the buoyant force supporting block 1. But both blocks are floating, so the buoyant forces supporting them are equal to their weights. So block 2 must be heavier than block 1.

- b. Evaluate the following statements (T/F/N). As always, explain your reasoning.

- (i) **Archimedes' Principle states that the buoyant force is the fluid pressure exerted on the underside of any part of any object immersed in that fluid.**

False. Archimedes' Principle states: "The buoyant force on any object is equal to the weight of the fluid it is displacing." In terms of forces, the buoyant force is the **net** pressure force (the sum of **all** pressure forces) exerted on an object that is at least partially immersed in a fluid.

- (ii) **For any 1 kg of incompressible fluid, its volume decreases when the pressure on it increases.**

False. An incompressible fluid is one which does not change its density (ratio of mass to volume) under the influence of pressure.

- (iii) **If you have two wooden blocks, A and B, with identical outer dimensions (L x W x H), but different densities ($\rho_A > \rho_B$), and both float in the same pool, the buoyant force on each block is the same.**

False. If the blocks have the same volumes but different densities, they must have different masses—and therefore different weights. And if they're both floating, the buoyant force on each must match that object's weight, so the buoyant forces must be different.

- (iv) **If a bucket of bricks is sitting (fully immersed) on a scale at the bottom of a pool of water, that scale reading is less if the bottom of the pool is 5 m deep than if it is 3 m deep.**

False. Assuming the bricks and bucket haven't changed their volume (i.e. the volume of the water they are displacing), then the buoyant force by the water (incompressible and hence of uniform density regardless of depth) will be the same at any depth.

- (v) **Any object that is 50% immersed in a fluid has a density that is half of the fluid's density.**

False. Suppose you're standing, half immersed in the shallow end of a swimming pool. Has your density suddenly become half that of water? (The % immersion indicator an object's density relative to the fluid is valid only for a **floating** object—when the buoyant force and the gravitational force are equal and opposite.)

- (vi) **If an object's volume increases by 10.0%, then its density decreases by less than 10.0%.**

True. Let $\rho_i = m_i/V_i$. Then $\rho_f = m_f/V_f = m_i/(1.1V_i) = (1/1.1)(m_i/V_i) = (1/1.1)\rho_i$
So: $\Delta\% \rho = 100 \cdot [(\rho_f - \rho_i)/\rho_i] = 100 \cdot \{[(1/1.1)\rho_i - \rho_i]/\rho_i\} = 100 \cdot [(1/1.1) - 1] = -9.09\%$

- (vii) **Any object that is 100% immersed in a fluid must have a density greater than the fluid's density.**

False. Suppose you're holding a ping-pong ball completely under water. Has its density suddenly become greater than that of water? (The % immersion indicator an object's density relative to the fluid is valid only for a **floating** object—when the buoyant force and the gravitational force are equal and opposite.)

- (viii) **As an ice cube melts in a glass of water, the uppermost tip of its exposed portion remains at the same height in the glass.**

False. Ice floats with the same % immersed (about 90%), so as the cube shrinks, its height above the water and its depth below the water both decrease.

1. c. (i) A 579-N lump of steel floats in mercury (Hg). What volume of Hg does it displace? ($\rho_{\text{Hg}} = 13,600 \text{ kg/m}^3$).

For any object floating in a fluid: $F_{G.obj} = F_B$

That is: $F_{G.obj} = F_{G.fluid.disp}$

Or: $F_{G.obj} = \rho_{fluid}(V_{fluid.disp})g$

Thus: $V_{fluid.disp} = F_{G.obj}/(\rho_{fluid}g)$
 $= 579/[(13,600)(9.80)] = 4.34 \times 10^{-3} \text{ m}^3 \text{ (4.34 liters)}$

- (ii) A 42.2-kg gold statue hangs by a thread, completely immersed in water. The thread tension is 390 N. The density of gold is $19,300 \text{ kg/m}^3$. Is the statue pure gold? Show all work and calculations to support your answer.

Strategy: Find the volume of the statue (by calculating the buoyant force exerted on it), then calculate its density (via $\rho = m/V$). If that density is 19,300, the statue is pure gold; otherwise, it isn't.

Write the vertical force equation for the statue: $F_T + F_B - F_G = 0$

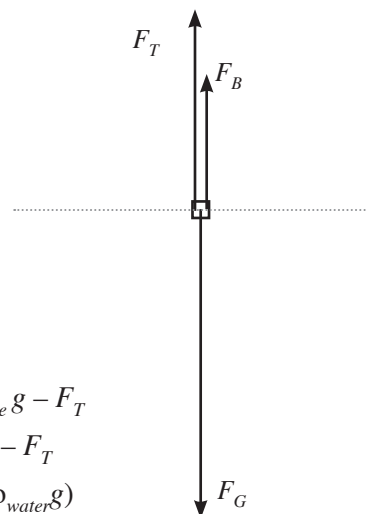
Isolate the buoyant force: $F_B = F_G - F_T$

Re-write the buoyant force and the weight: $\rho_{water}(V_{water.disp})g = m_{statue}g - F_T$

The statue is completely immersed: $\rho_{water}(V_{statue})g = m_{statue}g - F_T$

Solve for V_{statue} : $V_{statue} = (m_{statue}g - F_T)/(\rho_{water}g)$

Divide this into m_{statue} to get ρ_{statue} :

$$\begin{aligned}\rho_{statue} &= m_{statue}/V_{statue} \\ &= m_{statue}/[(m_{statue}g - F_T)/(\rho_{water}g)] \\ &= m_{statue}\rho_{water}g/(m_{statue}g - F_T) \\ &= (42.2)(1000)(9.80)/[(42.2)(9.80) - 390] \\ &= 17,553 \text{ kg/m}^3 \quad \text{That's not pure gold.}\end{aligned}$$


2. a. As you dive toward the bottom of a swimming pool, the pressure increases noticeably. Does the buoyant force also increase? (Assume that water is incompressible.) Explain your answer fully.

No. The buoyant force on you is the weight of the fluid you displace. So, assuming you don't change your volume (the volume of the water you are displacing), and assuming that that volume of water doesn't get any heavier (water being incompressible and hence of uniform density regardless of depth), the buoyant force doesn't change with depth.

1 pt.

- b. Would you float more easily (with less of your body immersed) in water on the moon than on earth?

No. For any floating object, the buoyant force (the weight of the fluid displaced) is equal to the weight of the object. Both of these weights are affected proportionally by a different local g value, so that g value is irrelevant: $\rho_{\text{water}}(V_{\text{water,disp}})g = \rho_{\text{you}}(V_{\text{you}})g$. Your percent immersion would still be the same ratio of your density to water's.

1 pt.

- c. A glass beaker, filled to the brim with water, is resting on a scale. A block is placed in the water, causing some of it to spill over. The water that spills is wiped away; the beaker is still filled to the brim. Explain fully how and why the initial and final readings on the scale compare if the block is made of...

(i) wood (ii) iron

In either case, the water spilled is the water displaced, and that water's weight is the buoyant force then being exerted on the object that displaced it. Therefore...

(i) For a floating object such as wood, the spilled water's weight (the F_B) is equal to the object's weight, so the scale reading would not change.

1 pt.

(ii) For a non-floating object such as iron, the spilled water's weight (the F_B) is less than the object's weight, so the scale reading would increase.

- d. You have two identical drinking glasses. One contains free-floating ice cubes, and the other contains no ice. You fill each glass completely to the brim with water. Which glass now weighs more? Explain your answer.

The glasses have equal weights. For any floating object, the buoyant force (the weight of the fluid displaced) is equal to the weight of the object. So the ice weighs the same as the water it displaces. Its weight could be replaced exactly by water sufficient to "fill in the hole" it makes in the water; the (rim) level would be unchanged.

1 pt.

- e. Two pencils have the same average density (500 kg/m^3). The mass of the mechanical pencil is twice the mass of the conventional pencil. They are both held completely submerged under the surface of water (1000 kg/m^3) and released. Which accelerates at the greater rate? Explain your answer fully.

For the mechanical pencil (1):

$$\Sigma F_{1,y} = m_1 a_{1,y}$$

$$F_{B,1} - F_{G,1} = m_1 a_{1,y}$$

$$\rho_{\text{water}}(V_{\text{water,disp}})g - m_1 g = m_1 a_{1,y}$$

$$\rho_{\text{water}}(V_1)g - m_1 g = m_1 a_{1,y}$$

$$\rho_{\text{water}}(m_1/\rho_1)g - m_1 g = m_1 a_{1,y}$$

$$\rho_{\text{water}}(1/\rho_1)g - g = a_{1,y}$$

For the conventional pencil (2):

$$\Sigma F_{2,y} = m_2 a_{2,y}$$

$$F_{B,2} - F_{G,2} = m_2 a_{2,y}$$

$$\rho_{\text{water}}(V_{\text{water,disp}})g - m_2 g = m_2 a_{2,y}$$

$$\rho_{\text{water}}(V_2)g - m_2 g = m_2 a_{2,y}$$

$$\rho_{\text{water}}(m_2/\rho_2)g - m_2 g = m_2 a_{2,y}$$

$$\rho_{\text{water}}(1/\rho_2)g - g = a_{2,y}$$

1 pt.

But: $\rho_1 = \rho_2$

So: $a_{1,y} = a_{2,y}$

The two pencils accelerate upward at the same initial rate.

2. f. A large aluminum salad bowl ($\rho_{\text{aluminum}} = 2700 \text{ kg/m}^3$), in the shape of a hemisphere of radius 18.0 cm, has a mass of 0.25 kg. You place this bowl, empty and upright (like a boat) into a vat containing pure water. Then you place pieces of wood ($\rho_{\text{wood}} = 680 \text{ kg/m}^3$) into the bowl until the bowl sinks. What volume of water is now displaced?

First, write the total weight of the loaded bowl at which the bowl's rim goes under (i.e. when the volume of water displaced by the bowl is equal to the bowl's hemispherical volume):

$$m_{\text{load}}(g) = (\rho_{\text{water}})(V_{\text{bowl}})(g)$$

Thus:

$$(m_{\text{bowl}} + m_{\text{wood}}) = (\rho_{\text{water}})(V_{\text{bowl}})$$

The mass of the wood that sinks the bowl:

$$\begin{aligned} m_{\text{wood}} &= (\rho_{\text{water}})(V_{\text{bowl}}) - m_{\text{bowl}} \\ &= (1000)[(2/3)\pi(0.18)^3] - 0.25 = 11.96 \text{ kg} \end{aligned}$$

The bowl sinks, but the wood floats—and displaces enough water to support its entire weight:

$$m_{\text{wood}}(g) = (\rho_{\text{water}})(V_{\text{disp.wood}})(g)$$

Solve this for $V_{\text{disp.wood}}$:

$$\begin{aligned} V_{\text{disp.wood}} &= m_{\text{wood}}/\rho_{\text{water}} \\ &= 11.96/1000 = 0.01196 \text{ m}^3 \end{aligned}$$

The sunken bowl displaces only the metal's volume:

$$\begin{aligned} V_{\text{alum}} &= m_{\text{bowl}}/\rho_{\text{aluminum}} \\ &= 0.25/2700 = 9.25 \times 10^{-5} \text{ m}^3 \end{aligned}$$

The total water displaced is the sum of these two:

$$\begin{aligned} V_{\text{disp.total}} &= V_{\text{disp.wood}} + V_{\text{alum}} \\ &= 0.01196 + 9.25 \times 10^{-5} = \mathbf{0.0121 \text{ m}^3} \end{aligned}$$

1 pt.

- g. A raft is made of 14 identical logs lashed together. Each log is 35.0 cm in diameter, 9.75 m long, and has a density of 760 kg/m³. How many 75-kg persons can the raft hold (floating in pure water) while still keeping everyone's feet dry?

The buoyant force is a weight. It's the weight of the fluid being displaced: $F_B = F_{G,\text{fl. disp}}$

Buoyancy is a matter of comparing weights. When an object floats, its entire weight, $F_{G,\text{obj}}$, is supported (opposed) by the buoyant force, F_B : $F_{G,\text{obj}} = F_B$ That is: $F_{G,\text{obj}} = F_{G,\text{fl. disp}}$

(But $F_{G,\text{obj}} = F_B$ is true only if the object is floating. If it is not floating—i.e. if it's sitting at the bottom of the fluid—then $F_B = F_{G,\text{fl}}$ is still true, but it's not supporting the entire $F_{G,\text{obj}}$; that is, $F_{G,\text{obj}} > F_{G,\text{fl. disp}}$.)

In this problem, $F_{G,\text{obj}} = F_{G,\text{raft}} + F_{G,\text{people}}$. And we can calculate F_B , the weight of the fluid displaced:

$F_B = F_{G,\text{fl. disp}} = (\rho_{\text{water}})(V_{\text{water. displ}})g = (\rho_{\text{water}})(V_{\text{raft}})g$, because in order to get the maximum buoyant force, the raft logs must displace the maximum amount of water—be completely immersed.

And $V_{\text{raft}} = 14V_{\text{log}}$, so $F_{G,\text{fl. disp}} = (\rho_{\text{water}})(14V_{\text{log}})g$

So: $(\rho_{\text{water}})(14V_{\text{log}})g = F_{G,\text{raft}} + F_{G,\text{people}}$

But: $F_{G,\text{raft}} = (\rho_{\text{log}})(V_{\text{raft}})g = (\rho_{\text{log}})(14V_{\text{log}})g$

And: $F_{G,\text{people}} = n(mg)$

(n is the number of persons; m is the mass of a person.)

Putting it all together: $(\rho_{\text{water}})(14V_{\text{log}})g = (\rho_{\text{log}})(14V_{\text{log}})g + n(mg)$

Solve that for n : $n = [14V_{\text{log}}(\rho_{\text{water}} - \rho_{\text{log}})]/m$

But $V_{\text{log}} = (\pi r_{\text{log}}^2)l_{\text{log}}$

So: $n = [14(\pi r_{\text{log}}^2)l_{\text{log}}(\rho_{\text{water}} - \rho_{\text{log}})]/m$

The numbers: $n = [14\pi(0.175^2)(9.75)(1000 - 760)]/75 = 42.025...$

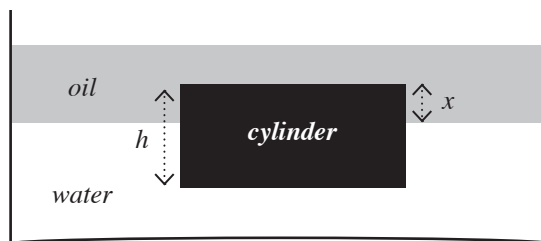
The raft will support up to 42 persons (assuming each person's mass is 75 kg).

1 pt.

2. h. A solid cylinder (radius = 0.254 m; height = 0.180 m) has a mass of 32.0 kg.

At first, the cylinder is floating freely (with its flat top and bottom horizontal) in a tub of pure water.

Then oil ($\rho_{oil} = 690 \text{ kg/m}^3$) is poured into the tub, and the cylinder comes to rest, now fully submerged—but partly in the water, partly in the oil—as shown here. It is still floating (i.e. not touching the bottom of the tub).



(SIDE VIEW – drawing not to scale)

How much of the cylinder (i.e. what part of its height, measured in meters) is in the oil?

A buoyant force is always equal to the weight of the fluid displaced:

$$F_B = W_{fl.disp}$$

In more detail:

$$F_B = \rho_{fl} V_{fl.disp} g$$

So:

$$F_{B.water} = \rho_{water} V_{water.disp} g$$

And:

$$F_{B.oil} = \rho_{oil} V_{oil.disp} g$$

The total buoyant force here is the sum of the two buoyant forces:

$$F_{B.T} = F_{B.water} + F_{B.oil}$$

Substituting:

$$F_{B.T} = \rho_{water} V_{water.disp} g + \rho_{oil} V_{oil.disp} g$$

The object is floating, so the total buoyant force on the cylinder is equal to the object's entire weight:

$$F_{B.T} = F_{G.obj}$$

Therefore:

$$F_{G.obj} = \rho_{water} V_{water.disp} g + \rho_{oil} V_{oil.disp} g$$

Or:

$$m_{obj} g = \rho_{water} V_{water.disp} g + \rho_{oil} V_{oil.disp} g$$

Simplifying:

$$m_{obj} = \rho_{water} V_{water.disp} + \rho_{oil} V_{oil.disp}$$

1 pt.

The portion of the cylinder immersed in the water is displacing that much water:

$$V_{water.disp} = V_{obj.water}$$

$$\text{Or: } V_{water.disp} = \pi r^2 (h - x)$$

The portion of the cylinder immersed in the oil is displacing that much oil:

$$V_{oil.disp} = V_{obj.oil}$$

$$\text{Or: } V_{oil.disp} = \pi r^2 x$$

$$\text{Substituting: } m_{obj} = \rho_{water} \pi r^2 (h - x) + \rho_{oil} \pi r^2 x$$

Now solve for x: $(\rho_{water} \pi r^2 h - m_{obj}) / (\rho_{water} \pi r^2 - \rho_{oil} \pi r^2) = x$

The numbers: $[(1000)\pi(.254)^2(.180) - 32.0] / [(1000)\pi(.254)^2 - (690)\pi(.254)^2] = x = \underline{.0713 \text{ m}}$

The cylinder is immersed in the oil for 7.13 cm of its total height.

(See notes on the next page for more discussion of buoyancy and Archimedes' Principle.)

Why Archimedes' Principle Works

The buoyant force is the net of all fluid forces acting on any part of an object that's in contact with the fluid. Because fluid pressure increases with depth, this net force is always upward, toward the surface of the fluid.

The old reliable way to calculate this net force, F_B , is as the weight of the fluid displaced (Archimedes' Principle), but the physics is always that the buoyant force is the net of the “underneath” fluid pressure forces (acting upward) vs. the “overhead” fluid pressure forces (acting downward); the lateral fluid forces all oppose and cancel one another.

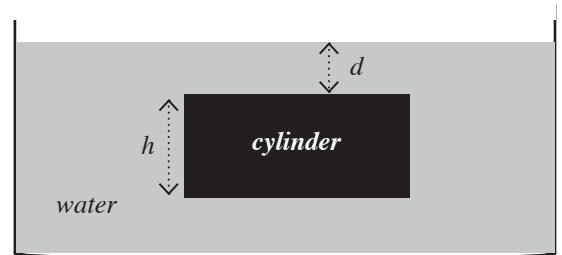
That physics principle is fairly easy to visualize if there's just one fluid and the object is fully immersed, so that we can calculate and compare the upward and downward fluid pressure forces—and the math for Archimedes Principle emerges fairly readily. See Case A below.

But it's not nearly so obvious why Archimedes' Principle still works mathematically when there's no fluid pressure on top at all (the object is only partly immersed). See Case B below.

And it's even less evident in Case C (this HW problem), when there's a combination of two fluids. (Indeed: How can the oil offer any upward forces on the cylinder when no oil contacts the under side? Answer: It doesn't. But the presence of the oil layer increases the water pressure on the underside sufficiently to behave as if it's supporting the weight of the oil that's been displaced.) Below is the solution “from scratch,” proving why using Archimedes' Principle is valid even here.

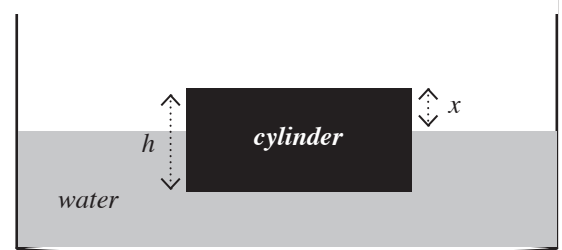
Case A:

$$\begin{aligned}
 F_B &= F_{\text{bottom}} - F_{\text{top}} \\
 &= P_{\text{bottom}}(A) - P_{\text{top}}(A) \\
 &= [P_{\text{atm}} + \rho_{\text{water}}g(h + d)](A) - [P_{\text{atm}} + \rho_{\text{water}}gd](A) \\
 &= [\rho_{\text{water}}gh](A) \\
 &= (\rho_{\text{water}}g)Ah \\
 &= \rho_{\text{water}}gV_{\text{water.displ}}
 \end{aligned}$$



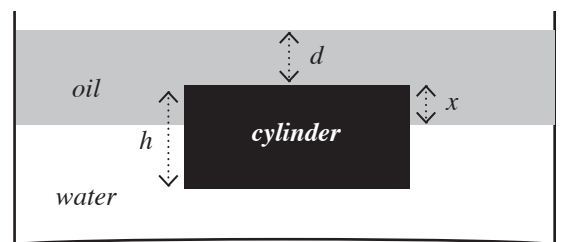
Case B:

$$\begin{aligned}
 F_B &= F_{\text{bottom}} - F_{\text{top}} \\
 &= P_{\text{bottom}}(A) - P_{\text{top}}(A) \\
 &= [P_{\text{atm}} + \rho_{\text{water}}g(h - x)](A) - P_{\text{atm}}(A) \\
 &= [\rho_{\text{water}}g(h - x)](A) \\
 &= (\rho_{\text{water}}g)[A(h - x)] \\
 &= \rho_{\text{water}}gV_{\text{water.displ}}
 \end{aligned}$$



Case C:

$$\begin{aligned}
 F_B &= F_{\text{bottom}} - F_{\text{top}} \\
 &= P_{\text{bottom}}(A) - P_{\text{top}}(A) \\
 &= [P_{\text{atm}} + \rho_{\text{oil}}g(d + x) + \rho_{\text{water}}g(h - x)](A) - [P_{\text{atm}} + \rho_{\text{oil}}gd](A) \\
 &= [\rho_{\text{oil}}gx + \rho_{\text{water}}g(h - x)](A) \\
 &= \rho_{\text{oil}}g[Ax] + \rho_{\text{water}}g[(A)(h - x)] \\
 &= \rho_{\text{oil}}gV_{\text{oil.displ}} + \rho_{\text{water}}gV_{\text{water.displ}}
 \end{aligned}$$



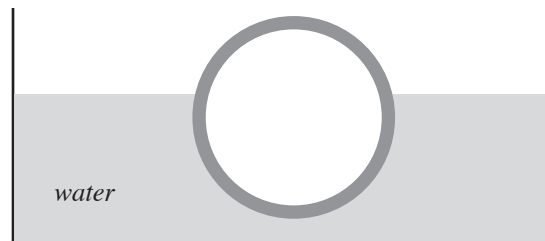
2. i. A hollow aluminum sphere (outer radius = 0.128 m) floats at rest in a tub of pure water.

tub

The sphere is 66.9% immersed.

The hollow chamber within the aluminum is also a sphere, but it is entirely empty (evacuated).

Find the radius of this hollow chamber.



The volume of the aluminum in the shell is the difference between the volume of the outer sphere (that's the entire object) and the volume of the inner sphere (that's the evacuated space): $V_{alum} = V_{outer} - V_{inner}$

In other words: $V_{inner} = V_{obj} - V_{alum}$

The volume of any sphere is $(4/3)\pi r^3$, where r is the radius of the sphere.

So: $(4/3)\pi r_{inner}^3 = (4/3)\pi r_{outer}^3 - V_{alum}$

Calculate V_{obj} : $V_{obj} = (4/3)\pi r_{outer}^3 = (4/3)\pi(0.128)^3 = 8.78453 \times 10^{-3} \text{ m}^3$

Therefore: $(4/3)\pi r_{inner}^3 = 8.78453 \times 10^{-3} \text{ m}^3 - V_{alum}$

Solve for r_{inner} : $r_{inner} = \{(3/4\pi)[8.78453 \times 10^{-3} - V_{alum}]\}^{1/3}$

So we need to find the volume of the aluminum shell, V_{alum} .

First, note that $\rho_{alum} = 2700 \text{ kg/m}^3$ (this would be given on an exam)

Then, note that $V_{alum} = m_{alum}/\rho_{alum}$

But the mass of the aluminum is the object's entire mass (there's nothing else—not even air): $m_{alum} = m_{obj}$

Therefore: $V_{alum} = m_{obj}/\rho_{alum} = m_{obj}/2700$

Now, notice: $\rho_{obj} = m_{obj}/V_{obj}$

Or: $m_{obj} = \rho_{obj}V_{obj}$

Notice also: $\rho_{obj}/\rho_{water} = 0.669$ (this density relationship applies, because the object is floating)

Thus: $\rho_{obj} = 0.669\rho_{water} = 669 \text{ kg/m}^3$

Substituting: $m_{obj} = 669V_{obj} = 669(8.78453 \times 10^{-3}) = 5.87685 \text{ kg}$

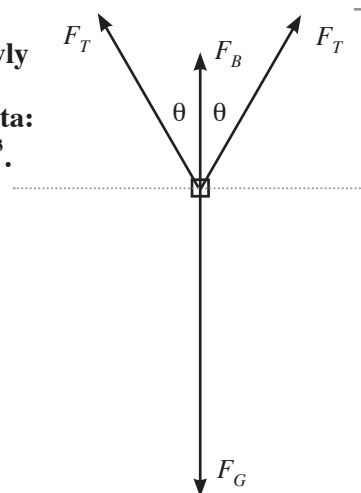
Therefore: $V_{alum} = m_{obj}/\rho_{alum} = 5.87685/2700 = 2.1766 \times 10^{-3} \text{ m}^3$

Finally: $r_{inner} = \{(3/4\pi)[8.78453 \times 10^{-3} - 2.1766 \times 10^{-3}]\}^{1/3} = \underline{0.116 \text{ m}}$

1 pt.

- j. In 1968, engineers retrieved a jettisoned nuclear missile in shallow water in Baffin Bay by attaching two cables to the tip of the missile and raising it slowly [slowly; hence, we ignore drag here—and focus on the initial a_y value]. Find the magnitude of the missile's [initial] upward acceleration with this data:

- The missile was completely submerged in seawater of density 1030 kg/m^3 .
- The missile's mass was 425 kg and its volume was 0.200 m^3 .
- The tension in each cable was 1750 N .
- Each cable made an angle of 30.0° with the vertical.



$$\Sigma F_y = ma_y$$

$$F_{T,y} + F_B + F_{T,y} - F_G = ma_y$$

$$2F_T \cos \theta + \rho V g - mg = ma_y$$

$$a_y = (2F_T \cos \theta + \rho V g - mg)/m$$

$$= [2(1750)\cos 30 + (1030)(.2)(9.80) - (425)(9.80)]/425 = \underline{2.08 \text{ m/s}^2}$$

1 pt.

3. a. Evaluate the following statements (T/F/N). As always, explain your reasoning.

(i) ρgh is a calculation of gravitational potential energy per unit volume.

True. It is the per-unit-volume simplification (in Bernoulli's equation) of the expression $mgh (= \rho Vgh)$.

(ii) At a certain depth in the Pacific Ocean, the total pressure is P . If you go to twice that depth (treating the seawater as static and incompressible), the total pressure is less than $2P$.

True. At the first depth (h): $P_{total} = P_{atm} + \rho gh$
At twice the depth ($2h$): $P_{total} = P_{atm} + 2\rho gh < 2(P_{atm} + \rho gh)$

(iii) A manometer can indicate negative gauge pressure.

True. If the fluid level in the tube on the side exposed to P_{atm} is lower than the one side exposed to the total pressure in the container, that must mean $P_{atm} > P_{container}$. And $P_{gauge} = P_{container} - P_{atm}$.

(iv) If a barometer is 2 m tall, its fluid must have a density less than water.

False (assuming we're talking about a barometer built to measure earth's surface P_{atm} ; if not, then there's not enough information). The fluid column in a barometer must be of height h such that $\rho_{fluid}gh = P_{atm}$. For earth's surface P_{atm} , and $\rho_{water} = 1000 \text{ kg/m}^3$, h would need to be greater than 10 m.

(v) Gauge pressure is never more than absolute pressure.

True. $P_{gauge} = P_{abs} - P_{atm}$

(vi) When you sip liquid through a drinking straw, you are creating a space in your mouth that has negative gauge pressure.

True. $P_{gauge} = P_{abs} - P_{atm}$

(vii) As you place an ice cube in a half-full glass of water (and no water spills out as a result), the water pressure on the bottom of the glass will increase; and then, as that ice cube melts, the water pressure on the bottom of the glass decreases.

False. The first part of the statement is true: The pressure on the bottom does increase because it's now deeper (i.e. the surface has risen higher). But the second part of the statement is false. When floating ice melts, the level of the water does not change.

(viii) If water were a compressible fluid, its static pressure at 1 m depth would be greater than it is as an incompressible fluid.

True. If water were compressible, then its density would increase with depth, and therefore the gravitational energy lost per unit volume (as you descend) would be greater, since more mass would be moving downward.

(ix) Neither a soda straw nor a vacuum cleaner would operate effectively on the moon.

True. Both actions require the surrounding atmosphere to push air into a low-pressure chamber, but there is no such atmosphere on the moon..

(x) "Suction" is a tension force.

False. "Suction" is not a pulling force. It seems to be "drawing" fluid (into a chamber where there is less pressure than the surrounding atmosphere), but it's actually an uneven contest of pushing (pressure) forces.

b. Evaluate the following statements (T/F/N). As always, explain your reasoning.

(i) Bernoulli's equation equates the mechanical energy density of any two points in a steady flow of incompressible fluid.

True. The equation was derived directly from mechanical energy accounting (assuming $W_{ext} = 0$), and then expressed on a per-volume basis. Hence: energy per unit volume, or energy density.

(ii) In any steady flow of incompressible fluid, a lower point in the flow always has higher pressure.

False. A lower point in a static fluid always has a higher pressure, but if the fluid is moving—faster at the lower point than the higher point—then it's possible that the lower point has lower pressure.

(iii) In a steady, vertical flow of incompressible fluid, the fluid pressure in a narrower section of pipe can be greater than in a wider section of pipe.

True. Smaller pipe diameter does mean faster fluid flow, and if the flow is horizontal, faster flow means lower pressure, but that won't necessarily be true if the faster flow is at a lower altitude.

(iv) In any steady horizontal flow of incompressible fluid, higher pressure is associated with smaller pipe diameter.

False. Smaller pipe diameter means faster fluid flow, and if the flow is horizontal, faster flow means lower pressure.

3. b. (v) **In any steady flow of incompressible fluid, the fluid pressure in a narrower section of pipe is always less than in a wider section of pipe.**

False. Smaller pipe diameter does mean faster fluid flow, and if the flow is horizontal, faster flow means lower pressure, but that won't necessarily be true if the faster flow is at a lower altitude.

- (vi) **If a strong wind were to blow across the top of a drinking straw sitting in a glass of water, then (assuming the water in the glass is sheltered from this wind) the water level in the straw would rise slightly.**

True. Faster fluid flow at the same level as a slower portion (of the same fluid) has lower pressure. (Even though air is a compressible fluid, there is a version of Bernoulli's Equation—modified for variable density—that applies here.) So the motionless air over the glass would have higher pressure than the air across the top of the straw, so the water level in the straw would rise to equalize.

- (vii) The mass of an object can also be expressed $\rho_{obj} V_{obj} g$.

False. $\rho_{obj} = m_{obj}/V_{obj}$, so $(\rho_{obj} V_{obj}) = m_{obj}$. The expression $\rho_{obj} V_{obj} g$ is the weight of the object.

- (viii) The density of an incompressible fluid can be significantly changed, but not by pressure.

True. This is the definition of “incompressible.”

- c. **What is the gauge pressure at the surface of an open-air swimming pool?**

Zero. $P_{gauge} = P_{total} - P_{atm}$, but in the open atmosphere, $P_{total} = P_{atm}$.

- d. **What is the gauge pressure in the chamber at the top of a barometer?**

-1 atm. $P_{gauge} = P_{total} - P_{atm}$, but in the chamber at the top of a barometer, $P_{total} = 0$.

- e. **Express energy density in standard (not base) SI units.**

Pa. Energy density has units of $\text{J/m}^3 = \text{N}\cdot\text{m/m}^3 = \text{N/m}^2 = \text{Pa}$.

- f. **Express 3.25 atm in fundamental (“base”) SI units.**

$$3.25 \cdot (1.01 \times 10^5 \text{ Pa}) = 3.28 \times 10^5 \text{ Pa} = 3.28 \times 10^5 \text{ N/m}^2 = \mathbf{3.28 \times 10^5 \text{ kg/(m}\cdot\text{s}^2)}$$

- g. **Calculate the fluid pressure potential energy (U_{FL}) contained in 1 cm^3 of pure water at a [static] depth of 1.00 m.**

$$U_{FL} = PV = (P_{atm} + \rho gh)(V) = [1.01 \times 10^5 + (1000)(9.80)(1.00)](1 \times 10^{-6} \text{ m}^3) = \mathbf{0.111 \text{ J}}$$

4. a. Match the expression on the left, with the one best type of measure that it represents, from the list on the right.

(i) $(1/2)\rho v^2$	Pressure
(ii) P/A	(Nothing meaningful)
(iii) $\rho A v$	Mass flow rate
(iv) $\rho g h$	Pressure
(v) PA	Force
(vi) $A v$	Volume flow rate
(vii) $\rho V g$	Weight force

- b. If you were to build a manometer, using pure water as the fluid, how tall would its column need to be to indicate a total pressure of 1.56 atm inside the pressurized container?

The column in a manometer measures the difference between the total pressure in the container and the pressure at the top of the (open) column, which is just atmospheric pressure.

In other words: $P_{\text{deep}} = P_{\text{shallow}} + \rho g h$ (where $P_{\text{deep}} = P_{\text{total}}$ and $P_{\text{shallow}} = P_{\text{atm}}$).

Thus: $\rho g h = P_{\text{total}} - P_{\text{atm}} = 1.56 \text{ atm} - 1 \text{ atm} = 0.56 \text{ atm}$

Solve for h : $h = (0.56 \text{ atm})/(\rho g) = [(0.56)(1.01 \times 10^5)]/[(1000)(9.80)] = \underline{5.77 \text{ m}}$

- c. If you were to build a barometer using a fluid with a density of $6,700 \text{ kg/m}^3$, how tall would its column need to be to measure an air pressure that is 10% greater than standard atmospheric pressure?

Since the bulb is evacuated ($P = 0$) at the top of the fluid column of a barometer, the height of that column represents (via $\rho g h$) the full value of P_{atm} : $P_{\text{deep}} = P_{\text{shallow}} + \rho g h$ (where $P_{\text{deep}} = P_{\text{atm}}$ and $P_{\text{shallow}} = 0$).

The column capacity must be able to measure 110% of standard P_{atm} , so:

$$\rho g h = 1.10 P_{\text{atm}}$$

Solve for h : $h = (1.10 P_{\text{atm}})/(\rho g) = [(1.10)(1.01 \times 10^5)]/[(6700)(9.80)] = \underline{1.69 \text{ m}}$

- d. You're practicing your scuba technique in an outdoor swimming pool (filled with pure water—they forgot to add chemicals). The open-air barometer by the pool reads exactly 1 atm before you dive. So you dive down to the bottom of the pool (4.0 m deep) and immediately note the pressure now reading on your pressure gauge (which read zero at the surface). You sit down there for say, a half-hour, then look at your pressure gauge again. Now its reading has increased by 1.25% (compared to the reading you noted a half-hour ago. What is the barometer's new reading (in atm)? [Note: Keep in mind that your gauge is always measuring against the original 1 atm reading you took before you dove; it cannot re-calibrate once you're submerged.]

Initially: $P_{\text{deep},i} = P_{\text{shallow},i} + \rho g h$ (And: $P_{\text{gauge},\text{deep},i} = P_{\text{deep},i} - 1 \text{ atm}$)

Finally: $P_{\text{deep},f} = P_{\text{shallow},f} + \rho g h$ (And: $P_{\text{gauge},\text{deep},f} = P_{\text{deep},f} - 1 \text{ atm}$)

We know: $(1.0125)(P_{\text{gauge},\text{deep},i}) = P_{\text{gauge},\text{deep},f}$

That is: $(1.0125)(P_{\text{deep},i} - 1 \text{ atm}) = P_{\text{deep},f} - 1 \text{ atm}$

Or: $(1.0125)(P_{\text{shallow},i} + \rho g h - 1 \text{ atm}) = P_{\text{shallow},f} + \rho g h - 1 \text{ atm}$

Thus: $(1.0125)(P_{\text{shallow},i} + \rho g h - 1 \text{ atm}) + 1 \text{ atm} - \rho g h = P_{\text{shallow},f}$
 $= (1.0125)[1 \text{ atm} + (1000)(9.80)(4.0) - 1 \text{ atm}] + 1 \text{ atm} - [(1000)(9.80)(4.0)] = 101,490 \text{ Pa}$
 $= 101,490/(1.01 \times 10^5) \text{ atm} = \underline{1.00(5) \text{ atm}}$

4. e. A storm gathers over the local neighborhood swimming pool (still filled with pure water). Previously the barometer read exactly 1 atm, but then it drops by 2.75%. By what percentage does the pressure drop at the bottom of the pool, where the depth is 3.00 m?

$$\Delta P_{\text{surface}} (= P_{\text{surface},f} - P_{\text{surface},i}) = -(0.0275)(1 \text{ atm}) = -.0275 \text{ atm}$$

$$\Delta P_{\text{bottom}} (= P_{\text{bottom},f} - P_{\text{bottom},i}) = \Delta P_{\text{surface}} \quad \text{This is Pascal's Principle}$$

Therefore: $\Delta P_{\text{bottom}} = -.0275 \text{ atm} = -(0.0275)(1.01 \times 10^5 \text{ Pa}) = -2777.5 \text{ Pa}$

By definition: $\Delta\%P_{\text{bottom}} = 100(\Delta P_{\text{bottom}}/P_{\text{bottom},i})$

That is: $\Delta\%P_{\text{bottom}} = 100[\Delta P_{\text{bottom}}/(P_{\text{shallow},i} + \rho g h)]$

The numbers: $\Delta\%P_{\text{bottom}} = (100)(-2777.5)/[1.01 \times 10^5 + (1000)(9.80)(3)] = \underline{-2.13\%}$

(The pressure at the bottom of the pool drops by 2.13%.)

4. f. An open tank holds pure water. The pressure at point X in the water is 2.00 atm when the pressure at the surface is 1.00 atm. Find the pressure at X (in atm) when the surface pressure has increased by 7.5%.

$$\Delta P_{\text{surface}} = 0.075(1 \text{ atm}) = 0.075 \text{ atm}$$

$$\Delta P_X = \Delta P_{\text{surface}} = 0.075 \text{ atm} \quad (\text{This is Pascal's Principle})$$

Therefore: $P_{X,f} = P_{X,i} + \Delta P_X = 2 \text{ atm} + 0.075 \text{ atm} = \mathbf{2.08 \text{ atm}}$

- g. Pure water and then oil are poured into this U-shaped tube, open at both ends, and, as shown here, they come to equilibrium. What is the oil's density?

The pressures at points A and B are equal: $P_A = P_B$ Any two points at the same level (same "altitude") in the same incompressible static fluid are at equal pressure.

Also, both the surface pressures above each respective column of fluid are just P_{atm} .

And by the equation for pressure at depth in a static fluid, we know that

$$P_A = P_{\text{atm}} + \rho_{\text{oil}} g h_{\text{oil}}$$

$$\text{and} \quad P_B = P_{\text{atm}} + \rho_{\text{water}} g h_{\text{water}}$$

Substituting, we have this:

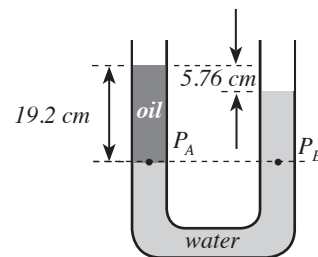
$$P_{\text{atm}} + \rho_{\text{oil}} g h_{\text{oil}} = P_{\text{atm}} + \rho_{\text{water}} g h_{\text{water}}$$

Just solve that for ρ_{oil} .

$$\rho_{\text{oil}} = (\rho_{\text{water}} h_{\text{water}}) / h_{\text{oil}}$$

Plug in the numbers:

$$\rho_{\text{oil}} = (1000)(.192 - .0576) / .192 = \mathbf{700 \text{ kg/m}^3}$$



5. a. In the situation shown (where two masses press on equal-height cylinders of fluid), if $d_2 = 3d_1$, and $m_2 = 8m_1$, then is m_2 rising or sinking? Explain (show your work).

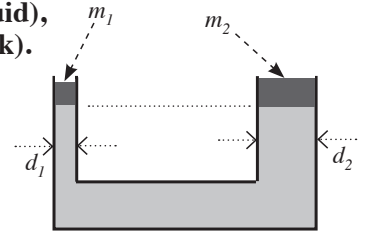
The fluid pressures just under each cylinder head:

$$P_1 = (m_1)(g)/[\pi(d_1/2)^2] + P_{atm} = 4(m_1)(g)/[\pi(d_1)^2] + P_{atm}$$

$$P_2 = P_{atm} + (m_2)(g)/[\pi(d_2/2)^2] = 8(m_1)(g)/[\pi(3d_1/2)^2] + P_{atm}$$

$$= (32/9)(m_1)(g)/[\pi(d_1)^2] + P_{atm}$$

Since $4 > 32/9$, $P_1 > P_2$, so m_2 is **rising** (because the fluid at point 1 is being pressed down on with more pressure than at point 2).



- b. Evaluate the following statement (T/F/N). Explain your reasoning. In the hydraulic lift arrangement shown at right, the two masses sit at rest on massless cylinder heads above a reservoir of incompressible hydraulic fluid. The chambers above the masses are sealed, with gas pressures P_1 and P_2 . If $d_2 = 3d_1$, $h_1 = 2h_2$, and $m_2 = 8m_1$, then $P_2 > P_1$.

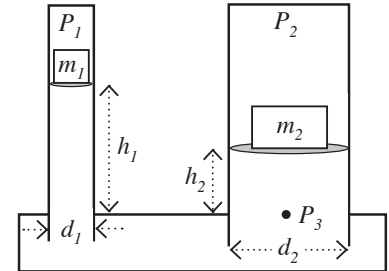
True: $P_3 = P_1 + m_1g/A_1 + \rho gh_1$ And: $P_3 = P_2 + m_2g/A_2 + \rho gh_2$

So: $P_1 + m_1g/A_1 + \rho gh_1 = P_2 + m_2g/A_2 + \rho gh_2$

$$= P_2 + 8m_1g/(9A_1) + (1/2)\rho gh_1$$

So: $P_2 - P_1 = [m_1g/A_1 + \rho gh_1] - [8m_1g/(9A_1) + (1/2)\rho gh_1] = (1/9)(m_1g)/A_1 + (1/2)\rho gh_1$

But: $(1/9)(m_1g)/A_1 + (1/2)\rho gh_1 > 0$ (all values are magnitudes). Thus: $P_2 > P_1$



- c. In the diagram shown at right (not to scale), everything is at rest. $F = 36.0 \text{ N}$ $m = 5,000 \text{ kg}$ $d_1 = 4.00 \text{ cm}$ $d_2 = 2.20 \text{ m}$ $h = 2.00 \text{ m}$ Find the density of the oil.

Let Point 1 be just beneath the left (smaller) plate.

Let Point 2 be just beneath the right (larger) plate

Then $P_1 = P_2 + \rho gh$

But: $P_1 = P_{atm} + F/[\pi(d_1/2)^2]$

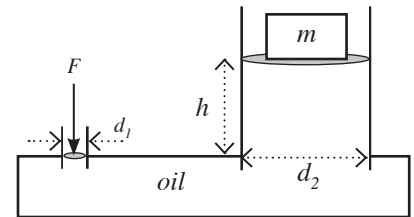
And: $P_2 = P_{atm} + mg/[\pi(d_2/2)^2]$

Therefore: $P_{atm} + F/[\pi(d_1/2)^2] = P_{atm} + mg/[\pi(d_2/2)^2] + \rho gh$

Simplify: $4F/(\pi d_1^2) = 4mg/(\pi d_2^2) + \rho gh$

Solve for ρ : $\rho = [4/(\pi gh)][(F/d_1^2) - (mg/d_2^2)] = \{4/[\pi(9.80)(2.00)]\}\{36.0/(.04^2) - [(5000)(9.80)/(2.20^2)]\}$

$$= 804 \text{ kg/m}^3$$



- d. Refer again to the diagram above, at right. Again, everything is at rest. Here is the data this time: $F = 36 \text{ N}$ $m = 5,000 \text{ kg}$ $d_1 = 4.00 \text{ cm}$ $d_2 = 2.20 \text{ m}$ $\rho_{oil} = 750 \text{ kg/m}^3$ By what factor would you need to increase (multiply) F in order to lift m (increase h) by 0.81 m (where everything would again be at rest)? Assume F is still applied in the manner shown.

First, find h_i . Let Point 1 be just beneath the left plate. Let Point 2 be just beneath the right plate.

Then $P_1 = P_2 + \rho gh$

But: $P_1 = P_{atm} + F_i/[\pi(d_1/2)^2]$

And: $P_2 = P_{atm} + mg/[\pi(d_2/2)^2]$

Therefore: $P_{atm} + F_i/[\pi(d_1/2)^2] = P_{atm} + mg/[\pi(d_2/2)^2] + \rho gh_i$

Simplify: $4F_i/(\pi d_1^2) = 4mg/(\pi d_2^2) + \rho gh_i$

Solve for h_i : $h_i = [4/(\pi \rho g)][(F_i/d_1^2) - (mg/d_2^2)]$

$$h_i = \{4/[(\pi)(750)(9.80)]\}\{(36/0.04^2) - [(5000)(9.80)/2.20^2]\} = 2.1439 \text{ m}$$

Therefore: $h_f = h_i + 0.81 = 2.9539 \text{ m}$

Now solve for F_f , using the simplified result from above: $4F_f/(\pi d_1^2) = 4mg/(\pi d_2^2) + \rho gh_f$

$$F_f = [(\pi d_1^2)/4][4mg/(\pi d_2^2) + \rho gh_f]$$

$$F_f = [(\pi)(.04)^2/4]\{4(5000)(9.80)/[(\pi)(2.20)^2] + (750)(9.80)(2.9539)\} = 43.48 \text{ N}$$

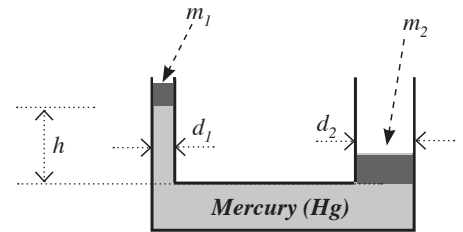
The factor increase is given by F_f/F_i : Factor = $43.48/36 = 1.21$

5. e. A dry wooden log (a cylinder: length = 4.26 m; diameter = 27.8 cm) is placed into a vat of preservative oil. Initially the log floats in the oil with 41.3% of its volume exposed above the level of the liquid. Then gradually, the floating log absorbs 39.1 kg of oil (with no change to the log's volume). As a result, 3/4 of the log's volume is now immersed. Find: (i) The initial mass of the (dry) log. (ii) The density of the oil.

(i) A floating object, %imm. = ρ_{obj}/ρ_{fluid} So: $\rho_{log,i} = (1 - 0.413)\rho_{oil}$ and: $\rho_{log,f} = 0.75\rho_{oil}$
 In other words: $m_{log}/V_{log} = (0.587)\rho_{oil}$ and: $(m_{log} + 39.1)/V_{log} = 0.75\rho_{oil}$
 Solve each for ρ_{oil} and equate: $(m_{log}/V_{log})/(0.587) = (m_{log} + 39.1)/[(0.75)V_{log}]$
 Solve for m_{log} : $m_{log} = (39.1)/[(0.75/0.587) - 1] = \mathbf{141\text{ kg}}$

(ii) $\rho_{log,i} = (1 - 0.413)\rho_{oil}$ Thus: $\rho_{oil} = \rho_{log,i}/0.587 = (m_{log}/V_{log})/0.587$
 $= \{140.808/[\pi(0.278/2)^2(4.26)]\}/0.587 = \mathbf{928\text{ kg/m}^3}$

- f. Everything is at rest in the initial situation shown. $h = 1.42\text{ m}$; $d_1 = .300\text{ m}$; $d_2 = .600\text{ m}$. Then m_1 is increased by 500 kg, and the system is allowed to come to rest again.

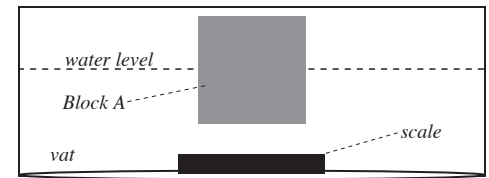


- (i) Which column of mercury (the column top) is now higher—and what is the height difference now between the two column tops?
 (ii) How much (and in what direction) has each column top moved?

(i) Let P_1 be the pressure at the top of column 1 of mercury (just under m_1); likewise for P_2 .
 Then initially: $P_{2,i} - P_{1,i} = \rho gh_i = (13,600)(9.80)(1.42) = 189,258\text{ Pa}$
 And finally: $P_{2,f} - P_{1,f} = \rho gh_f$
 But: $P_{2,f} = P_{2,i}$ and: $P_{1,f} = P_{1,i} + [(500)(9.80)]/[\pi(0.3/2)^2] = P_{1,i} + 69,321\text{ Pa}$
 Substitute: $P_{2,i} - (P_{1,i} + 69,321) = \rho gh_f$ That is: $(P_{2,i} - P_{1,i}) - 69,321 = \rho gh_f$
 That is: $189,258 - 69,321 = \rho gh_f$
 Thus: $h_f = (189,258 - 69,321)/[(13,600)(9.80)] = \mathbf{0.900\text{ m}}$
 (Note: We set up the equation assuming that m_1 would still be higher than m_2 . If h_f had turned out to be negative, that would have shown that our assumption was wrong; but we were correct.)

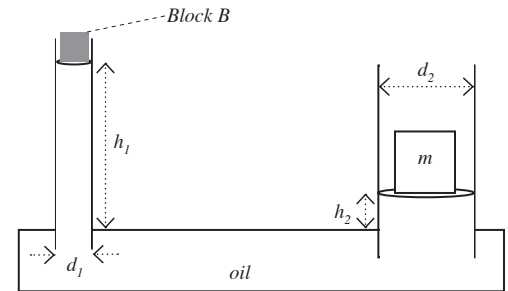
(ii) The magnitude of the depth change was: $|\Delta h| = |1.42 - 0.900| = 0.520\text{ m}$. But *both* columns must have adjusted (#1 down; #2 up)—because the fluid is incompressible. Thus: $|\Delta h_1| + |\Delta h_2| = 0.520$
 Also: $|\Delta V_1| = |\Delta V_2|$ That is: $A_1|\Delta h_1| = A_2|\Delta h_2|$
 In other words: $\pi(d_1/2)^2|\Delta h_1| = \pi(d_2/2)^2|\Delta h_2|$
 Simplify: $|\Delta h_1| = (d_2/d_1)^2|\Delta h_2|$ Numbers: $|\Delta h_1| = 4|\Delta h_2|$
 Solve (two equations, two unknowns): $|\Delta h_1| + |\Delta h_2| = 0.520$ and $|\Delta h_1| = 4|\Delta h_2|$
 Results: $|\Delta h_1| = 0.416\text{ m}$; $|\Delta h_2| = 0.104\text{ m}$ (m_1 sinks by 0.416 m; m_2 rises by 0.104 m)

5. g. You have two solid uniform blocks, A and B. As shown in the first diagram, block A (a cube, 0.870 m on each edge) is initially floating, 59.1% immersed in a shallow vat of pure water. Naturally, at the bottom of the vat is a kitchen scale. When block B is set atop block A, the stack of two blocks sinks to the bottom of the vat, coming to rest on the scale. At that point, block A is 87.2% immersed, while block B is still dry. The scale reads 850 N.



If you were to take block B and set it instead on the hydraulic lift mechanism in the position shown here (in the second diagram), what mass, m , could it support?

The tank is filled with hydraulic oil of density 892 kg/m^3 .
The two cylinders have diameters $d_1 = 0.185 \text{ m}$ and $d_2 = 0.952 \text{ m}$.
The (massless) platforms are at known heights above the top of the tank: $h_1 = 5.90 \text{ m}$; $h_2 = 1.82 \text{ m}$.



To answer the question (about the second diagram/situation), clearly we need to know W_B , the weight of block B. To find that, we'll need to use the first diagram/situations—so we'll need to know all about block A first.

$$V_A = (0.870)^3 = 0.658503 \text{ m}^3$$

$$\rho_A/\rho_{\text{water}} = 59.1\% = 0.591 \quad \text{or:} \quad \rho_A/1000 = 0.591 \quad \text{So:} \quad \rho_A = 591 \text{ kg/m}^3$$

$$\rho_A = m_A/V_A, \text{ so:} \quad m_A = \rho_A V_A = (591)(0.658503) = 389.18 \text{ kg}$$

$$\text{And:} \quad F_{G,A} = m_A g = (389.18)(9.80) = 3813.92 \text{ N}$$

Now do a FBD and analysis of block A after block B has been placed on it:

coord. axes	FBD of block A	y-analysis
		$\Sigma F_{A,y} = m_A a_{A,y}$ $F_{\text{Buoyant}} + F_N - F_{G,A} - F_{N,BA} = m_A a_{A,y}$ $\rho_{\text{water}}(0.872V_A)g + F_{\text{scale}} - m_A g - F_{N,BA} = 0$ $1000(0.872)(.658503)(9.80) + 850 - 3813.92 - F_{N,BA} = 0$ $F_{N,BA} = 1000(0.872)(.658503)(9.80) + 850 - 3813.92$ $= 2663.39 \text{ N}$

But $F_{N,BA} = F_{N,AB}$ (by Newton's Third Law), and since there are no other y-forces except gravity on Block B, clearly, $F_{G,A} = F_{N,AB} = 2663.39 \text{ N}$.

Now use what we know about block B and do a simple pressure equation for the second situation. Any two points in the tank of static fluid that are at the same height will have the same pressure. A convenient height to choose here is the top of the main tank, since that's where the two heights, h_1 and h_2 , are measured from.

$$P_{\text{atm}} + W_B/A_1 + \rho_{\text{oil}}(g)h_1 = P_{\text{atm}} + mg/A_2 + \rho_{\text{oil}}(g)h_2$$

$$\text{Simplify:} \quad W_B/A_1 + \rho_{\text{oil}}(g)(h_1 - h_2) = mg/A_2$$

$$\text{Solve for } m: \quad (A_2/g)[W_B/A_1 + \rho_{\text{oil}}(g)(h_1 - h_2)] = m$$

$$\text{But note:} \quad A_1 = \pi d_1^2/4 \quad \text{and} \quad A_2 = \pi d_2^2/4$$

$$\text{Substitute:} \quad m = [\pi d_2^2/(4g)][4W_B/(\pi d_1^2) + \rho_{\text{oil}}(g)(h_1 - h_2)]$$

$$\text{Calculate:} \quad m = \{\pi(.952)^2/[4(9.80)]\}\{4(2663.39)/[\pi(.185)^2] + (892)(9.80)(5.90 - 1.82)\} = \underline{9.79 \times 10^3 \text{ kg}}$$

6. a. If you're pumping water out of a 64 m^3 swimming pool using a hose 15.0 cm in diameter, and the water is flowing in the hose at a rate of 5.72 m/s , how long will it take you to empty the pool?

The volumetric flow rate (volume per second) is given by $Q = Av$, where A is the cross-sectional area of the hose, and v is the water speed as it flows through that cross-section. And then the time needed for a certain total volume flow, V , is: $t = V/Q$ Thus: $t = V/(Av) = V/[\pi(d/2)^2(v)] = 64/[\pi(0.15/2)^2(5.72)] = \underline{633 \text{ s}}$ (10 min, 33 s)

- b. The aorta has a radius of about 1.1 cm , and it carries blood away from the heart at a speed of about 40 cm/s . It branches eventually into a very large number of capillaries that distribute the blood to all cells throughout the body. A capillary typically has a radius of about $6 \times 10^{-4} \text{ cm}$, and blood travels through it at about 0.07 cm/s . Calculate the approximate number of capillaries in the human body.

Treating the blood as an incompressible fluid, use the Equation of Continuity for volumetric flow: $A_1 v_1 = A_2 v_2$

Here, the cross-sectional area of N (an unknown number of capillaries) forms the total cross-sectional area A_2 .

Thus: $A_a v_a = N A_c v_c$ Solve for N : $N = (A_a v_a)/(A_c v_c) = (\pi r_a^2 v_a)/(\pi r_c^2 v_c)$

Simplify: $N = (r_a^2 v_a)/(r_c^2 v_c) = [(0.11)^2(40)]/[(6 \times 10^{-4})^2(0.007)] = \underline{1.92 \times 10^9}$ (1.92 billion capillaries!)

- c. In a factory, a pipe that is 2 cm in diameter at ground level delivers water at a rate of 1.5 kg/s to a larger pipe located several meters above. If the gauge pressure at ground level is 2 atm (assign $h = 0$ at ground level), what is the total mechanical energy of one liter (1 L) of water flowing in the larger pipe above?

Bernoulli's equation comes directly from analysis of the total mechanical energy (the sum of all relevant kinetic and potential energy terms) of 1 m^3 of fluid. In other words, it sums the total mechanical energy density at any point in the flow. And since there is no work done between the two points in question, that energy density does not change: $E_{\text{mech.density},1} = E_{\text{mech.density},2}$ *This is what Bernoulli's equation is saying.*

So we do not need any information about the higher pipe to know the energy density of the fluid flowing in it.

We can just calculate the energy density of the fluid flowing in the lower pipe; the upper result would be the same.

Find the volumetric flow, Q , in the lower pipe: $Q = (1.5 \text{ kg/s})(1 \text{ m}^3/1000 \text{ kg}) = 0.0015 \text{ m}^3/\text{s}$

Now find the flow speed in the lower pipe: $v = Q/A = 0.0015/[\pi(0.01)^2] = 4.77465 \text{ m/s}$

Find the total pressure in the lower pipe: $P = P_{\text{gauge}} + P_{\text{atm}} = 3 \text{ atm} = 3.03 \times 10^5 \text{ Pa}$

Find $E_{\text{mech.density}}$ in the lower pipe: $E_{\text{mech.density}} = P + (1/2)\rho v^2 + \rho gh$
 $= 3.03 \times 10^5 + (1/2)(1000)(4.77465)^2 + (1000)(9.80)(0) = 3.144 \times 10^5 \text{ J/m}^3$

Convert 1 L (1000 cm^3) of water to m^3 : $(1000 \text{ cm}^3)(1 \text{ m} / 100 \text{ cm})^3 = 0.001 \text{ m}^3$

Now calculate the total mechanical energy in the 1 L of water:

$$E_{\text{mech}} = (E_{\text{mech.density}})V = (3.144 \times 10^5)(0.001) = \underline{314 \text{ J}}$$

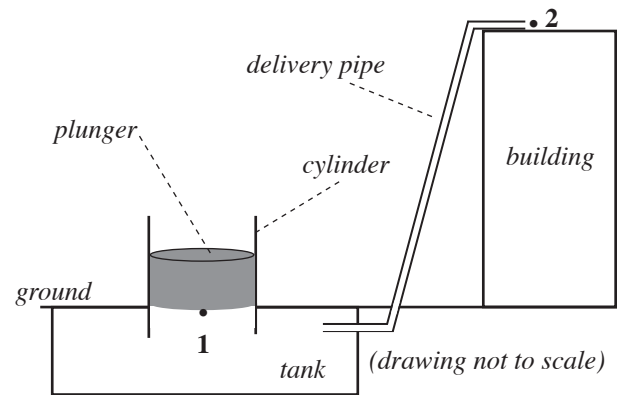
- d. Water flows horizontally through a large, uniform pipe of radius 0.68 m . The water fills the pipe completely, and the flow rate is $1.30 \text{ m}^3/\text{s}$. Assuming the same water speed throughout the flow, find the difference in energy contained in 1 L of water flowing at the center vs. 1 L of water flowing along the lowermost surface (the "floor") of the pipe.

The entire principle of Bernoulli's equation is that the total energy per unit volume everywhere in the flow is the same, provided that no work (e.g. pumping or friction) is done between the two points in question. Each side of Bernoulli's equation has units of energy density (also known as pressure)—and the equals sign in the equation states that the two points have equal energy densities. *So really, you don't need to do any calculations to answer this question.* A liter of water would have the same total mechanical energy at either point; the difference is 0 J.

- e. During the 2010 tornado in Aumsville, OR, the pressure outside was 99.0 kPa while the pressure inside the building was 101 kPa . What was the magnitude of the net force caused by this pressure difference on the building's door (which was 1.00 m wide and 2.50 m tall)?

$$F_{\text{Pressure.net}} = P_{\text{net}} \cdot A = (P_{\text{inside}} - P_{\text{outside}}) \cdot A = [(101 \times 10^3) - (99 \times 10^3)] \cdot [(2.50)(1.00)] = \underline{5.00 \times 10^3 \text{ N}}$$

6. f. Pure water is lifted from a (full) underground tank to the flat roof of a building by means of a cylinder-plunger system, as shown. The plunger has a diameter of 24.0 cm and a mass of 793 kg. Its top is open to the air. The bottom of the plunger is pressing on the water in the tank at ground level and is moving downward at 2.68 cm/s. At the roof level, the delivery pipe has a diameter of 5.10 cm, and it is emptying the water into an open-air basin. How tall is the building?



Incompressible fluid is moving—use Bernoulli's Equation, which relates pressure, speed and altitude for any two points in the flow:

$$P_1 + (1/2)\rho v_1^2 + \rho g h_1 = P_2 + (1/2)\rho v_2^2 + \rho g h_2$$

Choose two points where you have information (points **1** and **2** chosen shown here).

Identify what you know:

$$m = 793 \text{ kg}$$

$$r_1 = 0.12 \text{ m (radius of pipe at point 1)}$$

$$v_1 = .0268 \text{ m/s}$$

$$r_2 = .0255 \text{ (radius of pipe at point 2)}$$

$$P_1 = P_{atm} + F/A_1$$

$$F/A_1 = mg/(\pi r_1^2)$$

$$P_2 = P_{atm}$$

$$\text{Let } h_1 = 0$$

h_2 is the answer you seek.

But before using Bernoulli to solve for h_2 , you need v_2 . Use the Principle of Continuity for incompressible fluid flow: Mass flow rate and—even simpler—volume flow rate is constant throughout the flow: $A_1 v_1 = A_2 v_2$

Isolate v_2 :

$$v_2 = v_1(A_1/A_2)$$

Substitute expressions for area:

$$v_2 = v_1[(\pi r_1^2)/(\pi r_2^2)]$$

Simplify:

$$v_2 = v_1(r_1/r_2)^2$$

Now you can start solving the Bernoulli equation for h_2 (and recall that $h_1 = 0$):

Subtract $P_2 + (1/2)\rho v_2^2$:

$$P_1 - P_2 + (1/2)\rho(v_1^2 - v_2^2) = \rho g h_2$$

Divide by ρg :

$$[P_1 - P_2 + (1/2)\rho(v_1^2 - v_2^2)]/\rho g = h_2$$

Substitute expressions:

$$\{P_{atm} + mg/(\pi r_1^2) - P_{atm} + (1/2)\rho[v_1^2 - v_1^2(r_1/r_2)^4]\}/\rho g = h_2$$

Simplify:

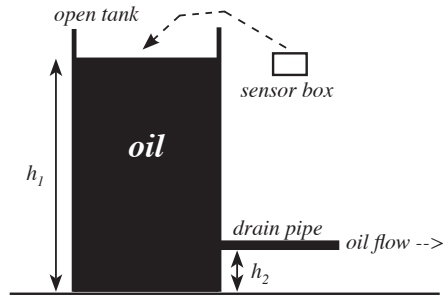
$$\{mg/(\pi r_1^2) + (1/2)\rho v_1^2[1 - (r_1/r_2)^4]\}/\rho g = h_2$$

Plug in the numbers:

$$h_2 = \{(793)(9.80)/[\pi(.12)^2] + (1/2)(1000)(.0268)^2[1 - (.12/.0255)^4]\}/[(1000)(9.80)] = \underline{\underline{17.5 \text{ m}}}$$

Note: g , h and i are ODAVEST items, but as HW problems, do only the V-E-S steps. (However, the solutions will show all seven steps.)

6. g.



A large tank holds incompressible oil to a depth of h_1 . The tank is being drained via a horizontal pipe (radius r), attached at a height h_2 above the tank bottom. Oil is flowing out through the pipe at a flow rate of Q . The descent speed of the oil level at the open-air tank top is negligible (≈ 0). The gauge pressure in the drain pipe is P_2 . A sealed box of sensor instruments, of density ρ_s , is to be placed into the oil. What percentage of that box's volume (if any) will remain above the oil surface?

Assume these are known values: $h_1, r, h_2, Q, P_2, \rho_s$

Objective:

A large, open tank contains an incompressible fluid of known depth.

The tank is draining via a small horizontal pipe connected to the side of the tank.

The drain pipe has a known radius and is located at a known height above the bottom of the tank.

The volumetric rate of fluid flowing out of the drain pipe is known.

The gauge pressure in the drain pipe is known.

The flow rate is such that the top level of fluid is moving very slowly—with negligible speed.

A sealed object of known density is placed into the fluid.

We want to find the percentage of that object (if any) that will remain above the fluid surface.

Data:

h_1	The depth of the fluid in the tank.
r	The radius of the drain pipe.
h_2	The height of the drain pipe above the bottom of the tank.
Q	The volumetric flow rate of the oil out of the drain pipe.
P_2	The gauge pressure in the drain pipe.
ρ_s	The density of the sealed object placed in the oil.
P_{atm}	The local atmospheric pressure.
g	The local magnitude of gravitational free-fall acceleration.

Assumptions:

Losses:

We will assume that the fluid has zero viscosity.

We will also assume that all pipe and tank surfaces are frictionless.

Drain pipe:

We will assume that the pipe has a uniform and circular cross-section.

We will also assume that the height h_2 measures the distance from the tank bottom to the center of the drain pipe.

We will also assume that the fluid pressure at the drainpipe outlet is exactly P_{atm} .

Pressure gauge:

We will assume that the pressure gauge reading P_2 has taken its reading at the radial center of the pipe at a point that is not near either end of the pipe.

Conditions:

We will assume that P_{atm} is uniform around the tank and environs.

We will also assume that g is uniform around the tank and environs.

Visual Rep(s): [FBD of floating object, with two forces: F_B upward and F_G downward]

[Sketch of tank and pipe, showing all dimensions and assumptions, and labeling point 1 at the fluid surface and point 2 in the pipe.

Equations:

- I. $Q = A_2 v_2 = (\pi r^2) v_2$
- II. $v_1 \approx 0$
- III. $P_{atm} + (1/2)\rho_{oil} v_1^2 + \rho_{oil} g h_1 = (P_2 + P_{atm}) + (1/2)\rho_{oil} v_2^2 + \rho_{oil} g h_2$
- IV. $\%_{exposed} = 100[1 - (\rho_s / \rho_{oil})]$

Solving:

Solve I for v_2 . Substitute that result into III.

Substitute II into III.

Solve III for ρ_{oil} . Substitute that into IV.

Solve IV for $\%_{exposed}$. (Cannot be negative; minimum value would be 0.)

Testing:

Dimensions: $\%_{exposed}$ should be a unit-less value between 0 and 100.

Dependencies:

Assuming all other data as given, if h_1 is greater, this would imply a lesser value of ρ_{oil} , which in turn would produce a lesser value of $\%_{exposed}$ (assuming that the object floats under the given data).

Assuming all other data as given, if r is greater, this would imply a lesser value of ρ_{oil} , which in turn would produce a lesser value of $\%_{exposed}$ (assuming that the object floats under the given data).

Assuming all other data as given, if h_2 is greater, this would imply a greater value of ρ_{oil} , which in turn would produce a greater value of $\%_{exposed}$ (assuming that the object floats under the given data).

Assuming all other data as given, if Q is greater, this would imply a greater value of ρ_{oil} , which in turn would produce a greater value of $\%_{exposed}$ (assuming that the object floats under the given data).

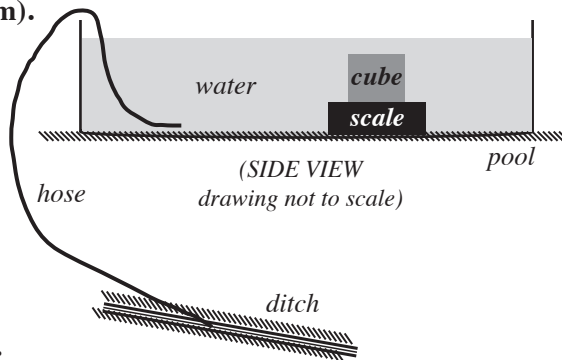
Assuming all other data as given, if P_2 is greater, this would imply a greater value of ρ_{oil} , which in turn would produce a greater value of $\%_{exposed}$ (assuming that the object floats under the given data).

Assuming all other data as given, if ρ_s is greater, the object would float lower in the oil (assuming that it floats with the given data), so $\%_{exposed}$ would be less.

Assuming all other data as given, if P_{atm} is greater, this would not affect the expected value of $\%_{exposed}$.

Assuming all other data as given, if g is greater, this would imply a lesser value of ρ_{oil} , which in turn would produce a lesser value of $\%_{exposed}$ (assuming that the object floats under the given data).

6. h. You're going to drain water from a wide cylindrical swimming pool (diameter d_{pool}) via a hose (diameter d_{hose}). First, you completely fill the hose with water, then close both its ends. You put one end in the ditch—a vertical distance h lower than the water level in the pool. You place the other end of the hose in the pool (near the bottom). A solid cube (side = s_{cube}) sits on a scale in the pool, completely immersed initially, with its top at a depth H below the surface. Initially, the scale reads $1/3$ of the cube's dry (rest) weight. You then open both ends of the hose and let the flow proceed *until the scale reading has doubled*. If you assume that the flow rate is the same for the entire process, how much time does this process take? (Ignore the volume displaced by the rubber of the hose in the pool, and assume the pool level lowers at a negligible speed.

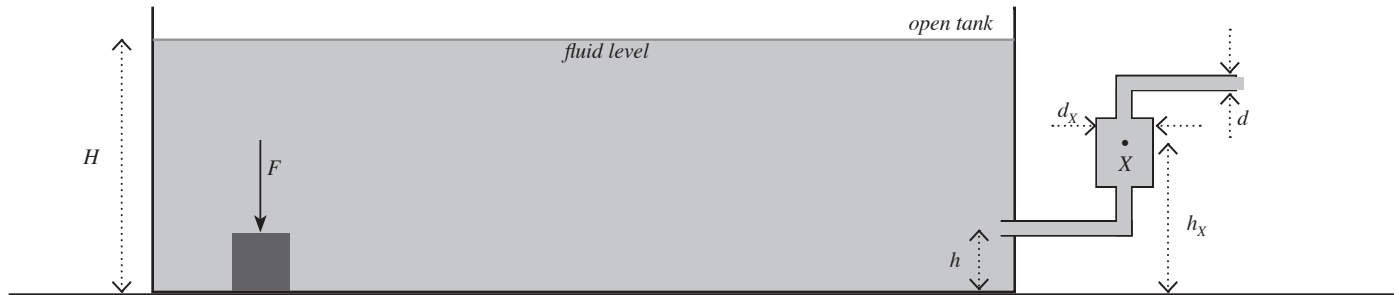


Consider these values as known: d_{pool} , d_{hose} , h , s_{cube} , H , ρ_{water} , g .

6. i. Refer to the diagram below. A large, open tank of fluid has a drain pipe leading from it, and fluid is draining through it at a volumetric flow rate Q . The pipe's outlet diameter, d , is known. Also known is the diameter, d_X , of the pipe at point X. The drain rate is such that the motion of the fluid level at the top of the tank is essentially zero. The surface of the fluid in the tank is at a height H (all heights measured above the tank bottom); the outlet pipe is connected to the tank at a height h , and point X is at height h_X .

Elsewhere in the tank (far away from the drain pipe; the fluid there is essentially motionless), a uniform block (a cube of side s and mass m) is pressed against the bottom of the tank by a force, F , applied vertically downward. As a result, the total pressure on the floor of the tank under the block is P . Find the gauge pressure of the fluid at point X.

Here is a summary of the known values: $Q, d, d_X, H, h, h_X, s, m, F, P, P_{atm}, g$



Objective: A large, open tank containing incompressible fluid to a depth H is draining via a small pipe. The volumetric rate of fluid flowing out of the pipe is known. The intake height (measured from the tank floor) and outlet diameter of the pipe are known. Also known are the height (again, measured from the tank floor) and the pipe diameter at a specific point X in the pipe. Far from the drain pipe intake, a cube of known size and mass rests on the tank floor. A steady force is exerted vertically downward on the cube. The total pressure on the tank floor under the cube is known. **We want to find the gauge pressure at point X in the drain pipe.**

Data:

Q	The volumetric flow rate of the fluid emerging from the drainpipe.
d, d_X	Drainpipe diameters (respectively): at the outlet and at point X.
H, h, h_X	Heights measured from the tank bottom (respectively): the fluid level in the tank, the drainpipe intake, and point X in the drainpipe.
s	The length of one side of the cube.
m	The mass of the cube.
F	The force being applied to the cube.
P	The total pressure on the tank floor under the cube.
P_{atm}, g	Local conditions (respectively): atmospheric pressure and gravitational acceleration.

Assumptions: *Losses:* We will assume that the fluid has zero viscosity.
 We will also assume that all pipe and tank surfaces are frictionless.

Pressures: We will assume that the pressure under the cube is uniform (that the force is evenly distributed across a perfectly level), so that P is a uniform value.
 We will also assume that the surface under the cube has not been evacuated—that the same static fluid pressure exists there just as elsewhere on nearby portions of the tank bottom.
 We will also assume that the fluid pressure at the drainpipe outlet is exactly P_{atm} .

Conditions: We will assume that P_{atm} is uniform around the tank and environs.
 We will also assume that g is uniform around the tank and environs.

Visual Rep(s):

Equations:

I. $A_{cube.side} = s^2$

II. $V_{fl.displ} = V_{cube} = s^3$

III. $P = F_N/A_{cube.side} + P_{atm} + \rho_{fluid}gH$

IV. $F_B = (\rho_{fluid})(V_{fl.displ})g$

[FBD of cube, with four forces:
 F_N and F_B directed upward;
 F and F_G directed downward]

V. $F_N + F_B - F - mg = 0$

VI. $Q = (\pi d_X^2/4)v_X$

[Sketch of tank and pipe, showing
 all dimensions, and labeling point 1
 at the fluid surface (and point X).]

VII. $P_{atm} + (1/2)\rho_{fluid}(0)^2 + \rho_{fluid}gH = P_X + (1/2)\rho_{fluid}v_X^2 + \rho_{fluid}gh_X$

VIII. $P_X = P_{X.gauge} + P_{atm}$

Solving:

Solve **I** for $A_{cube.side}$. Substitute that result into **III**.

Solve **II** for $V_{fl.displ}$. Substitute that result into **IV**.

Solve **III** for F_N (as an expression). Substitute that into **V**.

Solve **IV** for F_B (as an expression). Substitute that result into **V**.

Solve **V** for ρ_{fluid} . Substitute that result into **VII**.

Solve **VI** for v_X . Substitute that result into **VII**.

Solve **VII** for P_X . Substitute that result into **VIII**.

Solve **VIII** for $P_{X.gauge}$.

<u>Testing:</u>	<u>Dimensions:</u>	$P_{X,gauge}$ should have units of <u>pressure</u> (dimensions of mass/[length·time ²]).
	<u>Dependencies:</u>	<p>Assuming all other data as given, if Q is greater, v_X will be greater, so P_X (and thus $P_{X,gauge}$) will be <u>less</u>.</p> <p>Assuming all other data as given, d does not affect $P_{X,gauge}$.</p> <p>Assuming all other data as given, if d_X is greater, v_X will be less, so P_X (and thus $P_{X,gauge}$) will be <u>greater</u>.</p> <p>Assuming all other data as given, if H is greater, P_X (and thus $P_{X,gauge}$) will be <u>greater</u>.</p> <p>Assuming all other data as given, h does not affect $P_{X,gauge}$.</p> <p>Assuming all other data as given, if h_X is greater, P_X (and thus $P_{X,gauge}$) will be <u>less</u>.</p> <p>Assuming all other data as given, if s is greater, the pressure P would be less contributed by F (distributed over a larger area) and more reduced by F_B, implying that hydrostatic pressure would be contributing more, so ρ_{fluid} would need to be greater, which would produce a <u>greater</u> P_X (and thus $P_{X,gauge}$).</p> <p>Assuming all other data as given, if m is greater, the pressure P would be contributed more by F_G, implying that hydrostatic pressure would be contributing less, so ρ_{fluid} would need to be less, which would produce a <u>lesser</u> P_X (and thus $P_{X,gauge}$).</p> <p>Assuming all other data as given, if F is greater, the pressure P would be contributed more by F, implying that hydrostatic pressure would be contributing less, so ρ_{fluid} would need to be less, which would produce a <u>lesser</u> P_X (and thus $P_{X,gauge}$).</p> <p>Assuming all other data as given, if P is greater, this implies that hydrostatic pressure would be contributing more, so ρ_{fluid} would need be greater, which would produce a <u>greater</u> P_X (and thus $P_{X,gauge}$).</p> <p>Assuming all other data as given, if P_{atm} is greater, this implies that hydrostatic pressure would be contributing less, so ρ_{fluid} would need be less, which would produce a <u>lesser</u> P_X (and thus $P_{X,gauge}$).</p> <p>Assuming all other data as given, if g is greater, this would have multiple effects that are not clear without an explicit solution (or graph).</p>

