HW 2

Due: Tuesday, January 21, 2014, 5:00 p.m.

Print *your full* ***LAST*** *name:* ____

Print *your full* ***first*** *name:* ____

Print *your HW Grader's name:* ____________________________________

What is your HW Grader's box # (located outside of Wngr 234)? ________

"I affirm and attest that this HW assignment is my own work.
While I may have had help from (and/or worked with) others,
all the reasoning, solutions and results presented in final form
here are my own doing—and expressed in my own words."

Sign your name (full signature): ____________________________________

Print *today's date:* ____

General Instructions for HW 2

Requirements: The PH 212 HW graders will select **one** out of the six problems. They will then score that problem according to the points rubric posted with the HW 2 solutions (for a total possible 10 points for the entire HW assignment).

Format: The formats (type, length, scope) of the HW problems have been purposely created to closely parallel those of a typical exam (indeed, most HW problems have been taken from past exams).

You are required to include the completed, signed cover sheet—that's the first page of this file—as the front page of your HW submission when you turn it in. (Please staple together all pages, in order—cover sheet first—with one staple at upper left.)

Scoring: The rubrics (scoring guides—the points breakdowns) for HW problem solutions will closely match those for exam solutions. And in those rubrics, *when any item asks for an explanation (and most do), it means exactly what it says*. To get full credit, you must include a short but informative verbal explanation (in your own words) of your reasoning.

To get an idea of how best to approach various problem types (there are three basic types), refer to these example HW problems.

1. Evaluate the following statements (T/F/N). *As always, you must explain your reasoning with a valid mix of words, diagrams and/or calculations.*

a. The center of mass of an object is always located somewhere within the object (i.e. at a point occupied by part of the object).
False. Consider, for example, the center of mass of a uniform ring of material (which is hollow in the center).

b. If you double your distance from the rotation axis of the carousel you're riding on, you double your contribution to its total moment of inertia about that axis.
False. You *quadruple* your contribution, which is given by $I_{point.mass} = mr^2$

c. The units of moment of inertia could be expressed as N·m·s^2.
True. The SI units of I are kg·m^2, and N·m·s^2 = (kg·m/s^2)·m·s^2, which reduces to kg·m^2.

d. The moment of inertia of a solid sphere about its center is the same on the moon as on the earth.
True. Inertia depends only on an object's mass and how that mass is distributed. It has nothing to do with the gravitational force that may be acting on that mass.

e. Moment of inertia is a scalar quantity.
True. The moment of inertia is about mass and its distribution with respect to a given axis (not whether that axis points to the east or north, etc.). The sphere's resistance to angular acceleration about that axis is the same no matter which way that axis points.

f. The moment of inertia, *I*, of a rigid object depends on its angular acceleration as it rotates.
False. Inertia is a resistance to acceleration; it does not depend on the object's current velocity (at least not in Newtonian—classical physics). That would be like claiming that a car's mass depends on its speed.

g. The moment of inertia, *I*, of a rigid object varies according to the location of the rotation axis relative to the particles that form the object.
Not enough information. It's true most of the time, but there may be more than one location in an object that has the same moment of inertia. For example, each end of a simple rod can have the same moment of inertia about an axis through that end.

h. The moment of inertia, *I*, of a rigid object varies according to the orientation of the rotation axis relative to the particles that form the object.
Not enough information. It's true most of the time, but a uniform sphere has the same moment of inertia for any axis through its center.

2. a. **Moment of inertia is a measure of an object's <u>resistance to angular acceleration</u>.**

b. **Exercise 16, page 348: A 25-kg, solid door is 220 cm tall and 91 cm wide. Find the moment of inertia for its rotation along its hinges (i.e. how a door normally swings); then for a vertical axis within the door 15 cm from one edge.**

Using the I equations for the proper geometric form from page 318: $I_{edge} = (1/3)Ma^2$, where a is the width.

Thus: $I_{door.hinge} = (1/3)(25)(.91)^2 = \mathbf{6.90\ kg{\cdot}m^2}$

The other axis is located (91÷2) – 15 = 30.5 cm from the center of mass. And, using the parallel axis theorem, $I_{axis} = I_{cm} + Md^2$, where d is the distance from the center of mass to the desired axis, we get this:

$I_{axis} = (1/12)(25)(.91)^2 + (25)(0.305)^2 = \mathbf{4.05\ kg{\cdot}m^2}$

c. **A disk rotating about its center has a 1.00 kg mass attached 1.50 m from the axis. Where should you place a 2.50 kg mass so that you can remove the 1 kg mass without changing the total moment of inertia of the system?**

You want to replace the moment of inertia <u>contributed</u> by one point mass with that <u>contributed</u> by another.

In other words: $I_1 = I_2$ Or: $m_1r_1^{\ 2} = m_2r_2^{\ 2}$

Solve for r_2: $\sqrt{[(m_1r_1^{\ 2})/m_2]} = r_2 = \sqrt{[(1.00)(1.50)^2)/2.50]} = \underline{0.949\text{ m}}$

Place the 2.50 kg mass at a distance of 0.949 m from the axis.

d. **You're standing on a carousel 1.34 m from its axis of rotation. What minimum distance would you need to walk to arrive at a position where <u>your contribution</u> to the total moment of inertia has doubled?**

The moment of inertia of a point mass is calculated as $I = mr^2$.

Initial case: $I_i = mr_i^{\ 2}$ Final case: $I_f = mr_f^{\ 2}$

We want: $I_f = 2I_i$ That is: $mr_f^{\ 2} = 2mr_i^{\ 2}$

Solve for r_f: $r_f = \sqrt{2}(r_i) = \sqrt{2}(1.34) = 1.895$ m

Then the minimum distance, d_{min}, you need to walk to get to that new radial position is by walking directly <u>away</u> from the axis: $d_{min} = r_f - r_i = 1.895 - 1.34 = \underline{\mathbf{0.555\ m}}$

e. **Suppose you have a thin, uniform rod with a total mass of 3.00 kg and a total length of 2.00 m. By what percentage does the moment of inertia decrease if you rotate it about a point located 20% (0.40 m) in from one end—rather than about the very end of the rod?**

$$I_{end} = (1/3)ML^2 = (1/3)(3.00)(2.00)^2 = 4.00\text{ kg}{\cdot}\text{m}^2$$

For the other axis, use the Parallel Axis Theorem: $I_{axis} = I_{c.m.} + Md^2$, where d is the distance from the c.m. to the other (parallel) axis.

Thus: $I_{.40} = I_{c.m.} + Md^2 = (1/12)ML^2 + Md^2 = (1/12)(3.00)(2.00)^2 + (3.00)(0.60)^2 = 2.08\text{ kg}{\cdot}\text{m}^2$

And then $\Delta I\% = 100(I_f - I_i)/I_i = 100(2.08 - 4.00)/4.00 = \mathbf{-48.0\%}$

f. **A certain thin, uniform rod has a mass of 2 kg and length 3 m. To what shorter length must you trim it (discarding the smaller part) so that the remaining length has half the original moment of inertia about its end?**

We want this: $I_2 = (1/2)I_1$ where: $I_1 = (1/3)M_1L_1^{\ 2}$ *and* $I_2 = (1/3)M_2L_2^{\ 2}$

So: $(1/3)M_2L_2^{\ 2} = (1/2)(1/3)M_1L_1^{\ 2}$ Simplify: $M_2L_2^{\ 2} = (1/2)M_1L_1^{\ 2}$

But <u>the mass is proportional to the length</u> (the rod is uniform); you're trimming to the same fraction of mass as length: $M_2/M_1 = L_2/L_1$

Substitute for M_2: $(M_1L_2/L_1)L_2^{\ 2} = (1/2)M_1L_1^{\ 2}$ Simplify: $L_2^{\ 3} = (1/2)L_1^{\ 3}$

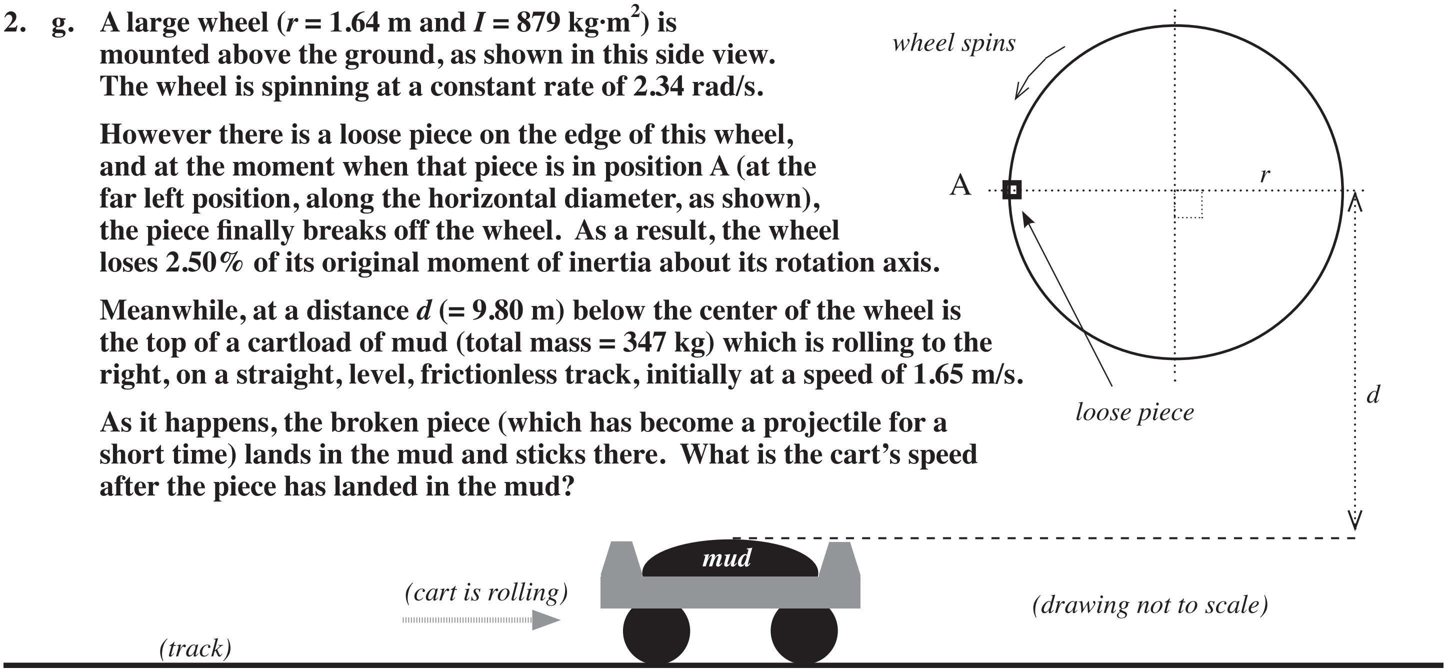
Solve for L_2: $L_2 = [(1/2)L_1^{\ 3}]^{1/3} = [(1/2)3^3]^{1/3} = [(1/2)3^3]^{1/3} = \underline{\mathbf{2.38\ m}}$

2. g. A large wheel (r = 1.64 m and I = 879 kg·m^2) is mounted above the ground, as shown in this side view. The wheel is spinning at a constant rate of 2.34 rad/s.

However there is a loose piece on the edge of this wheel, and at the moment when that piece is in position A (at the far left position, along the horizontal diameter, as shown), the piece finally breaks off the wheel. As a result, the wheel loses 2.50% of its original moment of inertia about its rotation axis.

Meanwhile, at a distance d (= 9.80 m) below the center of the wheel is the top of a cartload of mud (total mass = 347 kg) which is rolling to the right, on a straight, level, frictionless track, initially at a speed of 1.65 m/s.

As it happens, the broken piece (which has become a projectile for a short time) lands in the mud and sticks there. What is the cart's speed after the piece has landed in the mud?



I. Find the moment of inertia, I_P, of the loose piece:
<u>Use this equation</u>: $I_P/I_T = 0.025$
<u>Use these knowns</u>: $I_T = 879$ (given) <u>Solve for</u>: I_P **(21.975 kg·m^2)**

II. Find the mass, m_P, of the loose piece:
<u>Use this equation</u>: $I_P = m_P r_P^2$
<u>Use these knowns</u>: $I_P = 21.975$ (from **step I**) <u>Solve for</u>: m_P **(8.1704 kg)**
$r_P = 1.64$ (known)

III. Note the direction of the piece's velocity (and therefore its momentum) as it falls as a projectile:
Since the piece breaks off in position A (shown), its tangential velocity vector is straight down; it is "thrown" vertically downward from the wheel. *So it always has zero x-velocity:* $\boldsymbol{v_{P.x}} = \mathbf{0}$ <u>Therefore</u>: $\boldsymbol{P_{P.x}} = \mathbf{0}$

IV. Note the nature of the piece's collision with the mud cart:
The track/earth beneath the cart offers an upward impulse to the cart during the the collision, so the total initial y-momentum, $\boldsymbol{P_{T.i.y}}$, of the piece&cart system is NOT conserved; but the total x-momentum is: <u>Therefore</u>: $\boldsymbol{P_{T.i.x}} = \boldsymbol{P_{T.f.x}}$

V. Find the total initial x-momentum, $\boldsymbol{P_{T.i.x}}$, of the two-body system:
<u>Use this equation</u>: $\boldsymbol{P_{T.i.x}} = \boldsymbol{P_{P.i.x}} + \boldsymbol{P_{C.i.x}}$
<u>Use these knowns</u>: $\boldsymbol{P_{P.i.x}} = \boldsymbol{P_{P.x}} = 0$ (from **step III**) <u>Solve for</u>: $\boldsymbol{P_{T.i.x}}$ **(572.55 kg·m/s)**
$\boldsymbol{P_{C.i.x}} = m_C v_{C.i.x} = (347)(1.65)$ (given)

VI. Find v_f, the common final speed of the cart and piece together as one body:
<u>Use this equation</u>: $\boldsymbol{P_{T.i.x}} = \boldsymbol{P_{T.f.x}} = (m_C + m_P)v_f$
<u>Use these knowns</u>: $\boldsymbol{P_{T.i.x}} = 572.55$ (from **step V**) <u>Solve for</u>: v_f **v_f = <u>1.61 m/s</u>**
$m_C = 347$ (given)
$m_P = 8.1704$ (from **step II**)

3. Evaluate the following statements (T/F/N). *As always, you must explain your reasoning with a valid mix of words, diagrams and/or calculations.*

a. **$\Delta \boldsymbol{L}$ is a vector quantity.**
True. $\boldsymbol{L} = I\omega$, where ω is an object's angular velocity, which is a vector. So the difference, $\Delta \boldsymbol{L}$, between two vector values of $\boldsymbol{L}$ must also be a vector.

b. **If a spinning skater draws in her arms, she increases her angular momentum.**
False. She increases her angular velocity—by decreasing her moment of inertia. *An object cannot change its own angular momentum.* (That's just another way to state the principle of the Conservation of Angular Momentum.)

c. **An isolated rotating object (constant *m*) can change its own angular velocity.**
True. A figure skater does this, for example, when she extends her arms.

d. **If one object's angular momentum is changing, there must be another object whose angular momentum is changing.**
True. An object cannot change its own angular momentum—some other object must do so. But in the process, the second object's angular momentum is changed, too—***in the opposite direction***. (So this statement **d** is yet another way to state the principle of the Conservation of Angular Momentum.)

e. **An object of constant mass can change its own moment of inertia.**
True. A skater retains her mass value when extending her arms, but she definitely changes her moment of inertia.

f. **An object of constant moment of inertia about an axis can change its own angular velocity about that axis.**
False. That would imply that it could change its own angular momentum; it cannot.

g. **The angular momentum of a collection of objects is always conserved.**
False. That would be true only for an *isolated* collection of objects.

h. **An object's angular momentum may be changing even as its angular velocity remains constant.**
True. Consider a figure skater and her coach: If the skater were allowed to spin on her own—and extend her arms—her angular velocity would slow (due to conservation of angular momentum; she's an isolated system in that case). But suppose instead that, while she is extending her arms, her coach exerts a torque (say, pushing on her arms) so that she *maintains* her angular velocity? Thus, she'd have a greater moment of inertia but the same angular velocity—that's an increase in her angular momentum (caused, of course, by the torque from the external agent; when her coach intervened, she was no longer an isolated system).

4. a. Write the SI unit of angular momentum, using only fundamental ("base") SI units: **kg·m²/s** *1 pt.*

b. An ice skater is spinning in place on frictionless ice. Initially her angular velocity is 10.0 rad/s, but then she extends her arms, increasing her moment of inertia by 12.5%. What is her resulting angular velocity?

Use conservation of angular momentum: $\boldsymbol{L_i = L_f}$ Or: $I_i\omega_i = I_f\omega_f$ *1 pt.*

But: $I_f = 1.125(I_i)$ So: $I_i\omega_i = 1.125(I_i)\omega_f$ *1 pt.*

Solve for ω_f: $\omega_f = \omega_i/1.125 = 10_i/1.125 =$ **8.89 rad/s** *1 pt.*

c. A playground carousel ("merry-go-round") is free to rotate frictionlessly in the horizontal plane (and air resistance is negligible). Without riders, the carousel has a moment of inertia of 152 kg·m². But there is a single rider, initially standing 1.85 m from the axis of rotation, as the carousel turns at an angular speed of 0.640 rad/s. Then the person moves to another location, 0.75 m from the axis, and the angular speed is then 0.973 rad/s. Find the person's mass.

This is a case similar to a skater pulling her arms in as she rotates. No external influences interfere, so it's a simple application of the Conservation of Angular Momentum: $L_i = L_f$ or: $I_i\omega_i = I_f\omega_f$
The moment of inertia of the system changes, and therefore the angular velocity changes: $\omega_f = I_i\omega_i/I_f$ *1 pt.*

In this system, there are two objects, each with its own moment of inertia about the axis; the total moment of inertia is the sum of those two: $I = I_c + I_p$. The carousel's moment of inertia, I_c, is given. The person is standing with all of her mass essentially at one distance, r, from the axis of rotation. So her initial and final moments of inertia are simple to write: $I_{p.i} = m_p r_i^2$ and: $I_{p.i} = m_p r_i^2$ *1 pt.*

Thus, initially: $L_i = I_i\omega_i = (I_c + I_{p.i})\omega_i = (I_c + m_p r_i^2)\omega_i$ *1 pt.*

And finally: $L_f = I_f\omega_f = (I_c + I_{p.f})\omega_f = (I_c + m_p r_f^2)\omega_f$ *1 pt.*

$L_i = L_f$, so equate the two expressions: $(I_c + m_p r_i^2)\omega_i = (I_c + m_p r_f^2)\omega_f$ *1 pt.*

Now solve for m_p: $m_p = (I_c\omega_i - I_c\omega_f)/(r_f^2\omega_f - r_i^2\omega_i)$
$= [(152)(.640) - (152)(.973)]/[(.750)^2(.973) - (1.85)^2(.640)] =$ **30.8 kg** *1 pt.*

5. a. You're sitting in a chair that can rotate freely about a vertical axis. The moment of inertia of you and the chair (together as one object) about that axis is I_C. Initially, you and the chair are at rest. But you're holding a spinning disk whose axis of rotation aligns with the axis of the chair. The disk's moment of inertia about its axis is I_D. The disk's initial angular velocity is ω_D. You then use your free hand to stop the disk from spinning axis. As a result, you, the chair and the disk all rotate together with angular velocity ω_f. If $\omega_D/\omega_f = 10.0$, what is I_C/I_D?

Momentum is conserved in this angular (rotational) collision (and the pieces stay together afterward): $L_i = L_f$

But we know: $L_i = I_C\omega_C + I_D\omega_D = I_C(0) + I_D\omega_D = I_D\omega_D$ and $L_f = (I_C + I_D)(\omega_f)$

So: $I_D\omega_D = (I_C + I_D)(\omega_f)$ Rearrange: $\omega_D/\omega_f = (I_C + I_D)/I_D$

But $\omega_D/\omega_f = 10$, so: $(I_C + I_D)/I_D = 10$

Simplify: $I_C + I_D = 10I_D$ Or: $I_C = 9I_D$ Thus: $I_C/I_D =$ $\underline{9.00}$

b. You're sitting in a chair that can rotate freely about a vertical axis. The moment of inertia of you and the chair (together as one object) about that axis is I_C. Initially, you and the chair are at rest. But you're holding a disk whose axis of rotation aligns with the axis of the chair. The disk's moment of inertia about its axis is I_D. The disk is initially at rest. You then use your free hand to start the disk spinning on its axis at a velocity of ω_D. As a result, you and the chair also begin to rotate together, with an angular velocity ω_C. If $\omega_D/\omega_C = x$, what is I_D/I_C?

Momentum is conserved in this angular (rotational) explosion: $L_i = L_f$

The pieces (all as one object) were at rest beforehand: $L_i = 0$

And afterward: $L_f = I_C\omega_C + I_D\omega_D$

Therefore: $I_C\omega_C + I_D\omega_D = 0$ Or: $I_C\omega_C = -I_D\omega_D$

Rearrange: $I_C/I_D = -\omega_D/\omega_C$ But: $\omega_D/\omega_C = x$

Therefore: $I_C/I_D = -x$ Therefore: $I_D/I_C = \mathbf{-1/x}$

c. You're sitting in a chair that is free to rotate horizontally, but it's initially at rest. With one hand, you're holding the axle of a wheel so that the axle is aligned with the axis of your chair. Initially, the wheel is spinning with an angular velocity of 19.7 rad/s. Then you use your other hand to slow the wheel to 10.2 rad/s. As a result, you and the chair now have an angular velocity of 0.321 rad/s. (All velocities are measured with respect to the floor.)

(i) Assuming the wheel is a solid disk of mass 8.90 kg and an outer radius of 0.345 m, what is the moment of inertia of you and chair (together, as one object)?

(ii) If the same wheel as in part (i) were instead just spinning initially on a fixed axis set on the floor (without you and the chair), what mass would you then place on its outer rim to achieve the same slowing (19.7 rad/s to 10.2 rad/s)?

(i) This is a collision between two objects (chair+person and disk) that can rotate around a common axis.

Use conservation of angular momentum: $L_{i.TOTAL} = L_{f.TOTAL}$

The same two objects exist before and after: $I_C\omega_{i.C} + I_D\omega_{i.D} = I_C\omega_{f.C} + I_D\omega_{f.D}$

Rearrange to solve for I_C: $I_C\omega_{i.C} - I_C\omega_{f.C} = I_D\omega_{f.D} - I_D\omega_{i.D}$

$$I_C = I_D(\omega_{f.D} - \omega_{i.D})/(\omega_{i.C} - \omega_{f.C})$$

Calculate I_D: $I_D = (1/2)(8.90)(0.345)^2 = 0.52966$

<u>Therefore</u>: $I_C = (0.52966)(10.2 - 19.7)/(0 - 0.321) =$ $\underline{\mathbf{15.7\ kg{\cdot}m^2}}$

(ii) This is a collision between two objects (disk and point mass) that can rotate around a common axis.

Use conservation of angular momentum: $L_{i.TOTAL} = L_{f.TOTAL}$

In this case, two objects become one: $I_D\omega_{i.D} + I_P\omega_{i.P} = (I_D + I_P)\omega_f$

Simplify and expand ($\omega_{i.P} = 0$ and $I_P = m_P r_P^2$): $I_D\omega_{i.D} = (I_D + m_P r_P^2)\omega_f$

Rearrange to solve for m_P: $m_P = [(I_D\omega_{i.D}/\omega_f) - I_D]/r_P^2$

$$= [(0.52966)(19.7/10.2) - 0.52966]/0.345^2 = \underline{\mathbf{4.14\ kg}}$$

6. a. A flat, uniform, circular disk (R = 2.25 m, m = 84.0 kg) is initially at rest, but it is free to rotate in the horizontal plane about a vertical, frictionless axis through its center. A 45.0-kg person, standing 1.5 m from the axis, begins to run on the disk in a clockwise circular path, with a tangential speed of 2.00 m/s (measured with respect to the stationary ground—not the disk). Find the resulting angular velocity of the disk.

This is an explosion between two objects. Prior to the explosion, they were one object; afterwards they are two rotating objects. So, use the principle of Conservation of Angular Momentum: $L_i = L_f$

The moment of inertia of the initial system is the sum of its two parts: $I_i = I_d + I_p$ So: $L_i = (I_d + I_p)\omega_i$

After the collision, each object has its own moment of inertia and angular velocity: $L_f = I_d\omega_{f.d} + I_p\omega_{f.p}$

Combining these two facts into the fundamental equation above, we have: $(I_d + I_p)\omega_i = I_d\omega_{f.d} + I_p\omega_{f.p}$

From page 318 of the text, we know that the moment of inertia, I_d, of a solid, uniform disk is given by $I_d = (1/2)MR^2$, where M is the mass of the entire disk, and R is its outer radius. We also know that a single mass, m_p, located at a distance r_p from the axis of rotation has a moment of inertia about that axis given by $I_p = m_p r_p^2$.

Substituting these facts into the equation above, we get: $[(1/2)MR^2 + m_p r_p^2]\omega_i = (1/2)MR^2\omega_{f.d} + m_p r_p^2\omega_{f.p}$

But $\omega_i = 0$ and $\omega_{f.p} = v_{f.T.p}/r_p$: $0 = (1/2)MR^2\omega_{f.d} + m_p r_p^2 v_{f.T.p}/r_p$

Simplify and subtract $MR^2\omega_{f.d}$: $-MR^2\omega_{f.d} = 2m_p r_p v_{f.T.p}$

Solve for $\omega_{f.d}$: $\omega_{f.d} = -(2m_p r v_{Tp})/(MR^2) = -[2(45.0)(1.5)(2.00)]/[(84.0)(2.25)^2] = -$**0.635 rad/s**

The disk rotates at 0.635 rad/s in the direction opposite the person's motion.

b. Two uniform, solid disks are rotating freely about the same axis. Then they are linked together without the aid of any external influences. Before being connected, disk 1 had an angular velocity of 4.60 rad/s; disk 2 had an angular velocity of –8.50 rad/s. Both disks have the same mass (2.00 kg), but disk 1 has a radius of 12.0 cm, and disk 2 has a radius of 8.00 cm. At what angular velocity does the set of disks rotate after being connected?

This is a simple collision between two rotating objects—with no external interference—a simple application of the Conservation of Angular Momentum for the system: $L_i = L_f$ Or: $I_i\omega_i = I_f\omega_f$

Before the collision, each object has its own moment of inertia and angular velocity: $L_i = I_1\omega_1 + I_2\omega_2$

After the collision, there is just a single (combined) object, with a combined moment of inertia and a single angular velocity: $L_f = (I_1 + I_2)\omega_f$

Combining these two facts into the conservation equation above, we have: $I_1\omega_1 + I_2\omega_2 = (I_1 + I_2)\omega_f$

From page 318 of the text, we know that the moment of inertia, I, of a solid, uniform disk is given by $I = (1/2)MR^2$, where M is the mass of the entire disk, and R is its radius.

Substituting that fact into the equation above, we get this:

$$(1/2)M_1R_1^2\omega_1 + (1/2)M_2R_2^2\omega_2 = [(1/2)M_1R_1^2 + (1/2)M_2R_2^2]\omega_f$$

But $M_1 = M_2 = M$: $(1/2)MR_1^2\omega_1 + (1/2)MR_2^2\omega_2 = [(1/2)MR_1^2 + (1/2)MR_2^2]\omega_f$

Divide by $(1/2)M$: $R_1^2\omega_1 + R_2^2\omega_2 = (R_1^2 + R_2^2)\omega_f$

Solve for ω_f: $\omega_f = (R_1^2\omega_1 + R_2^2\omega_2)/(R_1^2 + R_2^2)$

$= [(.12)^2(4.60) + (.08)^2(-8.50)]/[(.12)^2 + (.08)^2]$ = **0.569 rad/s**

After being connected, the disks rotate together at (+)0.569 rad/s.

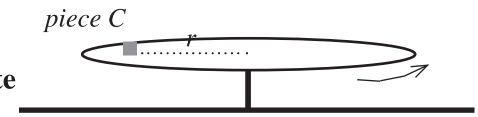
6. c. **An old firework shell from last year's July 4th show was launched but failed to explode. Instead, it fell and came to rest, hanging by a thread from a tree branch above a school playground. Its mass is 7.98 kg. Now (many months later), it finally explodes—into three pieces.**

Immediately after the explosion: **Piece A (m_A = 1.30 kg) is moving horizontally eastward at 32.4 m/s.**
Piece B (m_B = 0.65 kg) is moving horizontally westward at 64.8 m/s.

All three pieces land eventually somewhere on the playground. As it happens, Piece C lands on (and sticks to) the surface of the merry-go-round (I = 120 kg·m^2), at a distance r = 1.30 m from its center hub, as shown here.

The surface of the merry-go-round is 7.19 m lower in elevation than the position of the shell when it was hanging from the tree.

Just before piece C lands on it, the merry-go-round is rotating at a rate of 2.50 rad/s. At what rate is it rotating just after piece C lands on it?



Since the shell was hanging in the air, the pieces resulting from the explosion will all become projectiles (and we know the initial velocities of pieces A and B at the moment of their "launch"—right after the explosion).
So use conservation of momentum to find the initial velocity of piece C at the moment after the explosion:

The x-direction: $\boldsymbol{P_{T.i.x} = 0 = P_{T.f.x}}$ $\quad\quad$ $\boldsymbol{P_{T.f.x}} = \boldsymbol{P_{A.x}} + \boldsymbol{P_{B.x}} + \boldsymbol{P_{C.x}} = 0$

Solve for $P_{C.x}$: $\boldsymbol{P_{C.x}} = -(\boldsymbol{P_{A.x}} + \boldsymbol{P_{B.x}}) = -[(m_A)(\boldsymbol{v_{A.x}}) + (m_B)(\boldsymbol{v_{B.x}})] = -[(1.30)(32.4) + (0.65)(-64.8)] = \mathbf{0}$

The y-direction: $\boldsymbol{P_{T.i.y} = 0 = P_{T.f.y}}$ $\quad\quad$ $\boldsymbol{P_{T.f.y}} = \boldsymbol{P_{A.y}} + \boldsymbol{P_{B.y}} + \boldsymbol{P_{C.y}} = 0$

Solve for $P_{C.y}$: $\boldsymbol{P_{C.y}} = -(\boldsymbol{P_{A.y}} + \boldsymbol{P_{B.y}}) = -[(m_A)(\boldsymbol{v_{A.y}}) + (m_B)(\boldsymbol{v_{B.y}})] = -[(1.30)(0) + (0.65)(0)] = \mathbf{0}$

So the starting velocity of piece C is zero—in both the x- and y- directions. *Essentially it is being dropped from rest from the thread where it was hanging. It must have landed vertically on the merry-go-round.* That being the case, <u>mass C does not contribute any angular momentum</u> around the axis of the merry-go-round—because it was moving parallel to that axis (i.e. no part of its motion was tangent to any circle around that axis). The impact velocity of C is vertical and that value is irrelevant to the motion of the merry-go-round. The only effect piece C has is as a <u>point mass</u>—increasing the moment of inertia of the system (and thus slowing the rotation rate).

So use conservation of angular momentum before and after piece C lands: $\quad L_i = L_f \quad$ That is:

$$I_i\omega_i = I_f\omega_f$$

$$\omega_f = I_i\omega_i/I_f \quad = I_m\omega_i/(I_c + I_m) \quad = I_m\omega_i/(m_C r_C^2 + I_m)$$

$$= [(120)(2.50)]/[(7.98-1.30-0.65)(1.30^2) + 120] = \mathbf{2.30\ rad/s}$$