HW 5-6

Due: Monday, February 17, 2014, 5:00 p.m.

Print *your full* **LAST** *name:* ____

Print *your full* ***first*** *name:* ____

Print *your HW Grader's name:* ____

What is your HW Grader's box # (located outside of Wngr 234)? _________

"I affirm and attest that this HW assignment is my own work.
While I may have had help from (and/or worked with) others,
all the reasoning, solutions and results presented in final form
here are my own doing—and expressed in my own words."

Sign your name (full signature): ____

Print *today's date:* ____

General Instructions for HW 5-6

Requirements: The PH 212 HW graders will select **two** out of the twelve problems. They will then score each problem according to the points rubric posted with the HW 5-6 solutions (for a total possible 20 points for the entire HW assignment).

Format: ***You ARE required to include the completed, signed cover sheet—that's the first page of this file—as the front page of your HW submission when you turn it in.*** (Please staple together all pages, in order—cover sheet first—with one staple at upper left.)

Scoring: The rubrics (scoring guides—the points breakdowns) for HW problem solutions will closely match those for exam solutions. And in those rubrics, *when any item asks for an explanation (and most do), it means exactly what it says*. To get full credit, you must include a short but informative verbal explanation (in your own words) of your reasoning.

To get an idea of how best to approach various problem types (there are three basic types), refer to these example HW problems.

1. a. **Evaluate the following statements (T/F/N). As always, explain your reasoning.**

(i) **Light incident from air onto glass at 0° does not enter the glass.**
False. An incidence angle, remember is measured from the *normal* to the glass surface; light striking the glass surface perpendicularly certainly enters the glass.

(ii) **If two panes of glass (same material) have different thicknesses, then blue light could pass through the thicker pane in less time than red light passes through the thinner pane.**
Not enough information. Blue light will be transmitted more slowly, yes, so if you were to specify that each case has the light incident normally ($\theta_i = 0°$), then the claim would be false (i.e. the blue light would always "lose the race," since it's been handicapped both by speed and distance). But the incidence angles were not specified—and if you aim the (faster) red light at an angle sufficiently oblique so that its path is sufficiently longer than that of the (slower) blue light, red could end up "losing the race."

(iii) **White light can refract through glass without separating into colors.**
True. Send it through a pane of glass (both sides parallel) at an incidence angle of 0°.

(iv) **If you're viewing from underwater, the ceiling of an indoor swimming pool appears higher (farther from you) than it really is.**
True. With the rays entering the water and being bent toward the normal, they appear to be coming from points much higher above. Look at Fig. 23.14a (and take the point source case), in the text book and extrapolate upward—in straight lines—the rays reaching your eye in the water.

(v) **In the diagram, at least 2 rays are incorrectly drawn. True.** First, ray C simply could not happen (at least, not for positive n values; negative indices of refraction involve physics/math beyond this course). Then compare rays A and B: If $n_1 < n_2$, all rays should refract toward the normal (so ray A is correct; B is not). But if $n_1 > n_2$, all rays should refract away from the normal (so ray A is incorrect and ray B is correct).

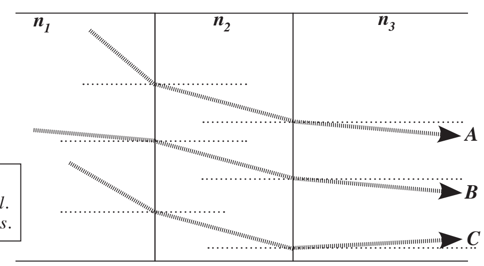
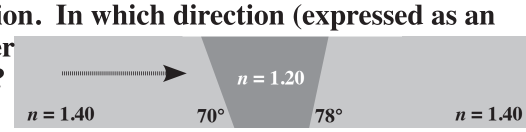


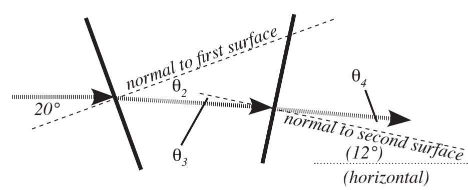
Diagram: All rays are the same (single) λ. Surfaces between the three regions are parallel. .... = normal to those surfaces.

(vi) **In the diagram, if ray B is correctly drawn, then $n_2 < n_1$.**
True. See argument in item (v), above.

b. **The light ray shown is traveling initially in the positive *x*-direction. In which direction (expressed as an angle measured from the positive *x*-axis) will it be traveling after refracting through (exiting on the right of) the darker material?**

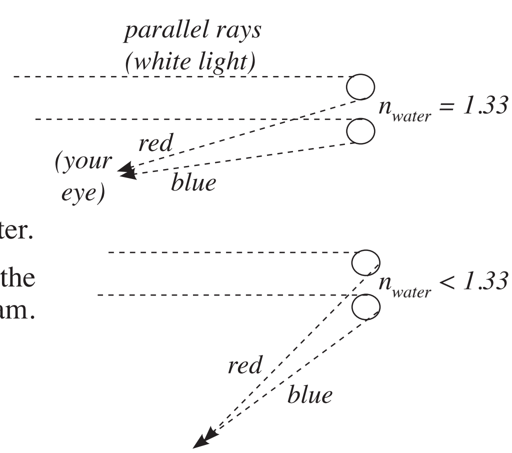


The normal to the first surface is +20°;
the normal to the second surface is –12°.
Snell's Law: $1.40\sin 20° = 1.20\sin\theta_2$. $\theta_2 = 23.517°$



So the ray refracts into slower medium at –3.517°
(i.e. 3.517° below horizontal).
So $\theta_3 = {}^-3.517 - ({}^-12) = 8.483°$.
Snell's Law: $1.20\sin 8.483° = 1.40\sin\theta_4$. $\theta_4 = 7.264°$
That's 7.264° above –12°, or **–4.74° with respect to the horizontal** (positive *x*-axis).

c. **If water were suddenly to have a lower index of refraction than it does now, how would this tend to change the appearance of a typical rainbow? Be specific in your explanation; use a diagram.**



A rainbow results from internal reflection and dispersion. If n_{water} were less: Internal reflection would work for a smaller fraction of any given raindrop; to get much reflection, the light would have to enter the drop even more obliquely (nearer the edge) than now. Less reflection = fainter.

Dispersion (a higher index of refraction for a higher frequency) causes the blue light to be reflected more severely back than red light—first diagram. A lower overall n_{water} would make all such reflection angles less severe—compare the second diagram to the first. So you'd see the rainbow **higher in the sky (and a wider arc—bigger circle).**

2. a. Evaluate the following statements (T/F/N). As always, explain your reasoning.

(i) A critical angle is an angle of incidence.
<u>True</u>. θ_C is the angle of incidence that produces an angle of refraction of 90°.

(ii) For light emerging into air, θ_C for water is less than θ_C for glass.
<u>False</u>. $\theta_C = \sin^{-1}(n_2/n_1)$, where $n_2 = n_{air} \approx 1$. Since $n_{water} \approx 1.33$, and $n_{glass} \approx 1.5$, we have

$$\theta_{C.water.air} = \sin^{-1}(1/1.33) = 48.8° \quad \text{and} \quad \theta_{C.glass.air} = \sin^{-1}(1/1.5) = 41.8°$$

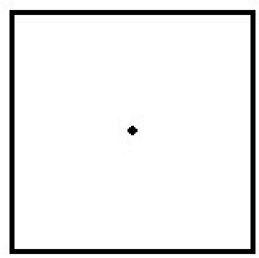
(ii) A diamond is less "sparkly" when immersed in water.
<u>True</u>. The "sparkle" of a gemstone comes from its ability to temporarily "trap" incident light (send it around a path of total internal reflection) until it encounters a surface at *a small enough angle of incidence to escape* (that surface being "aimed"—by the prudent cutting of facets in the stone—upward toward the eye). So the smaller the critical angle, θ_C, the more effective the gem is in re-directing light toward the eye; that is, less light "leaks" out via other faces in the stone.
Well, dipping a diamond in water *increases the critical angle*:

$$\theta_{C.diamond.air} = \sin^{-1}(1/2.4) = 24.6° \qquad \theta_{C.diamond.water} = \sin^{-1}(1.33/2.4) = 33.7°$$

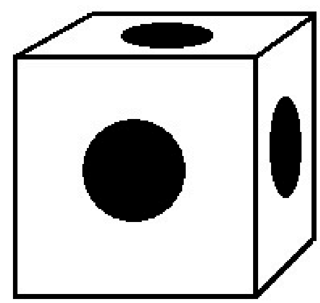
b. The glass core of an optical fiber has an index of refraction 1.60. The index of refraction of the cladding is 1.48. What is the maximum angle a light ray can make with the wall of the core if it is to remain inside the fiber?

We want total internal reflection at the core/cladding interface: $\theta_{C.core.cladding} = \sin^{-1}(1.48/1.60) = 67.7°$
That's the minimum *incidence* angle—measured from the *normal* to the core wall. So the maximum angle the ray can make, measured from the core wall itself is $90° - \theta_{C.core.cladding}$ = **<u>22.3</u>°**

c. A solid glass cube, of edge length 10.0 mm and index of refraction 1.75, has a small, dark spot at its exact center. Find the minimum radius of black paper circles that could be pasted at the center of each cube face to prevent the center spot from being seen, no matter what the direction of viewing.

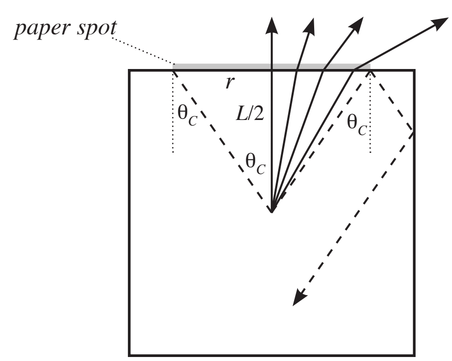


(before pasting paper circles)



(after pasting paper circles)

The spot needs to be only as big as is necessary to cover the rays that would emerge from the cube —*some rays will not emerge anyway, because of total internal reflection. (See side view here.)*



$\theta_{C.glass.air} = \sin^{-1}(1/1.75) = 34.85°$

The critical angle is part of the right triangle shown —with the radius of the minimum-size paper spot as one leg. Thus: $r/(L/2) = \tan 34.85°$

So $r = (L/2)\tan 34.85° = (.0100/2)\tan 34.85°$ = **<u>3.48 mm</u>**

3. a. Evaluate the following statements (T/F/N). As always, explain your reasoning.

(i) A diverging lens always produces a positive magnification.
<u>True</u>. For any negative focal length (that's a diverging lens) and any positive object distance, the thin lens equation will always produce a negative image distance—and thus a positive m value (since $m = -d_i/d_o$); the image will not be inverted with respect to the object.

(ii) For thin lenses, any real image is inverted.
<u>True</u>. Any real image has a positive image distance—producing a negative m value (since $m = -d_i/d_o$).

(iii) When a lens has a positive focal length and an object is placed at a distance equal to twice that focal length, its image will be virtual.
<u>False</u>. Using the thin lens equation, if $d_o = 2f$, then d_i will be <u>positive</u> (also $2f$), signaling a real image.

(iv) For a converging lens, the focal length for blue light is slightly shorter than the focal length for red light.
<u>True</u>. The index of refraction in any medium will be slightly higher for a higher frequency, which results in more pronounced refractive angles—thus, shorter focal lengths—for those higher frequencies. Blue light has a higher frequency than red light.

(v) For a thin lens, any virtual image is reduced (smaller than the object).
<u>False</u>. For a converging lens, if $d_o < f$, then the image will be virtual but <u>enlarged</u>.

(vi) The magnification produced by a diverging lens is always less than 1.
<u>True</u>. The image will always be virtual but non-inverted and <u>reduced</u> (thus $0 < m < 1$).

(vii) A diverging lens produces only virtual images.
<u>True</u>. The image will always be virtual but non-inverted and <u>reduced</u> (thus $0 < m < 1$).

(viii) A converging lens can produce an upright, enlarged image.
<u>True</u>. For a converging lens, if $d_o < f$, then the image will be upright and <u>enlarged</u>.

(ix) A lens that causes parallel light rays to all pass through one common focal point is thicker in its middle than at its edges.
<u>Not enough information</u>. This description (of a converging lens) is true so long at the medium surrounding the lens has an index of refraction less than the material forming the lens, but if the surrounding medium has a higher index, then "fatter-in-the-middle" would producing diverging, instead.

b. Which image distances are possible for a converging lens that has a focal length f?
As always, explain your reasoning fully.

Note: We are assuming here is that the question refers to a single converging lens creating an image from a single object—so only positive object distances are possible. If we allow for this lens to create its image by using another lens' image as its object, then that object distance could indeed be negative (as in part Vc of Lab 6), so the lens could produce any of these three image distances.

First, solve the thin lens equation ($1/f = 1/d_o + 1/d_i$) for d_o: $\quad d_o = (f)(d_i)/(d_i - f)$

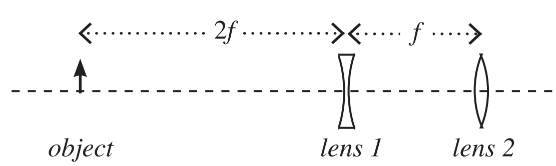
Now use this to test each d_i value below. If the result is a <u>positive</u> d_o, then the d_i is possible.

(i) $-2f$ $\quad d_o = (f)(-2f)/(-2f - f) = (2/3)f$ $\quad$ For a positive f, this is a positive d_o: **<u>Possible</u>.**

(ii) $(1/3)f$ $\quad d_o = (f)(f/3)/(f/3 - f) = -(1/2)f$ $\quad$ For a positive f, this is a negative d_o: **<u>Not possible</u>.**

(iii) $(5/4)f$ $\quad d_o = (f)(5f/4)/(5f/4 - f) = (5)f$ $\quad$ For a positive f, this is a positive d_o: **<u>Possible</u>.**

c. *Evaluate the following statement (T/F/N).*
As always, explain your reasoning.
If $|f_1| = |f_2| = f$ in the situation shown, then the overall lateral (or absolute) magnification, m, is –1.00.



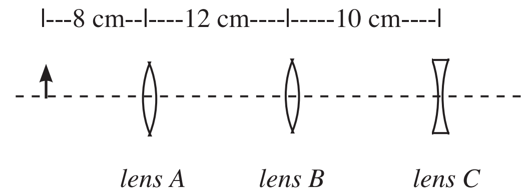
<u>**False**</u>.

<u>Lens 1</u>: $1/(-f) = 1/(2f) + 1/d_{i.1}$ $\quad$ Or: $-3/(2f) = 1/d_{i.1}$ $\quad$ Thus: $d_{i.1} = -2f/3$ $\quad$ And: $m_1 = -(-2f/3)/(2f) = 1/3$

<u>Lens 2</u>: $1/f = 1/(5f/3) + 1/d_{i.2}$ $\quad$ Or: $2/(5f) = 1/d_{i.2}$ $\quad$ Thus: $d_{i.2} = 5f/2$ $\quad$ And: $m_2 = -(5f/2)/(5f/3) = -3/2$

<u>So</u>: $\quad m_{total} = m_1 m_2 = (1/3)(-3/2) = -1/2$

4. In the drawing, the lenses have focal lengths of f_A = 4 cm, f_B = 12 cm, and f_C = –8 cm. The object is located at x = 0.00.



a. Find the total magnification (*m*) of this lens system. That is, find the ratio $h_{i,final}/h_o$.

b. What is the *x*-position of the final image?

Lens A: $1/4 = 1/8 + 1/d_i$ $d_i = 8$ $m_A = -(8/8) = -1$
The image of lens A (i.e. the object of lens B) is located 8 cm to the right of lens A (4 cm to the left of lens B). *1 pt.*

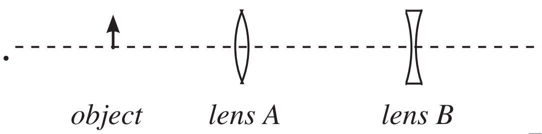
Lens B: $1/12 = 1/4 + 1/d_i$ $d_i = -6$ $m_B = -(-6/4) = 1.5$
The image of lens B (i.e. the object of lens C) is located 6 cm to the left of lens B (16 cm to the left of lens C). *1 pt.*

Lens C: $-1/8 = 1/16 + 1/d_i$ $d_i = -5.33$ $m_C = -(-5.33/16) = 0.333$ *1 pt.*

a. $m_T = m_A \cdot m_B \cdot m_C = (-1)(1.5)(0.333) =$ **–0.500**

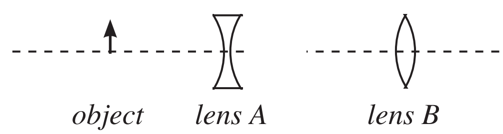
b. The final image position is 5.33 cm to the left of lens C. That's $x = (30 - 5.333) =$ **24.7 cm** *1 pt.*

c. The object shown here is located inside the focal length of lens *A*. Will be the resulting final image be virtual or real? Upright or inverted?



Since the do < f for the converging lens (A), the image of lens A (which becomes the object of lens B) is virtual—located somewhere to the left of the object—and it is upright. That object position (as with every object position for a diverging lens) will then produce a **virtual**, **upright** image (smaller and closer to lens B). *1 pt.*

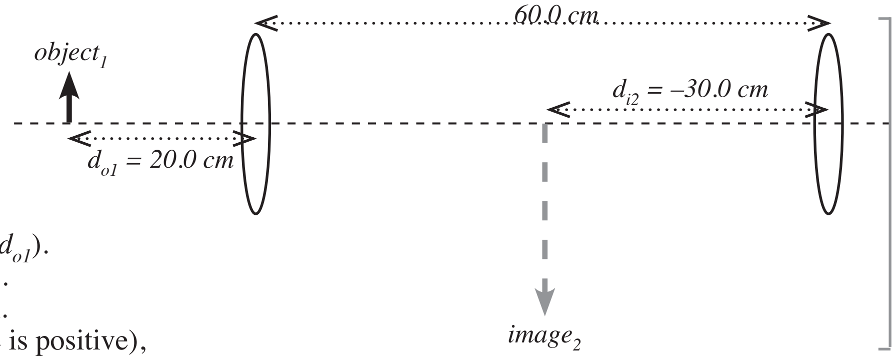
d. The two lenses shown here are separated by a distance equal to the focal length of lens *B*. The object distance is less than the lens separation distance. Will be the resulting final image be virtual or real? Upright or inverted?



The image of lens A (i.e. the object of lens B) is located somewhere between the object and lens A. This is outside the focal point of lens B, so lens B will produce a **real**, **inverted** image (somewhere to the right of lens B). *1 pt.*

4. e. Two lenses are placed 60.0 cm apart. When an object 4.00 cm tall is placed 20.0 cm in front of the first lens, the final image (from the second lens) is located halfway between the two lenses. If the magnification, m_1, of the first lens is –2.50, find the final image's height and also its orientation (upright or inverted) with respect to the original object.

The facts given are shown here (with distances drawn to scale but not necessarily the heights):

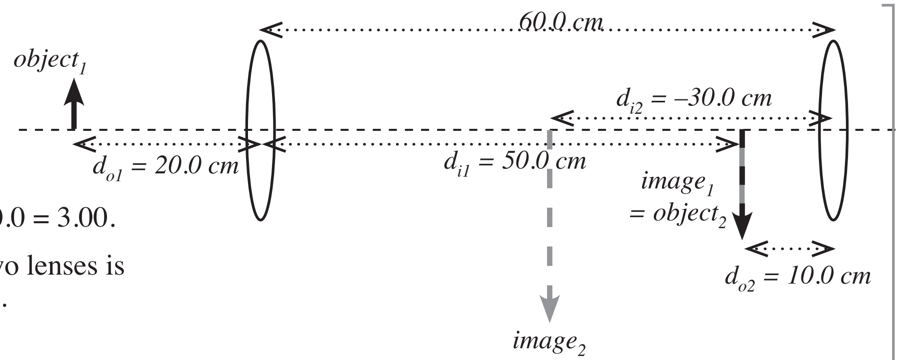


1 pt.

Also known: $m_1 = –2.50$, and $h_{o1} = 4.00$ cm.

To solve this, first note that $m_1 = –(d_{i1}/d_{o1})$.
Rearranging this, we get $d_{i1} = –m_1(d_{o1})$.
That is, $d_{i1} = –(−2.50)(20.0) = 50.0$ cm.
This is a real image (since the distance is positive), which means this is to the <u>right</u> of lens 1.

So here is the diagram with these new facts also shown:



1 pt.

Note that $m_2 = –(d_{i2}/d_{o2}) = –(−30.0)/10.0 = 3.00$.

Then the total magnification for the two lenses is $m_T = (m_1)(m_2) = (–2.50)(3.00) = –7.50$.

But by definition, $m_T = h_{i2}/h_{o1}$.

Therefore, $h_{i2} = m_T(h_{o1}) = (–7.50)(4.00) =$ **<u>–30.0 cm</u>**.

The final image height is 30.0 cm, oriented downward (inverted from the original orientation).

f. An object is 18 cm in front of a diverging lens that has a focal length of magnitude 12 cm. Calculate how far in front of the lens the object should be placed so that the size of its image is reduced by a factor of 2.0.

We want $m_B = m_A/2$ (same object height but half the image height).

First, <u>notice</u>: $1/f = 1/d_o + 1/d_i$ <u>So</u>: $d_o/f = 1 + d_o/d_i = 1 – 1/m$ <u>Thus</u>: $m = f / (f – d_o)$

Find m_A: $m_A = f / (f – d_{o,A}) = –12/(–12 – 18) = 0.4$

1 pt.

So we want: $m_B = m_A/2 = 0.2$

<u>Thus</u>: $0.2 = f / (f – d_o)$ <u>Or</u>: $0.2 = f / (f – d_o)$

<u>Thus</u>: $d_o = f – (f/0.2) = –12 –(–12/.2) =$ **<u>48.0 cm</u>**

1 pt.

5. **Evaluate the following statements (T/F/N). As always, explain your reasoning.**

   a. **A near-sighted person's near point is farther than is considered "normal."**
   <u>False</u>. A near-sighted person's near point is "normal." His/her far point is nearer than is considered "normal."

   b. **A near-sighted person's far point is nearer than is considered "normal."**
   <u>True</u>. *This is what needs correcting*. The person would like to be able to view objects at the normal far viewing limit ($d_{o.normal} = \infty$).

   c. **A far-sighted person's far point is farther than is considered "normal."**
   <u>False</u>. A far-sighted person's far point is "normal." His/her near point is farther than is considered "normal."

   d. **A diverging lens can correct nearsightedness.**
   <u>True</u>. The person would like to be able to view objects at the normal far viewing limit ($d_{o.normal} = \infty$), and a diverging lens does exactly what's needed: Produce a nearer, upright image from a farther upright object.

   e. **If you have both a contact lens and eyeglasses to correct your nearsightedness (wearing just one at a time), the contact lens (worn on the eye) must have a longer focal length than the eyeglass (worn 2 cm from the eye).**
   <u>True</u>. Compare the lens equation for each case ($d_{o.normal} = \infty$).

   <u>Contact lens</u>: $1/f = 1/\infty + 1/-(FP)$ Thus: $f = -FP$

   <u>Eyeglasses</u>: $1/f = 1/\infty + 1/-(FP - 2)$ Thus: $f = -(FP - 2)$

6. a. **You wear contact lenses to correct your vision. Your eyes are identical, and the lens for each eye has a refractive power of 2.75 diopters. Find the near point and far point for your eyes.**

A positive refractive power (diopters) indicates a positive f length, which is a converging lens—used to correct far-sightedness. A far-sighted person has a normal far point: $FP = \infty$

However, his/her near point (NP) is farther than is considered normal (i.e. $NP > 25$ cm).
And the lens needed to correct this (worn next to the eye) must satisfy $1/f$ (= RP) = $1/0.25 + 1/–(NP)$

Solving for NP: NP = **0.800 m** (or 80.0 cm)

b. **You wear contact lenses to correct your vision. Your eyes are identical, and the lens for each eye has a refractive power of –2.00 diopters. Find the near point and far point for your eyes.**

A negative refractive power (diopters) indicates a negative f length, which is a diverging lens—used to correct near-sightedness. A near-sighted person has a normal near point: NP = **0.250 m** (or 25.0 cm)

However, his/her far point (FP) is nearer than is considered normal (i.e. $FP < \infty$).
And the lens needed to correct this (worn next to the eye) must satisfy $1/f$ (= RP) = $1/\infty + 1/–(FP)$

Solving for FP: FP = **0.500 m** (or 50.0 cm)

c. **If your far point is 80.0 cm, but you use a contact lens to correct this, what image distance will you view for an object placed 45 cm from your eye?**

Your far point (FP) is nearer than is considered normal (i.e. $FP < \infty$).
And the lens needed to correct this (worn next to the eye) must satisfy $1/f = 1/\infty + 1/–(FP)$

Solving for f: f = –0.800 m (or –80.0 cm)

Then: $1/–80 = 1/45 + 1/d_i$

Solving for d_i: d_i = **–28.8 cm**

d. **After accidentally breaking his current eyeglasses, a certain professor must use an older pair (with an out-of-date correction) until new glasses are made. The refractive power of the old glasses is 1.660 diopters, and when wearing them (about 2.00 cm from his eyes), he must hold a newspaper at least 42.0 cm from his eyes in order to read it. What is his near point? (A "normal" near point is considered to be 25.0 cm.)**

Important points for this problem: First, we know that the image must be virtual (to the left of the lens —so d_i will be negative in the thin lens equation) and upright (not inverted), since his eye needs to view it in the correct orientation but at some distance longer than the 42 cm to the object (otherwise the lens would be unnecessary). We can use the thin lens equation to find d_i if we can first determine the focal length of the lens:

$f = 1/(r.p.) = 1/(1.66) = 0.60241$ m = 60.241 cm.

Now use the thin lens equation: $1/d_o + 1/d_i = 1/f$

Re-arrange to solve for d_i: $d_i = (fd_o)/(d_o - f)$

The numbers: $d_i = (60.241)(40)/(40 - 60.241) = -119$ cm

Since the distance from the image is 2 cm farther to his eye than to the lens, his near point, *NP*, must be 121 cm. (*NP* is an absolute value; it carries no sign.)

7. If a person's eyes need corrective lenses, they seldom both need exactly the same correction. The *type* of correction is often the same, of course—but not always. Suppose a person wears eyeglasses (2.00 cm from each eye) to properly correct his vision, as follows: The left lens has a refractive power of 2.00 diopters; the right lens has a refractive power of –1.25 diopters. Now suppose that he <u>removes his eyeglasses</u> to use a simple hand-held magnifier of focal length 3.50 cm. He is viewing a tiny plant specimen that is 0.500 mm long.

a. Calculate the <u>maximum</u> angular magnification he can achieve by (properly) using the magnifier with his <u>left</u> eye.

b. Calculate the <u>maximum</u> angular magnification he can achieve by (properly) using the magnifier with his <u>right</u> eye.

c. Approximately what *maximum viewing angle* (θ'), in radians, can he achieve by properly using the magnifier with his <u>left eye</u>?

d. Approximately what *maximum viewing angle* (θ'), in radians, can he achieve by properly using the magnifier with his <u>right eye</u>?

e. Looking at all of the above results (a-d), which eye should he use with the magnifier for the best detailed view of the specimen? *Fully explain your decision*.

f. <u>How far from the specimen</u> must he hold the magnifier properly for (i.e. next to) his near-sighted eye in order to get the most <u>comfortable</u> (i.e. the most distant-viewing) angular magnification?

a. First, find the near point of this far-sighted eye: $1/f_{eyeglass} = 1/d_{o.standard} + 1/[-(NP - .02)]$

$2.00 = 1/0.23 + 1/[-(NP - .02)]$ Thus: $NP = 0.4459$ m

Then for a properly used magnifier: $M_{max} = NP/f + 1$

$= .4459/.0350 + 1 = \underline{\mathbf{13.7}}$

b. We may assume that the near point of this near-sighted eye is "standard:" $NP = 0.25$ m

Then for a properly used magnifier: $M_{max} = NP/f + 1$

$= .25/.0350 + 1 = \underline{\mathbf{8.14}}$

c. By definition, $\theta' = M\theta$, where θ is the eye's best unaided view of the object.

In this case, that angle is : $\theta \approx h_o/NP = .0005/0.4459$

Thus: $\theta' = M\theta \approx 13.74(.0005/0.4459) = \underline{\mathbf{0.0154\ rad}}$

d. Again, by definition, $\theta' = M\theta$, where θ is the eye's best unaided view of the object.

In this case, that angle is : $\theta \approx h_o/NP = .0005/0.25$

Thus: $\theta' = M\theta \approx 8.143(.0005/0.25) = \underline{\mathbf{0.0163\ rad}}$

e. If we assume that the eye depths (distances from lens to retina) are <u>equal</u>; or that the near-sighted (right) eye depth is <u>greater</u> (and one or the other of these assumptions seems quite likely), then: We would choose the **<u>right eye</u>**, because the larger θ' it achieves would produce the larger *image height on the retina*.

(Thus: The angular magnification number, M, doesn't necessarily tell you which is the better angular view. It tells you which is the <u>most improved</u> angular view. The reason the far-sighted eye's M value is bigger is because it's starting with an inferior unaided view. A greater M value guarantees the better view only if the two unaided views are equal.)

f. The farthest away the image can be from his eye is its far point, so find it:

$1/f_{eyeglass} = 1/d_{o.standard} + 1/[-(FP - .02)]$

$-1.25 = 1/\infty + 1/[-(FP - .02)]$ Thus: $FP = 0.820$ m

Then for a properly used magnifier: $1/f = 1/d_o + 1/[-(FP)]$ Or: $1/(.035) = 1/d_o + 1/(-.820)$

Solving for d_o: $d_o = \underline{\mathbf{0.0336\ m}}$ (or: **<u>3.36 cm</u>**)

8. a. Evaluate the following statements (T/F/N). As always, explain your reasoning.

(i) If a lens can be used as a magnifier, its refractive power is positive.
True. A magnifier is a converging lens; its f is positive (and R.P. = 1/f).

(ii) If a lens can be used as a magnifier, then no matter how it's used, it always produces upright images.
False. A converging lens will produce inverted images whenever the object is placed beyond the focal point.

(iii) If a lens can be used as a magnifier, it is the sort of lens used as the eyepiece in a telescope or microscope.
True. Indeed, this is exactly the function and design of the eyepieces of those instruments: they are simply (very) powerful magnifiers.

(iv) The focal length of a magnifier can never be less than 1 cm.
False. The focal lengths of many microscope eyepieces, for example, are just a few millimeters.

1 pt.

(v) When used properly, a magnifier always produces positive magnification ($m > 0$).
True. When the object is placed at or inside the focal point, the image is non-inverted (positive m).

(vi) To use a magnifier conventionally, you must locate your eye right behind the lens and the object at the focal point of the lens.
True. This results in "comfortable viewing," where the image being placed "way out on the horizon (essentially at $-\infty$).

(vii) In order for a magnifier to produce maximum angular magnification, the object must be located at its focal point.
False. That would result in comfortable viewing (see item **vi**, above), but not maximized viewing. For the latter, you need to locate the object a small distance <u>inside</u> the focal point.

(viii) A glass magnifier's focal length in air is shorter than in water.
True. The focal length results from the severity of the refractive "bending" of the light coming through the lens. That bending (change of angle—computed by Snell's Law) is greater for a greater contrast between the the index of refraction of the lens and its surroundings. When you immerse a glass lens ($n \approx 1.50$) in water ($n \approx 1.33$), that contrast is less than for glass in air ($n \approx 1.00$). The bending is therefore greater in air, so the focal length is shorter.

1 pt.

b. Suppose you're unable to visually distinguish color, but you have two different lasers (red and green), mounted in parallel, side-by-side, aimed at a wall. Explain how a magnifying glass (not necessarily used conventionally) could help you find out which laser is which.

Place the magnifying glass in front of each lens, in turn, with the optical axis of the lens horizontal (and normal to the wall), and so that the parallel rays of the laser strike the same level on the upper portion of the lens. Then the spot that forms lower on the wall is the green laser; its path is "bent" more severely than the red laser.

1 pt.

c. A student wears contact lenses (2.25 diopters for each eye).

(i) What absolute magnification (m) do these lenses produce when she holds a newspaper at 35 cm from her eye?

(ii) What angular magnification (M) do these lenses produce when she holds a newspaper at 35 cm from her eye?

(i)

The focal length: $f\,(\text{meters}) = 1/\text{RP} = 1/2.25 = 0.444$ m

The image distance: $1/0.444 = 1/0.35 + 1/d_i$

Thus: $d_i = -1.647$

The magnification: $m = -(d_i/d_o) = -(-1.647/0.35) = \underline{\mathbf{4.71}}$

(ii)

The unaided view: *Could the student even view the newspaper at 35 cm with an unaided eye?*
<u>Find her near point</u>: $2.25 = 1/.25 + 1/(-\text{NP})$
Her near point os 0.5714 m. *<u>That's the best she can do</u>*. So: $\theta \approx h_o/0.5714$

The "aided" view: $\theta' \approx h_i/1.647) = (mh_o)/1.647$

The angular magnification: $M = \theta'/\theta = [(mh_o)/1.647]/(h_o/0.5714) = [(4.71h_o)/1.647]/(h_o/0.5714)$

$= \underline{\mathbf{1.63}}$

1 pt.

8. d. If your eye uses a contact lens of –1.25 diopters to properly correct your vision, and you're wearing that lens to view an object from a distance of 50.0 cm, …

(i) What is the absolute magnification (*m*) achieved in this situation?

(ii) What is the angular magnification (*M*) achieved in this situation?

(i) Find the focal length of this lens: f = 1/R.P. = 1/(-1.25) = –0.800 m = –80.0
Now locate the image for an object distance of 50 cm: $1/(-80) = 1/50 + 1/d_i$
Solving for d_i: $d_i = -30.769$ cm
Therefore: $m = -(d_i/d_o) = -(-30.769/50) = \underline{\mathbf{0.615}}(38)$ *1 pt.*

(ii) You're told to view the object from 50 cm, so your unaided viewing angle is given by $\theta \approx h_o/50$.
When wearing the lens—located next to your eye—your viewing distance is the same as d_i (except for the sign). Therefore, your viewing angle is given (approximately) by $\theta' \approx h_i/|d_i|$.
But $h_i = m(h_o)$. Substituting: $\theta' \approx m(h_o)/|d_i|$
Therefore: $M = \theta'/\theta = [m(h_o)/|d_i|]/(h_o/50) = 50m/|d_i| = 50(0.61538)/30.769 = \underline{\mathbf{1.00}}$ *1 pt.*

e. If you have normal vision and properly use a magnifying glass with a refractive power of 12.5 diopters, how far should you hold it from the object for maximum angular magnification? What if your near point is 40.0 cm?

If you have normal vision, your near point (*NP*) is 25 cm. And for the maximum viewing angle, you should view the image at your near point, 25 cm from your eye, which (since you hold the magnifier right next to your eye) means that the desired image distance is –25 cm.
Thus: $1/d_o + 1/(-.25) = 1/f = 12.5$ Solve for d_o: $d_o = \underline{\mathbf{0.0606\ m}}$ (or 6.06 cm)
Repeat for NP = 0.40 m: $1/d_o + 1/(-.40) = 1/f = 12.5$ $d_o = \underline{\mathbf{0.0667\ m}}$ (or 6.67 cm) *1 pt.*

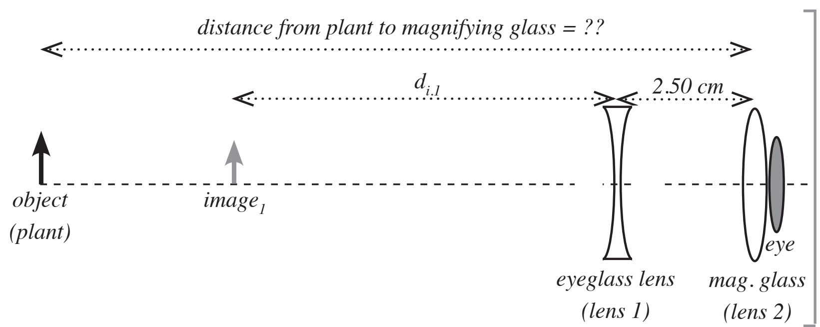
f. Compared to your best unaided view of an object, if you hold a magnifier (*f* = 8.00 cm) next to your eye (NP = 72.0 cm), what angular magnification (*M*) do you get by viewing the object 7.50 cm from your eye?

You can't view any object any closer than your near point (*NP*), 72 cm, so your best unaided viewing angle is given (approximately) by: $\theta \approx h_o/72$
Now locate the image produced by the magnifier: $1/f_o = 1/d_{o.o} + 1/d_{i.o}$
That is: $1/8.00 = 1/7.50 + 1/d_i$ Solve for d_i: $d_i = -120$ cm
Find the absolute magnification produced by the magnifier: $m = -(d_i/d_o) = -(-120/7.50) = 16.0$
Since the magnifier is located next to your eye, your viewing distance of its image is the same as d_i (except for the sign). Therefore, your viewing angle is given (approximately) by $\theta' \approx h_i/|d_i|$.
But: $h_i = m(h_o) = 16h_o$ Substituting: $\theta' \approx 16h_o/|d_i| \approx 16h_o/120 = 0.13333h_o$
Therefore: $M = \theta'/\theta = 0.13333h_o/(h_o/72) = 0.13333(72) = \underline{\mathbf{9.60}}$ *1 pt.*

g. You and a colleague are out looking for rare plants. Your colleague is nearsighted, but she wears eyeglasses to correct it. Her correction is -3.75 diopters for each lens. You have normal vision—no correction. You're trying to use a magnifying glass to view a small plant growing in rock crevice, but you can't get the large magnifying lens (*f* = 10.0 cm) close enough to the plant to use it as intended. Your colleague removes one lens from her eyeglasses and is able to place it so that your magnifier offers normal comfortable viewing for you. If the two lenses are 2.50 cm apart, what is the distance from the magnifying glass to the plant?

A nearsighted correction (negative diopters), lens (#1) is a diverging lens. And this makeshift arrangement allows comfortable use of the magnifier (lens_2), so its object (image_1) *must be located at its focal point*. **The distance from lens_2 to image_1 is 10.0 cm.** The two lenses are 2.50 cm apart, so $d_{i.1}$ = (–)7.50 cm. Now use the thin lens equation on lens 1: $1/d_{o.1} + 1/(-0.075) = 1/f_1 = -3.75$. So $d_{o.1}$ = 0.104348 m. *1 pt.*

This is the distance from the plant to lens 1 (and it's more than 10 cm—good, since it's hard to get a lens within 10 cm of the plant.) And then it's .025 m farther to the magnifier: 0.1043 + 0.025 = 0.129 m
With this makeshift arrangement, the distance from the plant to the magnifier is 12.9 cm.



1 pt.

9. a. Evaluate the following statements (T/F/N) for a properly operating, properly adjusted, conventional (2-lens) barrel microscope. As always, explain your reasoning.

(i) The image distance of the first lens is approximately equal to the object distance of the second lens.
False. The image distance of the first lens (the objective lens) consumes most of the barrel length; the object distance of the second lens (the eyepiece) is very short.

(ii) The microscope inverts the image for the viewer.
True. The objective lens creates a real image, which is inverted; then the eyepiece creates a virtual image, which does not re-invert the first image.

(iii) The image of the eyepiece is located at the focal point of the objective lens.
False. The image of the objective lens is located at the focal point of the eyepiece.

(iv) If L is the distance between the two lenses, and all lengths are expressed in meters, the refractive power of the objective lens could be correctly written as $1/(L - d_{o.eyepiece}) + 1/d_{o.objective}$.

True. The thin-lens equation, applied to the objective lens: $1/f_{objective} = 1/d_{o.objective} + 1/d_{i.objective}$

But if the lengths are expressed in meters, then: $1/f_{objective} = \text{R.P.}_{objective}$

And in a properly adjusted microscope: $d_{i.obj} = L - f_e$ and $f_e = d_{o.eyepiece}$

Therefore, in a properly adjusted microscope: $\text{R.P.}_{objective} = 1/d_{o.objective} + 1/(L - d_{o.eyepiece})$

b. For a properly operating, properly adjusted, conventional (2-lens) barrel microscope…

(i) Where should the object be placed? Just outside the focal point.

(ii) Where must the objective lens place its image? At the focal point of the eyepiece.

c. To read, you need to wear eyeglasses (2 cm from your eye) to correct your farsightedness (same for each eye). However, you remove those glasses when using your standard, two-lens microscope. The focal length of the objective lens of that microscope is 0.500 cm, and the distance between the two lenses is 16.5 cm. If you achieve a comfortable angular magnification of –3840 when you place the object 0.516 cm from the objective lens, what is the correction (in diopters) of your reading glasses?

First, locate the image formed by the objective lens: $1/f_o = 1/d_{o.o} + 1/d_{i.o}$

That is: $1/.500 = 1/.516 + 1/d_{i.o}$ Thus: $d_{i.o} = 16.125$ cm

But because the image of the objective lens is located at the focal point of the eyepiece, the distance L, between the two lenses is the sum of $d_{i.o}$ and f_e: $L = d_{i.1} + f_e$

Solving for f_e: $f_e = L - d_{i.1} = 16.5 - 16.125 = 0.375$ cm

Then we know: $M = -(NP)(L - f_e)/(f_o \cdot f_e)$

Solving for NP: $NP = -M(f_o \cdot f_e)/(L - f_e) = -(-3840)(0.500)(0.375)/(16.5 - 0.375) = 44.651$ cm

Now find the eyeglass focal length, f, that will correct that near point: $1/f_{glasses} = 1/d_{o.glasses} + 1/d_{i.glasses}$

That is: $1/f_{glasses} = 1/(25-2) + 1/-(NP - 2)$ Thus: $f_{glasses} = 49.92 \text{ cm} = 0.4992 \text{ m}$

Therefore: $RP = 1/f_{glasses\,(m)} = 1/(.4992) =$ **2.00 diopters**

10. a. Evaluate the following statements (T/F/n). ***As always, fully explain your reasoning.***

(i) The focal point of a telescope's objective lens is nearer to the eyepiece than to the objective lens.
True. The image distance of the first lens (the objective lens) is long—it consumes most of the barrel length—and that image distance is also the focal length of the objective lens; it forms its image right at its own focal point (because it is receiving essentially parallel rays arriving from a very distant object). So the two focal points ***coincide*** (see also items **b** and **c** below), but then the distance from that common point to the second lens (the eyepiece) is very short, since the eyepiece is a very powerful (very short-focal-length) magnifier.

(ii) For a properly adjusted, standard 2-lens telescope (where L is the distance between the two lenses), the angular magnification could be correctly written as $-(d_{o.eyepiece}/d_{i.objective})$.

False. In a properly adjusted telescope: $M = -f_o/f_e$ And: $f_e = d_{o.eyepiece}$

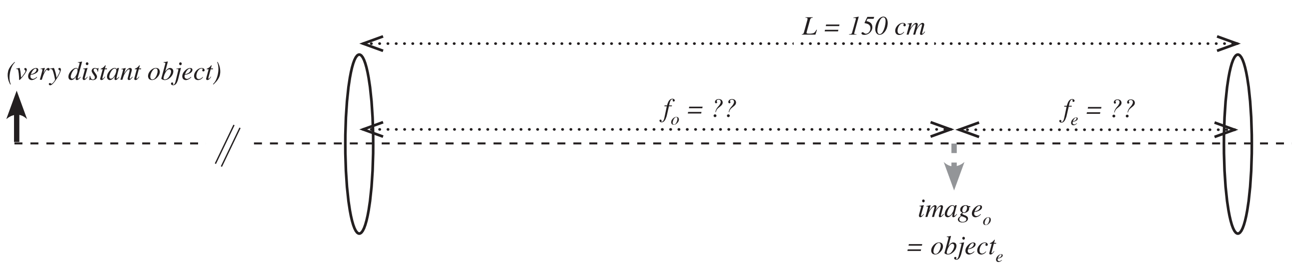
But since the rays from the distant object are essentially parallel, the objective lens lands its image right at its own focal point (which is also the focal point of the eyepiece, of course): $f_o = d_{i.objective}$

Therefore, in a properly adjusted telescope: $M = -(d_{i.objective})/d_{o.eyepiece}$

b. For a conventional telescope, where should the focal point of the objective lens be located?
At the focal point of the eyepiece.

c. A two-lens, refracting telescope has an angular magnification of –83.0. The length of the barrel (distance between lenses) is 1.50 m. Find the focal lengths of the two lenses.

The key point to realize about a telescope is that the focal points of the two lenses coincide; they're located at the same point. This is because the parallel light from the very distant object will form its image from right at the focal point of the objective lens; and that location needs to be very nearly (just inside) the focal point of the eyepiece. Hence the co-location:



Thus: $L \approx f_o + f_e$ Also, we know that for a telescope of this configuration: $M \approx -f_o/f_e$

Since we know both L and M (but neither f_o nor f_e), this is a simple case of two simultaneous equations with two unknowns, which we solve for f_e: $f_e = L/(1 - M)$

The numbers: $f_e = 150/[1 - (-83.0)] =$ **1.79 cm**

And therefore: $f_o = L - f_e = 150 - 1.79 =$ **148 cm**

The focal length of the objective is 148 cm. **The focal length of the eyepiece is 1.79 cm.**

10. d. A standard, two-lens telescope is to be modified with a third converging lens (the "inverter" lens—placed somewhere between the objective lens and the eyepiece) so that the comfortable angular magnification (*M*) of the telescope is changed from -120 to +120. If the inverter lens is identical to the eyepiece, and the length of the modified telescope (measured from objective lens to eyepiece) is 96.0 cm, find the focal length of the objective lens.

The angular magnification, M, of the un-modified scope is given by: $M = -(f_o/f_e)$

Thus: $-(f_o/f_e) = -120$ (or: $f_o/f_e = 120$)

In the modified version, the objective lens still lands its image at its own focal point (its d_o is still ∞), but that's no longer the focal point of the eyepiece. Rather it's at an intermediate point, where the inverter then uses it as its object, in turn forming its own (real) image at the focal point of the eyepiece. Thus the modified distance between the objective lens and the eyepiece is: $L_{modified} = f_o + d_{o.inv.} + d_{i.inv.} + f_e$

If the inverter lens isn't to change the overall magnification, it must simply <u>invert</u> the image *without altering its size*. That is: $m_{inv.} = -(d_{i.inv.}/d_{o.inv.}) = -1$ Which means: $d_{i.inv.} = d_{o.inv.}$

And of course we know this: $1/f_{inv.} = 1/d_{o.inv.} + 1/d_{o.inv.}$ Which means: $1/f_{inv.} = 2/d_{o.inv.}$

Therefore: $d_{o.inv.} = d_{i.inv.} = 2f_{inv} = 2f_e$ (because the inverter is identical to the eyepiece)

So: $L_{modified} = f_o + d_{o.inv.} + d_{i.inv.} + f_e = f_o + 5f_e$ That is: $f_o + 5f_e = 96.0$

Solve this and first equation above ($f_o/f_e = 120$) simultaneously for f_o: $f_o = 96.0/(1 + 5/120) =$ **<u>92.2 cm</u>**

10. e. Viewing the moon (a distance X away) with a telescope, you see two objects on the lunar surface. Here is some other information about the telescope:

- The telescope's barrel length (distance between its two lenses) is L.
- If you were to place its eyepiece alone next to your eye, it would properly correct your farsightedness.
- If you were to replace its objective lens with another lens identical to its eyepiece, the resulting microscope would have an approximate angular magnification of M.

The images of the two lunar objects are separated on your retina by a distance h_i. If your eye's depth (the distance from its lens to retina) is d_i, about how far apart are those objects (from each other) on the moon?

Known values: X, L, M, h_i, d_i

*Note: This is an ODAVEST items. Use the full 7-step problem-solving protocol—but keep in mind that **you're not being asked to actually solve for the final expression**. In fact, you're not being asked to do any math at all—**not even any algebra**. Rather, for the **S**olve step, you are to write a series of succinct instructions on how to solve this problem. Pretend that your instructions will be given to someone who knows math but not physics. And for the **T**est step, you should consider the situation and predict how the solution would change if the data were different—changing one data value at a time.*

Objective: Two objects on the moon's surface are separated by an unknown distance.

The objects are being viewed at a (large) known distance from earth via a telescope.

The telescope's barrel length (the distance between lenses) is a known value.

The telescope's eyepiece, if removed and placed next to the eye of this viewer, would properly correct the farsightedness of that eye.

If the telescope's objective lens were replaced with a lens identical to the eyepiece, the resulting microscope would offer this viewer's eye a known angular magnification.

The viewer's eye depth is a known distance.

The images of the two objects on the viewer's eye's retina are separated by a known distance.

We want to find the separation distance of the objects on the moon's surface.

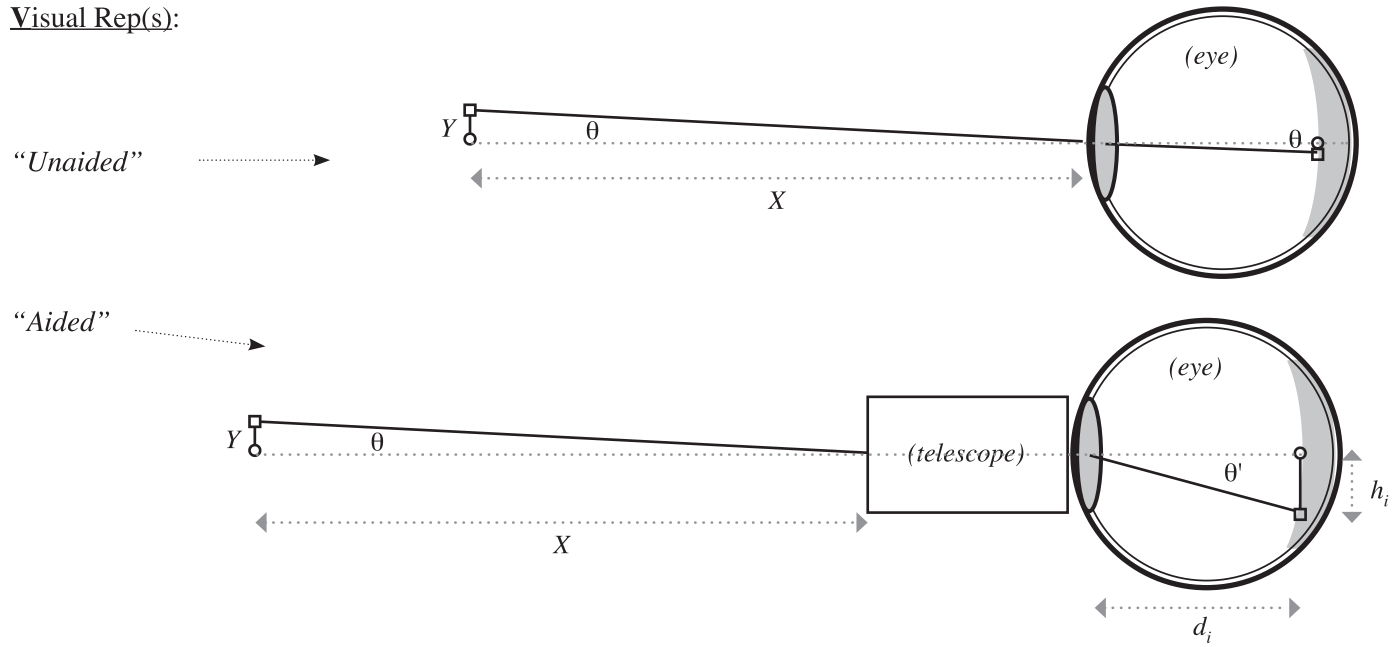
Data:

X	The viewing distance from earth to moon.
L	The barrel length (lens-to-lens distance) of the telescope.
M	The angular magnification that would be offered to the viewer's eye by a microscope constructed from lenses identical to the telescope's eyepiece.
h_i	The separation distance of the two images that form on the viewer's retina.
d_i	The depth of the viewer's eye.

Assumptions:

Object(s):	We will assume that the two objects are both at the same distance X from the viewer's telescope.
Microscope:	The M value given assumes that the distance between any object viewed and the objective lens' focal point is negligible.
Eye:	We will assume that the eye depth applies equally to each of the two images on the retina.
Conditions:	We will assume that the atmosphere does not magnify/reduce/distort the view.

Visual Rep(s):



Equations:

I. $1/f_e = 1/25 + 1/(-NP)$ (Correction equation for farsightedness, using the eyepiece.)

II. $M = -NP(L - f_e)/[(f_e)(f_e)]$ (Approximate angular magnification for a microscope using two identical copies of the eyepiece.)

III. $f_o = L - f_e$

IV. $M_{telescope} = -f_o/f_e$

V. $\theta \approx Y/X$

VI. $\theta' \approx h_i/d_i$

VII. $M_{telescope} = \theta'/\theta$

Solving:

Solve **I** for NP. Substitute the resulting expression into **II**.

Solve **II** for f_e. Substitute that result into **III** and **IV**.

Solve **III** for f_o. Substitute that result into **IV**.

Solve **IV** for $M_{telescope}$. Substitute that result into **VII**.

Substitute **V** into **VII**.

Solve **VI** for θ'. Substitute that result into **VII**.

Solve **VII** for Y.

Testing: Dimensions: Y should have units of length.

Dependencies:

Assuming all other data as given, if X were greater, this would imply a greater value of Y.

Assuming all other data as given, the effects of L on Y are not clear without graphing; the values of L, M, NP and f_e are co-dependent in 2nd- and 3rd- order polynomials.

Assuming all other data as given, the effects of M on Y are not clear without graphing; the values of L, M, NP and f_e are co-dependent in 2nd- and 3rd- order polynomials.

Assuming all other data as given, if h_i were greater, this would imply a greater value of Y.

Assuming all other data as given, if d_i were greater, this would imply a lesser value of Y (because the same h_i would imply a lesser θ').

11. a. As an avid bird-watcher, you're stalking the rare and elusive* North American Tinky Bird. Unfortunately, your assistant just dropped the binoculars into a pond. And now (wouldn't you know it?), 200 m away, standing 40 cm tall on the pond's opposite shore is an unidentified bird. *Is it… could it be…?* But at that distance, you can't see enough detail with your naked eye. Drat. So you decide to dismantle your assistant's eyeglasses (two identical lenses, each with a focal length of 28.29 cm) to try to enhance your view of the mystery bird.

(i) First you use the lenses to construct a standard-configuration 2-lens telescope (you have your assistant hold the lenses the right distance apart while you view the result). Make a quick sketch of this, showing relevant lengths and positions, and then explain why this doesn't enhance the detail in your view (plus the bird is upside-down—very annoying).

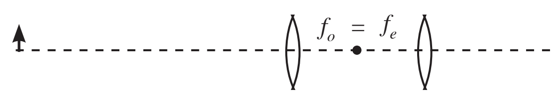
(ii) Then strikes The Bright Idea: Forget the normal assumptions of a telescope. You have your assistant change the distance, *L*, between the two lenses, and you also re-locate your eye so that *you get an <u>enlarged and upright</u> view of the bird*. In fact, depending on where you place your eye (it has normal uncorrected vision), you can get an angular magnification value as high as +40.0 with the lenses separated by that distance *L*. Calculate *L*.

(iii) At long last, you can positively identify it, get famous and retire: *This IS the fabled Tinky*. But you need a second person to corroborate your claim. So now you hold the lenses (same separation *L* as in part b) and your assistant then views the bird. How far should his eye be from the nearest lens in order to maximize his angular magnification?

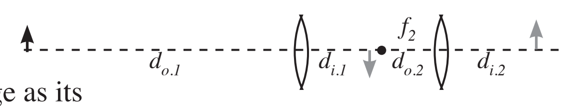
(iv) What is the maximum angular magnification <u>your assistant</u> gets with this makeshift system?

So rare and elusive, in fact, that it's never been seen. Its existence throughout North America is predicted theoretically by modern physics (but unfortunately, the search is being conducted only in a small region near the French-Swiss border).

(i) The angular magnification of this telescope is $M \approx -f_o/f_e = -28.29/28.29 = -1.00$ That's no improvement in view:



(ii) With two converging lenses (and $d_o > f$), the only way to create an upright final image is this: The first lens produces an inverted (real) image. The second lens then uses that image as its object, also inverting the resulting image. *Your eye must be somewhere to the right of the 2nd image to see it.*



Locate the first image: $1/f_1 = 1/d_{o.1} + 1/d_{i.1}$ Or: $1/28.29 = 1/20000 + 1/d_{i.1}$ So: $d_{i.1} = 28.3301$ cm.

Find the first magnification: $m_1 = -(d_{i.1}/d_{o.1}) = -(28.3301/20000) = -.0014165$

Find the first image height: $h_{i.1} = m_1(h_o) = -0.05666$ cm

Your <u>unaided</u> viewing angle of the bird is given by $\theta \approx 0.40/200 = 0.00200$ And your <u>best</u> <u>aided</u> view is 40 times better than this: $\theta' \approx 40(.002) = 0.08$. But $\theta' \approx h_{i.2}/(d_{view})$, and since you have normal vision, you'd be standing 25 cm to the right of the image to view it. So $0.08 = h_{i.2}/(25)$, so $h_{i.2} = 2.00$ cm.

Therefore: $m_2 = h_{i.2}/h_{o.2} = h_{i.2}/h_{i.1} = 2.00/(-0.05666) = -35.29818$

So now we know: $-(d_{i.2}/d_{o.2}) = -35.29818$. And, of course, $1/28.29 = 1/d_{o.2} + 1/d_{i.2}$

Solving simultaneously, we get: $d_{o.2} = 29.09$ and $d_{i.2} = 1029$

And therefore: $L = d_{i.1} + d_{o.2} = 28.3301 + 29.09 =$ <u>**57.4 cm**</u>

(iii) Your assistant can't view the above image as closely as you can; he's far-sighted (usually corrected with $+f$ eyeglasses worn 2 cm from his eyes). So find his near point—*the closest viewing distance he can use—to maximize his angular magnification:* $1/28.29 = 1/(25-2) + 1/-(NP-2)$ Solving: $NP = d_{view} = 125$ cm
So he must stand a distance of $125 + d_{i.2}$ from that nearer lens: $125 + 1029 =$ **1154 cm** $=$ **11.5 m**

(iv) Your assistant's best <u>unaided</u> viewing angle is the same as yours ($\theta \approx 0.40/200 = 0.002$). And the same lens configuration will produce the same final image height ($h_i = 2.00$ cm). But his viewing distance is 125 cm (rather than the 25 cm you could use). So for him, $\theta' \approx 2/125$; so his $M = \theta'/\theta = (2/125)/0.002 =$ <u>**8.00**</u>

11. b. Suppose you have four lenses with focal lengths 2.00 cm, 3.00 cm, 4.00 cm, and 36.0 cm, respectively.

(i) First, select among your four lenses to form the best conventional 2-lens telescope.

$M \approx -f_o/f_e$, so to maximize M, you want to maximize f_o and minimize f_e. So, let f_o = 36 cm, and let f_e = 2 cm.

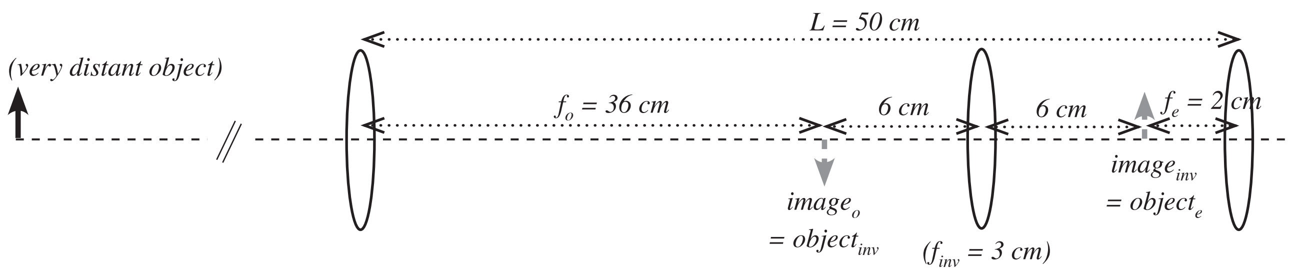
When you look through it, of course, your view is enlarged but inverted; you things upside down. So, how would you <u>modify</u> it—change its dimensions and place <u>one</u> of the other lenses somewhere between the objective lens and the eyepiece—to get a <u>non-inverted</u> view with the same angular magnification magnitude as before? (In selecting this third lens, keep the length of the telescope as short as possible.)

Use the 3 cm lens as the inverter lens and place it so that its object is the image of the objective lens, and its image is the object of the eyepiece (placed, of course, at the focal point of the eyepiece). Note that we want the magnification, m, of the inverter lens to be –1 (which flips the image with no change in size).

Thus we need to solve these two equations for the d_i and d_o of the inverter lens (whose f = 3 cm):

$-(d_i/d_o) = -1$ $\quad$ $1/3 = 1/d_o + 1/d_i$ $\quad$ <u>Results</u>: $d_o = d_i = 2f = 6$ cm

Draw a simple sketch of the resulting 3-lens instrument.



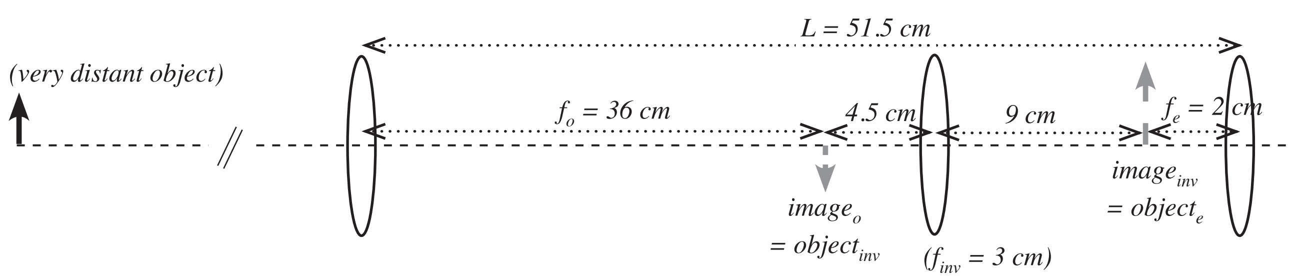
(ii) With your answer to part (a) as a starting point, keeping the relative order of those three lenses the same, how would you modify their distances from one another in order to approximately double the angular magnification of the overall instrument?

To roughly double the angular magnification of the instrument, we'd simply double the height of the image that the inverter lens delivers to the eyepiece. That is, now we want the magnification, m, of the inverter lens to be –2 (which flips the image while doubling the size).

Thus we need to solve these two equations for the d_i and d_o of the inverter lens (whose f = 3 cm):

$-(d_i/d_o) = -2$ $\quad$ $1/3 = 1/d_o + 1/d_i$ $\quad$ <u>Results</u>: $d_o = (3/2)f = 4.5$ cm $\quad$ $d_i = 3f = 9$ cm

Again, draw a simple sketch of your answer.



11. b. (iii) Same sort of question for a microscope: When you look through a conventional 2-lens microscope, your view is inverted—you see the object enlarged but upside down. First, select among your four lenses to form the best conventional 2-lens microscope that has a distance of 50.0 cm between the lenses.

$M \approx -NP(L - f_e)/(f_o \cdot f_e)$, so to maximize M, you want to minimize both f_o and f_e. And for a given L value, this equation would indicate that you'd want f_e to be the shortest of all (2 cm). So if L is to remain at 50 cm (and you could adjust the object distance accordingly), you must initially select f_o = 3 cm and f_e = 2 cm.

Now, assuming you'll be viewing an object located 3.20 cm from the objective lens, how would you then <u>modify</u> this instrument—change its dimensions and place <u>one</u> of the other lenses somewhere between the objective lens and the eyepiece—to get a <u>non-inverted</u> view with the same angular magnification magnitude? (You can change the overall length of the microscope, but keep it as short as possible.)

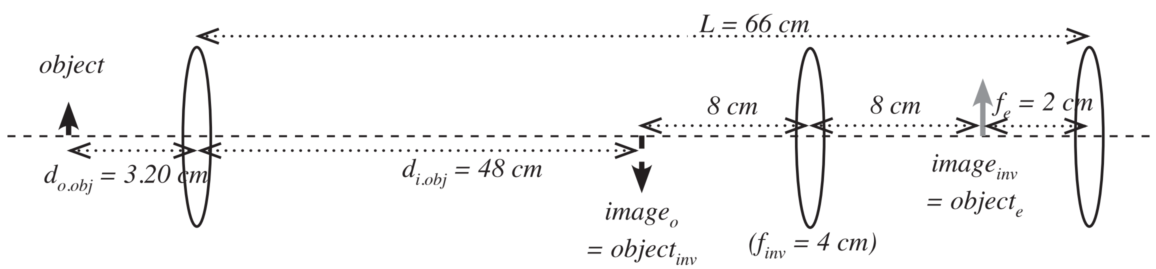
To invert the image delivered to the eyepiece—with no change in its size—the logic is exactly like part **a**: But the other short lens available in this case is the 4 cm lens—you'll have to use that as the inverter lens and place it so that its object is the image of the objective lens, and its image is the object of the eyepiece (placed, of course, at the focal point of the eyepiece). Note that we want the magnification, m, of the inverter lens to be –1 (which flips the image with no change in size).

Thus we need to solve these two equations for the d_i and d_o of the inverter lens (whose f = 4 cm):

$-(d_i/d_o) = -1$ $\quad$ $1/4 = 1/d_o + 1/d_i$ $\quad$ <u>Results</u>: $d_o = d_i = 2f = 8$ cm

Also, calculate the objective lens' image distance: 1/3 = 1/3.20 + 1/di $\quad$ <u>Result</u>: $d_i = 48$ cm

Draw a simple sketch of the resulting 3-lens instrument.

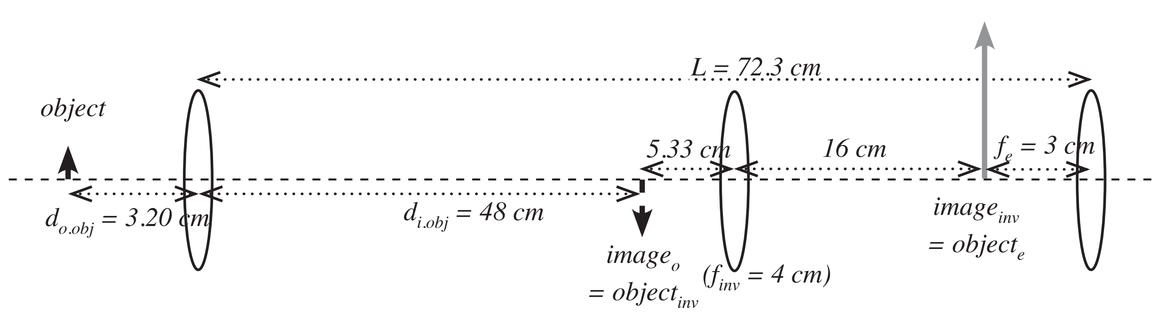


(iv) With your answer to part (iii) as a starting point, keeping the relative order of those three lenses the same, how would you modify their distances from one another in order to approximately triple the angular magnification of the overall instrument?

To roughly triple the angular magnification of the instrument, we'd triple the height of the image that the inverter lens delivers to the eyepiece. That is, now we want the magnification, m, of the inverter lens to be –3 (which flips the image while tripling the size). Thus we need to solve these two equations for the d_i and d_o of the inverter lens (whose f = 4 cm):

$-(d_i/d_o) = -3$ $\quad$ $1/4 = 1/d_o + 1/d_i$ $\quad$ <u>Results</u>: $d_o = (4/3)f = 5.33$ cm $\quad$ $d_i = 4f = 16$ cm

Again, draw a simple sketch as part of your answer.



12. You're on your way out of town for spring break when a sudden freak thunderstorm whisks you to a faraway desert island. Alas. Fortunately, washing up on the beach in the storm's aftermath is a watertight bag containing lots of food and your favorite beverages, a tub of industrial-strength sunscreen, a hammock and canopy, plus three converging lenses, a small flat mirror, a meter stick, a calculator and your PH 212 notes sheet. No smart phone. No dumb phone. No laptop. No iPad. No iPod. No iNothing. All there is to do is physics until you're rescued. Alas.

a. First you determine the focal lengths of the lenses (3.00 cm, 5.00 cm and 40.0 cm.). Explain how you do this (no equations or math necessary).

The sun's rays are parallel, and a converging lens concentrates (converges) parallel rays at its focal point. So take each lens in turn and experiment with it until you find how far from the lens is the point that is (very) hot and can start a fire or burn a hole in something. Use the meter stick to measure that distance.

b. Your notes sheet is seriously over-crammed with tiny writing—a magnifying glass would sure help. If the distance from your eye lens to your retina is 1.92 cm, and you have normal vision (i.e. not needing correction), then by using just one of the three lenses, what's the approximate height of the largest letter E you could form on your retina, if you wrote that E in your notes just 1.60 mm tall?

The best angular magnification you can get with a converging lens as a magnifying glass is: $M = NP/f + 1$

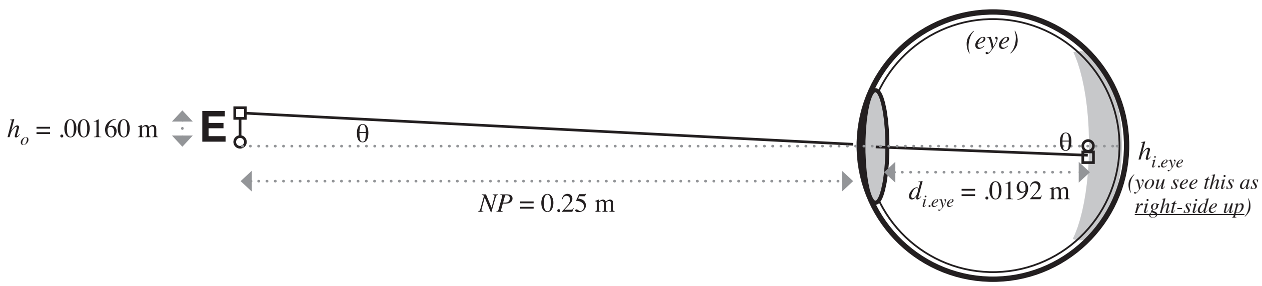
M is highest when the focal length is the shortest, so use the lens with $f = 3$ cm.

Thus: $M = 25/3 + 1 = 9.333$

The best unaided view, θ, you'll get of the E is from your near point (NP = 25 cm):

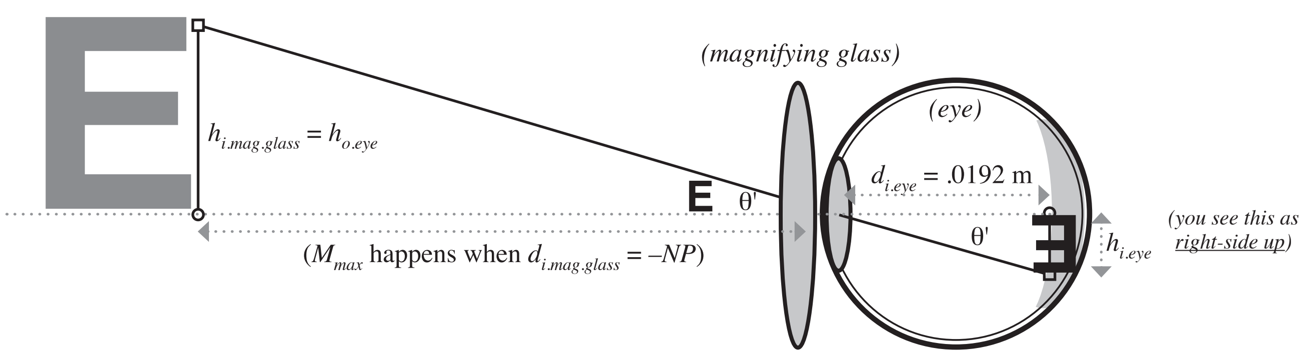
$$\theta \approx \frac{1.60 \times 10^{-3}}{0.25} \approx 6.40 \times 10^{-3} \text{ rad}$$

And this is the same angle formed by the image height, $h_{i.eye}$, on your retina, vs. the eye depth, $d_{i.eye}$:



But the magnified view, θ', is *M times* this good: $\theta' \approx M\theta \approx 9.333(6.40 \times 10^{-3} \text{ rad}) \approx 5.973 \times 10^{-2}$ rad

That's the angle formed by the image height, $h_{i.eye}$, on your retina, vs. the eye depth, $d_{i.eye}$: $\theta' \approx h_{i.eye}/0.0192$

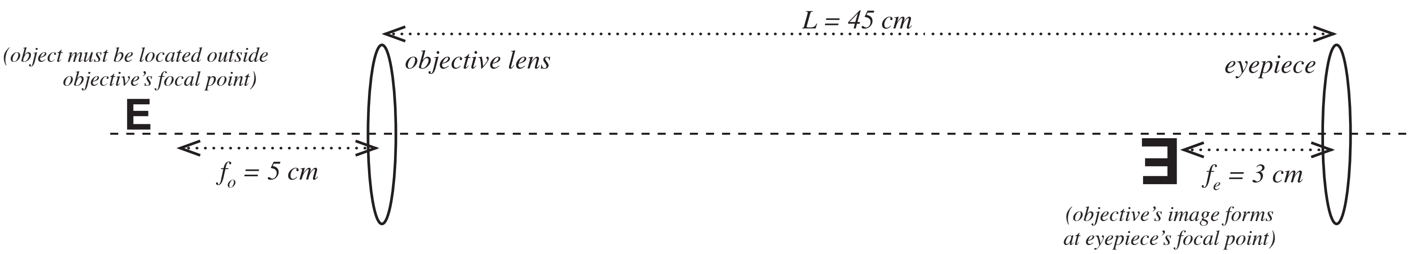


Solve for $h_{i.eye}$: $h_{i.eye} \approx 0.0192(\theta') \approx 0.0192(5.9733 \times 10^{-2}) \approx$ **$\underline{1.15 \times 10^{-3}\text{ m}}$** (or **$\underline{1.15\text{ mm}}$**)

12. c. Not satisfied with just a simple magnifying glass, you choose <u>two</u> lenses, holding one next to your eye and the other lens anywhere within your 45-cm reach. Some pairs of lenses produce better magnification results than other pairs. With the best such pair chosen, what's the size of the E on your retina now?

You're trying to form the best microscope you can with any two of the lenses you have. The angular magnification for a microscope is: $M = -NP(L - f_e)/(f_o f_e)$

This value is highest when L is longest, f_e is shortest, and f_o is the next-shortest. So use the 3-cm lens as the eyepiece; the 5-cm lens as the objective; and hold them at maximum separation—full arm's length (45 cm):

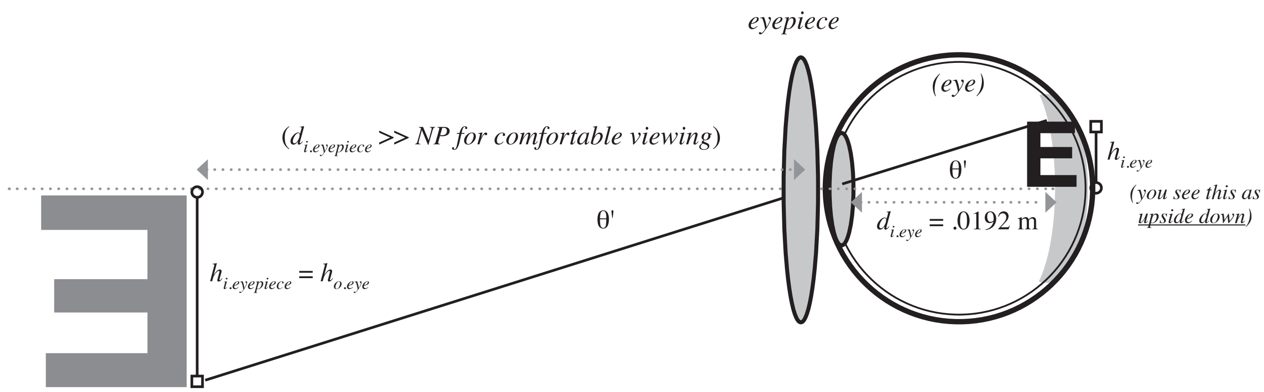


Then: $M = -NP(L - f_e)/(f_o f_e) = -25(45 - 3)/(3 \cdot 5) = -70.000$

Again, the best <u>unaided</u> view, θ, you'll get of the E is $\theta \approx 6.40 \times 10^{-3}$ rad, just as with the magnifying glass —see diagram of θ from part **(b)**.

But the magnified view, θ', is *M times* this good: $\theta' \approx M\theta \approx -70(6.4000 \times 10^{-3}\ \text{rad}) \approx -0.448$ rad

The same angle is formed by the image height, $h_{i.eye}$, on the retina, vs. the eye depth, $d_{i.eye}$: $\theta' \approx h_{i.eye}/0.0192$



Solve for $h_{i.eye}$: $h_{i.eye} \approx 0.0192(\theta') \approx 0.0192(-0.448) \approx$ –**<u>8.60 x 10</u>**$^{-3}$ **<u>m</u>** (or –**<u>8.60 mm</u>**)

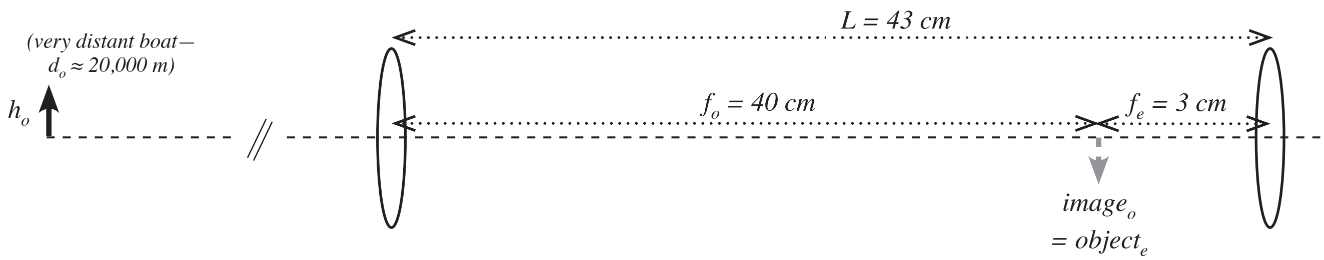
(The only problem: it's upside down.... Guess you'll have to stick with the magnifying glass to actually read your notes.)

12. d. Out on the horizon (which you know is ≈ 20 km away), something appears that might be a boat (!), but you can't tell for sure—it's just a tiny dot on your retina. Ah, but now that you can read your notes sheet, you can figure out the best pair of lenses to see more detail: Using that combination, you now see a 0.100 mm image on your retina of this vessel. About how tall is this boat?

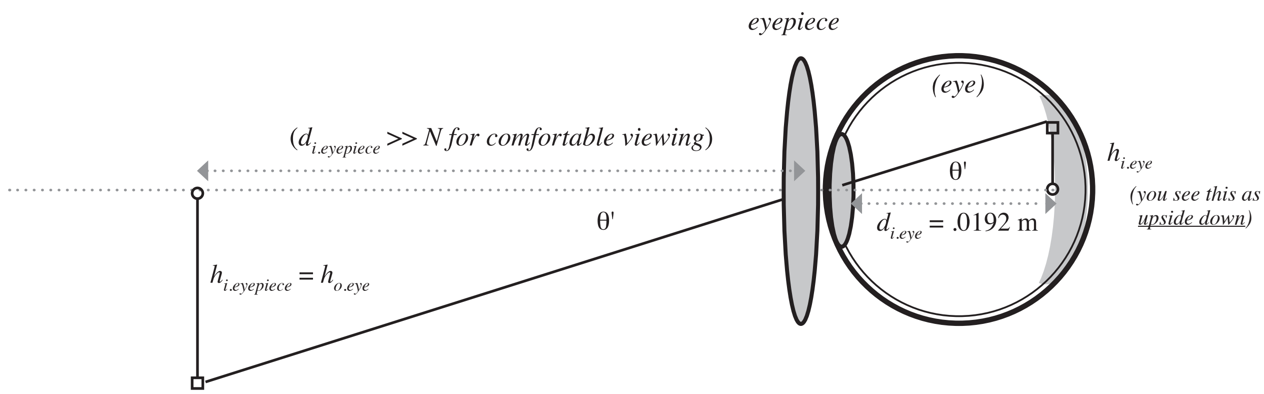
You're trying to form the best telescope you can with any two of the lenses you have.
The angular magnification for a telescope is: $M = -f_o/f_e$

This value is highest when f_e is shortest, and f_o is the longest—and note that the separation of these two lenses must equal the sum of their two focal lengths: $L = f_o + f_e$.

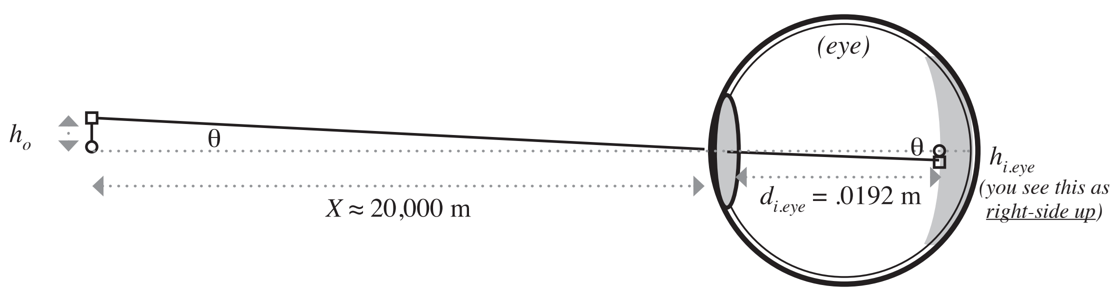
So use the 3-cm lens as the eyepiece; the 40-cm lens as the objective; and hold them at a separation equal to the sum of those two focal lengths (43 cm):



Thus: $M \approx -f_o/f_e = -40/3 = -13.333$



Compare the above (aided) situation to the unaided ("naked-eye") situation below:



By definition, $M = \theta'/\theta$, and for this situation: $M \approx [(h_{i.eye}/d_{i.eye})'/(h_o/d_o)]$

Solving for h_o: $h_o = (d_o h_{i.eye})/(M d_{i.eye})$

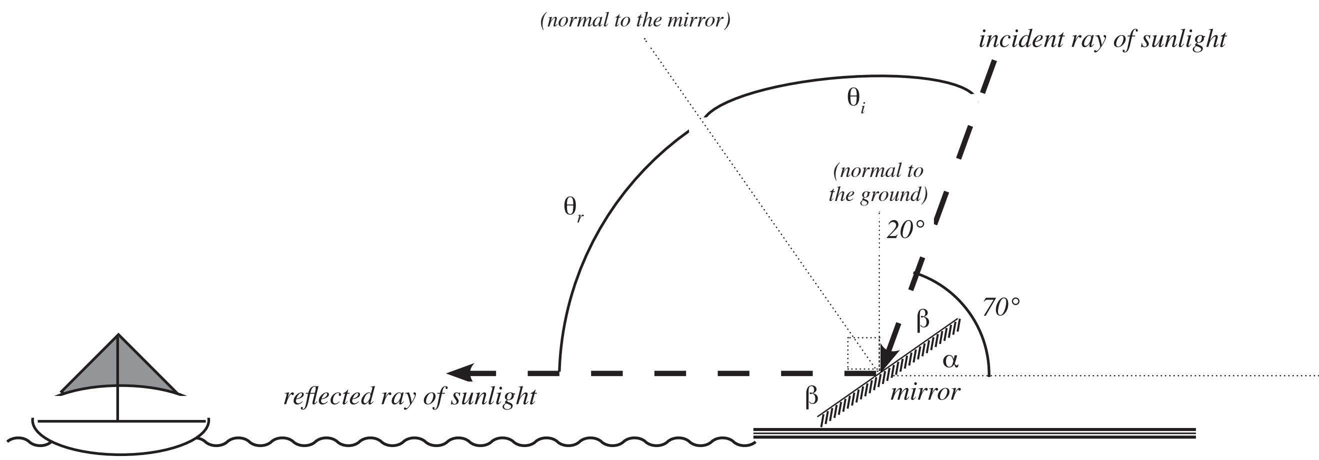
The numbers: $h_o = [(20{,}000)(0.100 \times 10^{-3})]/[(-13.333)(0.0192)] =$ **(–)7.81 m**

(Yes, it's upside down, but at least you can see that it's a boat—at least a decent-sized yacht, apparently: it rides about 25 feet out of the water.)

12. e. As you observe the boat, it's not moving toward you. Alas. Hmmm.... The sun is shining down on you, making an angle of 70° with the level ground—coming from slightly behind you (over your shoulder) as you face out to sea toward the boat. At what angle with respect to the ground should you hold the mirror to signal the boat?

You want the sunlight (angled as shown) to be incident on the mirror so that the reflected ray is parallel to the ground.

The key items are the normals to both the ground and the mirror--here's the complete picture:
The question is: What is the measure of the angle α?



As with all reflections, $\theta_i = \theta_r$.

And, using the plane of the level ground as 180°: $\theta_i + \theta_r + 70 = 180$

Solving these, we get: $\theta_i = \theta_r = 55°$

Then, note that $\beta + \theta_i = \beta + \theta_r = 90°$, so $\beta = 35°$.

But $\beta + \alpha = 70°$, so $\alpha = 35°$, also.

You should hold the plane of the mirror at an angle of 35° as measured from the ground.