CS 321: Assignment 6

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November 14, 2016

1. Answer

(a) Base Case:
$$w = \epsilon$$

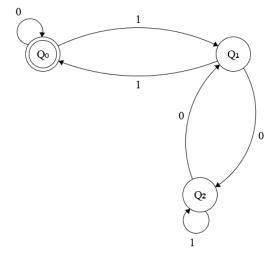
 $\delta^*(s, w) = \delta^*(s, \epsilon) = q$
 $A_s \to^* wA_q, A_S \to cA_q, A_s \to \epsilon A_q$

Inductive Step:

$$\begin{aligned} & \text{Let } w = xb \\ & \delta(\delta^*(s,x),b) = q \\ & \text{Let p} = \delta^*(s,x) \\ & \delta(p,b) = q \\ & A_s \to xA_p, A_p \to bA_q \\ & A_s \to^* xbA_q, A_s \to^* wA_q \end{aligned}$$

$$\begin{split} & \operatorname{L}(\operatorname{DFA}) = \operatorname{L}(\operatorname{CFL}) \\ & L(DFA) = \{w | \delta^*(s, w) \in F\} \\ & L(CFL) = \{w | A_s \rightarrow^* w\} \end{split}$$

An accepted string w must end in a terminal which means $w \in F$ as per the production rules of the given CFG.



(b)

Derivation of CFG:

- Starting nonterminal A_s
- $A_s \to 0 A_s |1A_1|\epsilon$
- $\bullet \ A_1 \to 1A_s | 0A_2$
- $\bullet \ A_2 \to 1A_2 | 0A_1$
- $\begin{array}{ccc} 2. & S \rightarrow aSddd|T \\ & T \rightarrow bTdd|R \end{array}$

 - $R \to c R |\epsilon$
 - (a) Eliminate the start symbol from right-hand sides
 - $S_0 \to S$
 - $S \rightarrow aSddd|T$
 - $T \rightarrow bTdd|\dot{R}$
 - $R \to c R |\epsilon$
 - (b) TERM: Eliminate rules with nonsolitary terminals

$$S_0 \to S$$

$$S_0 \rightarrow S \\ S \rightarrow S_1 S S_2 S_3 S_4 | T \\ T \rightarrow T_1 T T_2 T_3 | R$$

$$T \rightarrow T_1 T T_2 T_3 | R$$

$$R \to R_1 R | \epsilon$$

$$S_1 \to a$$

$$S_2 \rightarrow a$$

$$S_{1} \rightarrow d$$

$$S_{2} \rightarrow d$$

$$S_{3} \rightarrow d$$

$$S_{4} \rightarrow d$$

$$S_4 \rightarrow d$$

$$T_1 \rightarrow b$$

$$T_1 \to b$$

$$T_2 \to d$$

$$T_3 \to d$$

$$T_3 \to d$$

$$R_1 \to c$$

(c) BIN: Eliminate right-hand sides with more than 2 nonterminal

$$S_5 \rightarrow S_1 S$$

$$S_6 \rightarrow S_2S_3$$

$$S_7 \rightarrow S_5 S_6$$

$$S_5 \rightarrow S_1 S$$

$$S_6 \rightarrow S_2 S_3$$

$$S_7 \rightarrow S_5 S_6$$

$$S \rightarrow S_7 S_4 | T$$

$$S_1 \rightarrow a$$

$$S_1 \to a$$

$$S_2 \to d$$

$$S_3 \to d$$

$$S_4 \to d$$

$$S_2 \rightarrow c$$

$$S_A \to d$$

$$T_4 \rightarrow T_1 T$$

$$T_5 \rightarrow T_2 T_2$$

$$T_4 \to T_1 T$$

$$T_5 \to T_2 T_3$$

$$T \to T_4 T_5 | R$$

$$T_1 \to b$$

$$T_2 \to d$$
 $T_3 \to d$

$$T_3 \rightarrow d$$

$$R \to R_1 R | \epsilon$$

$$R_1 \to c$$

(d) **DEL:** Eliminate ϵ -rules

$$S_5 \rightarrow S_1 S$$

$$S_6 \rightarrow S_2 S_3$$

$$S_5 \rightarrow S_1 S$$

$$S_6 \rightarrow S_2 S_3$$

$$S_7 \rightarrow S_5 S_6$$

$$S \rightarrow S_7 S_4 | T$$

$$S_1 \to \epsilon$$

$$S_2 \rightarrow d$$

$$S_3 \rightarrow \epsilon$$

$$S_1 \to a$$

$$S_2 \to d$$

$$S_3 \to d$$

$$S_4 \to d$$

$$T_4 \rightarrow T_1 T$$

$$T_5 \rightarrow T_2 T_3$$

$$T_4 \rightarrow T_1 T$$

$$T_5 \rightarrow T_2 T_3$$

$$T \rightarrow T_4 T_5 | R$$

$$T_* \rightarrow b$$

$$T_1 \to b$$

$$T_2 \to d$$

$$T_3 \to d$$

$$T_3 \to d$$

$$R \to R_1 R | R_1$$

$$R_1 \to c$$

(e) UNIT: Eliminate unit rules

No unit rules

(f) Final Answer

$$S_5 \rightarrow S_1 S$$

$$S_6 \rightarrow S_2S_3$$

$$S_7 \rightarrow S_5 S_6$$

$$S_6 \rightarrow S_2 S_3$$

$$S_7 \rightarrow S_5 S_6$$

$$S \rightarrow S_7 S_4 | T$$

$$S_1 \rightarrow \epsilon$$

$$S_2 \rightarrow a$$

$$S_1 \to a$$

$$S_2 \to d$$

$$S_3 \to d$$

$$S_4 \to d$$

$$S_4 \to d$$

$$T_4 \rightarrow T_1 T$$

$$T_5 \rightarrow T_2 T_3$$

$$T_5 \rightarrow T_2 T_3$$

 $T \rightarrow T_4 T_5 | R$

$$T_{\cdot} \rightarrow l$$

$$T_1 \to b$$

$$T_2 \to d$$

$$T_3 \to d$$

$$T_3 \to d$$

$$R \to R_1 R | R_1$$

$$R_1 \to c$$

3. Answer

- (a) This language has the following cases:
 - $\bullet \ k=m, k\neq n$
 - $\bullet \ m=n, n\neq k$
 - $k = n, n \neq m$
 - $k \neq m \neq n$

(b)
$$S_0 \to SS_2$$

 $S \to aSa|bSb|cS_2$
 $S_2 \to aS_2|bS_2|\epsilon$

$$S_2 \to aS_2|bS_2|\epsilon$$