

Filters and Seismometers

(after F. Scherbaums book “Of Poles and Zeroes”)

Motivation

From source to receiver...

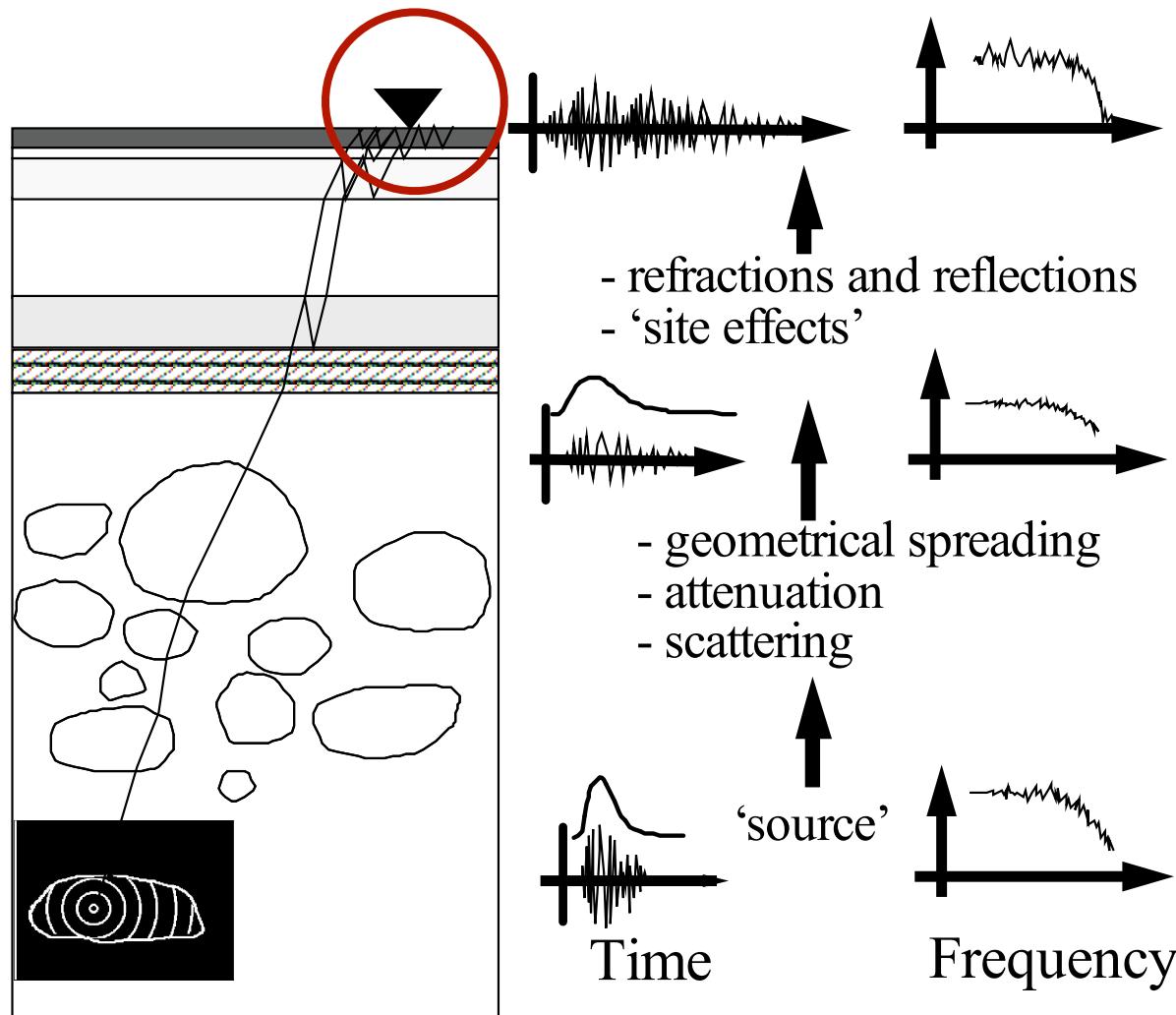


Fig. 1.1 Signal distortion during wave propagation from the earthquake source to the surface.

Influence of recording system

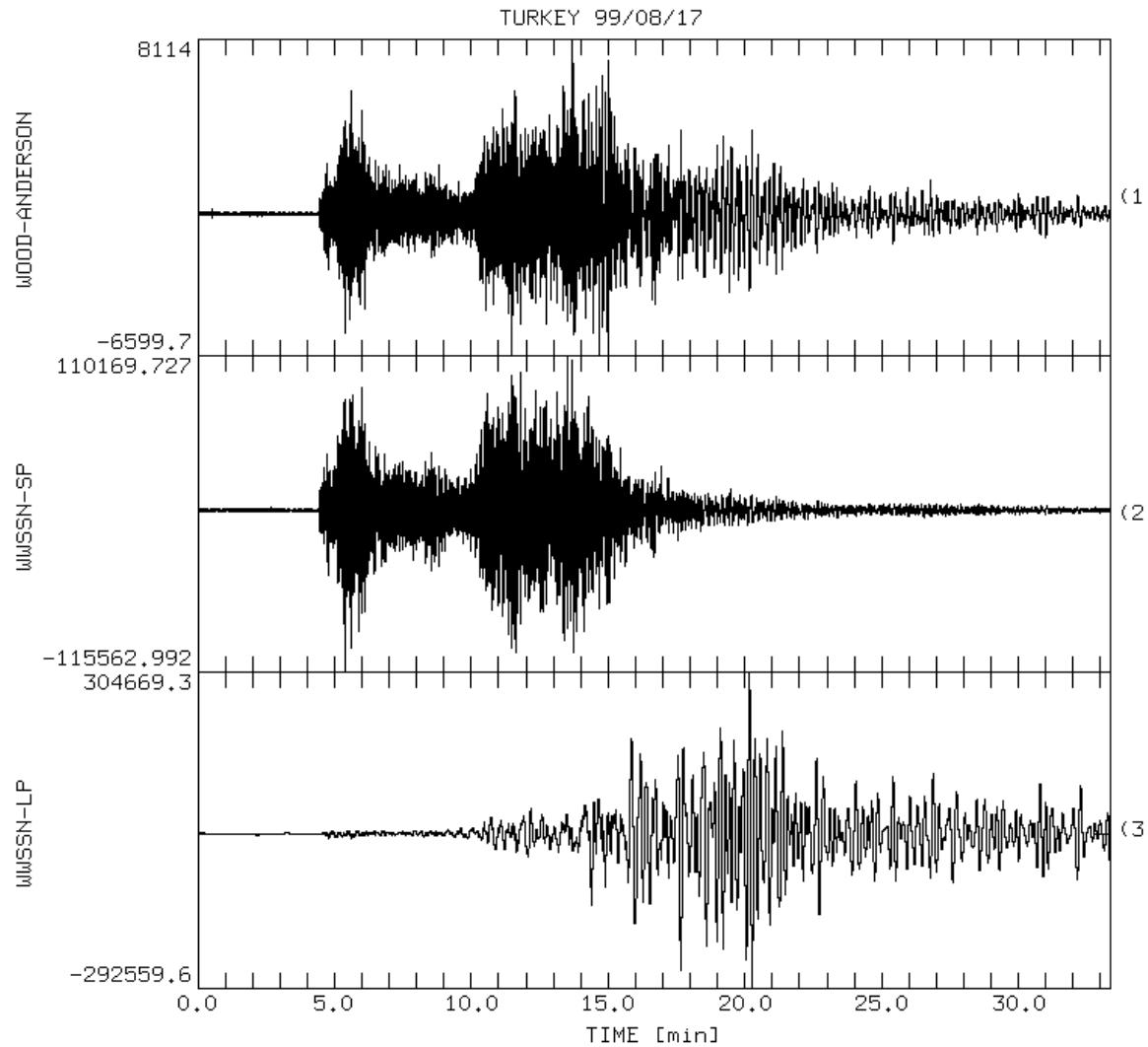


Fig. 1.2 Vertical component record of the Izmit earthquake in Turkey (1999/08/17) recorded at station MA13 of the University of Potsdam during a field experiment in Northern Norway. Shown from top to bottom are the vertical component records for a: Wood-Anderson, a WWSSN SP, and a WWSSN LP instrument simulation.

Displacement spectrum

spectral plateau

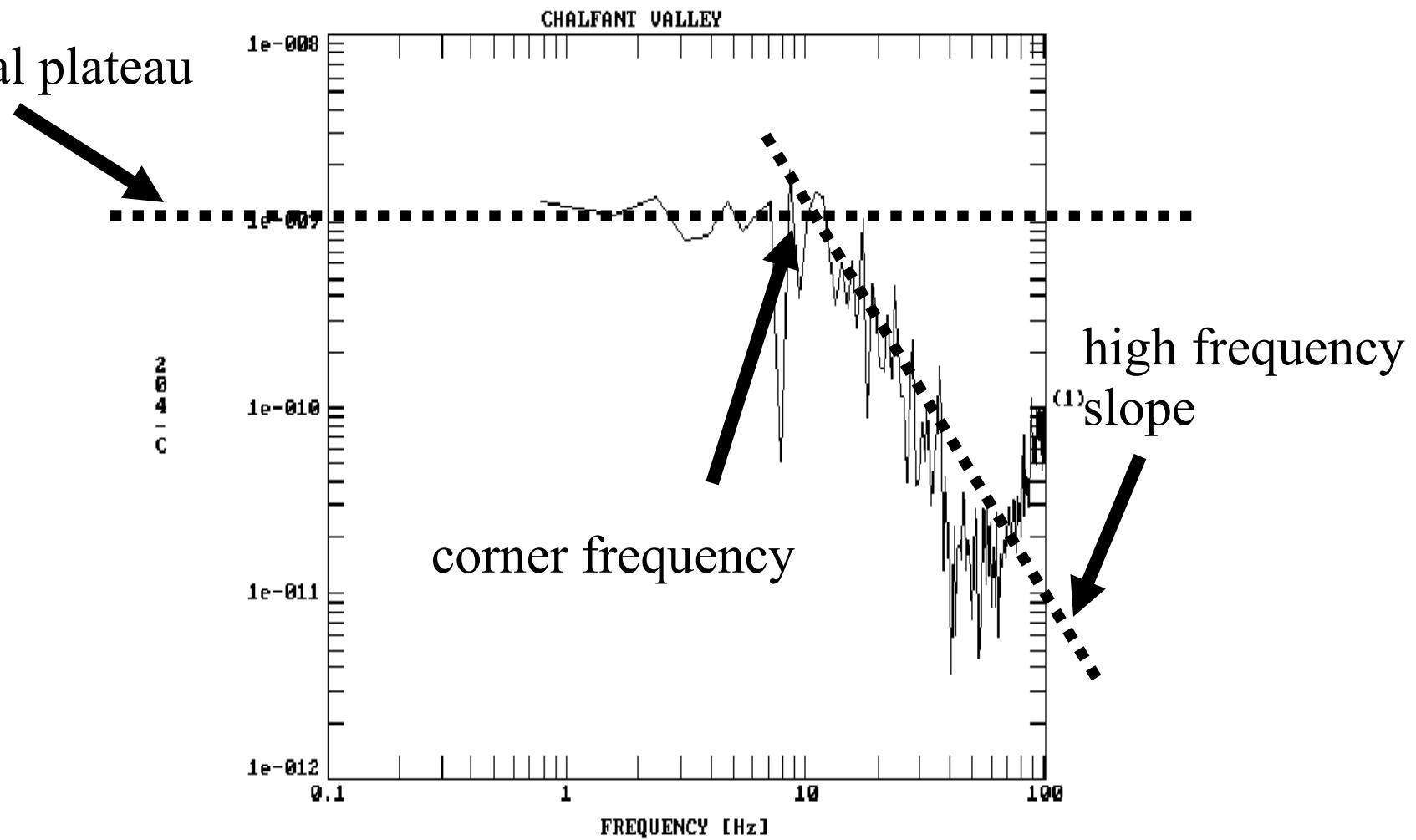


Fig. 1.8 Displacement spectrum for the P- wave portion of the instrument corrected displacement record of station 204 (top trace in Fig. 1.7).

Seismogram

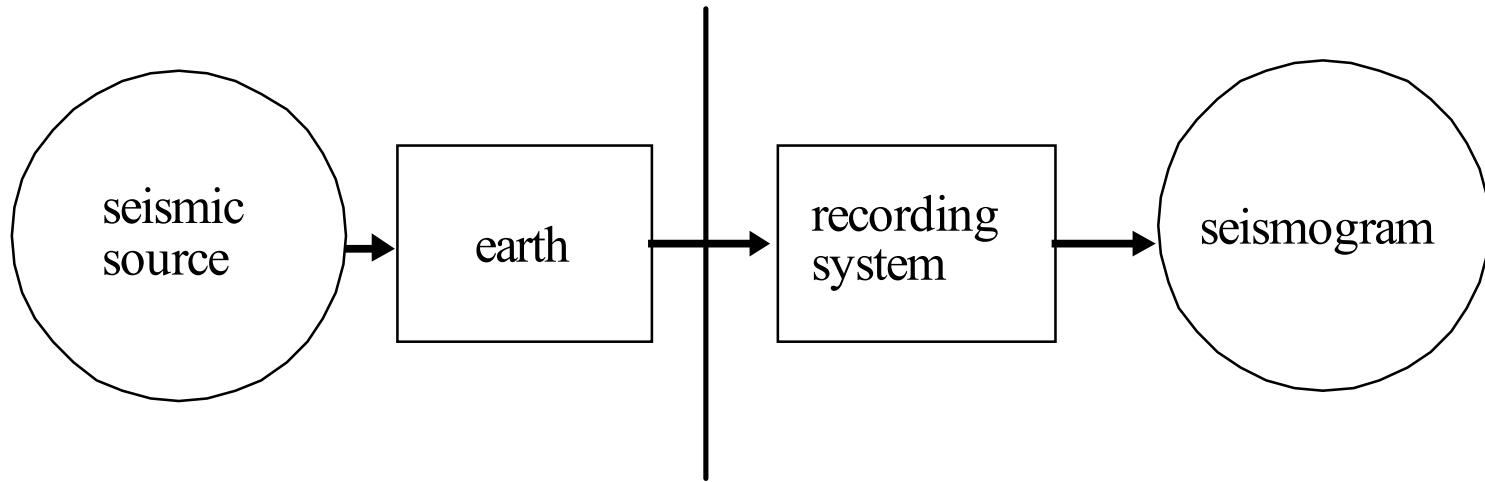
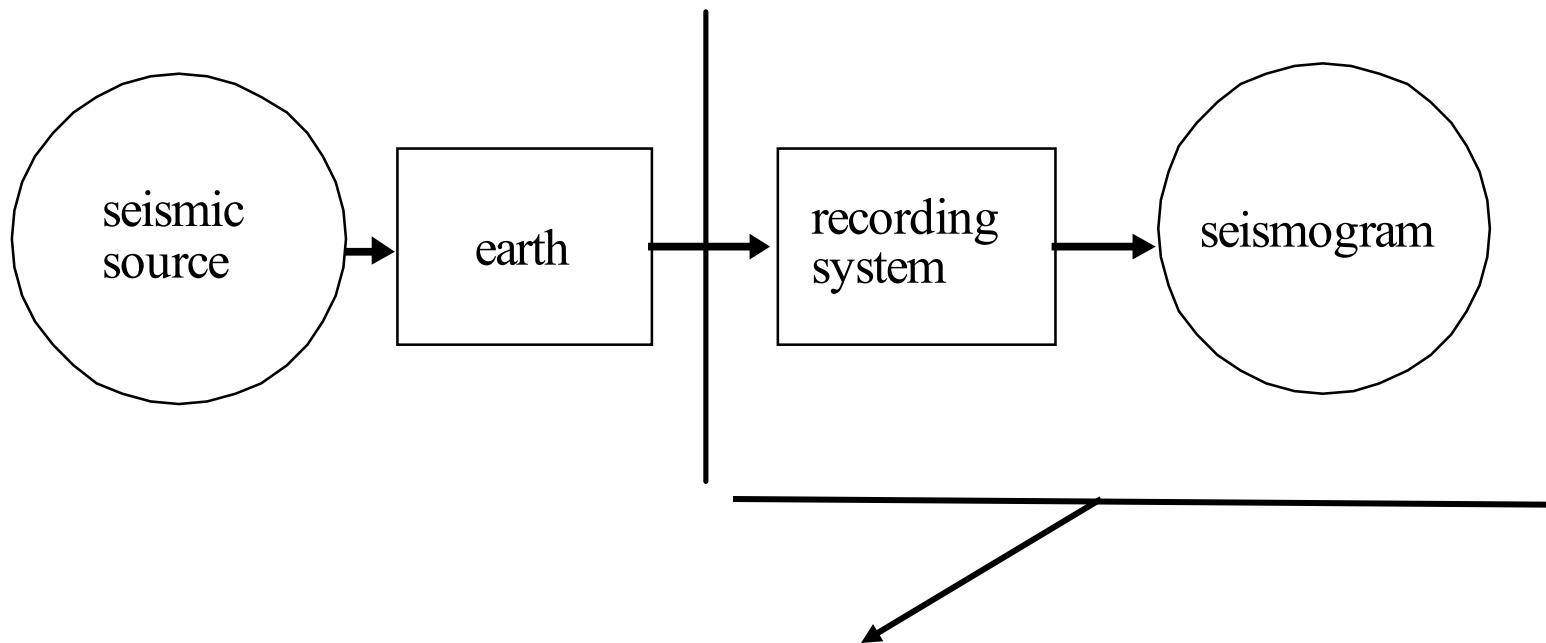
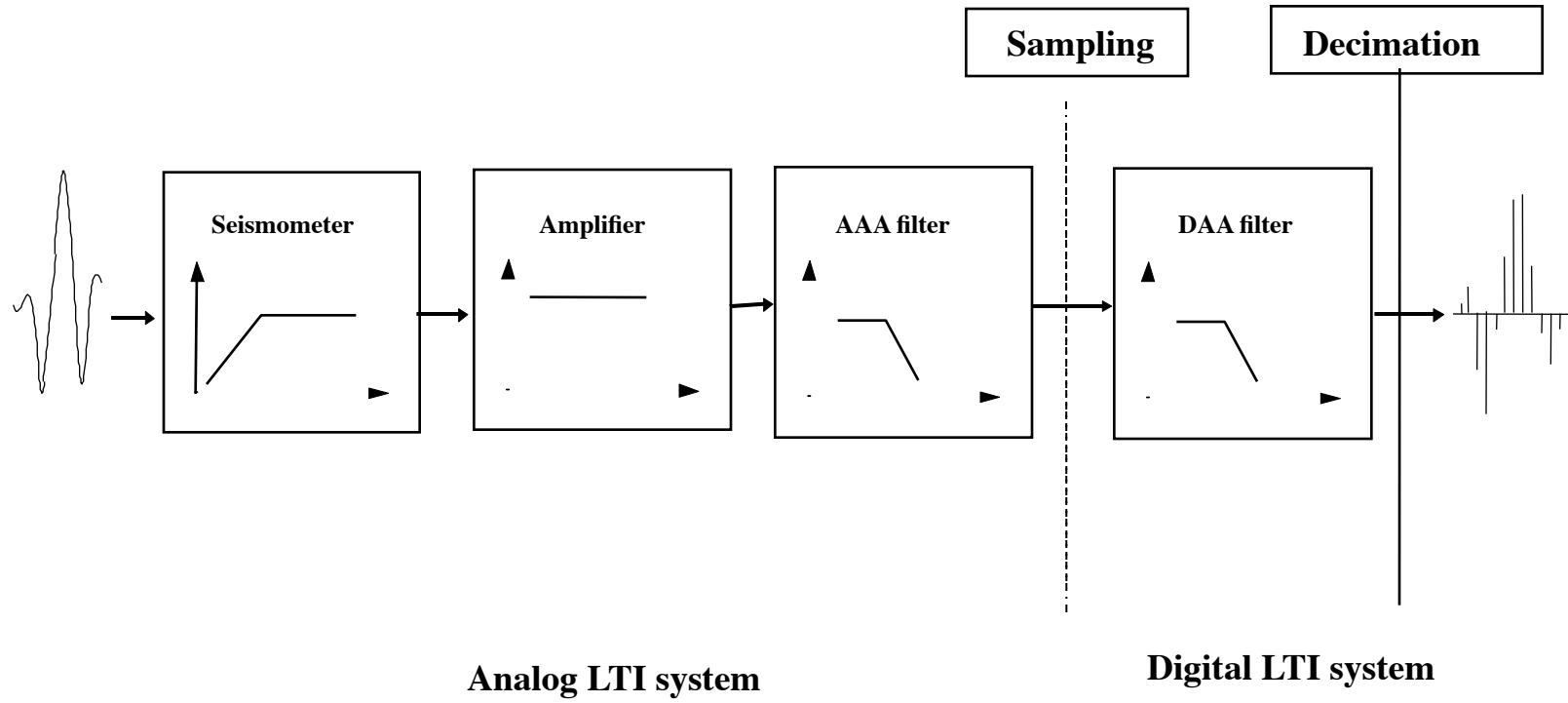


Fig. 1.11 System diagram of a seismogram

Seismogram



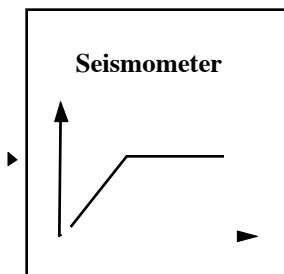
This part is covered by stationXML



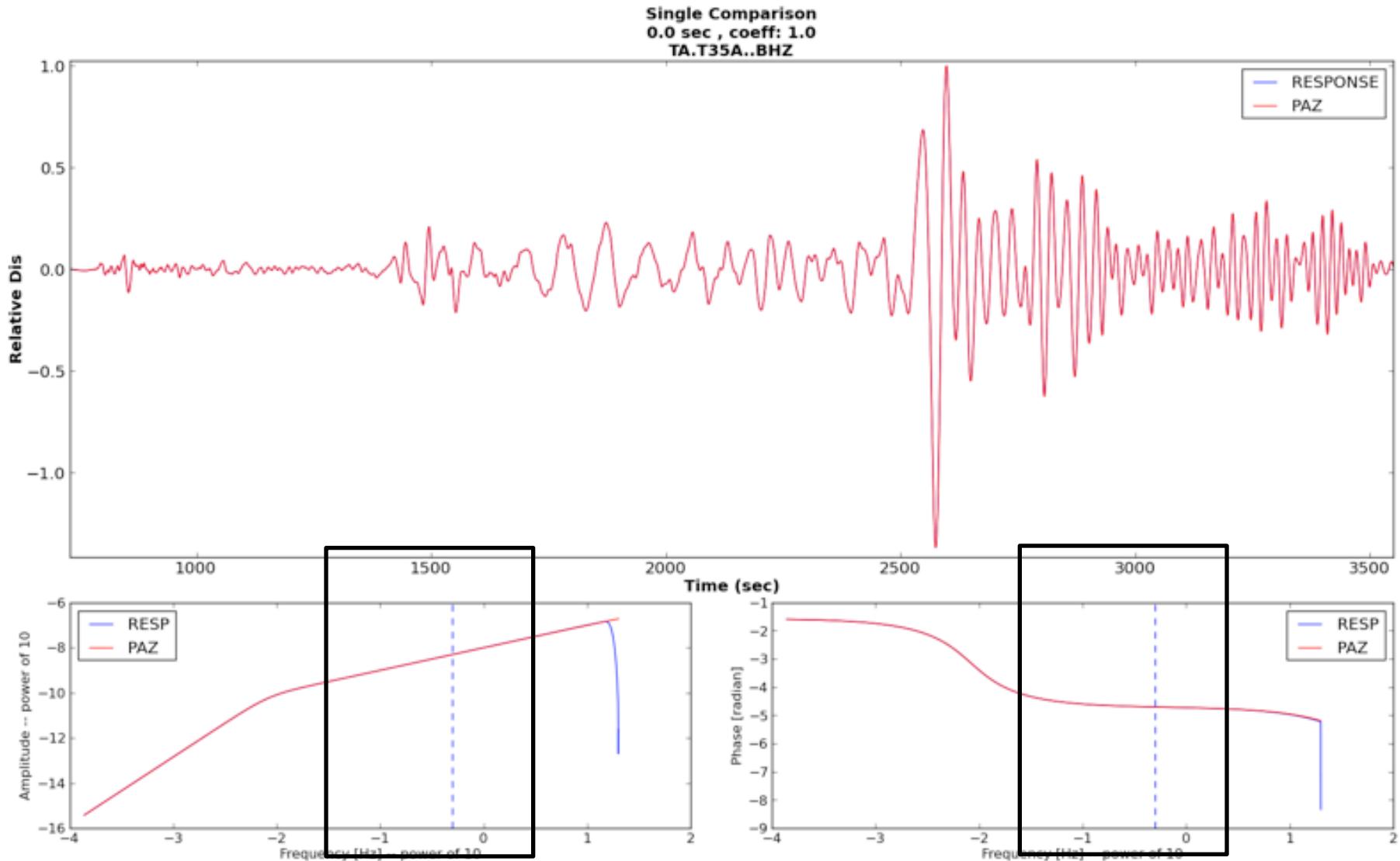
```

#      << IRIS SEED Reader, Release 4.4 >>
#
#      ===== CHANNEL RESPONSE DATA =====
B050F03 Station: RJOB
B050F16 Network: BW
B052F03 Location: ?
B052F04 Channel: EHZ
B052F22 Start date: 2007,199
B052F23 End date: No Ending Time
#
#      =====
#      +      +-----+      +
#      +      | Response (Poles & Zeros), RJOB ch EHZ |      +
#      +      +-----+      +
#
#      Transfer function type:          A [Laplace Transform (Rad/sec)]
B053F03 Stage sequence number:      1
B053F05 Response in units lookup:  M/S - Velocity in Meters per Second
B053F06 Response out units lookup: V - Volts
B053F07 A0 normalization factor:   6.0077E+07
B053F08 Normalization frequency:  1
B053F09 Number of zeroes:         2
B053F14 Number of poles:          5
#
#      Complex zeroes:
#      i real      imag      real_error  imag_error
B053F10-13 0 0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00
B053F10-13 1 0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00
#
#      Complex poles:
#      i real      imag      real_error  imag_error
B053F15-18 0 -3.700400E-02 3.701600E-02 0.000000E+00 0.000000E+00
B053F15-18 1 -3.700400E-02 -3.701600E-02 0.000000E+00 0.000000E+00
B053F15-18 2 -2.513300E+02 0.000000E+00 0.000000E+00 0.000000E+00
B053F15-18 3 -1.310400E+02 -4.672900E+02 0.000000E+00 0.000000E+00
B053F15-18 4 -1.310400E+02 4.672900E+02 0.000000E+00 0.000000E+00

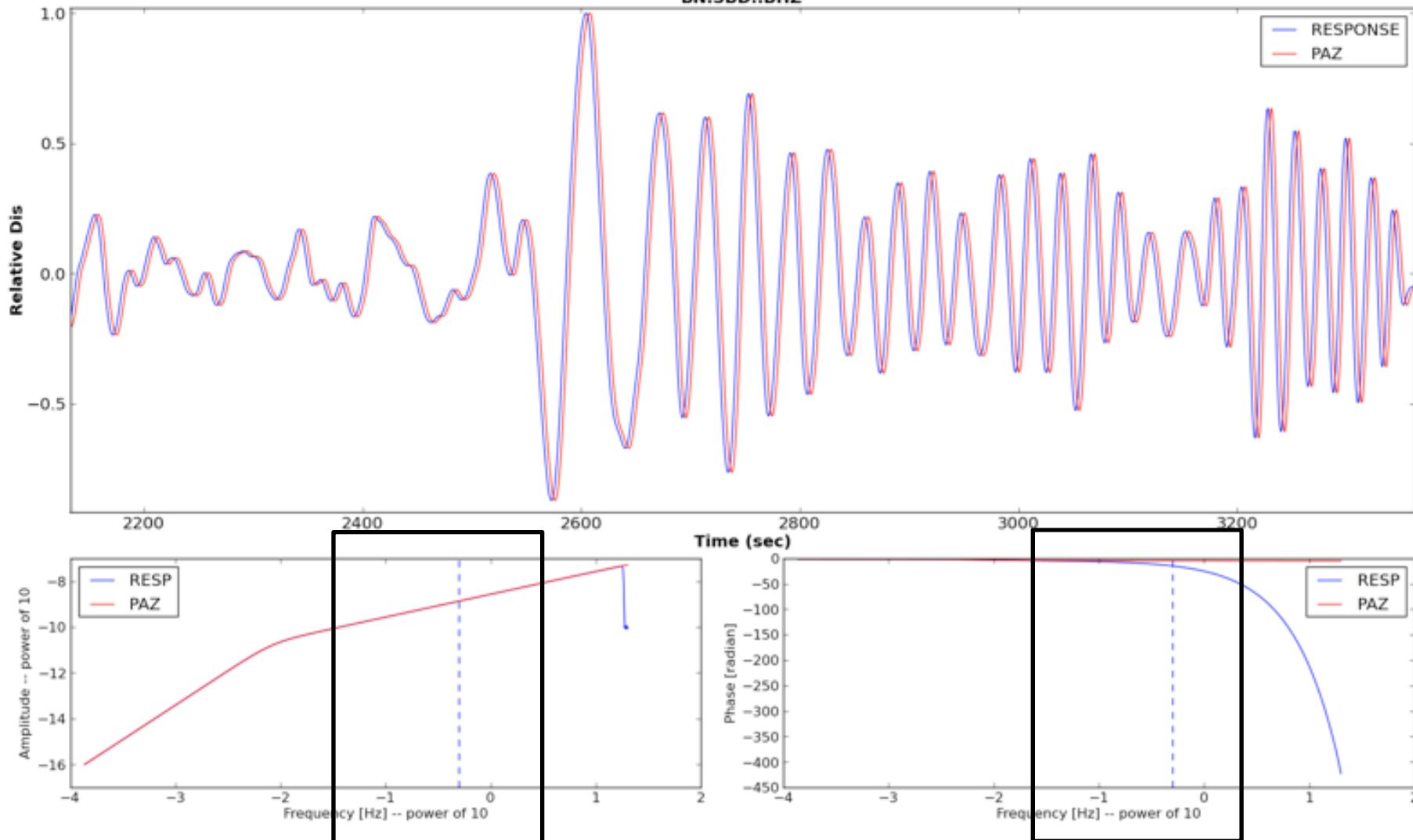
```



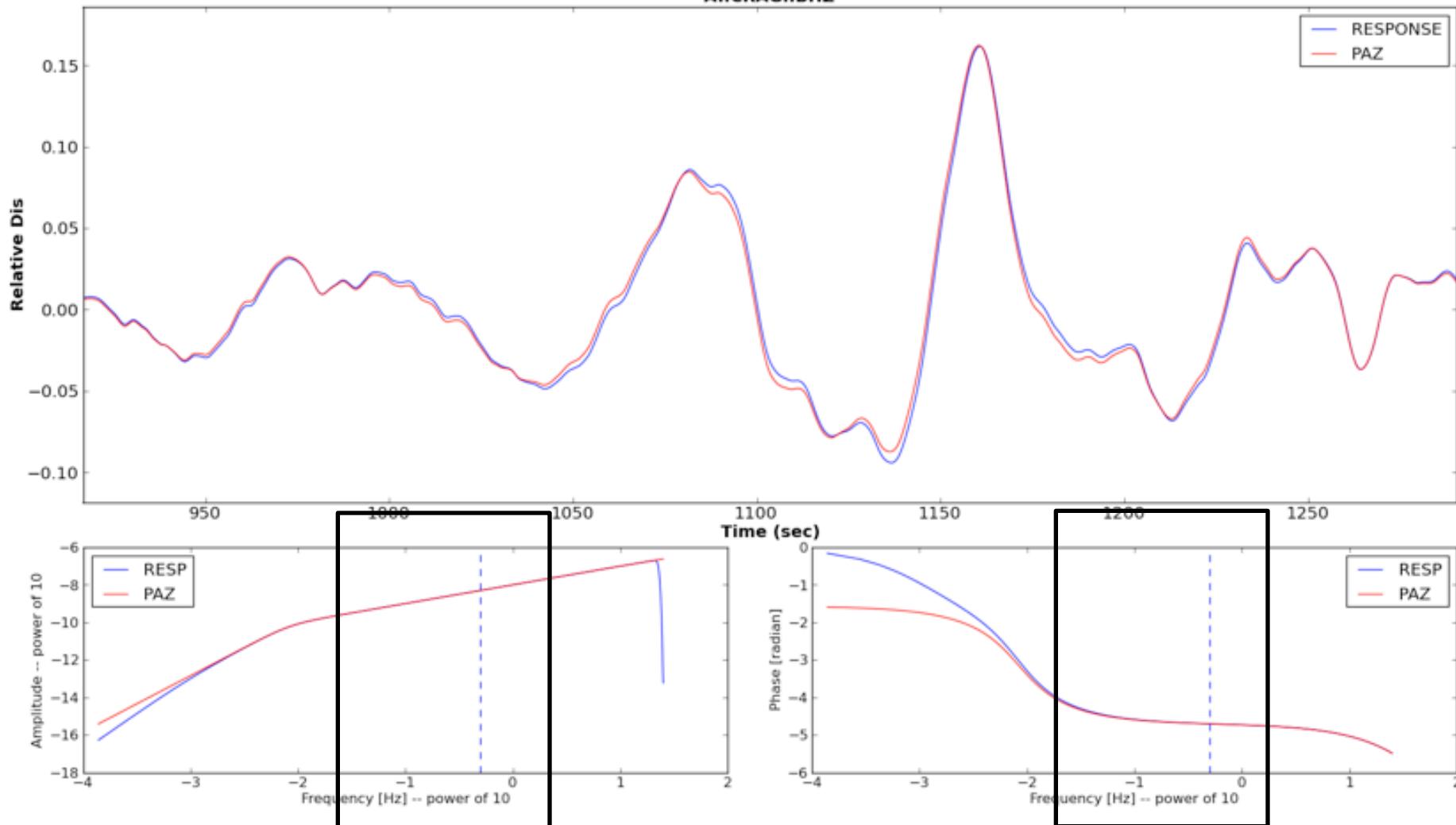
Automatic Data Retrieval Using obspyDMT



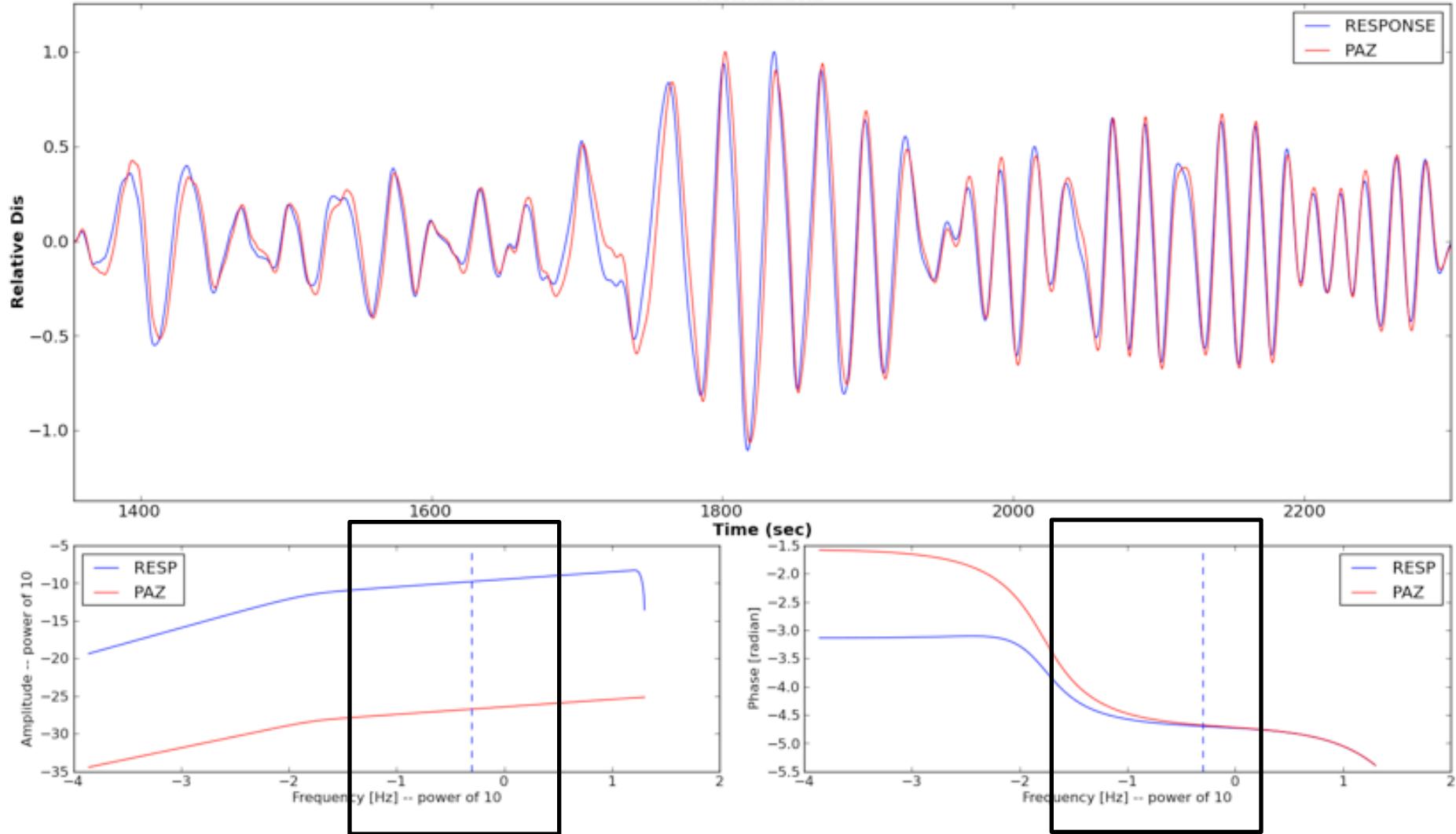
Single Comparison
-3.0 sec , coeff: 0.99858
BN.SBD..BHZ



Single Comparison
0.18 sec , coeff: 0.99941
AT.CRAG..BHZ



Single Comparison
-1.15 sec , coeff: 0.98025
AU.WR1..BHZ



Block diagram



Fig. 1.10 Block diagram of a system

Fourier Transformation

Definition:

$$\mathbf{F} \{x(t)\} = X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt$$

Common practice to write (with $\omega = 2\pi f$):

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

Back transformation:

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega$$

Eigenschaften der FT

(\Leftrightarrow indicates a transform pair, $x(t) \Leftrightarrow X(j\omega)$):

- *Time shifting* – $x(t - a) \Leftrightarrow X(j\omega) \cdot e^{-j\omega a}$

- *Derivative* – $\frac{d}{dt}x(t) \Leftrightarrow j\omega \cdot X(j\omega)$

- *Integration* – $\int_{-\infty}^{\infty} x(t)dt \Leftrightarrow \frac{1}{j\omega} \cdot X(j\omega)$

Faltung

$$g(t) = f(t) * h(t) = \int_{-\infty}^{\infty} f(\tau) \cdot h(t - \tau) d\tau$$

- Convolution Theorem —

$$f(t) * h(t) \Leftrightarrow F(j\omega) \cdot H(j\omega)$$

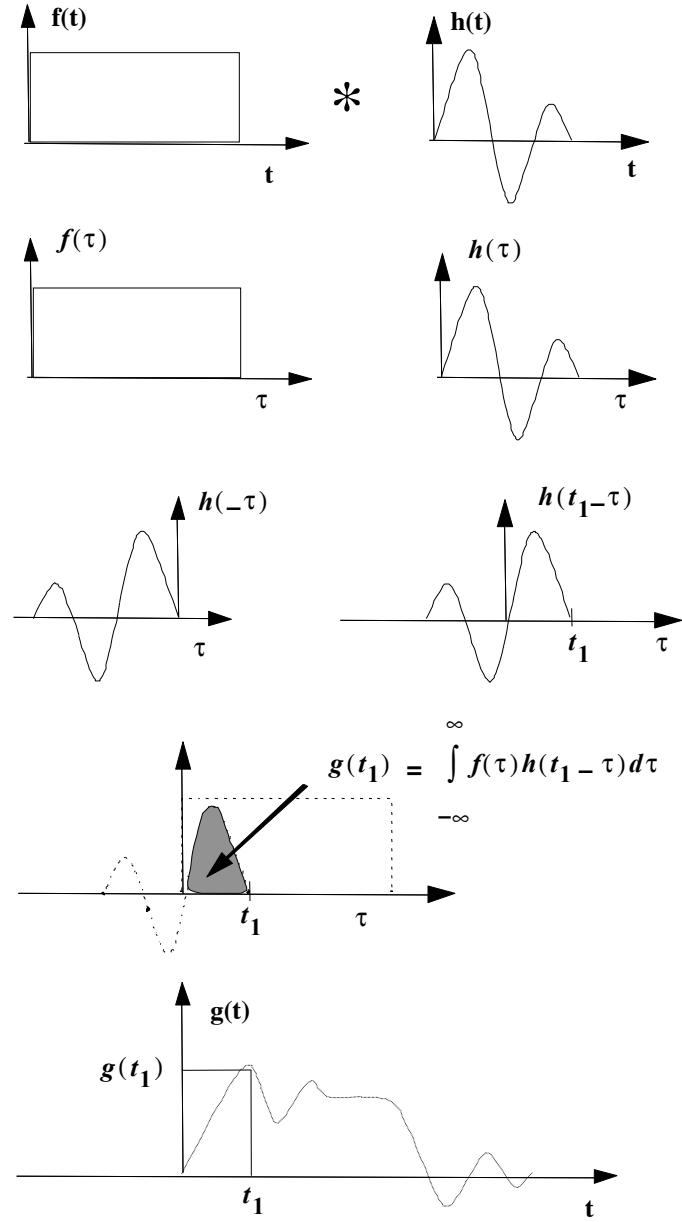


Fig. 2.5 Graphical interpretation of the convolution operation.

Harmonic input signals

$$RC\dot{y}(t) + y(t) - x(t) = 0 \quad \text{for harmonic input signal } x(t)$$

Harmonic input signal: $x(t) = A_i e^{j\omega t}$

Ansatz for the output signal: $y(t) = A_0 e^{j\omega t}$

$\dot{y}(t) = j\omega A_0 e^{j\omega t}$

$\longrightarrow A_0 e^{j\omega t} (RCj\omega + 1) = A_1 e^{j\omega t}$

$$\frac{A_o}{A_i} = \frac{1}{RCj\omega + 1} = T(j\omega)$$

Frequency response function

Input/output relation

$$A_o = T(j\omega)A_i$$

Transfer function and Laplace transform

Bilateral Laplace transform of $f(t)$: $L[f(t)] = \int f(t)e^{-st}dt$

with the complex variable $s = \sigma + j\omega$

$L[f(t)]$ will be written as $F(s)$

Property: $L[\dot{f}(t)] = sF(s)$

Transfer function and Laplace transform

Transforming equation $RC\dot{y}(t) + y(t) - x(t) = 0$

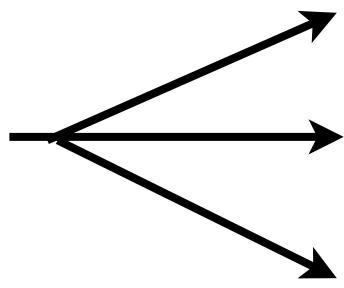
is becoming

$$RCsY(s) + Y(s) - X(s) = 0$$

with $Y(s)$ and $X(s)$ being the Laplace transforms of $y(t)$ and $x(t)$, respectively.

$$\rightarrow T(s) = \frac{Y(s)}{X(s)} = \frac{1}{1 + sRC} = \frac{1}{1 + s\tau} \quad \text{transfer function}$$

Filtering



$$y(t) = h(t) * x(t)$$

$$Y(j\omega) = H(j\omega)X(j\omega)$$

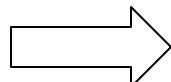
$$Y(s) = H(s)X(s)$$

General linear time invariant systems

Generalization of concepts

Rewrite the differential equation for the RC filter:

$$RC\dot{y}(t) + y(t) - x(t) = \alpha_1 \frac{d}{dt}y(t) + \alpha_0 y(t) + \beta_0 x(t) = 0$$

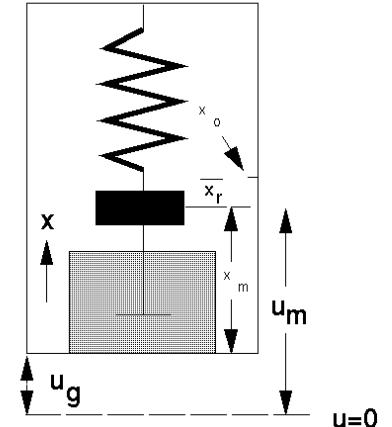
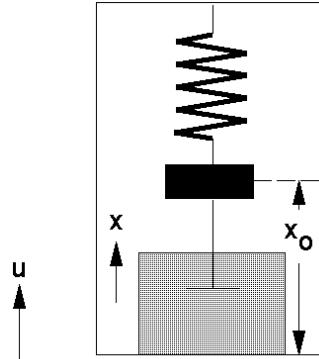


RC filter special case of
Nth order LTI system

$$\sum_{k=0}^N \alpha_k \frac{d^k}{dt^k} y(t) + \sum_{k=0}^L \beta_k \frac{d^k}{dt^k} x(t) = 0$$

WHAT IS IT GOOD FOR?????

Seismometer



- The inertia of the mass –

$$f_i = -m \ddot{u}_m(t)$$

- The spring –

$$f_{sp} = -kx_r(t)$$

k = spring constant (strength)

- The dashpot –

$$f_f = -D\dot{x}_m(t)$$

D = friction coefficient

$$-m \ddot{u}_m(t) - D\dot{x}_m(t) - kx_r(t) = 0 \quad \text{2nd order LTI !!}$$

Equilibrium – Conditions

$$-m\ddot{u}_m(t) - D\dot{x}_m(t) - kx_r(t) = 0$$

Since $u_m(t) = u_g(t) + x_m(t)$

→ $-m(\ddot{u}_g(t) + \ddot{x}_m(t)) - D\dot{x}_m(t) - kx_r(t) = 0$

With $\dot{x}_m(t) = \dot{x}_r(t)$

and $\ddot{x}_m(t) = \ddot{x}_r(t)$

→ $m\ddot{x}_r(t) + D\dot{x}_r(t) + kx_r(t) = -m\ddot{u}_g(t)$

$$\omega_0^2 = \frac{k}{m}; \quad 2h\omega_0 = \frac{D}{m}$$

$$\boxed{\ddot{x}_r(t) + 2h\omega_0\dot{x}_r(t) + \omega_0^2x_r(t) = -m\ddot{u}_g(t)}$$

Underdamped ($\omega_0 > \varepsilon$), Result:

$$x_r(t) = \frac{x_{r0}}{\cos \phi} e^{-\varepsilon t} \cos(\sqrt{\omega_0^2 - \varepsilon^2} t - \phi)$$

$$= \frac{x_{r0}}{\cos \phi} e^{-\varepsilon t} \cos(\omega t - \phi)$$

$$\phi = \arcsin \frac{\varepsilon}{\omega_0}$$

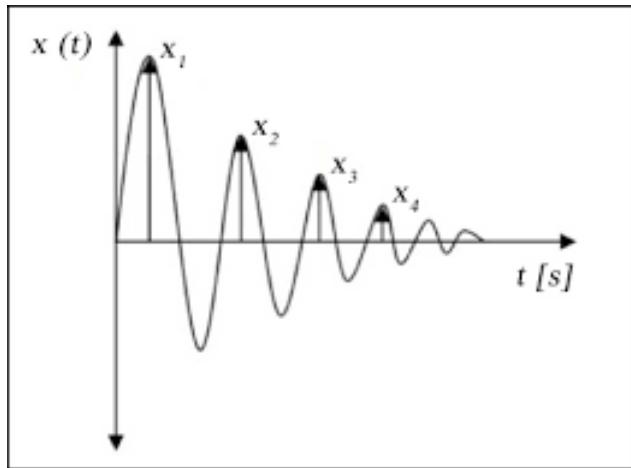
In the underdamped case ($h < 1$) the seismometer is oscillating with a period of $T = 2\pi/\omega$. This period is always larger than the undamped free periode T_0 .

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\omega_0^2 - \varepsilon^2}} = \frac{2\pi}{\omega_0 \sqrt{1 - \varepsilon^2 / \omega_0^2}} = \frac{2\pi}{\omega_0} \cdot \frac{1}{\sqrt{1 - h^2}}$$

$$= \frac{T_0}{\sqrt{1 - h^2}}$$

Most Simple Calibration

Release Test – Logarithmic Decrement



$$\ln \frac{x_n}{x_{n+1}} = \epsilon T = \Lambda$$

with

$$\Lambda = \epsilon T = \frac{\epsilon T_0}{\sqrt{1 - h^2}}$$

follows:

$$h = \frac{\Lambda}{\sqrt{4\pi^2 + \Lambda^2}}$$

Frequency-Transferfunction

$$\ddot{x}(t) + 2h\omega_0\dot{x}(t) + \omega_0^2x(t) = \ddot{u}_g(t)$$



$$-\omega^2X(j\omega) + j2h\omega_0\omega X(j\omega) + \omega_0^2X(j\omega) = \omega^2U(j\omega)$$

$$\frac{X(j\omega)}{U(j\omega)} = \frac{\text{Output}}{\text{Input}} = \frac{\omega^2}{\omega_0^2 - \omega^2 + j2\epsilon\omega} = T(j\omega)$$

mit $|T(j\omega)| = \frac{\omega^2}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\epsilon^2\omega^2}}$ Amplitude-Response

$$\Phi(\omega) = \arctan \frac{Im}{Re} = \arctan \left(\frac{-2\epsilon\omega}{\omega_0^2 - \omega^2} \right)$$
 Phase-Response

Transfer function

$$\ddot{x}(t) + 2h\omega_0\dot{x}(t) + \omega_0^2x(t) = \ddot{u}_g(t)$$

$$s^2X(s) + 2sh\omega_0X(s) + \omega_0^2X(s) = s^2U(s)$$

$$T(s) = \frac{X(s)}{U(s)} = \frac{s^2}{s^2 + 2sh\omega_0 + \omega_0^2}$$

Since a quadratic equation $x^2 + bx + c = 0$ has the roots
in the underdamped case:

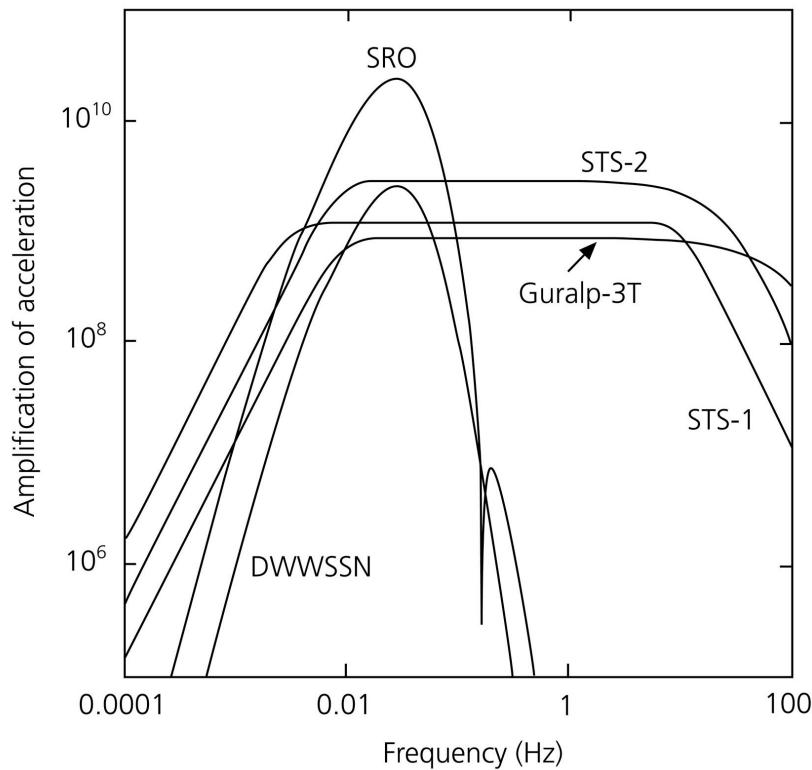
$$p_{1,2} = - (h \pm j\sqrt{h^2 - 1})\omega_0$$

Different Transferfunctions

Routine Data Processing in Earthquake Seismology
(2010) J. Havskov & L. Ottemöller, Springer

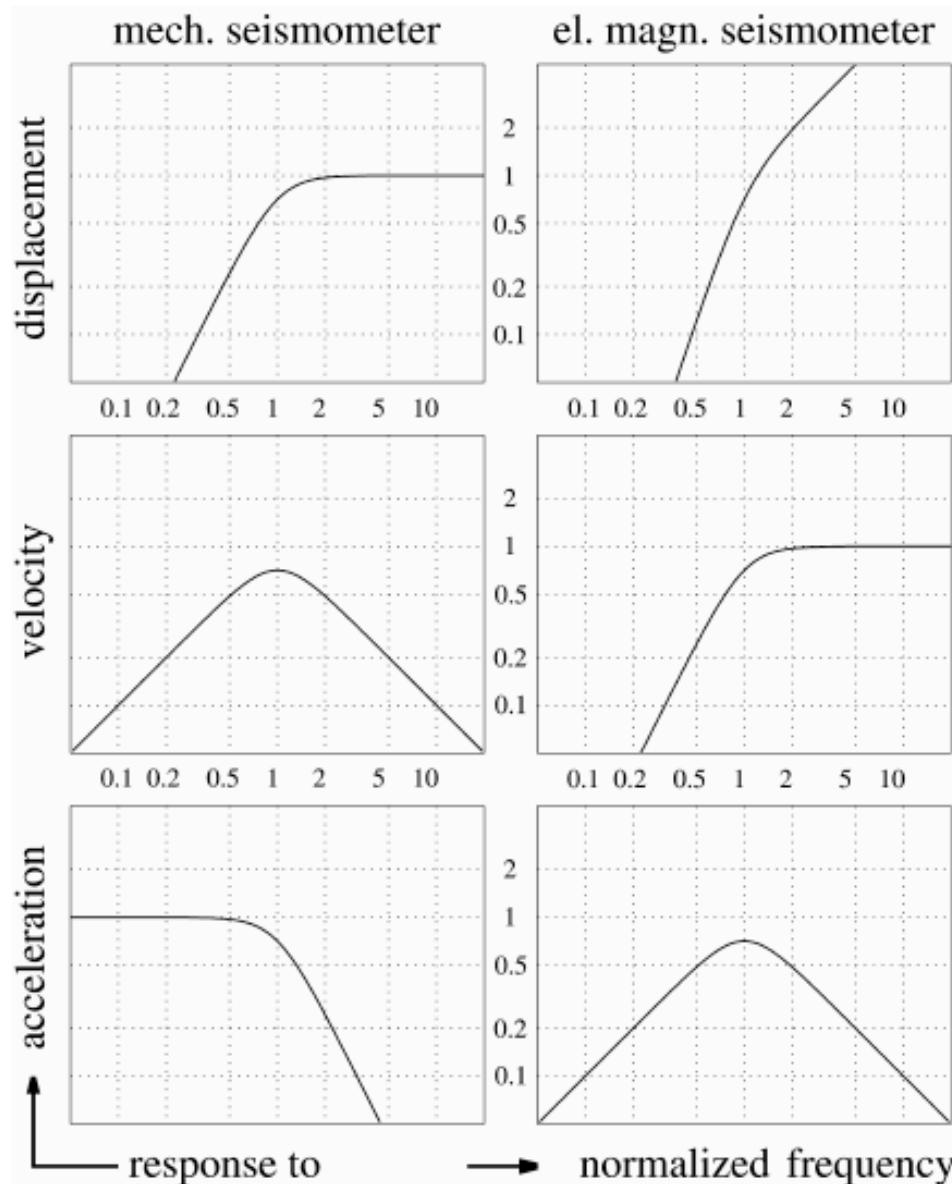
Band code	Band type	Sample rate	Corner period (s)
E	Extremely short period	≥ 80 to < 250	< 10
S	Short period	≥ 10 to < 80	< 10
H	High broadband	≥ 80 to < 250	≥ 10
B	Broad band	≥ 10 to < 80	≥ 10
M	Mid period	> 1 to < 10	
L	Long period	≈ 1	
V	Very long period	≈ 1	
U	Very long period	≈ 0.01	

Introduction to Seismology (2003) Stein & Wysession



The question to be answered
determines the seismometer to be used

Displacement, Velocity and Acceleration

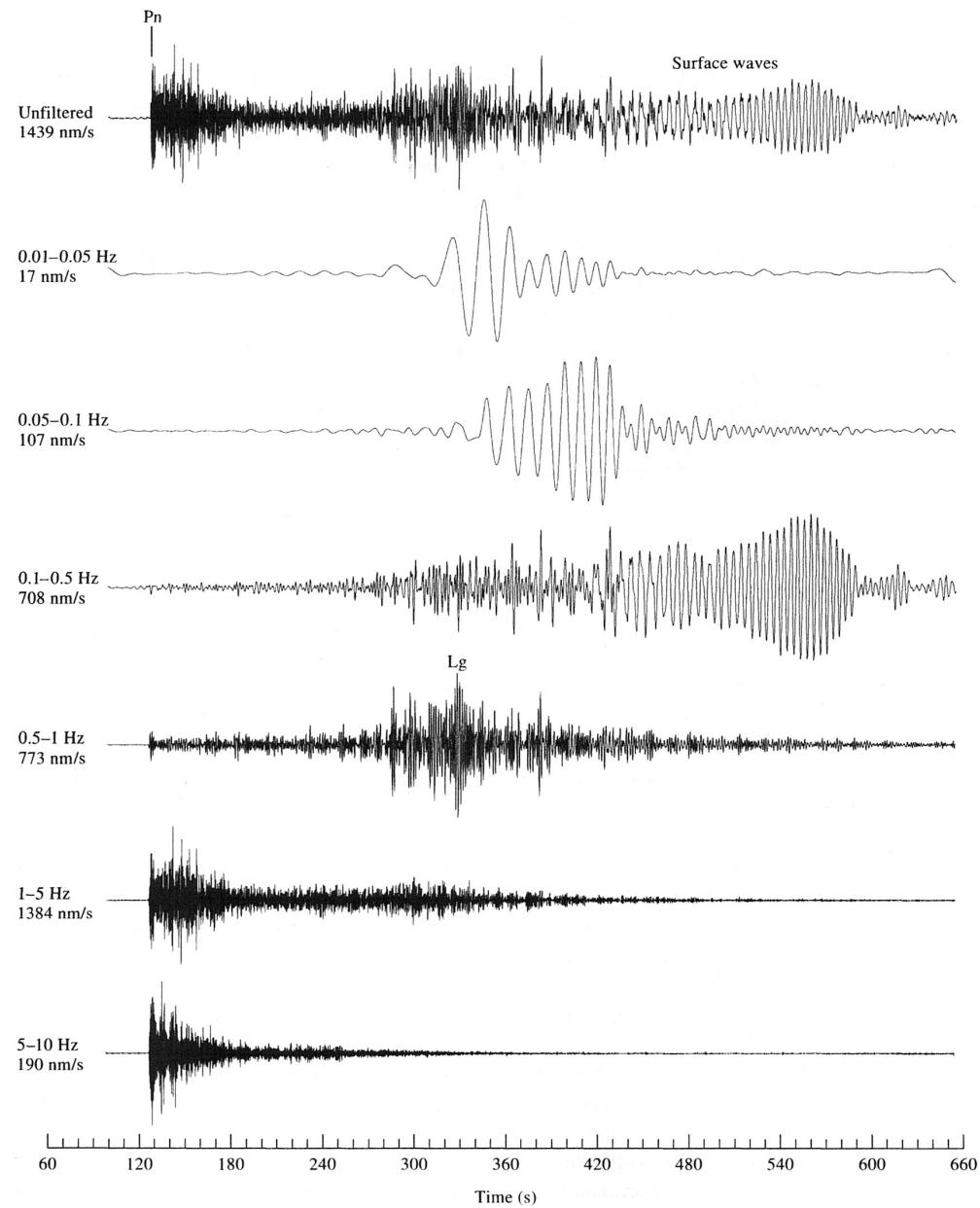
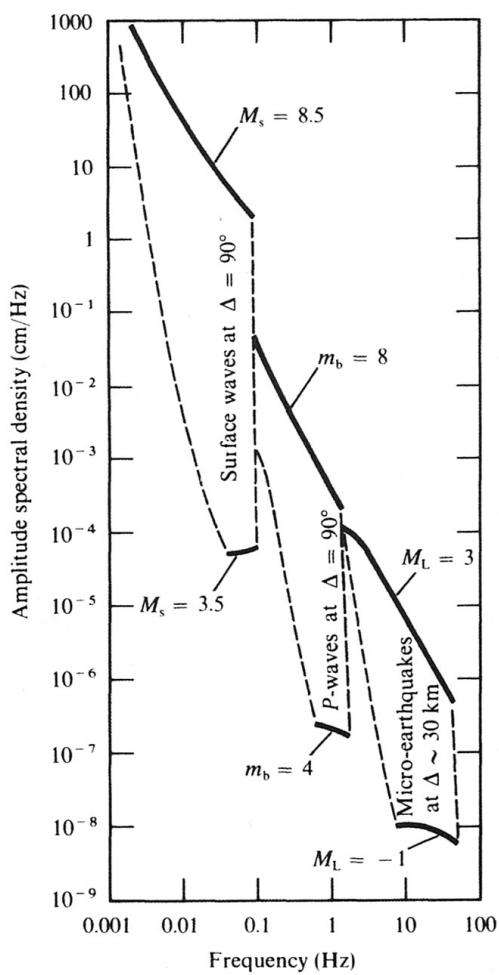


Differences in the building design causes differences in the output

Instrument classes

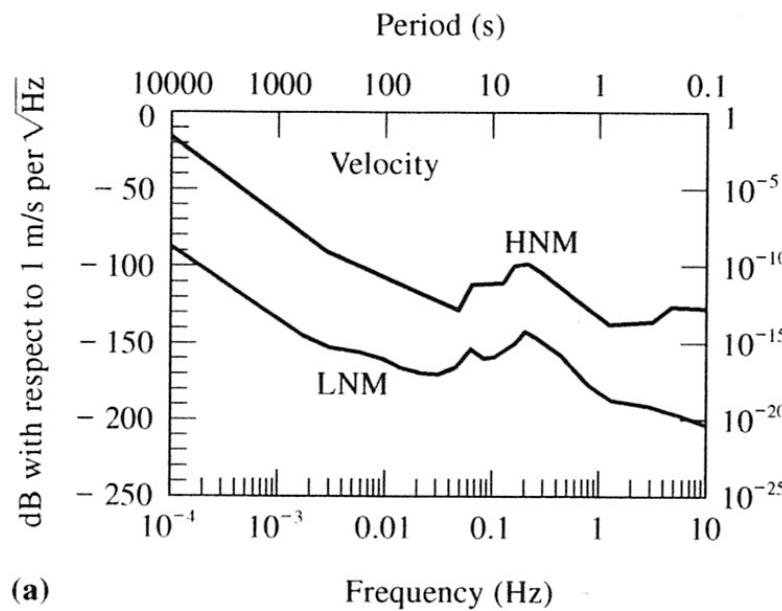
- High frequency telesismic body waves: SP-instruments (class A)
- LP body waves and telesismic surface waves: LP-instruments (class B)
- Regional body and surface waves: intermediate band (class C).
- Local magnitude: Wood-Anderson instrument.

Examples

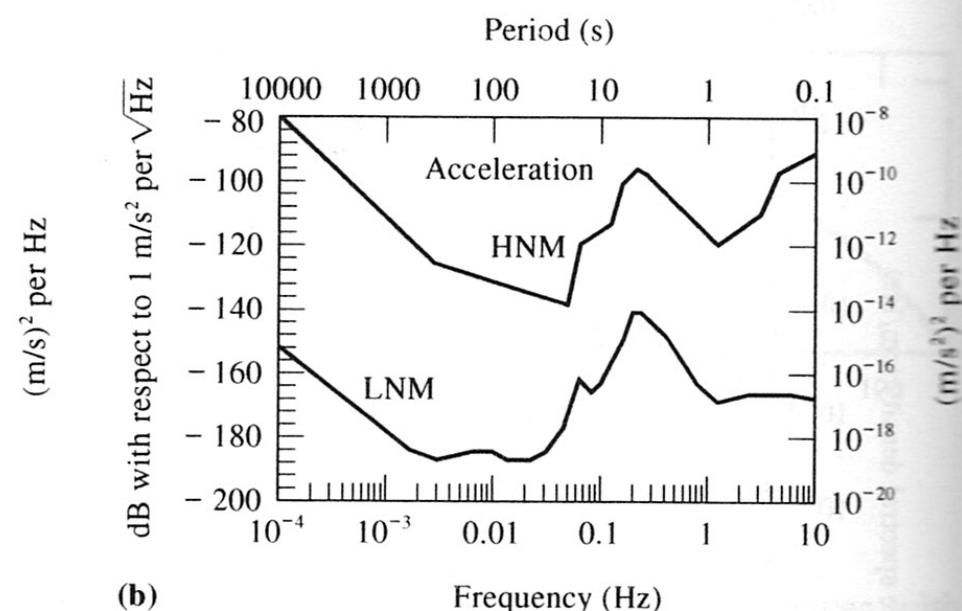


Noise

Low Noise Model / High Noise Model
Global average for comparing seismometer locations



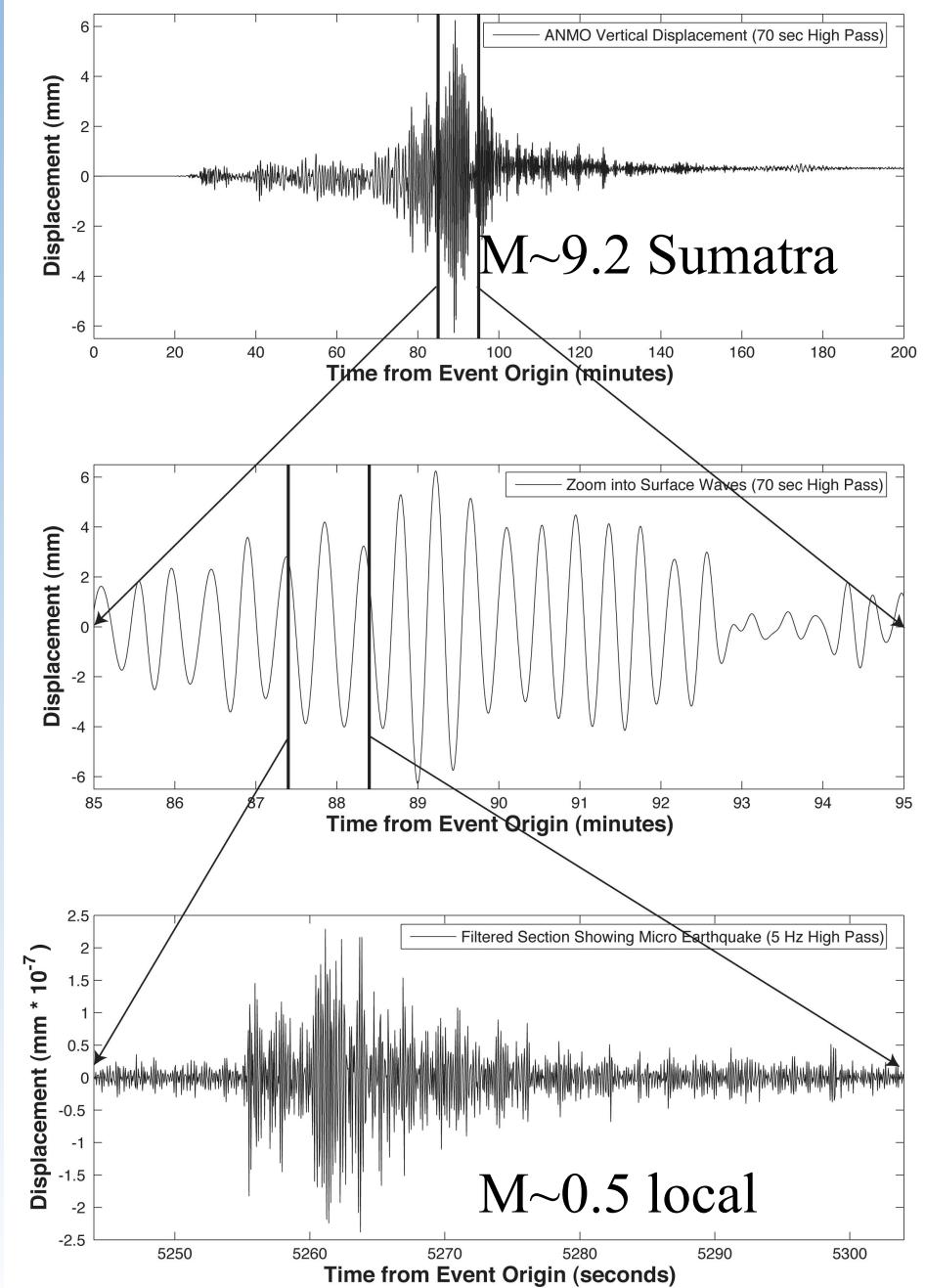
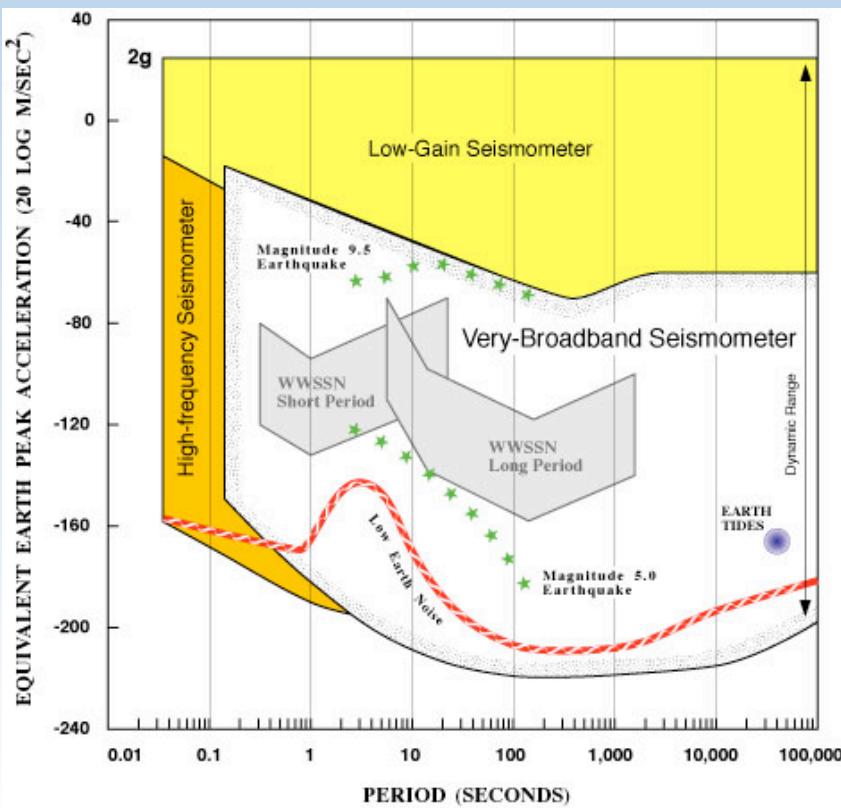
(a)



(b)

1. Microseisms - Peak: $\sim 12\text{-}20\text{ s}$
2. Microseisms - Peak : $\sim 5\text{-}10\text{ s}$

Bandwidth & Dynamic

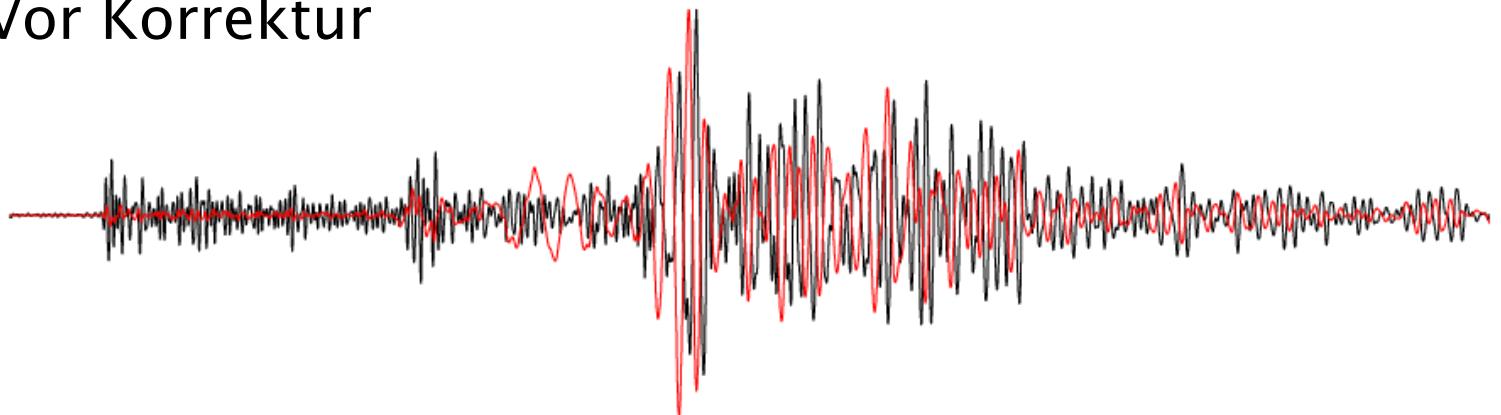


Inverse and simulation filtering of digital seismograms

NEXT: From filter problem to the simulation problem - the conversion of digital (broad-band) records into those from different seismograph systems.

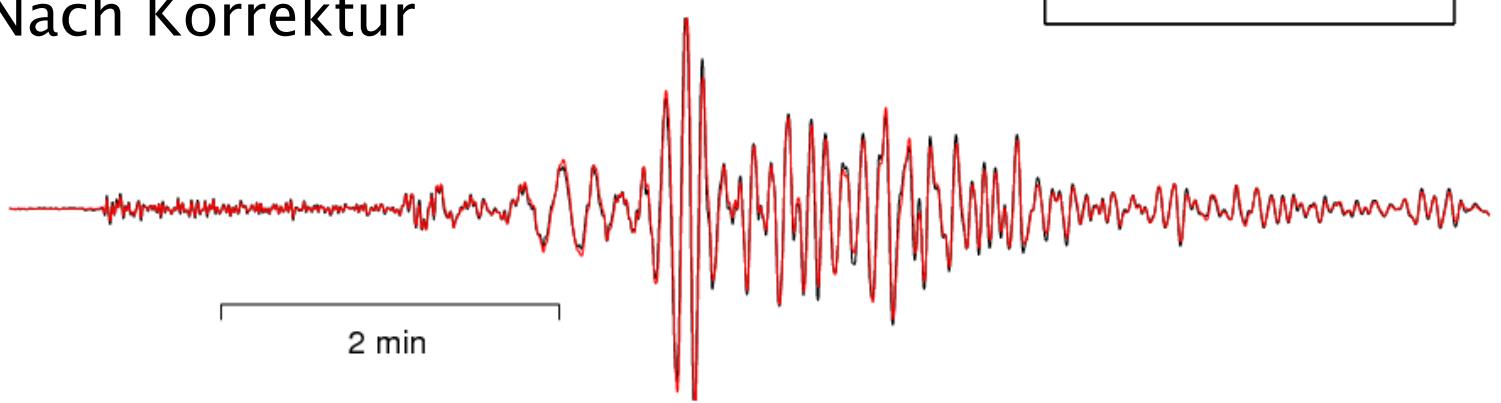
REASON: Signal amplitudes or onset time determination in a manner consistent with other observatories. Simulated systems will most commonly belong to the standard classes of instruments described by Willmore (1979) because **there is no single, optimum class of instruments for the detection and analysis of different types of seismic waves.**

Vor Korrektur



— S1
— Broadband

Nach Korrektur



Simulation = Deconvolution + Filtering

$$Y_{sim}(z) = \frac{T_{syn}(z)}{T_{act}(z)} \cdot Y_{act}(z) = T_{sim}(z) \cdot Y_{act}(z)$$

$T_{act}(z)$ = transfer function of actual recording system

$T_{syn}(z)$ = transfer function of the instrument to be synthesized

$Y_{act}(z)$ = z- transform of the recorded seismogram

$Y_{sim}(z)$ = z- transform of the simulated seismogram

The concept of instrument simulation

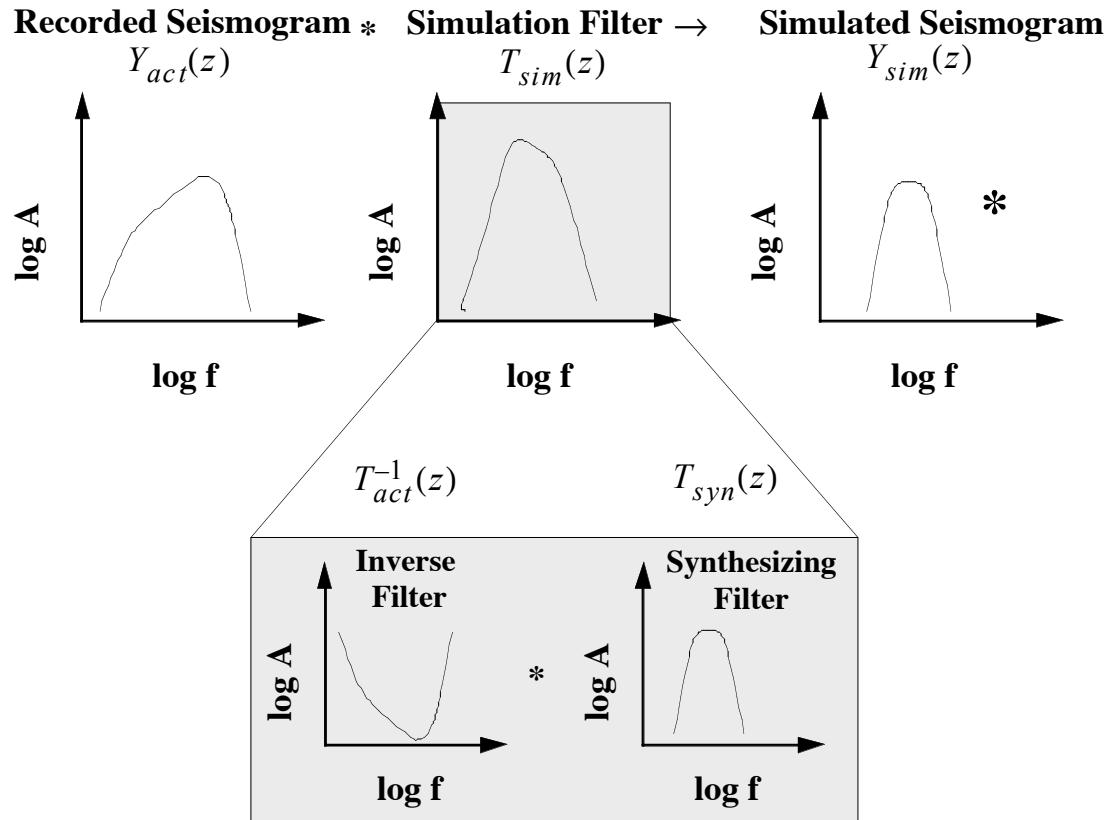


Fig. 9.2 The simulation of digital seismographs. The simulation filter can be thought of as a combination of an inverse filter for the actual recording system and a synthesizing filter for the simulated recording system. Displayed are schematic sketches of the amplitude frequency response functions of the contributing subsystems.

Stability problems

Noisefree situation

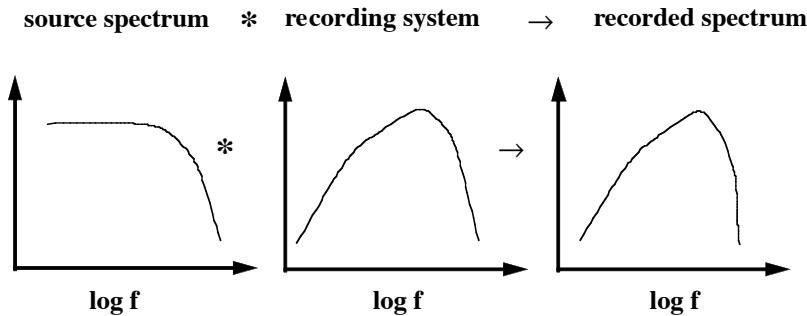


Fig. 9.3 Recording the displacement spectrum of an idealized earthquake source.

Recovery of source spectrum:

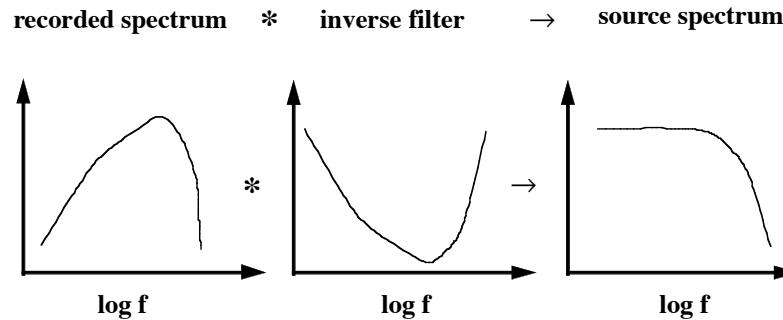


Fig. 9.4 Recovering the source spectrum by inverse filtering in the noisefree case.

Noisy situation

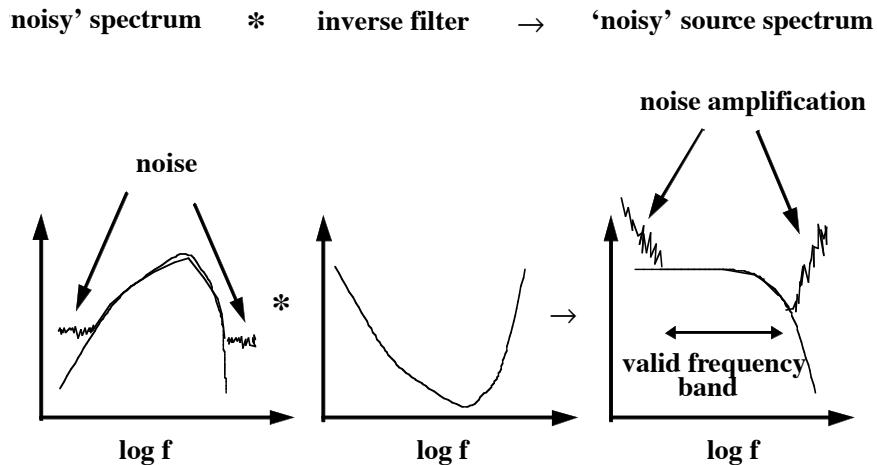


Fig. 9.5 Noise amplification by inverse filtering. The solid line in the left panel shows the signal plus noise while the noise-free signal is shown by the dashed line.

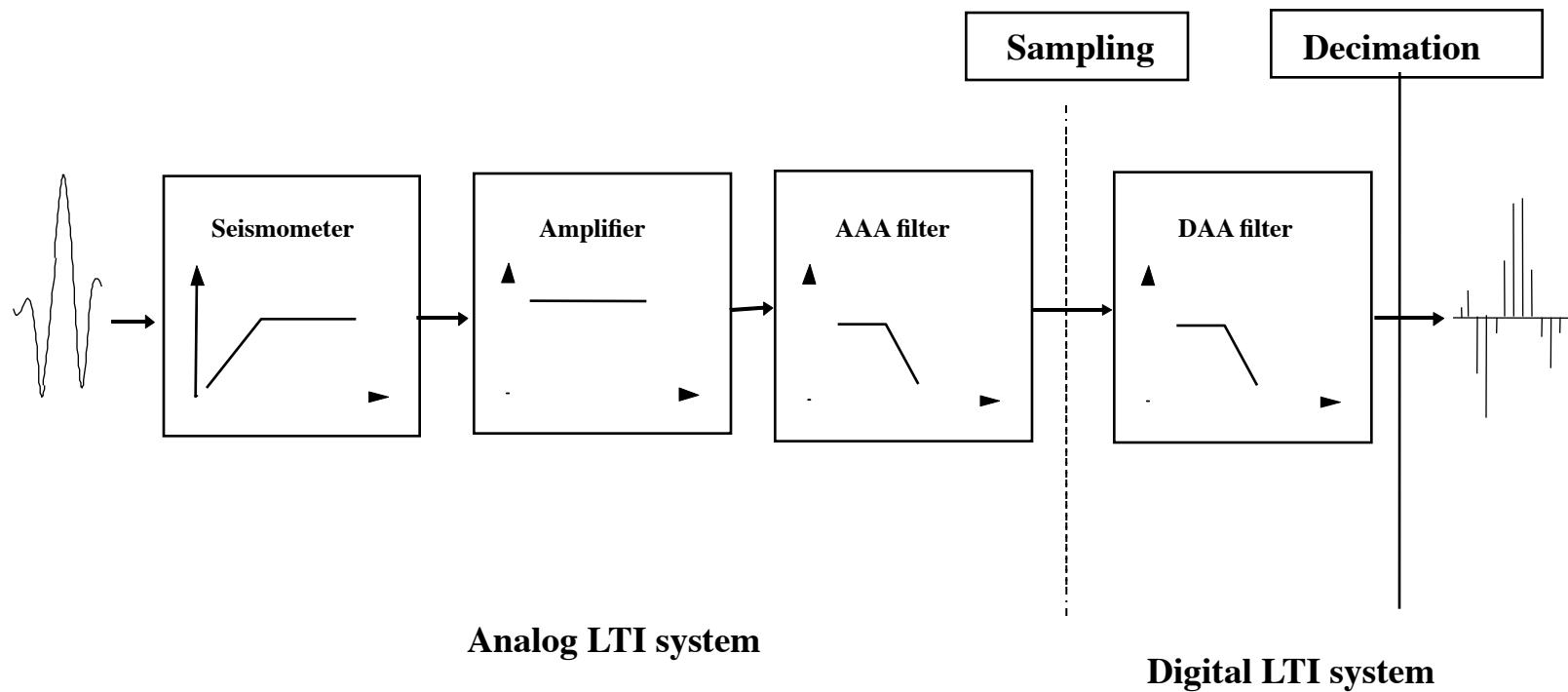
Problem:

- Decrease of signal-to-noise ratio (SNR) outside the passband of the recording instrument
- magnification of the inverse filter is largest where SNR is smallest. Thus, noise in this frequency band will be amplified (instability!).

Consequence:

- The instrument response can only be deconvolved within a certain *valid frequency band* in the presence of noise. The valid frequency band depends on both the signal-to noise ratio and the slope of the frequency response function of the recording systems.

FIR - Filter Effects



Why bothering?

What is the reason for doing FIR filtering and decimating?

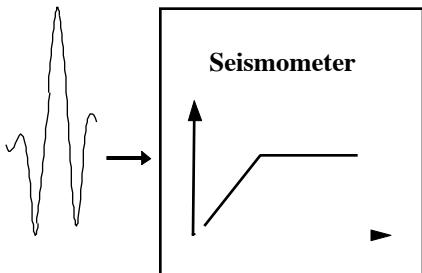
Nearly all seismic recorders use the oversampling technique to increase the resolution of recordings. In order to achieve an optimum valid frequency band, the filters are very steep.

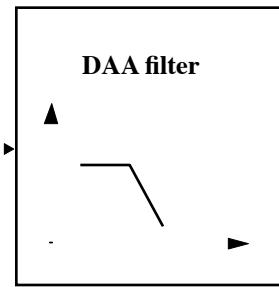
Besides its advantages this also bears new problems.

```

#      << IRIS SEED Reader, Release 4.4 >>
#
#      ===== CHANNEL RESPONSE DATA =====
B050F03 Station: RJOB
B050F16 Network: BW
B052F03 Location: ?
B052F04 Channel: EHZ
B052F22 Start date: 2007,199
B052F23 End date: No Ending Time
#
#      =====
#      +-----+ +-----+
#      + | Response (Poles & Zeros), RJOB ch EHZ | +-----+
#      +-----+ +-----+
#
#      B053F03 Transfer function type: A [Laplace Transform (Rad/sec)]
B053F04 Stage sequence number: 1
B053F05 Response in units lookup: M/S - Velocity in Meters per Second
B053F06 Response out units lookup: V - Volts
B053F07 A0 normalization factor: 6.0077E+07
B053F08 Normalization frequency: 1
B053F09 Number of zeroes: 2
B053F14 Number of poles: 5
#
#      Complex zeroes:
#      i real      imag      real_error   imag_error
B053F10-13 0 0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00
B053F10-13 1 0.000000E+00 0.000000E+00 0.000000E+00 0.000000E+00
#
#      Complex poles:
#      i real      imag      real_error   imag_error
B053F15-18 0 -3.700400E-02 3.701600E-02 0.000000E+00 0.000000E+00
B053F15-18 1 -3.700400E-02 -3.701600E-02 0.000000E+00 0.000000E+00
B053F15-18 2 -2.513300E+02 0.000000E+00 0.000000E+00 0.000000E+00
B053F15-18 3 -1.310400E+02 -4.672900E+02 0.000000E+00 0.000000E+00
B053F15-18 4 -1.310400E+02 4.672900E+02 0.000000E+00 0.000000E+00

```





```

#      +-----+
#      | FIR response, RJOB ch EHZ |
#      +-----+      +
#
B061F03 Stage sequence number:      3
B061F05 Symmetry type:            A
B061F06 Response in units lookup: COUNTS - Digital Counts
B061F07 Response out units lookup: COUNTS - Digital Counts
B061F08 Number of numerators:      96
#
# Numerator coefficients:
# i, coefficient
B061F09 0 3.767143E-09
B061F09 1 5.277283E-07
B061F09 2 2.184651E-06
B061F09 3 -5.639535E-06
B061F09 4 -1.233773E-06
B061F09 5 9.386712E-06
B061F09 6 4.859924E-06
B061F09 7 -1.644319E-05
...
#
#      +-----+
#      | Decimation, RJOB ch EHZ |
#      +-----+      +
#
B057F03 Stage sequence number:      4
B057F04 Input sample rate:        1.000000E+03
B057F05 Decimation factor:       5
B057F06 Decimation offset:        0
B057F07 Estimated delay (seconds): 1.490000E-01
B057F08 Correction applied (seconds): 0.000000E+00

```

Linear Difference Equation

$$\sum_{k=0}^N a_k y[n-k] = \sum_{l=0}^M b_l x[n-l]$$

Infinite Impulse Response: $a_k \neq 0$

Finite Impulse Response: $a_0 = 1$; $a_{k \neq 0} = 0$

- FIR filters :

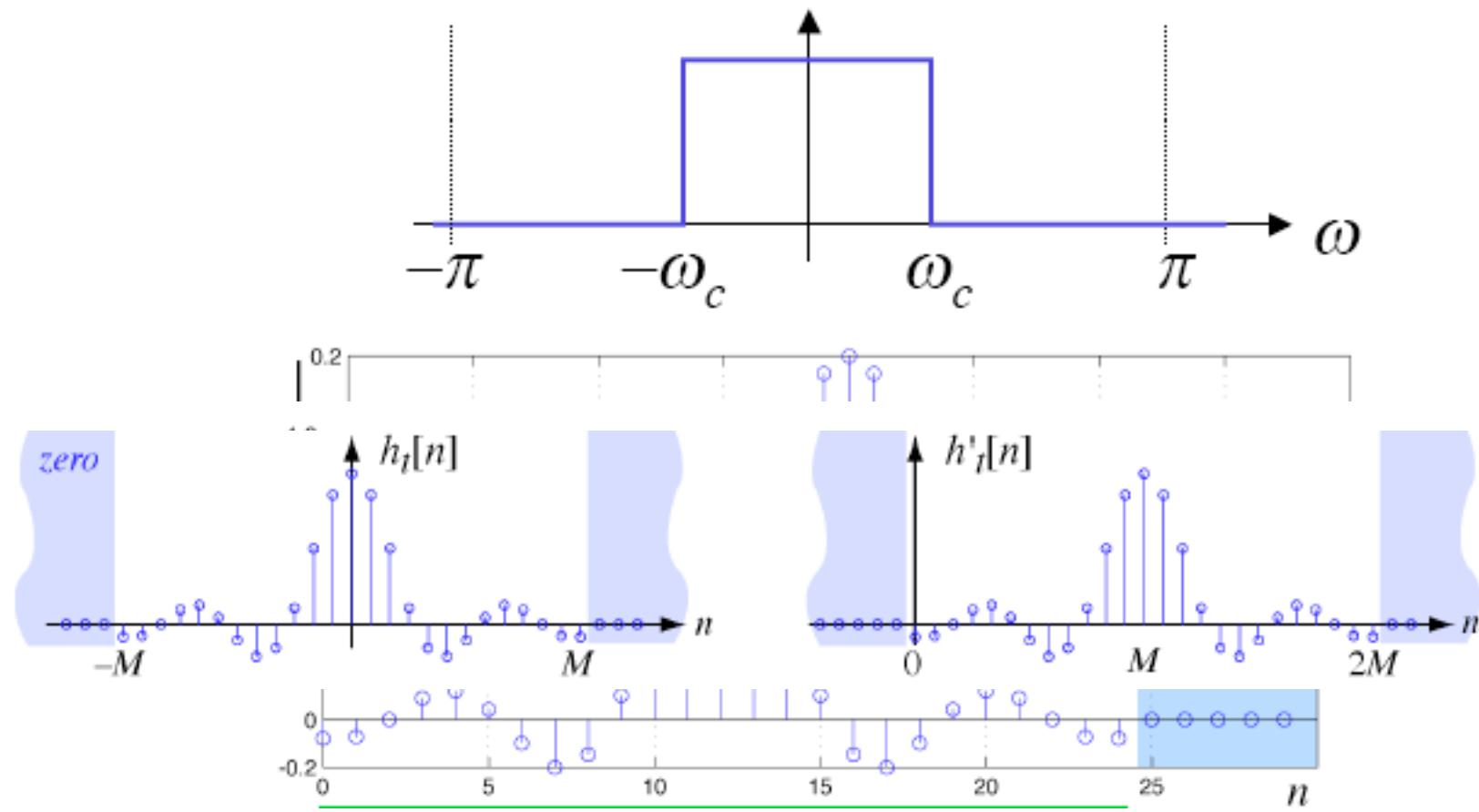
- + Always stable.
- Steep filters need many coefficients.
- + Both causal and noncausal filters can be implemented.
- + Filters with given specifications are easy to implement!

- IIR filters :

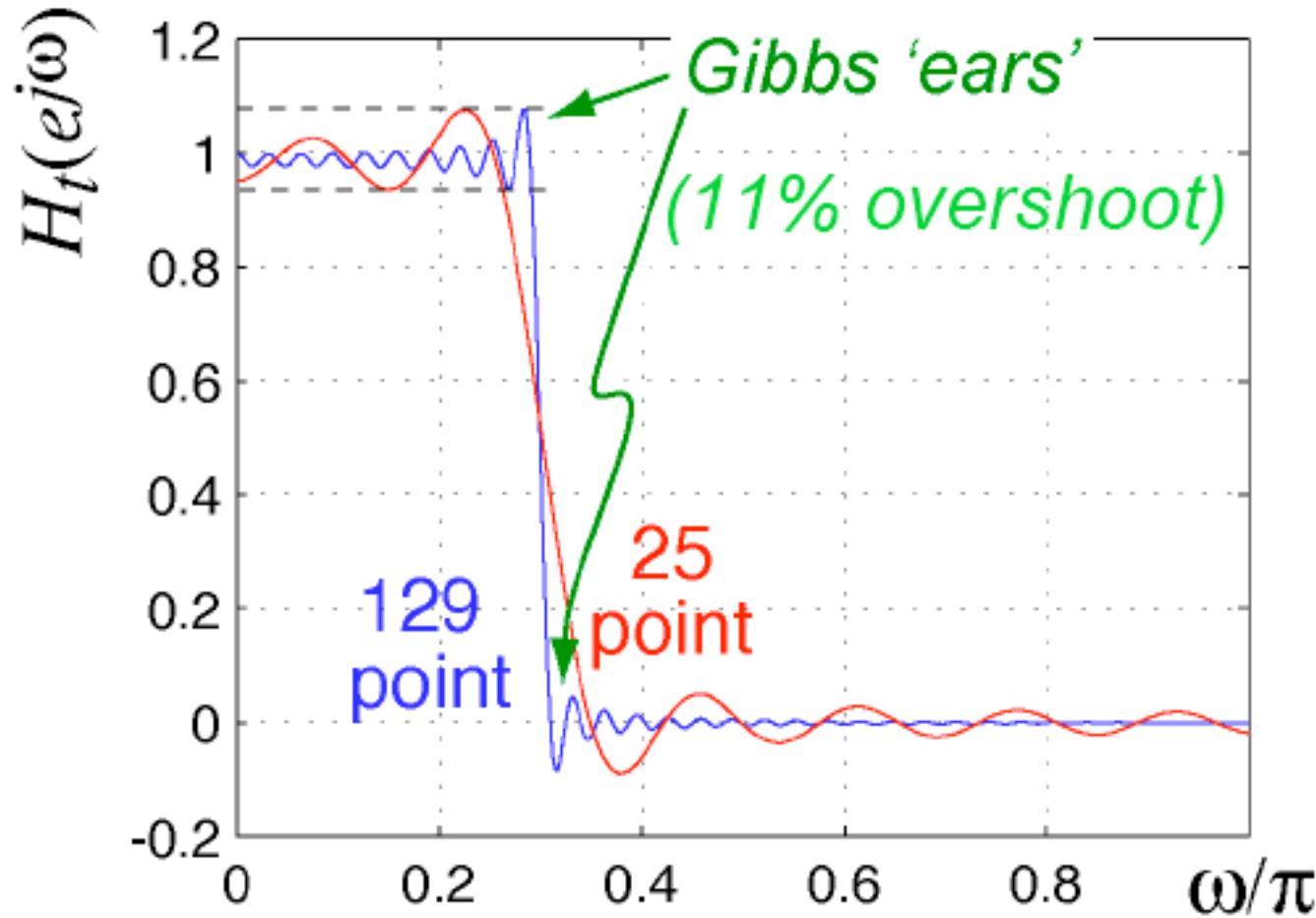
- Potentially unstable and subject to quantization errors.
- + Steep filters can easily be implemented with a few coefficients. Speed.
- Filters with given specifications are in general, difficult, if not impossible, to implement *exactly*(!).

In SEED the impulse response of the decimation filters are given.
But how to construct FIR filters?

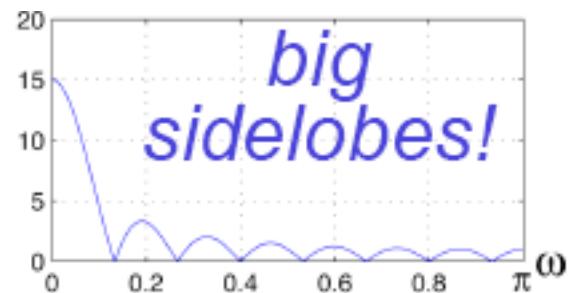
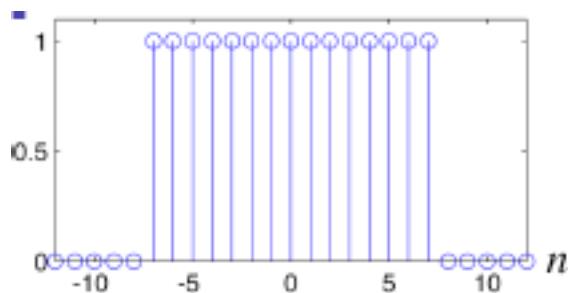
Easiest way: inverse DFT with selected spectral shape and phase
and truncate the (infinite) sequence to form a finite impulse
response



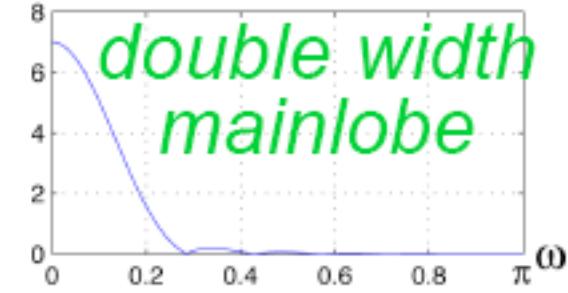
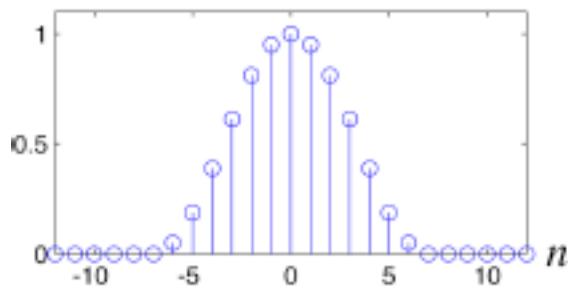
Filter length vs. steepness



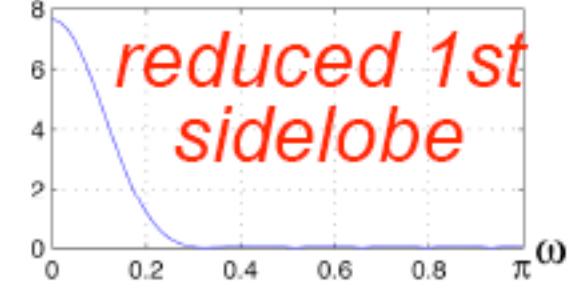
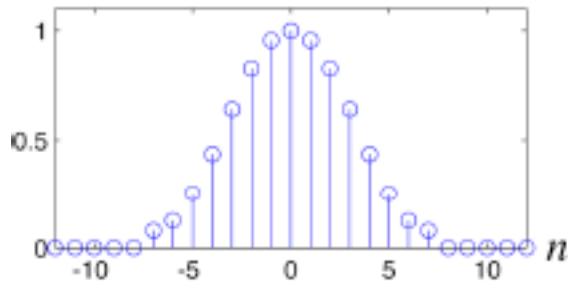
Rectangular



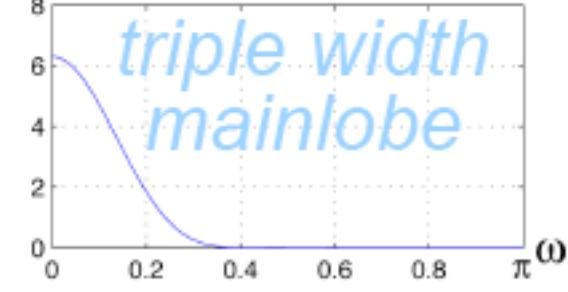
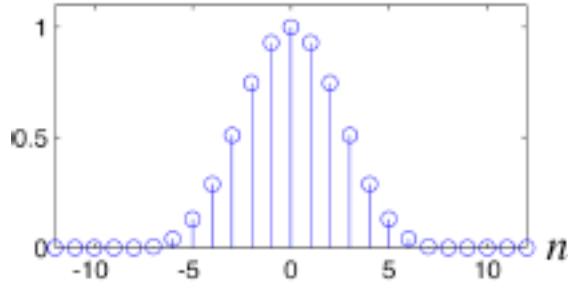
Hanning



Hamming



Blackman



Windowed FIR Filter Example:

1. Get ideal filter impulse response:

$$\omega_c = 0.15\pi \quad \Rightarrow h_d[n] = \frac{\sin 0.15\pi n}{\pi n}$$

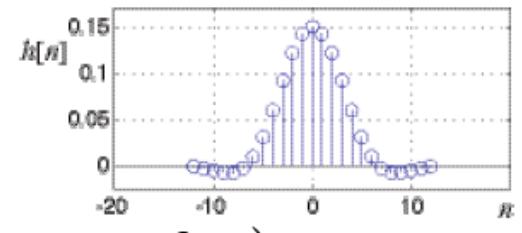
2. Get window function for truncation:

$$N = 25 \rightarrow M = 12 \quad (N=2M+1)$$

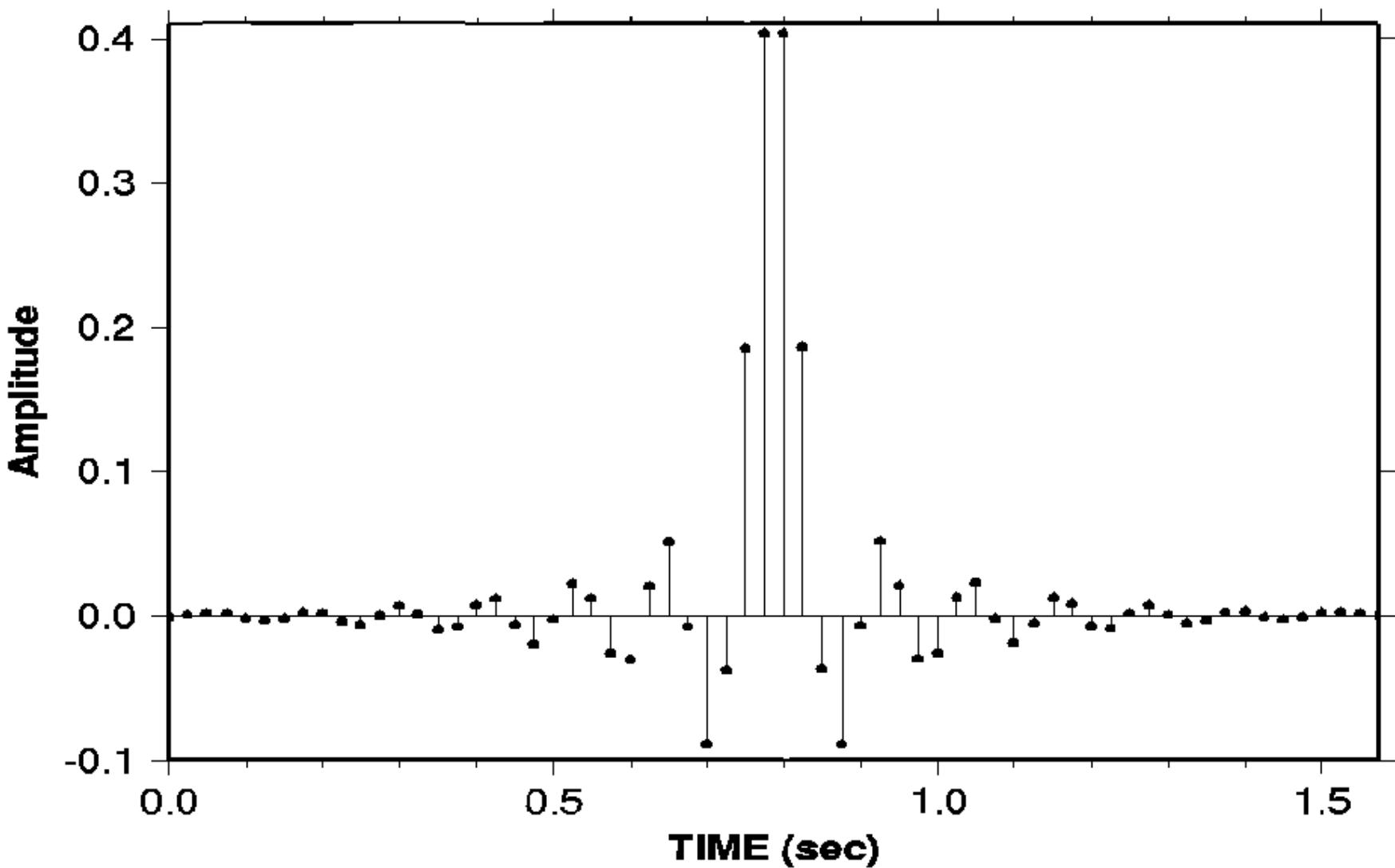
$$\Rightarrow w[n] = 0.54 + 0.46 \cos\left(2\pi \frac{n}{25}\right) \quad -12 \leq n \leq 12$$

3. Apply window:

$$\begin{aligned} h[n] &= h_d[n]w[n] \\ &= \frac{\sin 0.15\pi n}{\pi n} \left(0.54 + 0.46 \cos\frac{2\pi n}{25}\right) \quad -12 \leq n \leq 12 \end{aligned}$$

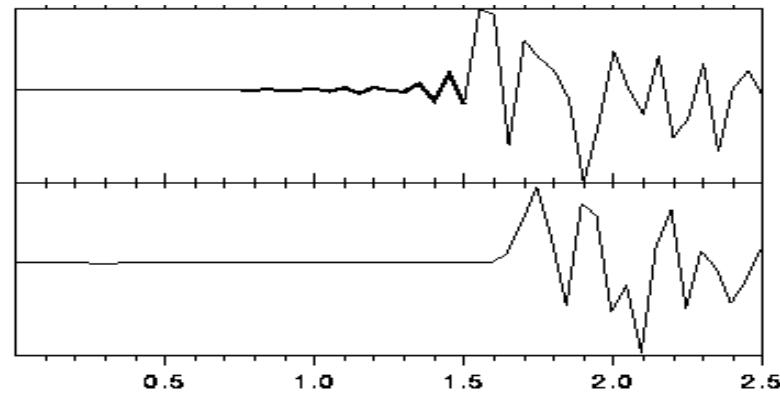


QDP 380 Stage 4



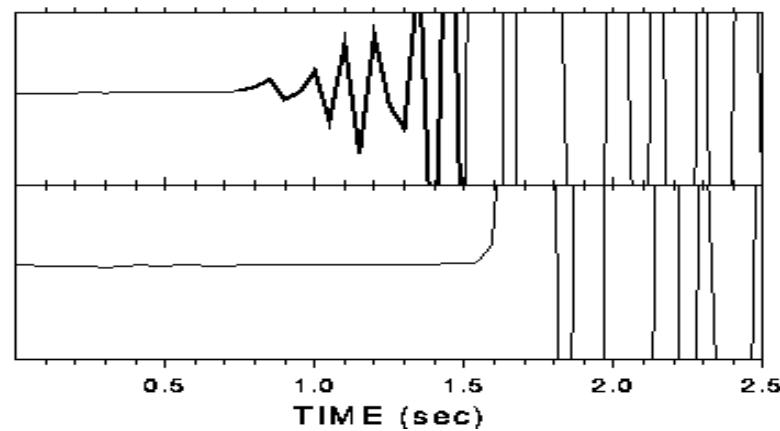
UC

CO



UC X 20

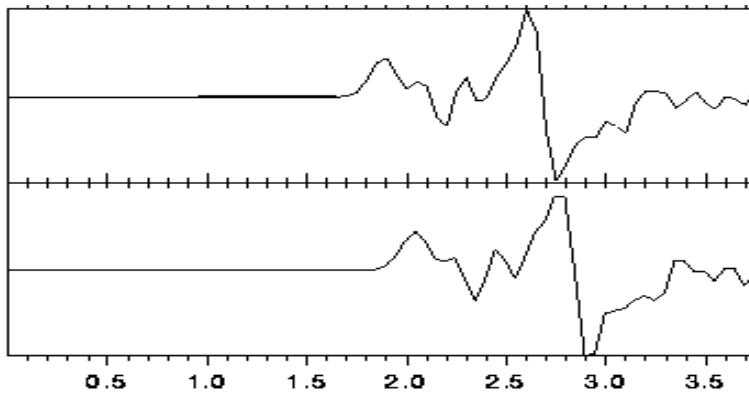
CO X 20



a)

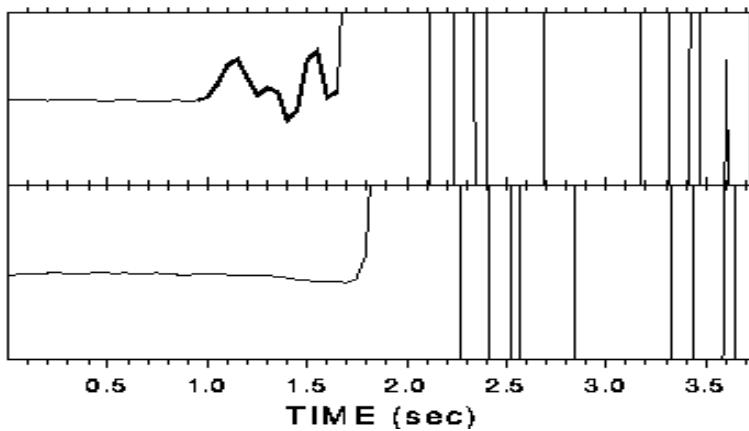
UC

CO



UC X 200

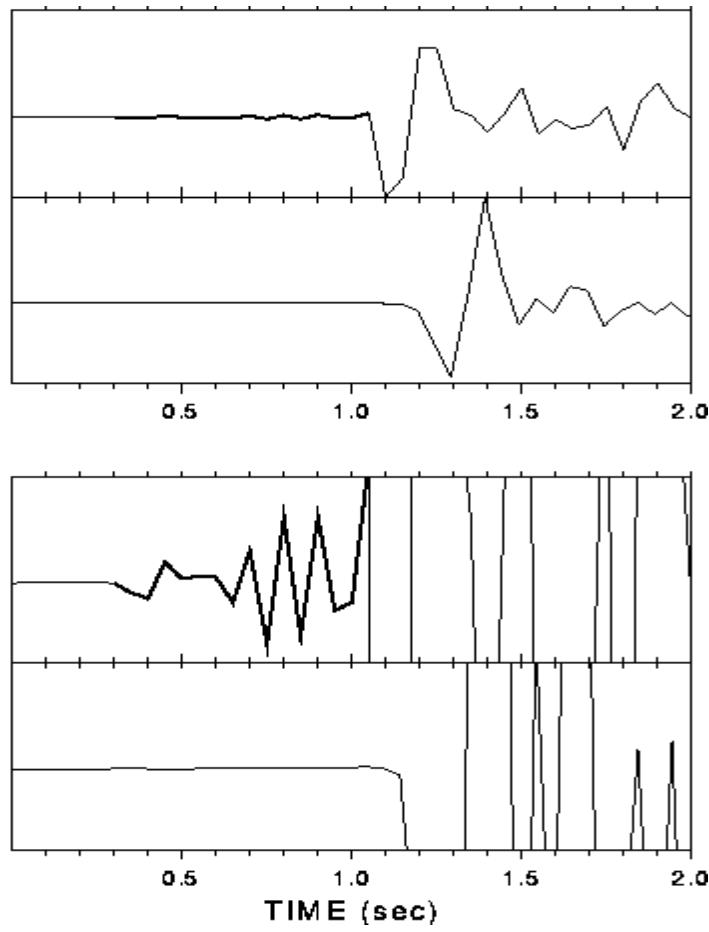
CO X 200



b)

UC

CO



UC X 30

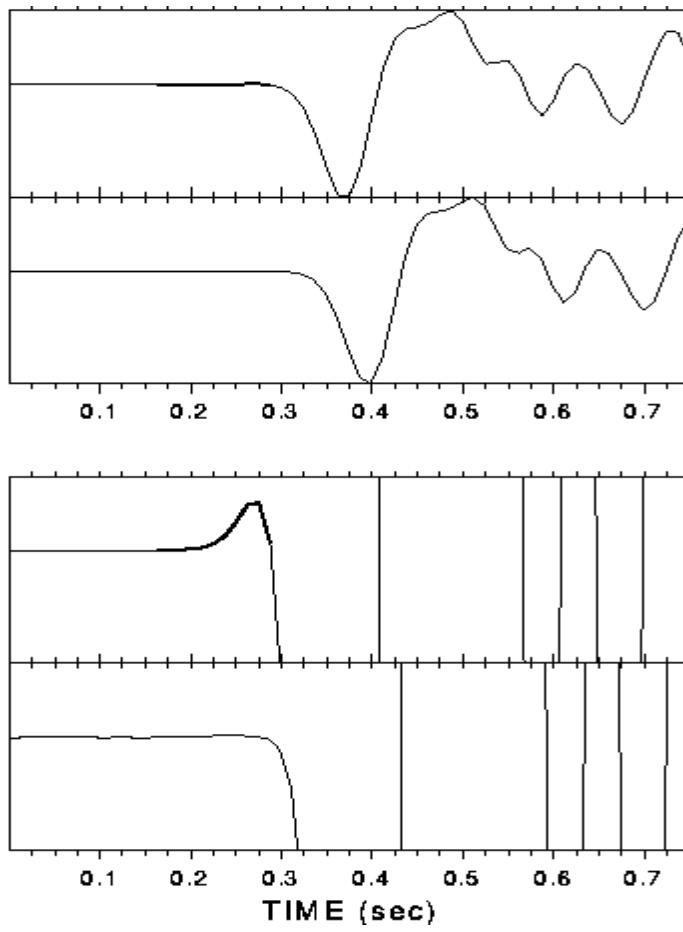
CO X 30

TIME (sec)

a)

UC

CO



UC X 50

CO X 50

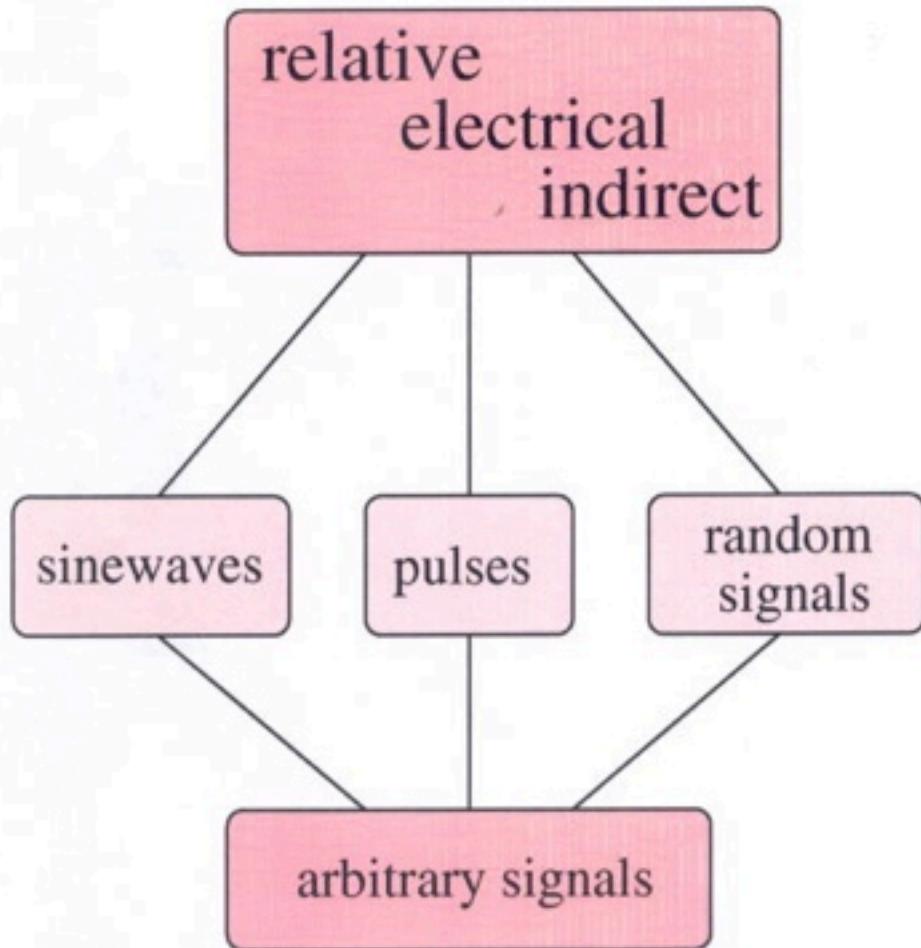
TIME (sec)

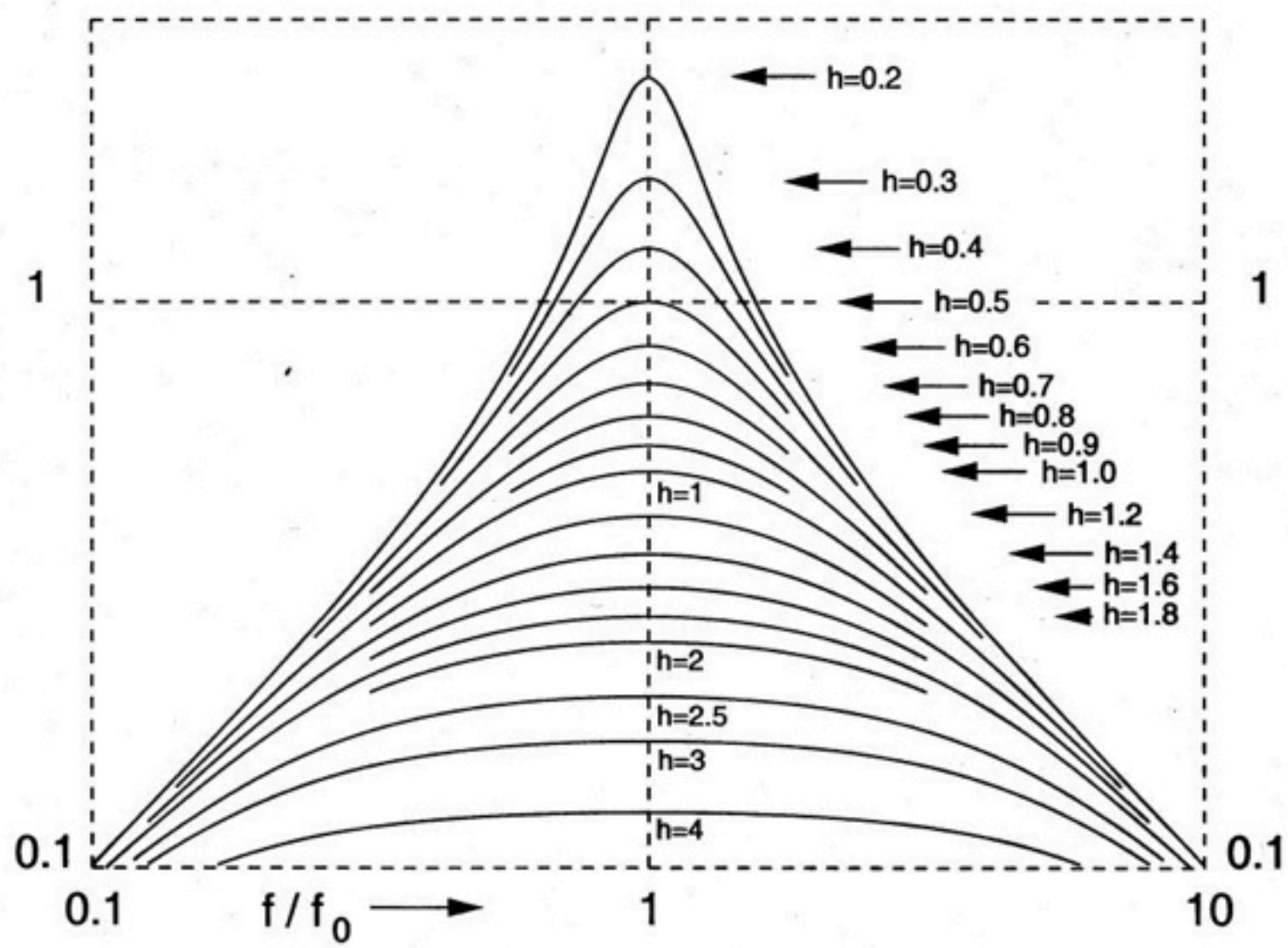
b)

Modern Seismometers Calibration

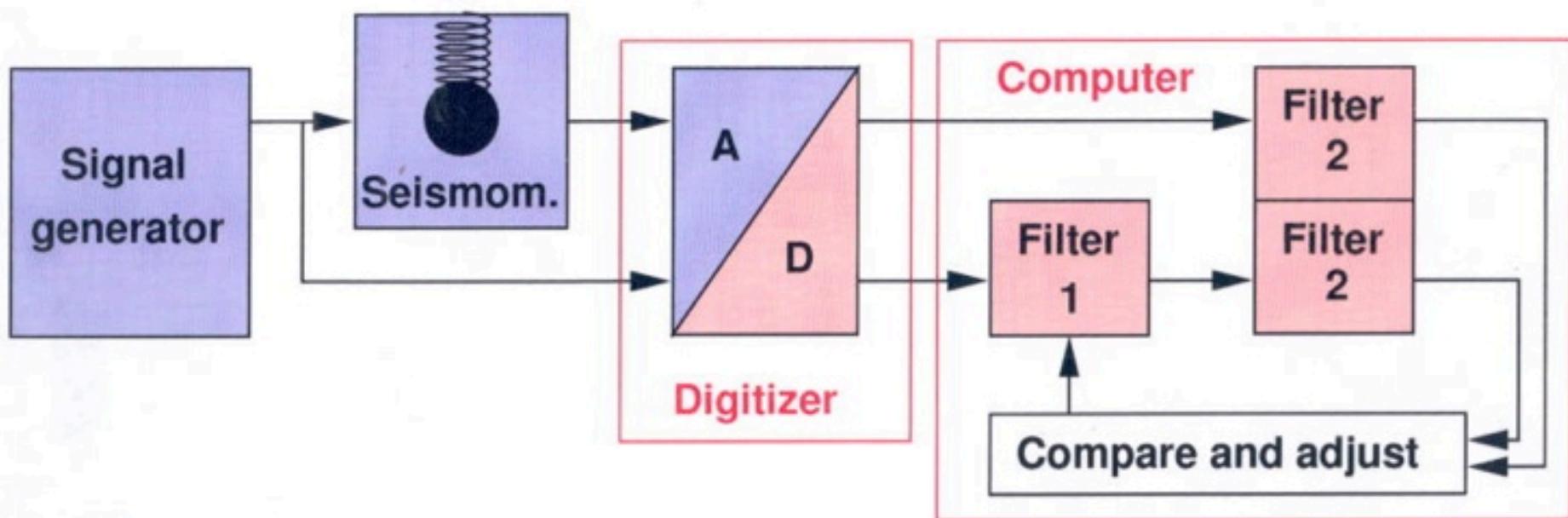
(Material of E. Wielandt)

Systematics of Seismometer calibration





Calibration with arbitrary signals

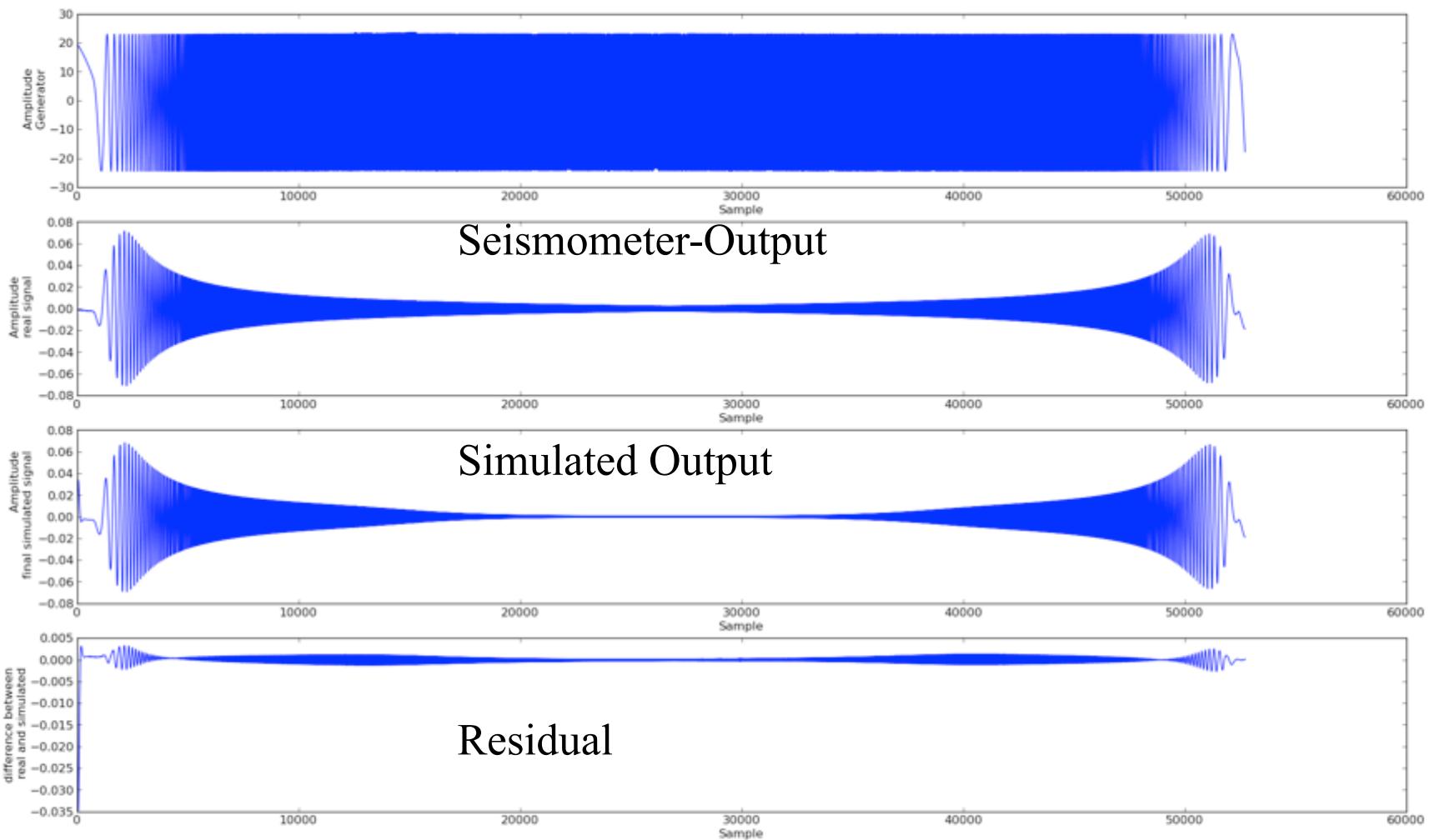


For a perfect fit, filter 1 must be an exact digital representation of the seismometer.

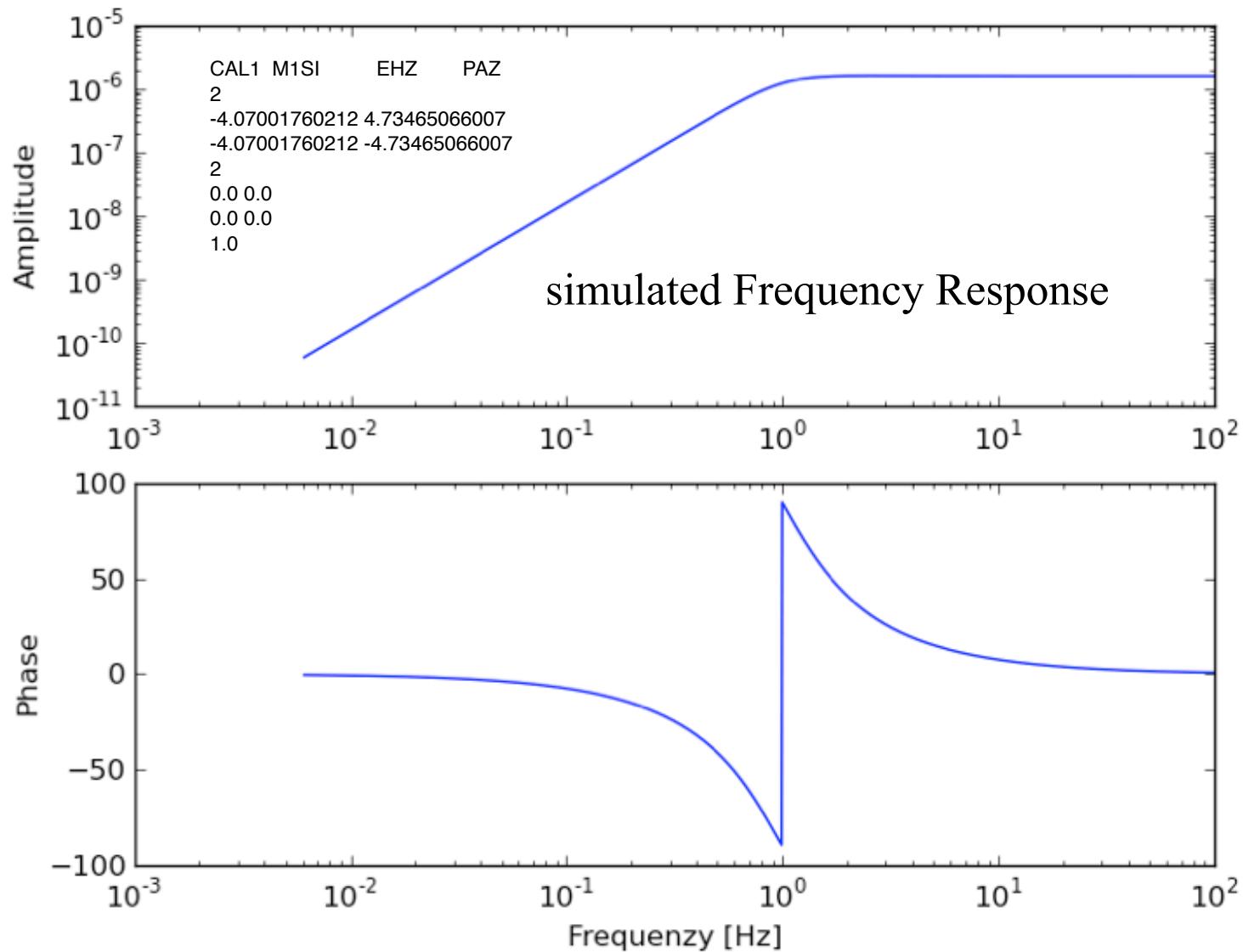
This is only possible when the passband is limited with the anti-alias-filter 2.

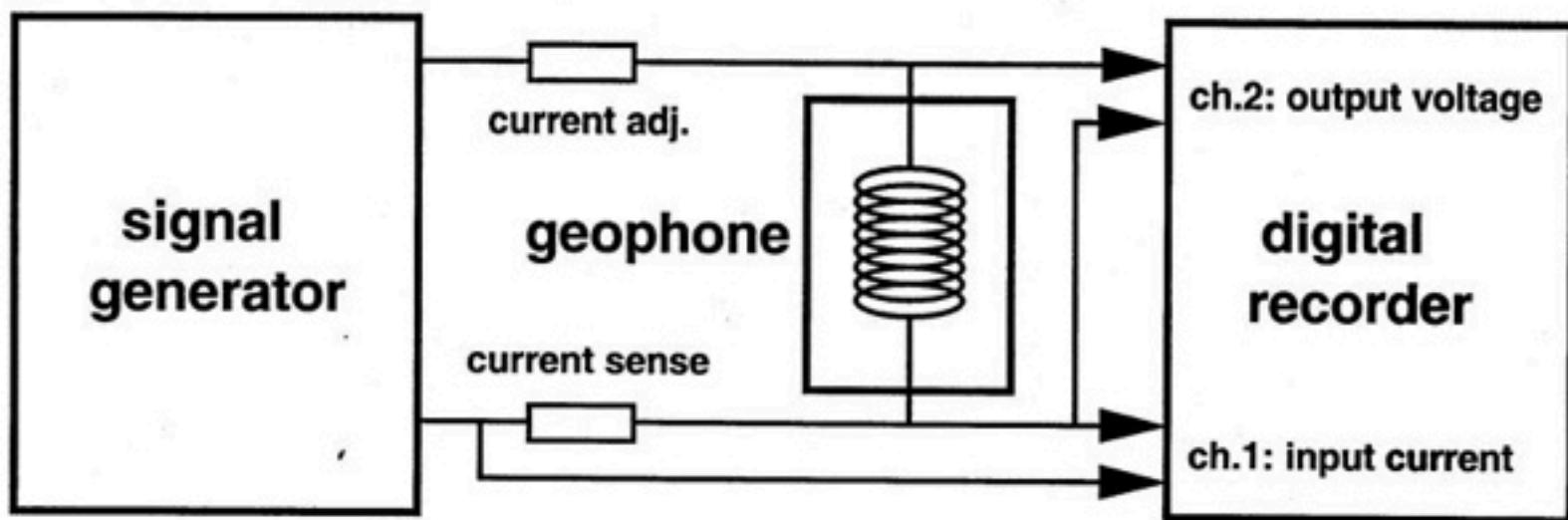
CALEX - ObsPy Version

Generator-Input



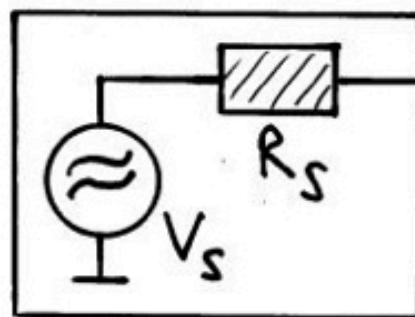
CALEX - ObsPy Version



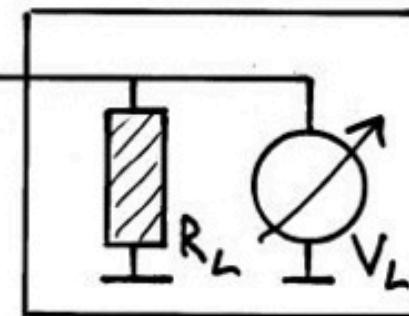


What can easily go wrong without being obvious:

- Polarity reversed
- Only one out of two signal wires connected
- One wire shorted
- One wire interchanged between different channels
- Signal reduction by source / receiver impedance



Seismometer

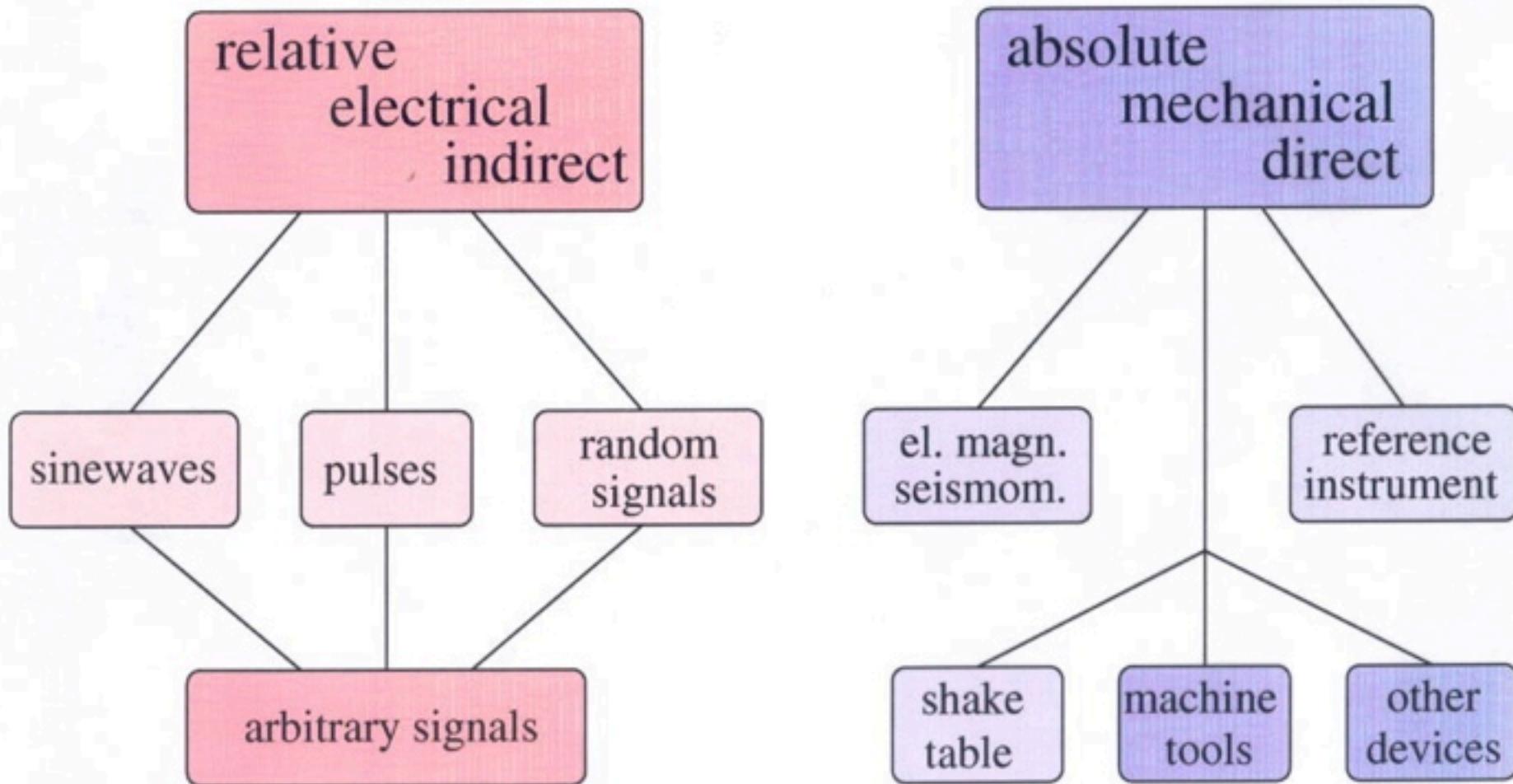


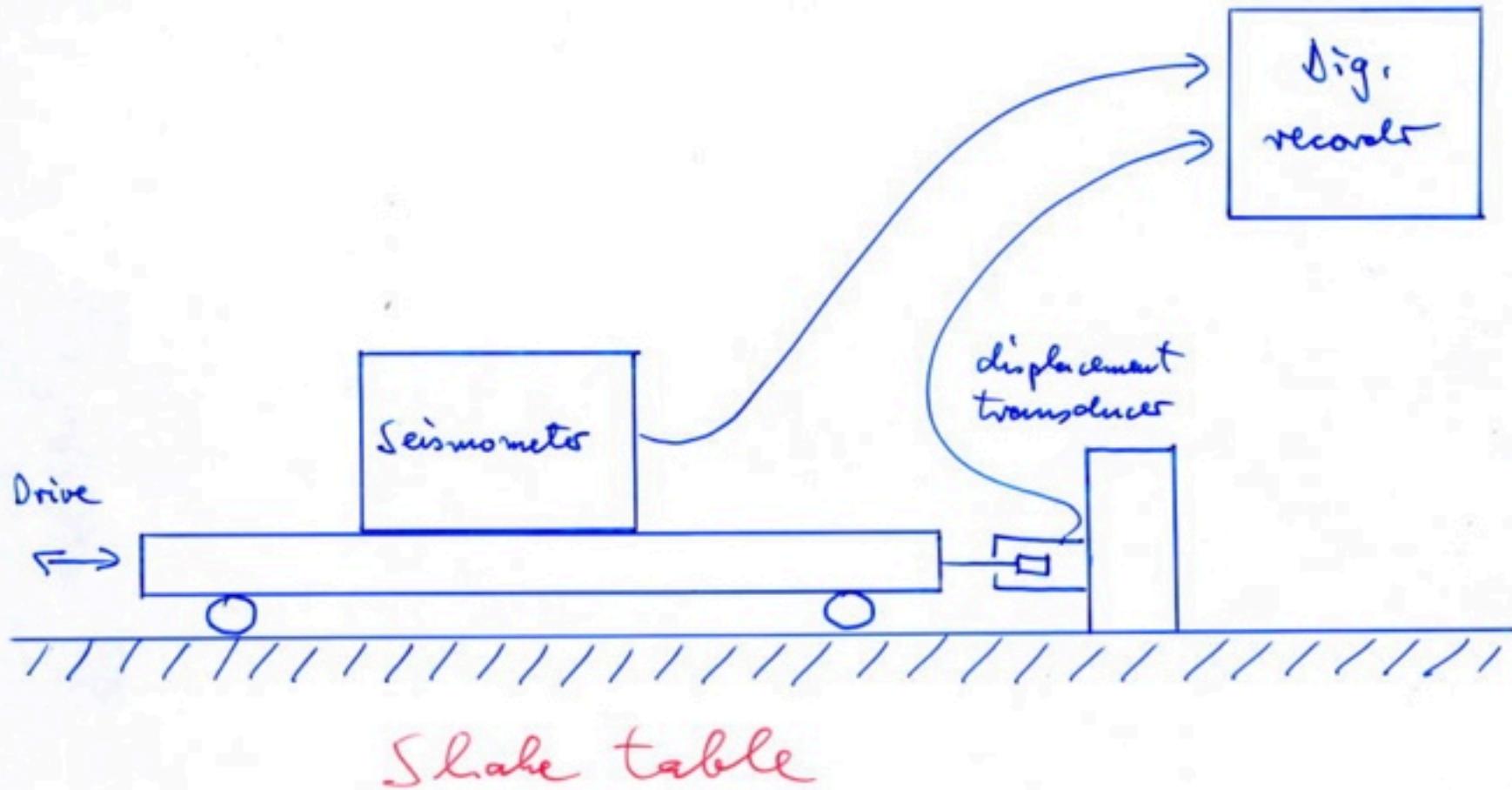
Recorder

The measured voltage is

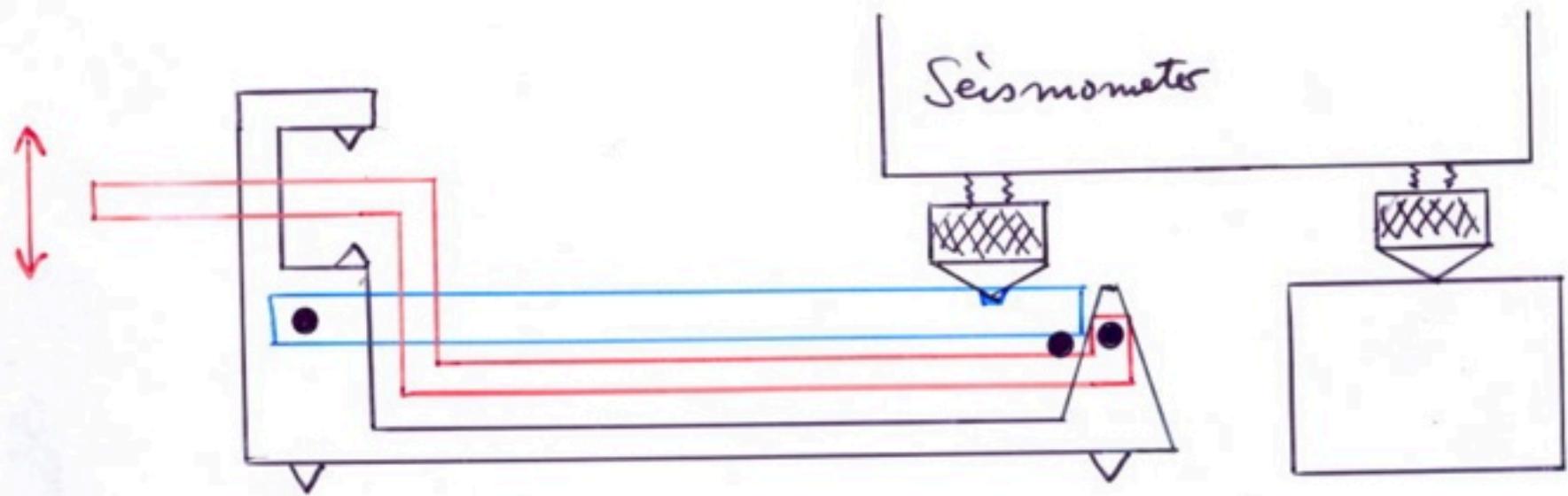
$$V_L = \frac{R_L}{R_L + R_s} V_s$$

Systematics of Seismometer calibration



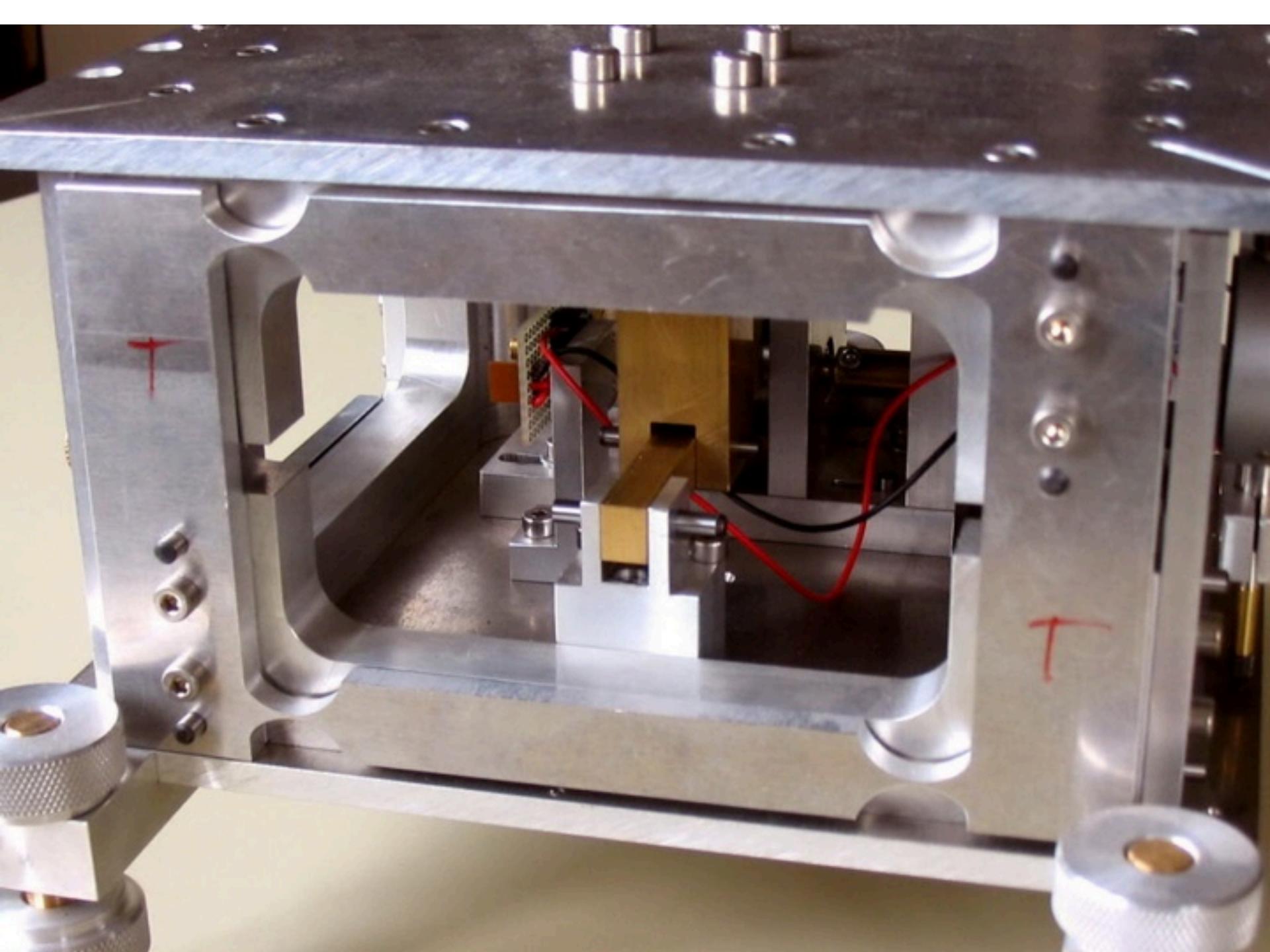






Tilt calibrator





HAHN & KOLB

Made in Germany

$\times 0.1\text{ mm}$

STOSSGESCHÜTZT

0,001 mm

60 40

50

80 40

30
70

90 10

0

80
20

70
30

60
40

50
30

40
20

30
10

20
0

10
0

0
0

1
0

2
1

3
2

4
3

5
4

6
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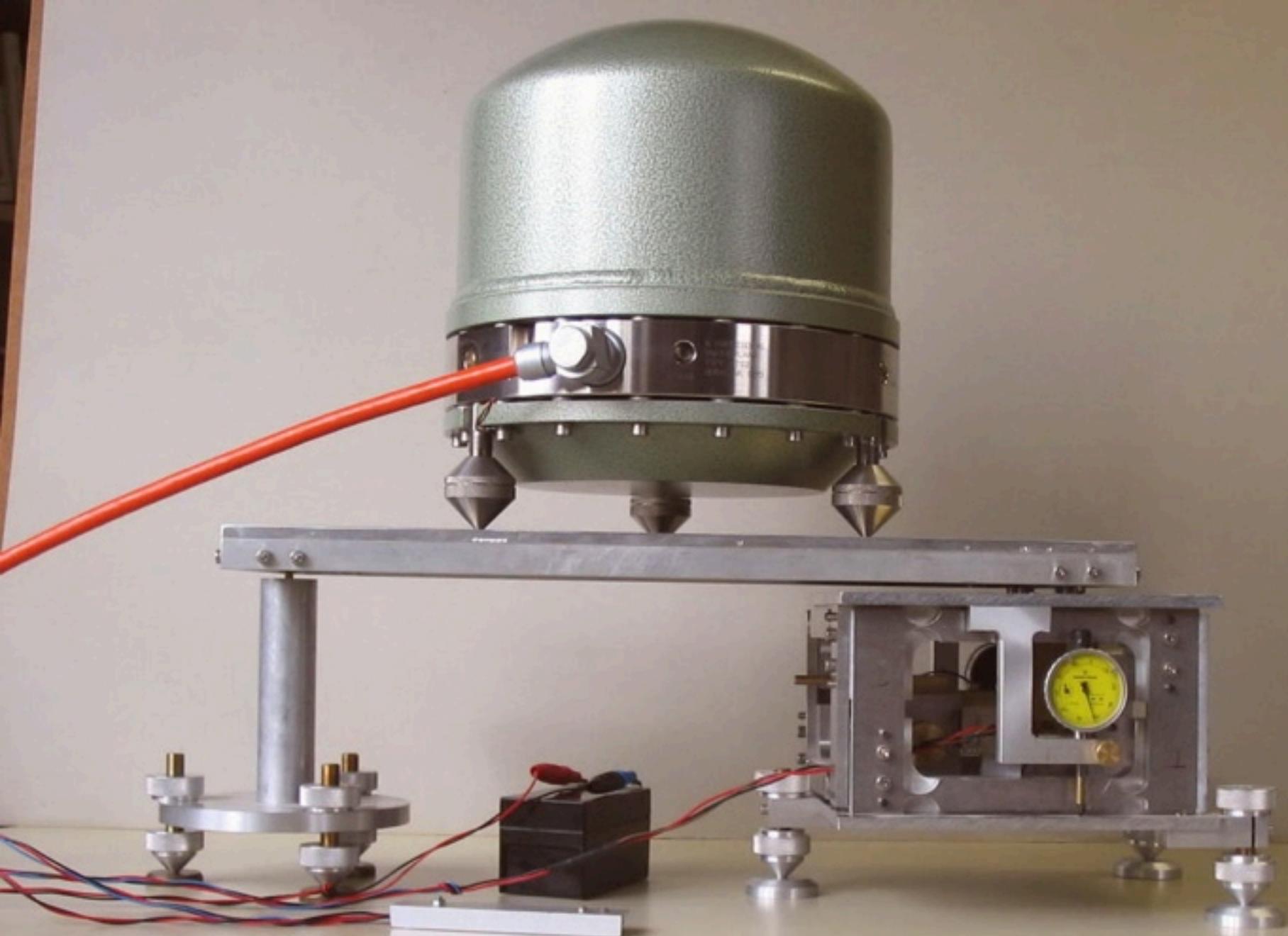
31
30

32
31

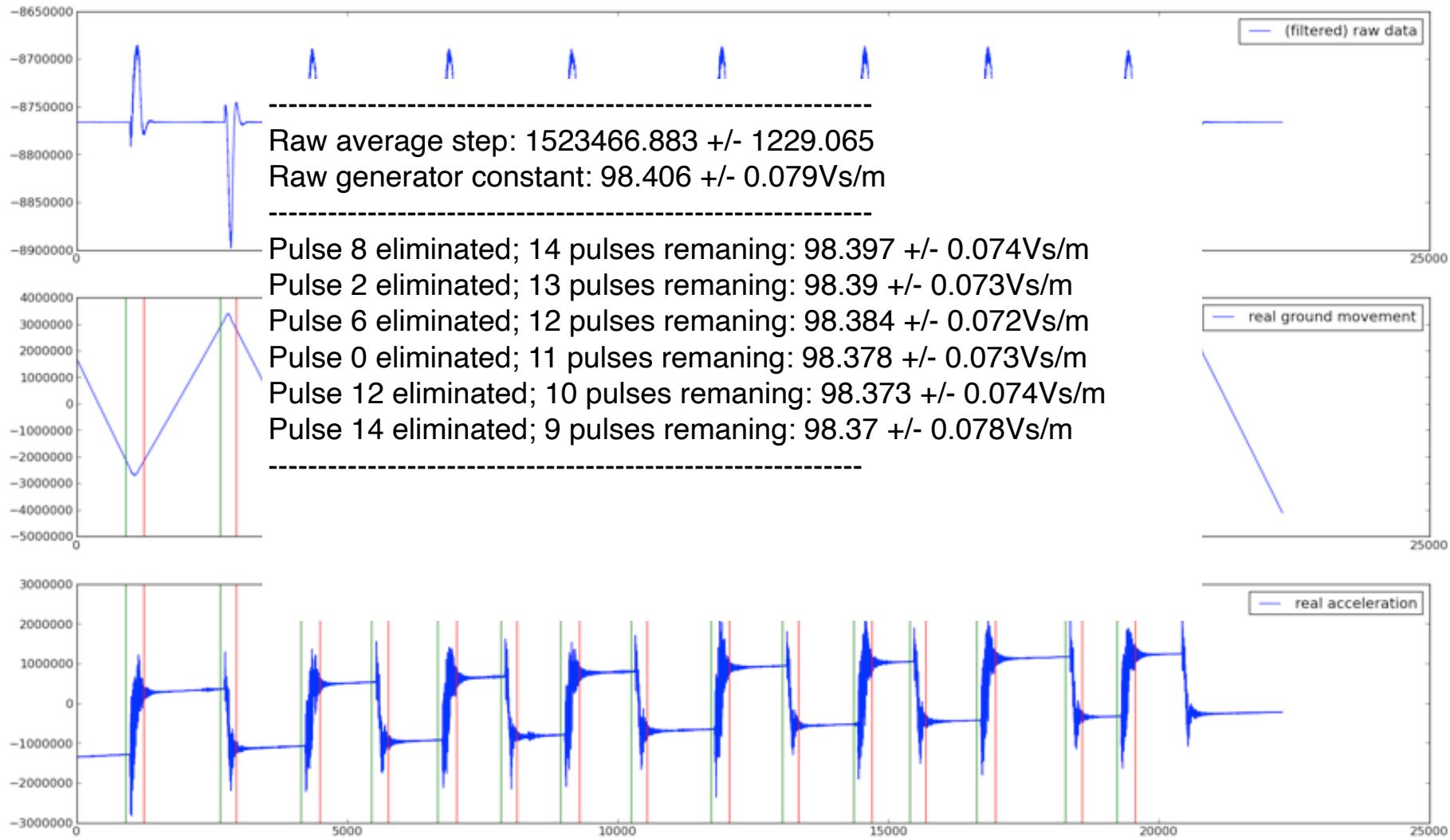


DISPCAL - ObsPy Version





TILTCAL - ObsPy Version



Absolute calibration of seismic sensors by displacement and tilt

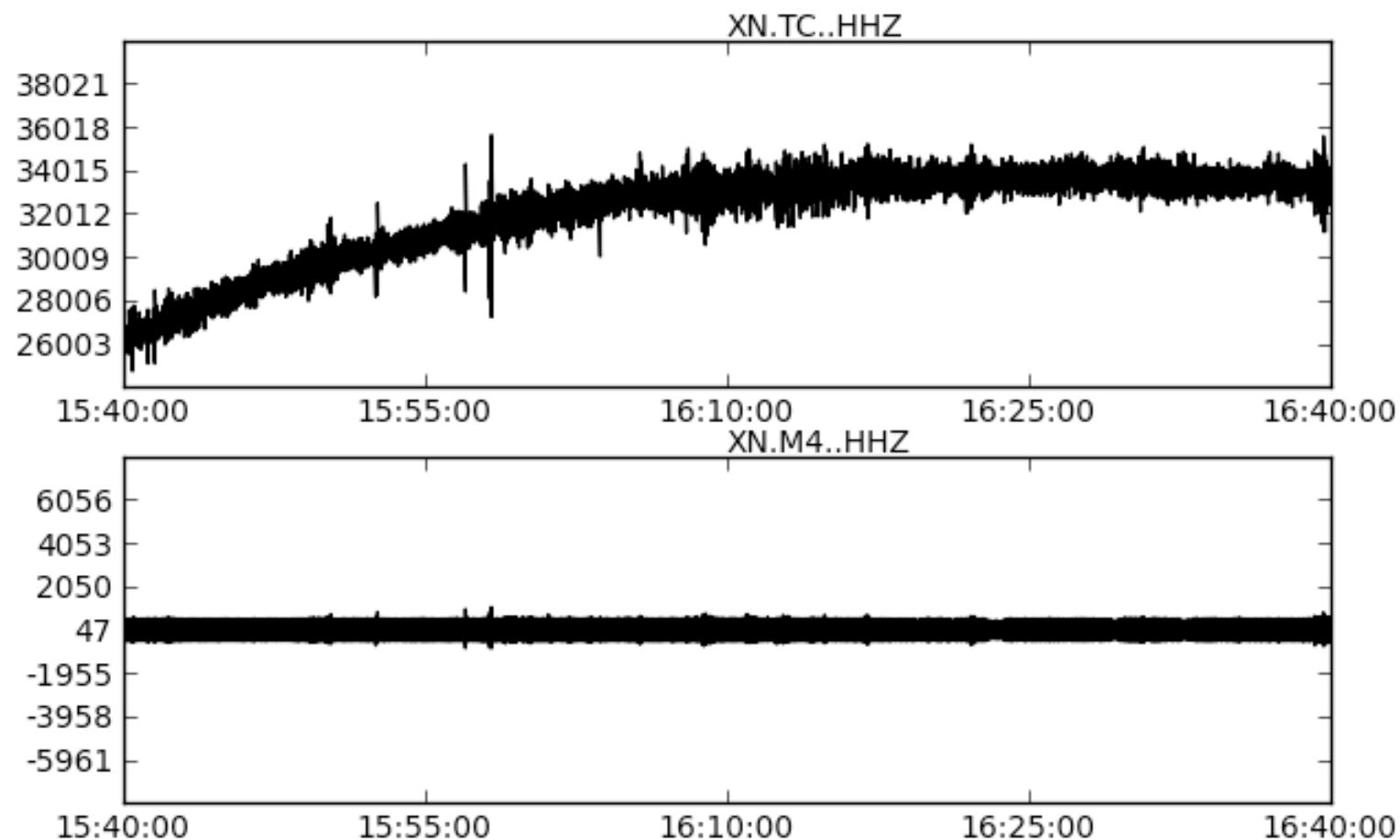
Sensor type	\	milling machine	balance	tilt lever
Streckeisen STS2 # 99113		X: 1484 +-3 Y: 1490 +-4 Z: 1502 +-6	Z: 1503 +-4	X: 1481 +-3 Y: 1480 +-4 Z: 1481 +-6
Sensonics Mk3A		H: 499 +-4		
SM4 geophone 10 Hz			Z: 28.5 +-0.1	
Specs were:	STS2	1500 Vs/m	Mk3A	500 Vs/m
				SM4 28.8Vs/m

Trillium

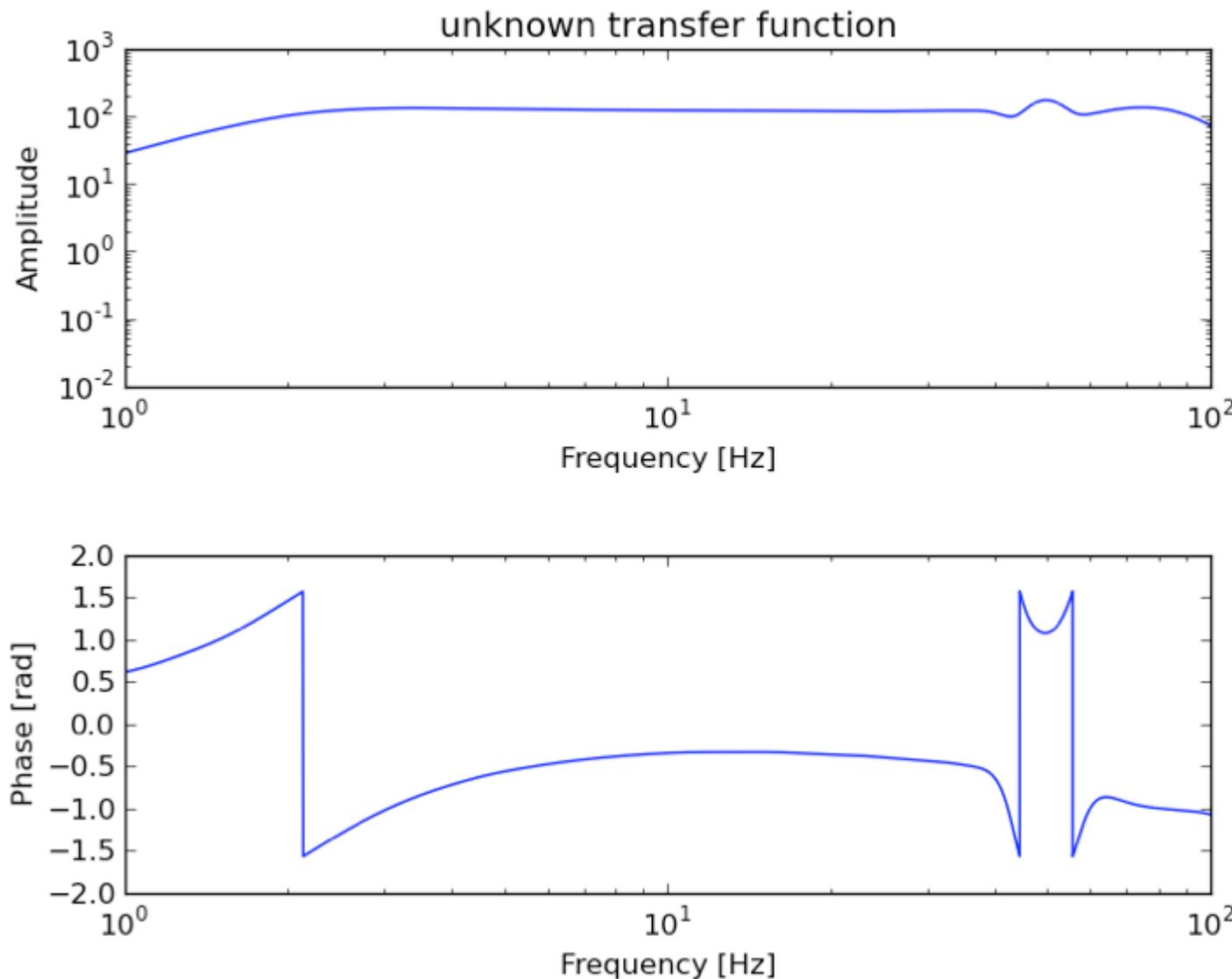
Perotti-Mechanics/N

NoiseCalib - ObsPy Version (RELCALSTACK)

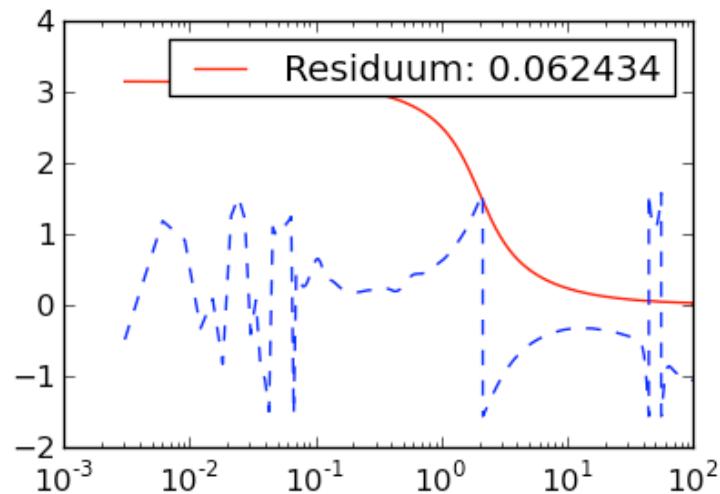
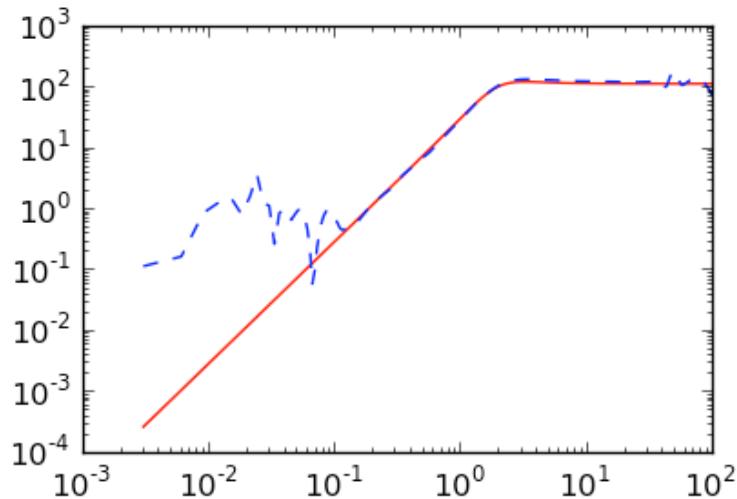
2011-12-15T15:40:00Z - 2011-12-15T16:40:00Z



NoiseCalib - ObsPy Version (RELCALSTACK)



NoiseCalib - ObsPy Version (PyQt_FitResp)



$f_0=2.0$ Hz, $h = 0.56$, $G=110$ V/m/s