

Simulation Study for Timing Model

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Global options for stan model runs

```
Chains = 4
options(mc.cores = Chains)
IT = 10^4
WarmUp = .5*IT
thin = 40
Sim_Iterations = 50
```

Global simulation sample sizes and follow-up time

```
N_PMQ = 800
N_AS = 200
N_CQ = 200
FUP_time = 300
```

Data simulation functions

We explore a few scenarios of increasing complexity to check:

- The model can recover correct parameters when the data generating process is correctly specified (sanity check)
- The effect of a mis-specified data generating process. For this we look at the impact of seasonality

Data simulation for model 1

This simulates data under the assumptions of Model 1.

We set the simulation parameters:

```
params_M1 = list(lambda = 1/1200,
                 gamma = 1/80,
                 lambda_recrud = 1/10,
                 logit_EarlyL = 0.5,
                 logit_c1_CQ = 0.01,
                 logit_c1_AS = 0.01,
                 logit_mean_p = logit(0.2),
                 logit_sd_p = (-logit(0.2)+logit(0.8))/1.96,
                 AS_shape = 10,
                 AS_scale = 21,
                 CQ_shape = 10,
                 CQ_scale = 42,
                 rate_decrease = .66)
```

Generate data with these parameters:

```
set.seed(475732)
out1 = simulate_dataset(N_PMQ = N_PMQ, N_CQ = N_CQ,
                       N_AS = N_AS, FUP_time = FUP_time,
```

```

        data_generation_function = generate_patient_data_Model1,
        params = params_M1,
        Study_Period = c(rep(1,N_PMQ/2),rep(2,N_PMQ/2 + N_AS + N_CQ)))
Simdata_Model1 = out1$Simdata
Simulation_truth1 = out1$Simulation_truth

```

Run Stan Model 1 on simulated data generated from Model 1

Load or compile the stan model 1

```

if(RECOMPILE_MODELS){
  writeLines('Compiling model 1....')
  source('../Timing_Model/StanModel1.R')
  save(Timing_Model1, file = '../RData/TimingModel/Timing_Model1.RData')
} else {
  load('../RData/TimingModel/Timing_Model1.RData')
}

```

Prior specification

```

# The hierachical parameters defining the prior distributions for model 1
Prior_params_M1 = list(Hyper_lambda_shape = 100,
  Hyper_lambda_rate = 100*(1/params_M1$lambda),
  Hyper_gamma_shape = 100,
  Hyper_gamma_rate = 100*(1/params_M1$gamma),
  Hyper_lambda_recrud_shape = 100,
  Hyper_lambda_recrud_rate = 100*(1/params_M1$lambda_recrud),
  Hyper_AS_shape_mean = params_M1$AS_shape,
  Hyper_AS_shape_sd = 1,
  Hyper_AS_scale_mean = params_M1$AS_scale,
  Hyper_AS_scale_sd = 1,
  Hyper_CQ_shape_mean = params_M1$CQ_shape,
  Hyper_CQ_shape_sd = 1,
  Hyper_CQ_scale_mean = params_M1$CQ_scale,
  Hyper_CQ_scale_sd = 2,
  Hyper_logit_mean_p_mean = params_M1$logit_mean_p,
  Hyper_logit_mean_p_sd = 1,
  Hyper_logit_sd_p_lambda = 1,
  Hyper_logit_c1_mean = params_M1$logit_c1_AS,
  Hyper_logit_c1_sd = .25,
  Early_L_logit_mean = params_M1$logit_EarlyL,
  Early_L_logit_sd = .5,
  Hyper_mean_rate_decrease = params_M1$rate_decrease,
  Hyper_sd_rate_decrease = 0.25)

```

Fit stan model to simulated data

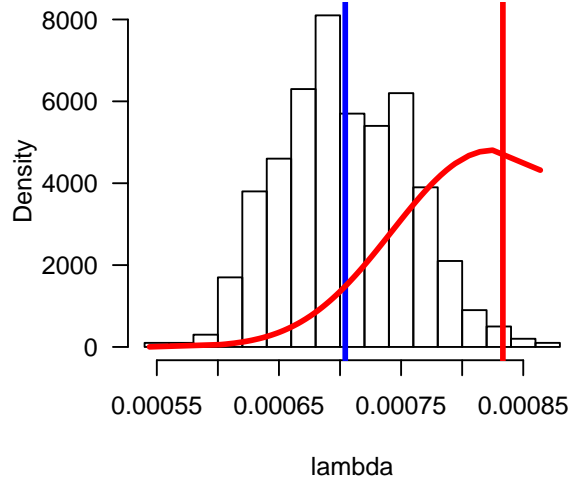
Plot output, comparing prior, ground truth and model estimate. This gives a qualitative assesement but for a single simulation run.

```

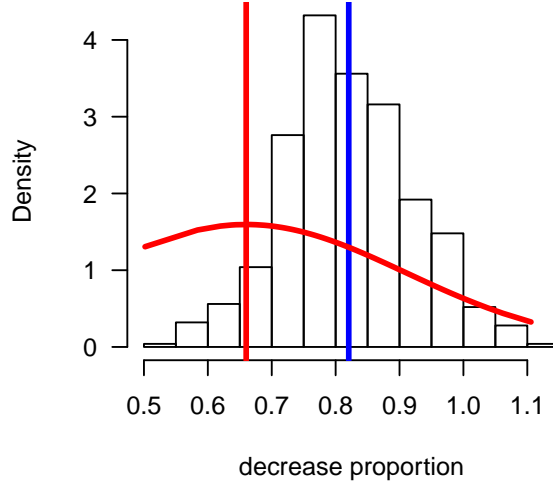
plot_output_model1(thetas_mod1,Simulation_truth1,
  Simdata_Model1,Prior_params_M1)

```

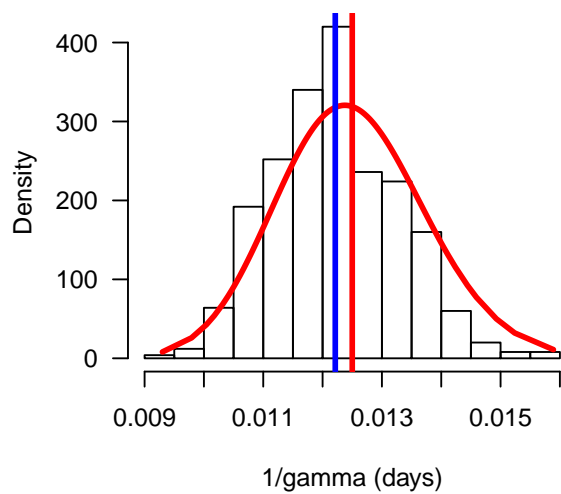
Reinfection rate (Period 1)



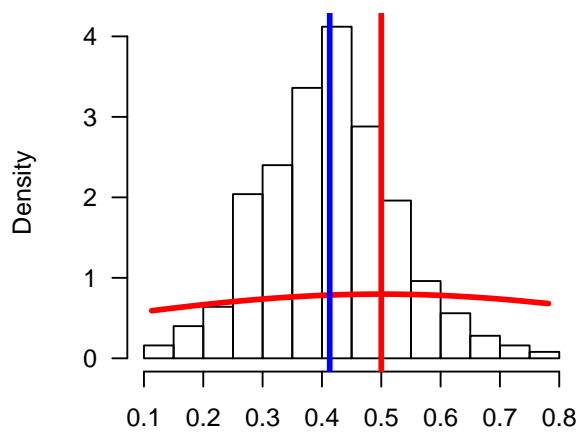
Decrease in reinfection rate (Period 2)

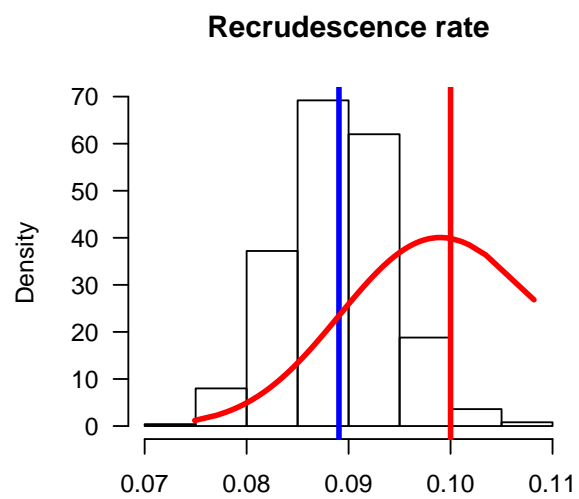
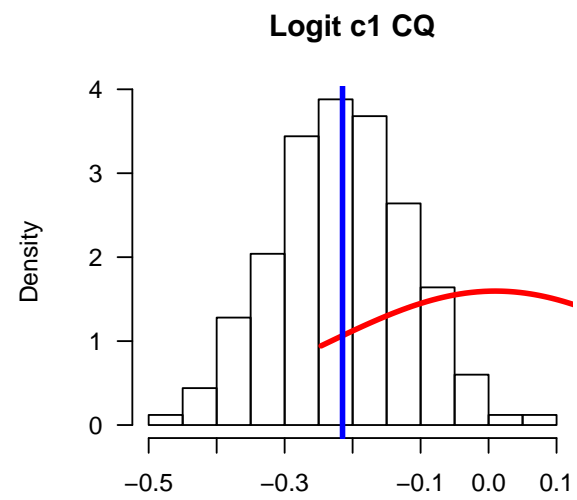
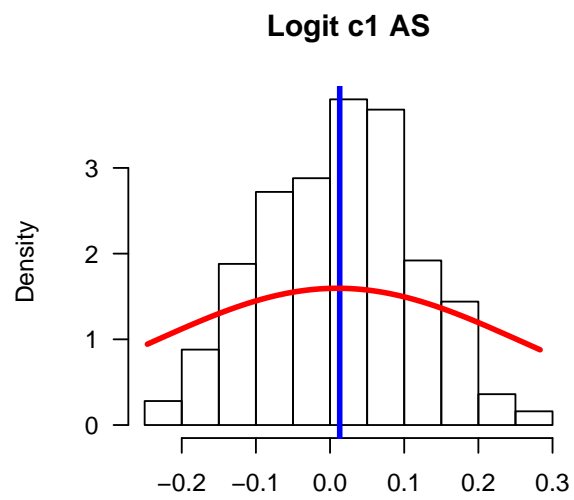


Mean time to late reLapse

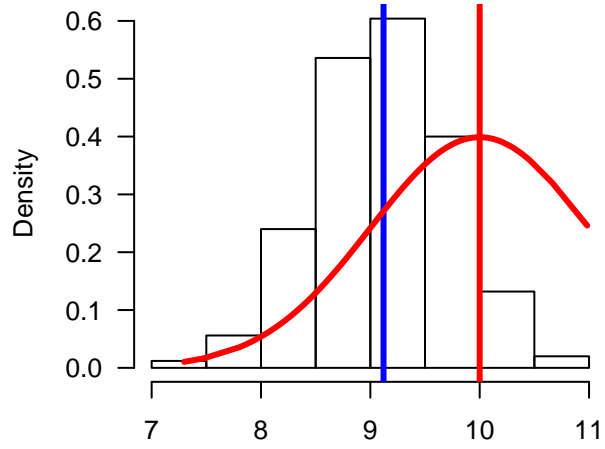


Logit early relapse

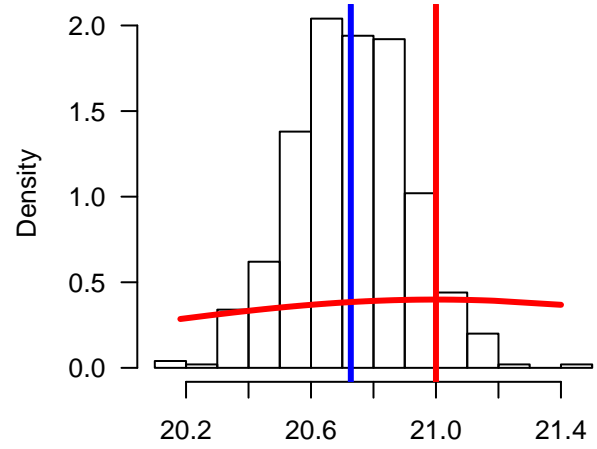




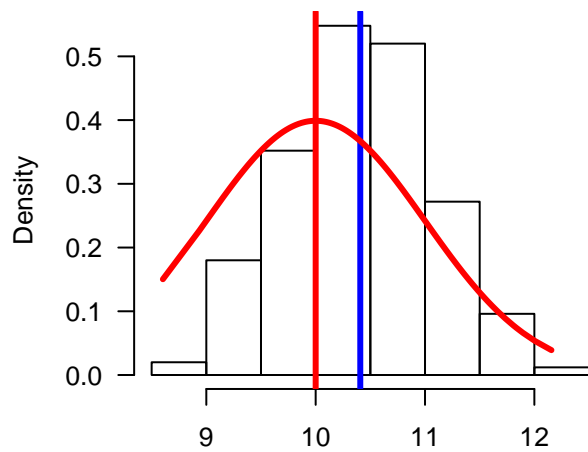
AS shape



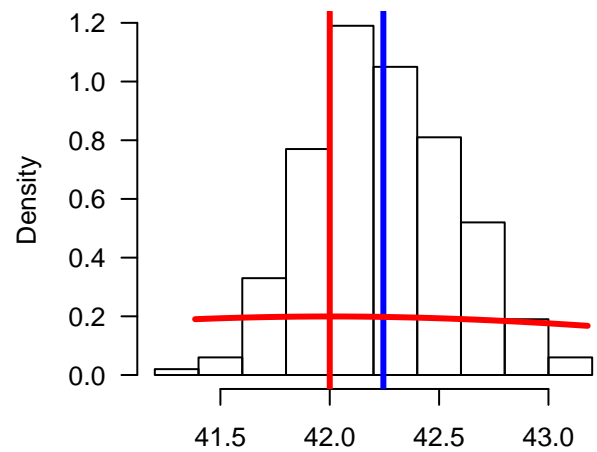
AS scale

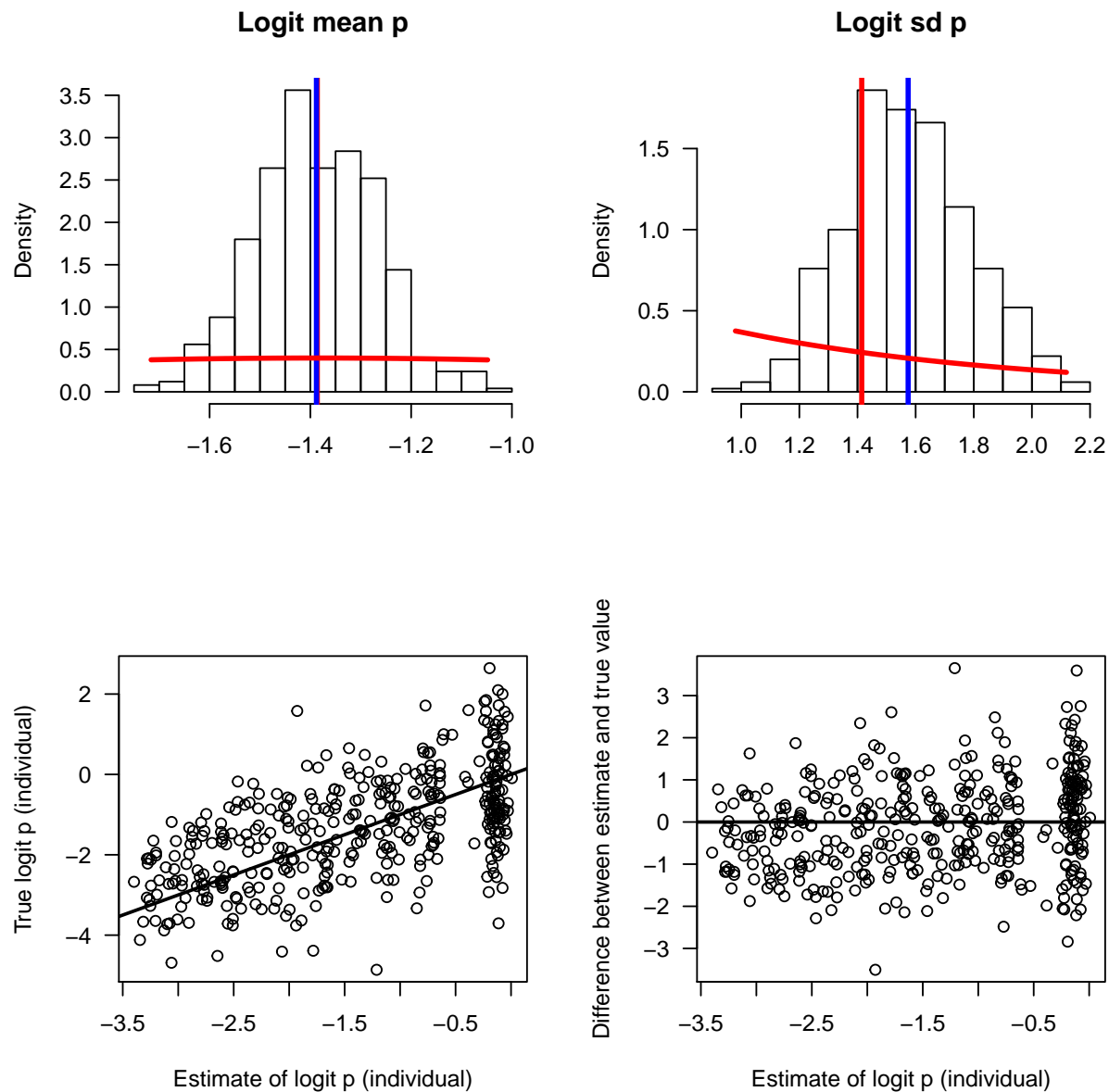


CQ shape



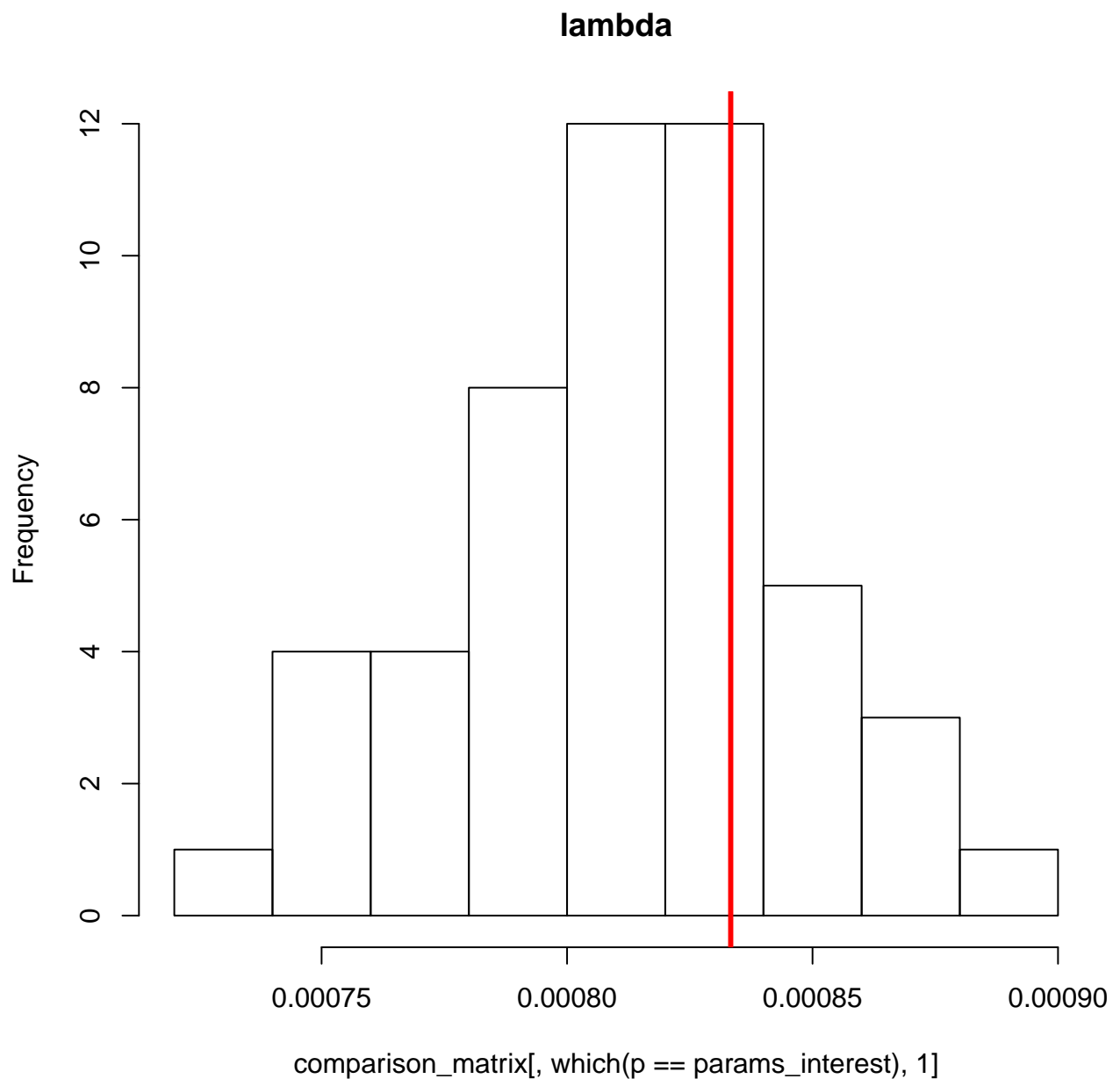
CQ scale

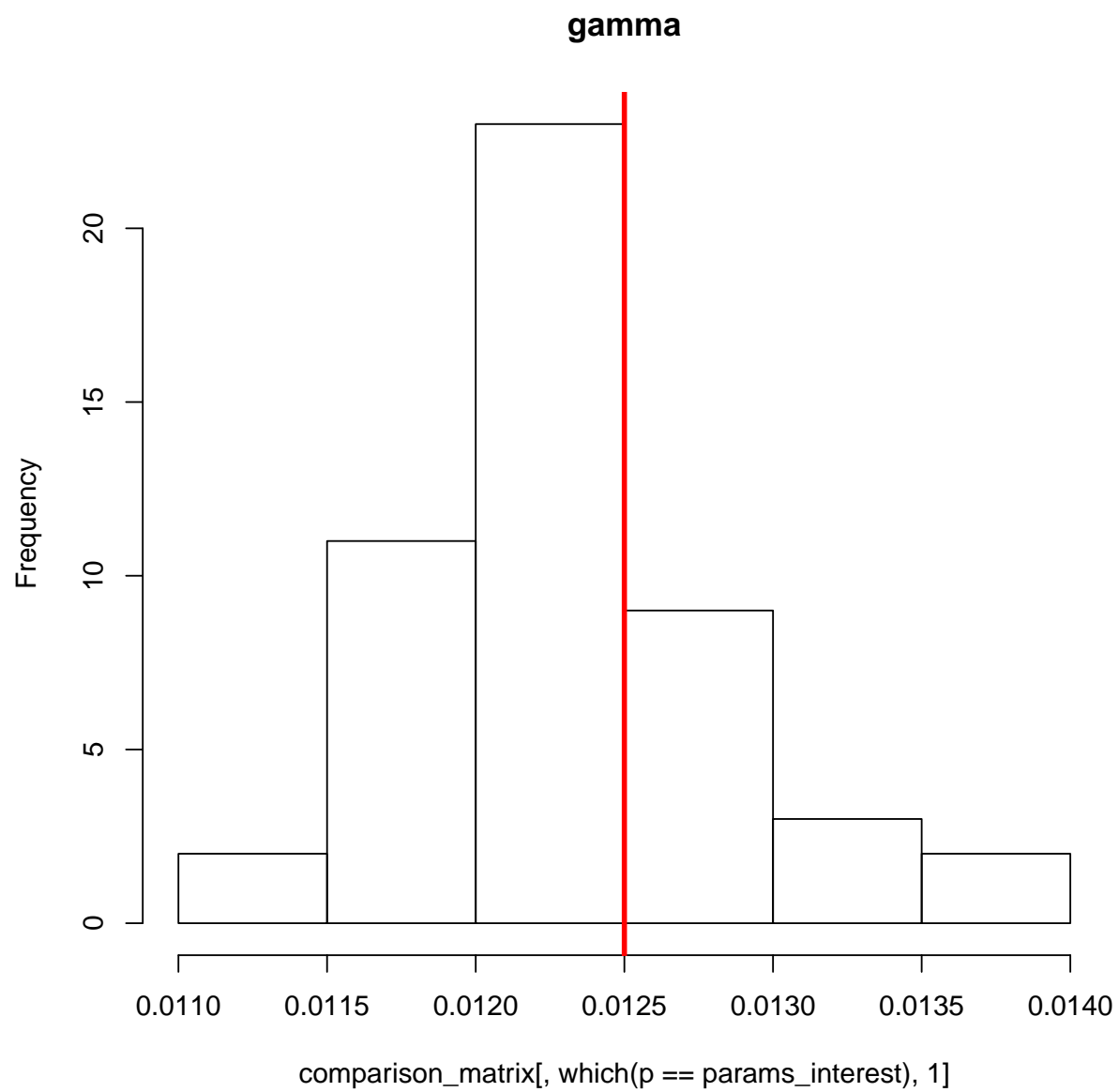


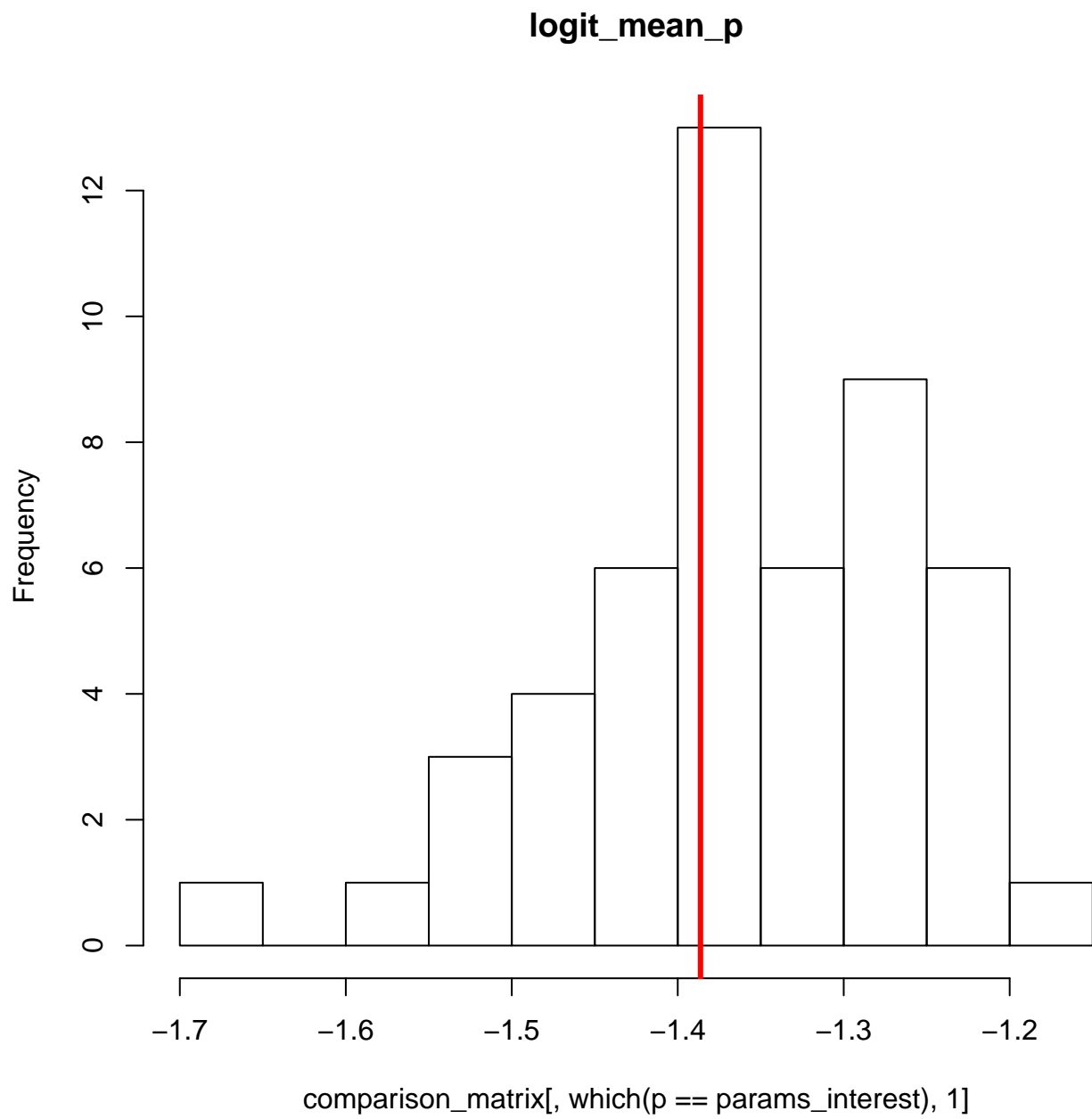


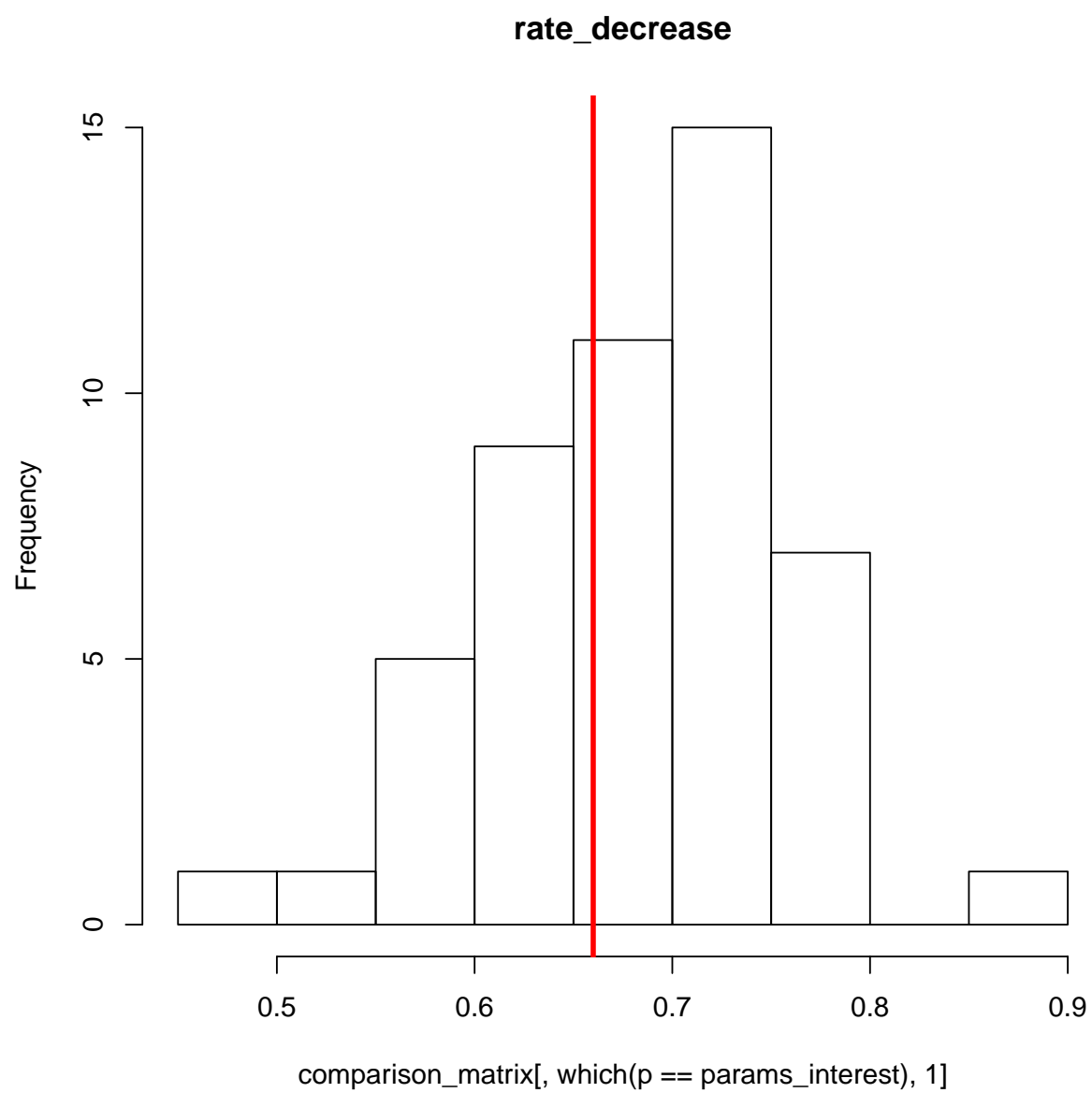
Multiple random simulations to assess systematic bias

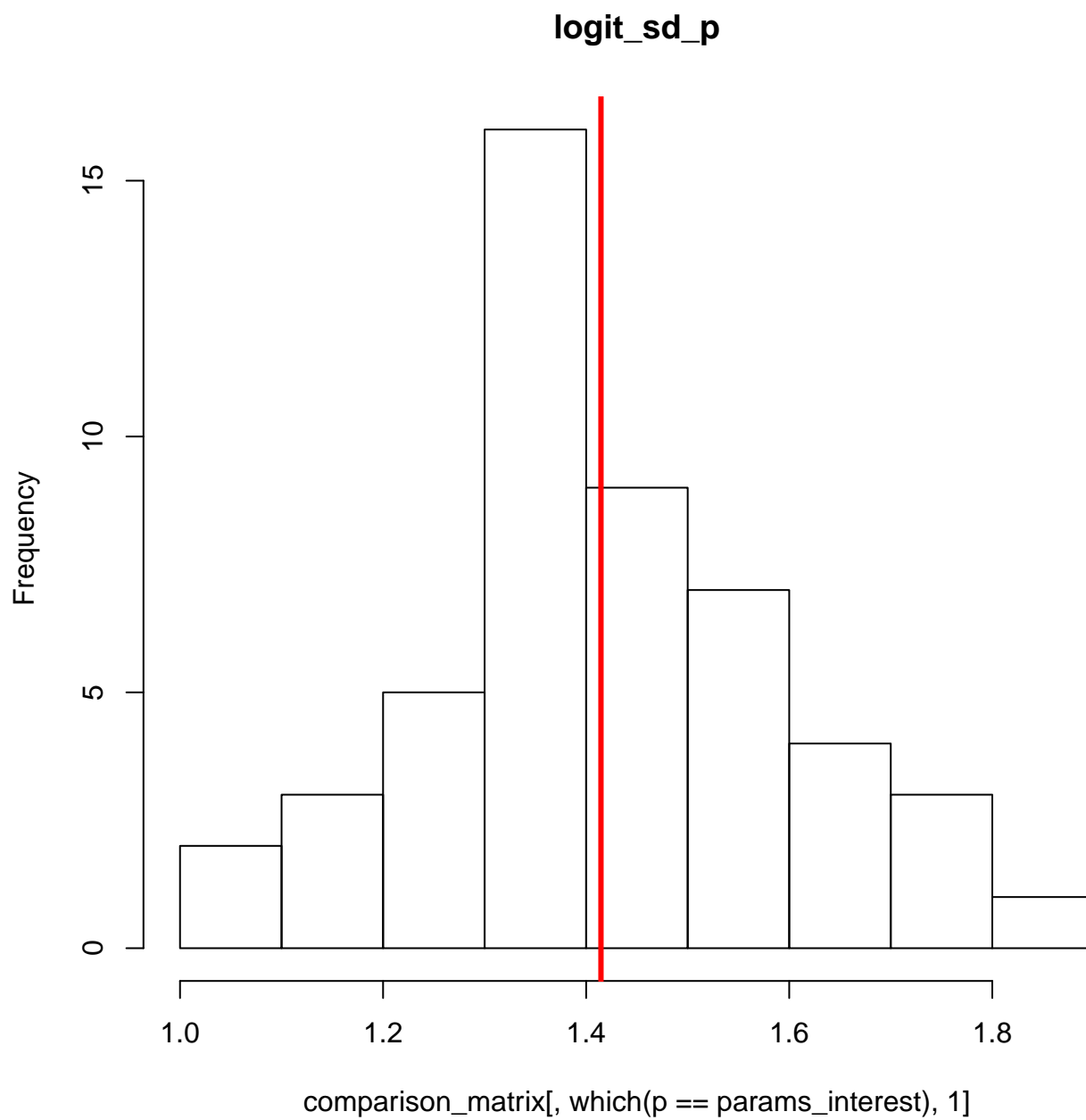
```
for(p in params_interest){
  hist(comparison_matrix[,which(p==params_interest),1], main=p)
  abline(v=unique(comparison_matrix[,which(p==params_interest),2]),
        lwd=3,col='red')
}
```

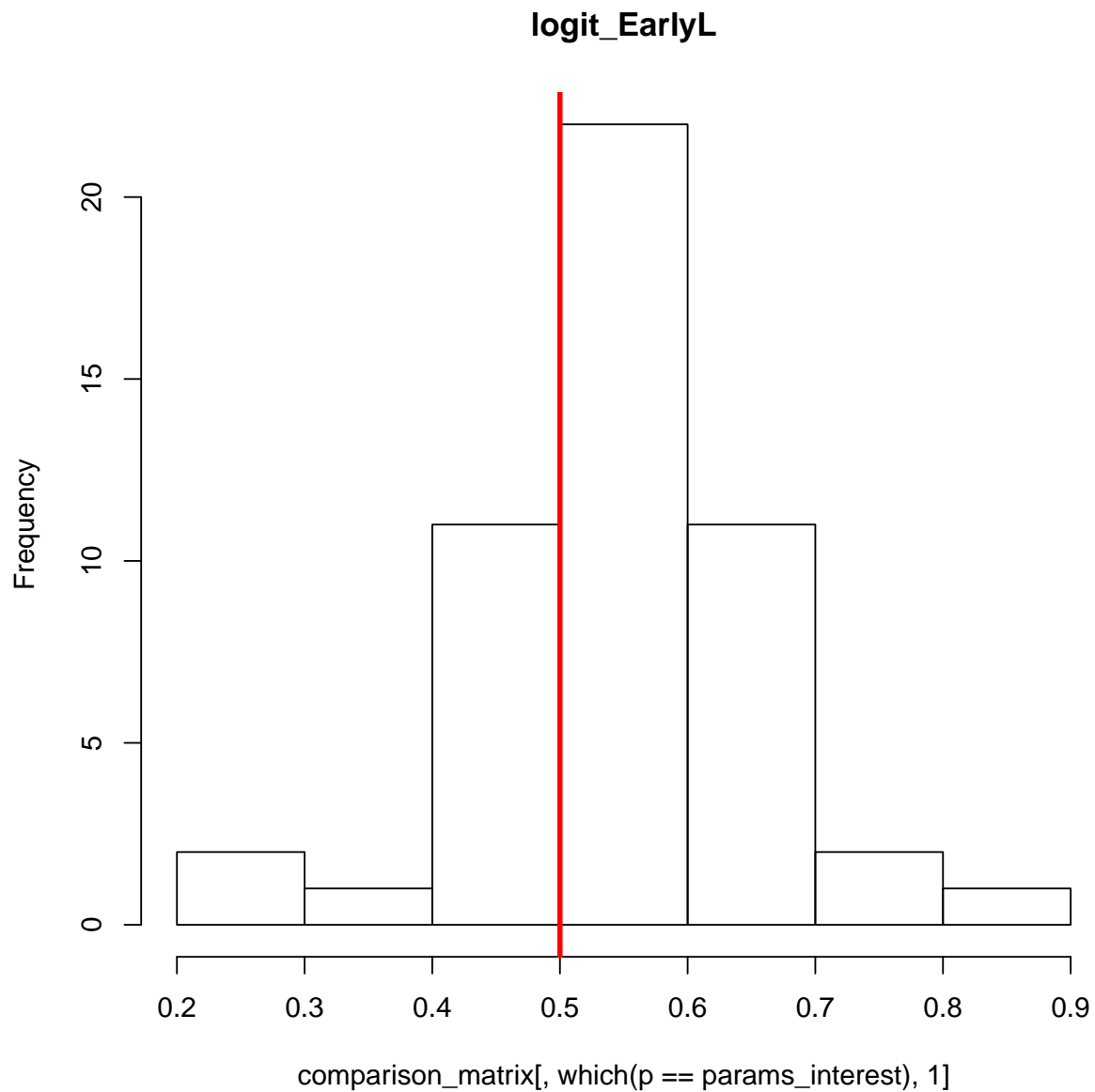












Data simulation for model 2

Model 2 does not assume 100% efficacy of primaquine.

Ground truth model parameters for simulated data:

```
params_M2 = c(params_M1,
  logit_mean_p_PMQ = logit(0.95),
  logit_sd_p_PMQ = (-logit(0.8)+logit(0.99))/1.96)
```

Generate simulated data under the assumptions of model 2

```
set.seed(7656728)
out2 = simulate_dataset(N_PMQ = N_PMQ,
```

```

      N_CQ = N_CQ,
      N_AS = N_AS,
      FUP_time = FUP_time,
      Study_Period = c(rep(1,N_PMQ/2),rep(2,N_PMQ/2 + N_AS + N_CQ)),
      data_generation_function = generate_patient_data_Model2,
      params = params_M2)
Simdata_Model2 = out2$Simdata
Simulation_truth2 = out2$Simulation_truth
table(Simulation_truth2$True_state, Simulation_truth2$Drug)

```

```

##
##           AS CHQ CHQ/PMQ
## EarlyRelapse 286 284    53
## LateRelapse  173 145    25
## Recrudescence 401 407    80
## Reinfection  167 168   946

```

Prior specification for stan fit:

```

Prior_params_M2 = c(Prior_params_M1,
  Hyper_logit_mean_pPMQ_mean = logit(0.95),
  Hyper_logit_mean_pPMQ_sd = (logit(0.95)-logit(0.7))/1.96,
  Hyper_logit_sd_pPMQ_lambda = 1)

```

Load or compile the stan model 2

```

if(RECOMPILE_MODELS){
  writeLines('Compiling model 2....')
  source('../Timing_Model/StanModel2.R')
  save(Timing_Model2, file = '../RData/TimingModel/Timing_Model2.RData')
} else {
  load('../RData/TimingModel/Timing_Model2.RData')
}

```

Fit stan model 2 to simulated data from model 2

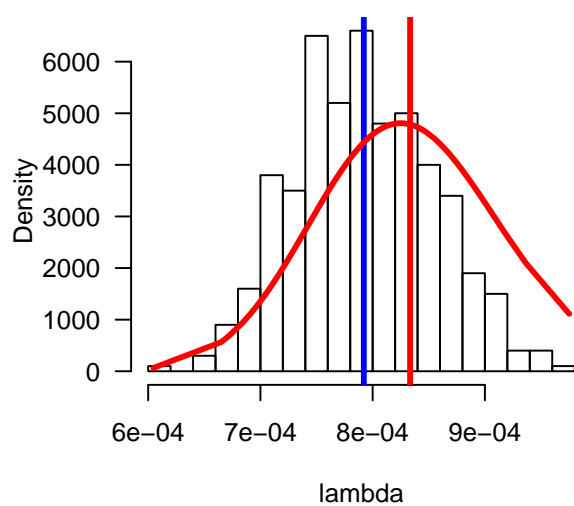
Plot output, comparing prior, ground truth and model estimate:

```

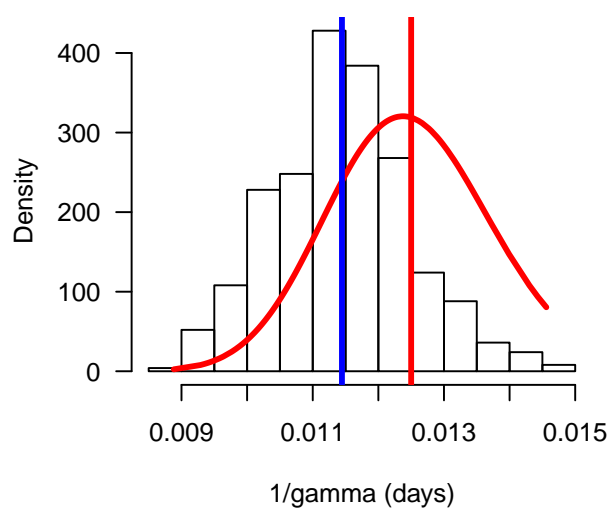
plot_output_model2(thetas_mod2,Simulation_truth2,Simdata_Model2,Prior_params_M2)

```

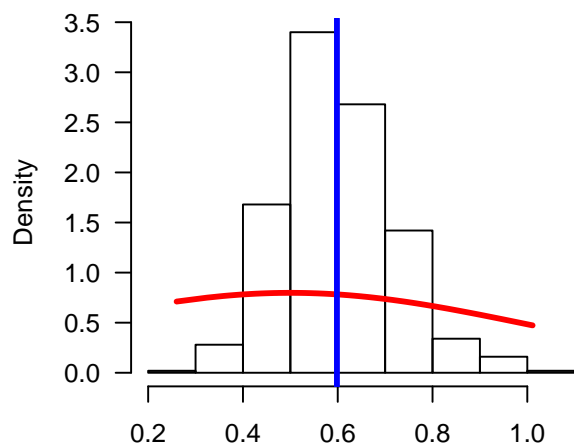
Reinfection rate

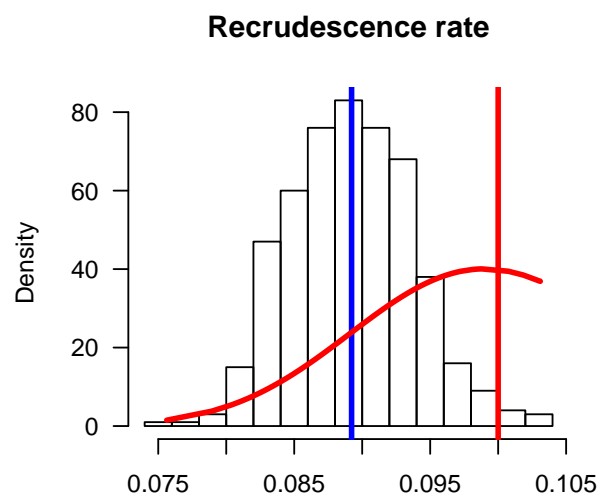
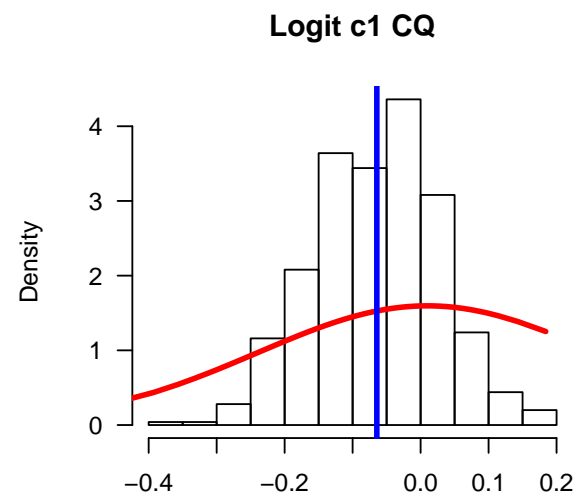
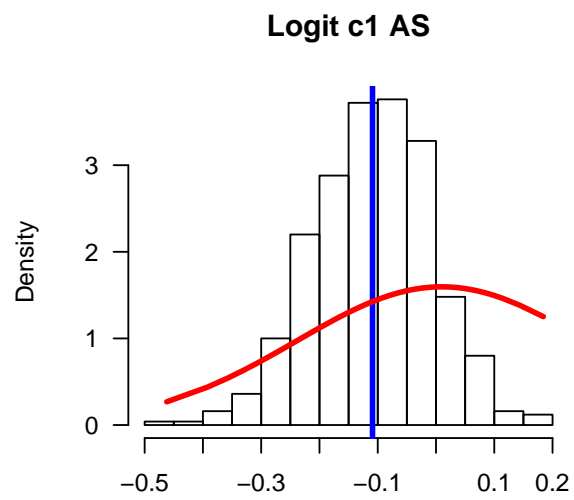


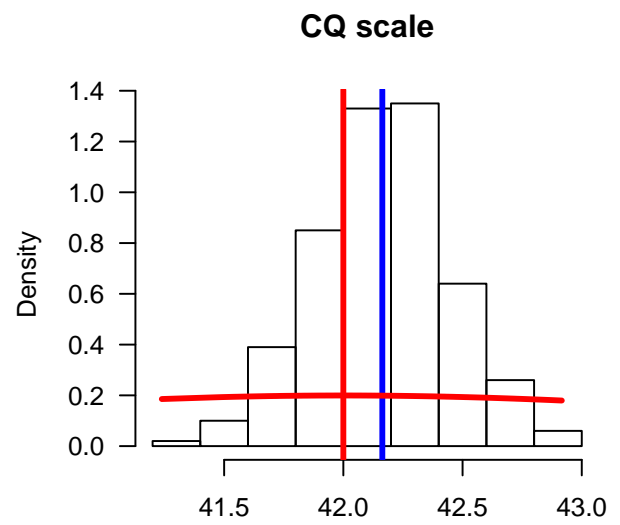
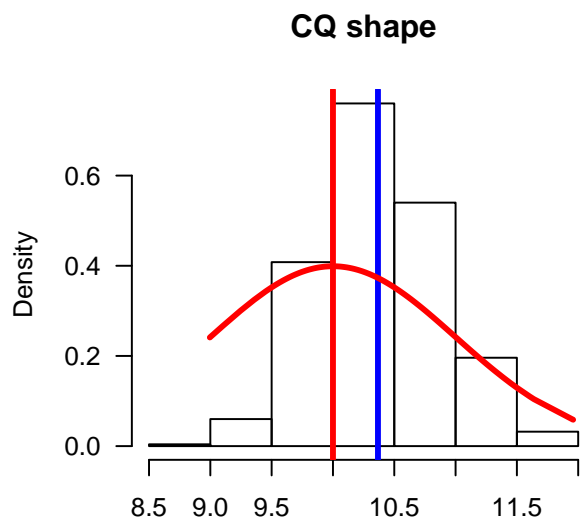
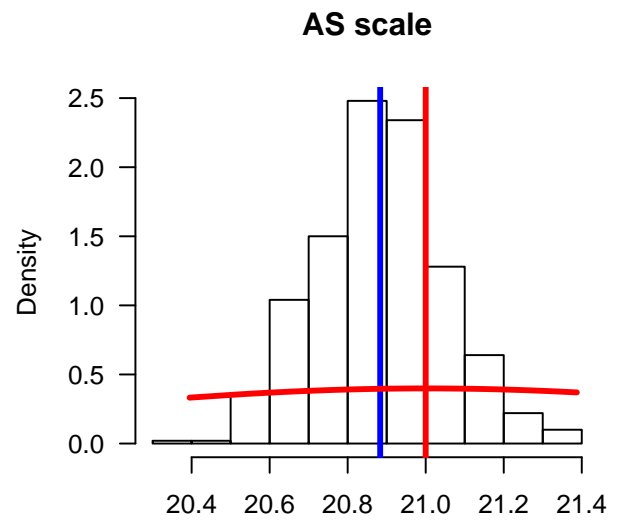
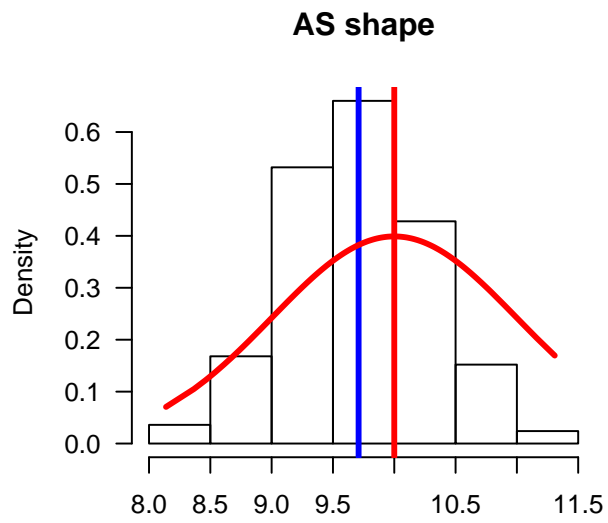
Mean time to late reLapse

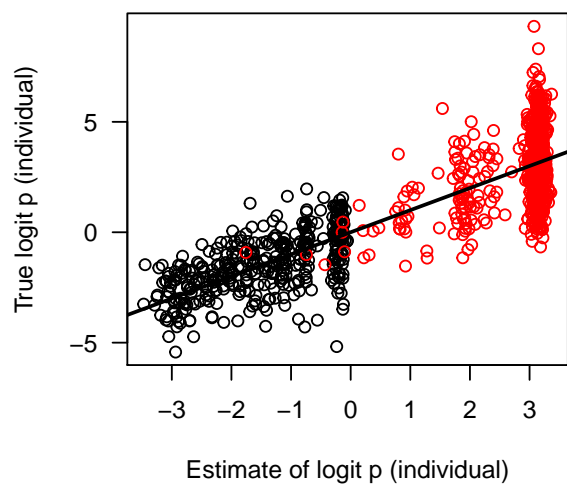
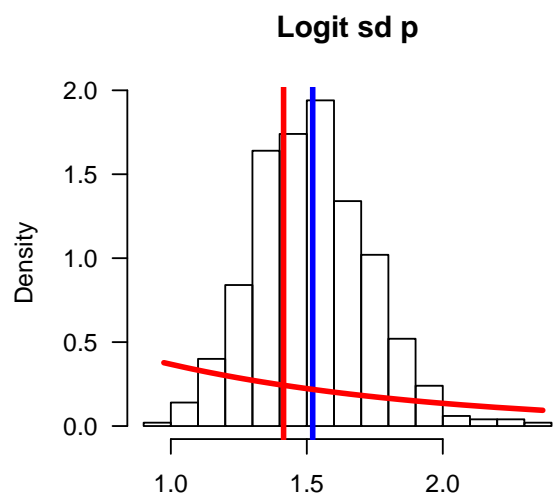
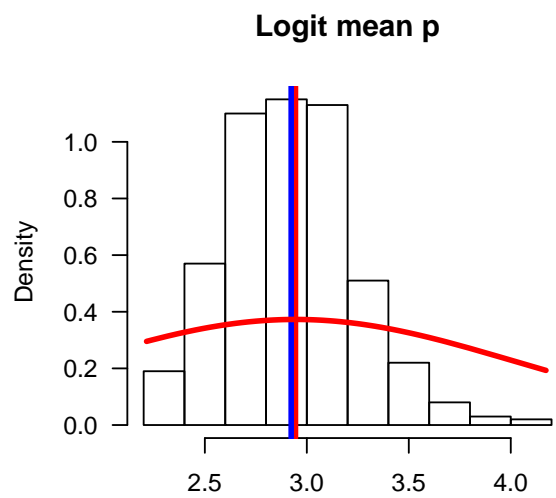
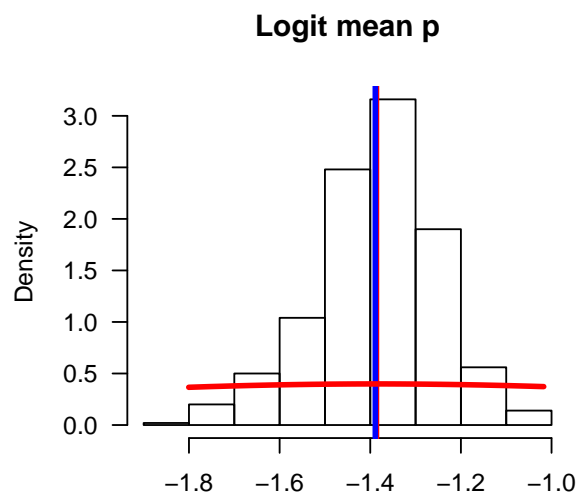


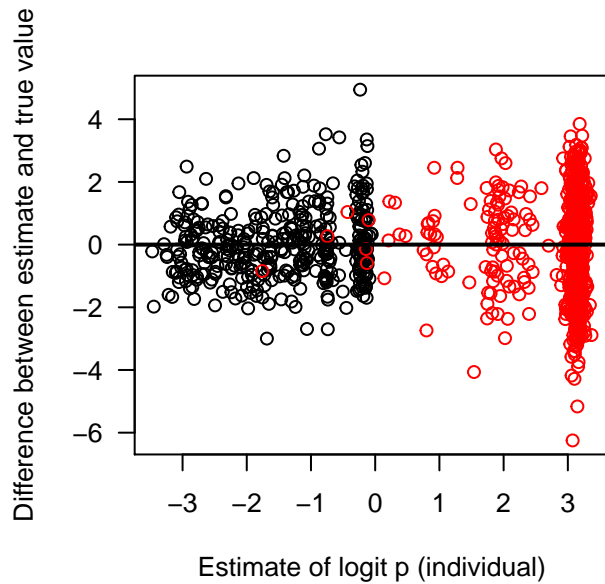
Logit early relapse











Compare summary statistics from simulated data and estimated:

```
# the true failure rate in the primaquine arm
sum(Simulation_truth2$True_state[Simulation_truth2$Drug=="CHQ/PMQ"] != 'Reinfection')/sum(Simulation_tr

## [1] 0.1431159

classification_mod2 = apply(thetas_mod2$prob_labels,c(2,3),quantile, probs=0.975)
1-sum(classification_mod2[Simdata_Model2$Drug==2,1])/sum(Simdata_Model2$Drug==2)

## [1] 0.07240821

classification_mod2 = apply(thetas_mod2$prob_labels,c(2,3),median)
1-sum(classification_mod2[Simdata_Model2$Drug==2,1])/sum(Simdata_Model2$Drug==2)

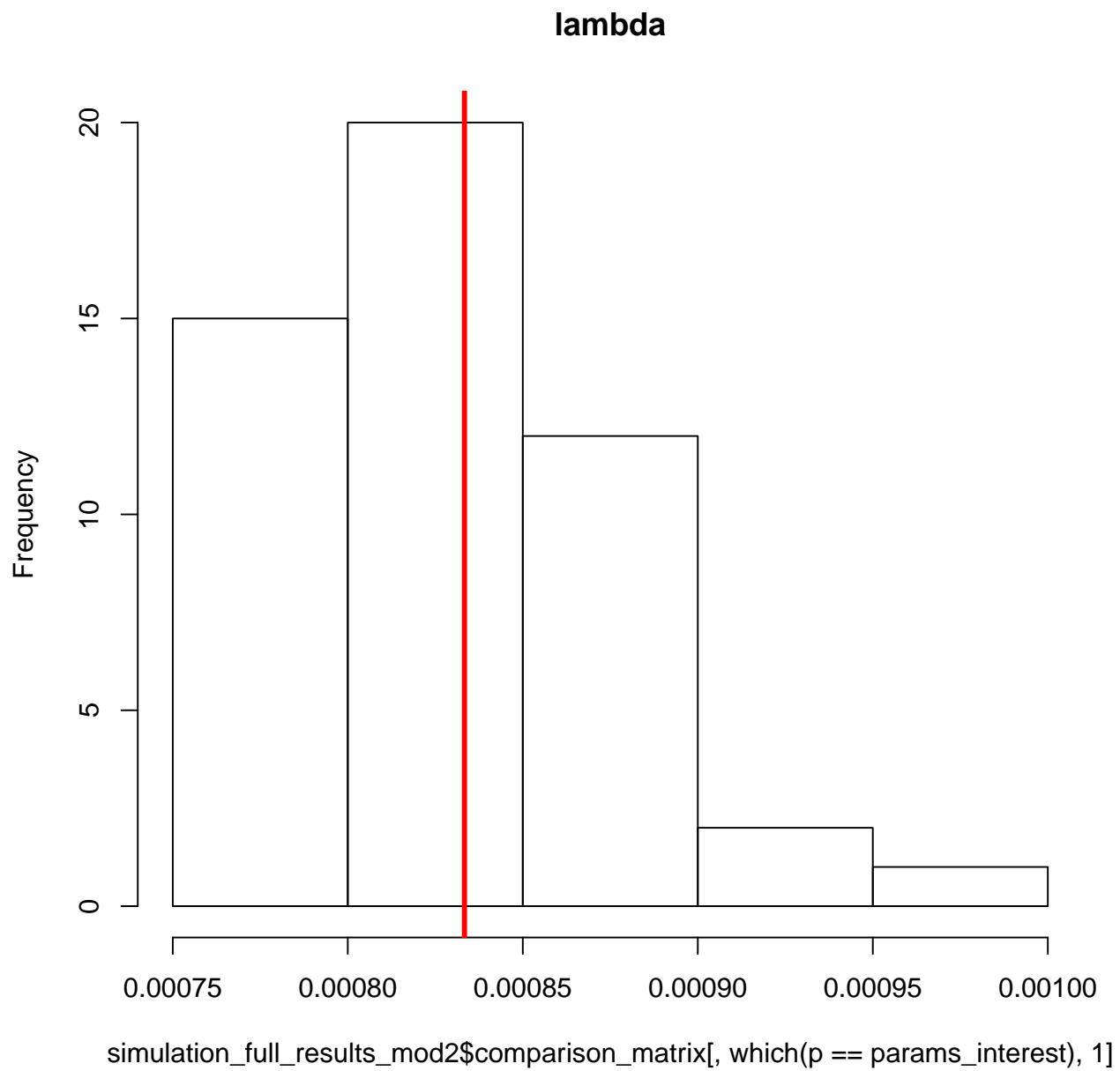
## [1] 0.1491698

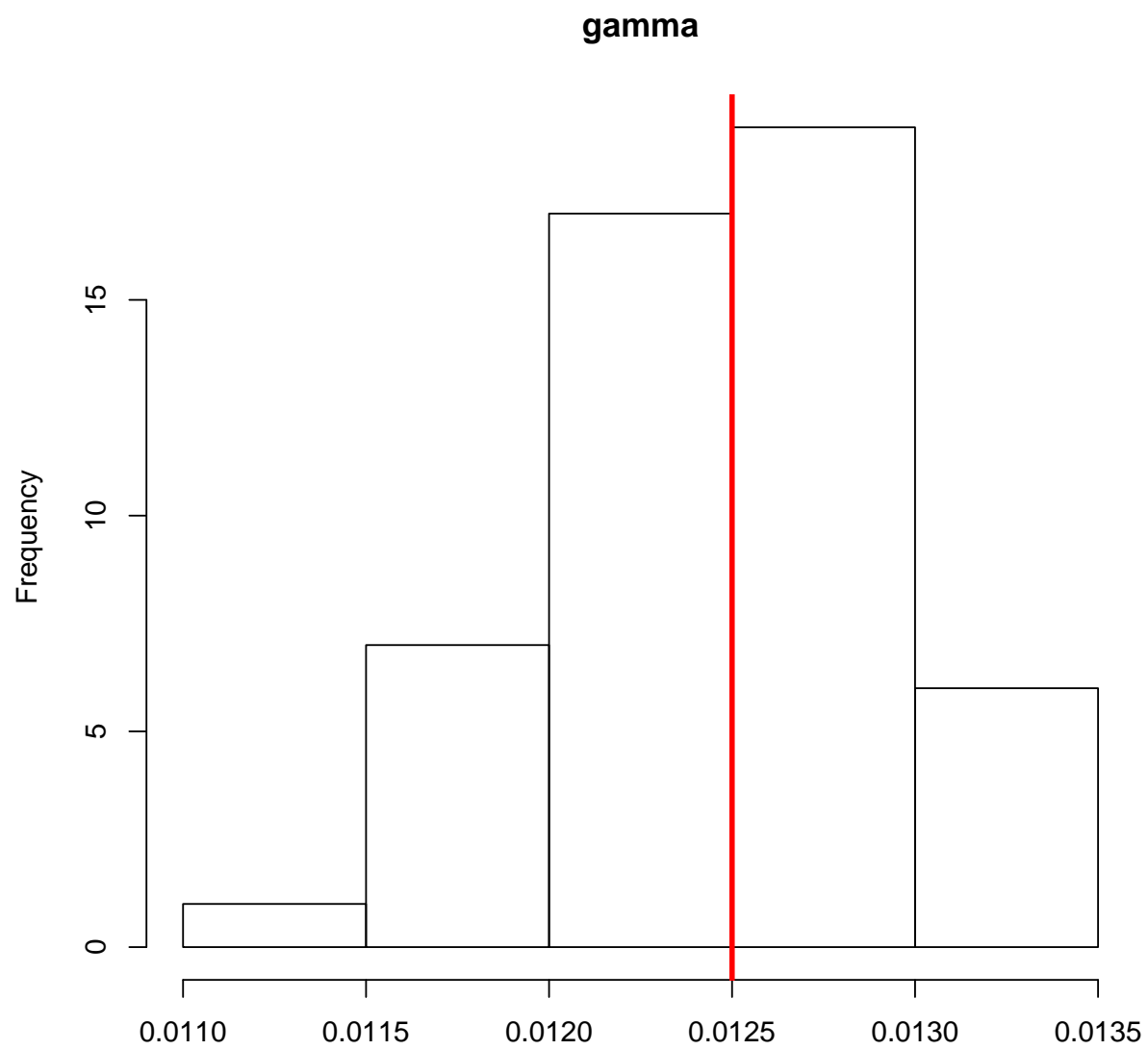
classification_mod2 = apply(thetas_mod2$prob_labels,c(2,3),quantile, probs=0.025)
1-sum(classification_mod2[Simdata_Model2$Drug==2,1])/sum(Simdata_Model2$Drug==2)

## [1] 0.2166946
```

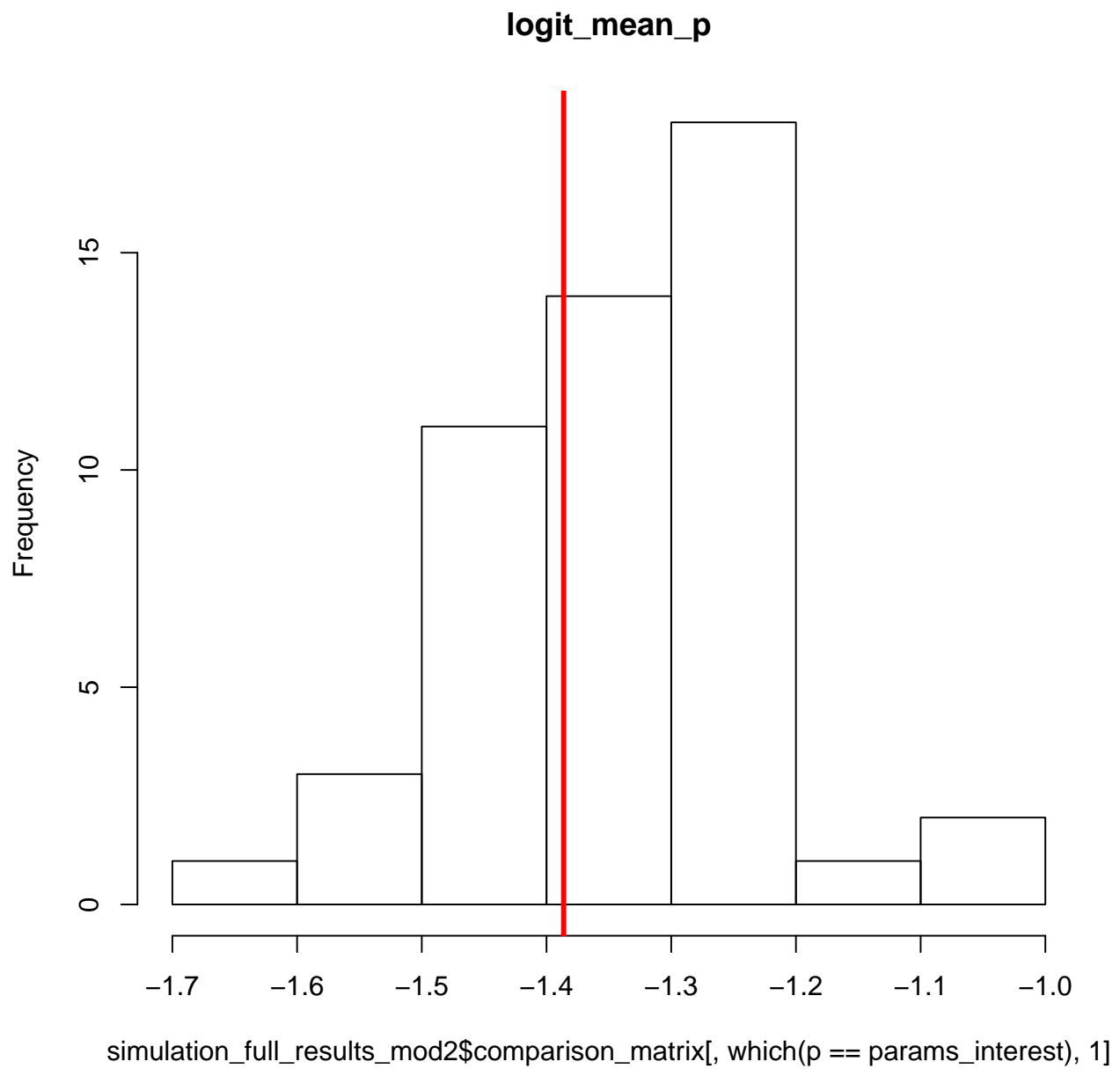
Multiple random simulations to assess systematic bias

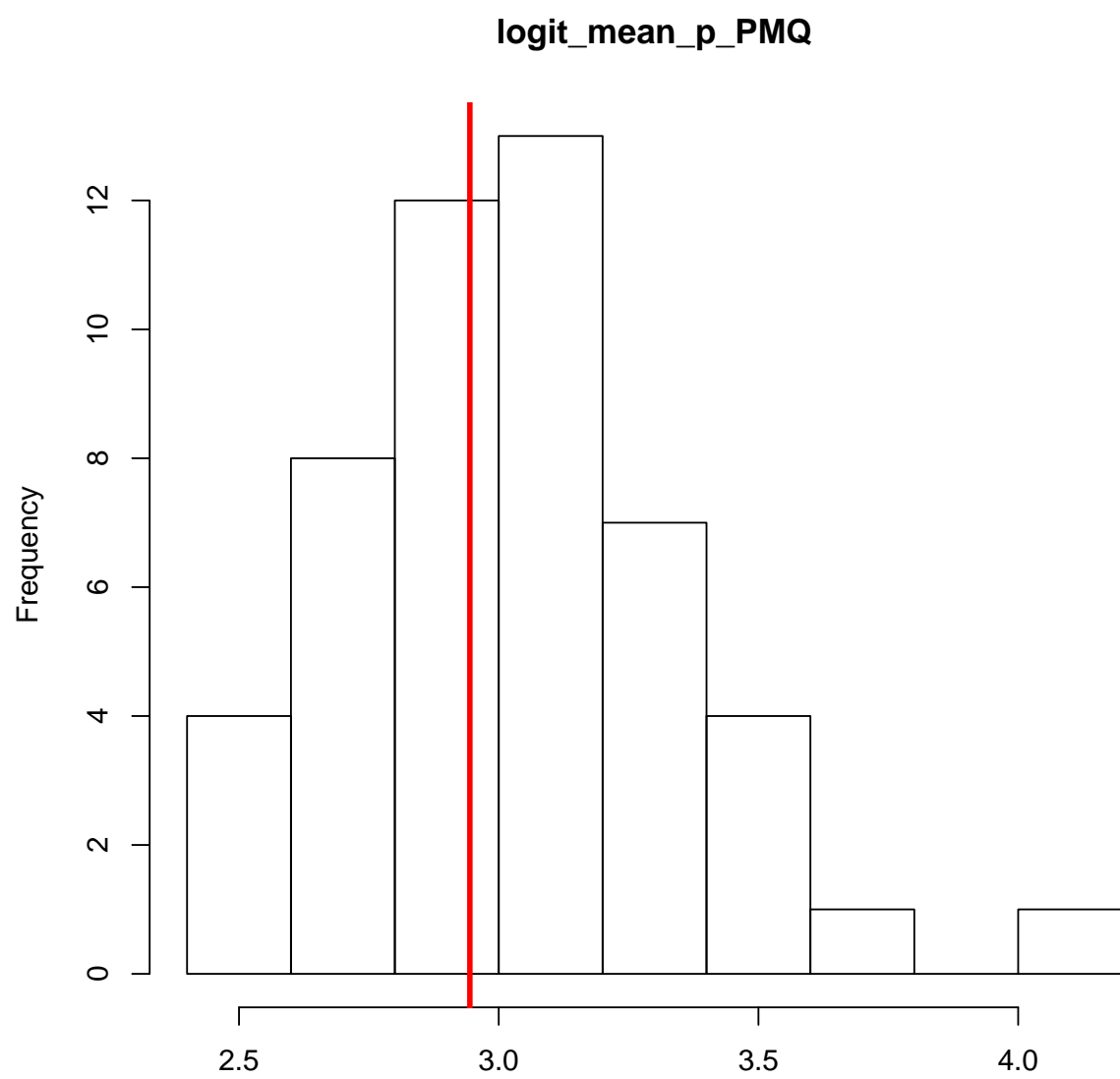
```
for(p in params_interest){
  hist(simulation_full_results_mod2$comparison_matrix[,which(p==params_interest),1], main=p)
  abline(v=unique(simulation_full_results_mod2$comparison_matrix[,which(p==params_interest),2]),col='red')
}
```



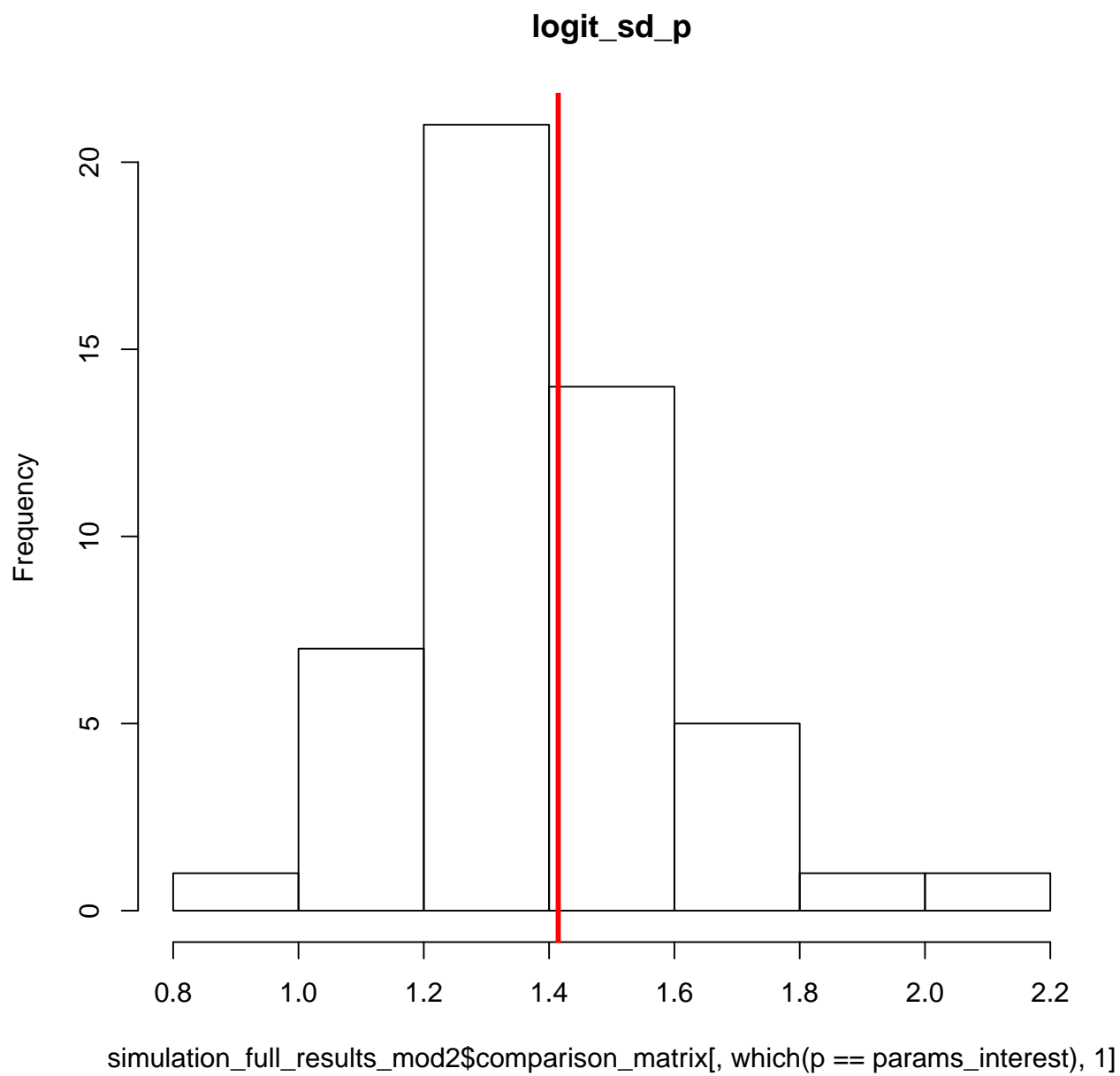


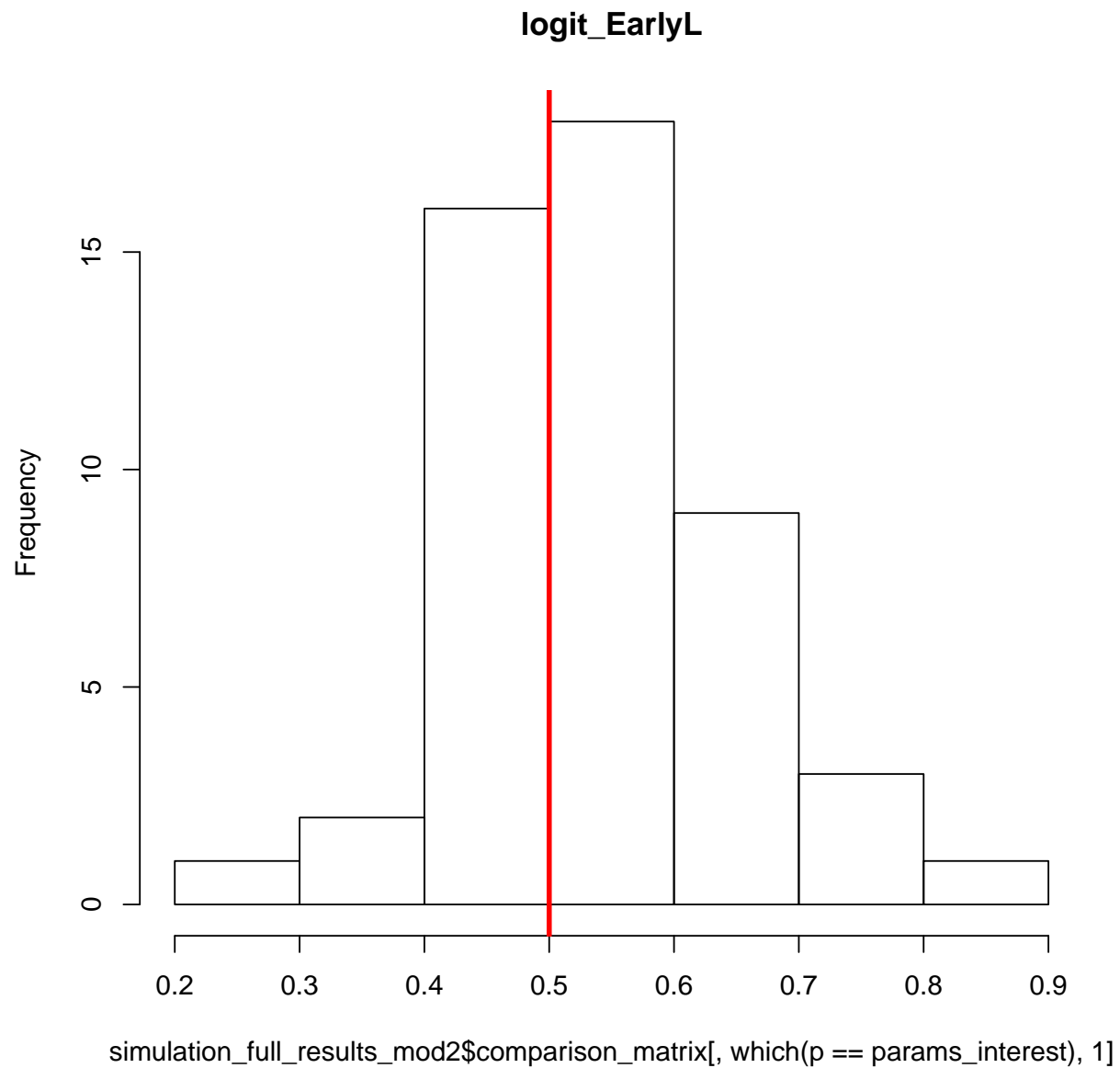
`simulation_full_results_mod2$comparison_matrix[, which(p == params_interest), 1]`



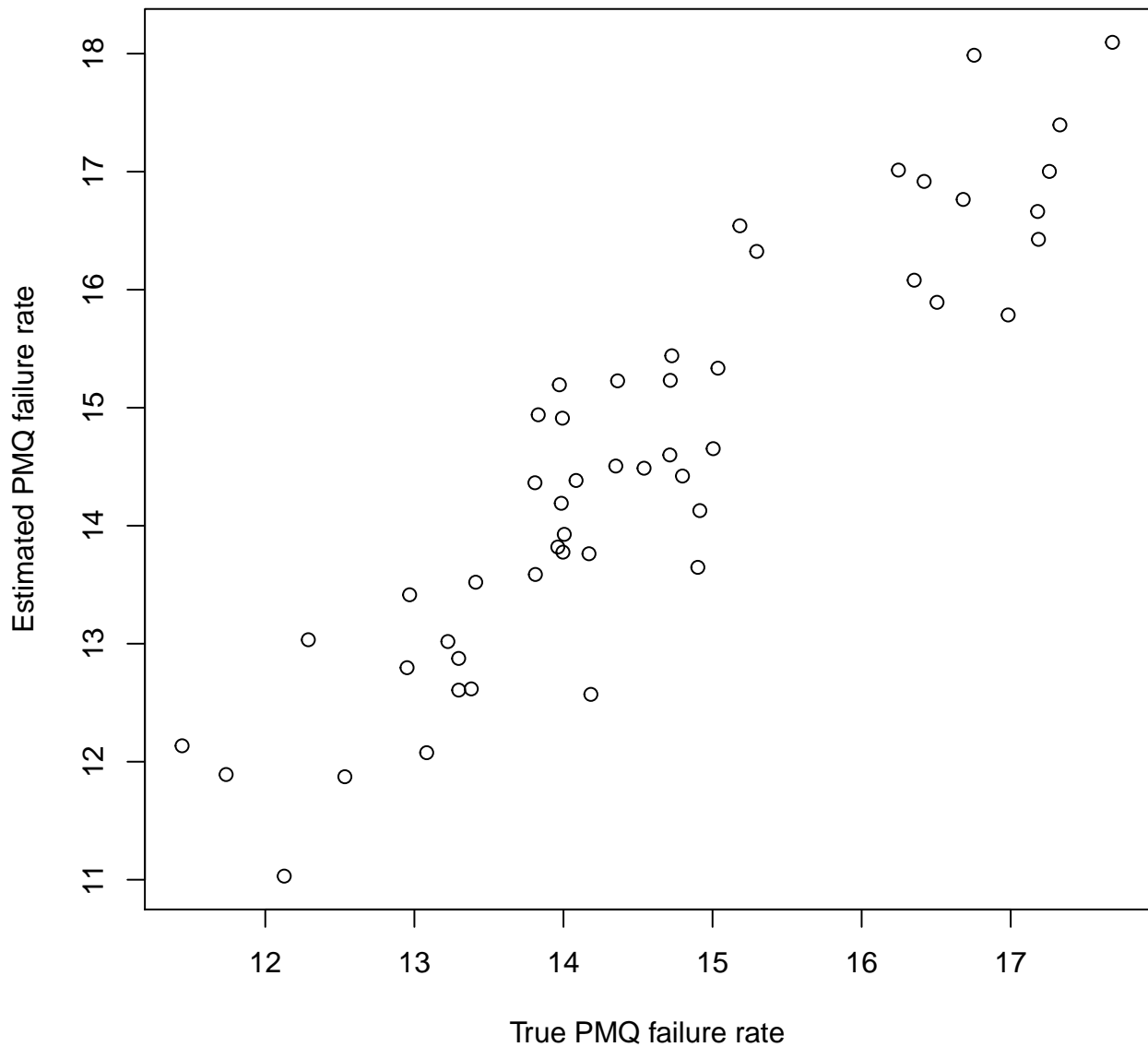


`simulation_full_results_mod2$comparison_matrix[, which(p == params_interest), 1]`





```
plot(100*simulation_full_results_mod2$failure_matrix[,1],
     100*simulation_full_results_mod2$failure_matrix[,2],
     xlab='True PMQ failure rate', ylab = 'Estimated PMQ failure rate')
lines(c(0,1),c(0,1))
```

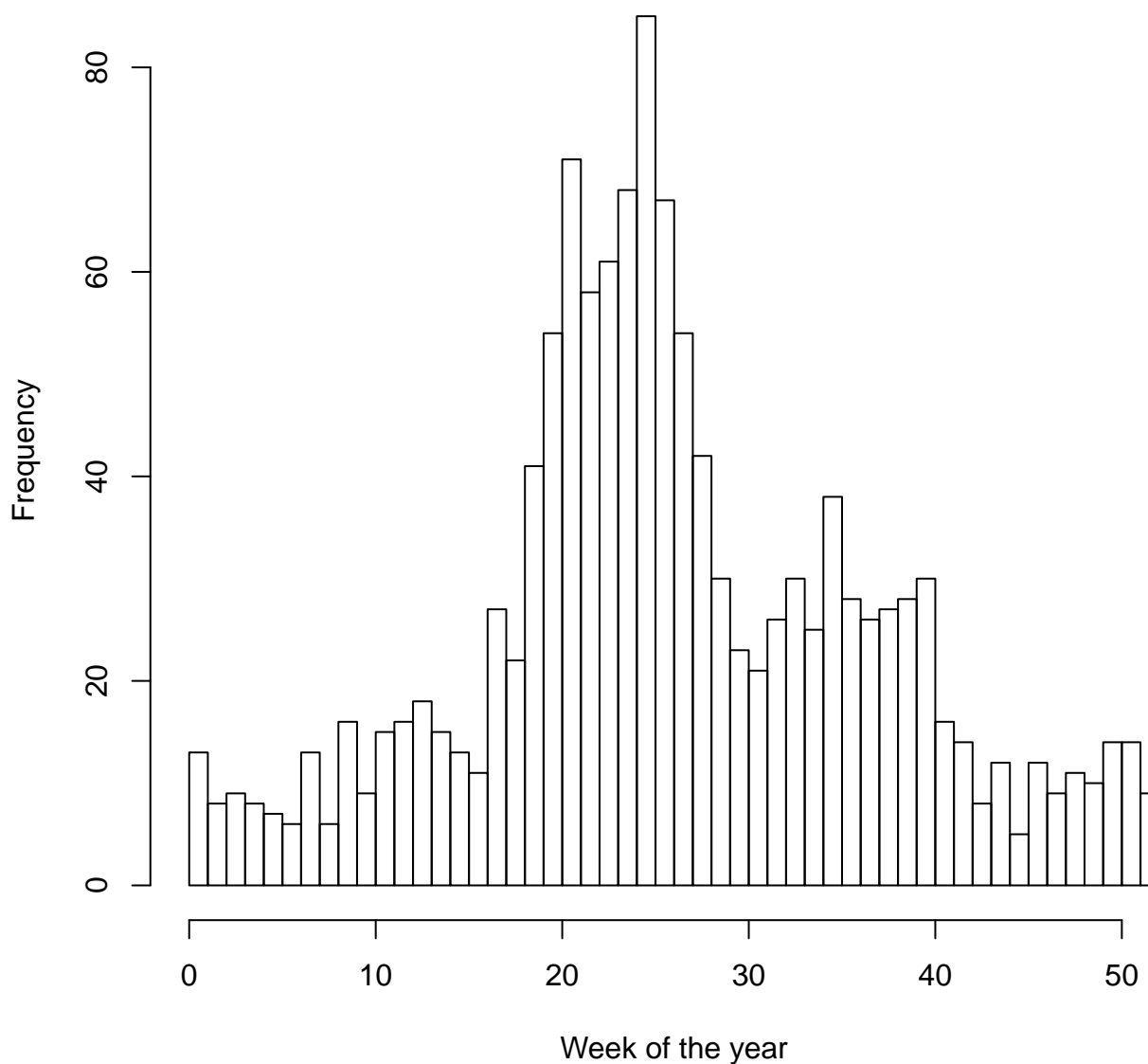



Simulating seasonal variation

Estimate the empirical seasonal reinfection distribution from enrollment episodes. This assumes that the majority of enrollment episodes are reinfections which should be approximately true.

```
load('../RData/TimingModel/Combined_Time_Event.RData')
Combined_Time_Data = filter(Combined_Time_Data, Censored == -1)
ind = Combined_Time_Data$WeekTime > 52
Combined_Time_Data$WeekTime[ind] = Combined_Time_Data$WeekTime[ind] - 52
hist(Combined_Time_Data$WeekTime, breaks = 0:52,
     main = 'Week of enrollment', xlab = 'Week of the year')
```

Week of enrollment



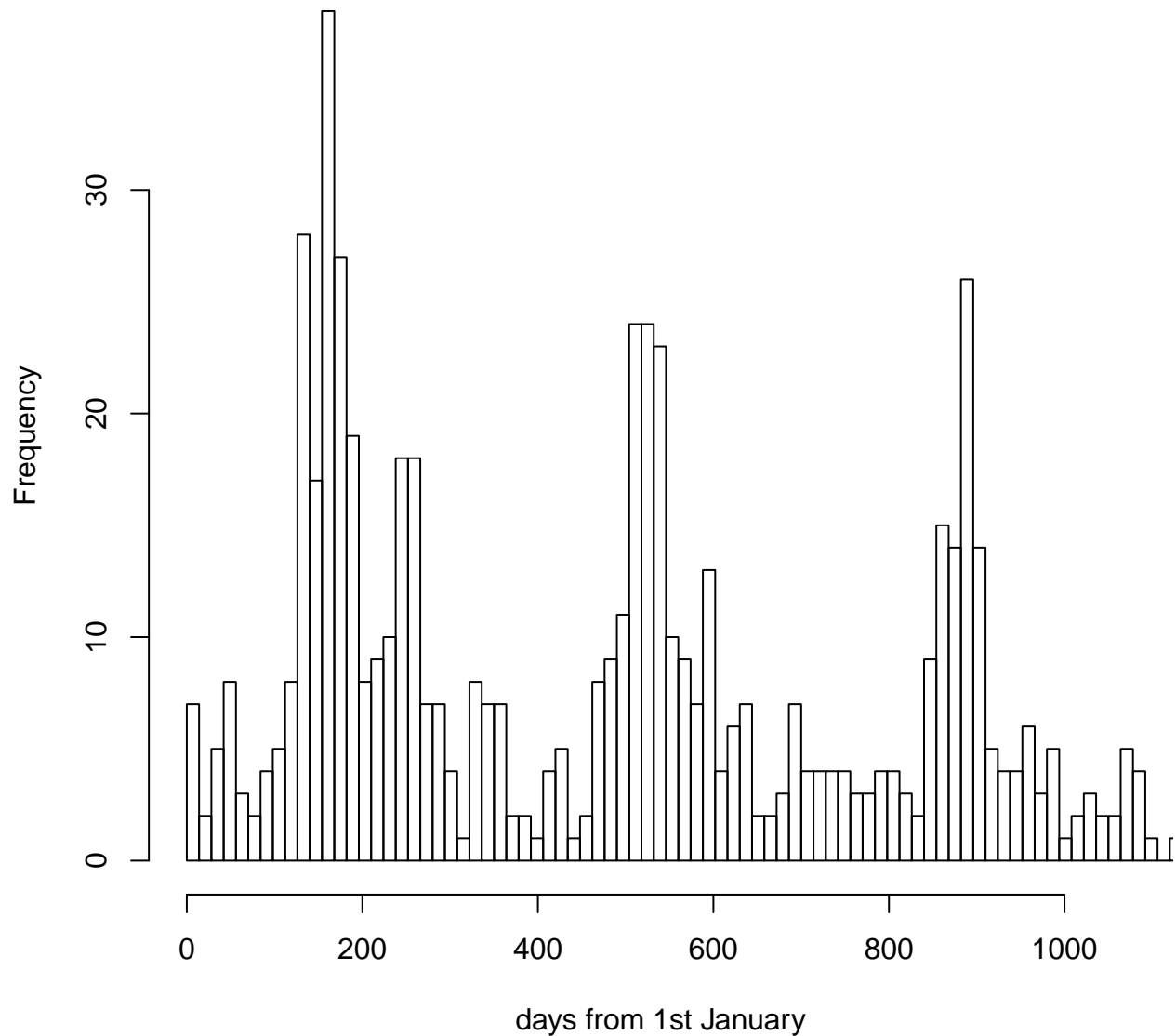
```
# the vector we can use for empirical distribution sampling
seasonal_sampling_vector = Combined_Time_Data$WeekTime
```

Generate patient data under the assumptions of model 1 with the additional assumption of seasonality for reinfection.

```
# test to show this does the right thing
set.seed(7576)
ys = sapply(1:1000, FUN = function(x, seasonal_sampling_vector){
  generate_reinfection_time_seasonal(params = params_M1,
    week_time = 0,
    seasonal_sampling_vector = seasonal_sampling_vector,
    Study_Period = 1)
}, seasonal_sampling_vector)
hist(ys, breaks = seq(0,20000,by=14), xlim = c(0,3*360),
```

```
main='Reinfection times',
xlab='days from 1st January')
```

Reinfection times



Generate data with these parameters:

```
set.seed(87678)
tic()
out3 = simulate_dataset(N_PMQ = N_PMQ, N_CQ = N_CQ,
                        N_AS = N_AS, FUP_time = FUP_time,
                        data_generation_function = generate_patient_data_Model2_Seasonal,
                        params = params_M2,
                        seasonal_sampling_vector = seasonal_sampling_vector,
                        Study_Period = c(rep(1, N_PMQ/2), rep(2, N_PMQ/2 + N_AS + N_CQ)))
toc()
```

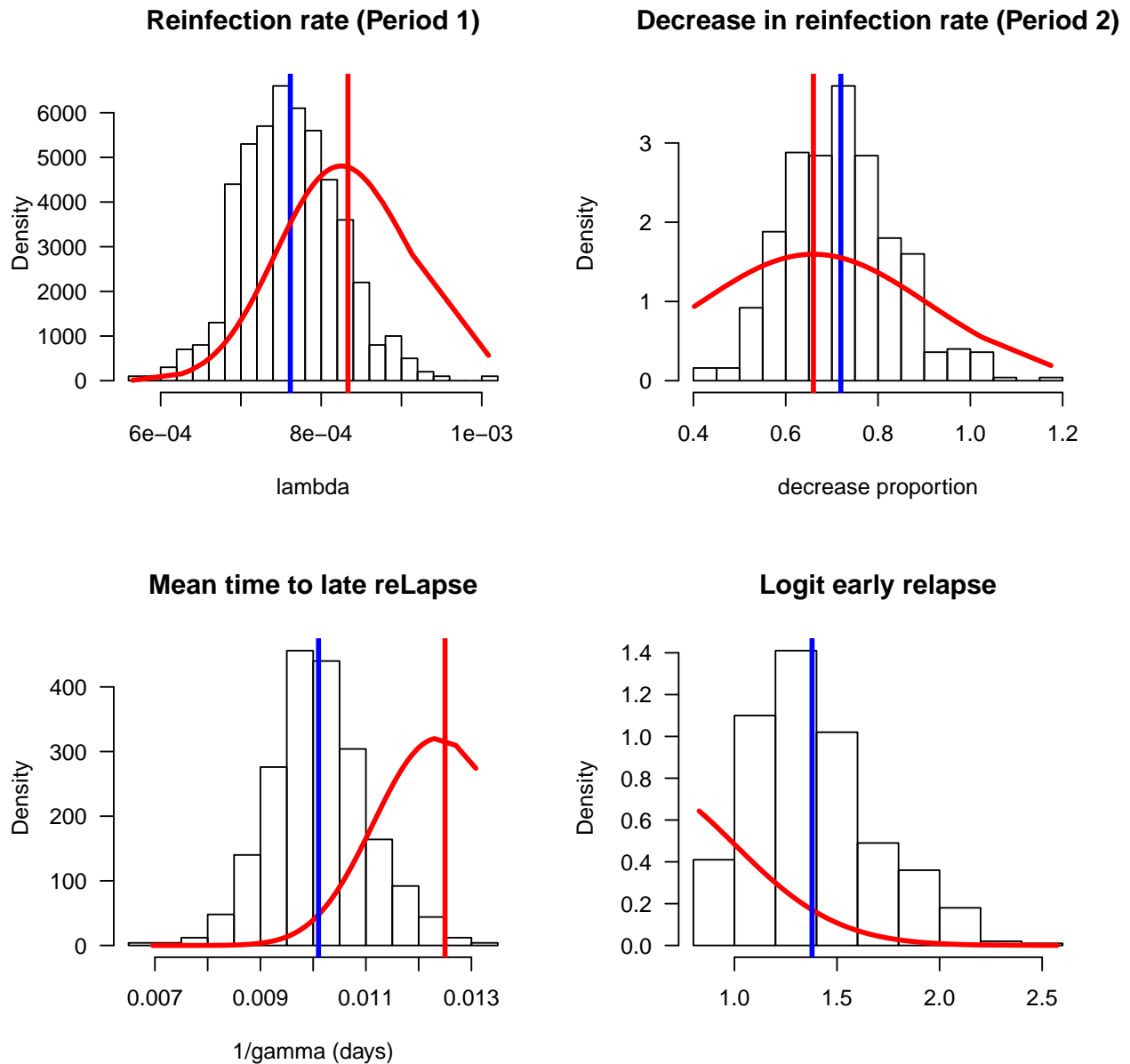
```
## 0.872 sec elapsed
```

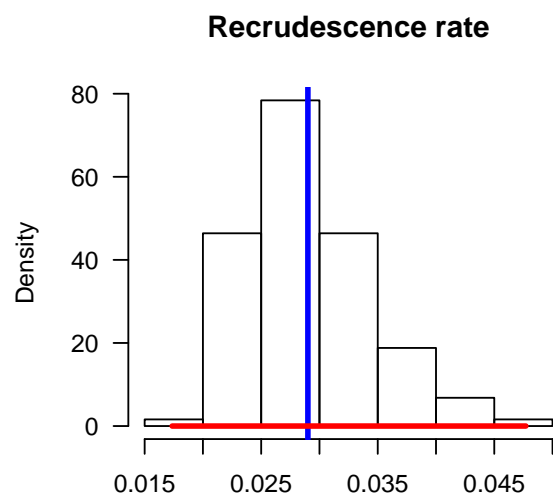
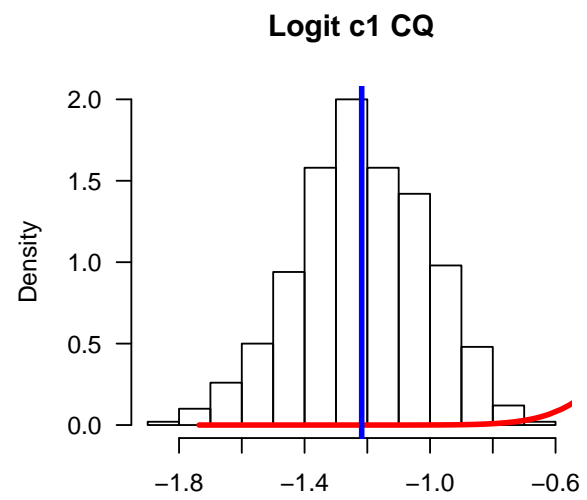
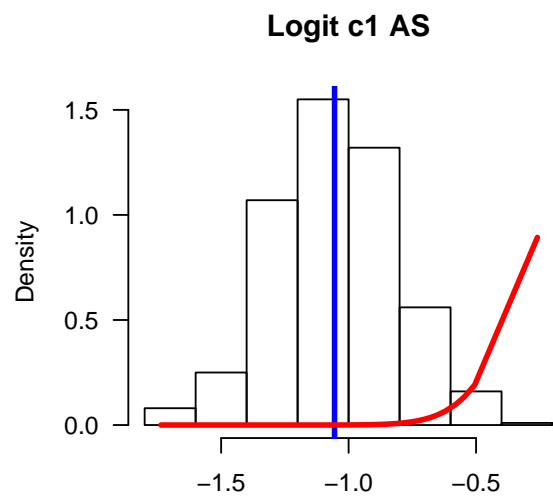
```
Simdata_Model_Seasonal = out3$Simdata  
Simulation_truth_Seasonal = out3$Simulation_truth
```

Fit stan model to simulated data where assumption of constant reinfection rate is wrong

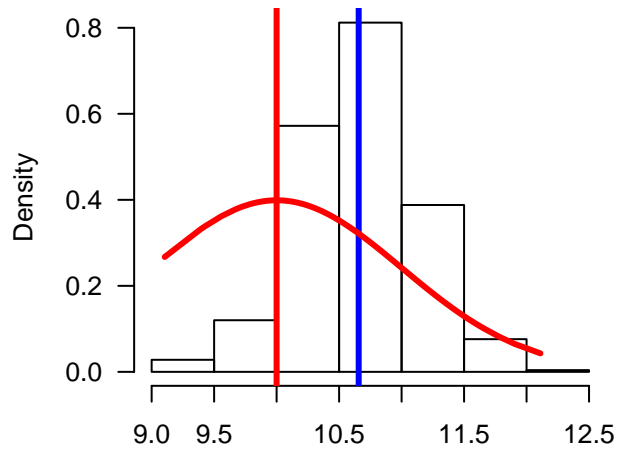
Plot output, comparing prior, ground truth and model estimate:

```
plot_output_model1(thetas_mod_seasonal,Simulation_truth_Seasonal,  
                   Simdata_Model_Seasonal,Prior_params_M1)
```

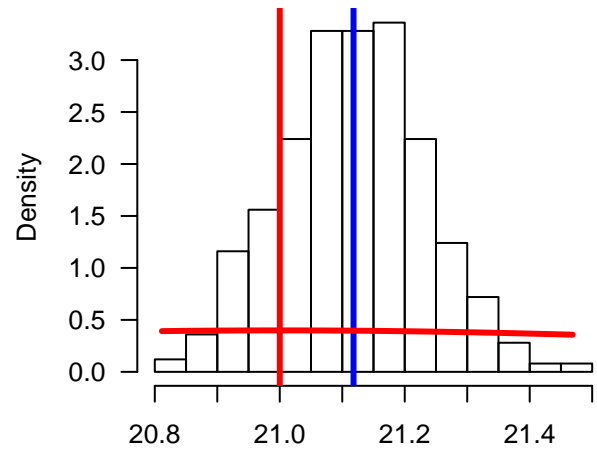




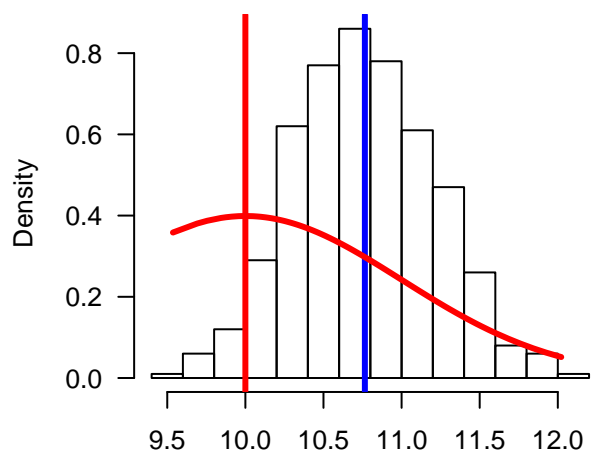
AS shape



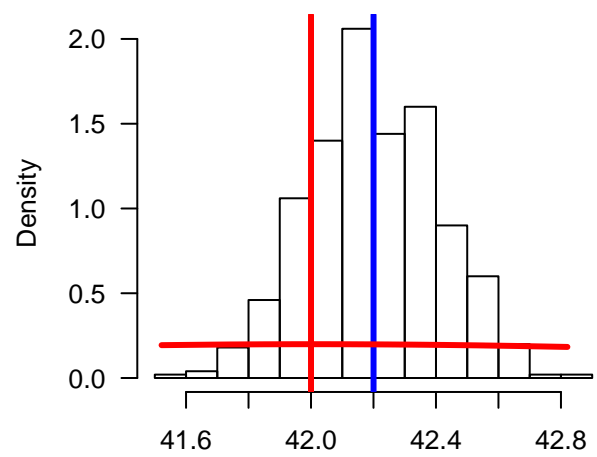
AS scale

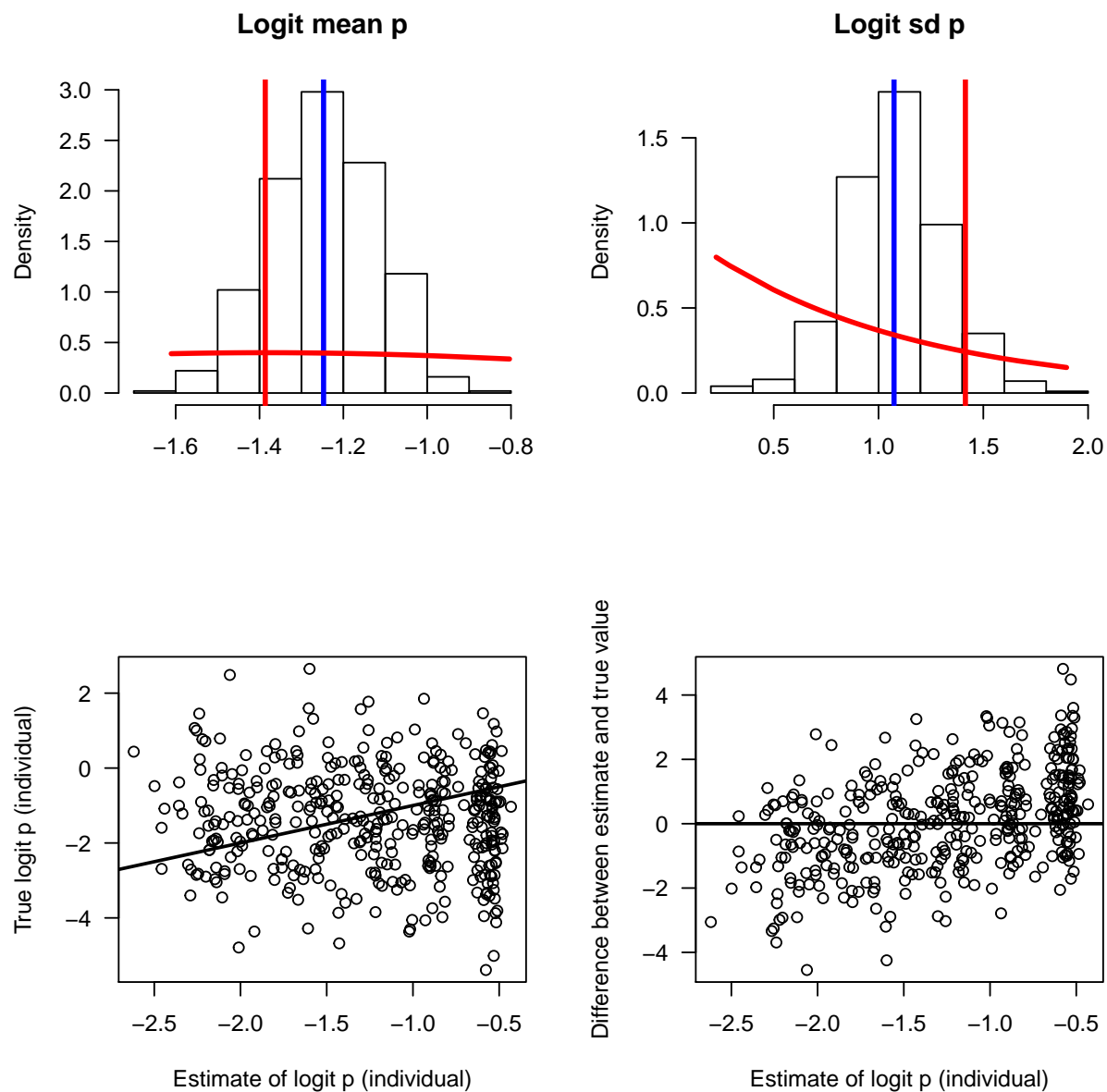


CQ shape



CQ scale

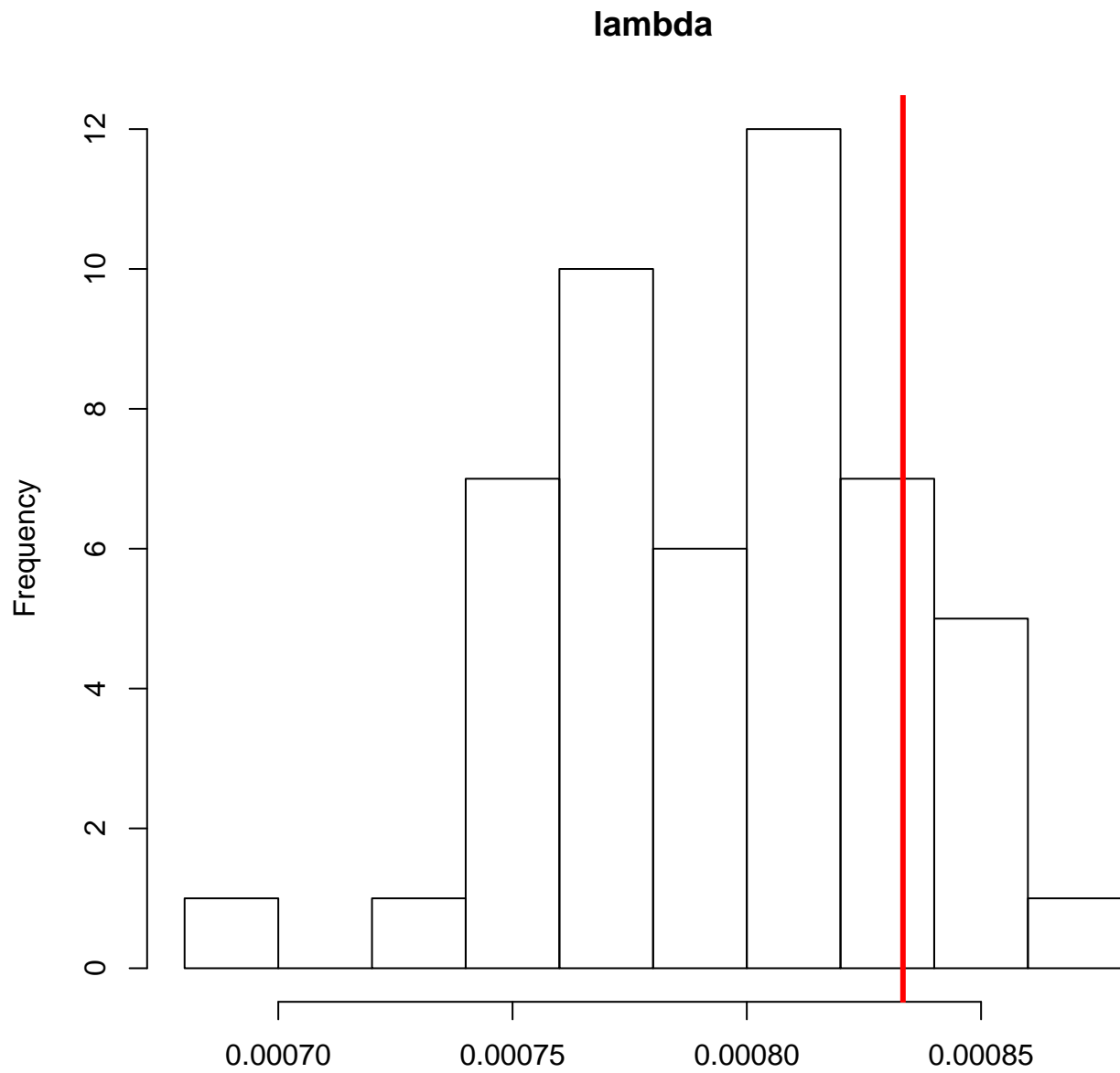




Run multiple random iterations to assess model fit and bias

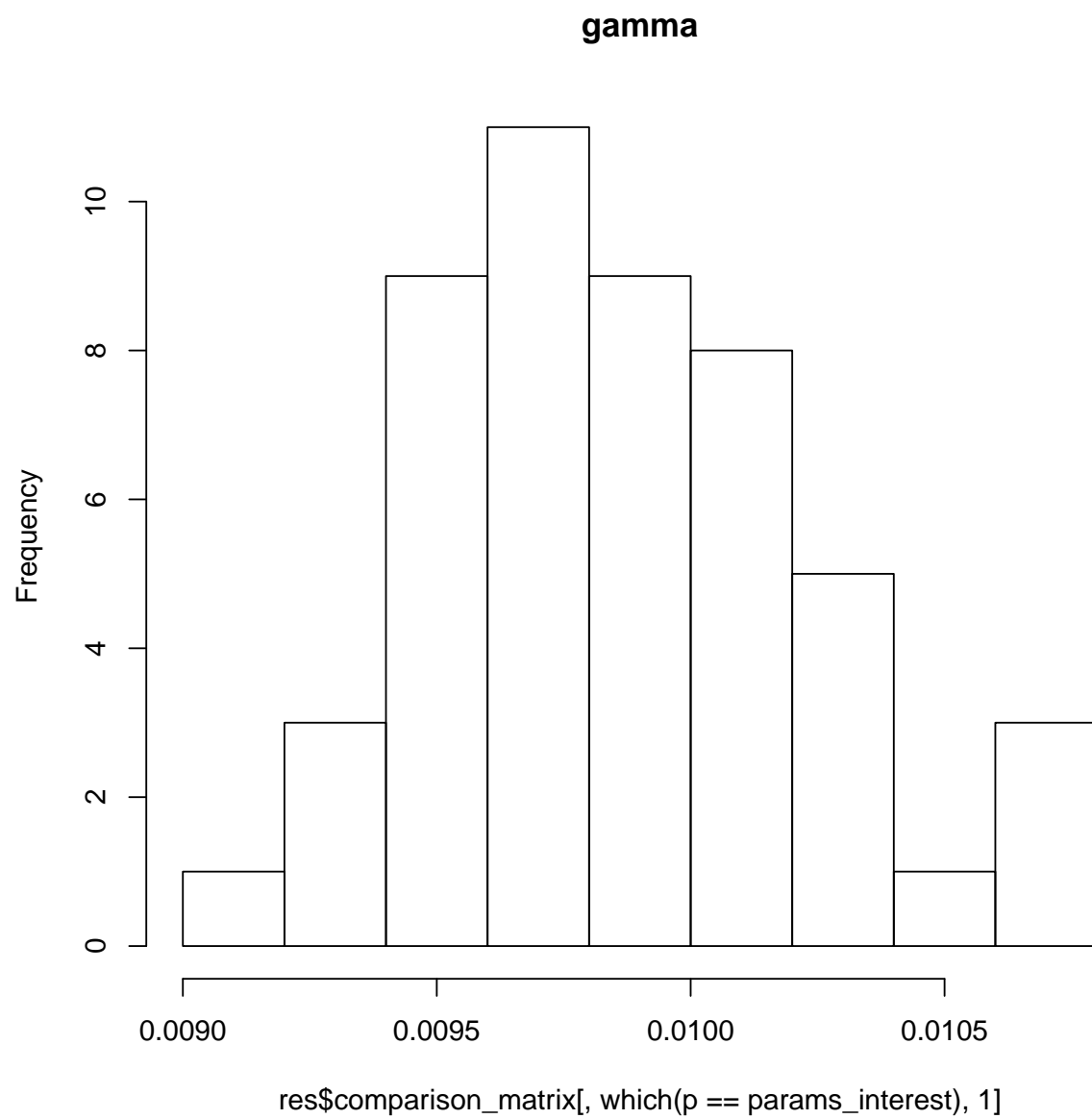
```
for(p in params_interest){
  writeLines(sprintf('For parameter %s, %s%% are below the true parameter',p,100*sum(res$comparison_matrix[,which(p==params_interest),1]), main=p)
  hist(res$comparison_matrix[,which(p==params_interest),1], main=p)
  abline(v=unique(res$comparison_matrix[,which(p==params_interest),2]),col='red',lwd=3)
}
```

For parameter lambda, 84% are below the true parameter

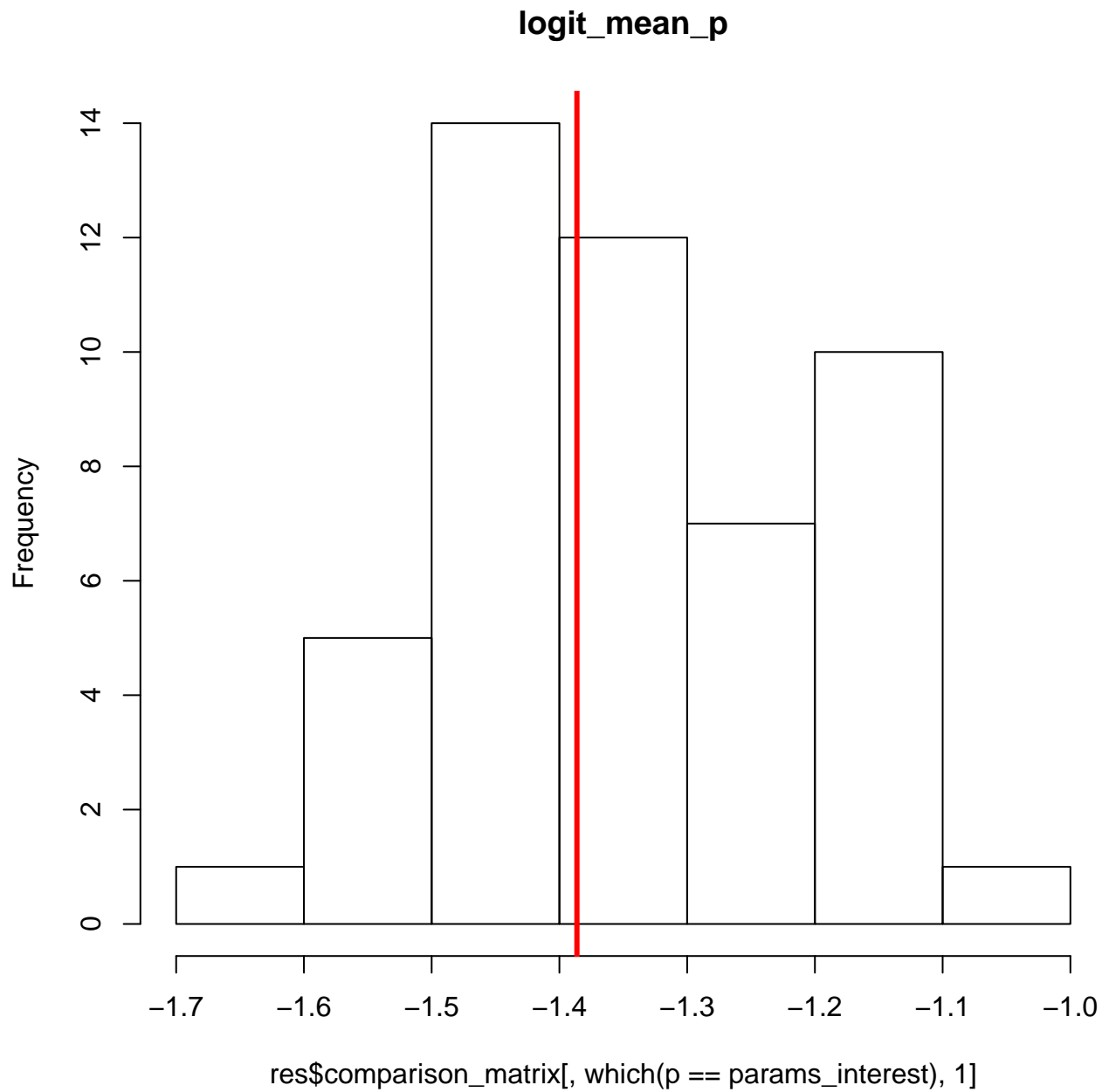


```
res$comparison_matrix[, which(p == params_interest), 1]
```

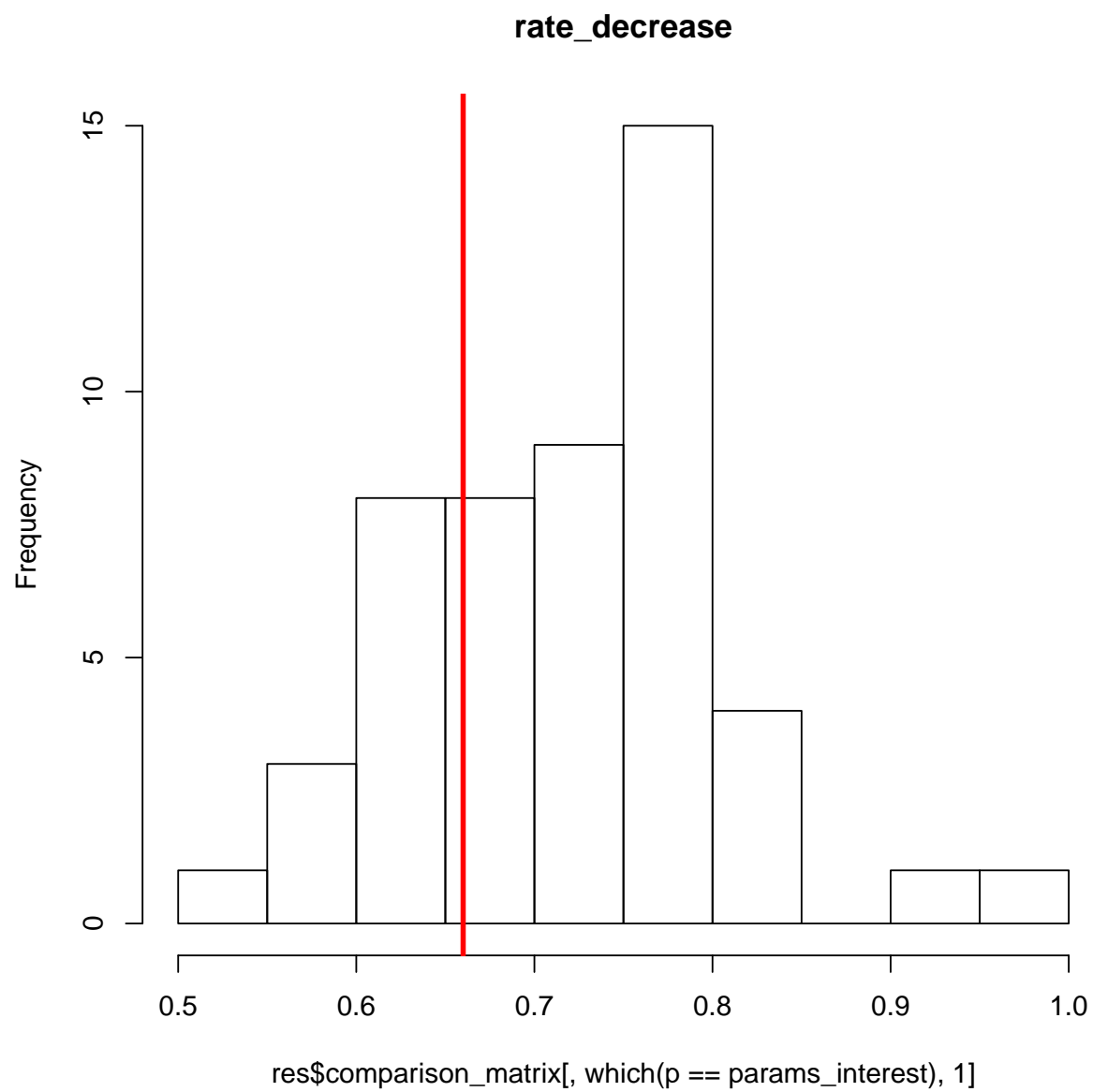
```
## For parameter gamma, 100% are below the true parameter
```

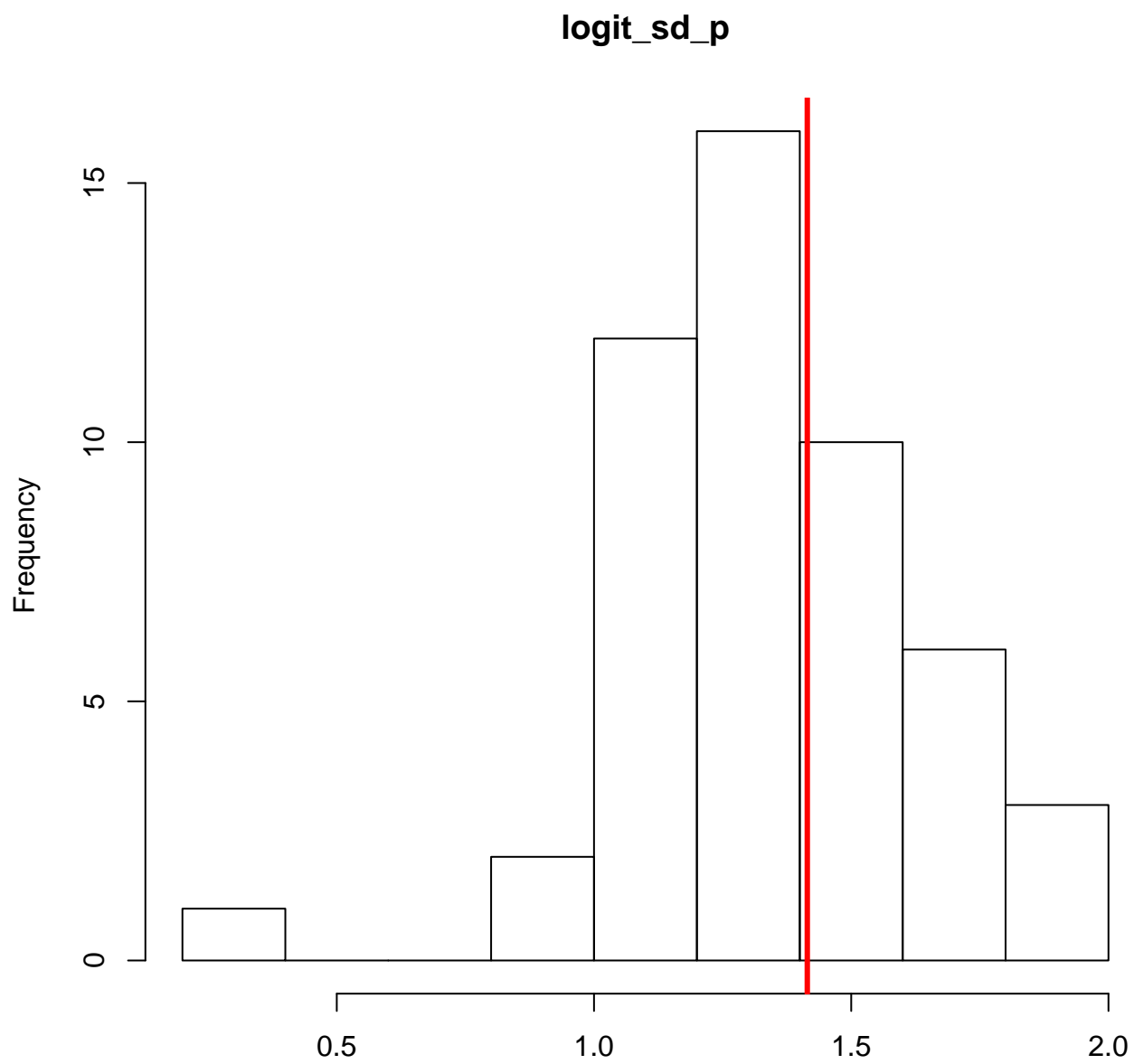
```
## For parameter logit_mean_p, 40% are below the true parameter
```



For parameter `rate_decrease`, 26% are below the true parameter

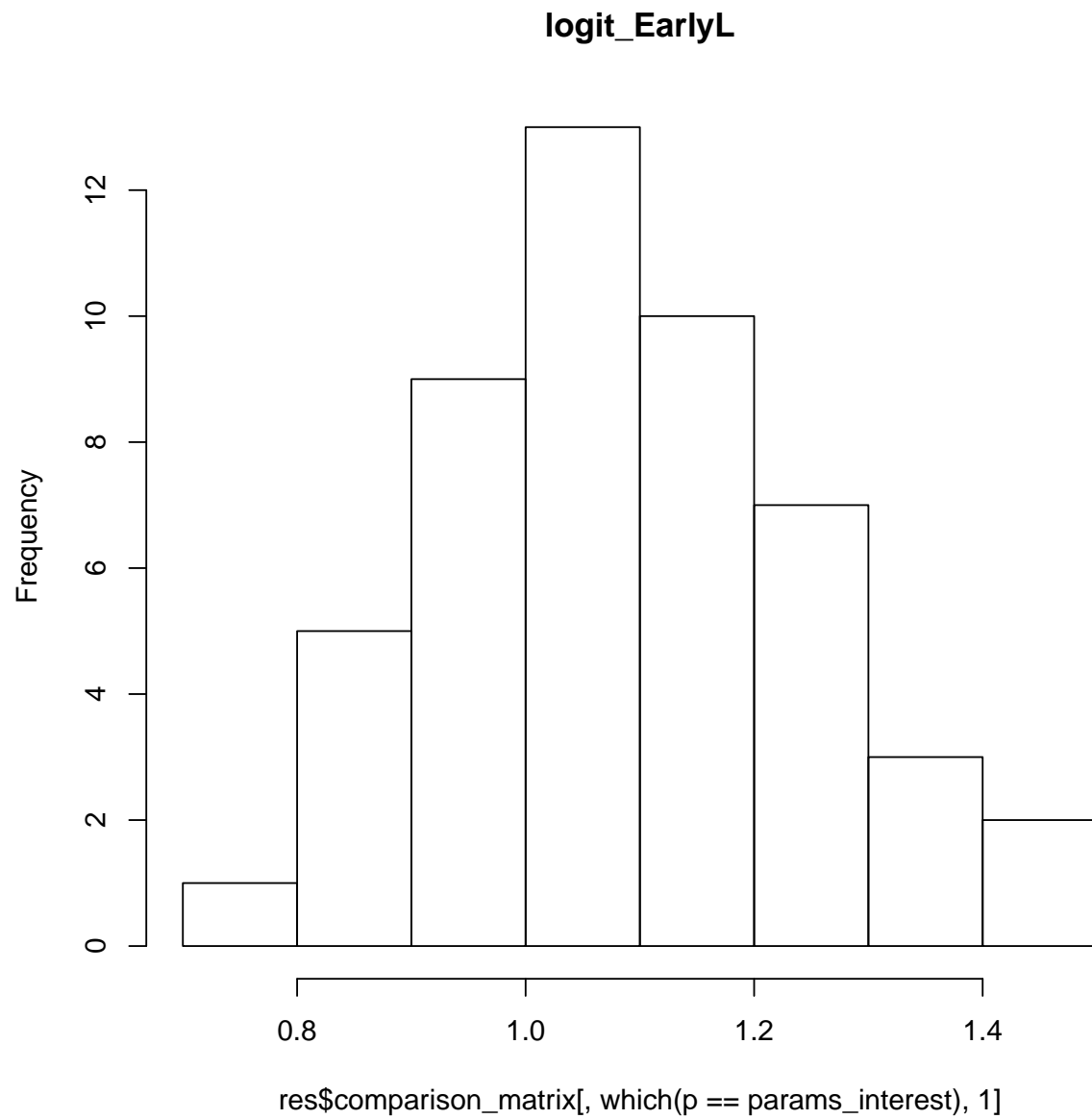


```
## For parameter logit_sd_p, 62% are below the true parameter
```

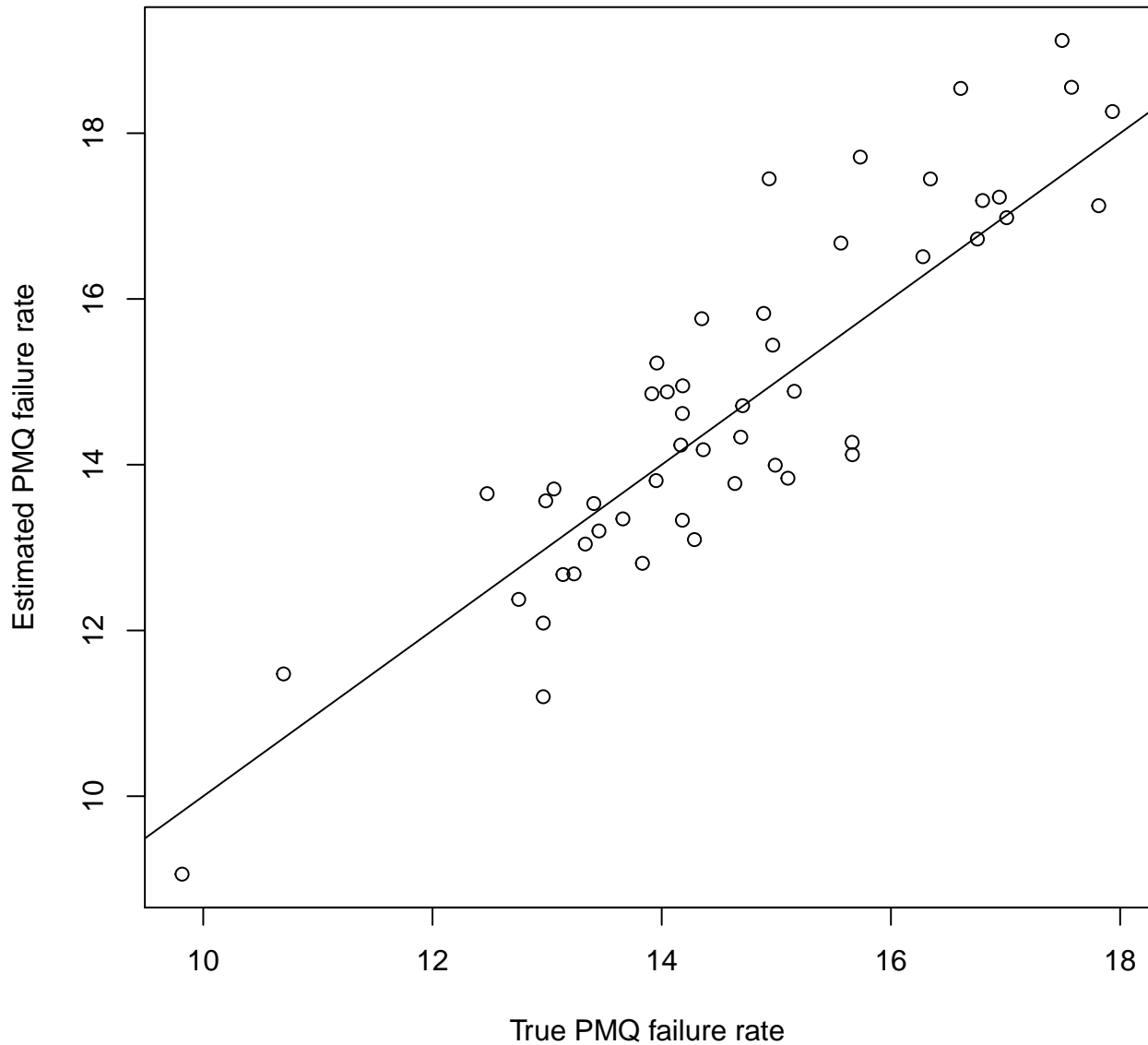


```
res$comparison_matrix[, which(p == params_interest), 1]
```

```
## For parameter logit_EarlyL, 0% are below the true parameter
```



```
plot(100*res$failure_matrix[,1],100*res$failure_matrix[,2],  
     xlab='True PMQ failure rate',ylab = 'Estimated PMQ failure rate')  
lines(c(0,100),c(0,100))
```



Posterior predictive p-values

In this section we simulate data from the posterior predictive for Model 2. We then apply a qualitative model check by comparing summary statistics from the posterior predictive with the true data summary statistics.

```
load('../RData/TimingModel/Posterior_theta_samples.RData')
load('../RData/TimingModel/Combined_Time_Event.RData')

PMQ_FUP = Combined_Time_Data$FU_time[!duplicated(Combined_Time_Data$patientid)&
  Combined_Time_Data$arm_num=='CHQ/PMQ']
CQ_FUP = Combined_Time_Data$FU_time[!duplicated(Combined_Time_Data$patientid)&
  Combined_Time_Data$arm_num=='CHQ']
AS_FUP = Combined_Time_Data$FU_time[!duplicated(Combined_Time_Data$patientid)&
  Combined_Time_Data$arm_num=='AS']
N_PMQ = sum(Combined_Time_Data$arm_num=='CHQ/PMQ' &
```

```

!duplicated(Combined_Time_Data$patientid))
N_CQ = sum(Combined_Time_Data$arm_num=='CHQ' &
!duplicated(Combined_Time_Data$patientid))
N_AS = sum(Combined_Time_Data$arm_num=='AS' &
!duplicated(Combined_Time_Data$patientid))
N_PMQ_BPD = sum(Combined_Time_Data$arm_num=='CHQ/PMQ' &
Combined_Time_Data$Study_Period==2 &
!duplicated(Combined_Time_Data$patientid))

set.seed(4543758)
params_M2_posterior = list()
i = sample(1:nrow(thetas2_matrix),1)
for(p in names(params_M2)){
  params_M2_posterior[[p]] = thetas2_matrix[i,p]
}

dat_posterior =
  simulate_dataset(N_PMQ = N_PMQ, N_CQ = N_CQ, N_AS = N_AS,
    FUP_time = c(sample(x = PMQ_FUP, size = length(PMQ_FUP), replace = T),
      sample(x = CQ_FUP, size = length(CQ_FUP), replace = T),
      sample(x = AS_FUP, size = length(AS_FUP), replace = T)),
    Study_Period = c(rep(2, N_PMQ_BPD), rep(2, N_PMQ - N_PMQ_BPD), rep(1, N_CQ + N_AS)),
    data_generation_function = generate_patient_data_Model2,
    params = params_M2_posterior)

AS_ind = dat_posterior$Simdata$Drug==0 &
  dat_posterior$Simdata$Censored==0
AS_ind_true = Combined_Time_Data$arm_num=='AS' &
  Combined_Time_Data$Censored==0
CQ_ind = dat_posterior$Simdata$Drug==1 &
  dat_posterior$Simdata$Censored==0
CQ_ind_true = Combined_Time_Data$arm_num=='CHQ' &
  Combined_Time_Data$Censored==0
PMQ_ind = dat_posterior$Simdata$Drug==2 & dat_posterior$Simdata$Censored==0
PMQ_ind_true = Combined_Time_Data$arm_num=='CHQ/PMQ' &
  Combined_Time_Data$Censored==0

par(las=1, mfrow=c(1,3))
*** Artesunate monotherapy: posterior predictive versus true data **
hist(Combined_Time_Data$Time_to_event[AS_ind_true],
  main = 'AS monotherapy',
  xlab='Days from last episode', col=alpha('blue',alpha = .4),
  breaks = seq(0,700,by=7), xlim = c(0,300))
hist(dat_posterior$Simdata$Durations[AS_ind],
  add=T, breaks = seq(0,700,by=7), col=alpha('red',alpha = .4))

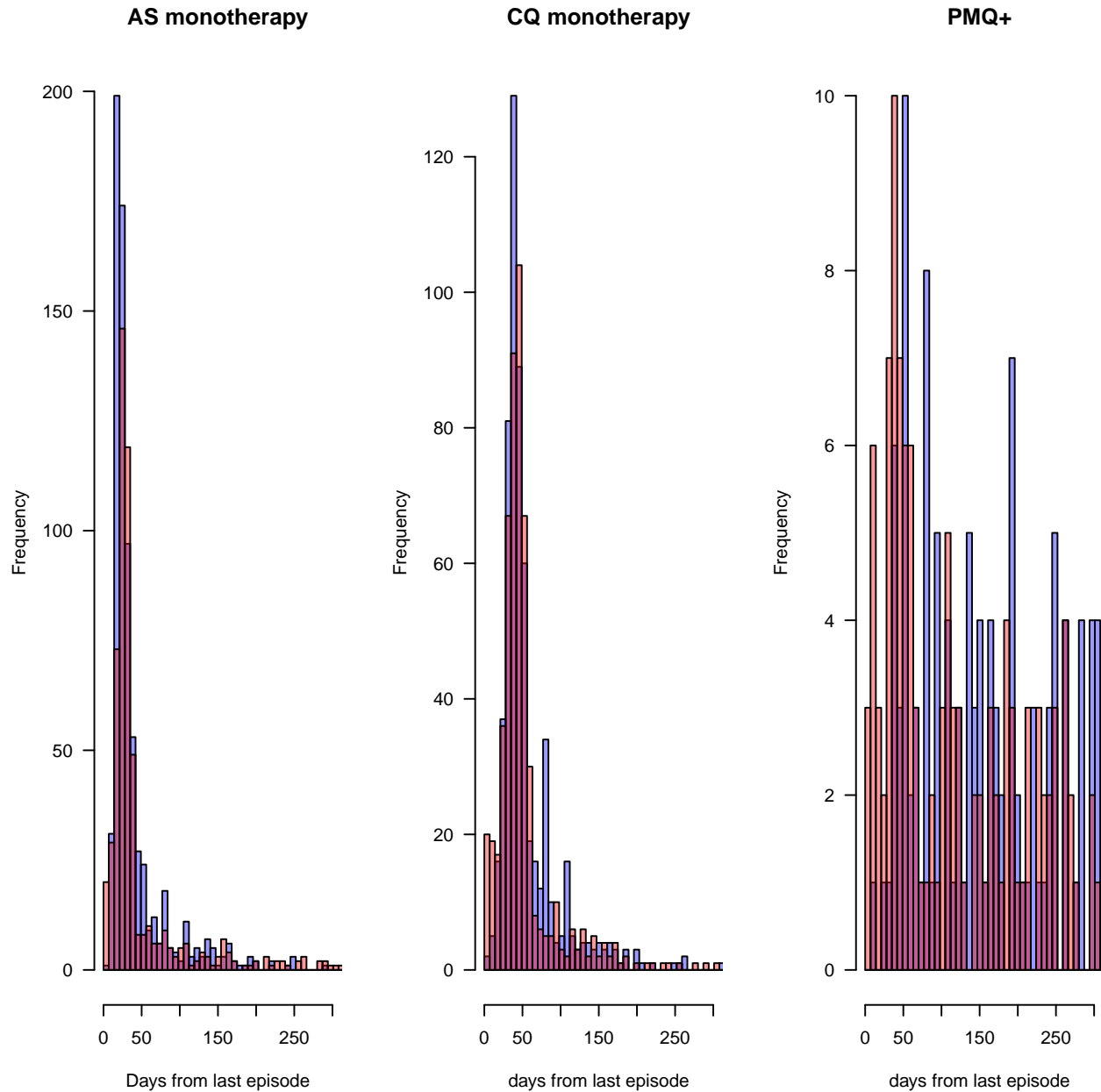
*** Chloroquine monotherapy: posterior predictive versus true data **
hist(Combined_Time_Data$Time_to_event[CQ_ind_true], main = 'CQ monotherapy',
  xlab='days from last episode', col=alpha('blue',alpha = .4),
  breaks = seq(0,700,by=7), xlim = c(0,300))
hist(dat_posterior$Simdata$Durations[CQ_ind],
  add=T, breaks = seq(0,700,by=7), col=alpha('red',alpha = .4))

```

```

*** Primaquine+ : posterior predictive versus true data **
hist(Combined_Time_Data$Time_to_event[PMQ_ind_true], main = 'PMQ+',
     xlab='days from last episode', col=alpha('blue',alpha = .4),
     breaks = seq(0,700,by=7), xlim = c(0,300))
hist(dat_posterior$Simdata$Durations[PMQ_ind],
     add=T, breaks = seq(0,700,by=7), col=alpha('red',alpha = .4))

```



```

# ***** Empirical cdfs *****
par(las=1, mfrow=c(1,3))
plot.ecdf(Combined_Time_Data$Time_to_event[AS_ind_true],
          col='blue',verticals = T, xlab='Days from last episode',
          main='Recurrence after AS monotherapy')
plot.ecdf(dat_posterior$Simdata$Durations[AS_ind],
          add=T,col='red',verticals = T)

```

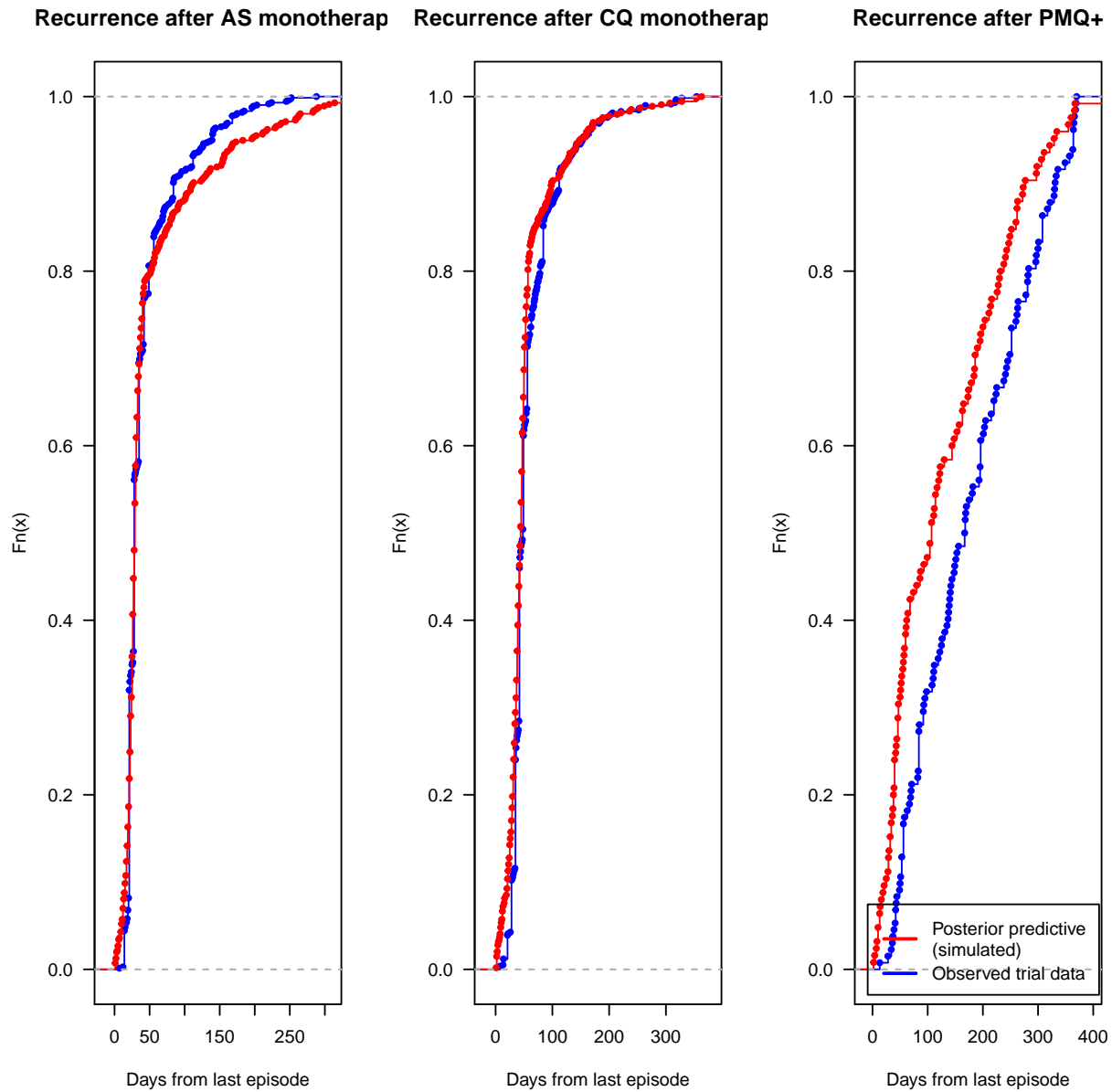


```

plot.ecdf(Combined_Time_Data$Time_to_event[CQ_ind_true],
          col='blue',verticals = T, xlab='Days from last episode',
          main='Recurrence after CQ monotherapy')
plot.ecdf(dat_posterior$Simdata$Durations[CQ_ind],
          add=T,col='red',verticals = T)

plot.ecdf(Combined_Time_Data$Time_to_event[PMQ_ind_true],
          col='blue',verticals = T, xlab='Days from last episode',
          main='Recurrence after PMQ+')
plot.ecdf(dat_posterior$Simdata$Durations[PMQ_ind],
          add=T,col='red',verticals = T)
legend('bottomright',col=c('red','blue'),
       legend = c('Posterior predictive \n(simulated)',
                   'Observed trial data'),lwd=2, inset = 0.01)

```



```

Sim_Iterations = 200
res = array(dim = c(Sim_Iterations,3))
pb = txtProgressBar(min=1, max = Sim_Iterations,style = 3)
for(Sim in 1:Sim_Iterations){

  params_M2_posterior = list()
  i = sample(1:nrow(thetas2_matrix),1)
  for(p in names(params_M2)){
    params_M2_posterior[[p]] = thetas2_matrix[i,p]
  }
  dat_posterior =
    simulate_dataset(N_PMQ = N_PMQ,N_CQ = N_CQ,N_AS = N_AS,
                     FUP_time = c(sample(x = PMQ_FUP,size = length(PMQ_FUP),replace = T),
                                   sample(x = CQ_FUP,size = length(CQ_FUP),replace = T),

```


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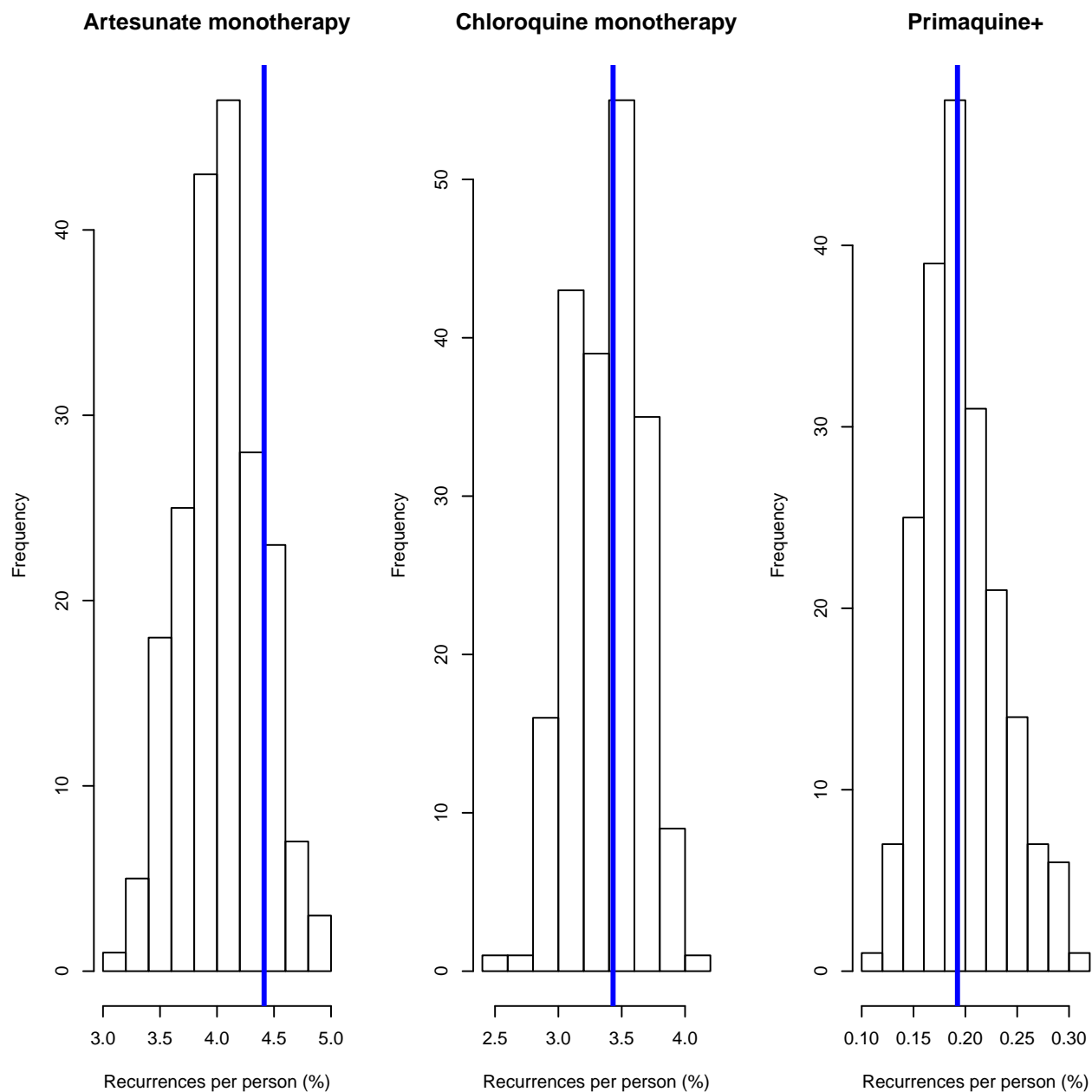
```

par(mfrow=c(1,3))
hist(res[,1], xlab = 'Recurrences per person (%)', main = 'Artesunate monotherapy')
true_stat_AS = length(Combined_Time_Data$Time_to_event[AS_ind_true])/
  (sum(Combined_Time_Data$Time_to_event[Combined_Time_Data$arm_num=='AS'])/360)
abline(v = true_stat_AS, col='blue',lwd=3)

hist(res[,2], xlab = 'Recurrences per person (%)', main = 'Chloroquine monotherapy')
true_stat_CQ = length(Combined_Time_Data$Time_to_event[CQ_ind_true])/
  (sum(Combined_Time_Data$Time_to_event[Combined_Time_Data$arm_num=='CHQ'])/360)
abline(v = true_stat_CQ, col='blue',lwd=3)

hist(res[,3], xlab = 'Recurrences per person (%)', main = 'Primaquine+')
true_stat_PMQ = length(Combined_Time_Data$Time_to_event[PMQ_ind_true])/
  (sum(Combined_Time_Data$Time_to_event[Combined_Time_Data$arm_num=='CHQ/PMQ'])/360)
abline(v = true_stat_PMQ,col='blue',lwd=3)

```

```
PP_pval_AS = 1-sum(res[,1]>true_stat_AS)/Sim_Iterations
writeLines(sprintf('The posterior predictive p value for the number of recurrences per person in the AS arm is %f', PP_pval_AS))

## The posterior predictive p value for the number of recurrences per person in the AS arm is 0.845

PP_pval_CQ = 1-sum(res[,2]>true_stat_CQ)/Sim_Iterations
writeLines(sprintf('The posterior predictive p value for the number of recurrences per person in the CQ arm is %f', PP_pval_CQ))

## The posterior predictive p value for the number of recurrences per person in the CQ arm is 0.525

PP_pval_PMQ = 1-sum(res[,3]>true_stat_PMQ)/Sim_Iterations
writeLines(sprintf('The posterior predictive p value for the number of recurrences per person in the PMQ+ arm is %f', PP_pval_PMQ))

## The posterior predictive p value for the number of recurrences per person in the PMQ+ arm is 0.535
```