In some situations, the reliability of a measurement is known to vary. For example, one could imagine a situation where the measurement mechanism goes through a "warm up" period, leading to less reliable measurements early in the measurement period but becoming more reliable in time. Statisticians call data with varying reliability *heteroscedastic*.

In the file `heteroscedastic.txt`, are 20 data points, $(t_i, y_i)$, $i = 1, ..., 20$. Presumably these are samples from a "true data function" that maps $\mathbb{R} \to \mathbb{R}$.

a. Choose a reasonable basis for this problem and construct the system matrix

b. Find the parameters characterizing the best-fit line in a least-squares sense.

c. Now suppose that you know, *a priori* that doubt about the reliability, or the uncertainty, in your data follows the function $\sigma(t) = 20e^{-t/2}$. Construct a diagonal $20 \times 20$ weighting matrix $S$ with entries $1/\sigma^2$ corresponding to the timestamp for each point in your dataset.

d. Use the above matrix $S$ to solve a weighted least squares problem ($\hat{y} = SA\theta$) and report the parameters characterizing it's best-fit line. Plot both the least squares and weighted least squares lines on a plot with the data so you can compare the fits.

e. Write a short summary of your observations, particularly considering how $S$ may relate to Fisher information. If you didn't have *a priori* knowledge about the reliability of your data, what could you do (if anything) to estimate it from the data itself? When would your scheme work well? When would it fail?