

# HW 16-17

## Imports

```
In [1]: import warnings

import numpy as np
import pandas as pd
from matplotlib import pyplot as plt

warnings.filterwarnings("ignore")
```

## HW 16

```
In [2]: # Setup: copied from HW 13
data: np.ndarray = pd.read_csv("uspopulation.txt", delimiter=" ").values.astype(float)

years = data[:,0]
pops = data[:,1]
```

```
In [3]: # A
tau = np.linspace(-1, 1, len(years)+1)
tau_pre = tau[:-1]
tau_20 = tau[-1]
```

```
In [4]: # B

# a design matrix is created as follows:
def design_mat(k: int, vec) -> np.ndarray:
    """ copied from HW 13 """
    A = np.stack([np.polynomial.legendre.Legendre.basis(i)(vec) for i in range(k+1)])
    return A

# for example, the design matrix given our data and a monomial basis up to k=11
des_mat = design_mat(11, tau_pre)
pd.DataFrame(des_mat)
```

Out [4]:

	0	1	2	3	4	5	6	7
0	1.0	-1.000000	1.000000	-1.000000	1.000000	-1.000000	1.000000	-1.000000
1	1.0	-0.833333	0.541667	-0.196759	-0.119309	0.336372	-0.414476	0.353133
2	1.0	-0.666667	0.166667	0.259259	-0.427469	0.305556	-0.017233	-0.240569
3	1.0	-0.500000	-0.125000	0.437500	-0.289062	-0.089844	0.323242	-0.223145
4	1.0	-0.333333	-0.333333	0.407407	0.012346	-0.333333	0.193416	0.165987
5	1.0	-0.166667	-0.458333	0.238426	0.274209	-0.273003	-0.145090	0.278912
6	1.0	0.000000	-0.500000	0.000000	0.375000	0.000000	-0.312500	0.000000
7	1.0	0.166667	-0.458333	-0.238426	0.274209	0.273003	-0.145090	-0.278912
8	1.0	0.333333	-0.333333	-0.407407	0.012346	0.333333	0.193416	-0.165987
9	1.0	0.500000	-0.125000	-0.437500	-0.289062	0.089844	0.323242	0.223145
10	1.0	0.666667	0.166667	-0.259259	-0.427469	-0.305556	-0.017233	0.240569
11	1.0	0.833333	0.541667	0.196759	-0.119309	-0.336372	-0.414476	-0.353133

In [5]: *# C: adapted from HW 13*

```

df = pd.DataFrame(
    {
        "K": [i+1 for i in range(11)],
        "sum of squared error": [0 for i in range(11)],
        "predicted population in 2020": [0 for i in range(11)],
        "condition number": [0 for i in range(11)]
    }
)

# Let's loop to solve
for i, row in df.iterrows():
    k = row["K"]
    A = design_mat(k, tau_pre)
    theta_hat, sse, rank, s = np.linalg.lstsq(A, pops)
    df.loc[i, "sum of squared error"] = sse
    df.loc[i, "predicted population in 2020"] = design_mat(k, np.array([tau_pre, 1])) @ theta_hat
    df.loc[i, "condition number"] = s[0]/s[-1]

# Here are our results
df

## What do you notice?
# I notice that the predicted population for 2020 still fluctuates as in hom

```

Out [5]:

	K	sum of squared error	predicted population in 2020	condition number
0	1	64833.089851	177.340667	1.000000
1	2	1211.729360	314.443621	1.756048
2	3	106.477333	342.047136	2.432760
3	4	105.948478	341.124798	3.311827
4	5	86.107178	332.065503	4.625532
5	6	65.260770	348.063864	6.922458
6	7	64.757088	352.746326	11.415503
7	8	27.127825	267.999303	21.459975
8	9	20.248611	181.742205	47.127184
9	10	7.605490	508.822167	124.648564
10	11	0.504120	-345.904083	419.481914

## HW 17

```
In [6]: # setup

data: np.ndarray = pd.read_csv("bias-variance-trade.txt", delimiter=" ", header=0)
x1 = data[:, 0]
y1 = data[:, 1]
x2 = data[:, 2]
y2 = data[:, 3]
```

```
In [7]: # A

# let's use the function and methods from HW 13 and 16 (copied/adapted from HW 16)
df = pd.DataFrame(
    {
        "K": [i+1 for i in range(11)],
        "sum of squared error": [0 for i in range(11)],
    }
)

theta_holder_1 = {}

# Let's loop to solve
for i, row in df.iterrows():
    k = row["K"]
    A = design_mat(k, x1)
    theta_hat, sse, rank, s = np.linalg.lstsq(A, y1)
    df.loc[i, "sum of squared error"] = sse
    theta_holder_1[k] = theta_hat

df
```

Out [7]:

	K	sum of squared error
0	1	3.041412
1	2	3.040519
2	3	3.040433
3	4	3.040249
4	5	2.952330
5	6	2.952087
6	7	2.778459
7	8	2.750194
8	9	0.994671
9	10	0.923089
10	11	0.142081

In [8]: # B

```

# let's use the function and methods from HW 13 and 16 (copied/adapted from
df = pd.DataFrame(
    {
        "K": [i+1 for i in range(11)],
        "sum of squared error": [0 for i in range(11)],
    }
)

theta_holder_2 = {}

# Let's loop to solve
for i, row in df.iterrows():
    k = row["K"]
    A = design_mat(k, x2)
    theta_hat, sse, rank, s = np.linalg.lstsq(A, y2)
    df.loc[i, "sum of squared error"] = sse
    theta_holder_2[k] = theta_hat

df

```

Out [8]:

	K	sum of squared error
0	1	1.331521
1	2	1.301472
2	3	1.239674
3	4	1.225391
4	5	1.110006
5	6	1.103873
6	7	1.025801
7	8	0.864047
8	9	0.479757
9	10	0.477410
10	11	0.165095

In [9]: # C

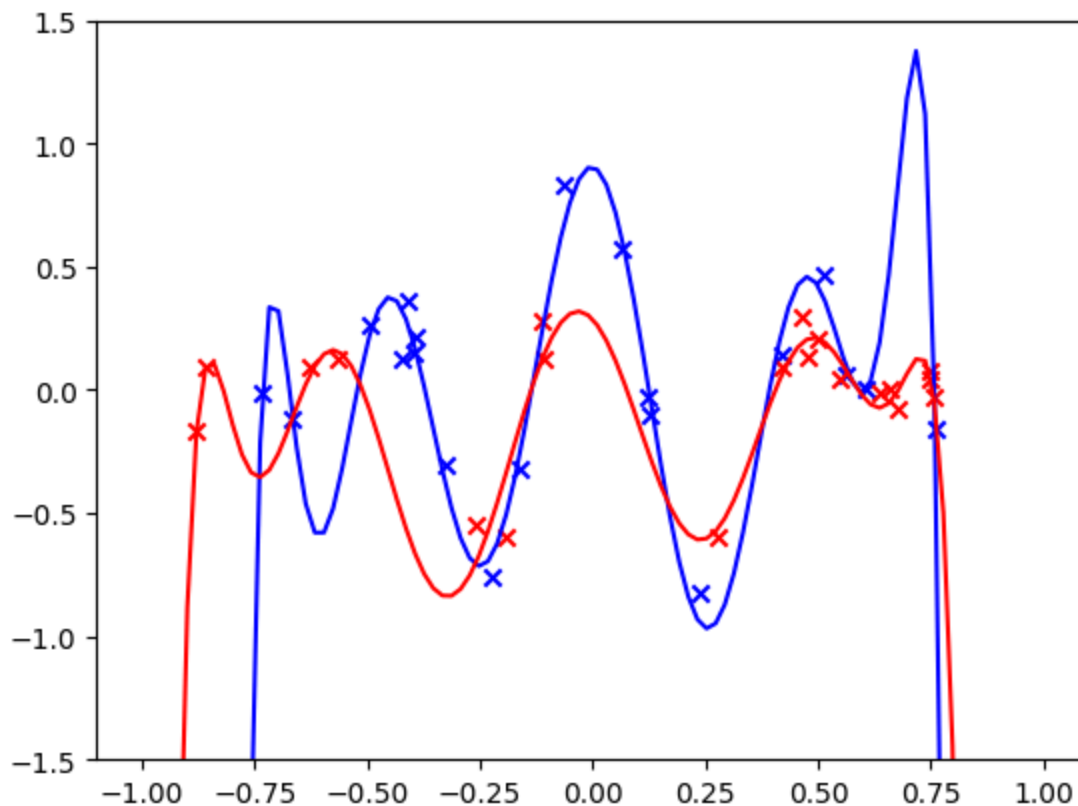
```
plt.scatter(x1, y1, color = "b", marker="x")
k1_to_plot = 11
bfx1 = np.linspace(-1, 1, 100)
ttp1 = theta_holder_1[k1_to_plot]
atp1 = design_mat(k1_to_plot, bfx1)
bfy1 = atp1@ttp1
plt.plot(bfx1, bfy1, color="b")

plt.scatter(x2, y2, color = "r", marker="x")
k2_to_plot = 11
bfx2 = np.linspace(-1, 1, 100)
ttp2 = theta_holder_2[k2_to_plot]
atp2 = design_mat(k2_to_plot, bfx2)
bfy2 = atp2@ttp2
plt.plot(bfx2, bfy2, color="r")

plt.ylim(-1.5, 1.5)

# How well does one model fit the data used to train the other model?
# not well -- the models are specific to their datas
```

Out [9]: (-1.5, 1.5)



## Acknowledgment

Work in this repository and with associated assignments and projects may be adapted or copied from similar files used in my prior academic and industry work (e.g., using a LaTeX file or Dockerfile as a starting point). Those files and any other work in this repository may have been developed with the help of LLM's like ChatGPT. For example, to provide context, answer questions, refine writing, understand function call syntax, and assist with repetitive tasks. In these cases, deliverables and associated work reflect my best efforts to optimize my learning and demonstrate my capacity, while using available resources and LLM's to facilitate the process.

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