

The file `aer.txt` contains data from the analysis of an enzyme reaction. Enzymes catalyze the rate of chemical reactions. In this data set, the dependent variable is the rate of the chemical reaction and the independent variable is the amount of enzyme.

The data should be fit to the four parameter model

$$y_\phi(u) = \frac{\phi_1(u^2 + \phi_2 u)}{u^2 + \phi_3 u + \phi_4}.$$

The parameters should each satisfy $\phi_i > 0$, so we take as our parameter vector:

$$\theta = (\log \phi_1, \log \phi_2, \log \phi_3, \log \phi_4)'.$$

- a. Which parameters could go to zero without resulting in a trivial model? Rewrite the model for each of these cases.
- b. Consider what happens when ϕ_2 is much larger than ϕ_1 . What would the numerator simplify to? Rewrite the model with this simplifying assumption.
- c. Repeat part b for ϕ_3 and ϕ_4 . Why did we ignore ϕ_1 ?
- d. For the full model, construct a cost surface running from $-10 \leq \log(\theta_i) \leq 10$. Produce six plots which show slices of the cost surface with respect to pairs of the model parameters.
- e. Identify regions on your cost surface plots which correspond to your simplified models
- f. Fit the full model to data. (Hint: At the best fit, I find that $\phi_1 \approx 0.2$.)
- g. Repeat Part f for each of your simplified models.
- h. Plot all of your models on the same set of axes with the data. What does this plot tell you about the identifiability of this model+parameter combination?