In a vector space, a "norm" is a way of measuring how "big" a particular vector is. In an earlier problem, you showed that matrices can be components of a vector space. In this problem, we will generalize the concept of a vector norm to matrices.

Below are four unit vectors which define the unit circle in some vector space $\mathcal{V}$:

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

(a) The following matrix is a linear transformation to the vector space $\mathcal{U}$. Apply the transformation to each of the unit vectors and plot them.

$$\begin{pmatrix} 1 & -\sqrt{3}/2 \\ \sqrt{3} & 1/2 \end{pmatrix}$$

(b) Which vector is the longest and what is it's norm? This is the $l-2$ norm of the matrix and it describes the maximum amount that a transformation can stretch a unit vector

(c) The l_2 norm is equivalently defined as the largest singular value. Using a programming language library of your choice (such as numpy.linalg), take the singular value decomposition of the matrix in part a and compare the singular values to the norm you found in part (b). You should find that they match

(d) Most linear algebra libraries allow you to calculate the norms of matrices. Learn how to do this for the programming library of your choice and explore the norms that are available. Which norms are not defined for matrices?