Consider the least squares error function:

$$C(\theta_1, \theta_2) = \frac{1}{2} \sum_{i=1}^{20} \left(f(t_i) - \theta_1 t - \theta_2 t^2 \right)^2$$

where t_i are 20 evenly spaced between 0 and 1 and $f(t) = t^2$.

a. Make a contour plot of C with θ_1 on the x-axis and θ_2 on the y axis.

b. Find the Fisher Information Matrix (FIM) for this model and calculate its eigenvalues and eigenvectors. Graphically confirm that the ellipses in your contour plot are oriented with its eigenvectors and the aspect ratio is given by the ratio of the square root of the eigenvalues.

c. Generate data according to $y_i = f(t_i) + \epsilon_i$ where ϵ is Gaussian noise with zero mean and standard deviation 0.1. Use Ordinary Least Squares (OLS) regression , fit your data to the model $\theta_1 t + \theta_2 t^2$.

d. Repeat your OLS fit 1000 times for different realizations of your data and plot your estimates as a scatter plot on top of your contour plot.

e. What does this problem show you about the FIM? Note that there is no "right" way to answer this question.