Recall the singular valued decomposition of a matrix $A = U\Sigma V'$. Let u_i and v_i denote the i^{th} columns of U and V respspectively and σ_i the i^{th} singular value ($\Sigma_{ii} = \sigma_i$). Assume the singular values are enumerated in decreasing order so that $\sigma_i \geq \sigma_{i+1}$ With these conventions, the SVD takes the form

$$A = \sum_i \sigma_i u_i v_i'.$$

Notice that each term in this sum is a rank-1 matrix. We can approximate the matrix A by truncating this sum, effectively ignoring the small singular values. Let's define A_n to be the matrix formed by retaining the first n terms of the singular value decomposition:

$$A_n = \sum_{i=1}^{n} \sigma_i u_i V_i'.$$

Import the grey-scale image, *weathered-face.jpg* as a matrix, A, and compute the singular value decomposition of $A = U\Sigma V'$.

a. How many non-zero singular values does A have?
b. What are the values of the five largest singular values?
c. Construct the approximations A_{10}, A_{20} and A_{40} and plot their corresponding images.
d. How many terms n do you do need to retain in the sum to recognize that the approxmation A_n is a face?