Collateralized Debt Networks with Lender Default

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Federal Reserve Board

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Motivation

- Collateralized debt market: Repo, margin/sec lending, Derivatives
- Networks: bilateral trades in OTC markets
- Reuse of collateral $\Rightarrow$ **lender default**

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- Borrowers' collateral value was 62.5% of Lehman's assets
- 5 years to resolve (e.g. Lehman 2008-2013, MF Global 2011-2015)
- Flip-side of the "securedness" of collateral

Research Question

- How do participants of the collateralized debt market borrow from each other when they can re-use collateral and default as lender?
- How is the underlying systemic risk determined?
- How do the structure of the market and systemic risk change under different market conditions, including a change in regulation?

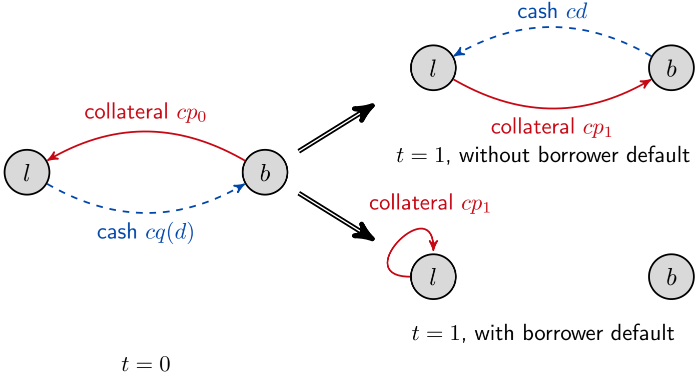
Related Literature

- General equilibrium with collateral:
  Geanakoplos (1997, 2003, 2010), Fostel and Geanakoplos (2015, 2016), Simsek (2013), Geerolf (2017)
- Financial networks:
  Acemoglu, Ozdaglar, Tahbaz-Salehi (2014, 2015), Allen and Gale (2000), Allen, Babus, Carletti (2012), Eisenberg and Noe (2001), Eliott, Golub, Jackson (2014), Eliott, Georg, Hazzell (2018), Erol (2018), Farboodi (2017), Glasserman and Young (2015), Jackson and Pernoud (2019), Zawadowski (2013)
- Market structure in repo markets:
  Baklanova et al. (2019), Copeland, Martin, Walker (2014), Gorton and Metrick (2012), Park and Kahn (2018), Gottardi, Maurin, and Monnet (2017), Infante (2015, 2017), Infante and Vardoulakis (2019)
- Central counterparty:
  Atkeson et al. (2015), Biais, Heider, Hoerova (2012), Frei et al. (2017), Duffie and Zhu (2011), Duffie, Scheicher, Vuillemey (2015), Glasserman, Moallemi, Yuan (2015), Paddrick and Young (2017), Singh (2011)

Contribution to the Literature

- New theoretical approach
  - Endogenous reuse of collateral: endogenize price, leverage, network
  - Model insights: counterparty risk vs leverage, dual loop of contagion
- Model predictions match the empirical literature
  - Weak relationship between haircuts and rates
  - Multiple haircuts for the same collateral
  - Reuse pattern changes during crisis
- Policy implications
  - There can be side effects from counterparty risk mitigation measures (e.g. central counterparties)

Collateralized Debt Contract



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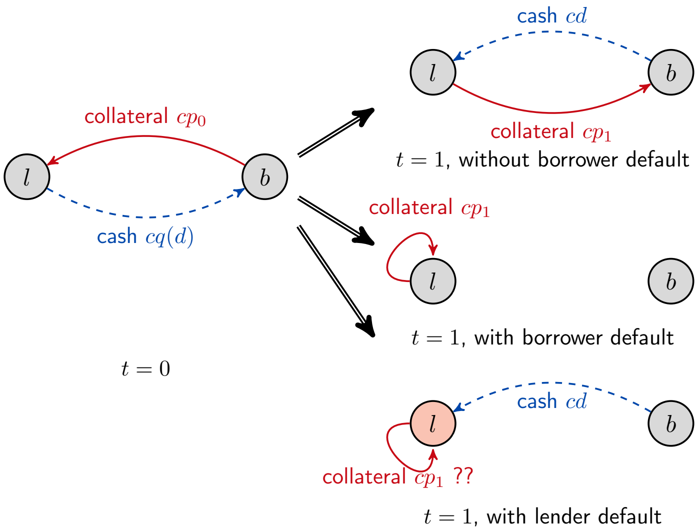

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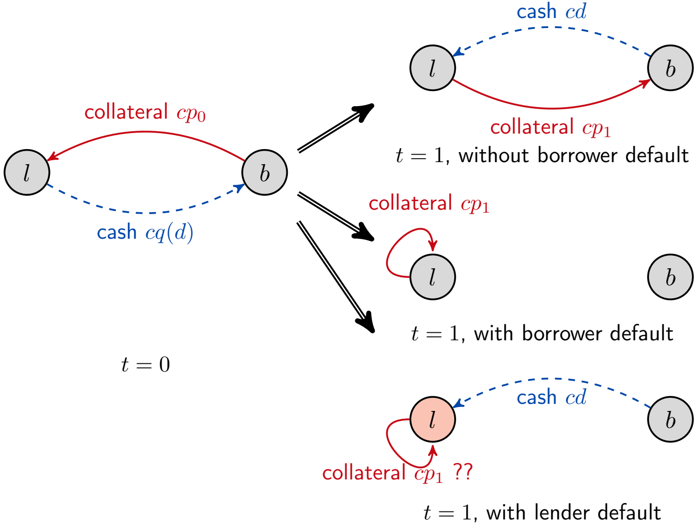
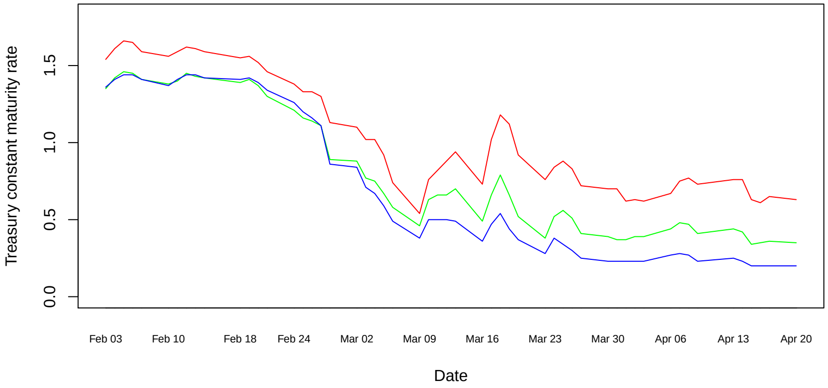


Two Contagion channels: Counterparty + Price (fire-sales)

Safe Asset as Collateral?



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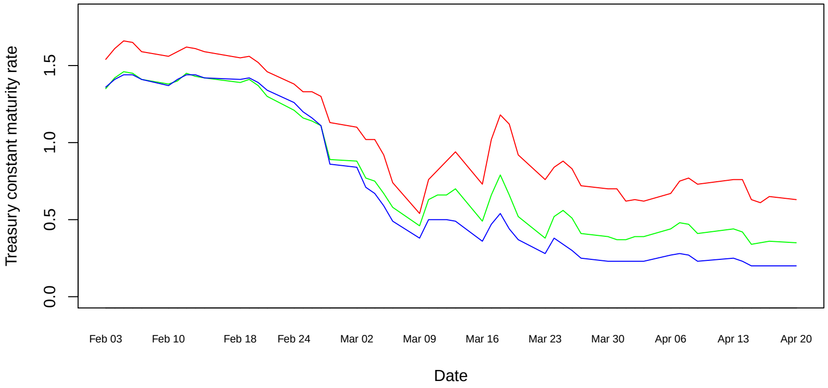
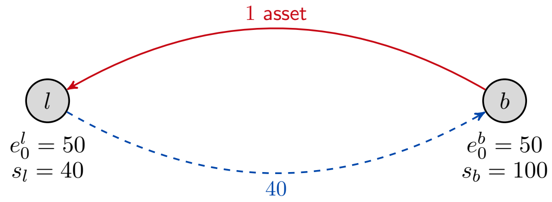


Small haircut, BUT large leverage! $\Rightarrow$ amplified payment swings

Re-use of Collateral



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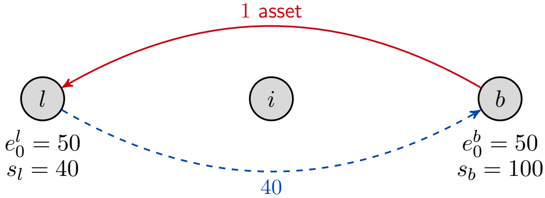

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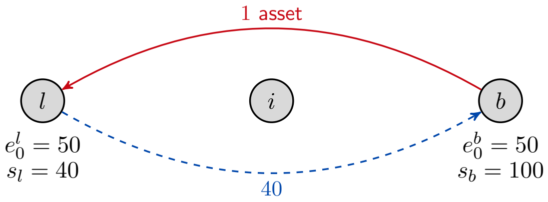

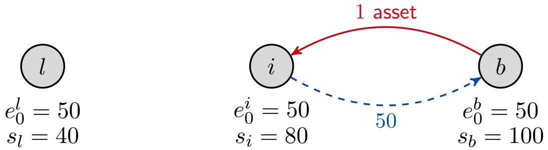

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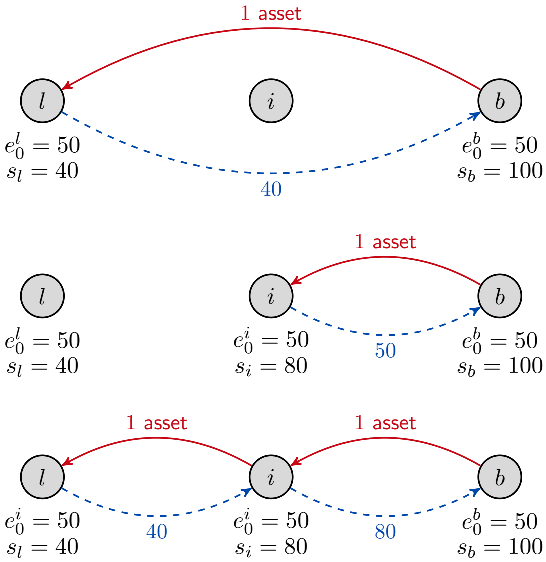

Methodology (GE + Financial Networks)

- Endogenous asset price
  - Walrasian markets (competitive + no asymmetric info)
- Endogenous leverage
  - No arbitrage contract prices (GE) + counterparty risk (network)
- Endogenous network formation (directed weighted graph)
  - Complex interest rate determination from all possible pairs and LTV
  - Portfolio decisions of each agent with given contagion structure
  - Amount borrowing and lending is the network (solve as a GE model)
  - Reduce the class of networks to consider (from $\mathbb{R}_+^{n^2} \times \mathbb{R}_+^{n^2}$)

Basic Setup

- Time: $t = 0, 1, 2$
- Two goods: traded in Walrasian market
  - Cash e is storable and the only consumption good
  - Asset a yields s unit of cash at $t = 2$
    - each agent has subjective belief on s at $t = 0$
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- Preference: Risk-neutral and determined by cash at $t = 2$
- Endowments: e_0 cash for each agent at $t = 0$

Heterogeneity of Agents

- Heterogeneous beliefs: Agent j believes $s = s_j$ with prob 1
- Type: Set $N = \{1, 2, \ldots, n\}$, where $s_1 > s_2 > \cdots > s_n$

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- Agents are competitive and know each other's type

Collateralized Debt per Unit of Collateral

Borrow cash using 1 unit of asset as collateral:

- d_{ij} : promised cash amount in $t = 1$ from j to i

- Nonrecourse contract : payment from j to i is

$$x_{ij} \equiv \min\{d_{ij}, \tilde{p}_1\}$$

- $q(d_{ij})$: amount of cash i lends to j in $t = 0$ (borrowing)

- $\dfrac{d_{ij}}{q(d_{ij})}$: gross interest rate $(1 + r)$

Network of Collateralized Debt

- c_{ij} : amount of collateral posted
- $c_{ij}d_{ij}$: amount of total promise
- $c_{ij}x_{ij} \equiv c_{ij}\min\{d_{ij}, \tilde{p_1}\}$: amount of total delivery
- $C = [c_{ij}]$: collateral matrix
- $D = [d_{ij}]$: contract matrix
- Debt network: directed weighted graph of (C, D) at $t = 0$

Lender Default Cost

- Lender/collateral holders can default if they go bankrupt (e.g. Lehman Brothers, MF Global)
- **Lender default cash cost**: $\Psi_{ij}(C)[p_1 - d_{ij}]^+$
- Interpretation: litigation cost, opportunity cost, congestion cost

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  1. (Zero Exposure) $\Psi_{ij}(C) = 0$ if $c_{ij} = 0$
  2. (No Net Loss) $\Psi_{ij}(C) \leq c_{ij}$ for any C
  3. (Congestion Effect) $\Psi_{ij}(C') \geq \Psi_{ij}(C)$ if $c'_{ij} = c_{ij}$ with $\sum_k c'_{ik} > \sum_k c_{ik}$
  4. (Share Effect) $\Psi_{ij}(\hat{C}) > \Psi_{ij}(C)$ if $\hat{c}_{ij} > c_{ij}$ with $\sum_k \hat{c}_{ik} = \sum_k c_{ik}$

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  1. $\dfrac{\partial \Psi_{ij}}{\partial c_{ij}} > 0$, $\dfrac{\partial^2 \Psi_{ij}}{\partial c_{ij}^2} > 0$ for any C with $c_{ij} > 0$
  2. $\dfrac{\partial \Psi_{ij}}{\partial c_{ij}} > \dfrac{\partial \Psi_{ij}}{\partial c_{ik}} \geq 0$ for any C with $c_{ij}, c_{ik} > 0$
  3. $\dfrac{\partial \Psi_{ij}}{\partial c_{kj}} = \dfrac{\partial \Psi_{ij}}{\partial c_{kl}} = 0$ for any C with distinct i, j, k, l

Timeline and Setup of the Model

n agents endowed with
e_0 cash and beliefs on asset payoff s,
trade due to different beliefs
$s_1 > s_2 > \cdots > s_n$

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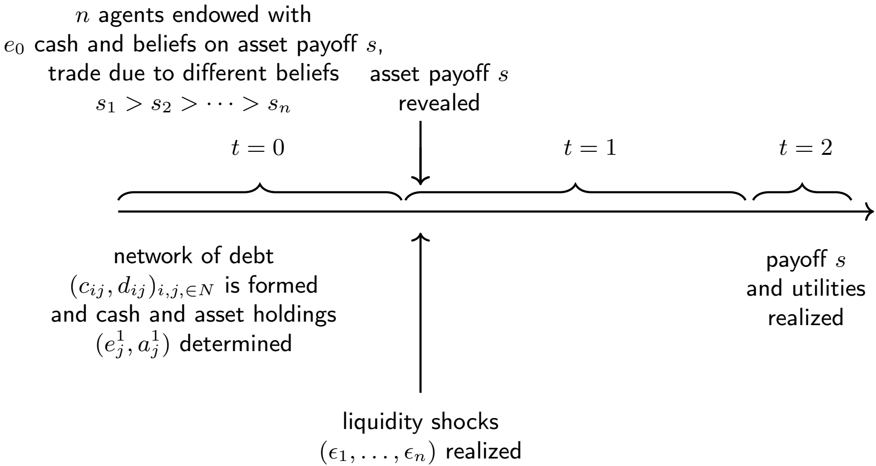

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network of debt
$(c_{ij}, d_{ij})_{i,j,\in N}$ is formed
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debt is paid back
some agents go bankrupt
and inflict lender default cost
$\Psi_{ij}(C)[p_1 - d_{ij}]^+$

payoff s
and utilities
realized

liquidity shocks
$(\epsilon_1, \ldots, \epsilon_n)$ realized

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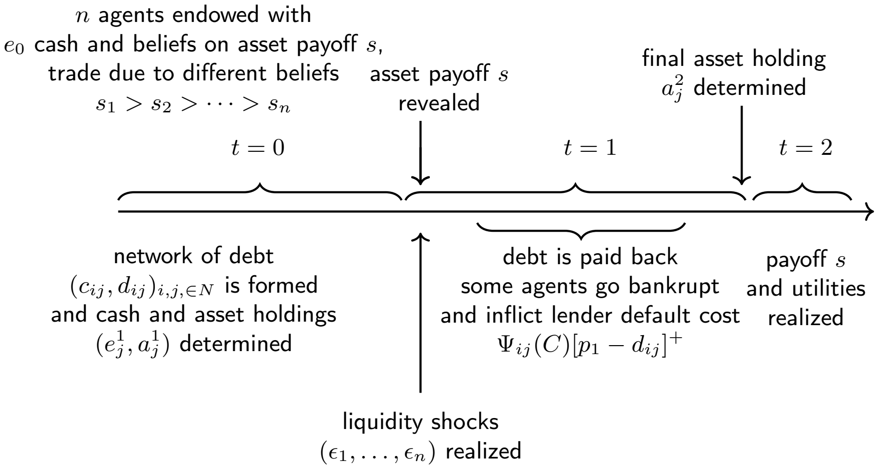

Payment Equilibrium at $t = 1$

- Agent j's wealth:

$$
\begin{aligned}
m_j(p_1) = e_1^j - \epsilon_j + a_j^1 p_1 \\
&+ \sum_{i \in N} \left(c_{ji} \min\{p_1, d_{ji}\} - c_{ij} \min\{p_1, d_{ij}\} \right) \\
&- \sum_{i : m_i < 0} \Psi_{ij}(C)[p_1 - d_{ij}]^+
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- Cash-in-the-market pricing (market clearing condition)

$$\sum_{j \in N} [m_j(p_1)]^+ = A p_1 \quad \text{if } 0 \le p_1 < s \quad \text{(liquidity constrained p)}$$

$$\sum_{j \in N} [m_j(s)]^+ \ge A s \quad \text{if } p_1 = s \quad \text{(fair p)}$$

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- Total wealth $\uparrow$ when $p_1 \uparrow$ and $\Psi \downarrow$
- (m^*, p_1^*) is a **payment equilibrium** if it clears the market

Network Equilibrium at $t = 0$

Maximization problem at $t = 0$:

$$\max_{\substack{e_j^1, \{c_{ij}, d_{ij}\}_{i \in N}, \\ a_j^1, \{c_{jk}, d_{jk}\}_{k \in N}}} E_j \left[[m_j(p_1)]^+ \frac{s}{p_1} \right]$$

$$\text{s.t. } a_j^1 + \sum_{k \in N} c_{jk} \geq \sum_{i \in N} c_{ij},$$

$$e^0 = e_j^1 - \sum_{i \in N} c_{ij} q_i(d_{ij}) + \sum_{k \in N} c_{jk} q_j(d_{jk}) + a_j^1 p_0,$$

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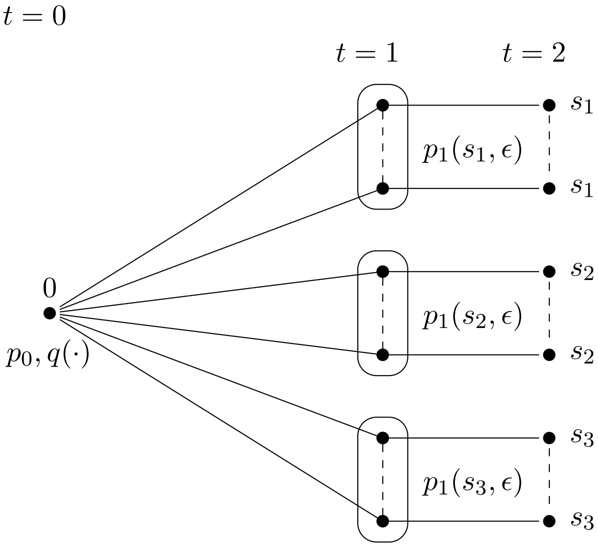
- **Network Equilibrium**: max EU under constraints and mkts clear

Expectations: Tree of States and Prices



Interaction of Network and Price Effects

- Market clearing price

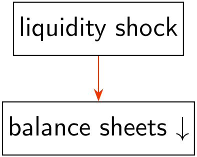
$$p_1 = \frac{\text{(Cash savings from } t = 0) - \text{(Destruction of cash)}}{\text{(Total fire-sales of the assets)}}$$

- Liquidity shock ϵ decreases price p_1 by
  1. destruction of cash by ϵ_j and Ψ
  2. greater amount of assets sold by bankrupt agents
- Second order bankruptcy which causes even more default cost

Dual Loops of Contagion

$\boxed{\text{liquidity shock}}$

Dual Loops of Contagion



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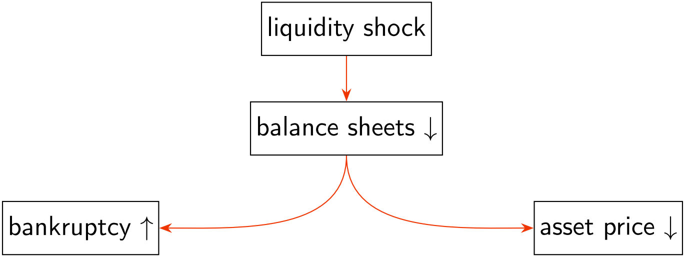

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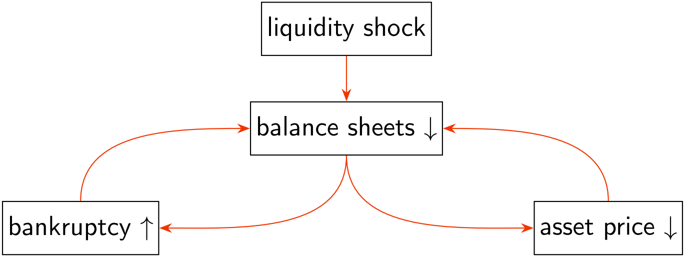
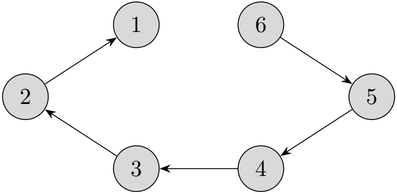


Counterparty and Price Channels of Propagation

- Counterparty channel propagation



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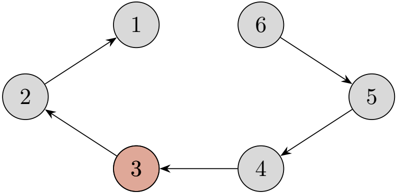

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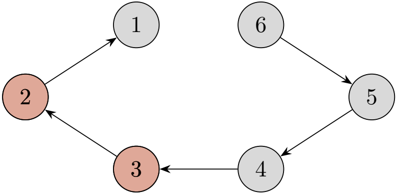

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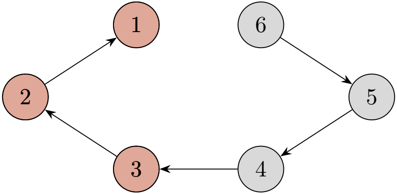

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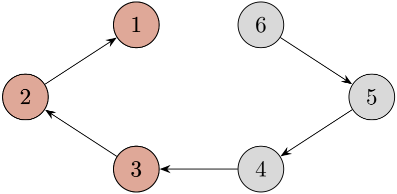
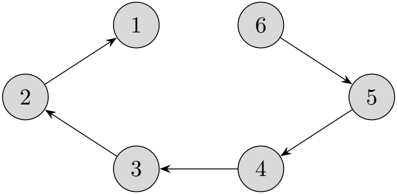


- Price channel amplification



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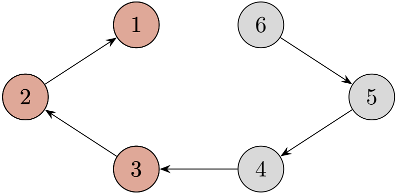

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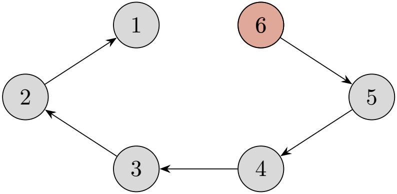

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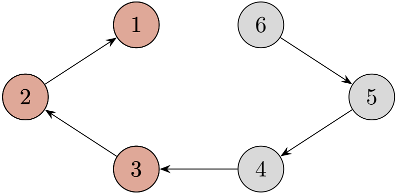

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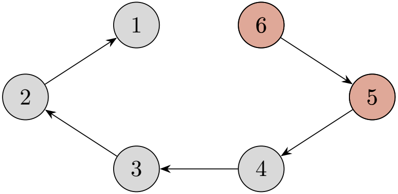

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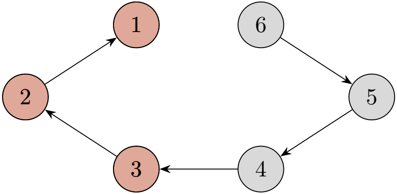

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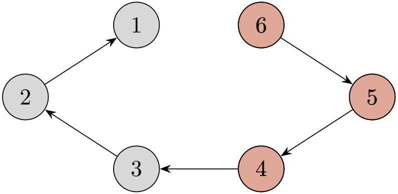
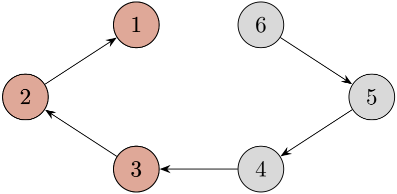


Counterparty and Price Channels of Propagation

- Counterparty channel propagation



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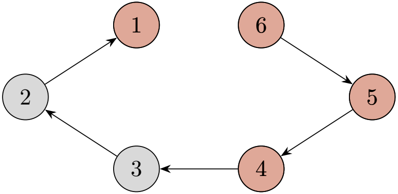
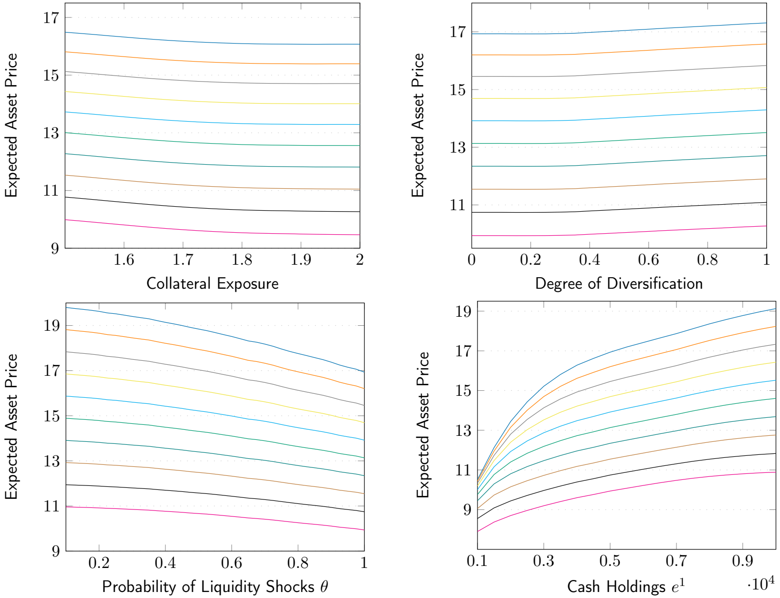


- "No major institution failed because of losses on its direct exposure to Lehman" (Upper, 2011)

Comparative Statics



Optimization Problem (Recap)

Agent j's maximization problem at $t = 0$:

$$\max_{\substack{e_j^1, \{c_{ij}, d_{ij}\}_{i \in N}, \\ a_j^1, \{c_{jk}, d_{jk}\}_{k \in N}}} E_j \left[[m_j(p_1)]^+ \frac{s}{p_1} \right]$$

$$\text{s.t. } a_j^1 + \sum_{k \in N} c_{jk} \geq \sum_{i \in N} c_{ij},$$

$$e^0 = e_j^1 - \sum_{i \in N} c_{ij} q_i(d_{ij}) + \sum_{k \in N} c_{jk} q_j(d_{jk}) + a_j^1 p_0,$$

Network Equilibrium

- For a given economy $(N, (s_j, \theta_j, e^0)_{j \in N}, A, \Psi, G)$, a septuple $(C^*, D^*, e^{1*}, a^{1*}, p_0^*, \tilde{p}_1^*, q^*)$ where $C^*, D^* \in \mathbb{R}_+^{n \times n}$, $e^{1*}, a^{1*} \in \mathbb{R}_+^n$, $p_0^* \in \mathbb{R}_+$, functions $p_1^* : \mathbb{R}_+^{n+1} \to \mathbb{R}_+$ and $q^* \equiv (q_1^*, \ldots, q_n^*)$ with $q_j^* : \mathbb{R}_+ \to \mathbb{R}_+$ is a **network equilibrium** if
  1. $(C^*, D^*, e^{1*}, a^{1*})$ solves the agent maximization problem with two constraints,
  2. markets are cleared as c_{ij}^* is optimal for both agent i and j for all $i, j \in N$,
  3. asset market clears as $\sum_{j \in N} a_j^1 = A$,
  4. asset price $\tilde{p}_1$ is determined by the payment equilibrium for each (ϵ, s),
  5. and asset price p_0 and contract prices q are determined by no arbitrage conditions.

Characterization of Network Equilibrium

- **Lemma.** (Positive Cash Holdings)
  If $\bar{\epsilon} > ne^0$, then $e_j^1 > 0$ for every $j \in N$ in any NE.

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- **Lemma.** (Maximum Leverage)
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- **Proposition.** (Existence)
  There exists a network equilibrium, if the maximum payment equilibrium is selected at $t = 1$, and the set of equilibria forms a complete lattice.

Theorem: Network and Contract Prices

- Any debt network from a NE is under *intermediation order*:

$$\sum_{\substack{i \in N \\ d_{ij} \geq \hat{d}}} c_{ij} \leq a_j^1 + \sum_{\substack{k \in N \\ d_{jk} \geq \hat{d}}} c_{jk} \quad \text{for any } \hat{d} \in \mathbb{R}^+ \text{ and } j \in N.$$

- The contract prices are determined by

$$q_j(d) = $$

$$q_{j+1}(s_{j+1}) + \frac{E_j\left[\min\left\{1, \frac{d}{p_1}\right\} - \min\left\{1, \frac{s_{j+1}}{p_1}\right\} - \frac{\partial \Psi_{j+1,j}(C)}{\partial c_{j+1,j}}\left[1 - \frac{s_{j+1}}{p_1}\right]^+ \mathbb{1}\{j+1 \in B(\epsilon)\}\right]}{E_j\left[\frac{1}{p_1}\right]}$$

- The asset price is determined by $p_0 = q_1(s_1)$.

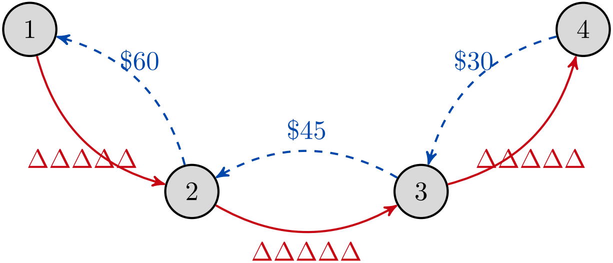
Main Result 1: Counterparty Risk vs Leverage

- In a NE, agent 1's returns from borrowing are

$$R_1^2 \equiv \frac{s_1}{q(s_1)-q(s_2)} E_1\left[1-\min\left\{1,\frac{s_2}{p_1}\right\}-\frac{\partial\Psi_{21}(C)}{\partial c_{21}}\left[1-\frac{s_2}{p_1}\right]^+ \mathbb{1}\{2\in B(p_1)\}\right]$$

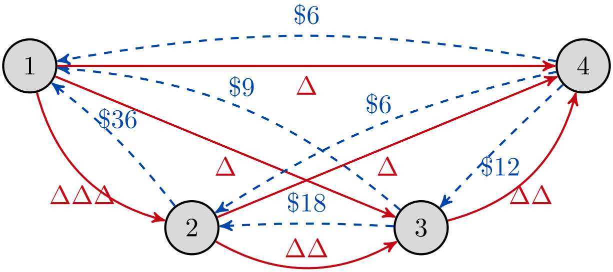
$$R_1^3 \equiv \frac{s_1}{q(s_1)-q(s_3)} E_1\left[1-\min\left\{1,\frac{s_3}{p_1}\right\}-\frac{\partial\Psi_{31}(C)}{\partial c_{31}}\left[1-\frac{s_3}{p_1}\right]^+ \mathbb{1}\{3\in B(p_1)\}\right]$$

- If cp risk is small, single intermediation lending chain is formed



Main Result 1: Counterparty Risk vs Leverage

- If cp risk is large, multi-chain network is formed
  - \# of counterparties $\uparrow$
  - reuse of collateral (intermediation) $\downarrow$
  - leverage and asset price $\downarrow$



Prediction Matches Empirical Observations

- Multiple haircuts for the same collateral (Baklanova et al., 2019)
- Weak relationship btw haircuts and rates (Baklanova et al., 2019)
- Network change after crisis (Lehman bankruptcy):
  - velocity (re-use) of collateral and leverage $\downarrow$ (Singh, 2011)
  - concentration of Prime Broker portfolio $\downarrow$ by 20% (Eren, 2015; Sinclair, 2020)
  - avg number of counterparties $\uparrow$ by 41% (Craig & von Peter, 2014)

Main Result 2: Not Enough Diversification

- **Proposition.** (Diversification and Systemic Risk) Suppose that $(C, D, e^1, a^1, p_0, \tilde{p}_1, q)$ is a network equilibrium. Then, there exists an allocation with higher expected asset prices and lower systemic risk under any agent's belief that is a diversification of agent j from C and paying respective contract prices q for the change in collateral matrix.

- Diversification has positive externalities:
  - concentration & intermediation $\downarrow \Rightarrow$ counterparty externality $\downarrow$
  - leverage $\downarrow \Rightarrow$ price externality $\downarrow$

- Agents diversify less than the socially optimal level in any eqm

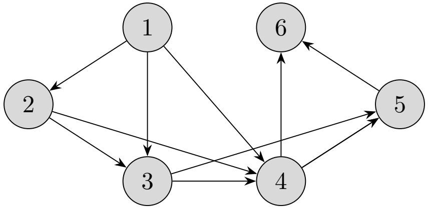
Central Counterparties (CCPs)

- After the crises (GFC and COVID-19), central clearing has been one of the most important market structure regulation issues (e.g. Dodd-Frank Act on swap markets)

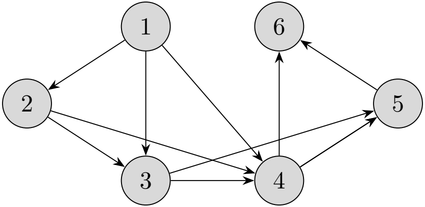
- Should we use CCP for collateralized debt market as well?

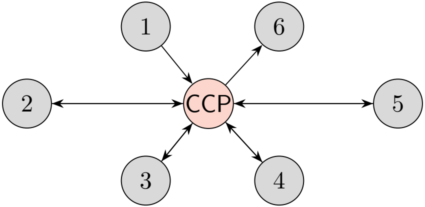
- Roles of CCP:
  - Novation: one contract $\Rightarrow$ two contracts w/ CCP
    $\Rightarrow$ agents deal with each other indirectly through CCP
  - Pooling: guarantee funds (γ from each agent) to cover default risk
    $\Rightarrow$ CCP bears the default costs instead
  - Netting: net out the obligations between the two
    $\Rightarrow$ CCP can reduce nominal exposure

Pooling and Netting

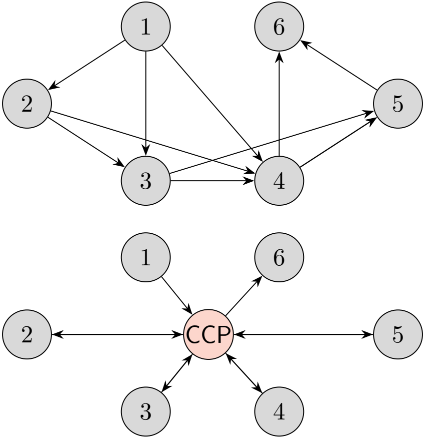


Pooling and Netting





Pooling and Netting



- Pooling can prevent second order bankruptcy
- But the lender default cost is still there
- Netting can reduce counterparty exposure (but not all of it)

CCP Procedure

- Add agent 0 as the CCP and add columns and rows for c_{0i} and c_{i0}
- Contract matrix is modified by adding another row and column with s_j for each $j-1$ row
- n client agents contribute γ amount of cash each so $e_1^0 = n\gamma$
- Define the new network with CCP as (C_{ccp}, D_{ccp})
- Netting of debt procedure:

$$\hat{c}_{ij}\hat{d}_{ij} = [c_{ij}d_{ij} - c_{ji}d_{ji}]^+$$

- Multilateral netting: $\hat{C}_{ccp} \circ \hat{D}_{ccp}$

CCP Procedure

- Netting of lender obligations procedure:
  1. For the given price p_1, compute the entry-by-entry indicator matrix $\Gamma \equiv \mathbb{1}(D = X)$.
  2. Compute the effective collateral matrix $C' \equiv C \circ \Gamma$.
  3. Perform the CCP netting procedure above to derive $\hat{C}'_{ccp}$.
  4. Redistribute the relevant collateral obligations from the updated $\hat{C}'_{ccp}$.

- CCP's nominal wealth after payments becomes

$$m_0(\epsilon|p_1) = n\gamma - \sum_{j\in N}\sum_{k\in N} \Psi_{jk}(C)[p_1 - d_{jk}]^{+}\mathbb{1}\left\{j \in B(\epsilon)\right\},$$

and CCP goes bankrupt when $m_0(\epsilon|p_1) = 0$

Agent's Optimization Problem under CCP

Agent j's problem becomes

$$\max_{\substack{e_j^1, \{c_{ij}, d_{ij}\}_{i \in N}, \\ \{c_{jk}, d_{jk}\}_{k \in N}}} E_j \Bigg[\Bigg(e_j^1 - \epsilon_j + a_j^1 p_1 + \sum_{k \in N} c_{jk} \min\{d_{jk}, p_1\} + \frac{m_0(\epsilon | p_1)}{\sum_{i \in N} \mathbb{1}\{i \notin B(\epsilon)\}}$$

$$- \sum_{i \in N} c_{ij} \min\{d_{ij}, p_1\} - \sum_{0 \in B(\epsilon)} \Psi_{ij}(C)[p_1 - d_{ij}]^+ \mathbb{1}\{i \in B(\epsilon)\} \Bigg) \frac{s}{p_1} \Bigg]^+$$

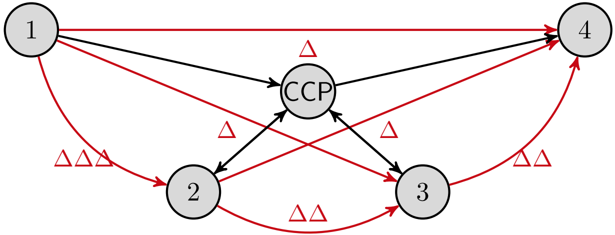
s.t.

$$a_j^1 + \sum_{k \in N} c_{jk} \geq \sum_{i \in N} c_{ij},$$

$$e^0 = e_j^1 + a_j^1 p_0 + \gamma - \sum_{i \in N} c_{ij} q(d_{ij}) + \sum_{k \in N} c_{jk} q(d_{jk}).$$

Main Result 3: Central Clearing & Systemic Risk

- Agents still trade bilaterally but should go through CCP

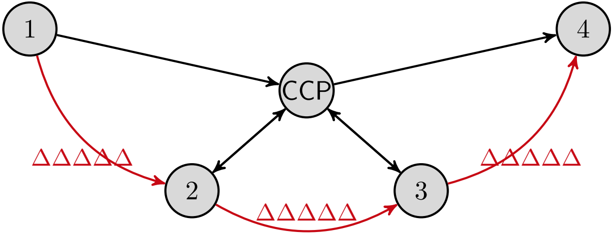


- If network doesn't change, addition of CCP reduces second order bankruptcies and also some first order bankruptcies through netting

Main Result 3: Central Clearing & Systemic Risk

- BUT network endogenously changes to extremely concentrated structure with high leverage which may cause more default costs

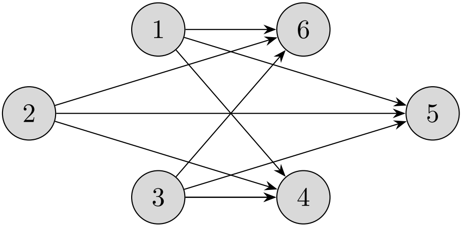
- CCP eliminates direct counterparty risk concern from agents (they concentrate counterparty risk to maximize leverage)



- Exogenous leverage or network model misses all these (absence of trade-off between counterparty risk and leverage)

Exogenous Leverage Case (Fixed Initial Margins)

- Agents are already fully diversifying



- **Irrelevance of CCP.**
  Both pooling and netting have zero effect on network formation under exogenous leverage.

Central Counterparty Trade-Off

- Benefit: Net counterparty exposure $\downarrow$ by multilateral netting
  + Prevention of second order bankruptcies

- Cost: Systemic risk increased by the endogenous network response

Central Counterparty Trade-Off

- Benefit: Net counterparty exposure $\downarrow$ by multilateral netting
  + Prevention of second order bankruptcies
- Cost: Systemic risk increased by the endogenous network response
- Not a normative result!
  Just a neglected side-effect of CCP that arises with the combination endogenous (price) + (leverage) + (network formation)

Policy Implications

- Supplementary Leverage Ratio (SLR):

$$\frac{\text{Tier 1 Capital}}{\text{Total Leverage Exposure}} \geq 3\%$$

- Network Supplementary Leverage Ratio:

$$\frac{\text{Tier 1 Capital}}{(c_{1i}^2 + c_{2i}^2 + \cdots + c_{ni}^2) \times \text{Total Leverage Exposure}},$$

  risk externality is included as weights of centrality
  (second order default and borrower/lender discipline)

- Network analysis before mandatory clearing (CCP)
  (e.g. CDS market)

Conclusion

- Developed a network + GE model
- Counterparty and price channels amplify each other
- Trade-off between leverage and counterparty risk
- Change in cp risk $\Rightarrow$ change in leverage, price, and network
- Positive externalities from diversification
- Changing cp interaction can have side-effects (CCP trade-off)

Thank you!