Calculated parameters for HBT-EP

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Calculated using SMath: https://smath.com

HBT-EP Parameters

$n\_0 := 2 \cdot 10^{19} \cdot \frac{1}{m^3}$

$B := 0.35\ T$

$kT\_e := 100 \cdot eV$

$kT\_i := 50 \cdot eV$

$r\_minor := 15\ cm$

$r\_major := 92\ cm$

$I\_p := 13\ kA$

Constants

$m_e = 9.1094 \cdot 10^{-31}\ kg$

$e = 1.6022 \cdot 10^{-19}\ C$

$Z := 1$ ion charge state

$\gamma := 1.4$ adiabatic index

$m_p = 1.6726 \cdot 10^{-27}\ kg$

$\mu_0 = 1.2566 \cdot 10^{-6}\ \frac{m\ T}{A}$

$eV := 1.602 \cdot 10^{-19}\ J$

$\varepsilon_0 = 8.8542 \cdot 10^{-12}\ \frac{F}{m}$

$m\_deu\_ion := 2.014\ m_p = 3.3687 \cdot 10^{-27}\ kg$

Derived Parameters

Frequencies

gyro/cyclotron frequencies

$$\omega\_ci := \frac{e \cdot B}{m\_deu\_ion} = 1.6646 \cdot 10^{7}\ \frac{rad}{s}$$

$$\omega\_ce := \frac{e \cdot B}{m_e} = 6.1559 \cdot 10^{10}\ \frac{rad}{s}$$

plasma frequencies

$$\omega\_pi := \sqrt{\frac{e^2 \cdot n\_0}{\varepsilon_0 \cdot m\_deu\_ion}} = 4.1488 \cdot 10^{9}\ \frac{rad}{s}$$

$$\omega\_pe := \sqrt{\frac{e^2 \cdot n\_0}{\varepsilon_0\ m_e}} = 2.5229 \cdot 10^{11}\ \frac{rad}{s}$$

upper hybrid frequency

$$\omega\_uh := \sqrt{\omega\_pi^2 + \omega\_pe^2} = 2.5233 \cdot 10^{11}\ \frac{rad}{s}$$

lower hybrid frequency

$$\omega\_lh := \sqrt{\omega\_ci^2 + \frac{\omega\_pi^2}{1 + \frac{\omega\_pe^2}{\omega\_ce^2}}} = 9.8358 \cdot 10^{8}\ \frac{rad}{s}$$

Velocities

thermal velocities

$v\_thi := \sqrt{\frac{2 \cdot kT\_i}{m\_deu\_ion}} = 68960.8598 \, \frac{m}{s}$

$v\_the := \sqrt{\frac{2 \cdot kT\_e}{m_e}} = 5.9306 \cdot 10^{6} \, \frac{m}{s}$

ion sound speed

$C\_s := \left(\frac{\gamma \cdot Z \cdot kT\_e}{m\_deu\_ion} \right)^{0.5} = 81595.5897 \, \frac{m}{s}$

alfven speed

$v\_A := \frac{B}{\sqrt{\mu_0 \cdot n\_0 \cdot m\_deu\_ion}} = 1.2029 \cdot 10^{6} \, \frac{m}{s}$

Length scales

gyroradius

$\rho\_i := \frac{v\_thi}{\omega\_ci} = 4.1427 \, mm$

$\rho\_e := \frac{v\_the}{\omega\_ce} = 0.0963 \, mm$

Debye length

$\lambda\_de := \sqrt{\frac{\varepsilon_0 \cdot kT\_e}{n\_0 \, e^{2}}} = 16.6219 \, \mu m$

Time scales

$t\_Alfven\_minor := \frac{r\_minor}{v\_A} = 124.7015 \, ns$

$t\_Alfven\_major := \frac{r\_major}{v\_A} = 764.836 \, ns$

Misc. parameters

number of particles in a debye sphere

$N := n\_0 \cdot \lambda\_de^{3} \cdot \frac{4 \cdot \pi}{3} = 3.8473 \cdot 10^{5}$

ion/electron mass ratio

$\mu := \frac{m\_deu\_ion}{m_e} = 3698.0115$

$\sqrt{\mu} = 60.8113$

Greenwald density limit

$n\_G := \frac{I\_p}{\pi \cdot r\_minor^{2}} \cdot \left(\frac{m^{2}}{10^{6} \, A} \cdot 10^{20} \, m^{-3} \right) = 1.8 \cdot 10^{19} \, m^{-3}$

Inverse tokamak aspect ratio

$\varepsilon := \frac{r\_minor}{r\_major} = 0.163$

Bohm diffusion coefficient

$D\_B := \frac{kT\_e}{16 \cdot \mathbf{e} \cdot B} = 1.0524 \cdot 10^{-18} \, m^{2} \, A$

Beta

$\beta := \frac{2 \, \mu_0 \cdot n\_0 \cdot kT\_e}{B^{2}} = 0.0066$

Collisions

Coulomb logarithm $ln\Lambda := \ln(4 \cdot \pi \cdot N) = 15.3913$

Electron Collision frequencies - [Fitzpatrick]

$$\tau\_ee := \frac{12 \cdot \sqrt{2} \cdot \pi^{\frac{3}{2}} \varepsilon_0^{\ 2} m_e^{\frac{1}{2}} \cdot kT\_e^{\frac{3}{2}}}{ln\Lambda\ e^4 \cdot n\_0} = 2.2345 \cdot 10^{-6}\ s$$

$$\tau\_ei := \frac{6 \cdot \sqrt{2} \cdot \pi^{\frac{3}{2}} \varepsilon_0^{\ 2} m_e^{\frac{1}{2}} \cdot kT\_e^{\frac{3}{2}}}{ln\Lambda\ e^4 \cdot n\_0} = 1.1172 \cdot 10^{-6}\ s$$

$$\nu\_ee := \frac{2 \cdot \pi}{\tau\_ee} = 2.8119 \cdot 10^{6}\ \frac{rad}{s}$$

$$\nu\_ei := \frac{2 \cdot \pi}{\tau\_ei} = 5.6239 \cdot 10^{6}\ \frac{rad}{s}$$

$\boxed{\nu\_e := \nu\_ee + \nu\_ei = 8.4358 \cdot 10^{6}\ \frac{rad}{s}}$ or $\boxed{\frac{\nu\_e}{2 \cdot \pi} = 1.3426 \cdot 10^{6}\ Hz}$

$\frac{\omega\_ce}{\nu\_e} = 7297.2897$ >> 1, which means the electrons ARE "magnetized"

Ion Collision frequencies - [Fitzpatrick]

$$\tau\_ii := \frac{12 \cdot \sqrt{2} \cdot \pi^{\frac{3}{2}} \varepsilon_0^{\ 2} \cdot m\_deu\_ion^{\frac{1}{2}} \cdot kT\_i^{\frac{3}{2}}}{ln\Lambda\ e^4 \cdot n\_0} = 4.812 \cdot 10^{-5}\ s$$

$$\tau\_ie := \frac{6 \cdot \sqrt{2} \cdot \pi^{\frac{3}{2}} \varepsilon_0^{\ 2} \cdot m\_deu\_ion \cdot kT\_i^{\frac{3}{2}}}{ln\Lambda\ e^4 \cdot n\_0\ m_e^{0.5}} = 0.0015\ s$$

$$\nu\_ii := \frac{2 \cdot \pi}{\tau\_ii} = 1.3057 \cdot 10^{5}\ \frac{rad}{s}$$

$$\nu\_ie := \frac{2 \cdot \pi}{\tau\_ie} = 4294.3807\ \frac{rad}{s}$$

$\boxed{\nu\_i := \nu\_ie + \nu\_ii = 1.3487 \cdot 10^{5}\ \frac{rad}{s}}$ or $\boxed{\frac{\nu\_i}{2 \cdot \pi} = 21464.8722\ Hz}$

$\frac{\omega\_ci}{\nu\_i} = 123.4278$ >> 1, which means the ions ARE "magnetized"

Mean free path (due to collision)

$$\lambda\_mfp\_e := v\_the \cdot \frac{2 \cdot \pi}{\nu\_e} = 4.4173\ m$$

$$\lambda\_mfp\_i := v\_thi \cdot \frac{2 \cdot \pi}{\nu\_i} = 3.2127\ m$$

Torodial transit frequency

$$\tau\_\varphi i := \frac{2\cdot\pi\cdot r\_major}{v\_thi} = 8.3823\cdot 10^{-5}\ \text{s} \qquad \nu\_\varphi i := \frac{2\cdot\pi}{\tau\_\varphi i} = 74957.4563\ \frac{\text{rad}}{\text{s}}$$

$$\tau\_\varphi e := \frac{2\cdot\pi\cdot r\_major}{v\_the} = 9.7469\cdot 10^{-7}\ \text{s} \qquad \nu\_\varphi e := \frac{2\cdot\pi}{\tau\_\varphi e} = 6.4464\cdot 10^{6}\ \frac{\text{rad}}{\text{s}}$$

$$\frac{\nu\_\varphi i}{\nu\_i} = 0.5558$$

The ions average about order 1 collision per toroidal transit. This means that the ions are largely oblivious to the poloidal variations to the B (due to tokamak curvature) and that the io think that B is more like a uniform cylinder

$$\frac{\nu\_\varphi e}{\nu\_e} = 0.7642$$

Same of electrons

Conductivity in a homogeneous B field - [Piel, page 91]

1. Spitzer conductivity (along B)

$$\sigma\_i := \frac{n\_0\ e^2}{m\_deu\_ion\cdot\nu\_ie} = 35490\ \frac{\text{S}}{\text{m}} \qquad \sigma\_e := \frac{n\_0\ e^2}{m_e\cdot\nu\_ei} = 1.0021\cdot 10^{5}\ \frac{\text{S}}{\text{m}}$$

2. Pederson current conductivity (current along electric field but perp to B)

$$\sigma\_i\_pederson := \sigma\_i\cdot\left(\frac{1}{1+\left(\frac{\omega\_ci}{\nu\_ie}\right)^2}\right) = 0.002362\ \frac{\text{S}}{\text{m}}$$

$$\sigma\_e\_pederson := \sigma\_e\cdot\left(\frac{1}{1+\left(\frac{\omega\_ce}{\nu\_ei}\right)^2}\right) = 0.0008\ \frac{\text{S}}{\text{m}}$$

3. Hall current conductivity (current orthogonalto B and E)

$$\sigma\_i\_hall := \sigma\_i\cdot\left(\frac{\left(\frac{\omega\_ci}{\nu\_ie}\right)}{1+\left(\frac{\omega\_ci}{\nu\_ie}\right)^2}\right) = 9.155\ \frac{\text{S}}{\text{m}}$$

$$\sigma\_e\_hall := \sigma\_e\cdot\left(\frac{\left(\frac{\omega\_ce}{\nu\_ei}\right)}{1+\left(\frac{\omega\_ce}{\nu\_ei}\right)^2}\right) = 9.1553\ \frac{\text{S}}{\text{m}}$$

Reference Conductivity/Resistivity values:

	Conductivity	Resistivity
Copper	6.0e7 S/m	1.7e-8 Ohm*m
Steel	1.4e6 S/m	7.1e-7 Ohm*m

References

* https://www.nrl.navy.mil/ppd/content/nrl-plasma-formulary
* [P.M. Bellan, Fundamentals of Plasma Physics (Cambridge University Press, 2008 Appendix C]
* [J. Wesson, Tokamaks, 3rd ed. (Oxford University Press, 2004), Chapter 14]
* [Figtzpatrick's online class notes, https://farside.ph.utexas.edu/teaching/plasma/Plasma/node41.html]
* [Piel, Plasma Physics, (Springer 2010)]