CVEN 306 Laboratory Exercise

Thermal Conduction in a Composite

Task 4 Solution

Task 4

Design a three-layer composite wall with the *minimum possible* effective heat transfer coefficient subject to the following engineering constraints:

- You must choose materials from among the three given in Table 1.
- $h_L = h_R = 5\,\mathrm{W\,m^{-2}\,K^{-1}}$.
- The wall must be 1 m thick.
- The overall density of the wall must be between $2.0\,\mathrm{g\,cm^{-3}}$ and $4.0\,\mathrm{g\,cm^{-3}}$.

Step 1: Write objective function

The ordering of the materials does not matter, so we will just assume that layer 1 is PP, layer 2 is stainless steel, and layer 3 is aluminum. The objective function in this case is Eq. (6):

$$\begin{aligned} R = \frac{1}{U_\mathrm{e}} &= \frac{1}{h_L} + \frac{L_1}{k_1} + \frac{L_2}{k_2} + \frac{L_3}{k_3} + \frac{1}{h_R} \\ &= \frac{1}{5} + \frac{L_1}{0.15} + \frac{L_2}{20} + \frac{1 - L_1 - L_2}{220} + \frac{1}{5} \\ &= 0.4 + 6.667L_1 + 0.05L_2 + 0.00455 - 0.00455L_1 - 0.00455L_2 \\ R &= 0.4046 + 6.6622L_1 + 0.0455L_2 \end{aligned} \tag{1}$$

Step 2: Write the constraints as linear inequalities

Constraint 1: The density must be at least $2.0\,\mathrm{g\,cm^{-3}}$, which can be written as

$$\begin{aligned} 0.9L_1 + 7.8L_2 + 2.7\,(1 - L_1 - L_2) &\geq 2 \\ 2.7 - 1.8L_1 + 5.1L_2 &\geq 2 \\ L_2 &\geq 0.3529L_1 - 0.1373 \end{aligned}$$

Constraint 2: The density must be no greater than $4.0\,\mathrm{g\,cm^{-3}}$, which can be written as

$$\begin{aligned} 0.9L_1 + 7.8L_2 + 2.7\,(1 - L_1 - L_2) &\leq 4 \\ 2.7 - 1.8L_1 + 5.1L_2 &\leq 4 \\ L_2 &\leq 0.3529L_1 + 0.2549 \end{aligned}$$

Constraint 3: The sum of the first two layer thicknesses cannot exceed the wall thickness, which means

$$L_2 \leq 1 - L_1$$

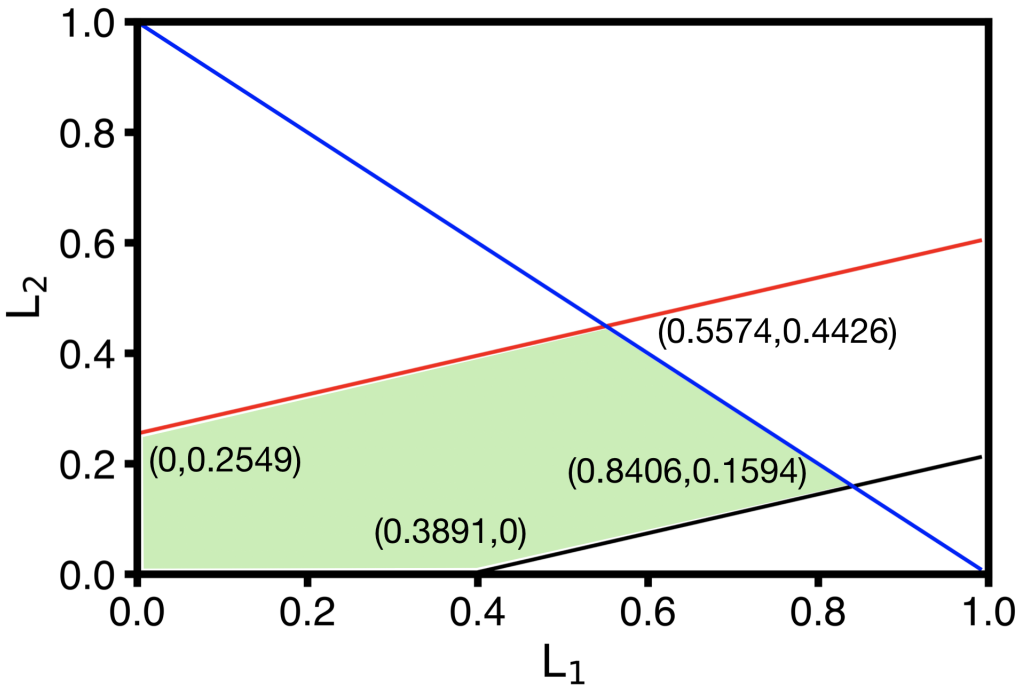
Constraints 4 and 5: None of the wall thicknesses can be negative:

$$L_1 \geq 0$$
$$L_2 \geq 0$$

To summarize, here are the constraints:

$$L_2 \geq 0.3529L_1 - 0.1373 \quad (2)$$
$$L_2 \leq 0.3529L_1 + 0.2549 \quad (3)$$
$$L_2 \leq 1 - L_1 \quad (4)$$
$$L_1 \geq 0 \quad (5)$$
$$L_2 \geq 0 \quad (6)$$

Step 3: Plot all the inequalities on the same graph



Step 3: Test the vertices

Vertex (L_1, L_2)	Value of $R = 1/U$	Diagnosis
$(0, 0)$	0.4046	Minimum
$(0, 0.2549)$	0.4162	
$(0.5574, 0.4426))$	4.1382	
$(0.8406, 0.1594)$	6.0121	Maximum
$(0.3891, 0)$	2.9969	

The maximum of the objective function is $R = 6.0121$ and it occurs when $L_1 = 0.8406$, $L_2 = 0.1594$, $L_3 = 0$. In this configuration, the miminum value of $U_\mathrm{e} = 1/R = 0.1663\,\mathrm{W\,m^{-2}\,K^{-1}}$, and the density is $2\,\mathrm{g\,cm^{-2}}$.