Example : Ear Decomposition

A block is a graph G such that $G - \{x\}$ is connected for every $x \in V(G)$ or $G = K_2$. Theorem 2.5.4 says a graph $G \neq K_2$ is a block if and only if every two edges are contained in a cycle, and if and only if every two vertices are contained in a cycle. In the figure below, the wheel with seven vertices is shown. It is clearly a block since any two edges are contained in a cycle (or since $G - \{x\}$ is connected for every $x \in V(G)$.

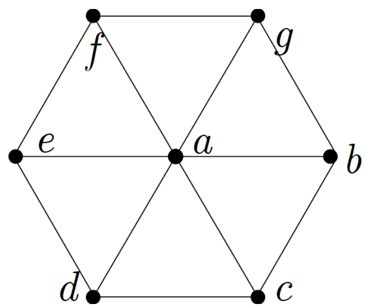

Figure 1: Wheel W with seven vertices

There are many ear decompositions of this graph W. Recall that an ear decomposition is a cycle C plus a collection of paths $P_1, P_2, \dots, P_k$ such that $W = C \cup P_1 \cup P_2 \cup \cdots \cup P_k$ such that P_i is a path in the graph $C \cup P_1 \cup \cdots \cup P_i$ of maximum length subject to having all its internal vertices of degree two (these paths are called ears). To get an ear decomposition of W, we work backwards: first we find an ear P_k of W. Since W has no vertices of degree two, every edge of W is a candidate for P_k. So let us take P_k to be the edge $\{a, b\}$. Now we remove $\{a, b\}$ from W, and search for an ear P_{k-1} in the resulting graph. Again there are many choices, so we pick one, say P_{k-1} is the path with edges $\{c, b\}$ and $\{b, g\}$. Then we remove b from the graph and repeat. Now the path with edges $\{f, g\}$ and $\{g, a\}$ is an ear, and we remove g from the graph. Then $\{a, c\}$ and $\{c, d\}$ form an ear, and we remove c from the graph. We can now remove $\{e, a\}$ and then we are left with the cycle C with $V(C) = \{a, d, e, f\}$.

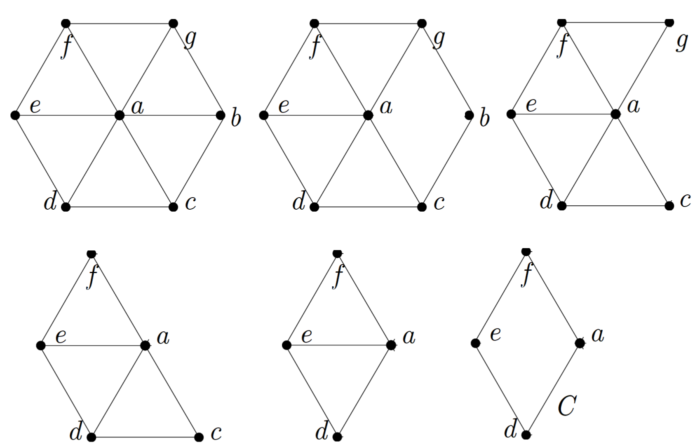

Figure 2: Ear decomposition of W

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