In the figure below, we represent states of the United States by vertices, and adjacency represents two states which share a state boundary.

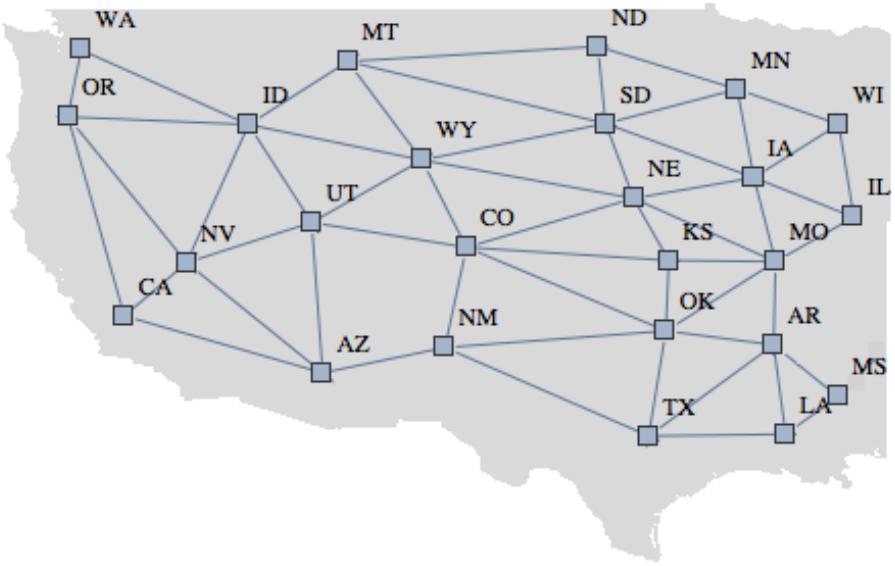

Figure 1: Adjacency of states in the U.S.

Suppose the states are ordered alphabetically by their initials, so AR is first, followed by AZ, CA, CO and so on. We find a breadth-first search tree rooted at AR.

Layer zero has AR only. Layer 1 consists of the neighbors of AR in alphabetical order, so we add the vertices (LA,MO,MS,OK,TX) in that order, and the edges $\{AR, LA\}, \{AR, MO\}$, and $\{AR, MS\}, \{AR, OK\}, \{AR, TX\}$ are added to the BFS tree in that order. So Layer 1 is (LA,MO,MS,OK,TX), in that order.

Next we consider the first added vertex to Layer 1. This is LA. Since all neighbors of LA are already in the tree, there is no neighbor of LA to add. The next vertex added to Layer 1 was MO. Since IA, IL, KS and NE are not already in the tree, we add (IA,IL,KS,NE) to Layer 2, in that order, and the edges $\{MO, IA\}, \{MO, IL\}, \{MO, KS\}, \{MO, NE\}$ are added to the BFS tree in that order. The next vertex added to Layer 1 was MS, which has no neighbors that are not already in the tree. Next we consider OK. Of the neighbors of OK, only NM and CO are not already in the tree, so we add CO then NM, and the edges $\{OK, CO\}$ and $\{OK, NM\}$ in that order. Finally, we consider TX, and see no neighbor of TX can be added. We conclude Layer 2 is (IA,IL,KS,NE,CO,NM) in that order.

The first vertex added to Layer 2 is IA, so we add (MN,SD,WI) in that order to Layer 3, and add the edges $\{IA, MN\}, \{IA, SD\}$ and $\{IA, WI\}$ to the tree so far. The next vertex added to Layer 2 is IL, but none of its neighbors are available to add. Next we look at KS, and add nothing. Next is NE, and we can add WY and the edge $\{NE, WY\}$. Next is CO, and we

can add UT and the edge $\{CO, UT\}$. Then we have NM, and we can add AZ and the edge $\{NM, AZ\}$. So Layer 3 is (MN,SD,WI,WY,UT,AZ) in that order.

The first vertex added to Layer 3 is MN, and we add ND and the edge $\{MN, ND\}$. Next is SD and we add MT and the edge $\{SD, MT\}$. Next is WI, but it has no available neighbors. Next is WY, and we can add ID and the edge $\{WY, ID\}$. Next is UT, and we can add NV and the edge $\{UT, NV\}$. Finally we have AZ, and we add CA and the edge $\{AZ, CA\}$. So Layer 4 is (ND,MT,ID,NV,CA) in that order.

The first vertex added to Layer 4 is ND, but it has no available neighbors, and next is MT, and it also has no available neighbors. ID is next, and it has two available neighbours, OR and WA, and then we add the edges $\{ID, OR\}$ and $\{ID, WA\}$. Now there are no vertices left in the graph, so we have our tree, with Layer 5 being (OR,WA) in that order. The tree is shown with black edges below, and its layers are shown by colors.

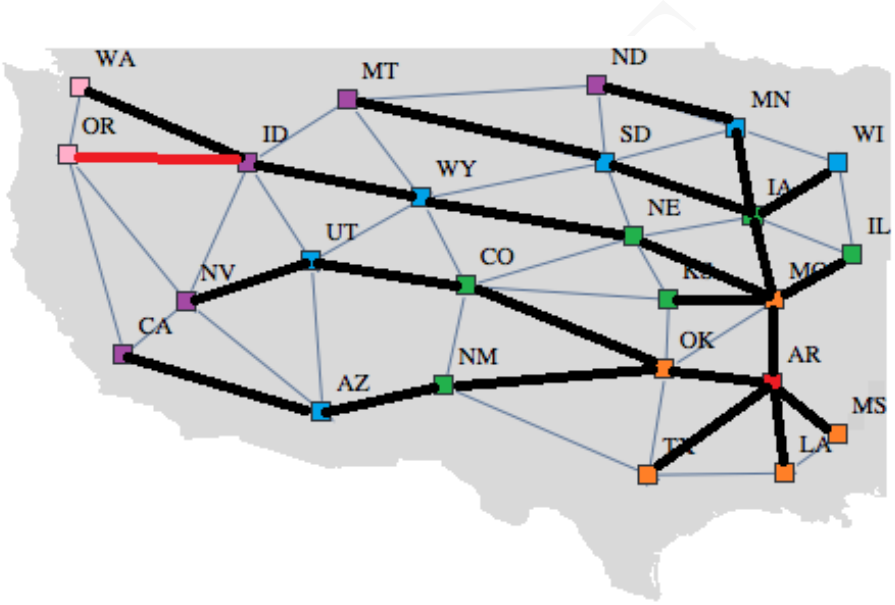

Figure 2: Adjacency of states in the U.S.

As an exercise, what is the height of the tree found? What do you think are the radius and diameter of this graph? What if we do the same for the entire map of the contiguous United States – then what is the radius and diameter?