Week 1 2018-01-19

Device electronics

Insulators - semi conductors - conductors - Superconductors

Insulator
- dimond $\sim 10^{14}$ Resistivity $\rho(\Omega\text{-cm})$
- glass $\sim 10^{11}$

Conductors
- silver 10^{-5}
- copper 10^{-6}

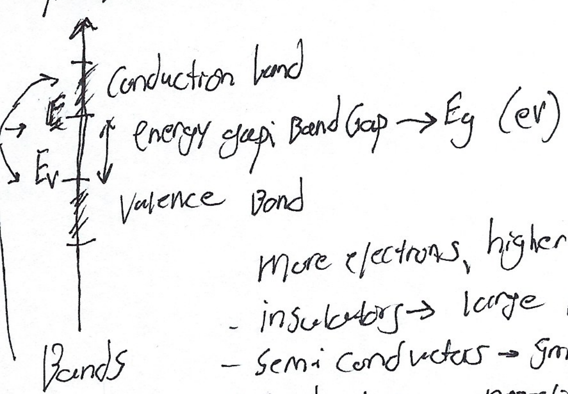
Semiconductors $10^{8} - 10^{-3}$

Ge
Si (group IV)
GaAs (group III - ~~IV~~ V)

Superconductors

0 (no resistivity)

electrons can only take on energy in certain bands. They can only take on energy from the higher level. ~~still part of the bands~~

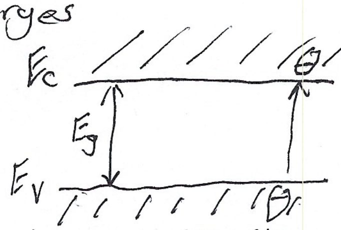


More electrons, higher the conductivity
- insulators → large bandgaps
- semi conductors → small band gaps
- conductors → negative band gaps

Quantifying different charges

$n(1/cm^3)$

$p(1/cm^3)$

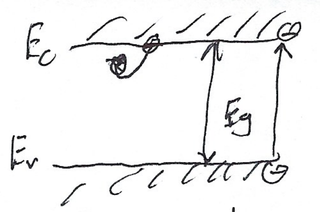


Pure material ↔ Intrinsic

$n = p = n_i(T)$

$\boxed{np = n_i^2}$ condition for thermal equilibrium

If you add impurities into the lattice



for every impurity added, increase the number of electrons by one.

In a cubic cm you have about 10^{10} electrons $\xrightarrow{\text{doping}}$ 10^{17}

about 10^{24} atoms (avogadro's #)

- only a very small fraction of these bonds are broken

small perturbation, and you change by 7 orders of magnitude

· ~~More positivity~~

an acceptor accepts ... and increased # of holes by 1.
(group 3 atom

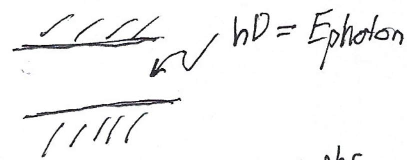
You have 10^{10}

Semi conductors where you introduce impurities are called Extrinsic Semiconductors

acceptor $p > n$

Intrinsic → if it is pure

extrinsic → if semiconductor is not pure.



Brillouin Zone graphs
- X axis - Wave function
- Y axis -

EK diagrams
- energy momentum relationships

∪ - Conduction band

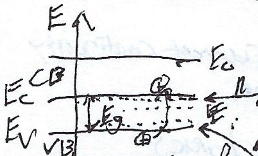
∩ - valance band

- Figure 6 - Valance band looks like a parabola
  - bottom of conduction band not lined ~~to~~ up but still a ~~para~~ parabola
  - indirect semiconductor

Week 2 2018-01-26 Device electronics

- how do electrons & holes & densities relate to each other, how do charges move
  - drift and diffusion current
- electric field and magnetic field
- relating charges to electric fields.
- understanding what charge neutrality is and how they relate to one another.

- the notion of energy bands:

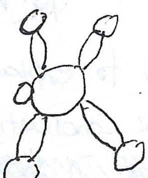


- Conduction band
- Valance band — Donors
- band gap
- E_0 vacume energy — acceptors
- CB Conduction band
- VB Valance band

Intrinsic
$n = p = n_i(T)$

- Can get temporary electrons & holes if you add

Add Impurities Extrinsic
$n >> p$



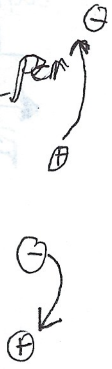
- You have a vacancy that is positively charged
- You have a mobile wandering charge refered to as a hole

how do electrons and holes coexist

Generation & Recombination:

What is the rate at which I can generate electrons, one is the rate

$$G = \frac{\text{number of generation per}}{\text{time and volume}}$$
(S) (cm³)



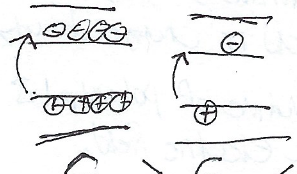
$R =$ "

Constant temperature, everything is steady state Thermal equilibrium

$R = G$

Generation G:

does G depend on electron/hole charge densities? If looking at rate charges are being generated, & temperature. If I want to understand generation ~~rates~~ rate.



$G_1 > G_2$ ←
or
$G_1 = G_2$

$G = \alpha_g(T)$

function dependent on temperature but not on n or p

→ What is the temperature and how much energy is being provided by that material.

$\alpha_r np = \alpha_g$

$R = \alpha_r(T)np$

Intrinsic
$n = p = n_i$
$np = n_i^2$
$\alpha_r np = \alpha_g$
$\alpha_r n_i^2 = \alpha_g$

→ Extrinsic ← add impurities
$\alpha_r np = \alpha_g$
$np = n_i^2$
→ $np = n_i^2$

Thermal equilibrium

- Why have a bunch of electrons and holes
- need knowledge of electric field as a minimum for motion of electrons

Donor atoms
acceptor atoms
electrons
holes

← What are all the charges that are present in this device
- ionized donors and acceptors
- mobile electrons and holes

$$\rho = q(\underbrace{p - n}_{\text{mobile}} + \underbrace{N_D^+ - N_A^-}_{\text{fixed}})$$

↑ charge density C/cm^3

@ room temperature: $N_D^+ \approx N_D$

Maxwell's Eqn:

$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon}$ Gauss Law

$\nabla \times \vec{E} = \frac{-d\vec{B}}{dt}$ Faradys Law

$\frac{dE}{dx} = \frac{\rho}{\epsilon}$

ampeers law

Faradays law used heavily in generators

- the rate of change of the flux is ~~apparent~~ approximately zero

$\nabla \times \vec{E} \approx 0$

$\vec{E} = -\nabla\phi$

$\nabla \times (\nabla\phi) \equiv 0$

Charges generate electric field, see this through gausses law.

- Derivitive of electric field is charge density

- Derivative of potential is the electric field.

the divergence of the gradient

$$\nabla\cdot(\nabla\phi) = \boxed{\nabla^2\phi = -\frac{\rho}{\epsilon}}$$

laplacian

Poisson equation

relating charges to voltages, and voltages to charges

$$\nabla^2\phi(x,y,z) = -\frac{\rho(x,y,z)}{\epsilon}$$

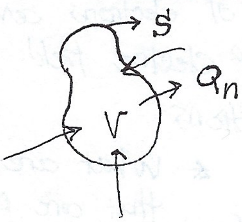
$$\nabla^2\phi = -\frac{q}{\epsilon}(-n+p+N_D^+-N_A^-) \qquad N_D^+ - N_A^- = D$$

$$\nabla^2\phi = \frac{q}{\epsilon}(n-p-D)$$

We need more equations since we have one equation and ~~three~~ ~~equations~~ three unknowns

need to understand <u>motion & current</u> + <u>Recombination and Generation</u>

→ This entering or exiting is called a ~~d~~ divergence



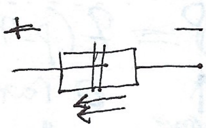
Can have electrons flowing in.

- or a situation charge density n going down as charges flow out

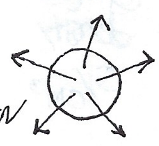
diverge <u>out</u> → charge density <u>down</u> ~~divergen~~

divergence <u>in</u> → charge density <u>up</u>

$R \sim (np - n_i^2) \to 0$



Current continuity equation



$$-\frac{dQ_n}{dt} = I_n + r_n$$

total recombination gen in that volume

$$Q_n = -q\int_{volume} n\,dV$$

"(R-G)" $1/cm^3\cdot sec$

$$I_n = \int_{surface} \vec{J}_n\cdot d\vec{s} \qquad r_n = -q\int_{vol} R\cdot dV$$

$$\frac{dQ_n}{dt} = -q\int_V \frac{dn}{dt}\,dV$$

$$r_n = -q\int_{vol} R\cdot dV$$

$$\Rightarrow q\int_V \frac{dn}{dt}\,dV = \int_V \nabla\cdot\vec{J}_n\,dV - q\int_V R\,dV$$

$$\boxed{\int_V \left[\frac{dn}{dt} - \frac{1}{q}\nabla\cdot\vec{J}_n + R\right]dV = 0}$$

$$\frac{dn}{dt} = \frac{-1}{q}\nabla\cdot\vec{J}_n - R$$

- Electron current continuity equation

$$\frac{dp}{dt} = \frac{-1}{q}\nabla\cdot\vec{J}_p - R$$

- hole current continuity equation

$R > 0$: Net recombination of charges

$R < 0$: Net generation of charges

$\vec{J}_n, \vec{J}_p(\check{n}, \check{p}, \vec{E} = \nabla\check{\phi})$

$R(n, \check{p})$

- need to explain $\vec{J}_n$ & $\vec{J}_p$ more concreatly, ~~what are things in the~~

Keeping track of things flowing in & out as well as recombination and generation.

★ <u>Can only</u> increase or decrease charge densities through the following ways.

- ex Ball, you ~~[illegible]~~ tilt the surface

Where are electrons ~~[illegible]~~ flowing, within the device

- leaf falling reaches a steady state velocity
- Same with electron

$\vec{E}$ → $\ominus$ $\vec{F}$ ← μ = Mobility $\oplus$ →

$$\vec{V}_{drift} = -\mu_n\vec{E} \qquad \vec{V}_{drift} = +\mu_p\vec{E}$$

$$\vec{J}_n^{dr} = -qn\vec{V}_n^{drift} = q\mu_n n\vec{E}$$

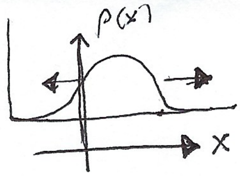
$$\vec{J}_p^{dr} = +qp\vec{V}_p^{drift} = q\mu_p p\vec{E}$$

$$\vec{J}^{drift} = \vec{J}_n^{dr} + \vec{J}_p^{dr}$$

$$\vec{J}^{drift} = \underbrace{(q\mu_n n + q\mu_p p)}_{\sigma\ \text{conductivity}}\vec{E}$$

$$\boxed{\vec{J}^{dr} = \sigma\vec{E}} \qquad \sigma = q(\mu_n n + \mu_p p)$$

Diffusion current



$$J_p^{diff} \sim \frac{dp}{dx}$$

$$J_p^{diff} = -qD_p \frac{dp}{dx}$$

Diffusivity

Diffusion Coefficient

$$\vec{J}_p^{\,diff} = -qD_p \nabla p$$

$$\vec{J}_n^{\,diff} = -qD_p \nabla p$$

$$\vec{J}_n = \vec{J}_n^{\,drift} + \vec{J}_{diff}^{\,diffusion} = q(-\mu_n n \nabla \phi + D_n \nabla n)$$

$$\vec{J}_p = \vec{J}_p^{\,drift} + \vec{J}_p^{\,diffusion} = -q(\mu_p p \nabla \phi + D_p \nabla p)$$

D_n & D_p are the diffusion constants,

thermal fluctuations allow

a property of the material and also effected by temperature

Week 3 Day 1 Device electronics

2018-02-02

$$\nabla^2 \phi = \frac{q}{\epsilon}(n-p-D)$$

$$\frac{\partial n}{\partial t} = \frac{1}{q}\nabla\cdot\vec{J}_n - R$$

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\nabla\cdot\vec{J}_p - R$$

$$\vec{J}_n = q(D_n \nabla n - \mu_n n \nabla\phi)$$

$$\vec{J}_p = -q(\underbrace{D_p \nabla p}_{\text{diffusion}} + \underbrace{\mu_p p \nabla\phi}_{\text{drift}})$$

$$\vec{E} = -\nabla\phi(x,y,z)$$

$R>0$: Recombination

$R<0$: Generation

$R=0$: T.E. $\longleftrightarrow$ $\vec{J}_n \equiv 0$, $\vec{J}_p \equiv 0$

pn-junction:

N_A^- $\qquad$ N_D^+

majority $p \simeq N_A^-$ $(10^{15} \cdot 10^{16})$ $\qquad$ $p = \frac{n_i^2}{N_D^+}$ minority

minority $n = \frac{n_i^2}{N_A^-}$ $\qquad$ $n = N_D^+$ majority

$p \gg n$ $\qquad$ $n \gg p$

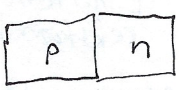
$n_i \sim 10^{10}$ 1/cm3

$\vec{E}=0$ $\qquad$ $\vec{E}=0$

- drift & diffusion current are zero
- ~~...~~ charge density is also zero

$$\rho = q(p-n+D)$$

Creat the junction



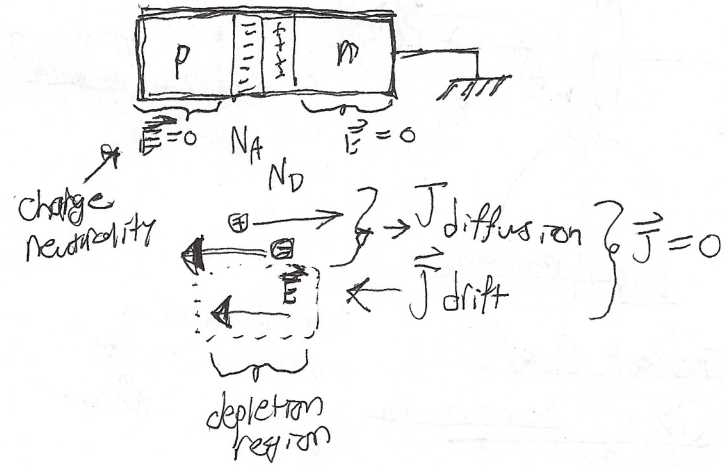
density of holes larger on p

density of electrons much higher

10^{16}

- You get a diffusion current.

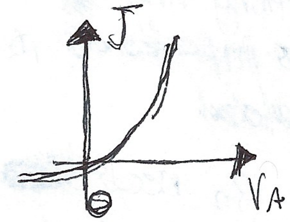
When electrons leave n side they leave positively charged fixed donors.

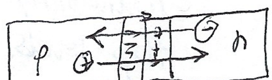


$V_A \neq 0$, ~~T.E.~~

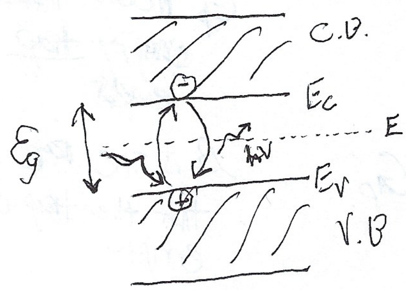
$V_A > 0$, $|\vec{E}|\downarrow \Rightarrow |\vec{J}_{drift}|\downarrow \Rightarrow \vec{J}\uparrow$

$V_A < 0$, $|\vec{E}|\uparrow \Rightarrow \vec{J} \approx 0$ (small)





electron reaches p region from n region it would recombine with an electron.



- have a material Shoot photons into it.

$$\varepsilon = h\nu$$

direct Recombination

1) Direct/optical G-R: generation and recombination

$R_n \sim np$

$R_n = \alpha_r np$

$G_n = \alpha_g$

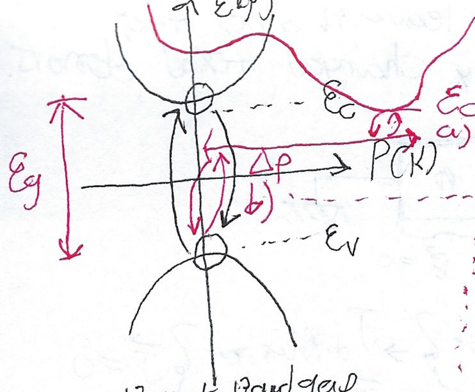
n_i^2 $\quad$ $\|$

T.E.: $R=0$, $R_n = G_n \Rightarrow \alpha_r np = \alpha_g$

$\alpha_r n_i^2 = \alpha_g$

In general: $R = R_n - R_G = \alpha_r np - \alpha_r n_i^2$

$$\boxed{R = \alpha_r(np - n_i^2)}$$



Direct bandgap

red silicon
- this is ~~not~~ a direct band gap
- it is an $\boxed{\text{indirect}}$ bandgap

mechanism a and b can be done by different

2) Indirect Rec/Gen:

CB ⊖ Recombination

(a) R_n $0<\beta<1$

CW R_p go down an anielate a hole ⊕

CB generation

G_n (d) anything that creats imperfections in the material

c) G_p

VB ⊕

a) $R_n = \alpha_n^r n(1-\beta)$

b) $R_p = \alpha_p^r p \beta$

R_n needs availability of trap levels &

d) $G_n = \alpha_n^g \beta$

c) $G_p = \alpha_p^g (1-\beta)$

G_p needs empty ~~trap~~ levels

In Thermal Equilibrium:

$n_o p_o = n_i^2$

$R = R_n - G_n = R_p - G_p$

$R = 0$

β is the rate that the trap is empty

$R_n - G_n = 0$

$R_p - G_p = 0$

$\therefore \alpha_n^r n_o (1-\beta) = \alpha_n^g \beta$

$\alpha_p^r p_o \beta = \alpha_p^g (1-\beta)$

$\Downarrow$

$\alpha_n^g = \alpha_n^r \boxed{n_o \frac{1-\beta}{\beta}} \rightarrow n_1$

$\alpha_p^g = \alpha_p^r \boxed{p_o \frac{\beta}{1-\beta}} \rightarrow p_1$

$$\boxed{\begin{aligned}\alpha_n^g &= \alpha_n^r n_1 \\ \alpha_p^g &= \alpha_p^r p_1\end{aligned}}$$

$R = R_n - G_n = R_p - G_p$

$R = \alpha_n^r [n(1-\beta) - n_1 \beta]$

$= \alpha_p^r [p\beta - p_1(1-\beta)]$

H.W.: $\beta = \ldots$

$\vdots$

$$R = \frac{np - n_i^2}{\left(\frac{1}{\alpha_p^r}\right)(n+n_1) + \left(\frac{1}{\alpha_n^r}\right)(p+p_1)}$$

τ_p = hole lifetime $\quad$ τ_n = electron lifetime

Indirected:

$$\boxed{R_{(n,p)} = \frac{(np - n_i^2)}{\tau_p(n+n_i) + \tau_p(p+n_i)}}$$

Schockley - Read - Hall (SRH)

TE in the context of D.D:

$\vec{J}_n = q(D_n \nabla n - \mu_n n \nabla \phi) = 0$

$\vec{J}_p = q(D_p \nabla p + \mu_p p \nabla \phi) = 0$

$D_n \nabla n = \mu_n n \nabla \phi$

$\frac{\nabla n}{n} = \frac{\mu_n}{D_n} \nabla \phi = \nabla\left(\frac{\mu_n}{D_n}\phi\right)$

$\nabla(\ln n) = \nabla\left(\frac{\mu_n}{D_n}\phi\right)$

$\ln n = \frac{\mu_n}{D_n}\phi + C_1$

$$\boxed{\begin{aligned} n &= C_n e^{\frac{\mu_n}{D_n}\phi} \\ p &= C_p e^{-\frac{\mu_p}{D_p}\phi}\end{aligned}}$$

$\frac{\mu_n}{D_n} = \frac{\mu_p}{D_p} = \frac{q}{KT} = \frac{1}{V_T}$

Einstein relationship

$n = C_n e^{\phi/V_T}$

$p = C_p e^{-\phi/V_T}$

$V_T \cong 25\text{ mV}$ @ room temperature.

$np = n_i^2$

$np = C_n C_p = n_i^2$

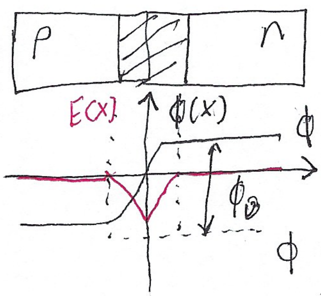
Reference point (x,y,z)

$n = p = n_i \quad \phi = 0$

$\Downarrow$

$\Rightarrow C_n = C_p = n_i$

$\Rightarrow$ $n = n e^{\phi/V_T}$
$p = n_i e^{-\phi/V_T}$



$n = N_D^+$

$\phi = V_T \ln\left(\frac{N_D^+}{n_i}\right)$

$\phi = -V_T \ln\left(\frac{N_A^-}{n_i}\right)$

$\phi_B = V_T \ln\left(\frac{N_D^+ N_A^-}{n_i^2}\right)$

$\vec{E} = -\nabla\phi^{(x,y,z)} + \text{Constant})$

$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon}$ $\phi_1(x,y,z)$

$\phi_1 = \phi_2(x,y,z) + 0$

Poisson equation:

$\nabla^2\phi = \frac{q}{\epsilon}(n - p - D)$

non-linear poisson eqn.

$\nabla^2\phi = \frac{q}{\epsilon}\left(n_i e^{\phi/V_T} - n_i e^{-\phi/V_T} - D\right)$

1D: $\frac{d^2\phi}{dx^2} = \frac{q}{\epsilon}\left(n_i e^{\phi/V_T} - n_i e^{-\phi/V_T} - D(x)\right)$

$D(x) = \begin{cases} N_D^+, & x > 0 \\ -N_A^-, & x < 0 \end{cases}$

- electron charge densities are exponentials of the functions

Reference Point: (x,y,z)

$n = p = n_i$

$\phi = 0$

$\Rightarrow C_n = C_p = n_i$

$\Rightarrow$ $n = n e^{\phi/V_T}$
$p = n_i e^{-\phi/V_T}$ T.E.

If not in thermal equilibrium what happens.

$n = n_i e^{\phi/V_T} \cdot f_n = e^{\ln f_n}$

$p = n_i e^{-\phi/V_T} \cdot f_p = e^{\ln f_p}$

$0 < i < \infty$

$\ln f_n \equiv \frac{-\psi_n(x)}{V_T}$

$\ln f_p \equiv \psi_p(x)/V_T$

$\Rightarrow$ then

$n = n_i e^{(\phi - \psi_n)/V_T}$

$p = n_i e^{-(\phi - \psi_p)/V_T}$

I like exponential dependence I want to keep it.

Quasi-Fermi Potentials: ψ_n, ψ_p

$\vec{J}_n = q(D_n \nabla n - \mu_n n \nabla\phi)$

$= q\left(D_n n_i \frac{1}{V_T} \nabla(\phi - \psi_n) e^{\frac{\phi - \psi_n}{V_T}} - \mu_n n_i e^{\frac{\phi - \psi_n}{V_T}} \nabla\phi\right)$

$\vec{J}_n = -q\mu_n n \nabla\psi_n$

$\vec{J}_p = -q\mu_p p \nabla\psi_p$

$n = n_i e^{(\phi - \psi_n)/V_T}$

$p = n_i e^{-(\phi - \psi_p)/V_T}$

Quasi-fermi potential allows us to express

$\rightarrow$ indirect recombination problem

– understand the quasi fermi potential

do the same problem for $\vec{J}_p$

ECE Device Electronics

2018-02-09 Week 4

T.E.: $\vec{J}_n \equiv 0$, $\vec{J}_p \equiv 0$

D.D. model $\Rightarrow$ $n = n_i e^{\phi/V_T}$ →

$p = n_i e^{-\phi/V_T}$ →

$V_T = \frac{KT}{q} \approx 25\,mV$ $\quad \underbrace{\qquad}_{np = n_i^2}$

$R(n,p) = 0$

$\cancel{T.E.}$ $J_n \neq 0$, $J_p \neq 0$, $R \neq 0$, $np \neq n_i^2$

→ $n = n_i e^{\phi/V_T} p(K) = n_i e^{\phi/V_T} e^{-\psi_n(x)/V_T}$

→ $p = n_i e^{-\phi/V_T} g(\lambda) = n_i e^{\phi/V_T} e^{+\psi_p(x)/V_T}$

ψ_n, ψ_p = Quasi Fermi potentials

$\vec{J}_n = -q\mu_n n \nabla \psi_n$

$\vec{J}_p = -q\mu_p p \nabla \psi_p$

} Semiconductors Drift + Diffusion

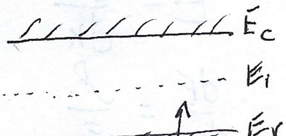
looks similar to:

$\vec{J} = q\mu_n \nabla \phi$ } Conductors only Drift

using the band diagram

$n = n_i e^{(\phi - \psi_n)/V_T}$

$p = n_i e^{(\phi - \psi_p)/V_T}$



(ϕ, ψ_n, ψ_p)

electrostatic potential

quasi fermi-potentials

Fermi energy/levels

Statistical Mechanics

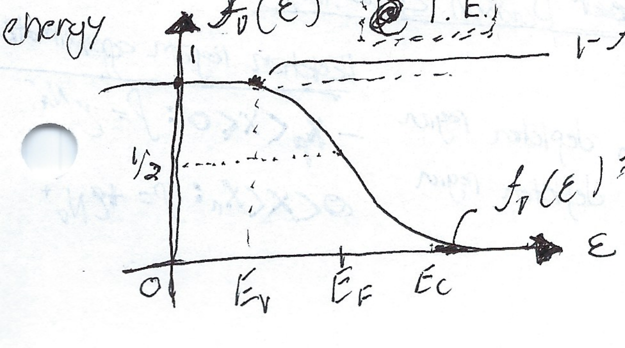
↓

fermi-Dirac Statistics

probability of finding particles at a given energy

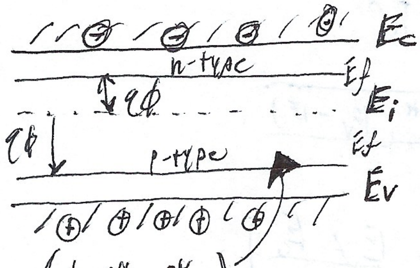
$f_D(E) = \frac{1}{1 + e^{(E-E_F)/KT}}$

T.E.



$f_D(E) \approx e^{-(E-E_F)/KT}$

(Fermions) → Pauli Exclusion principal



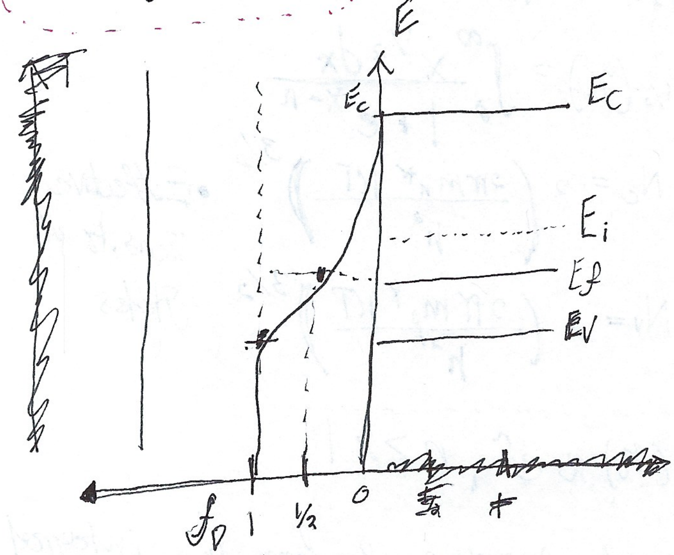
(ϕ, ψ_n, ψ_p)

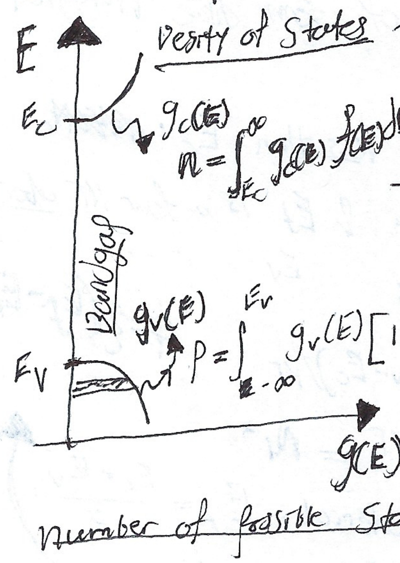
$1 - f = \frac{e^{(E-E_f)/KT}}{1 + e^{(E-E_f)/KT}} \approx e^{(E-E_F)/KT}$

Fermi Energy level

$\phi = \frac{1}{q}(E_f - E_i)$

P-type material





Density of States → g(E) - is a property of the semiconductor

$n = \int_{E_c}^{\infty} g_c(E) f(E) dE$

→ Density of States is zero in the bandgap

$p = \int_{-\infty}^{E_v} g_v(E)[1 - f(E)] dE$

number of possible States g(E) - Density of states

f(E) - probability of occupation

fermi energy

$$g_c(\mathcal{E}) = \frac{m_n^* \sqrt{2m_n^*(E-E_c)}}{\pi^2 \hbar^3}$$

$$g_v(\mathcal{E}) = \frac{m_p^* \sqrt{2m_p^*(E_v-E)}}{\pi^2 \hbar^3}$$

$$\Rightarrow n = N_c \mathcal{F}_{1/2}\left(\frac{E_f-E_c}{KT}\right)$$

$$p = N_v \mathcal{F}_{1/2}\left(\frac{E_v-E_f}{KT}\right)$$

fermi direct integral ~~as~~ of order of 1/2

$$\mathcal{F}_{1/2}(n) = \frac{2}{\sqrt{\pi}} F_{1/2}(n)$$

$$F_{1/2}(n) = \int_0^\infty \frac{x^{1/2}\,dx}{1+e^{x-n}}$$

$$N_c = 2\left(\frac{2\pi m_n^* KT}{h^2}\right)^{3/2}$$

$$N_v = 2\left(\frac{2\pi m_p^* KT}{h^2}\right)^{3/2}$$

• Effective Density of States

$$\boxed{\mathcal{F}_{1/2}(n) \approx e^n,\ n >> 1}$$

→ Study properties of fermi dirac integral function

if E_f is a few KT less than E_c: ~~E_c~~ E_c: $n \cong N_c e^{\frac{E_f-E_c}{KT}}$ & E_f is a few KT above E_v: $p \cong N_v e^{\frac{E_v-E_f}{KT}}$

$$\boxed{\phi = \frac{1}{q}(E_f - E_i)}$$

$$\Rightarrow np = N_c N_v e^{(E_v-E_c)/KT}$$

$$np = N_c N_v e^{-E_g/KT} = n_i^2$$

if we have intrinsic material: $E_f = \frac{E_c + E_v}{2}$:

$n = n_i$
$p = n_i$

$$n_i = N_c e^{-(E_c-E_i)/KT}$$

$$n_i = N_v e^{-(E_i-E_v)/KT}$$

Statistical mechanics ⇒

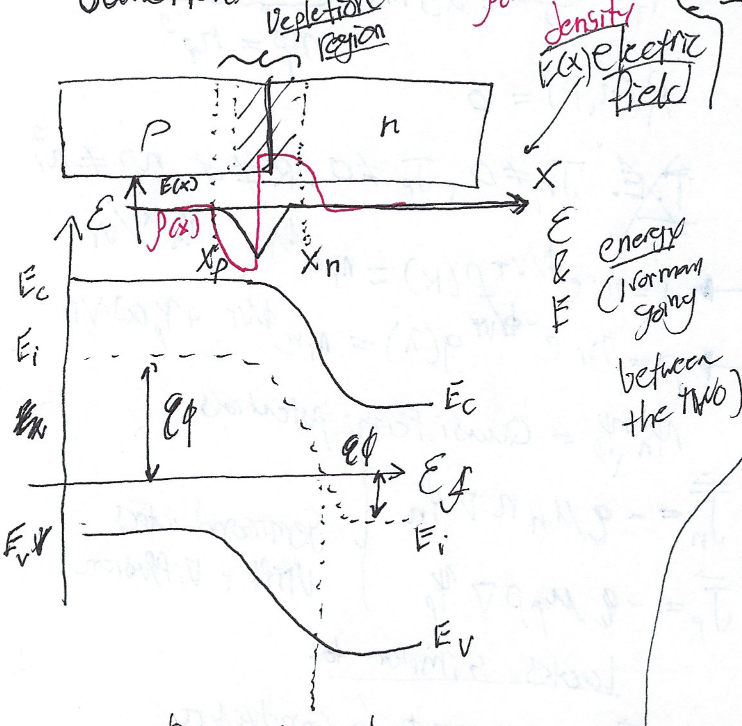
$$n = n_i e^{(E_f-E_i)/KT} = n = n_i e^{\phi/V_T}$$

$$p = n_i e^{(E_i-E_f)/KT} = p = n_i e^{-\phi/V_T}$$

From drift diffusion

Band diagram just another way of plotting the electrostatic potential (the Voltage)
– tied the fermi level to the potential

→ now we apply to the p-n junction



$\phi < 0$ $\phi = 0$ $\phi > 0$

$p >> n$ $n = p = n_i$ $n >> p$

~~n region~~:

$$n = N_D^+ = n_i e^{\phi/V_T}$$

$$\phi = V_T \ln\left(\frac{N_D^+}{n_i}\right)$$

$$E = -\frac{d\phi}{dx}$$

$$\frac{dE}{dx} = \frac{\rho}{\varepsilon}$$

p region:

$$p = N_A^- = n_i e^{-\phi/V_T}$$

$$\phi_p = -V_T \ln\left(\frac{N_A^-}{n_i}\right)$$

$$\phi_B = \phi_n - \phi_p$$

$$\phi_B = V_T \ln\left(\frac{N_D^+ N_A^-}{n_i^2}\right)$$

TE. $\nabla^2 \phi = \frac{q}{\varepsilon}\left(n_i e^{\phi/V_T} - n_i e^{-\phi/V_T} - D\right)$

Non-linear Poisson equation

[redacted]

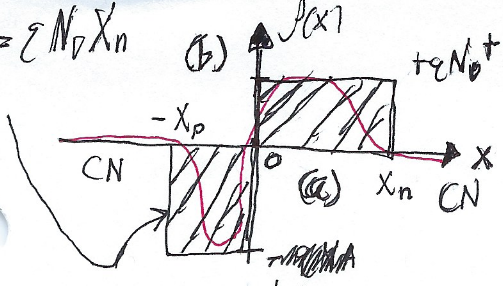
x_n depletion region
x_p depletion region

Depletion region approximate

$-x_p < x < 0$: $\rho = -qN_A^-$

$0 < x < x_n$: $\rho = +qN_D^+$

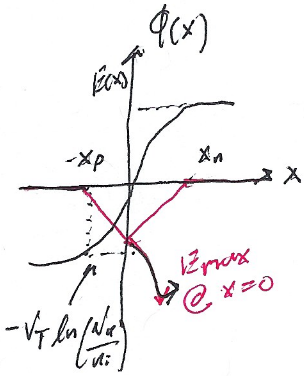
$Q = q N_D X_n$



(1) $\boxed{N_A^- X_p = N_D^+ X_n}$

a) $\frac{d^2\phi}{dx^2} = \frac{q}{\epsilon_s}(-N_D^+) = \frac{-qN_D^+}{\epsilon_s}$

b) $\frac{d^2\phi}{dx^2} = \frac{q}{\epsilon_s}(N_A^-) = \frac{qN_A^-}{\epsilon_s}$



c) $\frac{d\phi}{dx} = -\frac{qN_D}{\epsilon_s}x + C_1$

$\boxed{E_x = \frac{-d\phi}{dx} = \frac{qN_D(x - x_n)}{\epsilon_s}}$

$E_x(x_n) = 0$

$$\phi(x) = -\frac{qN_D}{2\epsilon_s}(x - x_n)^2 + C_2$$

$$= -\frac{qN_D}{\epsilon_s}(x - x_n)^2 + V_T \ln\left(\frac{N_D}{n_i}\right)$$

b) $\phi(x) = \frac{qN_a}{2\epsilon_s}(x + x_p)^2 - V_T \ln\left(\frac{N_a}{n_i}\right)$

$E_x(x) = -\frac{qN_a}{\epsilon_s}(x + x_p)$

$-x_p \le x \le 0$

$x_n = ?\quad x_p = ?$

$\phi(0^-) = \phi(0^+)$

$$\frac{qN_a}{2\epsilon_s}x_p^2 - V_T \ln\left(\frac{N_a}{n_i}\right) = \frac{-qN_D}{2\epsilon_s}x^2 + V_T \ln\left(\frac{N_D}{n_i}\right)$$

$$\boxed{N_a x_p^2 + N_D x_n^2 = \frac{2\epsilon_s}{q}\phi_B} \quad (2)$$

$$x_n = \sqrt{\frac{N_A}{N_D(N_A + N_D)}\frac{2\epsilon_s}{q}\phi_B}$$

$\Longrightarrow$

$$x_p = \sqrt{\frac{N_D}{N_A(N_A + N_D)}\frac{2\epsilon_s}{q}\phi_B}$$

$$W = x_n + x_p = \sqrt{\frac{N_D + N_A}{N_D N_A}\frac{2\epsilon_s}{q}\phi_B}$$

$$\boxed{\phi_B = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right)}$$

p^+n $\quad x_n \gg x_p$

$N_a \gg N_D$

n^+p $\quad x_n \ll x_p$

$N_A \ll N_D$

depletion region dominated by the lower doped side

The potential is continuous means $\rightarrow$

$E = \frac{-d\phi}{dx}$

implies $E(0) = \infty$ if there is a discontinuity, therefore the potential must be continuous.

ECE 6030
HW #1 Joseph Crandall
2018-02-08
Chapter 2: 1, 3, 6, 11, 14, 17, 21, 23
→ Pierret Chapters 1 & 2 reading

2.1) a)

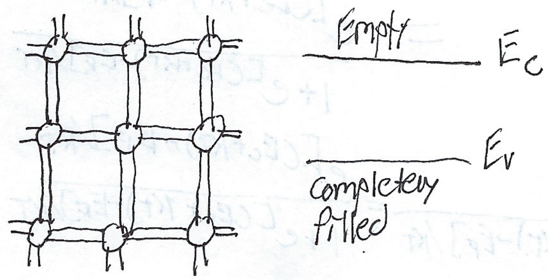
E_G vs T for Si

$E_{G300} = 1.1245$

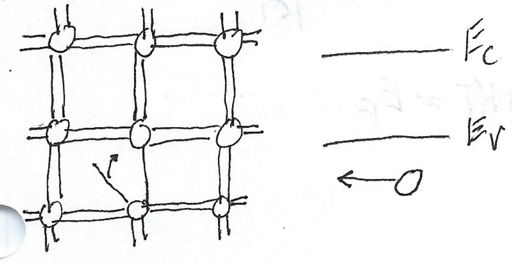
a) Solid line represents E_G vs T in the range of 0K to 600K.
E_G at $T = 300K$ is 1.1245 eV

b) dashed line represents E_G vs T at $T > 300K$. The plot is almost linear.

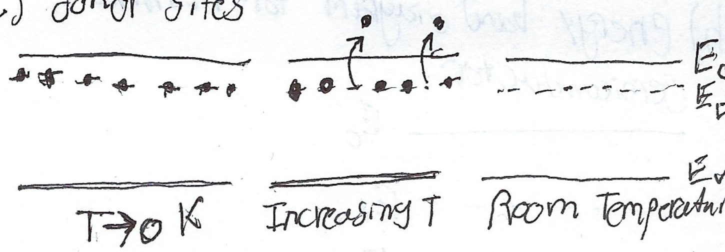
2.3
a) an electron



b) hole

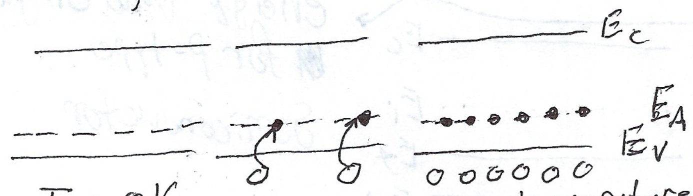


c) donor sites



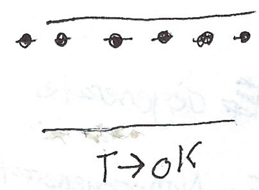
E_C: energy at conduction band
E_D: energy at donor band
E_V: energy at valence band

d) acceptor sites

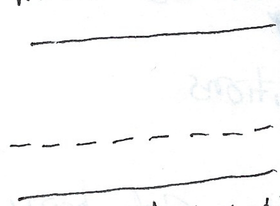


E_A: energy at acceptor band

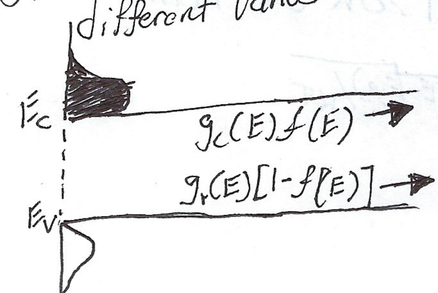
e) majority carriers electrons when $T = 0K$



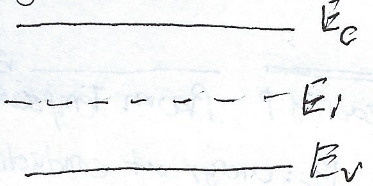
f) majority carriers holes when $T = 0K$



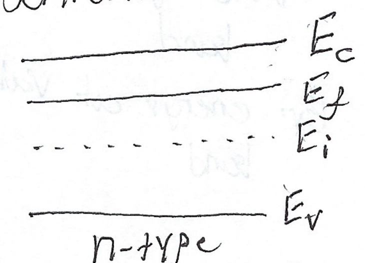
g) energy distribution of carriers in different bands



h) energy band diagram for intrinsic semiconductor

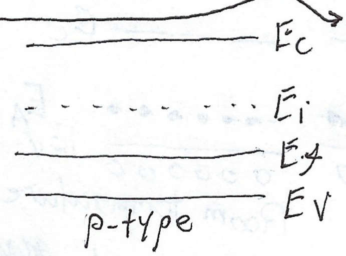


i) energy band diagram for n-type semiconductor



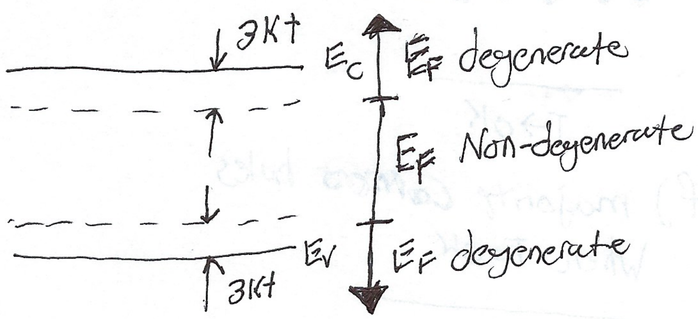
E_f: energy band at fermi level

j) energy band diagram for p-type semiconductor



k) (i)

energy band diagram for a non-degenerate/degenerate semiconductor



2.6) equilibrium conditions

$T > 0K$

a) probability of electron state being occupied, $T > 0K$ at Fermi level

$$f(E_F) = \frac{1}{1+e^{(E-E_F)/KT}}$$

$$E = E_F$$

$$\therefore f(E_F) = \frac{1}{1+e^0}$$

$$= \frac{1}{2}$$

b) E_F positioned at E_c

find probability of finding electrons in states $E_c + KT$

$$f(E_c+KT) = \frac{1}{1+e^{[(E_c+KT)-E_c]/KT}}$$

$$= \frac{1}{1+e^1}$$

$$= 0.269$$

c) determine location of fermi level when probability of state being filled at $E_c + KT$ is equal to probability of state being empty at $E_c + KT$

$$f(E_c+KT) = 1 - f(E_c+KT)$$

$$\frac{1}{1+e^{[(E_c+KT)-E_F]/KT}} = 1 - \frac{1}{1+e^{[(E_c+KT)-E_F]/KT}}$$

$$= \frac{e^{[(E_c+KT)-E_F]/KT}}{1+e^{[(E_c+KT)-E_F]/KT}}$$

$$\frac{e^0}{1+e^{[(E_c+KT)-E_F]/KT}} = \frac{e^{[(E_c+KT)-E_F]/KT}}{1+e^{[(E_c+KT)-E_F]/KT}}$$

$$0 = \frac{E_c+KT-E_F}{KT}$$

$$\therefore E_c + KT = E_F$$

2.11

~~[illegible]~~ equation for hole ~~[illegible]~~ concentration.

$$p = \int_{E_{bottom}}^{E_v} g_v(E)[1-f(E)]\,dE$$

→ density of states at energy E in the conduction and valence bands

$$g_v(E) = \frac{m_p^* \sqrt{2m_p^*(E_v - E)}}{\pi^2 \hbar^3} \quad (E \le E_v)$$

→ available state at energy E will be occupied by an electron

$$f(E) = \frac{1}{1+e^{\frac{(E-E_f)}{KT}}}$$

$$\therefore p = \frac{m_p^* \sqrt{2m_p^*}}{\pi^2 \hbar^3} \int_{E_{bottom}}^{E_v} \frac{\sqrt{E_v - E}\,dE}{1+e^{\frac{(E_F - E)}{KT}}}$$

assume

$$\eta = \frac{E_v - E}{KT} \qquad \eta_v = \frac{E_v - E_F}{KT} \qquad E_{bottom} \to -\infty$$

$$p = \frac{m_p^* \sqrt{2m_p^*}(KT)^3}{\pi^2 \hbar^3} \int_0^{\infty} \frac{\eta^{1/2}}{1+e^{\eta - \eta_v}}$$

→ remember that

$$F_{1/2}(\eta_v) \equiv \int_0^{\infty} \frac{\eta^{1/2}\,d\eta}{1+e^{\eta - \eta_v}}$$

$$N_v = 2\left[\frac{m_p^* KT}{2\pi \hbar^2}\right]^{3/2}$$

$$p = N_v \frac{2}{\sqrt{\pi}} F_{1/2}(\eta_v)$$

if semiconductor is non-degenerate such that $E_F \ge E_v + 3KT$

$$\eta_v \le -3$$

if Fermi dirac integral → $\eta \approx 0$

$$F_{1/2}(\eta_v) = \int_0^{\infty} \eta^{1/2} e^{-(\eta - \eta_v)}\,d\eta$$

$\vdots$

$$= \frac{\sqrt{\pi}}{2} e^{\frac{(E_v - E_F)}{KT}}$$

$$\therefore p = N_v e^{(E_v - E_F)/KT}$$

6.14

a) intrinsic carrier concentration

$$n_i = \sqrt{N_c N_v}\, e^{-E_G/2KT}$$

n_i = carrier concentration of Ge at room temperature $= (2 \times 10^{13})$

E_G = energy gap of Ge at room temperature $= 0.66\text{eV}$

$$KT = 0.0259 \text{ eV}$$

$$\sqrt{N_c N_v} = \frac{(2 \times 10^{13})}{e^{-[0.66/2(0.0259)]}}$$

$$= 6.831 \times 10^{18}$$

energy gap of Si at room temperature

$$E_G = 1.12 \text{ eV}$$

determine temperature at which the intrinsic carrier concentration of silicon is equal to germanium

$$n_i = \sqrt{N_c N_v}\, e^{-E_G/2KT}$$

$$2 \times 10^{13} = (6.831 \times 10^{18})\, e^{-1.12/2KT}$$

$$2.92 \times 10^{-6} = e^{-[1.12/2(KT)]}$$

$$-12.74 = -\frac{1.12}{2KT}$$

$$2KT = \frac{1.12}{12.74}$$

$$KT = 0.0439$$

$$T = \frac{0.0439}{8.617 \times 10^{-5}}$$

$$T = 510\ K$$

the temperature at which the intrinsic carrier concentration of silicon is equal to germanium.

ii) temperature at which GaAs the intrinsic carrier concentration in GaAs is equal to the room temperature (300K) intrinsic carrier concentration of Ge.

$$n_i = \sqrt{N_c N_v}\, e^{-E_G/(2KT)}$$

$$(2\times10^{13}) = (6.831\times10^{18})\, e^{-[1.42/2(KT)]}$$

energy gap of GaAs $E_G = 1.42\,eV$

$$2.92\times10^{-6} = e^{-[1.42/2(KT)]}$$

$$-12.74 = -\frac{1.42}{2KT}$$

$$2KT = \frac{1.42}{12.74}$$

$$T = \frac{0.0557}{(8.617\times10^{-5})}$$

$T = 646$ Kelvin

b) to determine the ratio of intrinsic concentrations ~~carrier effective masses~~ Ignore the differences in the carrier effective masses

$$\frac{n_{iA}}{n_{iB}} = \frac{e^{\frac{-E_{GA}}{2KT}}}{e^{\frac{-E_{GB}}{2KT}}}$$

$$= e^{\left[\frac{(E_{GB}-E_{GA})}{2KT}\right]}$$

$E_{GA} = 1\,eV \quad E_{GB} = 2\,eV$

$$KT = 0.0259$$

$$\frac{n_{iA}}{n_{iB}} = e^{\left[\frac{1}{2(0.259)}\right]}$$

$$= 2.42\times10^{8}$$

ratio of intrinsic concentrations.

2.17) determine equilibrium electron and hole concentrations inside a uniformly doped sample of Si

a) $T = 300\ K$

$N_A \ll N_D \quad N_D = 10^{15}\ cm^{-3}$

$$n_i = (9.15\times10^{19})\left(\frac{T}{300}\right)^2 e^{\frac{-0.5928}{KT}}$$

$K =$ Boltzman's constant $8.615\times10^{-5}\ eVK^{-1}$

$$n_i = (9.15\times10^{19})\left(\frac{300}{300}\right)^2 e^{\frac{-0.5928}{(8.615\times10^{-5})(300)}}$$

$$= (9.15\times10^{19})(1.0932\times10^{-10})$$

$$\cong 10^{10}\ cm^{-3}$$

hole concentration

$$p = \frac{n_i^2}{N_D} \quad n_i = 10^{10} \quad N_D = 10^{15}$$

$$p = \frac{(10^{10})^2}{10^{15}} = 10^{5}\ cm^{-3}$$

→

b) $N_A >> N_D$

$$P = N_A = 10^{16}\ cm^{-3}$$

$$n = \frac{n_i^2}{N_A}$$

$$n = \frac{(10^{10})^2}{10^{16}} = 10^{4}\ cm^{-3}$$

c) electron concentration

$$n = \frac{N_D - N_A}{2}\left[\left(\frac{N_D-N_A}{2}\right)^2 + n_i^2\right]^{1/2}$$

$$n = \frac{10^{16} - 9\times10^{15}}{2} + \left[\left(\frac{10^{16}-9\times10^{15}}{2}\right)^2 + (10^{10})^2\right]^{1/2}$$

$$= 5\times10^{14} + 5\times10^{14}$$

$$n = 10^{15}\ cm^{-3}$$

$$p = \frac{n_i^2}{n}$$

$$p = \frac{(10^{10})^2}{10^{15}} = 10^5\ cm^{-3}$$

d) intrinsic condition of Silicon at $T°K$

$$n_i = (9.15\times10^{19})\left(\frac{T}{300}\right)^2 e^{-\frac{0.5928}{KT}}$$

n_i @ 450 K

$$n_i = (9.15\times10^{19})\left(\frac{450}{300}\right)^2 e^{\frac{-0.5928}{(8.615\times10^{-5})(450)}}$$

$$= (9.15\times10^{19})\left(\frac{9}{4}\right)(2.2863\times10^{-7})$$

$$\cong 4.7\times10^{13}\ cm^{-3}$$

$$n = \frac{N_D + N_A}{2} + \left[\left(\frac{N_D - N_A}{2}\right)^2 + n_i^2\right]^{1/2}$$

$$n = \frac{10^{14} - 0}{2} + \left[\left(\frac{10^{14}-0}{2}\right)^2 + (4.7\times10^{13})^2\right]^{1/2}$$

$$n = 5\times10^{13} + 6.862\times10^{13}$$

$$n = 1.186\times10^{14}\ cm^{-3}$$

$$p = \frac{n_i^2}{n} = \frac{(4.7\times10^{13})^2}{1.186\times10^{14}} = 1.862\times10^{13}\ cm^{-3}$$

e) $$n_i = (9.15\times10^{19})\left(\frac{T}{300}\right)^2 e^{-\frac{0.5928}{KT}}$$

$T = 650\ K$

$$n_i = (9.15\times10^{19})\left(\frac{650}{300}\right)^2 e^{\frac{-0.5928}{(8.615\times10^{-5})(650)}}$$

$$n_i = (9.15\times10^{19})\left(\frac{13}{6}\right)^2 (2.526\times10^{-5})$$

$$\cong 1.085\times10^{16}\ cm^{-3}$$

$$n = \frac{N_D + N_A}{2} + \left[\left(\frac{N_D - N_A}{2}\right)^2 + n_i^2\right]^{1/2}$$

$$= \frac{10^{14} - 0}{2} + \left[\left(\frac{10^{14}-0}{2}\right)^2 + (1.085\times10^{16})^2\right]^{1/2}$$

$$= 5\times10^{13} + 1.085\times10^{16}$$

$$= 1.09\times10^{16}\ cm^{-3}$$

$$p = \frac{n_i^2}{n} = \frac{(1.085\times10^{16})^2}{1.09\times10^{16}} = 1.08\times10^{16}\ cm^{-3}$$

2.21

N_D VS N_A Matlab

2.23

a) fermi function

$$f(E) = \frac{1}{1 + e^{(E-E_F)/KT}}$$

E_F = Fermi level or Fermi energy

As $T \to 0K$, $(E-E_F)/KT \to -\infty$

for energies $E < E_F$

and $(E-E_F)/KT \rightarrow +\infty$ for all energies $E > E_F$

$\therefore f(E<E_F) =$

$$= \frac{1}{1+e^{-\infty}} = 1$$

$$f(E>E_F) = \frac{1}{1+e^{\infty}} = 0$$

all states below E_F will be filled and all states above E_F will be empty at $T \rightarrow 0$ kelvin

b) Si doped with acceptors then $N_A >> N_D$ and fermi energy gap

$$E_i - E_F = KT \ln\left(\frac{N_A}{n_i}\right)$$

$$N_A = 10^{14} \, cm^{-3}$$

$$E_i - E_F = KT \ln\left(\frac{10^{14}}{n_i}\right)$$

$$P = \frac{n_i^2}{n} = \frac{N_A - N_D}{2}\left[\left(\frac{N_A - N_D}{2}\right)^2 + n_i^2\right]^{1/2}$$

assuming $E_C - E_i = \frac{E_G}{2}$

& $E_i - E_V = \frac{E_G}{2}$ versus T

Matlab plot

$E_F - E_i$ vs T

c) As temperature increases the fermi level moves closer to the intrinsic level. For very high temperatures the fermi level may touch the intrinsic level.

$$E_i - E_F = KT \ln\left(\frac{n_i}{n_i}\right)$$

$$E_i - E_F = KT \ln(1)$$

$$= 0$$

$$\therefore E_i = E_F$$

d) N_A is progressively increased

Matlab plot

e) If Si doped with donors instead of acceptors

$$N_D = 10^{14} \, cm^{-3}$$

$$E_F - E_i = KT \ln\left(\frac{N_D}{n_i}\right)$$

$$E_i - E_F = -KT \ln\left(\frac{N_D}{n_i}\right)$$

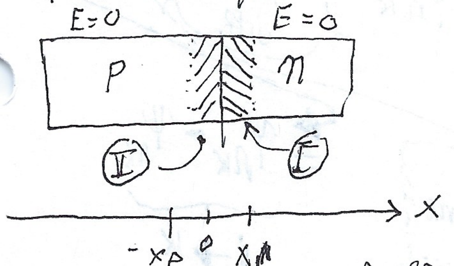
$$E_i - E_F = -KT \ln\left(\frac{10^{14}}{n_i}\right)$$

Matlab plot

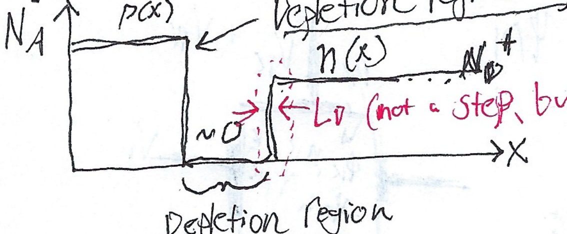
$$N_D = -10^{14} \, cm^{-3}$$

2018-02-16

- Device electronics
- Week 5 Day 1



Depletion region approximation



I $\phi(x) = \frac{-qN_D}{2\epsilon_s}(x-x_n)^2 + V_T \ln\left(\frac{N_D}{n_i}\right)$

$E(x) = \frac{qN_D}{\epsilon_s}(x-x_n)$

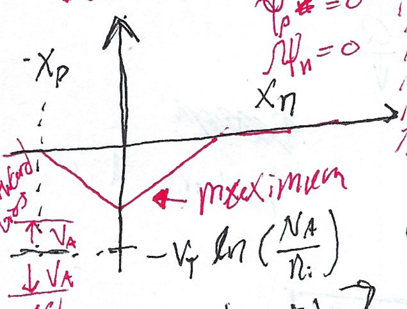
II $\phi(x) = \frac{qN_A}{2\epsilon_s}(x+x_p)^2 - V_T \ln\left(\frac{N_A}{n_i}\right)$

$E(x) = -\frac{qN_A}{\epsilon_s}(x+x_p)$

$x_n = \sqrt{\frac{N_A}{N_D(N_A+N_D)}\frac{2\epsilon_s}{q}\phi_B}$

$x_p = \sqrt{\frac{N_D}{N_A(N_A+N_D)}\frac{2\epsilon_s}{q}\phi_B}$

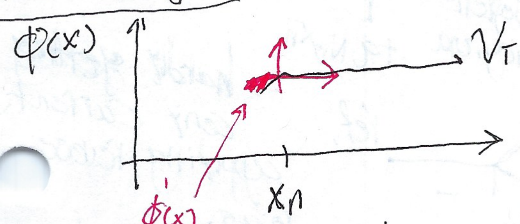
E(x) d(x)



$\phi_B = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right)$ ← Built in Barrier

$\left.\begin{array}{l}\phi(0^-) = \phi(0^+) \\ E(0^-) = E(0^+)\end{array}\right\}$

Debye Length:



→ interested in small change to the potential

$\phi'(x) = \phi_n - \phi(x)$ → $\phi(x) = \phi_n - \phi'$

Poisson equation

$$\frac{d^2\phi}{dx^2} = \frac{q}{\epsilon_s}(n - N_D^+)$$

$$\frac{d^2}{dx^2}(\phi_n - \phi'(x)) = \frac{q}{\epsilon_s}\left[n_i e^{\frac{\phi_n - \phi'}{V_T}} - N_D^+\right]$$

$$\frac{d^2\phi'}{dx^2} = \frac{q}{\epsilon_s}\left[N_D^+ - n_i e^{\phi_n/V_T} e^{-\phi'/V_T}\right]$$

$$= \frac{qN_D^+}{\epsilon_s}\underbrace{\left[1 - e^{-\phi'/V_T}\right]}_{\approx \phi'/V_T}$$

(first order approx) ← Via taylor series

$$\frac{d^2\phi'}{dx^2} \approx \frac{qN_D^+}{\epsilon_s V_T}\phi'$$

Characteristic equation

$A^2 - \frac{qN_D}{\epsilon_s V_T} = 0$

$\therefore A = \sqrt{\frac{qN_D}{\epsilon_s V_T}}$

$\phi'(x) = B\exp\left(\frac{-x}{L_D}\right)$

$L_D = \left(\frac{\epsilon_s V_T}{qN_D}\right)^{1/2}$ ← $\boxed{\text{Debye length}}$

once linear we can explicitly solve the equation

$L_D = \sqrt{\frac{1}{N_D}\cdot\frac{\epsilon_s}{q}\cdot V_T}$

$L_D \ll x_n$ or x_p

$\phi_B = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right)$

$\phi_B \gg V_T$

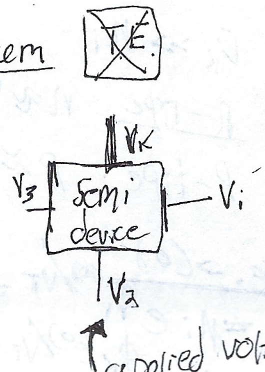
Non Thermal equilibrium $\boxed{\cancel{T.E.}}$

$n = n_i e^{(\phi - \psi_n)/V_T}$

$p = n_i e^{-(\phi - \psi_p)/V_T}$

$\vec{J}_n = -q\mu_n n \nabla\psi_n$

$\vec{J}_p = -q\mu_p p \nabla\psi_p$



(ϕ, n, p) Know values at contacts

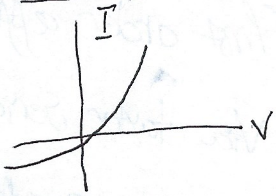
(ϕ, ψ_n, ψ_p)

$\Omega \rightarrow$ Contacts

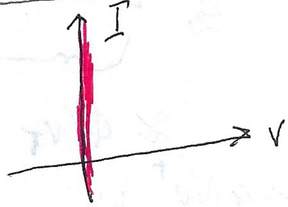
Ohmic Contact $\rightarrow$ apply voltage $\rightarrow$ free transport of carriers in any direction.

Shottky diode – Metal Semi contact $\rightarrow$
- get a diode
- a parasitic diode

Schottkey Contact



Ohmic Ω contact



~~Requirements~~ Requirements
- $\rho \approx 0$
- $np \approx n_i^2$

Maintain charge neutrality

maintain thermal equilibrium

@ Ω contact K:

T.E. $n_K p_K = n_i^2$

C.N. : $n_K - p_K - D_K = 0$

$$n_K = \frac{D_K}{2} + n_i\sqrt{\left(\frac{D_K}{2n_i}\right)^2 + 1}$$

$$p_K = -\frac{D_K}{2} + n_i\sqrt{\left(\frac{D_K}{2n_i}\right)^2 + 1}$$

$D_K \gg n_i$

$\rightarrow$ n-type : $n \approx N_D$, $p \approx \frac{n_i^2}{N_D}$

p-type : $p \approx N_A$, $n = \frac{n_i^2}{N_A}$

~~T.E.~~ $\rightarrow (0)$ applied voltages

$n_i = n_i e^{\phi_i^{(0)}/V_T} = n_i e^{(\phi_i - \psi_{n_i})/V_T}$

$n_K = n_i e^{\phi_K^{(0)}/V_T} = n_K e^{(\phi_n - \psi_{nK})/V_T}$

$\phi_i^{(0)} = \phi_i - \psi_{n_i}$

$\phi_K^{(0)} = \phi_K - \psi_{nK}$

$\underbrace{(\phi_K - \phi_i)}_{\text{non T.E. diff.}} - (\phi_K^{(0)} - \phi_i^{(0)}) = \underbrace{\psi_{nK} - \psi_{ni}}_{i \rightarrow K}$

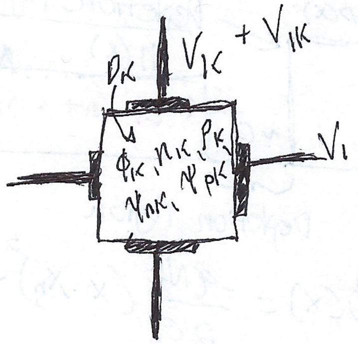
$V_{iK} = \psi_{nK} - \psi_{ni}$

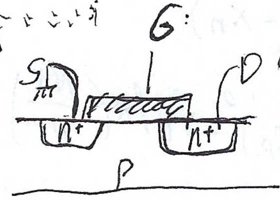
$V_{iK} = \psi_{pK} - \psi_{pi}$

$\downarrow$

$\psi_{nK} = V_K$

$\psi_{pK} = V_K$





• MOSFET Device

C-V Characteristics Capacitor & Voltage

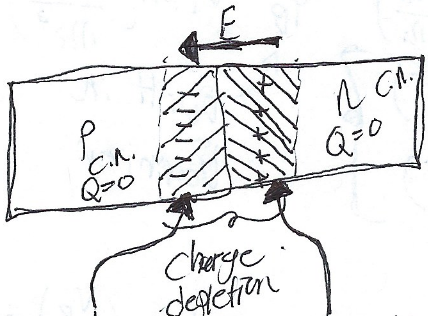
Capacitors store charge

$Q = CV \rightarrow C = \frac{Q}{V}$

$C = \frac{dQ}{dV}$

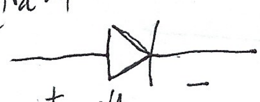
$Q(V_A)$

~~Lecture~~ Lecture 1



N_a region $+qN_Dx_n$

$-qN_ax_p$



ref. $\rightarrow$ hardly getting any current applying reverse bias.

$\rightarrow$ depletion region is going to expand

$$X_n(V) = \sqrt{\frac{N_A}{N_D(N_A - N_D)} \frac{2\epsilon_s}{q}(\phi_B - V_A)}$$

$$x_p(V_A) = \sqrt{\frac{N_D}{N_A(N_A + N_D)} \frac{2\epsilon_s}{q}(\phi_B - V_A)}$$

$$Q(V_A) = q N_D x_n(V_A) \cdot A$$

area

$$C = \epsilon_s \frac{A}{W}$$

Ex: $N_A \gg N_D$

$x_p \ll x_n$

$W = x_n + x_p \approx x_n$

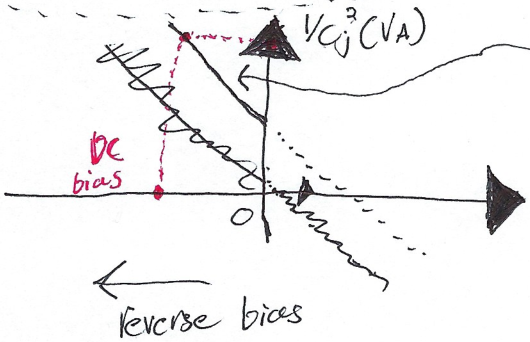
$$W = \sqrt{\frac{2\epsilon_s}{q N_D}(\phi_B - V_A)}$$

$$C = q N_D A \cdot \frac{dW}{dV_A}$$

junction capacitance / unit capacitance

$$C_j = \frac{C}{A} \qquad C_j = \sqrt{\frac{q N_D \epsilon_s}{2(\phi_B - V_A)}}$$

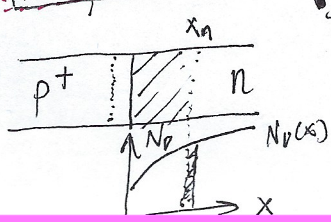
$$\boxed{\frac{1}{C_j^2} = \frac{2(\phi_B - V_A)}{q N_D \epsilon_s}}$$



Voltage controlled capacitors
— Varactors

HW: problem 1. in HW # 2

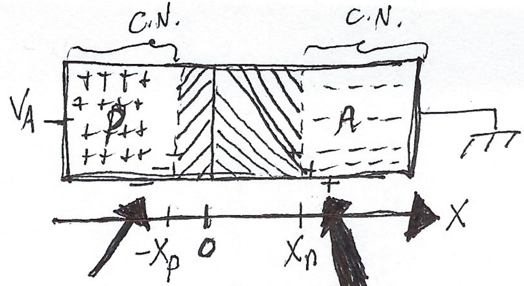
• Show that from slope



• You can get the doping density

$$N(x_n \text{ or } W) = \frac{2}{q\epsilon_s \frac{d(1/C_j^2)}{dV_A}}$$

• I-V Characteristics

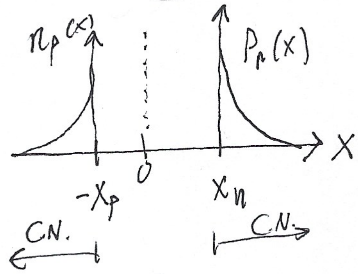


Minority carriers J_n

Majority carriers J_p

→ Need to calculate current resulting from minority carriers

$\Sigma \rightarrow J(V_A)$

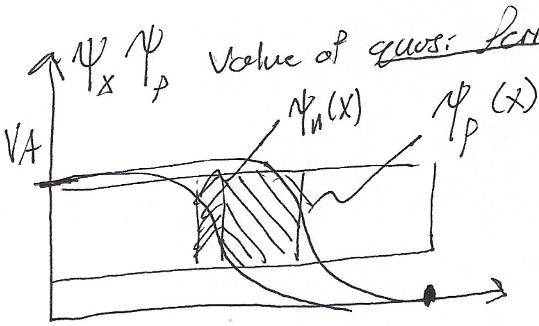


If region is charge neutral
— the drift current is zero in that region
— Electric field is zero

Need to calculate diffusion

$J_n^{diff}(x)$ $\quad$ $J_p^{diff}(x)$

→ Minority carriers are very interesting



Chapter 4 skipped

$CN \Rightarrow \vec{E} = 0 \Rightarrow J_{diff} = 0$

$$J_n = -q\mu_n n \frac{d\psi_n}{dx}$$

$$J_p = -q\mu_p p \frac{d\psi_p}{dx}$$

→ Done with chapter 5

Depl Region:

$$np = n_i e^{(\phi-\psi_n)/V_T} \cdot n_i e^{-(\phi-\psi_p)/V_T}$$

$$np = n_i^2 e^{(\psi_p - \psi_n)/V_T}$$

Dep. Reg ⇒ $np \approx n_i^2 e^{V_A/V_T}$

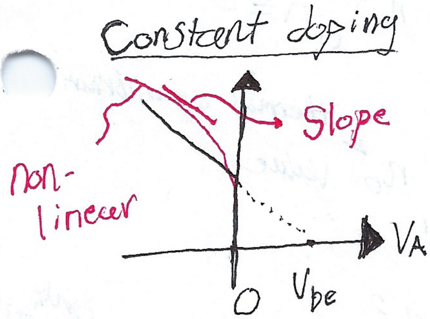
$R = (np - n_i^2)$

$V_A > 0 \Rightarrow R > 0 \Rightarrow$ Net Rec.

$V_A < 0 \Rightarrow R < 0 \Rightarrow$ Net generation

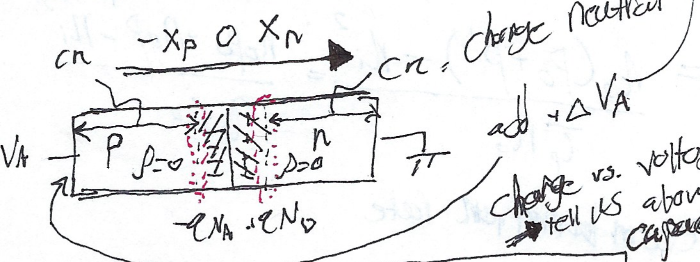
2018-02-23
Device electronics
Week 6 Day 1

Constant doping



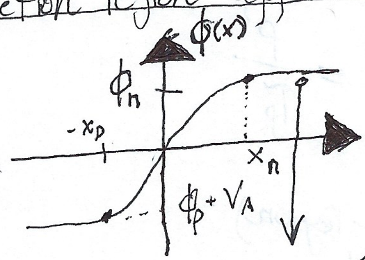
biasing potential

Review from last time



charge vs. voltage tell us about capacitance

$\boxed{\text{Depletion region approximation}}$:



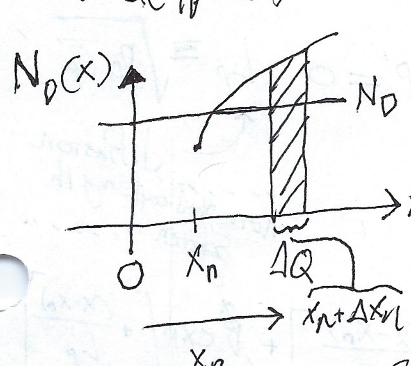
Debye length

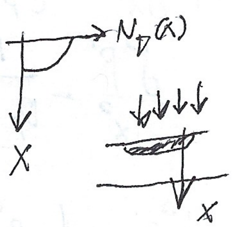
$x_n x_p \sim \sqrt{(\phi_B - V_A)}$

$$L_D = \sqrt{\frac{\epsilon_s V_T}{q N_D}}$$

$N_A >> N_D$: ($p^+ n$ diode)

$$C_j = \sqrt{\frac{q N_D \epsilon_s}{2(\phi_B - V_A)}} \qquad \frac{1}{C_j^2} = \frac{2(\phi_B - V_A)}{q N_D \epsilon_s}$$





$$C_j = \frac{\Delta Q}{\Delta V_A}$$

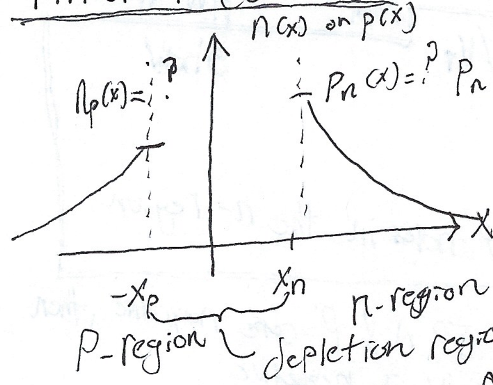
Show: $N_D(x) = \frac{2}{q\epsilon_s} \frac{1}{[d(1/C_j^2)/dV_A]}$ ← slope

What are the minority carriers doing

- Need to keep holes (electrons) coming from ~~right side~~ left side going right →

\# holes diffusing n side
\# electrons diffusing on p side

Minority carriers



current density, sum of drift and the diffusion

Diffusion (Start from thermal equilibrium)

T.E. $\vec{J_n} \approx$ small

$$J_n = q(\mu_n n \nabla\phi - D_n \nabla n) \approx 0$$

$$\nabla\phi \approx V_T \nabla \ln(n)$$

$① \to ②$ $\quad \phi_2 - \phi_1 = \ln\left(\frac{n_2}{n_1}\right)$

$$\to n_2 = n_1 e^{\frac{\phi_2 - \phi_1}{V_T}}$$

$x_1 \to x_n$

$x_2 \to x_p$

$$n(-x_p) = \underbrace{n(x_n)}_{N_D} e^{\frac{\phi(-x_p) - \phi(x_n)}{V_T}}$$

$$\phi(-x_p) = -V_T \ln\left(\frac{N_A}{n_i}\right) + V_A$$

$$\phi(x_n) = V_T \ln\left(\frac{N_D}{n_i}\right)$$

$$\Rightarrow n(-x_p) = N_D e^{\frac{V_A - \phi_B}{V_T}} = N_D \underbrace{e^{-\phi_B/V_T}}_{\frac{n_i^2}{N_D N_A}} e^{V_A/V_T}$$

↓

$$\phi_B = V_T \ln\left(\frac{N_D N_A}{n_i^2}\right)$$

$$\boxed{n(-x_p) = \frac{n_i^2}{N_A} e^{V_A/V_T}}$$ → minority density @ T.E.

remember: TE: $np = n_i^2$

in the p-region $p = N_A$

$n = \frac{n_i^2}{N_A}$

HW #2 Show

$$\boxed{p(x_n) = \frac{n_i^2}{N_D} e^{V_A/V_T}}$$

T.E. Value of holes in the n-region

$p'(x) = p(x) - p_0$

$p_0 = \frac{n_i^2}{N_D}$

If n & p' are negative, then V_A is negative

$n'(-x_p) = n(x) - n_0$

$n'(-x_p) = n_0(-x_p)[e^{V_A/V_T} - 1]$

$p'(+x_n) = p_0(x_n)[e^{V_A/V_T} - 1]$

@ TE When $V_A = 0$, $n' \equiv 0$, $p' \equiv 0$

Chapter 5 & 6

Ideal-Diode Analysis:

n-side, holes = ?

$$\frac{dp}{dt} = -\frac{1}{q}\nabla \cdot \vec{J}_p - R$$

$$\vec{J}_p = q(-\mu_p p \nabla\phi - D_p \nabla p)$$

Steady-State: $\Rightarrow \frac{d}{dt} = 0$

CN $\Rightarrow \vec{E} = 0$, $\nabla\phi = 0$

1D: $\nabla \rightarrow d/dx$

$$\frac{d}{dx}\left(D_p \frac{dp}{dx}\right) - R = 0 \Rightarrow \boxed{D_p \frac{d^2p}{dx^2} - R = 0}$$

$$R = \frac{np - n_i^2}{\tau_p(n + n_i) + \tau_n(p + n_i)}$$

@ T.E. ∴ $np = n_i^2$, $R \equiv 0$

T.E : n-side

$n = N_D \rightarrow n_0$ ← thermal equilibrium value

$p = p_0 + p'$

$$n_0 p_0 = N_D \frac{n_i^2}{N_D} = n_i^2$$

In n region only worry about majority carriers

$$R = \frac{n_0(p_0 + p') - n_i^2}{\tau_p n_0} = \frac{n_0 p_0 + n_0 p' - n_i^2}{\tau_p n_0}$$

≡ recombination rate

$$R = \frac{n_0 p'}{\tau_p n_0} = \frac{p'}{\tau_p}$$

$x \geq x_n$: (n-region)

$$D_p \frac{d^2 p}{dx^2} - \frac{p'}{\tau_p} = 0$$

$p = p_0 + p'$ constant

$$\frac{d^2 p}{dx^2} = \frac{d^2 p'}{dx^2}$$

$$D_p \frac{d^2 p'}{dx^2} - \frac{p'}{\tau_p} = 0$$

$$\frac{d^2 p'}{dx^2} - \frac{1}{D_p \tau_p} p' = 0 \qquad L_p \equiv \sqrt{D_p \tau_p}$$

diffusion length

$$\frac{d^2 p'}{dx^2} - \frac{1}{L_p^2} p' = 0$$

non-physical solution

$$p'(x) = A \exp\left[-\frac{(x - x_n)}{L_p}\right] + \cancel{B}^{\,0} \exp\left[+\frac{(x - x_n)}{L_p}\right]$$

$$p'(x) = A = p_0\left[e^{V_A/V_T} - 1\right] \exp\left[-\frac{(x - x_n)}{L_p}\right]$$

$$p'(x) = \frac{n_i^2}{N_D}\left[e^{V_A/V_T} - 1\right] e^{\left[\frac{-(x-x_n)}{L_p}\right]}$$

P-Side

$$R = \frac{n'}{\tau_n}$$

HW 2 show at home

$$n'(x) = \frac{n_i^2}{N_A}\left[e^{V_A/V_T} - 1\right]\exp\left[+\frac{(x+x_p)}{L_n}\right]$$

$$L_n = \sqrt{D_n \tau_n}$$

$$x \le -x_p$$

Calculate the current densities

hole diffuses into the n region

$$J_p = -qD_p \frac{dp}{dx} = -qD_p \frac{dp'}{dx}$$

$$J_n = qD_n \frac{dn}{dx} = qD_n \frac{dn'}{dx}$$

$$J_p(x) = qD_p \frac{n_i^2}{N_D} \cdot \frac{1}{L_p}\left[e^{V_A/V_T} - 1\right] e^{-(x-x_n)/L_p}$$

$x \ge x_n$.

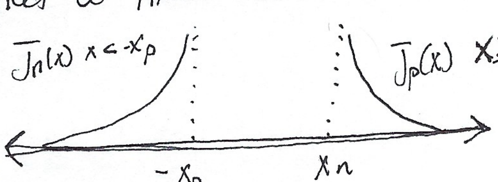
$$J_n(x) = qD_n \frac{n_i^2}{N_A} \cdot \frac{1}{L_n}\left[e^{V_A/V_T} - 1\right] e^{+(x+x_p)/L_p}$$

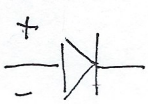
Can solve analytically with $x \le -x_p$

depletion region approximation

& charge neutrality

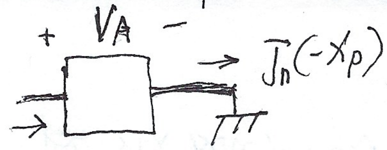
What do these current densities do?





have diode want to find current going through it.

~~how to solve~~ some for the current



A = cross-section area $J_p(x_n)$

→ hole density going down through recombination

$$I = A \cdot \left[J_p(+x_n) + J_n(-x_p)\right]$$

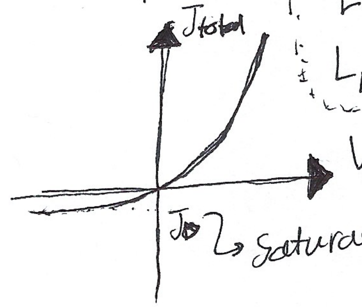
$$J_{total} = J_p(+x_n) + J_n(-x_p)$$

$$\boxed{J_{total} = q n_i^2 \underbrace{\left[\frac{D_p}{N_D L_p} + \frac{D_n}{N_A L_n}\right]}_{J_0}\left[e^{V_A/V_T} - 1\right]}$$

$$L_p = \sqrt{D_p \tau_p}$$

$$L_n = \sqrt{D_n \tau_n}$$

$$I = J_{total} \cdot A$$



↳ saturation current

- ideal diode equation

Current dominated by the lower doped side

$$J_{total} = q n_i^2 \left[\frac{1}{N_D}\sqrt{\frac{D_p}{\tau_p}} + \frac{1}{N_A}\sqrt{\frac{D_n}{\tau_n}}\right]\left[e^{V_A/V_T} - 1\right]$$

- large current, need efficient recombination process (short lifetime)

$N_A >> N_D$ p^+n diode

need to be able to understand and manipulate equations. need to make - how get high current out of P-N junction

Midterm stops here everything before on the exam.

- if carriers are recombining you get less current

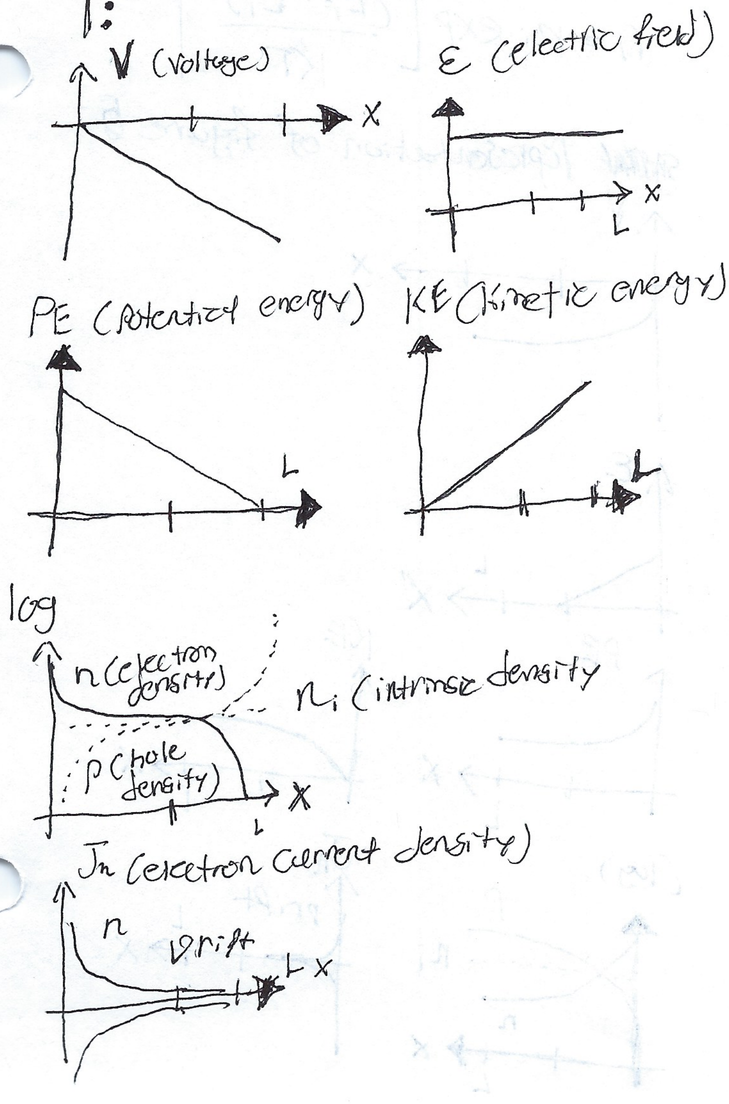
Homework # 2
Chapter 3 & 5
Ch3. 12, 15, 16, 17, 22
Ch5. 4, 10, 11, 13
Due : 2/27/2018
Joseph Crandall

Ch 3.) 12

Interpretation of Energy Band diagram

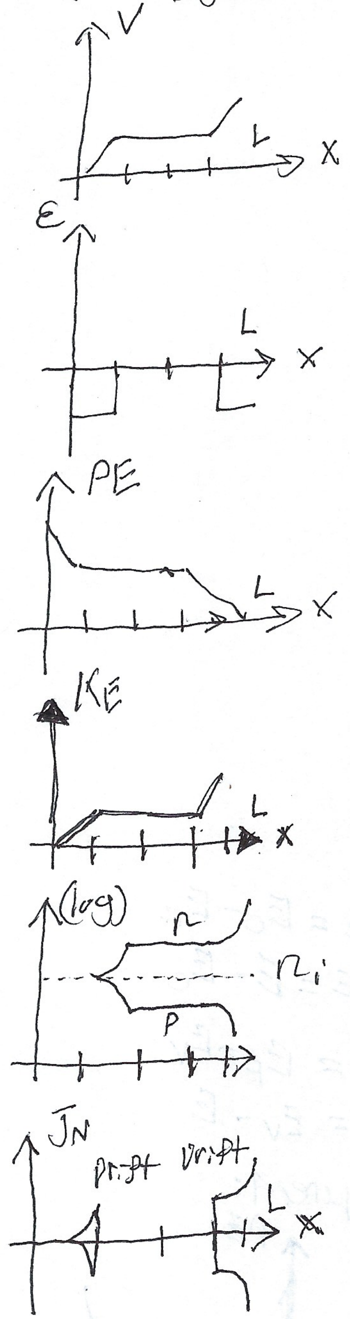
a) figures on page 144
Semiconductor in equilibrium because fermi-level has the same energy level as a function of the position. Since the value is constant, it is applicable for all cases.

graphical representation of figure 1:



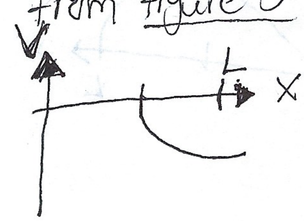
b) Potential V versus X form for the conduction band E_c form an upside down curve.

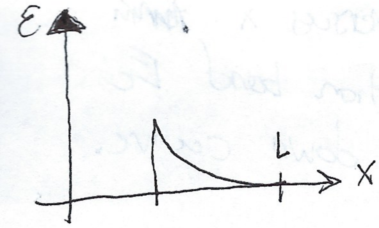
from figure 2

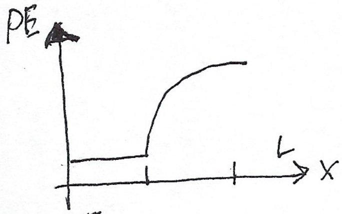


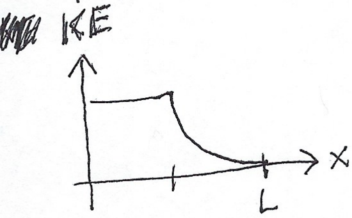
c) the electric field is the slope of the energy bands as a function of position.

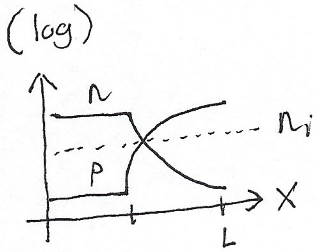
from figure 3

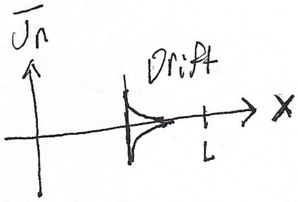












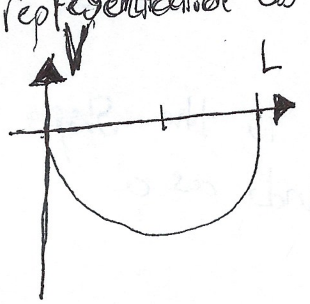
d.) for electrons $PE = E_C - E_F$

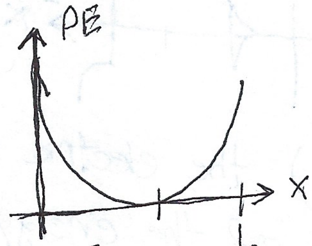
$KE = E - E_C$

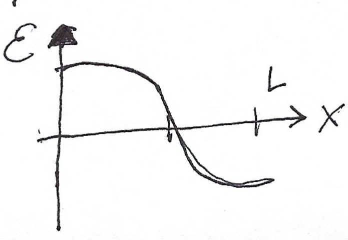
holes $PE = E_F - E_V$

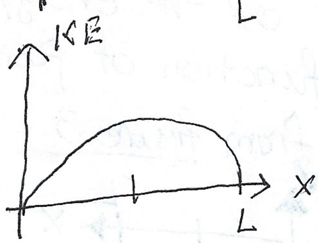
$KE = E_V - E$

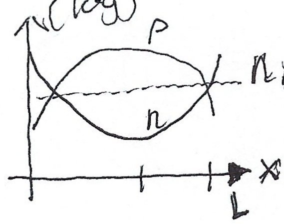
representation of figure 4:

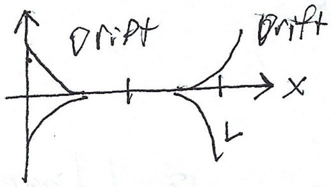












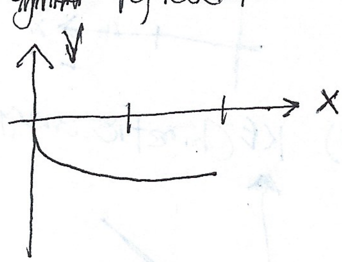
e) • universal carrier concentration for electrons when semiconductor is non-degenerate

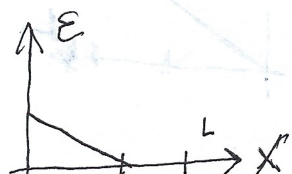
$$n = n_i \exp\left[\frac{(E_i - E_F)}{kT}\right]$$

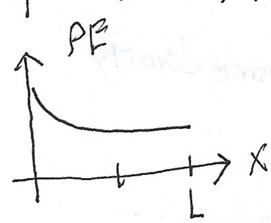
• universal carrier concentration for holes when semiconductor is non-degenerate

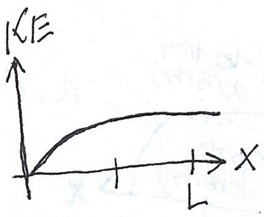
$$n = n_i \exp\left[\frac{(E_F - E_i)}{kT}\right]$$

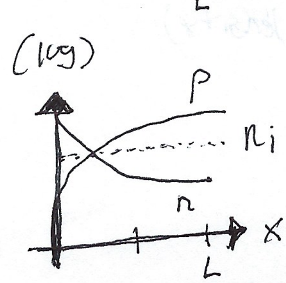
representation of figure 5:

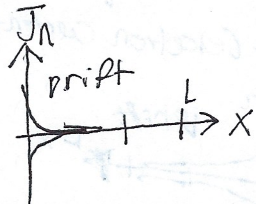












f) equilibrium conditions can be used as follows

$$J_n = J_n|_{drift} + J_n|_{diff} = 0$$

→ So, the expression can be

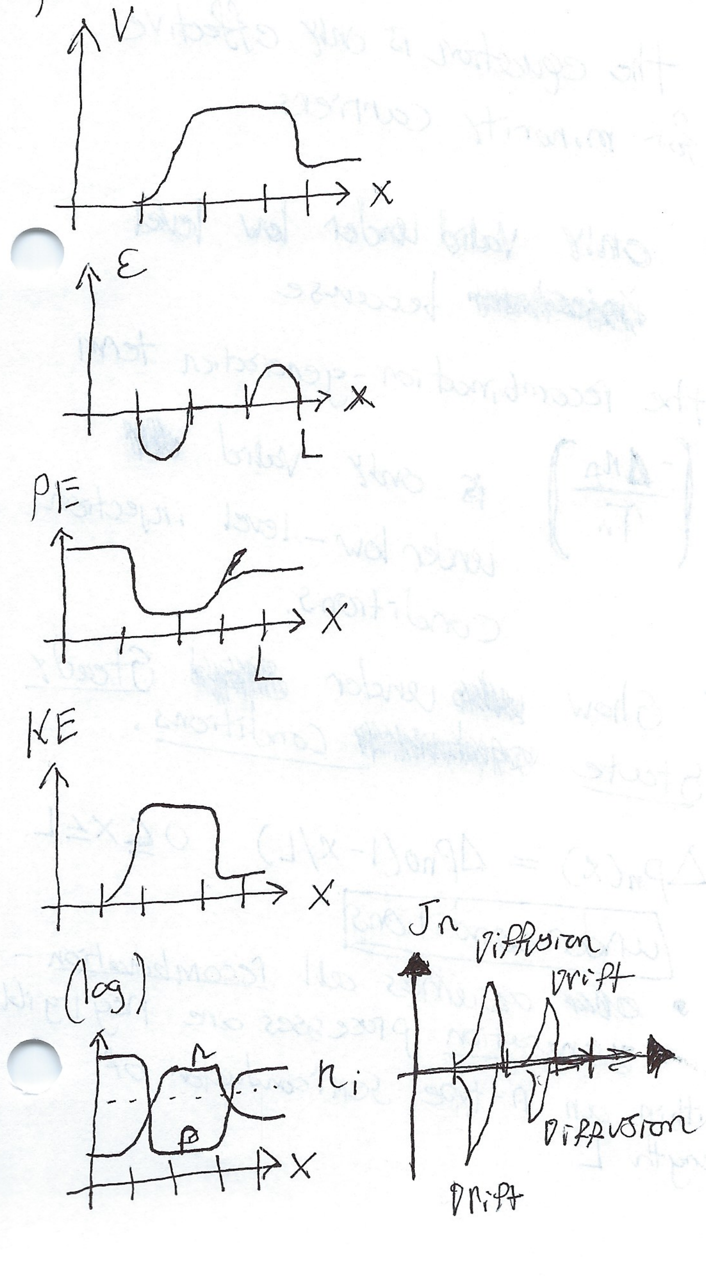
$$J_N|_{drift} = -J_N|_{diff}$$

use $E = -\frac{dV}{dx}$

$$E = \frac{1}{q}\frac{dE_c}{dx}$$

$$E = \frac{1}{q}\frac{dE_v}{dx}$$

representation of figure 6:



15) $E_T \cong E_i$

$n_1 \cong p_1 \cong n_i$

$\Delta n = \Delta p$

assume τ_n is comparable to t,

n electron carrier concentration

p hole concentration

n_0 equilibrium electron concentration

Δn deviation of electron concentration from equilibrium value

p_0 equilibrium hole concentration

Δp deviation of hole concentration from equilibrium value

R-G center relationship

$n = n_0 + \Delta n$

$p = p_0 + \Delta p$

$$\left.\frac{dp}{dt}\right|_{i-thermal\ R-G} = \left.\frac{dn}{dt}\right|_{i-thermal\ R-G}$$

$$= \frac{n_i^2 - np}{\tau_p(n+n_1) + \tau_n(p+p_1)}$$

$\Downarrow$

$$= \frac{n_i^2 - (n_0+\Delta n)(p_0+\Delta p)}{\tau_p(n_0+\Delta n+n_1)\tau_n(p_0+\Delta p+p_1)}$$

$$= \frac{n_i^2 - n_0p_0 - n_0\Delta p - p_0\Delta p - \Delta p^2}{\tau_p(n_0+\Delta p+n_i) + \tau_n(p_0+\Delta p+n_i)}$$

assume n-type semiconductor

$n_0p_0 = n_i^2$ and n_i^2 cancels $-n_0p_0$

↓

$n_o > p_o$ in n type semiconductor

$\therefore$ ~~[illegible]~~ $n_o \Delta p >> p_o \Delta p$

$n_o >> \Delta p$ with low level injection

$\therefore$ $n_o \Delta p >> \Delta p^2$

numerator is $n_o \Delta p$

$n_o >> \Delta p$ & $n_o >> n_i$

$\therefore$ $\tau_p(n_o + \Delta p + n_i) \cong \tau_p n_o$

$\tau_p \sim \tau_n$ & $n_o >> p_o + \Delta p + n_i$

$\therefore$ ~~[illegible]~~

$\tau_p n_o >> \tau_n(p_o + \Delta p + n_i)$

$\Delta p << n_o$ under low-level injection

$\therefore$ $n_o \Delta p >> \Delta p^2$

denomenator is effectively $\tau_p n_o$

$\therefore$

$$\frac{dp}{dt}\Big|_{i\text{-thermal R-G center Recombination}} =$$

$$= \frac{n_o \Delta p}{\tau_p n_o}$$

$$= -\frac{\Delta p}{n_o}$$

for holes in a ~~p-type~~ n-type material

$$\frac{dn}{dt}\Big|_{i\text{therm RG}} = \frac{n_o \Delta n}{\tau_n n_o} = -\frac{\Delta n}{\tau_n}$$

for electrons in a p-type material

16.

minority carrier diffusion equation for electrons

$$\frac{d\Delta n_p}{dt} = \frac{D_N\, d^2 \Delta n_p}{dx^2} - \frac{\Delta n_p}{\tau_n} + G_L$$

a) because the diffusion is considered to be the leading mode of the minority carrier transport

- the minority carrier drift current is insignificant compared to the diffusion current when setting up the equation.

b) the equation is only effective for minority carriers

c) only valid under low level ~~injection~~ because

the recombination-generation term $\left(\frac{-\Delta n_p}{\tau_n}\right)$ is only valid ~~[illegible]~~ under low-level injection conditions.

17. Show ~~[illegible]~~ under ~~[illegible]~~ steady state ~~[illegible]~~ conditions.

$\Delta p_n(x) = \Delta p_{no}(1 - x/L) \quad 0 \leq x \leq L$

$\boxed{\text{under conditions}}$

- ~~[illegible]~~ assumes all recombination-generation processes are negligible

within an n-type semiconductor of length L

• 2 one employs the boundary conditions $\Delta p_n(0) = \Delta p_{no}$ & $\Delta p_n(L) = 0$

Note: neglecting recombination-generation is an excellent approximation when L is much less than a minority carrier diffusion length.

• A $\Delta P(x)$ solution of the above type is frequently encountered in practical problems.

$$\frac{d\Delta p_n}{dt} = D_p \frac{d^2 \Delta p_n}{dx^2} - \frac{\Delta p_n}{\tau_p} + G_L$$

↑

hole minority carrier differential equation

D = diffusion coefficient

G_L = is the simplified symbol for the photogeneration rate

G_{Lo} is the photogeneration rate at $x=0$

① at steady state:

$$\frac{d\Delta p_n}{dt} = 0$$

Center

② When RG ~~center~~ Recombination is neglected

$$\therefore \frac{\Delta p_n}{\tau_p} = 0$$

↓

When there is no light ③

$G_L = 0$

$$\therefore \frac{d^2 \Delta p_n}{dx^2} = 0 \Leftarrow \text{deduced expression}$$

general solution expressed as follows:

$$\Delta p_n = A + Bx$$

Boundary conditions:

I $\Delta p_n(0) = \Delta p_{no}$

$\therefore \Delta p_n(0) = A$

II $\Delta p_n(L) = 0$

$= A + BL$

$$B = -\frac{A}{L}$$

$$= -\frac{\Delta p_{no}}{L}$$

$$\therefore \Delta p_n(x) = \Delta p_{no} - \frac{\Delta p_{no}}{L} x$$

$$\Delta p_n(x) = \Delta p_{no}\left(1 - \frac{x}{L}\right) \quad \text{for } 0 \leq x \leq L$$

3.22)

$G_L = 10^{15}$ electron-hole pairs/$cm^3 s$ for $x > 0$

$G_L = 0$ for $x < 0$

· Steady state

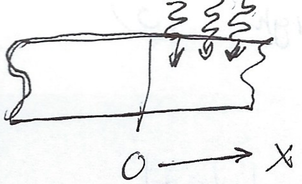
· Semiconductor Silicon

· uniformly doped $N_D = 10^{18}/cm^3$

entire bar

$\tau_p = 10^{-6}$ sec

$T = 300K$



a) there is no ~~perturbation~~ on the negative side of $x=0$ & hole concentration at $x=-\infty$ is

$$P(-\infty) = P_o = \frac{n_i^2}{N_D} = \frac{10^{20}}{10^{18}} = 10^2/cm^3$$

b) estimated that ~~a~~ concentration gradient there is a at $x=0$ due to diffusion, but ~~at~~ as $x \to \infty$ the gradient will disappear.

• at a distance from $x=0$, the carriers produced by light, will just interact with the carriers created by thermal Recombination–Generation Center recombination, ~~and~~ under steady state.

$$G_L = \frac{\Delta P_n(\infty)}{T_p}$$

$$\Delta P_n(\infty) = G_L T_p = (10^{15})(10^{-6}) = 10^9\ cm^{-3}$$

$$P(\infty) = P_o + \Delta P_n(\infty) \cong \Delta P_n(\infty) = 10^9\ cm^{-3}$$

c) → low level injection conditions prevail

• the largest ΔP_n occur at $x=\infty$

$$\Delta P_n(\infty) = \frac{10^9}{cm^3} \ll n_o = N_D = 10^{18}\ cm^{-3}$$

d) $\Delta P_n(x)$ for $x<0$

$$0 = D_p \frac{d^2 \Delta P_n}{dx^2} - \frac{\Delta P_n}{T_p}$$

• general solution of differential equation

$$\Delta P_n(x) = A e^{-\frac{x}{L_p}} + B e^{\frac{x}{L_p}}$$

① $\Delta P_n(-\infty) = 0$

$$\therefore A = 0$$

$$\Delta P_n(x) = B e^{\frac{x}{L_p}}$$

$$\frac{d\Delta P_n(x)}{dx} = \frac{B e^{\frac{x}{L_p}}}{L_p}$$

② Value of $\Delta P_n(x)$ for $x>0$

$$0 = D_p \frac{d^2 \Delta P_n}{dx^2} - \frac{\Delta P_n}{T_p} + G_L$$

$$\Delta P_n(x) = G_L T_p + A' e^{-x/L_p} + B' e^{x/L_p}$$

$$\Delta P_n(\infty) = G_L T_p$$

$$\therefore B' = 0$$

$$\Delta P_n(x) = G_L T_p + A' e^{-x/L_p}$$

$$\frac{d\Delta P_n(x)}{dx} = -\frac{A'}{L_p} e^{-x/L_p}$$

~~ll~~ $\Delta p_n(x)$ when $x=0$

$B = G_L T_p + A'$

↳ Continuity of $\Delta p_n(x)$

$\frac{B}{L_p} = \frac{A'}{L_p}$ continuity of $\frac{d\Delta p_n(x)}{dx}$

$B = A'$

$= \frac{G_L T_p}{2}$

~~therefore~~ Therefore

$\Delta p_n(x)$ for $x<0$ & $x>0$ & $x=0$

$$\Delta p_n(x) = \begin{cases} \frac{G_L T_p}{2} e^{x/L_p} & x \leq 0 \\ \frac{G_L T_p}{2}\left[1 - \frac{e^{-x/L_p}}{2}\right] & x \geq 0 \end{cases}$$

chapter 5

4) - Si ~~Step~~ junction

- maintained at room temperature
- equilibrium condition
- p-side doping of $N_A = 2\times10^{15}\ cm^{-3}$
- n-side doping of $N_D = 10^{15}\ cm^{-3}$

a) $V_{bi} = \frac{KT}{q} \ln\left(\frac{N_A N_D}{n_i^2}\right)$

n_i = intrinsic carrier concentration $= 10^{10}$

$K = 8.617\times10^{-5}\ eV/K$ Boltzman Constant

$q = 1.6\times10^{-19}\ C$

$$V_{bi} = \frac{(8.617\times10^{-5})(300)}{(1.6\times10^{-19})} \ln\left[\frac{(2\times10^{15})(10^{15})}{(10^{10})^2}\right]$$

$= (0.0254)(23.718)$

$= 0.614\ V$

b) p-side width of pn junction depletion region X_p

$$X_p = \left[\frac{2K_S \varepsilon_0}{q} \frac{N_D}{N_A(N_A+N_D)} V_{bi}\right]^{1/2}$$

K_S = Semiconductor dielectric Constant $= 11.8$

$\varepsilon_0 = 8.854\times10^{-14}$ farad/cm = Permitivity of free Space

$$X_p = \left[\frac{2(11.8)(8.854\times10^{-14})}{1.6\times10^{-19}} \frac{(10^{15})}{2\times10^{15}(2\times10^{15}+10^{15})} \cdot (0.614)\right]^{1/2}$$

$= 3.6557\times10^{-5}$ cm

n-side width of pn junction depletion region:

$$X_n = \left[\frac{2K_S \varepsilon_0}{q} \frac{N_A}{N_D(N_A+N_D)} V_{bi}\right]$$

$$x_p = \left[\frac{2(11.8)(8.85\times10^{-14})}{1.6\times10^{-19}} \frac{(2\times10^{15})}{(10^{15})\{(2\times10^{15}) + (10^{15})\}} (0.614) \right]^{1/2}$$

$$= 7.31\times10^{-5}\ cm$$

depletion width W

$$W = x_n + x_p$$

$$= 7.31\times10^{-5}\ cm + 3.65\times10^{-5}\ cm$$

$$= 1.10\times10^{-4}\ cm$$

c) electric field ($\mathcal{E}$) at $x=0$

$$\mathcal{E}(0) = -\frac{qN_D}{K_S\varepsilon_0} x_n$$

$$\mathcal{E}(0) = -\frac{(1.6\times10^{-19})(10^{15})}{(11.8)(8.85\times10^{-14})} (7.31\times10^{-5})$$

$$= -1.12\times10^{-4}\ V/cm$$

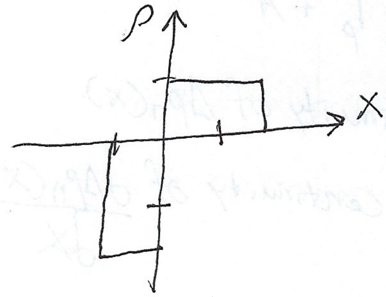
d) electrostatic potential at $x=0$

$$V(0) = \frac{qN_A}{2K_S\varepsilon_0} x_p^2$$

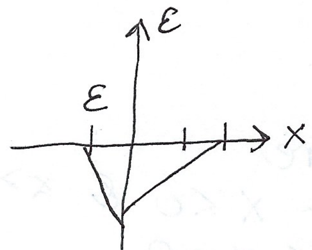
$$V(0) = \frac{(1.6\times10^{-19})(2\times10^{15})}{(2)(11.8)(8.85\times10^{-14})} (3.655\times10^{-5})^2$$

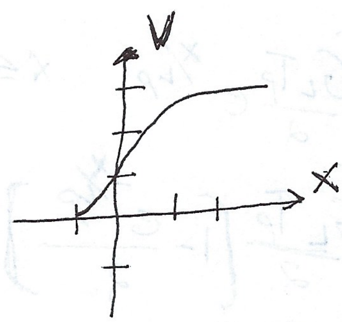
$$= 0.205\ V$$

e) charge density

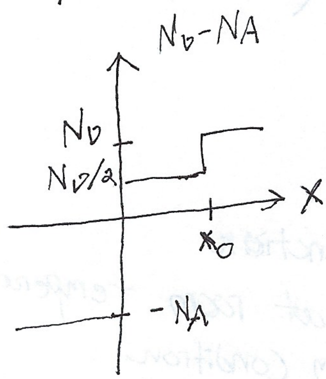


electric field





10)



assumption $x_n > x_0$ for applied biases of interest.

a) calculate built in voltage across junction → first calculate field intensity inside the depletion region

$$\mathcal{E} = -\frac{D_N}{\mu_n}\left(\frac{1}{n}\right)\frac{dn}{dx}$$

$D =$ diffusion coefficient

$\mu =$ carrier mobility

$\rho =$ resistivity $N =$ doping concentration

$$\rho = \frac{1}{q(\mu_n n + \mu_p p)}$$

$n =$ electron concentration

$p =$ hole concentration

n-type semiconductor

$$\rho = \frac{1}{q\mu_n N_D}$$

p-type semiconductor

$$\rho = \frac{1}{q\mu_p N_A}$$

electrostatic potential at a given point

$$V = -\frac{1}{q}(E_c - E_{ref})$$

Consequently electric field

$$\mathcal{E} = -\nabla V$$

$$= \frac{1}{q}\frac{dE_c}{dx} = \frac{1}{q}\frac{dE_v}{dx} = \frac{1}{q}\frac{dE_i}{dx}$$

E = energy

total net carrier currents in a semiconductor

$$J_p = J_p|_{drift} + J_p|_{diff} = q\mu_p p\mathcal{E} - qD_p \nabla p$$

$$J_N = J_N|_{drift} + J_N|_{diff} = q\mu_n n\mathcal{E} + qD_N \nabla n$$

$$J = J_N + J_p$$

einstein relationship for electrons and holes

$K =$ Boltzmann constant

$$\frac{D_N}{\mu_n} = \frac{D_P}{\mu_p} = \frac{KT}{q} = V_T$$

V_T = voltage equivalent of temperature

low level injection implies

$\Delta p \ll n_0$ $n \simeq n_0$ in n-type material

$\Delta n \ll p_0$ $p \simeq p_0$ in p-type material

n_0, p_0 electron & hole concentrations in material under analysis when equilibrium conditions prevail

n, p electron & hole concentrations in metal under arbitrary conditions

$$\Delta n = n - n_0$$

$$\Delta p = p - p_0$$

deviations in carrier concentrations from equilibrium values, where positive corresponds to carrier excess & negative to carrier deficit

N_T number of R-G centers/cm^3

C_n & C_p positive proportionality constant

for holes in n-type material

for indirect thermal recombination generation

$$\left.\frac{dp}{dt}\right|_{\substack{i\text{-thermal} \\ R-G}} = \left.\frac{dp}{dt}\right|_R + \left.\frac{dp}{dt}\right|_G$$

$$\left.\frac{dp}{dt}\right|_R = -C_p N_T p$$

$$\left.\frac{dp}{dt}\right|_G = C_p N_T p_0$$

$$\left.\frac{dp}{dt}\right|_{\substack{i\text{-thermal} \\ R-G}} = -C_p N_T(p - p_0)$$

$$= -C_p N_T \Delta p$$

for electrons in p-type material

$$\left.\frac{dn}{dt}\right|_{\substack{i\text{-thermal}\\ R-G}} = -C_n N_T \Delta n$$

τ_p & τ_n time constants

$$\tau_p = \frac{1}{C_p N_T}$$

$$\tau_n = \frac{1}{C_N N_T}$$

~~therefore~~ therefor

for holes in n-type material

$$\left.\frac{dp}{dt}\right|_{\substack{i\text{-thermal}\\ R-G}} = \frac{-\Delta p}{\tau_p}$$

for electrons in p-type material

$$\left.\frac{dn}{dt}\right|_{\substack{i\text{-thermal}\\ R-G}} = \frac{-\Delta n}{\tau_n}$$

Continuity equations

$$\frac{dn}{dt} = \frac{1}{q}\nabla\cdot\vec{J}_N + \left.\frac{dn}{dt}\right|_{\substack{\text{thermal}\\ R-G}} + \left.\frac{dn}{dt}\right|_{\substack{\text{other}\\ \text{processes}}}$$

$$\frac{dp}{dt} = -\frac{1}{q}\nabla\cdot\vec{J}_p + \left.\frac{dp}{dt}\right|_{\substack{\text{thermal}\\ R-G}} + \left.\frac{dp}{dt}\right|_{\substack{\text{other}\\ \text{processes}}}$$

Minority carrier diffusion equations

G_L simplified symbol for the photogeneration rate

$$\frac{d\Delta n_p}{dt} = D_N\frac{d^2\Delta n_p}{dx^2} - \frac{\Delta n_p}{\tau_n} + G_L$$

subscript on carrier concentrations denote that equations are only valid for minority carriers

applying to electrons in p-type material and to holes in n-type materials

$$\frac{d\Delta p_n}{dt} = D_p\frac{d^2\Delta p_n}{dx^2} - \frac{\Delta p_n}{\tau_p} + G_L$$

$\boxed{\text{Back to the problem 5.10}}$ a)

$$\mathcal{E} = -\frac{KT}{nq}\frac{dn}{dx}$$

built in potential across a depletion layer

$$V_{bi} = -\int_{-x_p}^{x_n}\mathcal{E}\,dx$$

$$= -\int_{-x_p}^{x_n} -\frac{KT}{nq}\frac{dn}{dx}\,dx$$

$$= \frac{KT}{q}\int_{-x_p}^{x_n}\frac{dn}{n}$$

$\ln(x_n) - \ln(-x_p)$

$$= \frac{KT}{q}\ln\left(\frac{n(x_n)}{n(-x_p)}\right)$$

$$n(x_n) = N_D$$

$$\frac{n_i^2}{N_A} = n(-x_p)$$

$$V_{bi} = \frac{KT}{q}\ln\left[\frac{N_D}{\frac{n_i^2}{N_A}}\right]$$

$$V_{bi} = \frac{KT}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right)$$

b) Charge density in semiconductors

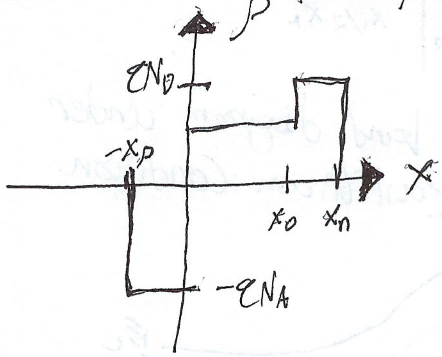
$$\rho = q(p - n + N_D - N_A)$$

in depletion layer approx

n-type charge = p-type charge

$\rho = q(N_D - N_A) \quad -x_p \le x \le x_p$

Charge density across the depletion layer



c) Charge densities through out the depletion region

$$\rho = \begin{cases} 0 & x > x_n \text{ and } x < -x_p \\ -qN_A & -x_p \le x \le 0 \\ \dfrac{qN_D}{2} & 0 \le x \le x_0 \\ qN_D & x_0 \le x \le x_n \end{cases}$$

Poisson equation for electric field intensity

$$\frac{d\mathcal{E}}{dx} = \begin{cases} \dfrac{qN_D}{2K_s\varepsilon_0} & 0 \le x \le x_0 \\ \dfrac{-qN_A}{K_s\varepsilon_0} & -x_p \le x \le 0 \\ \dfrac{qN_D}{K_s} & x_0 \le x \le x_n \end{cases}$$

for n-region that is lightered doped $\rightarrow 0 \le x \le x_0$

space charge density $= \dfrac{qN_D}{2}$

$\rightarrow$ for $x_0 \le x \le x_n$ space charge density $= qN_D$

electric field at arbitrary point between 0 and x_0

$$\mathcal{E}(x) = \int_x^{x_0} \left(-\frac{qN_D}{2K_s\varepsilon_0}\right) dx' \quad 0 \le x \le x_0$$

$$= \frac{-qN_D}{2K_s\varepsilon_0}(x_0 - x)$$

electric field at arbitrary point between x_0 and x_n

$$\mathcal{E}(x) = \int_{x_0}^{x_n} \left(-\frac{qN_D}{K_s\varepsilon_0}\right) dx'$$

$$= \frac{-qN_D}{K_s\varepsilon_0}(x_n - x_0)$$

$\therefore$

$$\mathcal{E}(x) = -\frac{qN_D}{2K_s\varepsilon_0}(x_0 - x) - \frac{qN_D}{K_s\varepsilon_0}(x_n - x_0)$$

$$= \frac{-qN_D}{K_s\varepsilon_0}\left(\frac{x_0}{2} - \frac{x}{2} + x_n - x_0\right)$$

$$\mathcal{E}(x) = \frac{-qN_D}{K_s\varepsilon_0}\left(x_n - \frac{x}{2} - \frac{x_0}{2}\right)$$

for $0 \le x \le x_n$

electric field in p-side depletion region

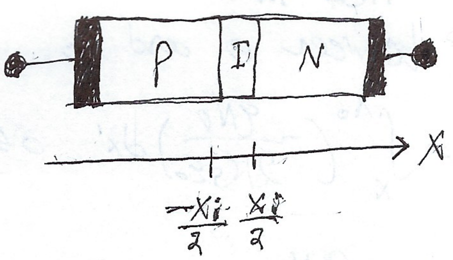
$$\mathcal{E}(x) = \int_{-x_p}^{x} \left(\frac{-qN_A}{K_s\varepsilon_0}\right) dx' \quad -x_p \le x \le 0$$

$$= -\frac{qN_A}{K_s\varepsilon_0}\int_{-x_p}^{x} dx'$$

$$\mathcal{E}(x) = \frac{-qN_A}{K_s\varepsilon_0}(x + x_p) \quad -x_p \le x \le 0$$

5.11

a)

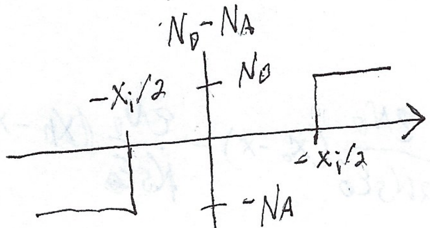


~~middle~~ middle region is intrinsic (lightly doped) and narrow

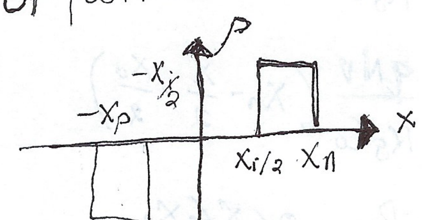
- P & n regions are uniformly doped $N_D - N_A = 0$ in I region.

When $N_A > N_D$

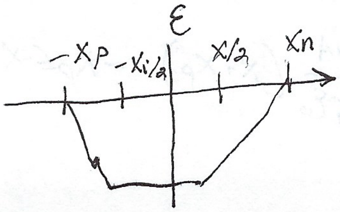
- Step junction profile ~~[illegible]~~ as a function of position

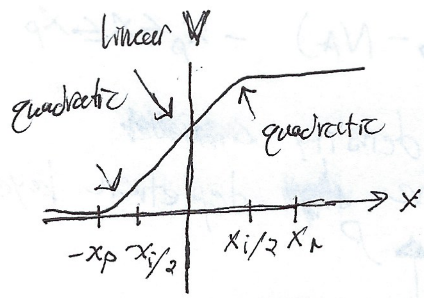


- Charge density as a function of position

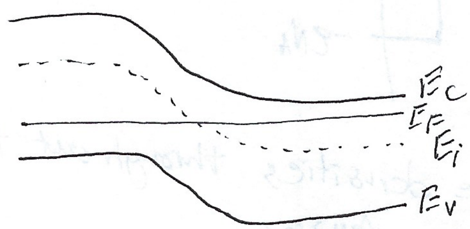


- $\rho = 0$ in the I region
  - electric field is constant for region
  - expect a step-ramp condition outside the i-region





energy band diagram under ~~eq~~ equilibrium condition



b) - derivation of built in voltage is applicable to an arbitrary doping profile

- $n(X_n)$ & $n(-X_p)$ $= \frac{n_i^2}{N_A}$ for the pin diode

- assume P & n regions are non degenerate

$$V_{bi} = \frac{KT}{q} \ln\left[\frac{N_A N_D}{n_i^2}\right]$$

c) deduced expression for depletion approximation

$$\rho = \begin{cases} 0 & X < -X_p \\ -qN_A & -X_p \le X \le -\frac{X_i}{2} \\ 0 & -\frac{X_i}{2} \le X \le \frac{X_i}{2} \\ qN_D & \frac{X_i}{2} \le X \le X_n \\ 0 & X > X_n \end{cases}$$

Poisson's equation

$$\frac{d\mathcal{E}}{dx} = \frac{\rho}{K_S \varepsilon_0}$$

$$\frac{d\mathcal{E}}{dx} = \begin{cases} -\dfrac{qN_A}{K_S\varepsilon_0} & -x_p \le x \le -\dfrac{x_i}{2} \\ 0 & -\dfrac{x_i}{2} \le x \le \dfrac{x_i}{2} \\ -\dfrac{qN_D}{K_S\varepsilon_0} & x > x_n \end{cases}$$

- Seperation of variables
- integrate for specific condition for the ~~depletion area~~ boundaries where $E=0$ to the arbitrary points in the n and p-regions gives the same relationships and results as in the step junction analysis.
- In i-region deduced expression can be seen as follows:

$$\mathcal{E} = \mathcal{E}\left(-\frac{x_i}{2}\right)$$

$$\mathcal{E}(x) = \begin{cases} -\dfrac{qN_A}{K_S\varepsilon_0}(x_p + x) & -x_p \le x \le -\dfrac{x_i}{2} \\ -\dfrac{qN_A}{K_S\varepsilon_0}\left(x_p - \dfrac{x_i}{2}\right) & \dfrac{x_i}{2} \le x \le -\dfrac{x_i}{2} \\ -\dfrac{qN_D}{K_S\varepsilon_0}(x_n - x) & \dfrac{x_i}{2} \le x \le x_n \end{cases}$$

Set $\mathcal{E}(x) = -\dfrac{dV}{dx}$

- Separate variables
- integrate from depletion region edges to arbitrary points in n & p regions
- results in step junction analysis

Introduce

$$\mathcal{E}\left(-\frac{x_i}{2}\right) = \mathcal{E}_i$$

$$V\left(-\frac{x_i}{2}\right) = V_i$$

in the i region the deduced expression is

$$\frac{dV}{dx} = -\mathcal{E}_i$$

- integrate both sides

$$\int_{V_F}^{V(x)} dV = -\mathcal{E}_i \int_{-\frac{x_i}{2}}^{x} dx'$$

- Simplified expression

$$V(x) = V_i - \mathcal{E}_i\left(x + \frac{x_i}{2}\right)$$

therefor

$$V(x) = \begin{cases} \dfrac{qN_A}{2K_S\varepsilon_0}(x_p + x)^2 & -x_p \le x \le -\dfrac{x_i}{2} \\ \dfrac{qN_A}{(2K_S\varepsilon_0)}\left(x_p - \dfrac{x_i}{2}\right)\left(x_p + \dfrac{x_i}{2} + 2x\right) & -\dfrac{x_i}{2} \le x \le \dfrac{x_i}{2} \\ V_{bi} - V_A - \left(\dfrac{qN_D}{(2K_S\varepsilon_0)}(x_n - x)^2\right) & \dfrac{x_i}{2} \le x \le x_n \end{cases}$$

• $E(x)$ and $V(x)$ must be continuous at $\left(x = \frac{x_i}{2}\right)$ to determine x_n and x_p

• equate electric field at $x = \frac{x_i}{2}$

$$\frac{x_i}{2} \le x \le \frac{-x_i}{2} \text{ and } \frac{x_i}{2} \le x \le x_n$$

$$N_A\left(x_p - \frac{x_i}{2}\right) = N_D\left(x_n - \frac{x_i}{2}\right)$$

• equating ~~electric~~ electric potential expressions at $x = \frac{x_i}{2}$

$$\frac{qN_A}{2K_S\varepsilon_0}\left[\left[x_p - \frac{x_i}{2}\right]\left(x_p + \frac{3x_i}{2}\right)\right] =$$

$$= V_{bi} - V_A - \left(\frac{qN_D}{2K_S\varepsilon_0}\right)\left(x_n - \frac{x_i}{2}\right)^2$$

$$\therefore \left(x_n - \frac{x_i}{2}\right) = \frac{N_A}{N_D}\left(x_p - \frac{x_i}{2}\right)$$

$\therefore$

$$\left(x_p - \frac{x_i}{2}\right)^2 + \frac{2N_D}{N_A + N_D}\left(x_p - \frac{x_i}{2}\right)$$

$$- \frac{2K_S\varepsilon_0}{q}\frac{N_D}{N_A(N_A+N_D)}(V_{bi} - V_A) = 0$$

<u>Solve quadratic equation</u>

$$= -\frac{N_D x_i}{N_A + N_D} + \left[\left(\frac{N_D x_i}{N_A + N_D}\right)^2 + \frac{2K_S\varepsilon_0}{q}\frac{N_D}{N_A(N_A+N_D)}(V_{bi} - V_A)\right]^{1/2}$$

• therefor a positive root has been chosen

• Since $\left(x_p - \frac{x_i}{2}\right)$ should be greater than zero

13) ran out of time

2018-03-02

Week ~~7~~ 7 Device electronics

Joseph Crandall

• <u>~~Final~~ project abstract</u> of what I am going to work on ~~...~~ after the midterm
 • review paper

<u>Pn junction details</u>: (last content on the ~~...~~ midterm)

• depletion region ~~...~~ approximation

• ideal diode equation

$$I = I_0\left(e^{V_a/V_T} - 1\right)$$

 • minority carriers
 • assume regions are charge neutral on the basis that we have low level injection
 • diffusion current only

• entire voltage V_a is felt across the depletion region

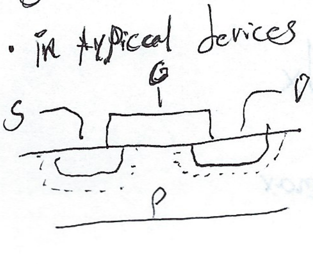
• factor to add to <u>ideal diode equation</u> ($n=1$) n which is the diode ideality factor

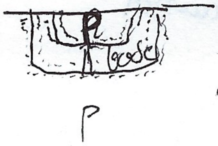
$$I = I_0 e^{V_a/nV_T} - 1$$

Where $1 \le n < 2$

• derived under basis that we have ideal diode

• long diode versus short diode
 • in typical devices





Pit

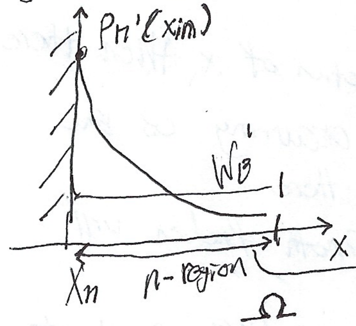
• depletion region on both sides of the

<u>Recall</u>:

– indirect recombination in the charge neutral region

$$J_0 = q n_i^2\left(\frac{D_p}{N_D L_p} + \frac{D_n}{N_A L_n}\right)$$

<u>Short diode model</u>:

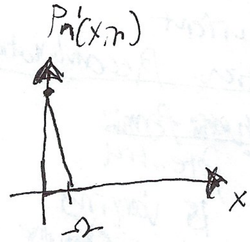


at ohmic contact
$np = n_i^2$
$p_n'(\Omega\ \text{contact}) \equiv 0$

short at limit

Limit of a short C.N. region:

$$p_n'(x) = p_n^0\underbrace{\left(e^{V_a/V_T} - 1\right)}_{0}\left[1 - \frac{(x - x_n)}{W_D'}\right]$$



$$J_p = -q D_p \frac{dp}{dx}$$

$$J_p = -q D_p \frac{n_i^2}{N_D W_D'}\left(e^{V_a/V_T} - 1\right)$$

replace L_p with a smaller number

$$J_p = -q D_p \frac{dp}{dx}$$

$$J_p = q D_p \frac{n_i^2}{N_D W_D'}\left(e^{V_a/V_T} - 1\right)$$

Constant function of space

• We have a short charge neutral region, then we have a limit

Current Continuity Equation

$$\frac{dp}{dt} = \frac{1}{q}\nabla \vec{J}_p - R = 0$$

$$\frac{dJ_p}{dx}$$

- Constructed problem so that long region cannot exist. Forcing a linear decay
- Consistent with neglecting recombination
- reverse saturation current might be shorter than you expect.
- if constant function of x, then there is no recombination occurring as you go deeper and deeper there
- at the limit any smooth function will be a line
  - → density will be ~~[illegible]~~ constant
- decrease the volume where recombination occurs
  - therefor we see a ~~[illegible]~~ ~~[illegible]~~ in the increase in the current

Depletion region: & Generation Recombination

Quasi fermi potential:

$$\vec{J}_n = -q\mu_n n \frac{d\psi_n}{dx}$$

$$\vec{J}_p = -q\mu_p p \frac{d\psi_p}{dx}$$

p = very large

P = very small

$$\left.\begin{aligned}\vec{J}_n &- q\mu_n n \frac{d\psi_n}{dx} \\ \vec{J}_p &= q\mu_p p \frac{d\psi_p}{dx}\end{aligned}\right\} \text{Current densities don't vary too much}$$

$$n = n_i e^{\frac{\phi - \psi_n}{V_T}}$$

$$p = n_i e^{-(\phi - \psi_p)/V_T}$$

$$np = n_i^2 \exp\left[\frac{\psi_p - \psi_n}{V_T}\right] \approx n_i^2 e^{V_a/V_T}$$

- in the depletion region

$$R = \frac{np - n_i^2}{\tau_p(n+n_i) + \tau_n(p+n_i)}$$

$$\approx \frac{n_i^2(e^{V_a/V_T} - 1)}{\tau_p(n+n_i) + \tau_n(p+n_i)}$$

if $V_a > 0 \Rightarrow R > 0 \Rightarrow$ net recombination

if $V_a < 0 \Rightarrow R < 0 \Rightarrow$ net generation

- if i am generating electron hole pairs
- this diode when off will not be as off as we might think it is.
- ~~reverse~~ bias current will be larger
- → reverse bias diode
  - current source
  - Use
    - → can be a photodetector under reverse bias
  - ~~if reverse bia~~ puts a lower limit on what you can detect.

J_r = recombination current

- → decrease in the current
- the divergence of the current is equal to r

$$\frac{1}{q}\frac{dJ}{dx} - R = 0 \quad \leftarrow S.S.$$

$$\Rightarrow J_r = q\int_{-x_p}^{+x_n} R\,dx$$

$$J_r \leq q X_d \cdot R_{max}$$

$$X_d = (x_n + x_p)$$

$$R \cong \frac{n_i^2(e^{V_a/V_T}-1)}{\tau(n+p+2n_i)}$$

$\frac{dR}{dn}=0$ under the condition that
$np = n_i^2 e^{V_a/V_T}$

Show @ home

$$\Rightarrow \boxed{p=n=n_i e^{V_a/2V_T}} \leftarrow \text{Worst case}$$

$$R \;\cancel{\ldots} = \frac{np-n_i^2}{\tau_p(n+n_i)+\tau_n(p+n_i)}$$

$$\approx \frac{n_i^2(e^{V_a/V_T}-1)}{\tau(n+n_i)+\tau(p+n_i)}$$

max

$$\rightarrow \frac{n_i^2(e^{V_a/V_T}-1)}{2\tau n_i(e^{V_a/2V_T}+1)}$$

$e^{V_a/2V_T}$

$$\underbrace{\approx \frac{n_i^2 e^{V_a/V_T}}{2\tau n_i e^{V_a/2V_T}} = \frac{n_i}{2\tau}e^{V_a/2V_T}}_{R_{max}}$$

Reduction due to recombination When $V_a>0$

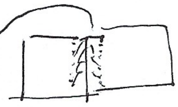
$$\rightarrow J_r \le \frac{q X_d n_i}{2\tau} e^{V_a/2V_T}$$

ideal

$$\boxed{1 \le n \le 2}$$

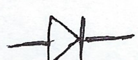
all recombination case

$V_a - I(R_n+R_p) \leftarrow$



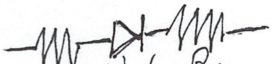
$$I = I_0\left(e^{\frac{V_a-IR}{V_T}}-1\right)$$

include ohmic loss
→ less current



+ $V_A - IR$

V_A



+ R_p ideal diode R_n −

• how much does it decrease When we have ~~...~~ high levels of injection ~~...~~

• ~~grow holes as~~ • limited

→ $p_n \approx n_i e^{V_a/2V_T}$

→ $n = n_i e^{V_a/2V_T}$

• Understanding ~~...~~ the analytical ~~effects~~ equations

2060

The George Washington University
School of Engineering and Applied Science
Department of Electrical and Computer Engineering
ECE 225 – Device Electronics
Spring 2010

MIDTERM EXAMINATION
2.5 Hours
Closed Book and Closed Notes (You may use the attached *Cheat Sheet*)
3/12/2010

1) This problem requires brief but to the point answers. Consider a semiconductor device where the drift-diffusion model accurately describes the operation of the device:
   a) The device can be analyzed in one dimension, and the electrostatic, electron and hole quasi-Fermi potentials, respectively, are given as follows:
   $$\phi(x) = 0.8/[\,1+\exp(x/xd)\,] + 1.6 \text{ Volts}$$
   $$\psi_n(x) = 2 \text{ Volts}$$
   $$\psi_p(x) = 2 \text{ Volts.}$$
   Calculate the resulting electron and hole current densities? Is the device in thermal equilibrium? Explain.
   b) Calculate the built-in electrostatic potential for a *p*-type semiconductor with an ionized acceptor density of $N_A = 2\text{x}10^{16}$ cm^{-3} . Calculate the difference (in electron volts) of the Fermi level and the mid-gap of the semiconductor? Repeat these calculations at 0K.
   c) We learned in class that there are two mechanisms of current transport in semiconductors: drift and diffusion. Explain why diffusion takes place in semiconductors and why it does not take place in conductors.

2) Electrons and holes are injected across an abrupt *pn*-junction with doping densities N_a for x<0 and N_d for x>0. Consider device parameters such that these uniformly doped regions are much longer than Ln, the electron diffusion length and Lp, the hole diffusion length, respectively (the long base diode case). Keeping the n-sided grounded, a bias V_a is applied to the p-side.
   a) Employing the drift-diffusion model show that the excess hole charge density in the "*n-region*" is given by the following expression: $p_n'(x) = p_{n0}(e^{\,qVa/kT} - 1)\, e^{-(x-xn)/Lp}$. Here, the variable definitions are as given in class. The *pn*-junction is at x=0.
   b) Calculate the hole current density and state the assumption you use, if any.
   c) Is this the total current density in the device? If not, state (but do not necessarily derive) an expression for the total current density.

3) In this problem we will consider a *pin*-diode and assume that device characteristics can be accurately calculated by one-dimensional analysis. The doping profile is as follows: $N_a = 10^{16}$ cm^{-3} for $\boldsymbol{x \leq -0.5}$, $N_d = N_a = 0$ for $\boldsymbol{-0.5 \leq x \leq +0.5}$, and $N_d = 3\text{x}10^{16}$ cm^{-3} for $\boldsymbol{0.5 \leq x}$. (the spatial units are in microns).
   a) Plot the net charge density, *ρ(x)*, at thermal equilibrium.
   b) Compute and plot the electric field and electrostatic potential that correspond to this charge density. Clearly indicate the length of the depletion region on each side of the junction.
   c) Determine the depletion capacitance for the *pin* diode as a function of the reverse bias voltage (negative V_a is applied on the *p*-side and the *n*-side is grounded).
   d) Would the peak value of the electric field be lower or higher if the undoped region were to be removed? Based on your answer, explain the purpose of this undoped region.

1)

2010 #1

a) electrostatic potential

$$\phi(x) = \frac{0.8}{\left[1+\exp\left(\frac{x}{x_d}\right)\right]} + 1.6V$$

electron quasi fermi potential

$\psi_n(x) = 2$ volts

hole quasi-fermi potential

$\psi_p(x) = 2$ volts

~~electron carrier concentration~~

→ When device in T.E., current density in context of Drift Diffusion model is:

$$\vec{J}_n = q(D_n \nabla n - \mu_n n \nabla \phi) = 0$$

$$\vec{J}_p = q(D_p \nabla p + \mu_p p \nabla \phi) = 0$$

→ If not in T.E.

$$n = n_i e^{(\phi - \psi_n)/V_T}$$

$$p = n_i e^{-(\phi - \psi_p)/V_T}$$

$$\vec{J}_n = -q\mu_n n \nabla \psi_n$$

$$\vec{J}_p = -q\mu_p p \nabla \psi_p$$

$$\vec{J}_n \sim \Delta \psi_n$$

$$\vec{J}_p \sim \Delta \psi_p$$

∴ ψ_n and ψ_p are constant values, the derivative

$$\Delta \psi_n = 0$$

$$\Delta \psi_p = 0$$

∴ $\vec{J}_n = 0$

$\vec{J}_p = 0$

In T.E $\vec{J}_n = 0$ & $\vec{J}_p = 0$

b) $\phi_B = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right)$

$$\phi_{Built\,p} = -V_T \ln\left(\frac{N_A^-}{n_i}\right)$$

$N = 2\times10^{16}\ cm^{-3}$

$n_i = 10^{10}\ cm^{-3} = 1.45\times10^{10}$ for SI

$$V_T = \frac{kT}{q} = \frac{1.38\times10^{-23}(300)}{1.6\times10^{-19}}$$

$= 0.026\ V$

$$\phi_{Built\,p} = -0.026 \ln\left(\frac{2\times10^{16}}{1.45\times10^{10}}\right)$$

$= 0.36\ V$

difference of fermi level and midgap

$$\phi = \frac{1}{q}(E_f - E_i)$$

$$\phi q = (E_f - E_i)$$

$= 0.36(1.6\times10^{-19})\ J$

e) OK, no ionization

∴ $E_f = E_i$.

intrinsic conditions prevail

c) $\vec{J}$ = ~~[illegible]~~

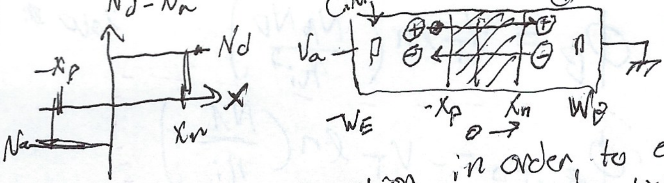
$$\vec{J}_p^{\,diff} = -qD_p \nabla p$$

$$\vec{J}_n^{\,diff} = qD_n \nabla p$$

Diffusion current is due to the gradient of the charge densities (electron & hole). In conductors charge is uniformly distributed, Thus no diffusion current.

d)

a)



assumption in order to evaluate the minority carrier densities at the edge of the quasi-neutral regions $-x_p$ & x_n

T.E.

$\vec{J}_n$ is small

$$\vec{J}_n = q(\mu_n \nabla\phi - D_n \nabla n) \approx 0$$

$V_a < 0$ reverse bias

$V_a > 0$ forward bias

reduces barrier across junction

$$\phi_2 = V_T \ln(n_2)$$

$$\phi_1 = V_T \ln(n_1)$$

$$\phi_2 - \phi_1 = V_T \ln\left(\frac{n_2}{n_1}\right)$$

$$\frac{n_2}{n_1} = e^{\frac{\phi_2 - \phi_1}{V_T}}$$

$$n_2 = n_1 e^{\frac{\phi_2 - \phi_1}{V_T}}$$

Set $x_1 = x_n$
$x_2 = x_p$

$$n(-x_p) = n(x_n) e^{\frac{\phi(-x_p) - \phi(x_n)}{V_T}}$$

$$= N_D e^{\frac{\phi(-x_p) - \phi(x_n)}{V_T}}$$

remember that

$$\phi(-x_p) = -V_T \ln\left(\frac{N_A}{n_i}\right) + V_a \leftarrow \text{p side bias voltage}$$

$$\phi(x_n) = V_T \ln\left(\frac{N_D}{n_i}\right)$$

$$\therefore n(-x_p) = N_D$$

$$= N_D \exp\left[\frac{-V_T \ln\left(\frac{N_A}{n_i}\right) + V_a - V_T \ln\left(\frac{N_D}{n_i}\right)}{V_T}\right]$$

$$= \exp\left[\frac{V_a - \left(V_T \ln\left(\frac{N_a}{n_i}\right) + V_T \ln\left(\frac{N_D}{n_i}\right)\right)}{V_T}\right]$$

remember Built in potential

$$\phi_B = V_T \ln\left(\frac{N_A N_D}{n_i^2}\right)$$

$$\therefore n(-x_p) = N_D \exp\left[\frac{V_a - \phi_B}{V_T}\right]$$

$$= N_D e^{V_a/V_T} e^{-\phi_B/V_T}$$

note that

$$e^{-\phi_B/V_T} = \frac{n_i^2}{N_d N_a}$$

$$\therefore n(-x_p) = \frac{n_i^2}{N_a} e^{V_a/V_T}$$

in the p-region, (in thermal equilibrium and charge neutral region)

$$np = n_i^2, \quad p = N_a \Rightarrow n = \frac{n_i^2}{N_a}$$

recall $n_0(-x_p)$ denotes value at T.E.

$$n_0(-x_p) = \frac{n_i^2}{N_a}$$

$$\therefore n(-x_p) = n_0(-x_p) e^{V_a/V_T}$$

note: minority carrier densities increase at the edge of the charge neutral regions for $V_a > 0$

define: excess carrier due to applied voltage

$$n' \equiv n - n_0$$

$$\therefore n'(-x_p) = n(-x_p) - n_0(-x_p)$$

$$= n_0(-x_p) e^{V_a/V_T} - n_0(-x_p)$$

$$n'(-x_p) = n_0(-x_p)\left(e^{V_a/V_T} - 1\right)$$

↑ Current voltage characteristics (for n)

↓ Ideal-diode analysis

- Consider the excess holes injected into the n-region

use current continuity equation for holes

$$\frac{dp}{dt} = -\frac{1}{q} \nabla \cdot \vec{J}_p - R$$

note steady state implies $\rightarrow \frac{dp}{dt} = 0$

$\vec{J}_p = q(-\mu_p p \nabla\phi - D_p \nabla p)$

~~[illegible]~~ therefor

$$0 = -\frac{1}{q}\nabla\cdot\left(q(-\mu_p p \nabla\phi - D_p \nabla p)\right) - R$$

$$0 = \nabla\cdot(\mu_p p \nabla\phi + D_p \nabla p) - R$$

recall in charge neutral regions

$\vec{E} \approx 0 \quad \therefore \quad \nabla\phi = 0$

$$\therefore \quad 0 = \frac{d}{dx}\left(D_p \frac{dp}{dx}\right) - R$$

recall recombin term R:

$$R = \frac{(np - n_i^2)}{\tau_p(n + n_i) + \tau_n(p + n_i)}$$

recall @ T.E. $np = n_i^2$
so that $R = 0$

Consider small applied voltages and perturbations of the minority carrier density near $x = x_n$

$n = n_0$

$p = p_0 + p'$

recall in n-region $n >> p$ and $n >> n_i$

$$\therefore R \simeq \frac{(np - n_i^2)}{\tau_p(n_0)}$$

$$\simeq \frac{n_0(p_0 + p') - n_i^2}{\tau_p(n_0)}$$

$$= \frac{n_0 p_0 + n_0 p' - n_i^2}{\tau_p n_0}$$

$$R \approx \frac{\cancel{p'}}{\cancel{n_0}} = \frac{p'}{\tau_p}$$

$\therefore$ near $x > x_n$

$$D_p \frac{d^2 p}{dx^2} - \frac{p'}{\tau_p} = 0$$

recall $p = p_0 + p'$ 2do-~~[illegible]~~ 2

for constant doping density

$$\cancel{[illegible]} = \cancel{x}\ 0 = \frac{dp_0}{dx}$$

$$\therefore \quad D_p\left(\frac{d^2 p_0}{dx^2} + \frac{d^2 p'}{dx^2}\right) - \frac{p'}{\tau_p} = 0$$

$$D_p\left(\frac{d^2 p'}{dx^2}\right) - \frac{p'}{\tau_p} = 0$$

$$\frac{d^2 p'}{dx^2} - \cancel{[illegible]}\ \frac{1}{D_p \tau_p} p' = 0$$

Define $L_p = \sqrt{D_p \tau_p}$ = diffusion length of holes in p-type material

$$\therefore \quad \boxed{\frac{d^2 p'}{dx^2} - \frac{1}{L_p^2} p' = 0}$$

general solution for $x \geq x_n$

$$p'(x) = A \exp\left[-\frac{(x - x_n)}{L_p}\right] + B \exp\left[\frac{x - x_n}{L_p}\right]$$

recall long diode conditions

$W_B >> L_p$

B not physical $\therefore$ cause increase

$\therefore A = p'(x_n)$

$$p'(x) = p'(x_n) e^{-(x - x_n)/L_p}$$

recall from current voltage characteristics
(still need to do p)

$$p'(x_n) = p_0(x_n)\left[e^{V_a/V_T} - 1\right]$$

$$\therefore \quad \boxed{p'(x) = p_0(x_n)\left[e^{V_a/V_T} - 1\right] e^{-(x - x_n)/L} \quad \text{for } x \geq x_n}$$

b) $J_p = -qD_p \frac{dp}{dx}$

$$\boxed{J_p(x) = qD_p \frac{p_o(x_n)}{L_p}\left[e^{V_a/V_T}-1\right]e^{-(x-x_n)/L_p} \qquad x \ge x_n}$$

c) No. Total current is the sum of electrons and hole current densities

$$J_{total} = J_p(x_n) + J_n(-x_p)$$

$$= qD_p \frac{p_o(x_n)}{L_p}\left[e^{V_a/V_T}-1\right]e^{-(x_n-x_n)/L_p}$$

$$+ qD_n \frac{n_o(-x_p)}{L_p}\left[e^{V_a/V_T}-1\right]e^{(-x_p+x_p)/L_n}$$

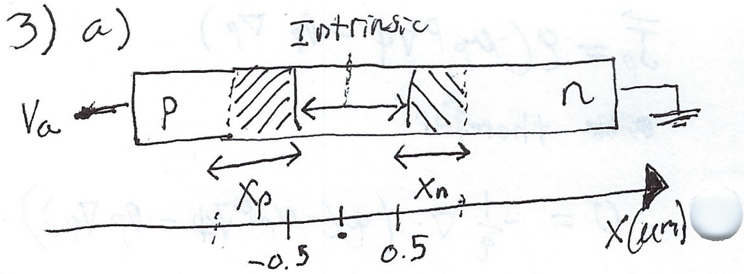
$$= \frac{qD_p}{L_p}\frac{n_i^2}{N_d}\left[e^{V_a/V_T}-1\right]$$

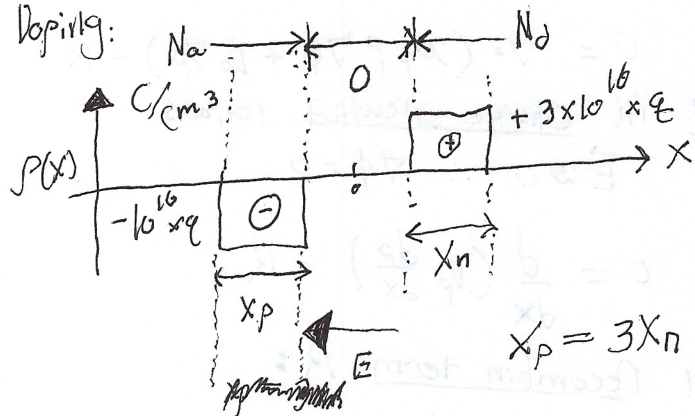
$$+ \frac{qD_N}{L_n}\frac{n_i^2}{N_a}\left[e^{V_a/V_T}-1\right]$$

$\therefore$

$$J_T = qn_i^2\left(\frac{D_p}{L_pN_d}+\frac{D_N}{L_nN_a}\right)\left(e^{V_a/V_T}-1\right)$$

3) a)

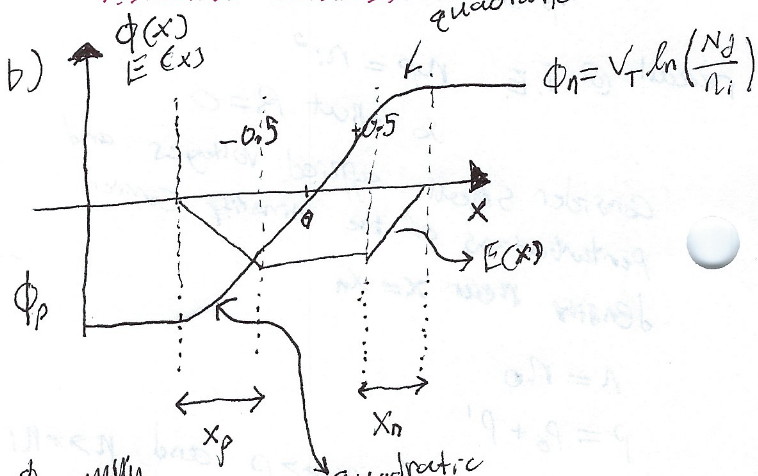




$x_p = 3x_n$

i.e. $\boxed{qN_ax_p = qN_Dx_n}$

b)



$\phi_p = -V_T\ln\left(\frac{N_a}{n_i}\right)$

$\because \frac{d^2\phi}{dx^2} = -\frac{\rho(x)}{\epsilon_s}$

& $\because E(x) = -\frac{d\phi}{dx}$

$\therefore \frac{dE}{dx} = \frac{\rho(x)}{\epsilon_s}$

Assume the p and n region to be non-degenerate.

$$\boxed{V_{bi} = \frac{kT}{q}\ln\left(\frac{N_aN_D}{n_i^2}\right)}$$

$$\rho = \begin{cases} 0 & x < -x_p \\ -qN_A & -x_p \le x \le -0.5 \\ 0 & -0.5 \le x \le 0.5 \\ qN_D & 0.5 \le x \le x_n \\ 0 & x_n < x \end{cases}$$

· Refer poisson equation

$$\frac{d\mathcal{E}}{dx} = \frac{\rho}{K_S \varepsilon_0}$$

$$\frac{d\mathcal{E}}{dx} \begin{cases} -\frac{qN_A}{K_S\varepsilon_0} & -x_p \le x \le -0.5 \\ 0 & -0.5 \le x \le 0.5 \\ \frac{qN_D}{K_S\varepsilon_0} & 0.5 \le x \le x_n \end{cases}$$

Same ranges ⇓

$$\mathcal{E}(x) = \begin{cases} \int_{-x_p}^{x} -\frac{qN_a}{K_S\varepsilon_0} = \frac{-qN_a}{K_S\varepsilon_0}(x+x_p) \\ -\frac{qN_a}{K_S\varepsilon_0}(x_p-0.5) \\ \int_{x}^{x_n} \frac{qN_D}{K_S\varepsilon_0} = -\frac{qN_D}{K_S\varepsilon_0}(x_n - x) \end{cases}$$

to be Continuous

Key Concept:
seperating variables and integrating from the depletion region edges where $\mathcal{E}=0$ to arbitrary points in the n- and the p- regions yield the same relationships and results as in the step junction analysis. In the i-region $\mathcal{E}$ = constant

2010年?

Setting $\mathcal{E}(x) = -\frac{dV}{dx}$, seperate variables, and integrating from the depletion region edges to arbitrary points in the n- and p- regions again yields the same relationships and results in the step junction analysis.

Introduce

$$\mathcal{E}(-0.5) = \mathcal{E}_i$$
$$V(-0.5) = V_i$$

Note that in i-region $\frac{dV}{dx} = -\mathcal{E}_i$

$$\int_{V_i}^{V(x)} dV = -\mathcal{E}_i \int_{-0.5}^{x} dx'$$

$$V(x) = \begin{cases} -\int_{-x_p}^{x} -\frac{qN_A}{K_S\varepsilon_0}(x_p+x) \\ = \frac{qN}{K_S\varepsilon_0}\frac{1}{2}(x_p+x)^2 \\ \vdots \end{cases}$$

c) Determine depletion Capacitance

$$C = \frac{dQ_n}{dV_a} \text{ or } \frac{dQ_p}{dV_a}$$

$$Q_n = qx_n N_d$$
$$Q_p = qx_p N_A$$

d) Peak $\vec{E}$ field would be higher if "i" region removed
↓
- Purpose is to increase break-down (reverse bias) voltage of junction

2008 O

The George Washington University
School of Engineering and Applied Science
Department of Electrical and Computer Engineering
ECE 225 – Device Electronics
Spring 2008

MIDTERM
2.5 Hours
Closed Book and Closed Notes (You may use the attached *Cheat Sheet*)
March 14, 2008

1) Consider the drift-diffusion model.
   a) The current density for electrons and holes is given by the following expression:
   $$\vec{J}_n = -q\mu_n n\nabla\psi_n \text{ and } \vec{J}_p = -q\mu_p p\nabla\psi_p.$$
   Based on this plot the quasi-Fermi potentials across a *pn*-junction under 1 V reverse bias. Clearly mark the junction and the edges of the depletion region.
   b) The depletion region of a *pn*-junction acts like a current source under reverse bias. Explain why this is the case. (You may want to use the results from part (a)).
   c) State the conditions for a metal-semiconductor junction to act like an Ohmic contact. Explain why the difference between the electrostatic and quasi-Fermi potential is independent of the applied bias at the Ohmic contacts.

2) Consider an abrupt junction *pn*-diode under reverse bias. The doping profile is as follows: constant acceptor doping density N_a for $x \leq 0$ and constant donor type doping density N_d for $x \geq 0$. Assume that the device can be analyzed in 1D (*x*-direction).
   a) Compute and plot the built-in electric field and electrostatic potential. Indicate the length of the depletion region on each side.
   b) Determine the depletion capacitance for the *pn* diode as a function of the reverse bias voltage (negative V_a applied on the p-side and ground on n-side).
   c) Describe a simple experiment where one can measure N_d if $N_a >> N_d$.

3) Consider an abrupt *pn*-junction with doping N_a for $x<0$ and N_d for $x>0$. Assume that these uniformly doped regions are much longer than *Ln*, the electron diffusion length and *Lp*, the hole diffusion length, respectively (the so-called long base diode). There is an applied bias V_a to the *p*-side and the *n*-side is reference ground.
   a) Employing the drift-diffusion model show that the excess electron charge density in the "*p*-region" is given by the following expression: $n_p'(x) = n_{p0}(e^{qVa/kT} - 1)\, e^{(x+xp)/Ln}$. Note that the usual variable definitions apply.
   b) Determine the resulting electron current density.
   c) State but do not derive the corresponding equation for the excess holes in the n-side of the junction.

CHEAT SHEET

The following equations may be helpful: Poisson equation:

$$\nabla^2 \phi = \frac{q}{\epsilon}(n - p - D) \,, \quad D = N_d - N_a$$

Current-continuity equations:

$$\frac{\partial n}{\partial t} = \frac{1}{q}\nabla \cdot \vec{J}_n - R$$

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\nabla \cdot \vec{J}_p - R$$

Electron and hole current densities (the drift-diffusion model):

$$\vec{J}_n = q\left(\mu_n n \vec{E} + D_n \nabla n\right)$$

$$\vec{J}_p = q\left(\mu_p p \vec{E} - D_p \nabla p\right)$$

$$\vec{E} = -\nabla \phi$$

Einstein relationship:

$$V_T = \frac{kT}{q} = \frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} \approx 0.026 \text{ V at room temperature}$$

Physical Constants

(in units frequently used in semiconductor electronics)

Electronic charge	q	1.602×10^{-19} C
Speed of light in vacuum	c	2.998×10^{10} cm s^{-1}
Permittivity of vacuum	ϵ_0	8.854×10^{-14} F cm^{-1}
Free electron mass	m_0	9.11×10^{-31} kg
Planck's constant	h	6.625×10^{-34} J s
		4.135×10^{-15} eV s
Boltzmann's constant	k	1.38×10^{-23} J K^{-1}
		8.62×10^{-5} eV K^{-1}
Avogadro's number	A_0	6.022×10^{23} molecules (g mole)$^{-1}$
Thermal voltage	$V_t = kT/q$	
at 80.6°F (300K)		0.025860 V
at 68°F (293K)		0.025256 V

Device Electronics I
Midterm 2008

1) consider drift-diffusion model

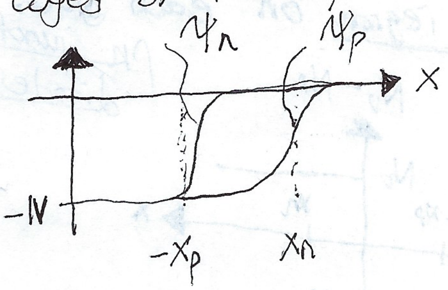
a) current density for electrons
current ~~density~~ for holes
density

$$\vec{J}_n = -q\mu_n n \nabla\psi_n$$

$$\vec{J}_p = -q\mu_p p \nabla\psi_p$$

→ Plot the quasi-Fermi potential across a pn-junction under $\boxed{1\,V}$ reverse bias.

→ Clearly mark the junction and the edges of the depletion region.



q = charge
μ_n = electron mobility
μ_p = hole mobility
n = equilibrium electron concentration
p = equilibrium hole concentration
ψ_n = n-type quasi fermi potential
ψ_p = p-type quasi fermi potential

b) The depletion region of a PN-junction acts like a current source under reverse bias. Explain why this is the case.

b) $R(n,p) \sim (np - n_i^2)$ 2008!

• for several Generation - ~~Recombination~~ Recombination Mechanisms.

• The more general steady state result, valid

$$\left.\frac{dp}{dt}\right|_{\substack{i\text{-thermal}\\ R-G}} = \left.\frac{dn}{dt}\right|_{\substack{i\text{-thermal}\\ R-G}} = \frac{n_i^2 - np}{\tau_p(n+n_1) + \tau_n(p+p_1)}$$

n_i = intrinsic semiconductor concentration under equilibrium condition

τ_p = time constant unit 1/time

$$= \frac{1}{C_p N_T}$$

N_T = number of R-G centers
C_p = positive proportionality constant

$$\tau_n = \frac{1}{C_n N_T}$$

V_T = thermal voltage $= \frac{kT}{q}$

• general relations

$$n = n_i e^{(\phi - \psi_n)/V_T}$$

$$p = n_i e^{-(\phi - \psi_p)/V_T}$$

ϕ = electrostatic potential inside the semiconductor component of an MOS device

$$np = n_i^2 e^{(\psi_p - \psi_n)/V_T}$$

for the applied bias V_a at the p side ⇒

$$np \equiv n_i^2 e^{(\psi_p - \psi_n)/V_T} \cong n_i^2 e^{V_a/V_T}$$

$$\Rightarrow (np - n_i^2) = n_i^2\left(e^{V_a/V_T} - 1\right) = R(n,p)$$

if $V_a < 0$ Regeneration < 0

⇒ results in net generation

c) Ω - Contact conditions are:

State the conditions for a metal - Semiconductor junction to act like an ohmic contact.

Explain why the difference between ~~the difference between~~ the electrostatic and quasi-Fermi potential is independent of the applied biasis at the ohmic contacts.

→ ~~ohmic~~

Ω ohmic conditions are:

Charge neutrality ⇒ $\rho = q(p - n + D) = 0$

q = charge, ρ = charge density, D = doping profile

$$\rho = q(p - n + N_D - N_A)$$

N_D = Donor doping concentration

N_A = acceptor doping concetration

$N_D - N_A$ = net doping concentration (doping profile)

Recombination is zero or Thermal equilibrium

→ $np - n_i^2 = 0$

→ n and p are only functions of doping density at Ω contacts, regardless of applied voltage.

Since, $n = n_i e^{\phi_{bi}/V_T}$

$p = n_i e^{-\phi_{bi}/V_T}$

⟹ $(\phi - \psi_n)$ and $(\phi - \psi_p)$ must be at ϕ_{bi} regardless

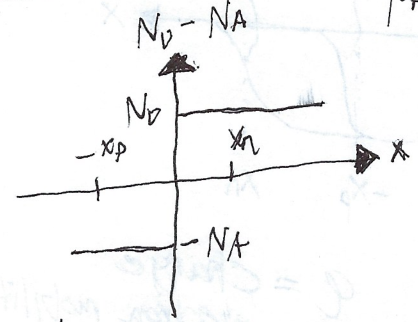
2) Consider an abrupt junction pn-diode under reverse bias. The doping profile is as follows:

- Constant acceptor doping density N_A for $x \le 0$
- Constant donor type doping density N_d for $x \ge 0$

Assume that the device can be analyzed in 1D (x-direction)

a) Compute and plot the built in electric field and electrostatic potential. Indicate the length of the depletion region on each side.

pn junction diode



$\mathcal{E} = -\dfrac{dV}{dx}$ $\quad\mathcal{E}$ = electric field

Integrating across the depletion region gives

$$-\int_{-x_p}^{x_n} \mathcal{E}\, dx = \int_{V(-x_p)}^{V(x_n)} dV$$

$$= V(x_n) - V(-x_p)$$

$$= V_{bi}$$

Note: reverse bias is the direction of little or no current flow

∴ under equilibrium condition

$$J_N = \frac{-D_N}{\mu_n}\frac{dn}{dx}\left(\frac{1}{n}\right) = \frac{-KT}{q}\frac{dn}{dx}\frac{1}{n}$$

q = Charge

μ_n = n type mobility

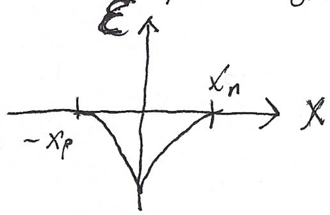
D_n = n type diffusion coefficient

n = electron concentration

→ Solve for <u>electric field</u> & plot

$$J_N = q\mu_n n \mathcal{E} + qD_N \frac{dn}{dx} = 0$$

$$\mathcal{E} = -\frac{D_n}{\mu_n}\frac{1}{n}\frac{dn}{dx}$$



Solve for built in electrostatic potential

$$V_{bi} = -\int_{-x_p}^{x_n} \mathcal{E}\, dx$$

⇒

<u>use einstein relationship</u>

& for electrons $\frac{D_N}{\mu_n} = \frac{KT}{q}$

for holes $\frac{D_P}{\mu_p} = \frac{KT}{q}$

K = boltzmann constant

T = temperature (at 300K)

q = charge of electron

V_T = thermal voltage $= \frac{KT}{q}$

$$V_{bi} = -\int_{x_p}^{x_n} \mathcal{E}\, dx$$

$$= \frac{KT}{q} \int_{x_p\, n(-x_p)}^{n(x_n)} \frac{dn}{dn}$$

$$= \frac{KT}{q} \ln\left[\frac{n(x_n)}{n(-x_p)}\right]$$

• nondegenerately doped step junction where

N_D = n-side doping concentration

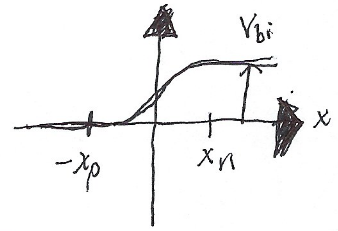
N_A = p-side doping concentration

$$n(x_n) = N_D$$

$$n(-x_p) = \frac{n_i^2}{N_A}$$

$$V_{bi} = \frac{KT}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right)$$

$n_i = 10^{10}$ intrinsic carrier concentration



Coming from chapter 7.

(might not be on midterm)

b) Determine the depletion layer capacitance for the pn diode as a function of the reverse bias voltage (negative V_a applied on the p-side and ground on the n-side)

($\frac{1}{C_d^2}$ vs. V_a plot)

p. 301

C) small signal junction capacitance can be measured, and, from above result, relate that to N_d ~~[illegible]~~

3). Consider an abrupt pn-junction with doping N_a for ~~[illegible]~~ $x < 0$ and N_d for $x > 0$

- Assume that these uniformly doped regions are much longer than

→ L_n, the electron diffusion length

→ L_p, the hole diffusion length

respectively (the so-called long base diode)

There is an applied bias V_a to the p-side and the n-side is reference ground.

a) Employing the drift-diffusion model show that the excess electron charge density in the "p-region" is given by the following expression : ~~[illegible]~~

$$n_p'(x) = n_{po}(e^{qV_a/kT} - 1)e^{(x+x_p)/L_n}$$

- Note that the usual variable definitions apply.

→ the creation (or appearance) of an excess of minority carriers along a given plane in a semiconductor, the subsequent diffusion of the excess carrier concentration characterized by a decay length (L_p) — occurs often enough in semiconductor analysis

$L_p \equiv \sqrt{D_p \tau_p}$ associated with the minority carrier holes in the n-type material.

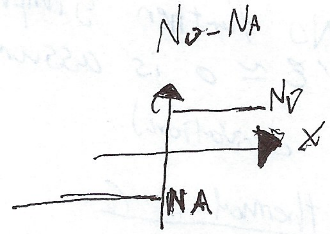
$L_N \equiv \sqrt{D_N \tau_n}$ associated with the minority carrier electrons in a p-type material.

→ referred to as minority carrier diffusion lengths

$\therefore$

$$L_p \equiv \sqrt{D_p \tau_p} = \sqrt{\left(\frac{kT}{q}\right)\mu_p \tau_p}$$

Quasi-Fermi Levels



Ideal diode analysis

on the p-side

- What is happening minority carrier (electrons)

$$\frac{dn}{dt} = ~~[illegible]~~$$

$$= \frac{1}{q}\nabla \cdot \vec{J}_N + \frac{dn}{dt}\bigg|_{\substack{\text{Thermal}\\ \text{R-G}}} + \frac{dn}{dt}\bigg|_{\substack{\text{other}\\ \text{process}}}$$

$$\vec{J}_N = q\mu_n n \mathcal{E} + qD_N \nabla n$$

- ~~assume steady state : $\frac{dn}{dt} = 0$~~

- ~~assume charge neutral~~

$\therefore$

Common Diffusion equation ~~[illegible]~~

Simplifications minorities

~~Steady State~~ $\frac{d\Delta n_p}{dt}$ change with respect to time, to the change of the electron concentration in a p-type material

next page

Steady State: electrons in $\leftarrow$ p-type | holes in n-type

$$\frac{d\Delta n_p}{dt} \to 0 \qquad \frac{d\Delta p_n}{dt} \to 0$$

No concentration gradient or no diffusion current

$$D_N \frac{d^2 \Delta n_p}{dx^2} \to 0$$

$$D_p \frac{d^2 \Delta p_n}{dx^2} \to 0$$

No drift current or

$\mathcal{E} = 0$

No further simplification ($\mathcal{E} \simeq 0$ is assumed in the derivation)

No thermal R-G

$$\frac{\Delta n_p}{\tau_n} \to 0 \quad \left(\frac{\Delta p_n}{\tau_p} \to 0\right)$$

No light

$G_L \to 0$

---

assumptions

Steady state : $\frac{d}{dt} = 0$

charge neutral : $\vec{E} = 0$

$\nabla \phi = 0$

1D: for $x < 0$

Minority carrier diffusion equations

$$\cancel{\frac{d\Delta p_n}{dt} = D_p \frac{d^2 \Delta p_n}{dx^2} - \frac{\Delta p_n}{\tau_p} + G_L}$$

electron concentration in p-type material

$$\cancel{\frac{d\Delta p_n}{dt} = D_p \frac{d^2 \Delta p_n}{dx^2} - \frac{\Delta p_n}{\tau_p} + G_L}$$

$$\frac{d\Delta n_p}{dt} = D_N \frac{d^2 \Delta n_p}{dx^2} - \frac{\Delta n_p}{\tau_n} + G_L$$

in 1D with no illumination

$$= D_N \frac{d^2 \Delta n_p}{dx^2} - \frac{\Delta n_p}{\tau_n}$$

$$\left.\frac{dn}{dt}\right|_{\text{i-thermal R-G}} = -\frac{\Delta n_p}{\tau_n}$$

$$= \frac{n_i^2 - np}{\tau_p(n+n_1) + \tau_n(p+p_1)}$$

- not in thermal equilibrium
- on p-region

$p = N_A \to p_0$

$n = n_0 + n'$

$$n_0 p_0 = N_A \frac{n_i^2}{N_A} = n_i^2$$

$$\therefore -\frac{\Delta n_p}{\tau_n} = \frac{n_i^2 - p_0(n_0 + n')}{\tau_n p_0}$$

in p region only majority majority carrier

$$-\frac{\Delta n_p}{\tau_n} = \frac{p_0 n'}{\tau_n p_0} = \frac{n'}{\tau_n}$$

$$D_n \frac{d^2 n'}{dx^2} - \frac{q n'}{q \tau_n} = 0$$

$$\frac{d^2 n'}{dx^2} - \frac{n'}{D_N \tau_N} = 0$$

$$L_n \equiv \sqrt{D_N \tau_n}$$

$$\frac{d^2 n'}{dx^2} - \frac{1}{L_n^2} n' = 0$$

General Solution of differential equation

$$n'(x) = A \exp\left[\frac{-(x - x_p)}{L_n}\right] + B \exp\left[\frac{(x - x_p)}{L_n}\right]$$

$A \rightarrow$ non-physical length Solution

$$n'(x) = n_0\left[e^{V_A/V_T} - 1\right] e\left[\frac{(x-(-x_p))}{L_p}\right]$$

$$n'(x) = \frac{n_i^2}{N_A}\left[e^{V_A/V_T} - 1\right] \exp\left[\frac{(x+x_p)}{L_n}\right]$$

$$n'(x) = \frac{n_i^2}{N_A}\left[e^{V_A/V_T} - 1\right] \exp\left[\frac{(x+x_p)}{L_n}\right]$$

$L_n = \sqrt{D_n \tau_n}$

$x \leq -x_p$

2008 4

b) Same process for electron ~~current~~ density

- electrons diffuse into the p-region

$$J_n = q D_n \frac{dn}{dx} = q D_n \frac{dn'}{dx}$$

$$J_n(x) = q D_n \frac{n_i^2}{N_A}\left(\frac{1}{L_n}\right)\left[e^{V_A/V_T} - 1\right] \exp\left[\frac{(x+x_p)}{L_n}\right]$$

c) excess holes in the n-side of the junction

$$p'(x) = \frac{n_i^2}{N_D}\left[e^{V_A/V_T} - 1\right] e\left[\frac{-(x - x_n)}{L_p}\right]$$

Department of Electrical and Computer Engineering
ECE 6030 – Device Electronics

Spring 2018 - MIDTERM EXAMINATION
2.5 Hours - Closed Book and Closed Notes
(You may use the attached *Cheat Sheet*)
3/9/2018

1) A *pn*-diode has uniform doping densities where in the *p*-region $\mathbf{N_a} = 10^{15}$ cm^{-3} and in the *n*-region is $\mathbf{N_d} = 10^{16}$ cm^{-3}.
   a) Plot the net charge density, $\rho(x)$, at thermal equilibrium employing the depletion region approximation.
   b) For the above result, compute and plot the electric field, the electrostatic potential and the energy-band diagram that correspond to this charge density.
   c) Compute and clearly indicate the length of the depletion region on each side of the junction.
   d) A negative DC-bias V_a is applied on the *p*-side and the *n*-side is grounded. Determine the depletion capacitance for this diode as a function of the reverse bias voltage. Explain how the resulting capacitance vs. DC bias plot can be used to determine the built-in potential from experimental small-signal capacitance measurements.

2) The answers to this problem are expected to be brief but to the point. Consider a semiconductor device where the drift-diffusion model is employed to describe the operation:
   a) Let us assume that the device can be accurately described where the electron and hole charge densities are related to the electrostatic potential by the following expressions: $n=n_i \exp(\phi/v_T)$ and $p=n_i \exp(-\phi/v_T)$, respectively, and that the electrostatic potential in the device is computed to be:

   $$\phi(x) = 0.35[\exp(x/x_0) - \exp(-x/x_0)]/[\exp(x/x_0) + \exp(-x/x_0)] \text{ Volts}, \; -4x_0 \le x \le 4x_0$$

   Is there a location in the device that has intrinsic charge density conditions? If so, where? Calculate the resulting electron and hole current densities. Briefly explain your answer.
   b) We normally consider the ionized doping density to be equal to the total doping density. State one physical condition where this assumption would *not* be valid and briefly justify why.
   c) The current density for electrons and holes is given by the following expression:

   $$\vec{J}_n = -q\mu_n n \nabla \psi_n \text{ and } \vec{J}_p = -q\mu_p p \nabla \psi_p.$$

   Based on what you know of charge and current densities across a *pn*-junction, plot the quasi-Fermi potentials across a *pn*-junction under 3 V reverse bias. Clearly mark the junction, the Ohmic contacts and the edges of the depletion region. Based on this result, where would you expect charge generation to take place?

3) Under some DC applied DC bias V_A on the p-side of an abrupt *pn*-junction electrons and holes are injected across the depletion region. (The n- side is grounded). Consider doping densities $\mathbf{N_a}$ for x<0 and $\mathbf{N_d}$ for x>0, and device parameters such that the length of the uniformly doped *n*-region, ***Wn***, is much shorter than ***Lp***, the so-called hole diffusion length, and that the length of the uniformly doped *p*-region, ***Wp***, is much longer than ***Ln***, the electron diffusion length, respectively (the short- and long-diode cases, respectively).
   a) Based on the drift-diffusion model show that the excess electron charge density in the p-region is given by the expression: $n_p'(x) = n_{p0}(e^{qVa/kT} - 1)\, e^{(x+xp)/Ln}$. Note that the variables are as defined in the class, and the *pn*-junction is at *x*=0.
   b) Employing the drift-diffusion model and (an appropriate modification) of the previous result, derive an expression for the excess hole charge density in the n-region.
   c) Derive an expression for the total current density in the device.

CHEAT SHEET

The following equations may be helpful: Poisson equation:

$$\nabla^2 \phi = \frac{q}{\epsilon}(n - p - D) \ , \quad D = N_d - N_a$$

Current-continuity equations:

$$\frac{\partial n}{\partial t} = \frac{1}{q} \nabla \cdot \vec{J}_n - R$$

$$\frac{\partial p}{\partial t} = -\frac{1}{q} \nabla \cdot \vec{J}_p - R$$

Electron and hole current densities (the drift-diffusion model):

$$\vec{J}_n = q\left(\mu_n n \vec{E} + D_n \nabla n\right)$$

$$\vec{J}_p = q\left(\mu_p p \vec{E} - D_p \nabla p\right)$$

$$\vec{E} = -\nabla \phi$$

Einstein relationship:

$$V_T = \frac{kT}{q} = \frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} \approx 0.026 \text{ V at room temperature}$$

Physical Constants

(in units frequently used in semiconductor electronics)

Electronic charge	q	1.602×10^{-19} C
Speed of light in vacuum	c	2.998×10^{10} cm s^{-1}
Permittivity of vacuum	ϵ_0	8.854×10^{-14} F cm^{-1}
Free electron mass	m_0	9.11×10^{-31} kg
Planck's constant	h	6.625×10^{-34} J s
		4.135×10^{-15} eV s
Boltzmann's constant	k	1.38×10^{-23} J K^{-1}
		8.62×10^{-5} eV K^{-1}
Avogadro's number	A_0	6.022×10^{23} molecules (g mole)$^{-1}$
Thermal voltage	$V_t = kT/q$	
at 80.6°F (300K)		0.025860 V
at 68°F (293K)		0.025256 V

Device electronics

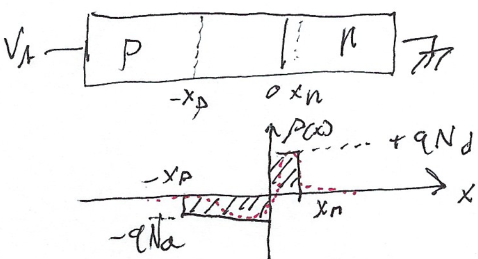
May ~~4th~~ 14th Final Exam

Midterm review

1a) pn-diode

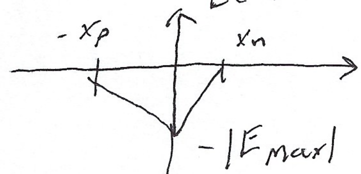
$N_a = 10^{15}\ cm^{-3}$

$N_d = 10^{16}\ cm^{-3}$



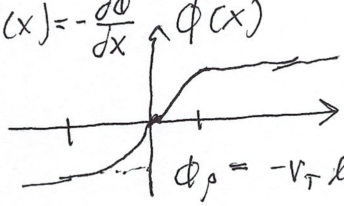
... charge density

b)



$$\frac{dE}{dx} = \frac{\rho}{\epsilon}$$

c) $E(x) = -\frac{d\phi}{dx}$



$\phi_n = V_T \ln\left(\frac{N_d}{n_i}\right)$

$\phi_p = -V_T \ln\left(\frac{N_a}{n_i}\right)$

$$\therefore \frac{d^2\phi}{dx^2} = \frac{-\rho}{\epsilon}$$

$\rho = q(p - n + D)$

$D = N_d^+ - N_a^-$

$x_n, x_p \rightarrow E(0^-) = E(0^+)$

$\phi(0^-) = \phi(0^+)$

d) $C = \frac{dQ}{dV_a}$

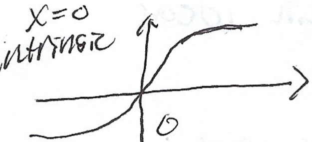
Midterm review

2) a) $n = n_i e^{\phi/V_T}$

$p = n_i e^{-\phi/V_T}$

~~$\phi(x)$ zero~~

$$\phi(x) = 0.35\,\frac{e^{x/x_0} - e^{-x/x_0}}{e^{x/x_0} + e^{-x/x_0}}$$



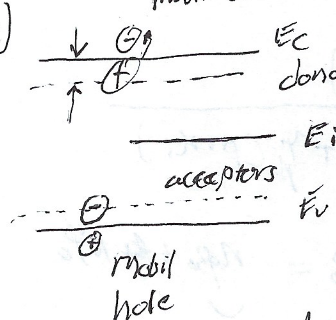
$\phi(0) = 0$

$np = n_i^2$

$\Rightarrow J_n = 0$

$J_p = 0$

b)



$N_d^+ < N_d$

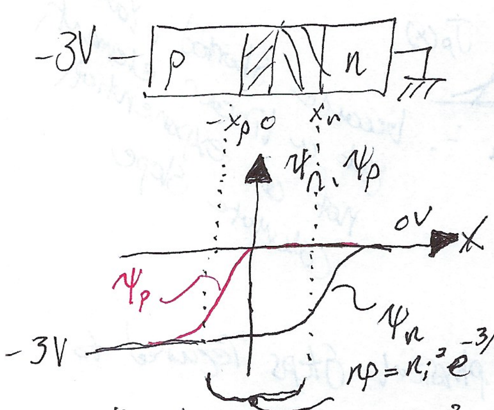
$N_a^- = N_a$

As if the material has not been doped, ~~low~~ low temperature

c) $J_n = -q\mu_n n \nabla \psi_n$

$J_p = -q\mu_p p \nabla \psi_p$

derivatives



$np = n_i^2 e^{-3/V_T}$

$n = n_i e^{(\phi - \psi_n)/V_T}$

$p = p_i e^{-(\phi - \psi_p)/V_T}$

$np = n_i^2 e^{\frac{(\psi_p - \psi_n)}{V_T}}$

$\psi_n(x) =$

$\psi_p(x) =$

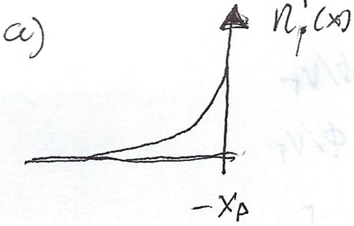
$np \ll n_i^2$

$R < 0$

$\Rightarrow$ generation

$R \sim (np - n_i^2)$

3a)



$$n_p'(x) = n_{p0}\left(e^{V_a/V_T} - 1\right)e^{(x+x_p)/L_n}$$

$$V_T = \frac{kT}{q}$$

main ideas

1) $J_{drift} = 0$

only diffusion current

$$\Rightarrow \frac{d^2n}{dx^2}$$

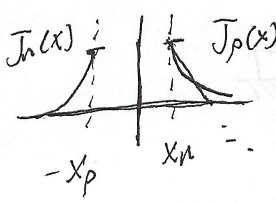
2) $$R = \frac{(np - n_i^2)}{\tau_p(n + n_i) + \tau_n(p + n_i)}$$

$$np = (n_0 + n')p_0 = \underbrace{n_0 p_0}_{n_i^2} + n' p_0$$

$$R = \frac{n' p_0}{\tau_n p_0} = \frac{n'}{\tau_n}$$

b) Basic requirements

c)



$\therefore$ because shorted, you get a linear element, not an exponential, can calculate slope

highlight key physical steps required to get to the answer

2018-03-23

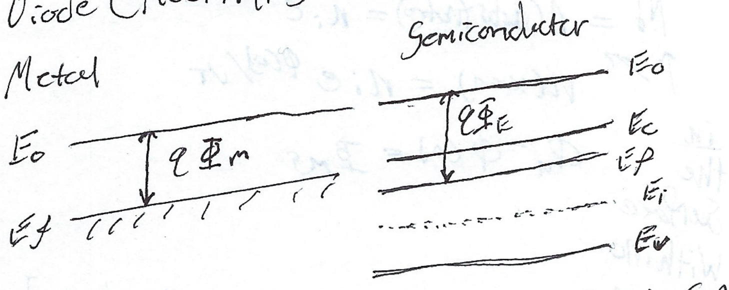
Week 10 Metal Semiconductor Junction

- metal semiconductor junctions
  - diode
  - ohmic contacts
- FET's
  - Junction FETS
  - MOS Capacitor
  - MOSFET
- BJT's
- additional topics

Metal Semiconductor Junction

- usually you get a diode
- more careful, get an ohmic contact

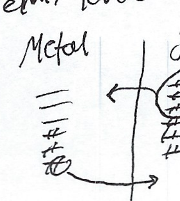
Diode (Rectifying Contact)



n-type Semi-Con

$\Phi_m \neq \Phi_s$

Fermi levels being equal $\Rightarrow$ T.E. Condition



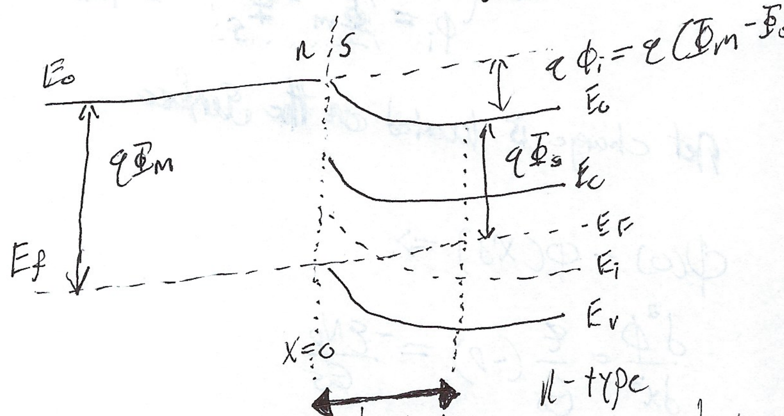
fermi dirac statistics

$$f_{FD} = \frac{1}{1 + e^{\frac{E - E_f}{KT}}}$$

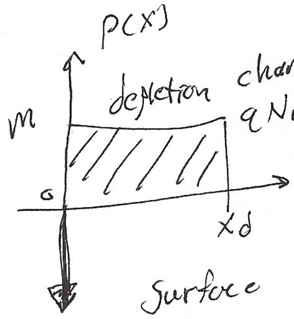
electrons in the semiconductor are in more energetic side transfered to less energetic side

- at some point the two fermi levels are going to ~~balance~~ out & we will have balance

amount of band bending



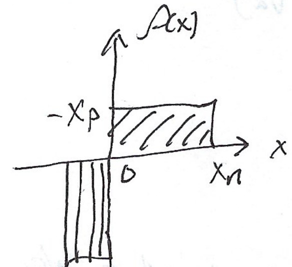
n-type semiconductor



$\underbrace{-qN_Dx_d\,\delta(x)}$

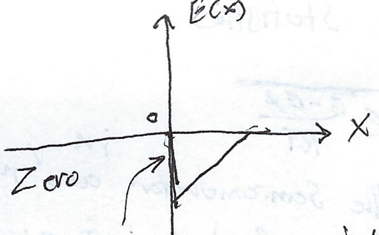
charge deposited on the metal surface

delta: dirac delta function

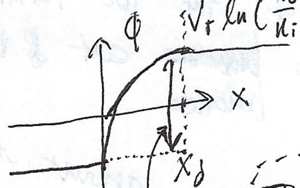


Very narrow, idealized as a dirac delta function

band bending required to make two fermi levels equal ~~to one~~ to one another.



- integrate a delta function & You get a step



$E = \frac{-d\phi}{dx}$

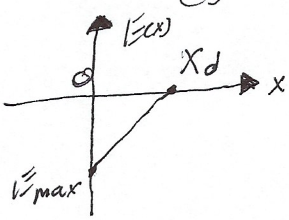
$\phi_i = \boxed{\Phi_m - \Phi_s} = \Phi_{ms}$

Net charge is located on the Surface

$\phi(0) - \phi(X_d) \Rightarrow$

$\frac{d^2\phi}{dx^2} = \frac{q}{\epsilon_s}(-D) = \frac{-qN_D}{\epsilon_s}$

$E_{max} = -\frac{qN_D}{\epsilon_s} X_d$



$\Rightarrow X_d = \sqrt{\frac{2\epsilon_s}{qN_D}\Phi_{ms}}$

apply V_A on the metal side

$X_d = \sqrt{\frac{2\epsilon_s}{qN_D}(\Phi_{ms} - V_a)}$

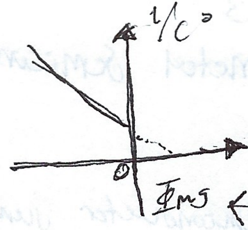
Charge:

$Q_s = qN_d X_d A$

Surface of metal

$C = \frac{dQ_s}{dV_a}$ • Similar to what we did with the pn-junction

Similar to pn-junction



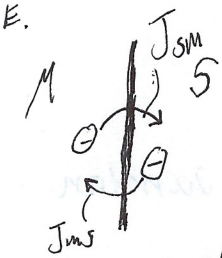
$\frac{d(1/C^2)}{dV_A} = \frac{-2}{q\epsilon_s N_d}$

difference of fermi level on two sides of the junction

Tendency of electrons going
- on reverse bias You hardly get any current

Current:

T.E.



- in Thermal Equilibrium

Two are equal $\Rightarrow$ net current is zero once initial transfer takes place transfer is equal

proportional to $\sim n_s = n(0)$

$\boxed{n_s = N_d \cdot e^{-\Phi_{ms}/V_T}}$ Surface electron density

$n = n_i e^{\phi/V_T}$

$N_d = n(\text{Substrate}) = n_i e^{\phi_n/V_T}$

$n(x=0) = n_i e^{\phi(0)/V_T}$

in the Surface Within no depletion

$\phi_n - \phi(0) = \Phi_{ms}$

- estimate of charge going back and forth during Thermal equilibrium

At T.E.

$|J_{ms}| = |J_{sm}| = K n_s = K N_d e^{-\Phi_{ms}/V_T}$

apply $V_A \leftrightarrow$ T.E.

J_{sm} — does <u>not</u> change from T.E.

J_{ms} = changes

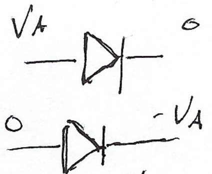
electrons going from S to M:

n_S

$n = n_i e^{\phi/V_T}$: T.E.

$n = n_i e^{(\phi - \psi_n)/V_T}$: Not T.E.

$= n_i e^{\phi/V_T} (e^{-\psi_n/V_T})$



$\psi_n = -V_A$

$\vec{J}$

$n_S = \begin{pmatrix}\text{T.E.}\\ \text{value}\end{pmatrix} e^{-\frac{\psi_n}{V_T}} \leftarrow \psi_n = -V_a$

$= \begin{pmatrix}\text{T.E.}\\ \text{value}\end{pmatrix} e^{V_a/V_T}$

increases as e^{V_A/V_T}

invariant with resp to V_A

$J = J_{ms} - J_{sm}$

$= K(N_d e^{-\Phi_{ms}/V_T}) e^{V_a/V_T} -$

$K(N_d e^{-\Phi_{ms}/V_T})$

$\therefore$

$J = J_o(e^{V_a/V_T} - 1)$

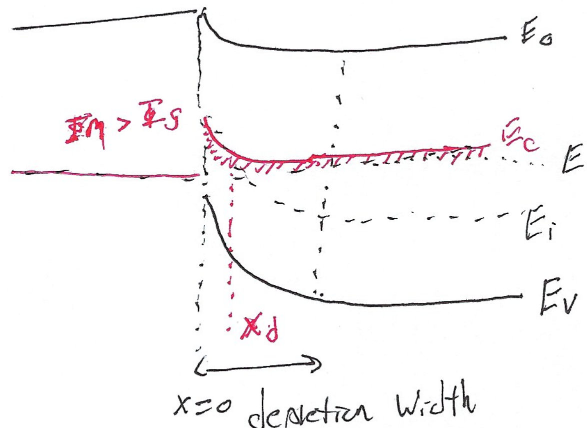
$J_o = K N_d e^{-\Phi_{ms}/V_T}$

ideal diode equation for MS-junction

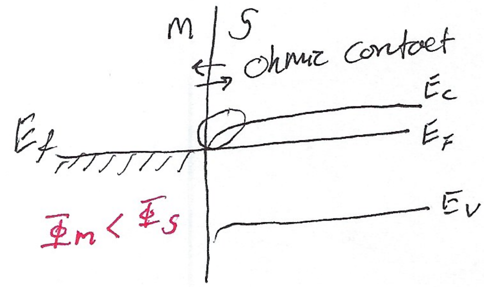
$J = J_{ms} - J_{sm}$

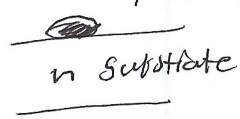
increase N_d: What happens

E_c will get



tunnelling ohmic contact, free transfer of electrons from metal to semiconductor





metal whose work function is less than semiconductor work function (ohmic contact)

2018-03-23

Week 10 Metal Semiconductor junction

- metal Semiconductor junctions
  - diode
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  - MOS Capacitor
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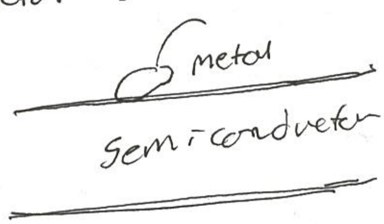
- additional topics

Metal Semiconductor junction



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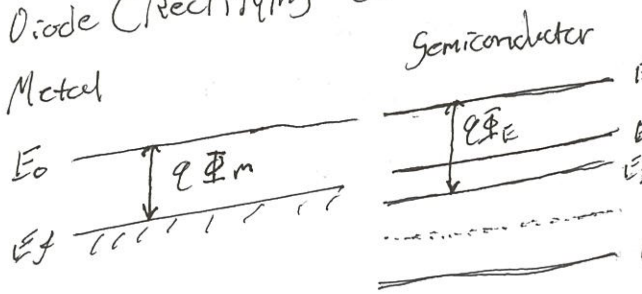
Diode (Rectifying Contact)



$\Phi_m \neq \Phi_s$ n type Semi-con

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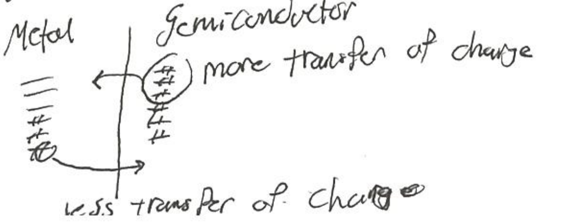

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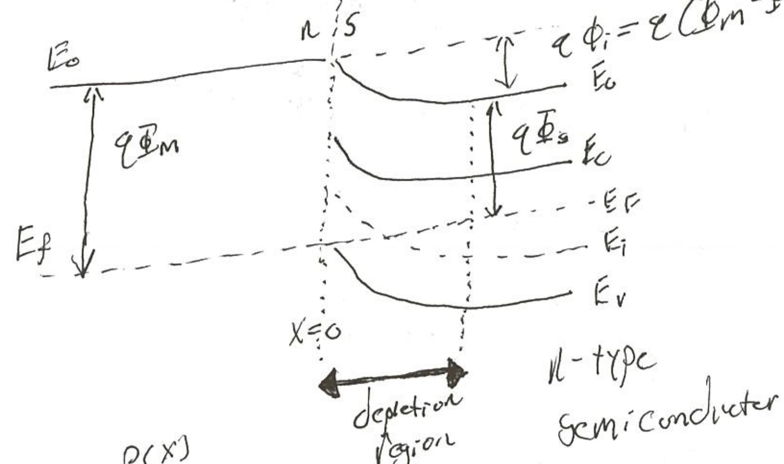
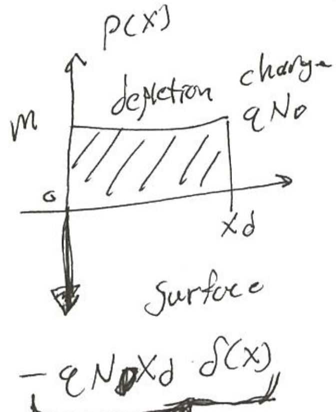
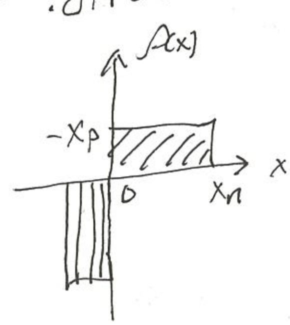




charge deposited on the metal surface

~~delta~~: dirac delta function



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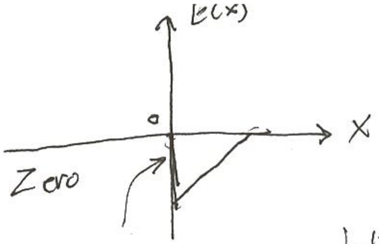

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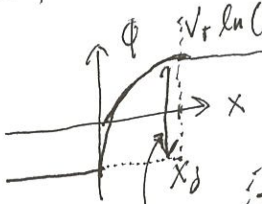

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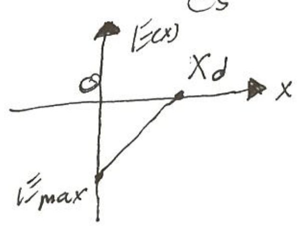

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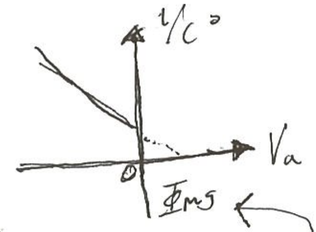
Surface of metal

$C = \frac{dQ_s}{dV_a}$ • Similar to what We did with the pn-junction

Similar to pn-junction



$$\frac{d(1/C^2)}{dV_A} = \frac{-2}{q\epsilon_s N_d}$$

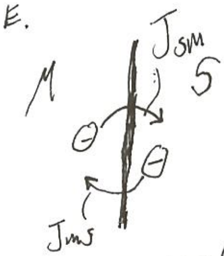
difference of fermi level on two sides of the junction

Tendency of electrons going

- on reverse bias You hardly get any current

Current:

T.E. - in Thermal Equilibrium



Two are equal $\Rightarrow$ net current is Zero

once initial transfer takes place transfer is equal

proportional to $\sim n_s = n(0)$

$$\boxed{n_s = N_d \cdot e^{-\Phi_{ms}/V_T}}$$

Surface electron density

$n = n_i e^{\phi/V_T}$

$N_d = n(\text{substrate}) = n_i e^{\phi_n/V_T}$

$n(x=0) = n_i e^{\phi(0)/V_T}$

in the Surface Within no depletion

$\phi_n - \phi(0) = \Phi_{ms}$

- estimate of charge going back and forth during Thermal equilibrium

At T.E.

$|J_{ms}| = |J_{sm}| = K n_s = K N_d e^{-\Phi_{ms}/V_T}$ → increase N_d : What happens

E_c will get

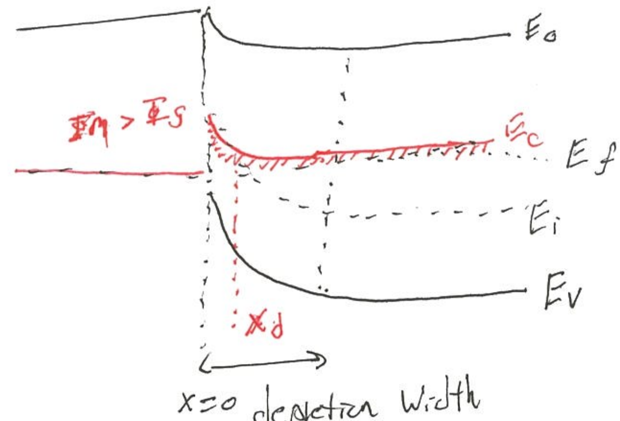
apply $V_A \leftrightarrow$ $\cancel{T.E.}$

J_{sm} — does not change from T.E.

J_{ms} = changes

electrons going from S to M :

n_s

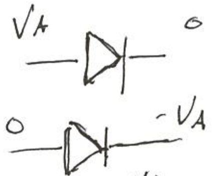


$n = n_i e^{\phi/V_T}$: T.E.

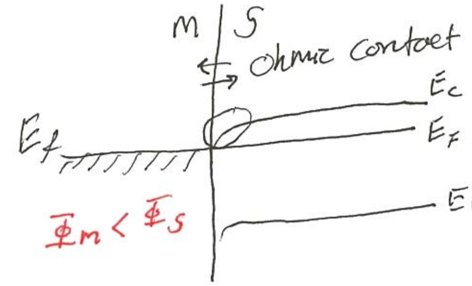
$n = n_i e^{(\phi - \psi_n)/V_T}$: Not T.E.

$= n_i e^{\phi/V_T} (e^{-\psi_n/V_T})$

tunnelling ohmic contact, free transfer of electrons from metal to semiconductor

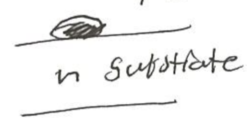


$\overrightarrow{J}$ $\psi_n = -V_A$



$n_s = \begin{pmatrix} T.E. \\ \text{value} \end{pmatrix} e^{-\frac{\psi_n}{V_T}}$ ← $\psi_n = -V_a$

$= \begin{pmatrix} T.E. \\ \text{value} \end{pmatrix} e^{V_a/V_T}$



Metal whose work function is less then semiconductor work function (ohmic contact)

increases as e^{V_A/V_T}

invariant with resp to V_A

$J = J_{ms} - J_{sm}$

$= K(N_d e^{-\Phi_{ms}/V_T}) e^{V_a/V_T} - K(N_d e^{-\Phi_{ms}/V_T})$

∴

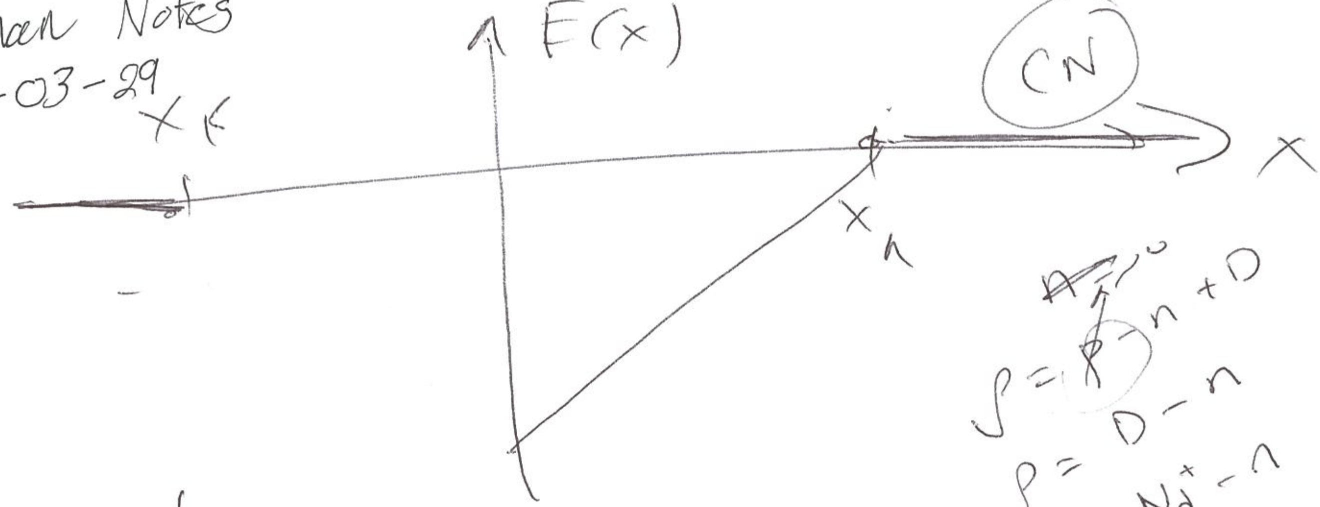
$J = J_0 (e^{V_a/V_T} - 1)$

$J_0 = K N_d e^{-\Phi_{ms}/V_T}$

ideal diode equation for MS-junction

$J = J_{ms} - J_{sm}$

Korman Notes
2018-03-29



$\rho = p - n + D$

$\rho = D - n$ ($N_a =$

$\rho = N_d^+ - n$

$\rho =$

$$\frac{d\phi}{dx} = -\frac{qN_d}{\epsilon_s}\cancel{x} + C_1$$

$D = N_d^+ - N_a^-$

$n = N_D^+ = n_i e^{\phi/V_T}$

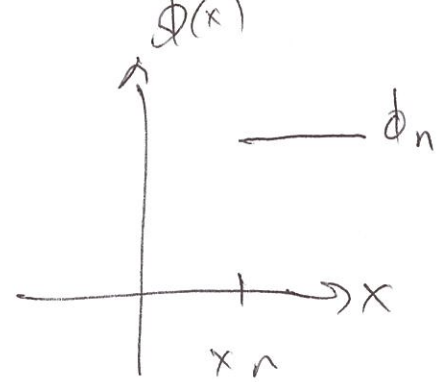
$\frac{N_D}{n_i} = e^{\phi/V_T}$

$V_T \ln\left(\frac{N_D}{n_i}\right)$

$$E = -\frac{d\phi}{dx} = \frac{qN_d}{\epsilon_s}x - C_1$$

$$E(x=x_n) = 0 = \frac{qN_d}{\epsilon_s}x_n - C_1 = 0$$

$$C_1 = \frac{qN_d}{\epsilon_s}x_n$$



$$E(x) = \frac{qN_d}{\epsilon_s}(x - x_n)$$

$$\phi(x) = -\frac{qN_d}{2\epsilon_s}(x - x_n)^2 + C_2$$

$$\phi(x=x_n) = V_T \ln\left(\frac{N_d}{n_i}\right) = \phi_n = C_2$$

$$J_n = q\left(\mu_n n \nabla\phi + D_n \nabla n\right) = 0$$

$$D_n \nabla n = -\mu_n n \nabla \phi$$

$$\frac{\nabla n}{n} = -\underbrace{\frac{\mu_n}{D_n}}_{\frac{1}{V_T}} \nabla \phi$$

$$\frac{\nabla n}{n} = -\nabla\left(\frac{\phi}{V_T}\right)$$

$$\nabla(\ln n) = -\nabla\left(\frac{\phi}{V_T}\right)$$

$$\ln n = -\frac{\phi}{V_T} + C$$

$$e^{\ln n}$$

$$n = C_2 e^{-\phi/V_T}$$

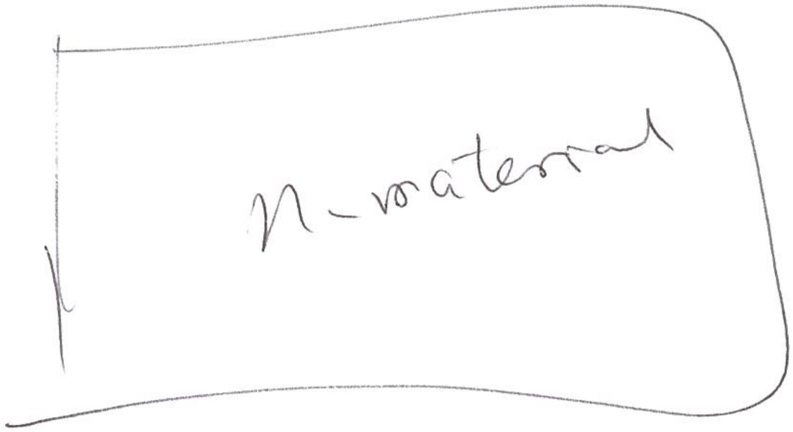
$\cancel{np = n_i^2}$

$$p = \frac{n_i^2}{n} = \frac{n_i^2}{N_d} \ll N_d \approx n$$

$$\phi = 0,\quad n = n_i \Rightarrow C_2 = n_i$$

$$n = n_i e^{\phi/V_T},\quad p = n_i e^{-\phi/V_T}$$

$$\Rightarrow np = n_i^2$$



N_D^+

$np = n_i^2$

$p = \frac{n_i^2}{n} = \frac{10^{20}}{10^{15}} = 10^5$

$p \sim 10^5$

$n \sim 10^{15}$ $\quad np = 10^{20} = n_i^2$

CN: $\rho = q(p - n + D) = 0$ in.

T.E. $\quad np = n_i^2$ T.E.

$p = \frac{n_i^2}{n}$

$\frac{n_i^2}{n} - n + D = 0$

$n_i^2 - n^2 + Dn = 0$

$n^2 - Dn - n_i^2 = 0$

$n = \frac{D \pm \sqrt{D^2 + 4n_i^2}}{2}$

$n =$

$= \frac{D}{2}$

HomeWork 3
Read Chapter
Pierret 6, 7, 8, 9, & 14

problems
Ch 6. 7, 8, 23
Ch 7. 2, 5
Ch 8. 2
Ch 9. Read only
Ch 14. 2, 3, 6, 7

Due: 3pm April 6th

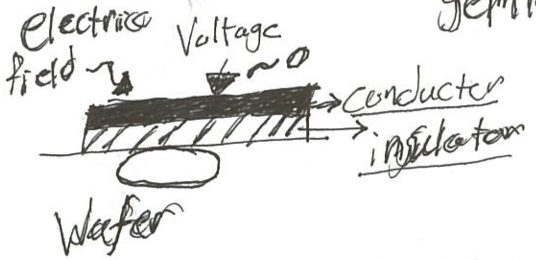
Joseph Crandall

2018-03-30

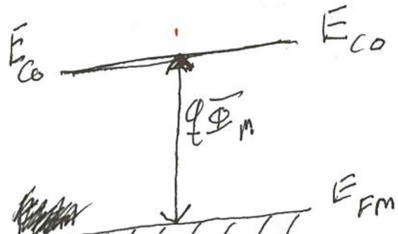
Week 11

- Device electronics
- Metal - Oxide - Silicon (MOS)

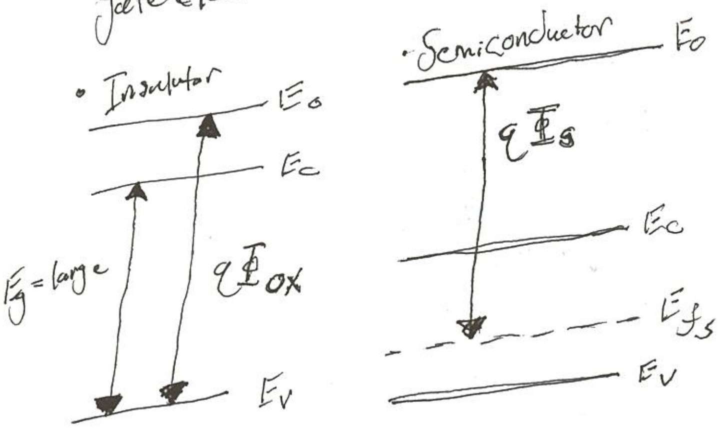
Conductor - Insulator - Semiconductor



MOS has applications outside of MOSFET



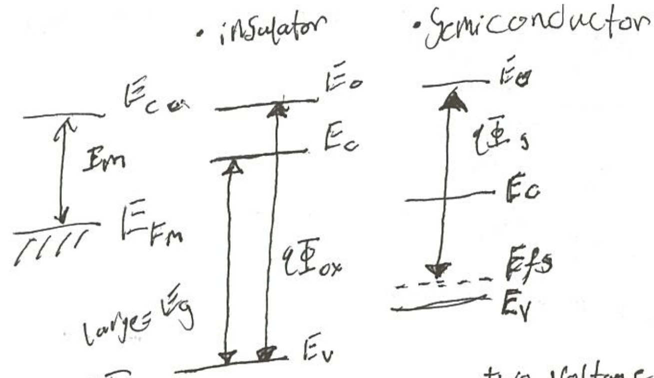
"metal" conducting gate electrode

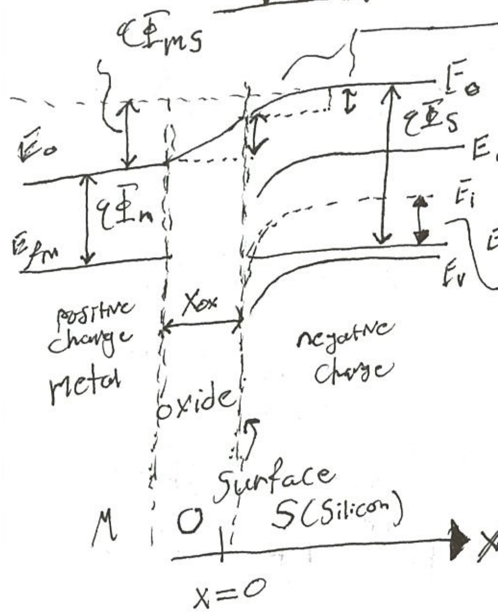


Example Schematic:

$\Phi_m < \Phi_s$

relationship of work functions





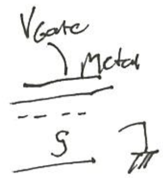
two voltage drops qV_{ox} (in oxide) & qV_s (Band bending)

$q\phi_p$

$\phi_p = -V_T \ln\left(\frac{N_a}{n_i}\right)$

p-type

$\Phi_{ms} = \Phi_m - \Phi_s$



need to apply voltage

$V_G = \Phi_{ms}$

$V_{FB}^o = \Phi_{ms}$

Flat Band potential

→ difference between the metal and semiconductor work function

- top level of Silicon near the oxide is negatively charged

→ only control is the voltage we apply between the substrate & the gate how does the gate voltage control the semiconductor

$\boxed{\text{Q - midterm 1d} \quad \text{- old final exams for practice}}$

~~flat band potential~~

$V_G - V_{FB}$

0

accumulation

if you apply more potential bands will bend up.

Surface subscripts indicate quant @ $x = 0$

Onset of Strong Inversion

Intrinsic

Strong Inversion

accumulation | Flat Band | Depletion | Weak Inversion

$V_G - V_{FB}$

0

$p_s > N_a$ $p_s < N_a$

$p_s = N_a$

$\phi_s = 0$

$p_s = n_i$

$n_s = n_i$

Increase hole charge density even above the

$p_s < n_i$

$n_s > n_i$

semi more n-type than p-type

$n_s = N_a$

$\phi(x) = \frac{1}{q}(E_f - E_i)$

$\phi(x=0) = \phi_s$

Negative number

Flat Band

$\phi_s = \phi_p$

$\phi_s < \phi_p$

$p_s > N_a$

Intrinsic

$n_s > N_a$

Onset of inversion

$\phi_s < 0$ $\phi_s > 0$

ϕ_s

$\phi_p = -V_T \ln\left(\frac{N_a}{n_i}\right)$ 0 $n_s > p_s$ $|\phi_p| = +V_T \ln\left(\frac{N_a}{n_i}\right)$

How whole system is controlled by gate potential

Poisson equation:

$V_{FB}^{\circ} = \Phi_{ms} + \ldots + \ldots$

oxide charge   interface charge

SiO_2 / "O": (oxide)

$$\frac{d^2\phi}{dx^2} = \frac{-\rho_{ox}}{\epsilon_{ox}}$$

$Si - SiO_2$ (interface):

$Si - SiO_2$ interface:

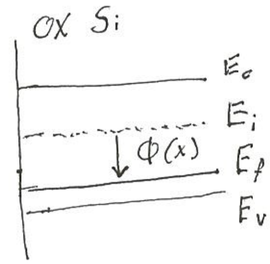
$$\epsilon_s E_s - \epsilon_{ox} E_{ox} = Q_s$$

Surface or interface charge

Silicon:

$$\frac{d^2\phi}{dx^2} = \frac{-\rho}{\epsilon_s} = \frac{q}{\epsilon_s}(n - p - D)$$

Flat Band:

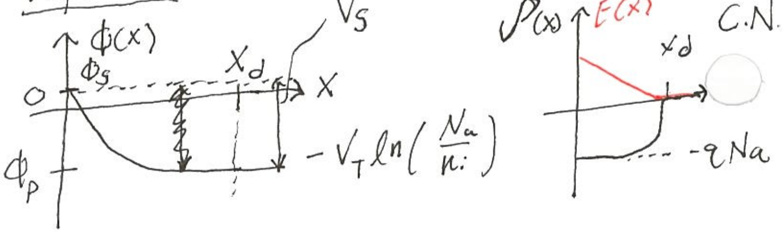


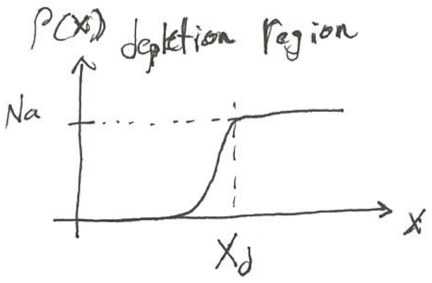
$\phi(x) = $ Constant

$E = -\frac{d\phi}{dx} = 0$

$n = \frac{n_i^2}{N_a}$, $p = N_a$ everywhere

Depletion:





$$X_d = \sqrt{\frac{2\epsilon_s}{q N_a} V_s}$$

$$\phi(x) = \phi_p + \frac{V_s}{X_d^2}(x - X_d)^2$$

$$0 \le x \le X_d$$

$$E_{silicon}(x) = -\frac{d\phi}{dx} = \frac{-2V_s}{X_d^2}(x - X_d)$$

$$E_s(x=0) = \frac{2V_s}{X_d} = \sqrt{\frac{2 q N_a}{\epsilon_s} V_s}$$

Total Voltage Drop $= V_G - V_{FB}^{\circ}$
$= V_{total}$

(ignoring ρ_{ox} & V_g)

$$V_s = \phi_s - \phi_p$$

In oxide: $\dfrac{d^2\phi}{dx} = 0$

$\left(\begin{matrix}\rho_{ox} = 0 \\ \text{for now}\end{matrix}\right)$ $E_{ox} = \dfrac{V_{ox}}{X_{ox}}$ (constant)

B.C. at $x = 0$:

$\epsilon_s E_s - \epsilon_{ox} E_{ox} = 0$ ($Q_s = 0$ for now)

$$\epsilon_s \sqrt{\frac{2 q N_a V_s}{\epsilon_s}} - \epsilon_{ox}\frac{V_{ox}}{X_{ox}} = 0$$

$$V_{ox} + V_s = V_G - V_{FB}^{\circ}$$

$$V_{ox} = (V_G - V_{FB}^{\circ}) - V_s$$

$$\sqrt{2 q N_a \epsilon_s V_s} - \frac{\epsilon_{ox}}{X_{ox}}\left([V_G - V_{FB}^{\circ}] - V_s\right) = 0$$

total band bending of

2 times ~~[illegible]~~ $|\phi_p|$ from flat band to onset of strong ~~[illegible]~~ inversion

$$0 \le V_s \le -2\phi_p = 2|\phi_p|$$

$$\phi_p \le \phi_s \le -\phi_p = |\phi_p|$$

F.B. ↑ onset of strong inversion ↑

$$V_s = \frac{V_N}{2} + [V_G - V_{FB}^{\circ}] - \sqrt{V_N}\sqrt{\frac{V_N}{4} + (V_G - V_F}$$

$$V_N = \frac{2 q N_a \epsilon_s}{C_{ox}^2}$$

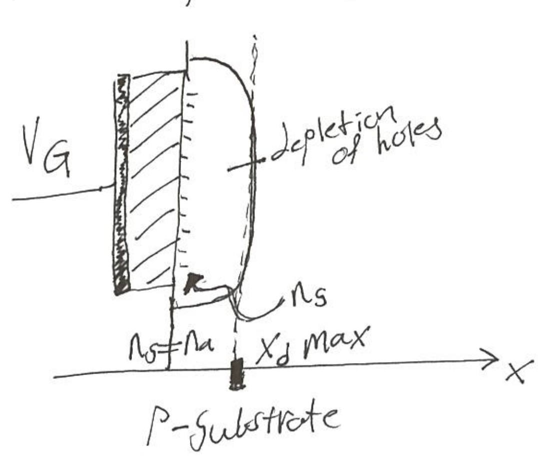
$$C_{ox} \equiv \frac{\epsilon_{ox}}{X_{ox}}$$

Question: $V_G = ? = V_{th} = ?$

for $V_s = 2|\phi_p|$ ← onset of strong [redacted] inversion

$$\Rightarrow V_{th} = V_{FB}^{\circ} + 2|\phi_p| + \frac{1}{C_{ox}}\sqrt{4\epsilon_s q N_a |\phi_p|}$$

Maximum depletion: $V_s = 2|\phi_p|$



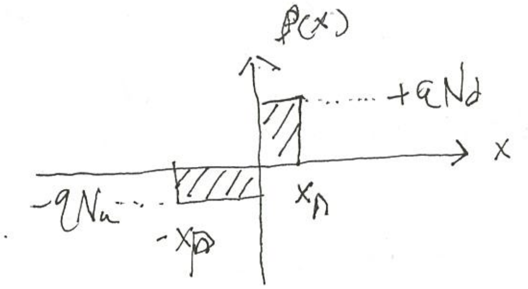
the voltage at which the depletion region has reached its maximum

• Device electronics midterm

1) Solutions. 2018 Spring 2018/03/31

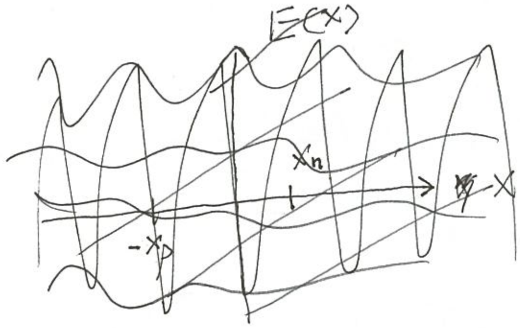
a) $N_a = 10^{15}\ cm^{-3}$ PN diode

$N_d = 10^{16}\ cm^{-3}$

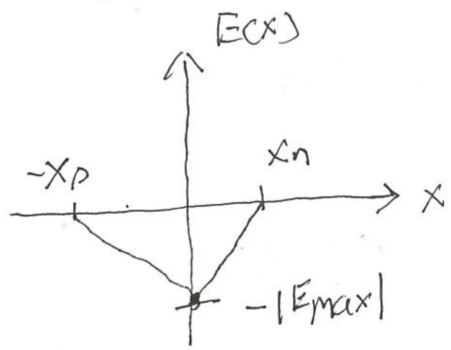


Charge Density $\rho\ [Cm^{-3}]\ [ASm^{-3}]$

b)

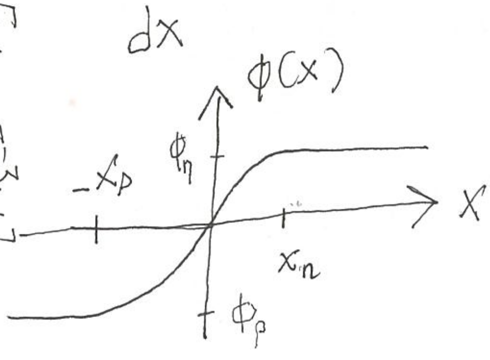


$$\frac{dE}{dx} = \frac{\rho}{\epsilon}$$



Electric Field $E\ [NC^{-1}]\ [Vm^{-1}]\ [Kgms^{-3}A^{-1}]$

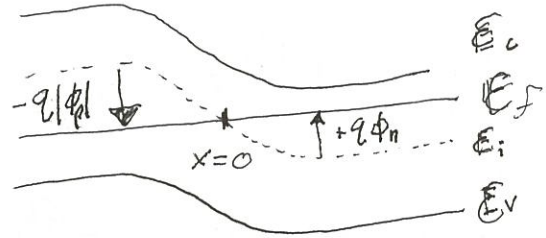
$$\frac{d\phi}{dx} = -E$$



~~ϕ_n~~ ✗

$$\phi_n = V_T \ln\left(\frac{N_d}{n_i}\right)$$

$$\phi_p = -V_T \ln\left(\frac{N_a}{n_i}\right)$$



$\phi\ [V]\ [Kg\,m^2A^{-1}S^{-3}]$

electrostatic potential

$\mathcal{E}\ [J]\ [Kg\,m^2S^{-2}]$

energy

c) $-X_p < x < 0 : \rho = -qN_A^-$

$0 < x < X_n : \rho = +qN_D^+$

$$-q(-N_A)X_p = N_D^+ X_n q$$

$$\frac{d^2\phi}{dx^2} = -\frac{qN_D^+}{\epsilon_s} \qquad 0 < x < X_n$$

$$\frac{d^2\phi}{dx^2} = \frac{qN_A^-}{\epsilon} \qquad -X_p < x < 0$$

Permitivity $\epsilon\ [Fm^{-1}]\ [S^4A^2m^{-3}kg^{-1}]$

$$\frac{d\phi}{dx} = -\frac{qN_D}{\epsilon}x + C_1$$

$$E_x = -\frac{d\phi}{dx} = \frac{qN_D}{\epsilon}x - C_1$$

$$E(x = x_n) = 0 = \frac{qN_D}{\epsilon}x_n - C_1$$

$$\therefore C_1 = \frac{qN_D}{\epsilon}x_n$$

$$E(x) = \frac{qN_D}{\epsilon}x - \frac{qN_D}{\epsilon}x_n$$

$$\boxed{E(x) = \frac{qN_D}{\epsilon}(x - x_n) \quad 0 < x < x_n}$$

$$\frac{d\phi}{dx} = -E$$

$$\phi = -\frac{qN_D}{2\epsilon}(x - x_n)^2 + C_2$$

$$\phi(x = x_n) = V_T \ln\left(\frac{N_d}{n_i}\right)$$

$$\therefore C_2 = V_T \ln\left(\frac{N_d}{n_i}\right)$$

$$\phi(x) = -\frac{qN_D}{2\epsilon}(x - x_n)^2 + V_T \ln\left(\frac{N_d}{n_i}\right)$$

$$\therefore \phi_n = V_T \ln\left(\frac{N_d}{n_i}\right)$$

$$\boxed{\phi(x) = -\frac{qN_D}{2\epsilon}(x - x_n)^2 + \phi_n \quad 0 < x < x_n}$$

$$-x_p < x < 0 \qquad \rho = -qN_A$$

$$\frac{dE}{dx} = \frac{\rho}{\epsilon}$$

$$\frac{dE}{dx} = \frac{-qN_A}{\epsilon}$$

$$E = \frac{-qN_A x}{\epsilon} + C_1$$

$$E(x = -x_p) = 0$$

$$E(-x_p) = \frac{-qN_A(-x_p)}{\epsilon} + C_1 = 0$$

$$\therefore C_1 = -\frac{qN_A(x_p)}{\epsilon}$$

$$E(X) = \frac{-qN_a x}{\epsilon} - \frac{qN_A(x_p)}{\epsilon}$$

$$\boxed{E(X) = \frac{-qN_a}{\epsilon}(X + X_p) \quad -X_p < x < 0}$$

$$\frac{d\phi}{dX} = -E$$

$$\phi = \frac{qN_a}{2\epsilon}(X + X_p)^2 + C_2$$

$$\phi(-X_p) = -V_T \ln\left(\frac{N_a}{n_i}\right)$$

$$\phi(X) = \frac{qN_a}{2\epsilon}(X + X_p)^2 - V_T \ln\left(\frac{N_A}{n_i}\right)$$

$$\phi_p = V_T \ln\left(\frac{N_A}{n_i}\right)$$

$$\boxed{\phi(X) = \frac{qN_a}{2\epsilon}(X + X_p)^2 - \phi_p \quad -X_p < X < 0}$$

$$\phi(0^-) = \phi(0^+)$$

$$E(0^-) = E(0^+)$$

$$\frac{qN_a}{2\epsilon}(X + X_p)^2 - \phi_p = \frac{-qN_D}{2\epsilon}(X - x_n)^2 + \phi_n$$

$$\frac{q}{2\epsilon}\left(N_a(X + X_p)^2 + N_D(X - x_n)^2\right) = \phi_p + \phi_n$$

$$\boxed{\phi_B = V_T \ln\left(\frac{N_a N_d}{n_i^2}\right) = \phi_n + \phi_p}$$

$$N_a(x + x_p)^2 + N_D(x - x_n)^2 = \frac{2\epsilon}{q}\phi_B$$

$$E(0^-) = E(0^+)$$

$$-\frac{qN_A}{\epsilon}(X + X_p) = \frac{qN_D}{\epsilon}(X - X_n)$$

$$\cancel{\frac{qN_A}{\epsilon}(X + X_p) + \frac{qN_D}{\epsilon}(X - X_n) = 0}$$

$$\cancel{\frac{q}{\epsilon}(N_A X + N_A X_p + N_D X - N_D X_n) = 0}$$

$$-N_A X - N_A X_p = N_D X - N_D X_n$$

$$N_D X_n - N_A X_p = (N_D + N_A)\, X$$

$0 < x < x_n$

$$E(x) = \frac{qN_D}{\epsilon}(x - x_n)$$

$$\phi(x) = \frac{-qN_D}{2\epsilon}(x - x_n)^2 + \phi_n$$

$$\phi_n = V_T \ln\left(\frac{N_d}{n_i}\right)$$

$-x_p < x < 0$

$$E(x) = -\frac{qN_A}{\epsilon}(x + x_p)$$

$$\phi(x) = \frac{qN_A}{2\epsilon}(x + x_p)^2 - \phi_p$$

$$\phi_p = V_T \ln\left(\frac{N_A}{n_i}\right)$$

$$\phi(0^-) = \phi(0^+)$$

$$N_a(0 + x_p)^2 + N_d(0 - x_n)^2 = \frac{2\epsilon}{q}\phi_B$$

$$N_a x_p^2 + N_d x_n^2 = \frac{2\epsilon}{q}\phi_B$$

$$E(0^-) = E(0^+)$$

$$-\frac{qN_a}{\epsilon}(0 + x_p) = \frac{qN_D}{\epsilon}(0 - x_n)$$

$$-N_a x_p = -N_D x_n$$

$$N_a x_p = N_D x_n$$

$$\therefore \quad x_p = \frac{N_D x_n}{N_a}$$

$$\& \quad x_n = \frac{N_a x_p}{N_D}$$

$$N_a\left(\frac{N_D x_n}{N_a}\right)^2 + N_d x_n^2 = \frac{2\epsilon}{q}\phi_B$$

$$\frac{N_D^2 x_n^2}{N_a} + N_d x_n^2$$

$$x_n^2\left(\frac{N_D^2}{N_a} + N_D\right) = \frac{2\epsilon}{q}\phi_B$$

$\cancel{X_n^p \pm Na^{2+}}$

$$X_n^2\left(\frac{N_D^2}{N_a}+\frac{N_D N_a}{N_a}\right)=\frac{2\epsilon}{q}\phi_B$$

$$X_n^2\left(\frac{N_D(N_D+N_a)}{N_a}\right)=\frac{2\epsilon}{q}\phi_B$$

$$\boxed{X_n=\sqrt{\frac{N_a}{(N_D(N_D+N_a))}\frac{2\epsilon}{q}\phi_B}}$$

$$N_a X_p^2 + N_d\left(\frac{N_a X_p}{N_D}\right)^2=\frac{2\epsilon}{q}\phi_B$$

$$N_a X_p^2 + \cancel{N_d}\,\frac{N_a^2 X_p^2}{N_D^{\cancel{2}}} =$$

$$N_a X_p^2 + \frac{N_a^2 X_p^2}{N_D}$$

$$X_p^2\left(N_a+\frac{N_a^2}{N_D}\right)=$$

$$X_p^2\left(\frac{N_D N_a}{N_D}+\frac{N_a^2}{N_D}\right)=$$

$$X_p^2\left(\frac{(N_D+N_a)N_a}{N_D}\right)=$$

$$\boxed{X_p=\sqrt{\frac{N_D}{N_a(N_D+N_a)}\frac{2\epsilon}{q}\phi_B}}$$

d) $\phi_B \Rightarrow \phi_B - V_A$

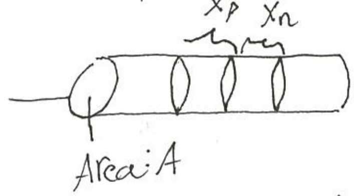
A: area ϕ_B: built in potential

$A = X_n q N_d = (-X_p)(-N_a)(q)$

$A = X_p N_a\ Q$

Q = total charge

A: cross sectional area



Volume = $A \cdot X_p$

Total charge = Volume X charge density

$Q = A \cdot X_p\, q\, N_a$?

Why does the q disapear

$Q = A \cdot x_p N_a$

Capacitance $C = \frac{dQ}{dV_a}$

$$Q = AN_a\sqrt{\frac{N_D}{N_a(N_D+N_a)}\frac{2\epsilon}{q}(\phi_B - V_a)}$$

$$Q = A\sqrt{\frac{N_a N_D}{(N_D+N_a)}\frac{2\epsilon}{q}(\phi_B - V_a)}$$

$$\frac{dQ}{dV_a} =$$

$(-V)^{1/2}$

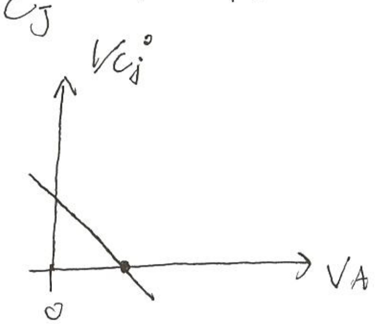
$\frac{1}{2}(-V)^{-1/2}$

$$C = \frac{dQ}{dV_a} = \frac{1}{2} A \sqrt{\frac{N_D N_A}{(N_D + N_A)} \frac{2q}{\epsilon} \frac{1}{(\phi_B - V_A)}}$$

Depletion Capacitance

Depletion layer Capacitance : C_J

$$\frac{1}{C_J^2} = \frac{1}{A^2} \frac{(N_D + N_A)}{N_D N_A} \frac{2\epsilon}{q} (\phi_B - V_A)$$



$V_A = 0$

ϕ_B x intercept

2. a)

$n = n_i e^{\phi/V_T}$

$p = n_i e^{-\phi/V_T}$

$\therefore$ the device is in thermal equl.

$\therefore$ $J_n = 0$

$J_p = 0$

Due to given $\phi(x)$ and T.E.

$np = n_i^2$ $n = n_i$ & $p = n_i$

at $x = 0$ $\phi(0) = 0$

b) Typically the dopants are a few kT away from the edges Conduction and Valence bands (E_c & E_v)

~~The ionization & activation are~~

$\therefore$ The ionization and activation are inherently thermal

?

$\therefore$ If low thermal energy then there is less ionization

this occurs at low T

c) $J_n drift = q\mu_n n \mathcal{E}$

$\mathcal{E}$ electric field

$J_n\ drift = -q\mu_n n \nabla \psi_n$

ψ_n = electrostatic potential

$\frac{d\mathcal{E}}{dx} = \frac{\rho}{\epsilon}$ ρ = charge density

$\mathcal{E}$ = electric field

$\frac{d\phi}{dx} = -\mathcal{E}$ ϕ = electric potential

$\frac{dE_i}{dx} = q\mathcal{E}$ E_i = intrinsic energy

q = Electric Charge

ϵ = Vacuum Permitivity / Permitivity of free space

$\rho[cm^{-3}]$ $[Asm^{-3}]$

$\mathcal{E}[NC^{-1}]$ $[Kg\,m\,s^{-3}A^{-1}]$

$[Vm^{-1}]$

$\epsilon[Fm^{-1}]$ $[s^4A^2m^{-3}Kg^{-1}]$

$$[Kg\,m\,s^{-3}A^{-1}] = \int \frac{[Asm^{-3}]}{[A^2s^4m^{-3}Kg^{-1}]}\,dx$$

$$= \int [A^{-1}s^{-3}Kg]\,dx$$

$$= [A^{-1}s^{-3}Kg\,m]$$

NIST order of Base SI units

[m Kg s A K mol cd]

c)

$$J_n = q\mu_n n E$$

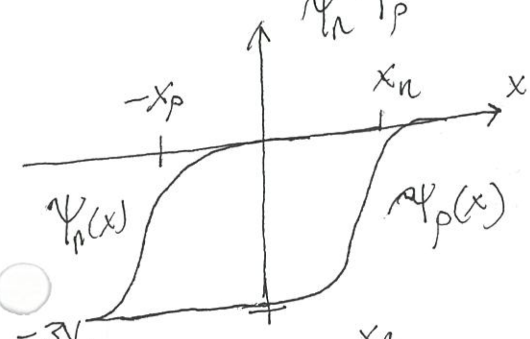
$$= -q\mu_n n \nabla\psi$$

quasi fermi potential

$$= -q\mu_n n \nabla\phi$$

$\psi_n\ \psi_p$

being used inter-changeably adding confusion



$-x_p$ 0 x_n

C.N. p-side — depletion region — C.N. n-side

$$n = n_i e^{\frac{\Phi - \psi_n}{V_T}}$$

$$p = n_i e^{-\frac{\phi - \psi_p}{V_T}}$$

$$np = n_i^2 e^{\frac{\psi_p - \psi_n}{V_T}}$$

R = recombination

$$R = \frac{n_i^2 - np}{\tau_p(n+n_i) + \tau_n(p+p_i)}$$

$$= \frac{-e^{\frac{\psi_p-\psi_n}{V_T}}}{\quad}$$

...

In $-x_p < x < x_n$

$$\psi_n > \psi_p$$

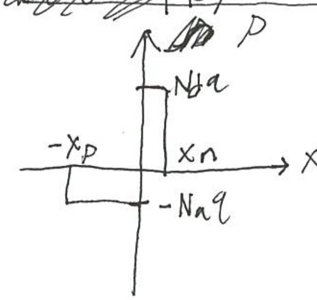
Why

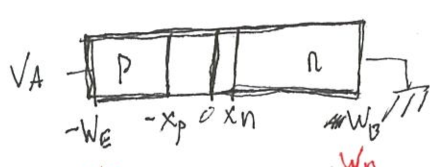
$\boxed{R<0}$

Why

therefor generation occurs

3a)





L_n, L_p diffusion length

$W_n < L_p$ Short diode

$W_p > L_n$ long diode

Consider the excess electron Charge density in the p region

current continuity equation for electrons:

$$\frac{dn}{dt} = \frac{1}{q} \nabla \cdot \vec{J_n} - R$$

$\boxed{\because}$ uniformly doped

$\therefore$ Steady State regime

$$\frac{dn}{dt} = 0$$

electron current density (drift, diff)

$$J_n = q(-\mu_n P \nabla \phi + D_n \nabla n)$$

$\uparrow$ drift $\qquad$ $\uparrow$ diffusion

$\therefore$

$$0 = \nabla(-\mu_n P \nabla\phi + D_n \nabla n) - R$$

$\boxed{\because}$ C.n. region $\mathcal{E} \approx 0 \therefore \nabla\phi \approx 0$

$$\therefore 0 = D_n \nabla^2 n - R$$

In 1D

$$0 = D_n \frac{d^2 n}{dx} - R$$

Recombination term

$$R = \frac{(np - n_i^2)}{\tau_p(n + n_i) + \tau_n(p + n_i)}$$

1 $\boxed{\because}$ TE. $np = n_i^2 \therefore R = 0$

Consider Small applied Voltage & Perturbation of minority Carrier density $x = x_p$

2 $\boxed{\because}$ $p = p_0$ majority carrier

$n = n_0 + n'$ minority carrier

3 $\boxed{\because}$ In p region $p >> n$ $p >> n_i$

&

$$R = \frac{np - n_i^2}{\tau_p n + \tau_p n_i + \tau_n p + \tau_n n_i}$$

3

$$\therefore R = \frac{np - n_i^2}{\tau_n p}$$

2

$$\therefore R = \frac{p_0(n_0 + n') - n_i^2}{\tau_n p_0}$$

$$= \frac{p_0 n_0 + p_0 n' - n_i^2}{\tau_n p_0}$$

1

$$= \frac{p_0 n'}{\tau_n p_0}$$

$$R = \frac{n'}{\tau_n}$$

$$0 = D_n \frac{d^2 n}{dx} - \frac{n'}{\tau_n}$$

2

$$0 = D_n \frac{d^2 n_0}{dx} + \frac{d^2 n'}{dx} - \frac{n'}{\tau_n}$$

constant doping density

$$\frac{dn_0}{dx} = 0$$

$$\therefore \quad 0 = D_n \frac{d^2 n'}{dx} - \frac{n'}{\tau_n}$$

$$0 = \frac{d^2 n'}{dx} - \frac{1}{D_n \tau_n} n'$$

Define diffusion length $L_n = \sqrt{D_n \tau_n}$

$$\therefore \quad 0 = \frac{d^2 n'}{dx} - \frac{1}{L_n^2} n'$$

General Solution $x \leq -x_p$

$$n'(x) = A \exp\left(-\frac{(x+x_p)}{L_n}\right) + B \exp\left(\frac{(x+x_p)}{L_n}\right)$$

A is not physical because as x decreases away from $-x_p$ it would result in an increased minority carrier density

$$\therefore \quad n'(x) = B \exp\left(\frac{(x+x_p)}{L_n}\right)$$

$$n'(-x_p) = B$$

$$\therefore \quad n'(x) = n'(-x_p) \exp\left(\frac{x+x_p}{L_n}\right)$$

Define excess carrier density due to applied voltage

$$n' \equiv n - n_0$$

$$p' \equiv p - p_0$$

$$n'(-x_p) = n_0(-x_p)\left[e^{V_a/V_T} - 1\right]$$

$$p'(x_n) = p_0(x_n)\left[e^{V_a/V_T} - 1\right]$$

$$\therefore \quad n'(x) = n_0(-x_p)\left[e^{V_a/V_T} - 1\right] e^{\frac{x+x_p}{L_n}}$$

$$V_T = \frac{kT}{q}$$

$$\therefore \quad n'(x) = n_0(-x_p)\left[e^{V_a q/kT} - 1\right] e^{\frac{x+x_p}{L_n}}$$

b) Short diode case
excess hole charge density in the n-region

$$\frac{dp}{dt} = -\frac{1}{q} \nabla \cdot \vec{J}_p - R$$

$$\vec{J}_p = q(-\mu_p p \nabla\phi - D_p \nabla p)$$

$$\because \text{C.N. } \mathcal{E} = 0 \quad \therefore \nabla\phi = 0$$

$$\therefore \quad J_p = -q D_p \nabla p$$

∵ uniformly dope ∴ Steady State

$$\frac{dp}{dt} = 0$$

$$0 = \nabla^2 D_p p - R$$
$$= D_p \nabla^2 p - R$$

$$R = \frac{np - n_i^2}{\tau_p (n + n_i) + \tau_n (p + n_i)}$$

3 ∵ T.E. $np = n_i^2$ ~~R~~

∵ majority carrier

$n = n_0$ majority

$p = p_0 + p'$ minority

2 ∵ In n region

$n >> \cancel{n_p}\ p$

$n >> n_i$

1 ∴ $R = \dfrac{n_0(p_0 + p') - n_i^2}{\tau_p n_0 + \tau_p n_i + \tau_n p + \tau_n n_i}$

∴ $= \dfrac{n_0 p_0 + n_0 p' - n_i^2}{\tau_p n_0}$

3 ∴ $= \dfrac{n_0 p'}{\tau_p n_0}$

$$R = \frac{p'}{\tau_p}$$

(in 1-D)

$$0 = D \frac{d^2 p}{dx} - \frac{p'}{\tau_p}$$

$$0 = \frac{d^2 p'}{dx} - \frac{1}{\tau_p D_p} p' = 0$$

Diffusion length $L_p = \sqrt{\tau_p D_p}$

$$\frac{d^2 p'}{dx} - \frac{1}{L_p^2} p' = 0$$

General Solution

$$p'(x) = A \exp\left(-\frac{x - x_n}{L_p}\right) + B \exp\left(\frac{x - x_n}{L_p}\right)$$

~~is not physical because as $x >> x_n$ the minority carrier density goes~~

~~$p'(x) = A \exp\left(-\frac{x - x_n}{L_p}\right)$~~

~~$p'(x_n) \neq A$~~

~~$p'(x) \neq p'(x_n) \exp\left(-\frac{x - x_n}{L_p}\right)$~~

for short diode, make linear approximation

$$p'(x) = A + B \left(\frac{x - x_n}{L_p}\right)$$

∵ Ohmic contact ∴ charge neutral $p'(W_B) = 0$

Lecture note

~~$0 = A + B$~~

∵ $p'(x_n) = p_0(x_n)[e^{V_a/V_T} - 1]$

∴ $A = p_0(x_n$

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6

Examination
BOOK

NAME Joseph Crandall DATE 2018-03-09

SUBJECT Device electronics

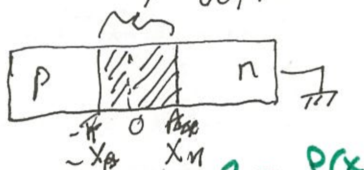
STUDENT NO. G25127483

11/30

"I, Joseph Crandall,
affirm that I have completed this assignment/examination in accordance with the CODE OF ACADEMIC INTEGRITY."

included
↓
b & c

depletion region



energy band diagram for problem

5/10

1.5/2 a)

Doping level plot & charge density curve (without approximat)

?? $\rho(x)$?

$q N_d$

$-q N_a$

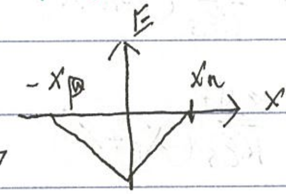
$N_d = 10^{16}\ cm^{-3}$

$N_a = ?$

- net charge density $\rho(x)$

b)

Electric field Plot



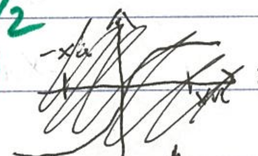
In thermal equilibrium, and uniformly doped (have ohmic contact)

• $np = n_i^2$

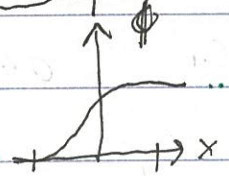
You also have charge neutral regions outside of the depletion zone

∴ $\rho = q(n - p + N_D - N_a) = 0$

1/2 b)



electric potential plot



Band diagram??

c)? relationship between charge density, electric field, & electrostatic potential (In 1 Dimension)

label your answers!

$$\frac{d\phi}{dx} = -E$$ ∴

careful!

$$\rho = \begin{cases} \cancel{-qN_a} & 0 \le x \le x_n \quad +qN_d \\ \cancel{qN_d} & -x_p \le x \le 0 \quad -qN_a \end{cases}$$

2.5/3

$$\frac{dE}{dx} = \frac{\rho}{\varepsilon_s}$$

$\boxed{\text{must be continuous}}$

For T.E. also $qN_ax_p = qN_dx_n$

error

error propagates

$$\frac{dE}{dx} = \begin{cases} -\frac{qN_a}{K\varepsilon_s} & 0 \le x \le x_n \\ \frac{qN_d}{K\varepsilon_s} & -x_p \le x \le 0 \end{cases}$$

$$E = \begin{cases} \int_{x_n}^{x} -\frac{qN_a}{K\varepsilon_s}\,dx = \frac{qN_a}{K\varepsilon_s}(x_n - x) & 0 \le x \le x_n \\ \int_{-x_p}^{x} \frac{qN_d}{K\varepsilon_s}\,dx = \frac{qN_d}{K\varepsilon_s}(x_p + x) & -x_n \le x \le 0 \end{cases}$$

$$\frac{d\phi}{dx} = \begin{cases} * -\frac{qN_a}{K\varepsilon_s}(x_n - x) & 0 \le x \le x_n \\ -\frac{qN_d}{K\varepsilon_s}(x_p + x) & -x_n \le x \le 0 \end{cases}$$

$$\phi(x) \equiv \begin{cases} -\frac{qN_a}{2K\varepsilon_s}(x_n - x)^2 + C_1 & 0 \le x \le x_n \\ -\frac{qN_d}{2K\varepsilon_s}(x_p + x)^2 + C_2 & -x_n \le x \le 0 \end{cases}$$

$$C_1 = V_T \ln\left(\frac{N_a}{n_i}\right)$$

$$C_2 = V_T \ln\left(\frac{N_d}{n_i}\right)$$

$$q\, x_n N_D = q\, x_p N_a$$

$\because$ $N_D = 10^{16}\ cm^{-3}$ $\quad\therefore \boxed{x_n \cdot 10 = x_p}$

$N_A = 10^{15}\ cm^{-3}$

Must be continuous therefor $\phi(0^-) = \phi(0^+)$

$$-\frac{qNa}{2K\varepsilon_s}(X_n)^2 + V_T \ln\left(\frac{Na}{n_i}\right) = -\frac{qNd}{2K\varepsilon_s}X_p^2 + V_T \ln\left(\frac{Nd}{n_i}\right)$$

· recall that $\phi_B = V_T \ln\left(\frac{N_A N_d}{n_i^2}\right)$

$$\frac{q}{2K\varepsilon_s}\left(\cancel{X} X_p^2 - X_n^2\right) + V_T\left(\ln\left(\frac{Na}{n_i}\right) + \ln\left(\frac{Nd}{n_i}\right)\right) = 0$$

$$\ldots + V_T \ln\left(\frac{N_a N_i}{n_i^2}\right) = 0$$

$$+ \phi_B = 0$$

recall that $X_n = \frac{X_p}{10}$

Also note: $V_T = \frac{KT}{q}$ thermal Voltage

$\therefore$

$$X_p^2 - \frac{X_p^2}{\cancel{100}\,10^2} = \phi_B\left(\frac{2K\varepsilon_s}{q}\right)$$ X

?

$$X_p = \sqrt{\phi_B\left(\frac{2K\varepsilon_s}{q}\right) - \left(1 - \frac{1}{100}\right)}$$? X

Following same process from X_p to solve for X_n

$$\therefore X_n = \sqrt{\phi_B\left(\frac{2K\varepsilon_s}{q}\right) - (100-1)}$$

?

$X_p = X_n 10$

d) When a negative bias is inserted. $V_a < 0$, ~~otherwise know as a~~ it has the effect of decreasing the output voltage ??

Answer?

0/3

$\frac{1.5}{10}$

2 $\phi(x) = 0.35 \frac{[\exp(x/x_0) - \exp(-x/x_0)]}{\exp(x/x_0) + \exp(-x/x_0)]}$ V $-4x_0 < x \leq 4x_0$

a)

$\frac{1.5}{3}$

drift diffusion model ~~is~~ employed

$n = n_i e^{(\phi/v_T)}$

$p = n_i e^{(-\phi/v_T)}$

In thermal equilibrium

$np = n_i^2$

where is it intrinsic?

also since there is a uniform carrier distribution in the device and since we know that with charge neutrality in the device that the tradition drift diffusion equation for the problem can be simplified

$\vec{J} = q(\mu_n n \vec{E} + D_n \nabla n)$ $\rho = q(n - p + N_d - N_a) = 0$

$\therefore \Delta n = 0$

$J_n = q \mu_n n \phi E$

$J_p = q \mu_p p E$

$\longrightarrow$

Because the carrier concentrations are uniformely distributed, there is ~~no~~ is no resulting electron and hole current densities

$\vec{J}_n(x) \approx 0$

$\vec{J}_p(x) \approx 0$

~~np~~ $np = n_i^2$ Intrinsic charge density condition is achieved at thermal equilibrium when $np = n_i^2$

b) The total doping densit D is normally described as the ~~m~~ by the donnor and acceptor donor concentrations that make it up $D = N_d + N_a$

0/3

If we were to increase the number of acceptors in the material (N_a) and did not have any knowledge about the donor concentration then it

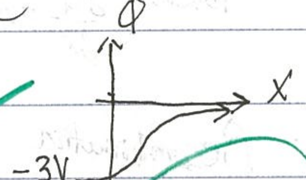
??

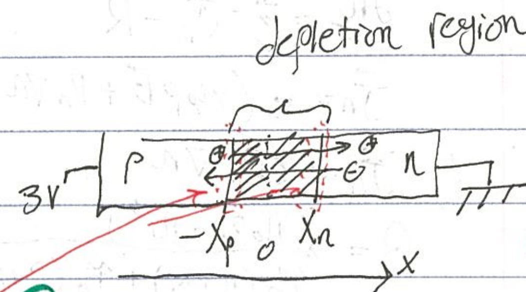
Would be inappropriate to refer to the total doping density as equivalent to the ionized doping density because we would loose

?? information, specifically the concentration the donor density.

C

0/4





Ohmic Contacts

?

?

reverse bias $V_a < 0$

$\therefore V_a = -3$

ohmic contacts have the following properties

- In thermal equilibrium
- charge neutral
- there is no charge recombination at the ohmic contacts

∴ It is likely that the generation would occur via the minority carriers outside of the depletion region

X

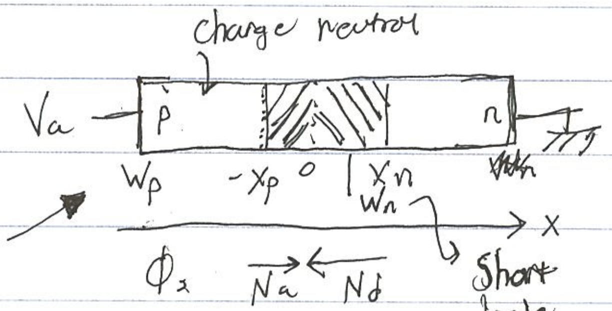
~~I remember the parts of this problem but I cannot seem to put them together~~

4.5/10

Show excess electron charge density

3

3.5/6



long diode case

Short diode case

- N_a $x < 0$
- N_d $x > 0$
- abrupt pn junction

$$\frac{dn}{dt} = \frac{1}{q}\cdot \vec{J_n} - R$$

$\vec{J_n} =$

$$\vec{J_n} = q(\mu_p p \vec{E} + D_n \nabla n)$$

however $\frac{d\vec{\Phi}}{dx} = 0 \therefore \vec{E} = 0$

$$\therefore \vec{J_n} = qD_n \nabla n$$

$$R = \frac{(n_o p_o - n_i^2)}{\tau_n (n + n_i) + \tau_p (p + p_i)}$$

Recombination term because ~~because~~ in the p region

ok

Recombination has to equal zero because in thermal equilibrium $np = n_i^2 \therefore R = 0$

$$n_o(x) = n_o(x_n)$$

– in p region

$p >> p_i$

$R =$??

? I am having trouble putting the parts of the problem together

~~[illegible]~~

~~p~~ $n' = n_0 + n$

$\frac{d^2 p_0}{dx^2} = 0$ X ?

$n'(x) = ?$

? $\frac{d^2 n'}{dx^2} - n' \frac{1}{L_n^2} = 0$ $L_n = \sqrt{\tau_n D_n}$

Answer?

General Solution

$n'(x) = A e^{(x_p + x)/L_n} + B e^{-(x_p + x)/L_n}$

as $x \to -\infty$ $e^{-(x_p + x)/L_n} \to \infty$

(a) this is not physical

$\therefore n'(-x_{p0}) = A e$

$\therefore A = n_{p0}$

b) ~~[illegible]~~ excess hole charge density in n region

1/2

$p'_n(x) = p_{n0}(e^{V_a/V_T} - 1)e^{-(x - x_n)/L_p}$ ok

but about short-diode?

c) ~~[illegible]~~

0/2

$\vec{J}_{total} = \vec{J}_n + \vec{J}_p = ?$

Answer?

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—PREAMBLE

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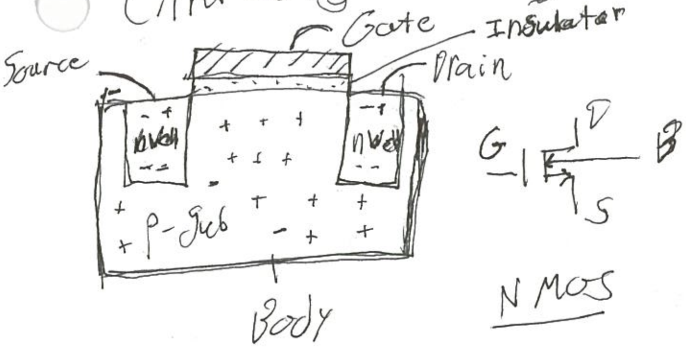
Week 12
Device Electronics
2018-04-06
(Ahmadi's lecture)

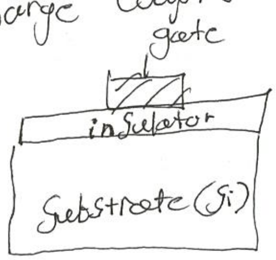


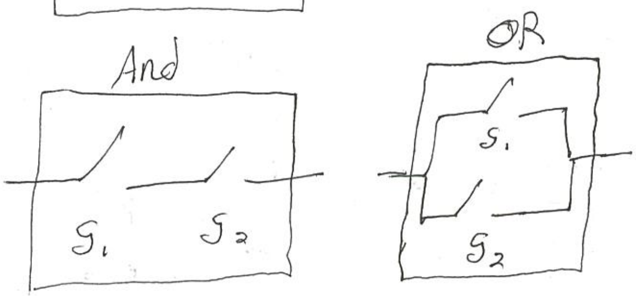
NMOS

~~Technology~~ - minimum width the technology can fabricate

Technology

Charge Coupled Devices (CCD)





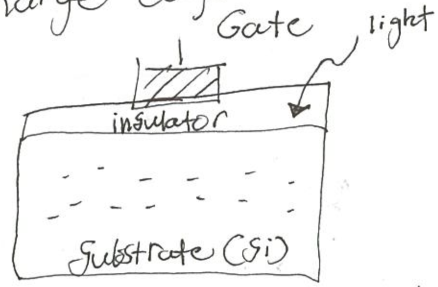
MOSFET

- majority carrier
- minority carrier

- in off electron can not go through barrier
- in on, minority carrier are pulled up ~~to~~ towards insulator to creat channel for the electrons to travel ~~travel~~ from n-well to next n-well (channel under gate)

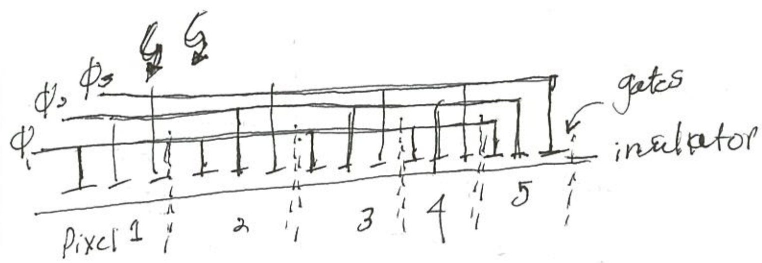
Without voltage diffusion occurs, and electrons (minority carriers, diffuse back into the bulk material

Charge Coupled device



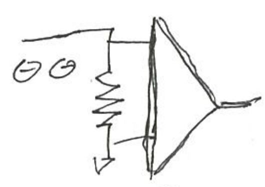
carries then bunch under the gate
When $V_G > V_T$

- CCD, combinations of Rows & Columns, each grid point is a pixel
- have to access electrons individually and collect current.

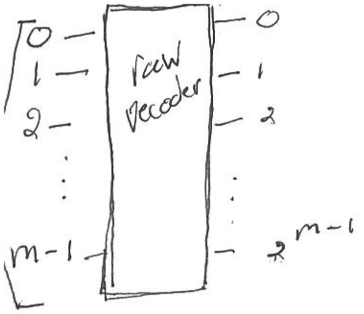


$\phi_1 = 1,\ \phi_2,\ \phi_3 = 0$
$\phi_1\ \phi_2 = 1,\ \phi_3 = 0$
$\phi_1 = 0,\ \phi_2 = 1,\ \phi_3 = 0$
$\phi_1 = 0,\ \phi_2 = 1,\ \phi_3 = 1$
$\phi_1 = 0,\ \phi_2 = 0,\ \phi_3 = 1$

moving Wells



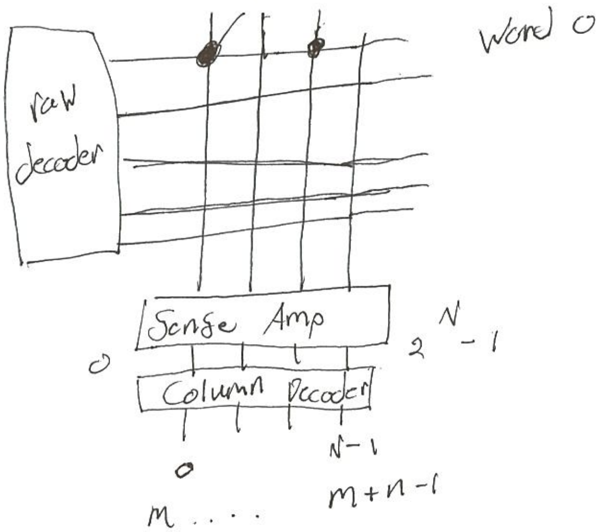
Memory?
S-RAM (Static)
D-RAM (Dynamic)



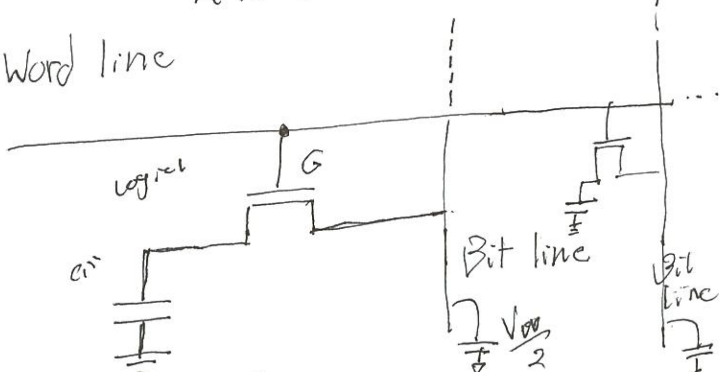
0	00
0	01
0	10
0	11
1	00
1	01

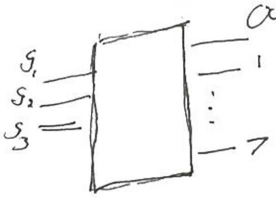
m-input

1 node stored 1 Bit



Word line



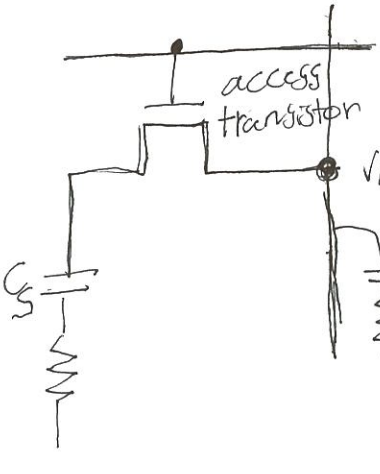


- all have parasitic effects
- You have pre charged your bit line to $\frac{V_{DD}}{2}$

S_2 S_1 S_0

1 0 0

→ charge sharing equation, how much voltage will change



$\Delta V('1') = ?$

$\Delta V('0') = ?$

ΔV ?

$Q = CV$

total charge in the system before access transistor is on:

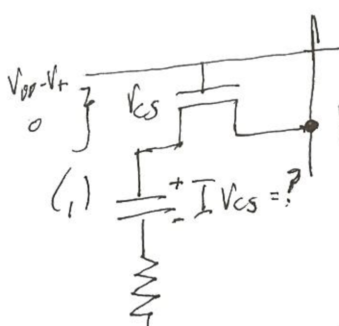
$C_S \cdot V_{CS} + C_B \frac{V_{DD}}{2} = 0$

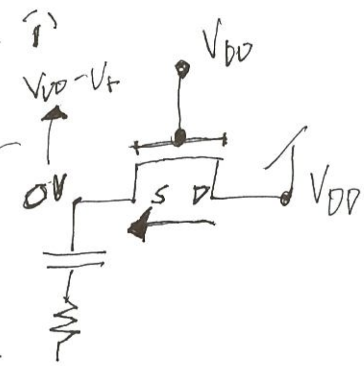
$\Delta V(C_S + C_B) = C_S\left(V_{CS} - \frac{V_{DD}}{2}\right)$

$\Delta V = \frac{C_S}{C_S + C_B}\left(V_{CS} - \frac{V_{DD}}{2}\right)$

Write process

Write '1' into the SRAM





$$I_D = \begin{cases} 0 & V_{gs} < V_t \\ \frac{1}{2} K'\left(\frac{W}{L}\right)(V_{gs} - V_t)^2 & \text{Sat} \\ K'\left(\frac{W}{L}\right)\left[(V_{gs} - V_t)V_{ds} - \frac{1}{2} V_{ds}^2\right] \end{cases}$$

triode region

$K' = \mu C_{ox}$

highest voltage $V_{DD} - V_t$

$$\Delta V(1) = \frac{C_S}{C_S + C_B}\left(V_{DD} - V_t - \frac{V_{DD}}{2}\right)$$

$$= \frac{C_S}{C_S + C_B}\left(\frac{V_{DD}}{2} - V_t\right)$$

Very small compared to C_B

$$\approx \frac{C_S}{C_B}\left(\frac{V_{DD}}{2} - V_t\right)$$

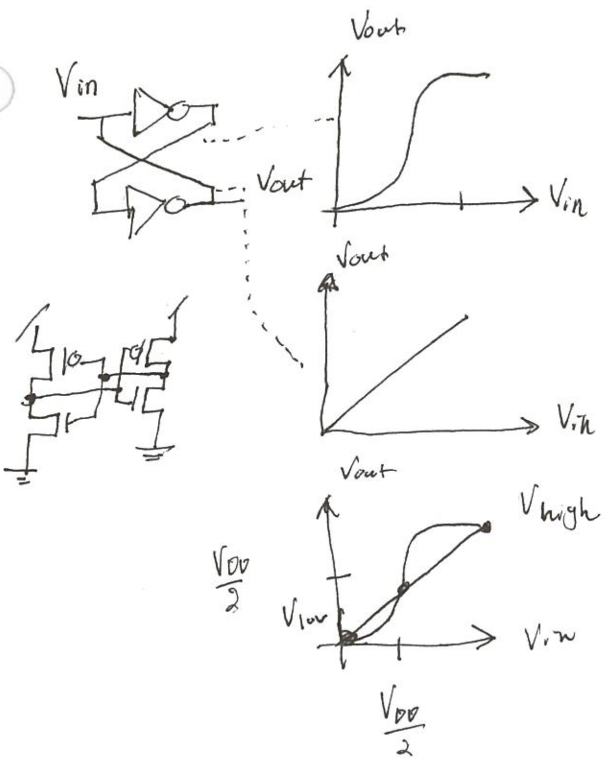
$$= \frac{1}{30}\left(\frac{5}{2} - 1\right) = \frac{1}{30}(1.5)$$

$$= \frac{1.5}{30} = 0.05 = 50 \text{ mV}$$

$$\Delta V(0) \approx \frac{C_S}{C_B}\left(0 - \frac{V_{DD}}{2}\right)$$

$$= \frac{1}{30}(-2.5)$$

$$= -83 \text{ mV}$$



$\frac{V_{DD}}{2}$ is an unstable state

(With noise, never exactly at $\frac{V_{DD}}{2}$

USEFUL DESIGN PARAMETERS (simplified)

Name	Symbol	0.18 μm NMOS	0.18 μm PMOS	0.13 μm NMOS	0.13 μm PMOS	Units
Channel Length (rounded for convenience)	L	200	200	100	100	nm
Supply Voltage	V_{DD}	1.8	1.8	1.2	1.2	V
Oxide Thickness	t_{ox}	35	35	22	22	Å
Oxide Capacitance	C_{ox}	1.0	1.0	1.6	1.6	μF/cm^2
Threshold Voltage	V_{T0}	0.5	−0.5	0.4	−0.4	V
Body-Effect Term	γ	0.3	0.3	0.2	0.2	V$^{1/2}$
Fermi Potential	$2\|\phi_F\|$	0.84	0.84	0.88	0.88	V
Junction Capacitance Coefficient	C_{j0}	1.6	1.6	1.6	1.6	fF/μm^2
Built-In Junction Potential	ϕ_B	0.9	0.9	1.0	1.0	V
Grading Coefficient	m	0.5	0.5	0.5	0.5	—
Nominal Mobility (low vertical field)	μ_0	540	180	540	180	cm^2/V-s
Effective Mobility (high vertical field)	μ_e	270	70	270	70	cm^2/V-s
Critical Field	E_c	6×10^4	24×10^4	6×10^4	24×10^4	V/cm
Critical Field × L	E_cL	1.2	4.8	0.6	2.4	V
Effective Resistance	R_{eff}	12.5	30	12.5	30	kΩ/□

Name	Symbol	Value	Units
Gate Capacitance Coefficient	C_g	2	fF/μm
Self Capacitance Coefficient	C_{eff}	1	fF/μm
Wire Capacitance Coefficient	C_w	0.1–0.25	fF/μm
Al Wire Resistance	$R_{\square}$	25–60	mΩ/□
Cu Wire Resistance	$R_{\square}$	20–40	mΩ/□
Wire Inductance	L_{eff}	40–50	pH/μm

HW #3 Device Electronics

Chapter 6: 7, 8, 22

Chapter 7: 4, 5

Chapter 8: 2

Chapter 9: Read

Chapter 14: 2, 3, 6, 7

Joseph Crandall 2018-04-06

Chapter 6: 7) a)

ideal Si p^+-n step junction diode

- area: $A = 10^{-4}\,[cm^2]$
- donor doping concentration: $N_D = 1.0 \times 10^{16}\,[cm^{-3}]$
- p-type time constant unit: $\tau_p = 10^{-6}\,[s]$

— Majority carrier mobility versus doping at room temperature (from ex. 3.1)

- carrier mobility: $\mu\,[cm^2 V^{-1} s^{-1}]$

$$\mu = \mu_{min} + \frac{\mu_0}{1+(N/N_{ref})^{\alpha}}$$

- doping concentration: $N\,[cm^{-3}]$

α is a fit parameter

— ~~model~~ temperature dependence, one additionally employs

$$A = A_{300}\left(\frac{T}{300}\right)^{\eta}$$

A represents μ_{min}, μ_0, N_{ref}, or α

A_{300}: 300 K value of parameter

$T\,[K]$: temperature

$\eta\,[\,]$: temperature exponent

— n_i fit relationship

$$n_i = (9.15 \times 10^{19})\left(\frac{T}{300}\right)^2 e^{-\frac{0.5928}{KT}}$$

$T\,[K]$

Boltsman constant: $K = 8.617 \times 10^{-5}\,\frac{eV}{K}$

b) I_0 versus T for $300\,K \le T \le 400\,K$

c) From figure 1, the reverse saturation current increases with increases in temperature

- the I-V characteristic of the diode for different temperature values can be observed
- the reverse saturation current increases with increase in temperature

8) a)

→ Photo generation rate: G_L

→ hole minority carrier lifetime: τ_p

→ hole diffusion coefficient: D_p

— excess minority carrier diffusion equation for the hole is given

$$\frac{d\Delta p_n}{dt} = D_p \frac{d^2\Delta p_n}{dx^2} - \frac{\Delta p_n}{\tau_p} + G_L$$

- Steady State

$$\frac{d\Delta p_n}{dt} = 0 \qquad \frac{d^2\Delta p_n}{dx^2} = 0$$

— valid for the region far away from n-side of the junction

$$\frac{d\Delta p_n}{dt} = D_p \frac{d^2\Delta p_n}{dx^2} - \frac{\Delta p_n}{\tau_p} + G_L$$

Calculate $\Delta p_n (x \to \infty)$

$$0 = \frac{\Delta p_n (x \to \infty)}{\tau_p} + G_L$$

$$\Delta p_n = (x \to \infty) = G_L \tau_p$$

Hence, the excess minority carrier diffusion equation $= G_L \tau_p$

b) Obtain IV characteristic of the p^+-n diode

- Boundary Conditions

$$\Delta p_n (x'=0) = \left[\frac{n_i^2}{N_D}\right](e^{qV_A/kT} - 1)$$

$$\Delta p_n (x' \to \infty) = G_L \tau_p$$

- lightly doped n side of the pn junction

$$0 = D_p \frac{d^2 \Delta p_n}{dx^2} - \frac{\Delta p_n (x \to \infty)}{\tau_p} + G_L$$

- General Solution

$$\Delta p_n (x') = G_L \tau_p + A_1 e^{-x'/L_p} + A_2 e^{x'/L_p}$$

Since $e^{x'/L_p} \to \infty$

- Second boundary condition is satisfied when the amplitude A_2 is zero

– so calculate first B.C.

$$\Delta p_n (x'=0) = G_L \tau_p + A_1$$

$$= \left[\frac{n_i^2}{N_D}\right](e^{qV_A/kT} - 1)$$

Number of donor atoms: N_D

Applied direct current Voltage: V_A

Calculate A_1

$$A_1 = \left[\frac{n_i^2}{N_D}\right](e^{qV_A/kT} - 1) - G_L \tau_p$$

$$\Delta p_n (x'=0)$$

$$\Delta p_n (x'=0) = G_L \tau_p + \left[(n_i^2/N_D)(e^{qV_A/kT} - 1) - G_L \tau_p\right] e^{-x'/L_p}$$

Calculate the associated current density. $J_p(x')$

$$J_p(x') = -qD_p \frac{d\Delta p_n}{dx'}$$

$$= q\frac{D_p}{L_p}\left[\left(\frac{n_i^2}{N_D}\right)(e^{qV_A/kT} - 1) - G_L \tau_p\right] e^{-x'/L_p}$$

hole majority diffusion length: L_p

hole coefficient : D_p

Calculate p^+-n diode current : I

$$I = AJ$$

$$= A\left[J_n(x = -x_p) + J_p(x = -x_n)\right]$$

$$\cong AJ_p(x'=0)$$

$$= qA\frac{D_p}{L_p}\frac{n_i^2}{N_D}(e^{qV_A/kT} - 1) - qA\frac{D_p \tau_p}{L_p} G_L$$

$$\because D_p \tau_p = L_p^2$$

$$I = I_0(e^{qV_A/kT} - 1) + I_L$$

c) – ideal diode characteristic

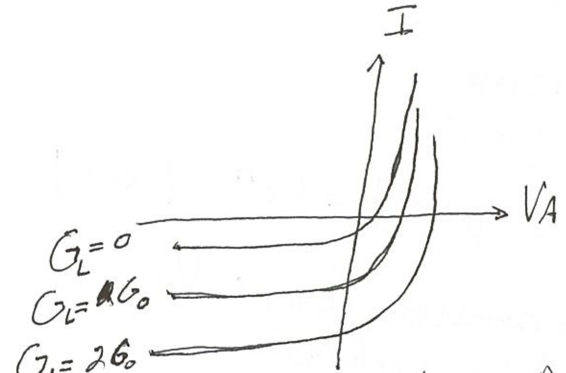
$$G_L = 0$$

– $G_L \neq 0$ Subtraction is done from I values

– dark curve is made downwards and shifted by an amount equal to I_L

$I_L \propto G_L$

- So the downward translation is increased as the value of G_L increases.



I-V characteristic graph

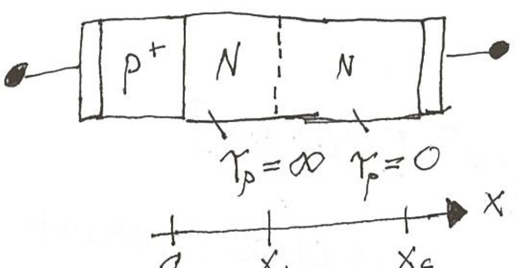
22) Silicon p+-n

$\tau_p = \infty \quad 0 \le x \le x_b$

$\tau_p = 0 \quad x_b \le x \le x_c$

room temperature I-V

depletion width (W) never exceeds x_b for all biases of interest



Δp in n type material

$\Delta p = 0 \quad x_b \le x \le x_c$

τ has the hole minority carrier lifetime $(\tau_p = 0)$

→ The boundary condition is

$\Delta P_n = 0$ at $(x = x_b)$

— p^+-n step junction, only n side of the junction established, depletion width totally on n side of the junction

- $\tau_p = \infty$ at $0 \le x \le x_b$

no recombination I_{R-G} current

- Static state

$$\frac{d\Delta p_n}{dt} = 0$$

- photo-generation rate $G_L = 0$ so there is no light $\frac{d\Delta p_n}{\tau_p} \to 0$

$\because (\tau_p = \infty)$

→ minority carrier diffusion equation reduced to

$$\frac{d^2 \Delta p_n}{dx^2} = 0 \quad [W \le x \le x_b] \quad \text{depletion width } W$$

→ Boundary Condition

$\Delta p_n(x_b) = 0$ &

$\Delta p_n(W) = \frac{n_i^2}{N_D}(e^{qV_A/kT} - 1)$ number of donor atoms N_D $[cm^{-3}]$ & intrinsic carrier concentration is n_i

→ General Solution

$\Delta p_n(x) = A_1 + A_2 x$

B.C. $0 = A_1 + A_2 x_b$

$\Delta p_n(W) = A_1 + A_2 W$

$\therefore$

$\Delta p_n(W) = -A_2(x_b - W)$

$A_2 = -\Delta p_n(W)/(x_b - W)$

Variable A_1

$A_1 = A_2 X_b$

$= \Delta p_n(W)\left(\dfrac{X_b}{X_b - W}\right)$

$\Delta p_n(x)$ is given

$$\Delta p_n(x) = \Delta p_n(W)\left(\frac{X_b - x}{X_b - W}\right)$$

$$= \frac{n_i^2}{N_D}\left(\frac{X_b - X}{X_b - W}\right)\left(e^{qV_A/kT} - 1\right)$$

$[W \leq x \leq X_b]$

Boltzmann Constant : k

Temperature : T

Applied direct current voltage : V_A

Current density J_p

$$J_p = -qD_p \frac{d\Delta p_n}{dx}$$

$$= q\frac{n_i^2}{N_D}\frac{D_p}{x_b - W}\left(e^{qV_A/kT} - 1\right)$$

Current I

$$I \triangleq AJ_p$$

$$= qA\frac{n_i^2}{N_D}\frac{D_p}{X_b - W}\left(e^{qV_A/kT} - 1\right)$$

Area A

IV characteristic diode I

$$qA\frac{n_i^2}{N_D}\frac{D_p}{x_b - W}\left(e^{qV_A/kT} - 1\right)$$

Chapter 7: 2)

- Assume a p^+-n junction
- $N_B(x) = N_D(x)$
- Why is it necessary to specify

$m > -2$

~~Per power law~~

Per power law

$[N_B(x) = N_D(x)] = bx^m \quad [x > 0]$

Bulk semiconductor doping : N_B

Number of donor atoms concentration : N_D

$\rho \triangleq qN_D$

$= qbx^m \quad [0 \leq x \leq x^n \triangleq W]$

W = Depletion width

Poisson equation

$$\frac{d\mathcal{E}}{dx} = \frac{\rho}{K_S\varepsilon_0}$$

$\rho = qbx^m$

$$\frac{d\mathcal{E}}{dx} = \frac{qb}{K_S\varepsilon_0}x^m \quad [0 \leq x \leq W]$$

Semiconductor dielectric constant : K_S

Permittivity of free space : ε_0

electric field $\mathcal{E}(x)$

$$\int_{\mathcal{E}(x)}^{0} d\mathcal{E}' = \int_x^W \frac{qb}{K_S\varepsilon_0}(x')^m dx'$$

$$-\mathcal{E}(x) = \frac{dV}{dx}$$

$$= \frac{qb}{K_s \varepsilon_0} \frac{(x)^{m+1}}{m+1} \Big|_x^W$$

$$= \frac{qb}{(m+1) K_s \varepsilon_0} (W^{m+1} - x^{m+1})$$

Seperation of variables & integrate across the depletion region

$$\int_0^{V_{bi} - V_A} dV = \frac{qb}{(m+1) K_s \varepsilon_0} \int_0^W [W^{m+1} - x^{m+1}] dx$$

$$V_{bi} - V_A = \frac{qb}{(m+1) K_s \varepsilon_0} \left[W^{m+1} x - \frac{x^{m+2}}{m+2} \right] \Big|_0^W$$

$$= \frac{qb}{(m+2) K_s \varepsilon_0} \cancel{\left[W^{m+1} x - \frac{x^{m+2}}{m+2} \right] \Big|_0^W} \; W^{m+2}$$

$$\boxed{W = \left[\frac{(m+2) K_s \varepsilon_0}{qb} (V_{bi} - V_A) \right]^{\frac{1}{m+2}} \quad \text{for } m > -2}$$

chapter 7) 5)

(7.12) $N_B(x) = \dfrac{2}{q K_s \varepsilon_0 A^2 |d(1/C_J^2)/dV_A|}$

(7.13) $x = \dfrac{K_s \varepsilon_0 A}{C_J}$

total charge : Q_N on n side of depletion region

$$Q_N = A \int_0^{x_n \cong W} \rho(x) d(x)$$

$$= qA \int_0^W N_D(x) d(x)$$

Depletion Width : W

Area : A

Junction capacitance C_J is a follows

$$C_J = \frac{dQ_P}{dV_A}$$

$$= \frac{-dQ_N}{dV_A}$$

$$= -qA \frac{d}{dV_A} \int_0^W N_D(x) dx$$

$$C_J = -qAN_D W \frac{dW}{dV_A}$$

also represented as follows

$$C_J = \frac{K_s \varepsilon_0 A}{W}$$

Permitivity: ε_0 dielectric constant : K_s

Semiconductor

derive $\dfrac{dW}{dV_A}$:

$$C_J = \frac{K_s \varepsilon_0 A}{W}$$

$$\frac{dC_J}{dV_A} = -\frac{K_s \varepsilon_0 A}{W^2} \frac{dW}{dV_A}$$

$$\frac{dW}{dV_A} = \frac{-W^2}{K_s \varepsilon_0} \frac{dC_J}{dV_A}$$

$$\frac{dW}{dV_A} = \frac{K_s \varepsilon_0 A}{C_J^2} \frac{dC_J}{dV_A}$$

non-degenerate donor Concentration

$N_D(W)$

$$C_J = -qAN_D W \frac{dW}{dV_A}$$

$$= -qAN_D W \frac{K_s \varepsilon_0 A}{C_J^2} \frac{dC_J}{dV_A}$$

$$= -\frac{qN_D W K_s \varepsilon_0 A^2}{C_J^3} \frac{dC_J}{dV_A}$$

$$N_D(W) = \frac{1}{qK_S\varepsilon_0 A^2[(dC_J/dV_A)/C_J^3]}$$

$$\frac{dC_J/dV_A}{C_J^3} = \frac{-2}{C_J^3}\frac{dC_J}{dV_A}$$

Width : W

Synchronization with distance: X from junction

$$\boxed{N_D(x) = \frac{2}{qK_S\varepsilon_0 A^2 |d(1/C_J^2)/dV_A|}}$$

derive distance X

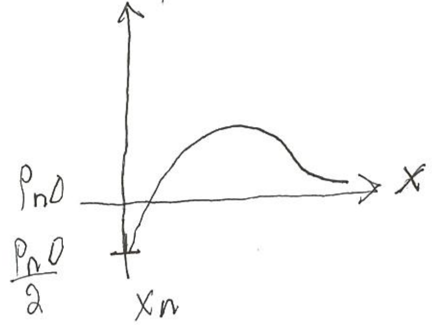
$$C_J = \frac{K_S\varepsilon_0 A}{W}$$

$$W = \frac{K_S\varepsilon_0 A}{C_J}$$

$$\boxed{X = \frac{K_S\varepsilon_0 A}{C_J} \quad [W \cong X]}$$

Chapter 8 ~~question~~ 2 2)

hole concentration on the n-side of a pn step junction diode



a) The junction is reversed biased since the deviation of the carrier concentration from the equilibrium position

$$\Delta p_n(x,t) = \Delta p_n(x_n,t) - p_{n0} < 0$$

Carrier deficit for the junction diode at its edges makes the junction reversed biased.

b) Law of Junction

$$n(x_n)\,p(x_n) = N_D p_{n0}/2$$
$$= n_i^2/2$$
$$= n_i^2 e^{qV_A/kT}$$

donor atom concentration : N_D

intrinsic concentration : n_i

electron charge : q 1.6×10^{-19} [C]

Boltzmann's constant : k 8.617×10^{-5} eV/K

temperature : T 300 [K]

$$V_A = (kT/q)\ln\left(\tfrac{1}{2}\right)$$
$$= \left(\frac{8.617\times10^{-5}\,\text{eV}\ 300\,\text{K}}{1.6\times10^{-19}\,\text{C}}\right)\ln\left(\tfrac{1}{2}\right)$$
$$= -0.018\,\text{V}$$

$$\boxed{V_A = -0.018\,\text{V}}$$

c) $$\left.\frac{d\Delta p_n}{dx}\right|_{x=x_n} = \begin{cases}\text{slope } \Delta p_n(x) \\ \text{or } p_n(x) \\ \text{versus } x \text{ plot} \\ \text{at } X = x_n\end{cases}$$

$$= -\frac{i}{qAD_p}$$

hole diffusion coefficients is D_p

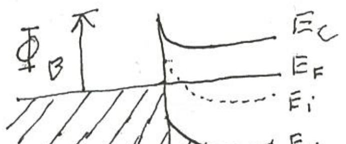
$$\frac{d\Delta p_n}{dx}\bigg|_{x=x_n} = \frac{dp_n}{dx}\bigg|_{x=x_n} > 0$$

for $i<0$ the current flowing is reversed bias

chapter 14 2)

a)

Metal | n-type semiconductor



energy band diagram for ideal MS constants between a metal and an n-type semiconductor $\Phi_M > \Phi_S$ system under equilibrium conditions

b) Combine B, D, F and have been labeled as N_A doped

$$\Phi_S = \chi + (E_C - E_F)_{FB}$$

$$\cong \chi + \frac{E_G}{2} - (E_F - E_i)_{FB}$$

$$(E_F - E_i)_{FB} = \begin{cases} KT\ln\left(\frac{N_D}{n_i}\right) & n\text{-type} \\ -KT\ln\left(\frac{N_A}{n_i}\right) & p\text{-type} \end{cases}$$

A n-type ~~materials~~

Material Ge

~~Type~~

Doping ~~n~~ 10^{16}

$n_i = 2.5\times10^{13}$

$E_F - E_i = 0.155$

$\frac{E_G}{2} = 0.33$

$\Phi_S = 4.18$

$\therefore$

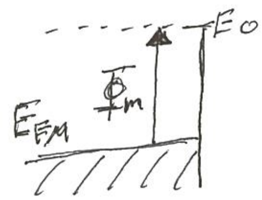
$\Phi_M > \Phi_S$

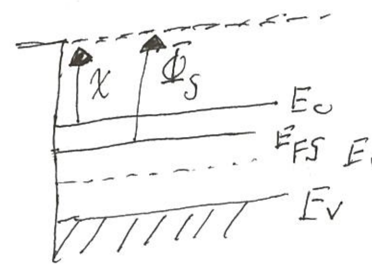
Rectifying ideal nature

When energy band diagram is characterised by the ideal contact

b) Surface induced energy band diagram for a metal (a) and ~~and~~ n-type (semi conductor)

(a) (b)





3) $\Phi_M = 5.6$ eV $\chi(Si) = 4.03$ eV

$N_D = 10^{15}$ cm^{-3}

a) $\Phi_B = \Phi_M - \chi$

$= 5.10$ eV $- 4.03$ eV

$= 1.07$ eV

b) Built-in Voltage V_{bi}

$$V_{bi} = \frac{1}{q}\left[\Phi_B - (E_c - E_F)_{FB}\right]$$

$$(E_c - E_F)_{FB} = \frac{E_G}{2} - (E_F - E_i)_{FB}$$

$$= \frac{E_G}{2} - kT \ln\left(\frac{N_D}{n_i}\right)$$

$$= 0.56 - (0.0259)\ln\left(\frac{10^{15}}{10^{10}}\right)$$

$$= 0.26 \text{ eV}$$

$$\boxed{V_{bi} = 1.07\text{V} - 0.26\text{ V} = 0.81\text{ V}}$$

c)
$$W = \left[\frac{2K_S\varepsilon_0}{qN_D}(V_{bi} - V_A)\right]^{1/2}$$

$$= \left[\frac{2(11.8)(8.85\times10^{-14})(0.81-0)}{(1.6\times10^{-19})(10^{15})}\right]^{1/2}$$

$$= 1.03\times10^{-4}\text{ cm}$$

d)
$$|\mathcal{E}|_{max} = |\mathcal{E}_{x=0}|$$

$$= \frac{qN_D}{K_S\varepsilon_0}W$$

$$= \frac{(1.6\times10^{-19})(10^{15})(1.03\times10^{-4})}{(11.8)(8.85\times10^{-14})}$$

$$= 1.58\times10^{4}\text{ V cm}^{-1}$$

b) Current for a set of electrons

$$I_{S\to M} = -qA\int_{-\infty}^{-v_{min}} v_x\, n(v_x)\, dv_x$$

$$n(v_x) = \left(\frac{4\pi kT m_n^{*2}}{h^3}\right) e^{\left\{\frac{(E_F - E_c)}{kT_e} - \frac{m_n^*}{2kT}v_x^2\right\}}$$

$$I_{S\to M} = -qA\int_{-\infty}^{-v_{min}} v_x\left[\left(\frac{4\pi kT m_n^{*2}}{h^3}\right) e^{\left[\frac{(E_F-E_c)}{kT_e} - \frac{m_n^* v_x^2}{2kT}\right]}\right] dv_x$$

$$= qA\left[\frac{4\pi kT m_n^{*2}}{h^3}\right] e^{\frac{(E_F-E_c)}{kT}} \int_{-\infty}^{-v_{min}} v_x e^{\frac{-m_n^* v_x^2}{2kT}} dv_x$$

$$= \text{''} \quad \text{''}\int_{v_{min}}^{\infty}\text{''}$$

$$= qA\left[\frac{4\pi kT m_n^{*2}}{h^3}\right] e^{\frac{(E_F-E_c)}{kT}}\left[\frac{-kT}{m_n^*}\right] e^{\frac{-m_n^*}{2kT}v_x^2}\Big|_{v_{min}}^{\infty}$$

$$= \text{''} \quad \text{''}\left[0 - e^{\frac{-m_n^*}{2kT}v_{min}^2}\right]$$

$$= \text{''} \quad \text{''}\left[\frac{kT}{m_n^*}\right]\left[e^{\frac{m_n^*}{2kT}v_{min}^2}\right]$$

$$v_{min}^2 = \frac{2q}{m_n^*}(V_{bi} - V_A)$$

$$I_{S\to M} = qA\left(\frac{4\pi kT m_n^{*2}}{h^3}\right) e^{\frac{(E_F-E_c)}{kT}} e^{\frac{-q(V_{bi}-V_A)}{kT}}$$

$$= qA\left[\frac{4\pi kT m_n^{*2}}{h^3}\right] e^{\frac{(E_F-E_c)}{kT}} e^{\frac{-qV_{bi}}{kT}} e^{\frac{qV_A}{kT}}$$

$$= \text{''} \quad \text{''}\, e^{\frac{(E_F-E_c)}{kT} - \frac{qV_{bi}}{kT}} e^{\frac{qV_A}{kT}}$$

$$\frac{qV_{bi}}{KT} = \frac{\Phi_B}{KT} + \frac{(E_F - E_c)}{KT}$$

$$\frac{(E_F - E_c)}{KT} - \frac{qV_{bi}}{KT} = \frac{-\Phi_B}{KT}$$

$$qA\left(\frac{4\pi K^2 T^2 m_n^*}{h^3}\right) = A\left[\frac{m_n^*}{m_0}\right]\left[\frac{4\pi q m_0 K^2}{h^3}\right]T^2$$

$$= AA^*T^2$$

Where $A^* = \left[\frac{m_n^*}{m_0}\right]\left[\frac{4\pi q m_0 K^2}{h^3}\right]$

$$\boxed{I_{S\to M} = AA^*T^2 e^{-\frac{\Phi_B}{KT}} e^{\frac{qV_A}{KT}}}$$

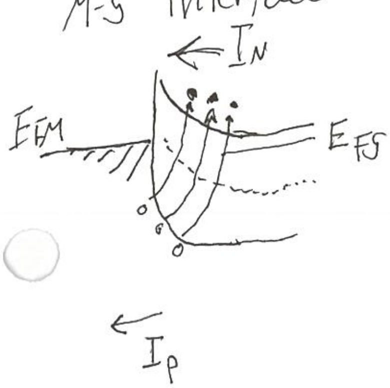
7) a)

→ diode terminals short circuited

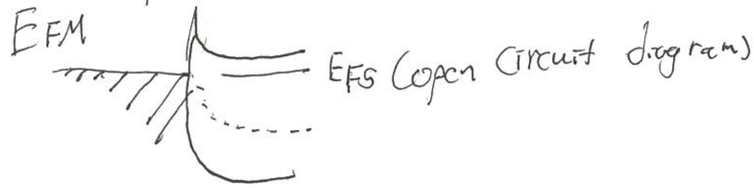
→ photogenerated carriers created in semiconductor near MS interface

E_{FM} same as E_{FS} ∵ short circuit

F_N & F_p deviate from E_{FS} near M-S interface



b) forward beveled under open open circuit



Negative going photocurrent balanced by positive going thermometric emission current

c) $(I_L) = -q$ multiplied by electro hole pair photo generated per second in volume

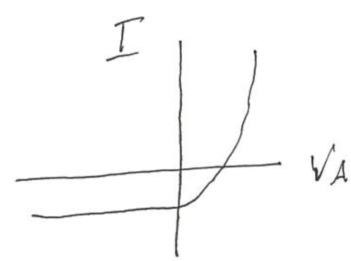
$W + L_p = A$

$I_L = -qA(W+L_p)G_L$

d) Constant value subtracted from dark I-V characteristics to obtain the light on characteristic

$I < 0$ when device is shorted

$(V_A = 0)$ & $V > 0$



2018-04-13

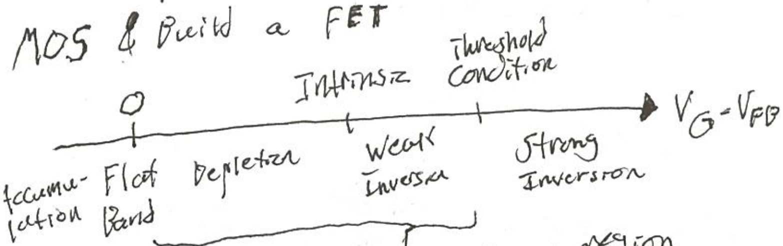
Device Electronics

- ~~Presentation~~ Hand in report on May 4^rd

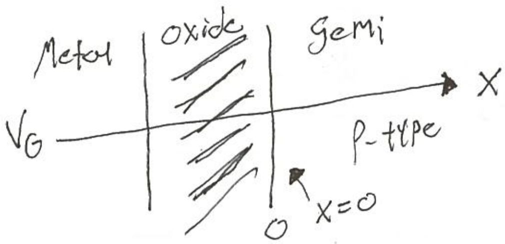
~~Monday~~

- HW #4 (final HW)

MOS & Build a FET



$\phi_s = \phi(x)|_{x=0}$



Flat Band

0

$\phi_s = \phi_p$

$p_s = N_a$

(easiest part)

Threshold condition

$\phi_s = -\phi_p = +V_T \ln\left(\frac{N_a}{n_i}\right)$

$\Updownarrow$

$n_s = N_a$

accumulation

$\phi_s < \phi_p$

$p_s > N_a$

Strong Inversion

$n_s > N_a$

$\underbrace{\phi_s > -\phi_p}_{\text{Today}}$

Depletion

$n_i < p_s < N_a$

Intrinsic

$p_s = n_i$

$n_s = n_i$

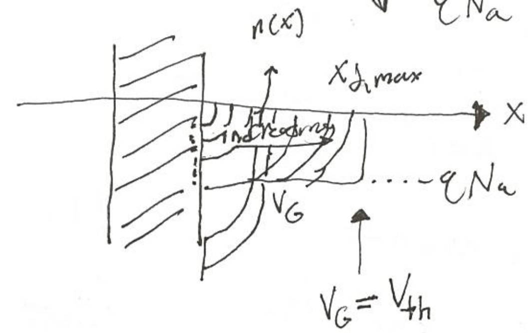
Weak Inversion

$n_i < n_s < N_a$

$p_s < n_i$

$V_{TH} = V_{FB}^0 + 2|\phi_p| + \frac{1}{C_{ox}}\underbrace{\sqrt{4\epsilon_s q N_a |\phi_p|}}_{\text{Depletion Charge}}$

$X_d|_{@V_a = V_{th}} = X_{d\,max} = \sqrt{\frac{4\epsilon_s|\phi_p|}{qN_a}}$

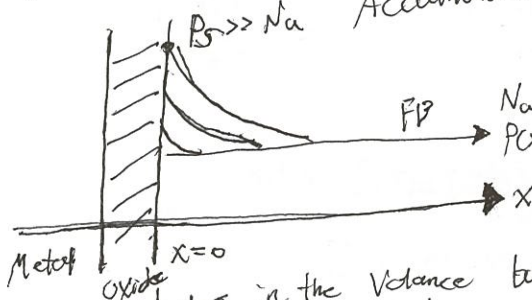


$V_G > V_{TH}$

n_s can increase ~~almost~~ without bounds

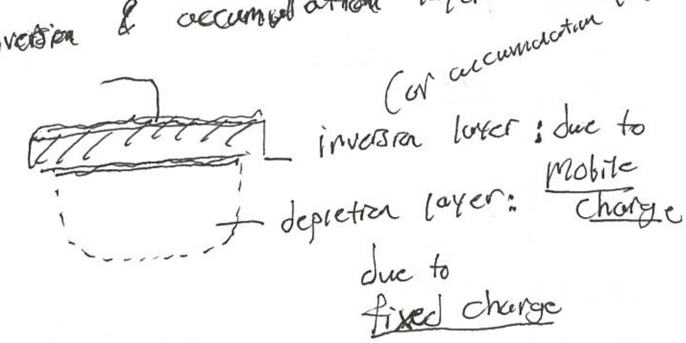
$V_G \le V_{FB}$ Flat band potential

Accumulation



holes in the valance band, not limited by any doping density due to mobile charges,

· inversion & accumulation layer



~~accumulation~~ of holes how thin is accumulation/inversion layer

$X_d|_{@V_G = \cancel{V_{th}}} = x_{d\,max} = \sqrt{\frac{4\epsilon_s|\phi_p|}{qN_a}}$

$2|\phi_p|$

$\frac{d^2\phi}{dx^2} = \frac{-\rho}{\epsilon_s} = \cancel{\ldots} \frac{q}{\epsilon_s}(n - p - D)$

Accumulation: ✓

$p \gg N_a$ $\quad \cong \frac{q}{\epsilon_s}(x - p \quad x) \quad (\cancel{A} - p - 0) \quad (n - p - V)$

Inversion:

$n \gg N_a$ $\quad \cong \frac{q}{\epsilon_s}(n \times x)$

(at or near $x = 0$) $= \frac{q n_i}{\epsilon_s} e^{\phi/V_T}$ ← !!!

In Strong inversion only
$n \gg N_a$ I think

Accumulation example

Accum: $\frac{d^2\phi}{dx^2} = \frac{-q n_i}{\epsilon_s} e^{-\phi/V_T}$

Recall that: $p = n_i e^{-\phi/V_T}$

=> ??

$$\frac{d^2\phi}{dx^2} = \frac{-q n_i}{\epsilon_s}\frac{p_s}{p_s} e^{-\phi/V_T} = \frac{q p_s}{\epsilon_s} e^{-(\phi - \phi_s)/V_T}$$

$p_s = n_i e^{-\phi_s/V_T}$

$\phi_s \ll -V_T \ln\left(\frac{N_a}{n_i}\right) \Rightarrow p_s \gg N_a$

ϕ_s: Surface potential [V]

ϕ_p: Potential of the Substrate deep in the Bulk [V]

$\phi' = \phi - \phi_s$

$$\frac{d^2\phi'}{dx^2} = \frac{-q p_s}{\epsilon_s} e^{-\phi'/V_T}$$

$$E = \frac{-d\phi}{dx} = \frac{-d\phi'}{dx}$$

$$\frac{dE}{dx} = \frac{q p_s}{\epsilon_s} e^{-\phi'/V_T}$$

Chain rule

$$\frac{dE}{dx} = \frac{dE}{d\phi'}\frac{d\phi'}{dx} = -E\frac{dE}{d\phi'}$$

$$E\frac{dE}{d\phi'} = -\frac{q p_s}{\epsilon_s} e^{-\phi'/V_T}$$

← know this trick

$$\boxed{E\frac{dE}{d\phi'}} = -K e^{-\phi'/V_T}$$

$\hookrightarrow \frac{1}{2}\frac{d}{d\phi'}(E^2)$

$$\frac{1}{2}\frac{d}{d\phi}(E^2) = 2E\frac{dE'}{d\phi'}\cdot\frac{1}{2}$$

$E(\infty)$ ← deep in the Substrate

$$\frac{1}{2}\int_{E(x)}^{E(\infty)} d(E^2) = -K\int_{\phi'(x)}^{\phi(\infty)} e^{-\phi/V_T}\, d\phi'$$

$$\frac{1}{2}\left[\cancel{E^2(\infty)}^{\,0} - E^2(x)\right] = 0$$

$$-K\int_{\phi'(x)}^{\phi'(\infty)} e^{-\phi'/V_T}\, d\phi' \quad ?$$

$$-E^2(x) = 2KV_T\left[e^{-\frac{\phi'(\infty)}{V_T}} - e^{\frac{-\phi'(x)}{V_T}}\right]$$

$$E^2(x) = 2KV_T\, e^{-\phi'(x)/V_T}$$

$$E(x) = -\sqrt{2KV_T}\; e^{\frac{-\phi'(x)}{2V_T}}$$

$$E(x) = -\sqrt{\frac{2p_s q V_T}{\epsilon_s}}\; e^{-\frac{\phi'(x)}{2V_T}}$$

$$-\frac{d\phi'}{dx} = \nearrow$$

$$\int_{\phi'(0)=0}^{\phi'(x)} e^{\phi'/2V_T}\, d\phi' = \sqrt{\quad}\int_0^x dx = \sqrt{\quad}\cdot x$$

$$2V_T\left[e^{\frac{\phi'(x)}{2V_T}} - 1\right] = \sqrt{\quad}\cdot x$$

$$e^{\phi'/2V_T} = \frac{x}{\sqrt{2}L_D} + 1$$

$$L_D = \sqrt{\frac{\epsilon_s V_T}{q p_s}} = \text{Debye length}$$

$$\rho(x) = q p(x) = q p_s e^{-\phi'/V_T}$$

$$\Rightarrow \boxed{\rho(x) = \frac{q p_s}{\left(\frac{x}{\sqrt{2}L_D} + 1\right)^2}} \leftarrow !!!$$

charge density

- Charge density varies with respect to x & characteristic distance of L_D which is Debye length

Depletion:

$$X_d = \sqrt{\frac{\epsilon_s(\text{built in potential})}{q(\text{Doping})}}$$

Depletion

$$X_d = \sqrt{\frac{\epsilon_s(\text{built in potential})}{q(\text{doping})}}$$

doping density 5.6 of Mag greater than n_i

Accum:

$$\frac{d^2\phi}{dx^2} = -\frac{q n_i}{\epsilon_s} e^{-\phi/V_T}$$

Recall: $\rho = n_i e^{-\phi/V_T}$

⇒ ??

$$\frac{d^2\phi}{dx^2} = -\frac{q n_i}{\epsilon_s}\frac{\rho_s}{\rho_s} e^{-\phi/V_T}$$

$$= -\frac{q\rho_s}{\epsilon_s} e^{-(\phi-\phi_s)/V_T}$$

$$\phi' = \phi - \phi_s$$

$$\rho_s = n_i e^{-\phi_s/V_T}$$

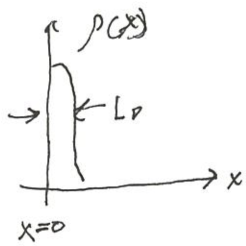
$$\phi_s << -V_T \ln\left(\frac{N_a}{n_i}\right) \Rightarrow \rho_s >> N_a$$

Home work due for the Strong inversion layer

problem #1

repeat for $n_s >> N_a$ for mobile electrons ~~p-type sheet~~ inversion layer show charge density electrons at surface

$$q(x) = \frac{qN_s}{\left(\frac{x}{\sqrt{2}L_D}+1\right)^2}$$

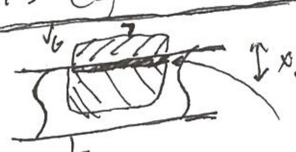


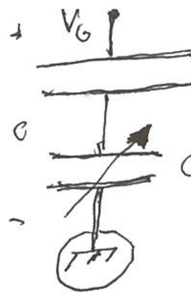
for analog

- ~~layout~~ schematic
- simulation
- layout (drc checks)

$$\rho(x) = \frac{q n_s}{\left(\frac{x}{\sqrt{2}L_D}+1\right)^2} \qquad L_D = \sqrt{\frac{\epsilon_s V_T}{q n_s}}$$

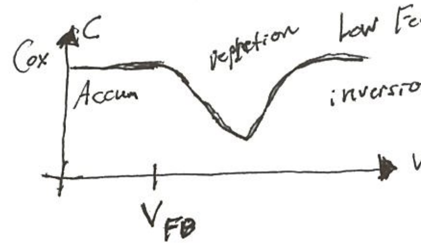
MOS Capacitance:



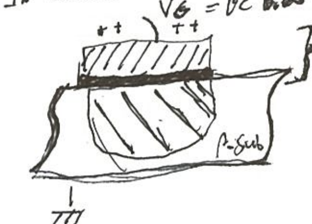


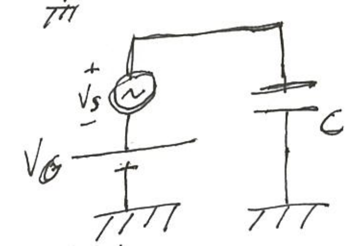
MOS system looks like two capacitors in series

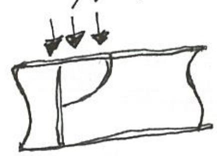
$$\frac{1}{C} = \frac{1}{C_{ox}} + \frac{1}{C_s(V_a)}$$



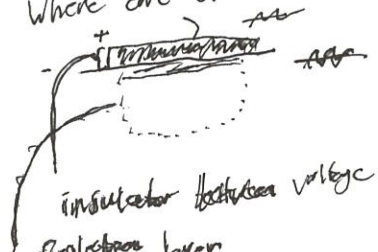
In accumulation ⇒ capacitor this thick







Where are electrons coming from in p-type Substr



Electrons created by generation →

generation recombination parameters, lifetime

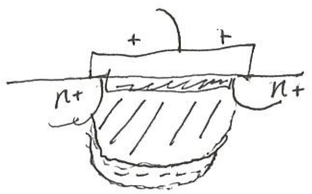
$$R = \frac{np - n_i^2}{\tau_p(n+n_i) + \tau_p(p+n_i)}$$

Can only add a negative ~~the~~ depletion layer

needs to go down a little bit & get less wide

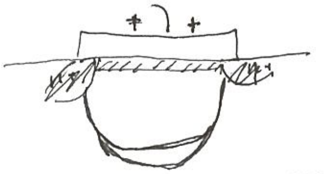
If you have a thick capacitor you have a small capacitance in MOS

Transform into MOSFET

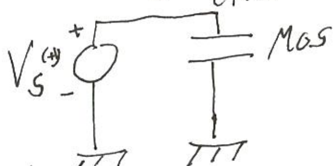


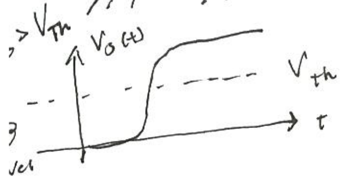
follows low frequency curve for the MOSFET

If you have MOS device

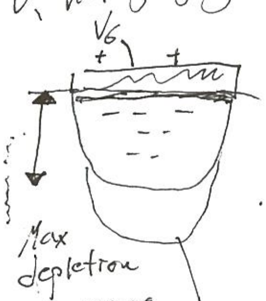


$$x_{d\,max} = \sqrt{\frac{\epsilon_s 4|\phi_p|}{q N_a}}$$





If, nothing going on



• Voltage

depletion charge forms

holes in depletion layer

• then inversion layer forms

full of electrons

as generation takes place, the IL starts going up back to steady state max.

For a while thickness is greater than Max based on timescales

$\tau \sim 10^{-5}$ sec

region depleted of holes and no electrons are being generated yet via generation

Shine light → generate electrons & holes, electrons go up to inversion layer holes go away

• layer of electrons in proportion to the intensity of the light

Pixels in charge coupled device

- concept Deep depletion

depleting MOS, should have inversion, transient not enough time for inversion layer to form, depletion layer is greater than Max

CCD camera

electron lifetime

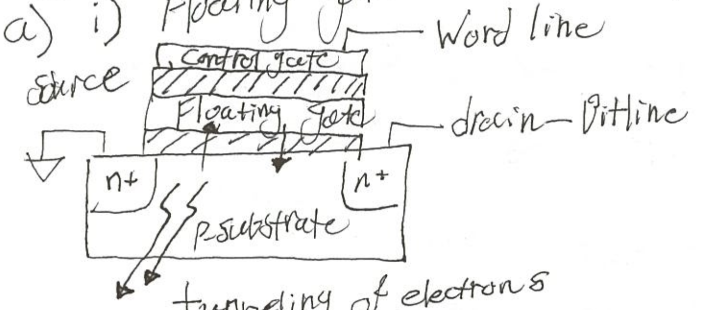
- originally used for a shift register

original pattern, shift register, now bars for camera

Device Final 2014
Joseph Crandall 2018-04-09

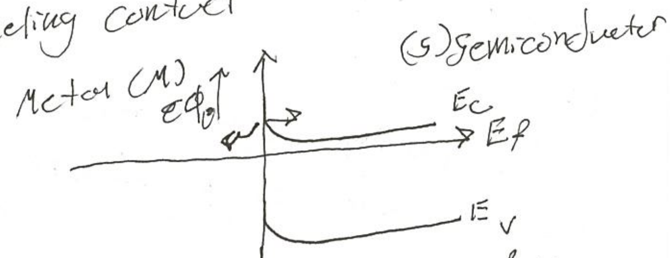
1) Non-Volatile MOS type Memory

a) i) Floating gate is one example



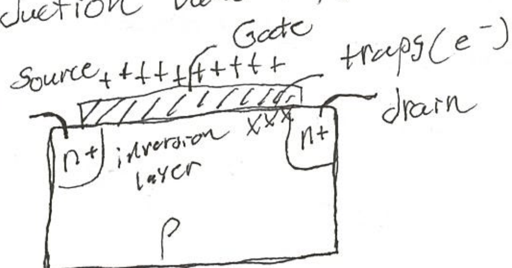
→ Charge tunnels between the substrate and the floating gate; this action is controlled by the word line (control gate)

ii) Ohmic Contact: Consider Schottky tunneling contact

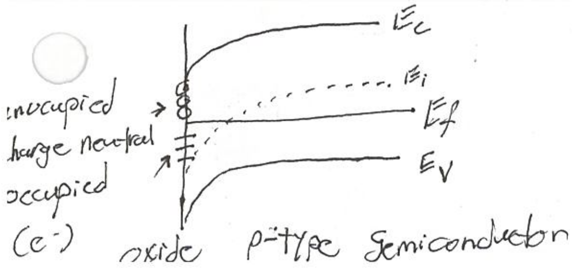


electrons ~~run~~ tunnel from Conduction Band to/from Metal

b)

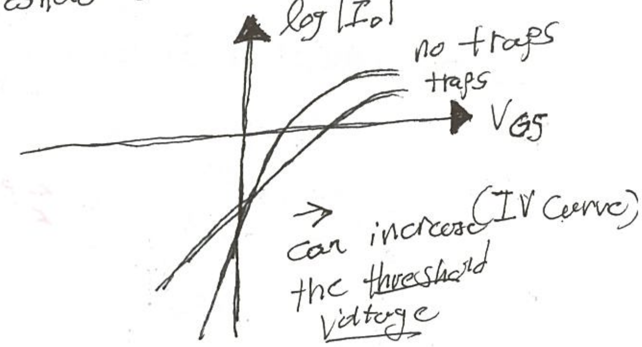


traps have a higher probability of ~~occ~~ occupation when the fermi-level is above the trap energy level. Consider the following MOS band diagram:

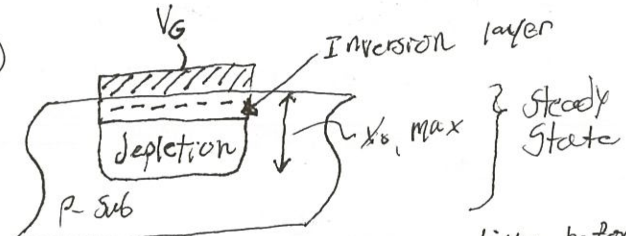


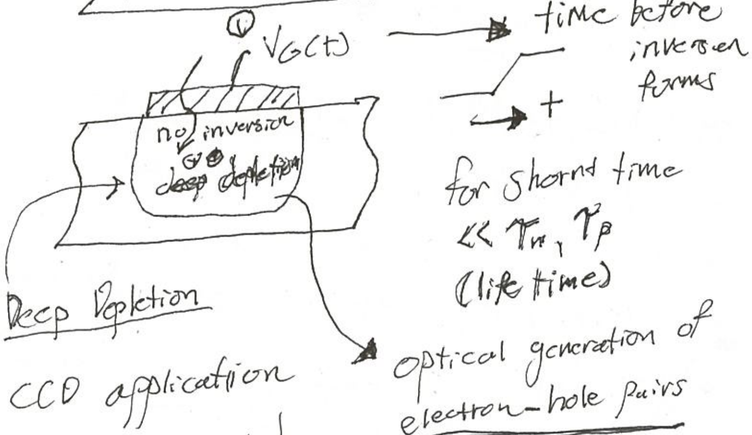
Traps bellow E_f are occupied

→ These electrons will shield some of the + charges on the gate → Decrease the rate of formation of the inversion layer → decrease the slope of the subthreshold curve.



c)





Deep Depletion

CCD application (charge coupled device)

2) a) Band diagram

Φ_M ~~ϕ_m~~ : metal work function [V]

Φ_S [V] : Semiconductor work function

E_c [J] : Conduction band energy [$kg\, m^2 s^{-2}$]

E_0 [J] [$kg\, m^2 s^{-2}$] :

E_f [J] [$kg\, m^2 s^{-2}$] : fermi energy

E_i [J] [$kg\, m^2 s^{-2}$] : intrinsic energy

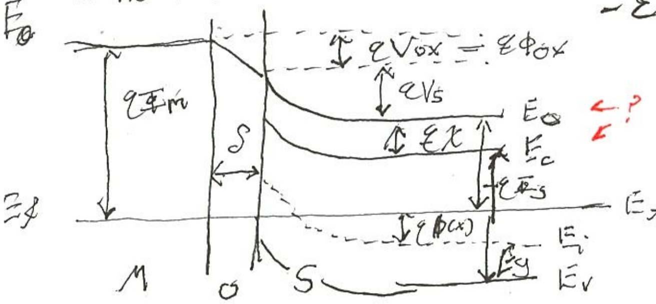
E_v [J] [$kg\, m^2 s^{-2}$] : Valence band energy

V_S [V][$kg\,m^2A^{-1}s^{-3}$] : total voltage drop in the semiconductor

Conditions

- $\Phi_M > \Phi_S$
- no V_A between n-type substrate (doping N_D) and gate
  - no trap charges in oxide & in interface

$-\mathcal{E} = \mathcal{E}V$



M: Metal minimum energy to make free electron

O: Oxide

S: Semiconductor E_0 [J] : Vacuum ~~potential~~ energy

χ [V] : electron affinity

E_g [J][$kg\,m^2s^{-2}$] : energy band gap

E_i [J][$kg\,m^2s^{-2}$] : energy midgap

δ [m] : oxide thickness

ϕ_{ox} [V][$kg\,m^2A^{-1}s^{-3}$] : Potential drop ~~in~~ of oxide

$\mathcal{E}$ [C][AS] : charge (potentially charge of electron)

$\phi(x)$ [V][$kg\,m^2A^{-1}s^{-3}$] : Potential drop in semiconductor

V_s [V] : total voltage drop in semiconductor

b) n-type substrate
not accumulation Factors $\Phi_m - \Phi_s$ could be depending on either depletion or inversion

$\mathcal{E}\phi(x)$: $E_f - E_i$

electrostatic potential

(C) Inversion layer

- ~~total~~ voltage drop in semiconductor is V_S

$$V_S + V_{ox} = \Phi_{ms} = \Phi_m - \Phi_s$$

$$\frac{d^2\phi}{dx^2} = \frac{-\rho}{\epsilon_s} = \frac{q}{\epsilon_s}(n - p - D)$$

ϕ : electric potential [V]

ρ ~~[cm^{-3}] charge~~ [$C\,m^{-3}$]: charge density

ϵ: permittivity of free space 8.854×10^{-12}

q: electron charge [C] 1.6×10^{-19}

n: electron density [m^{-3}]

p: hole density

D: $N_d^+ - N_a^-$

N_d^+: doping donor density [m^{-3}]

N_a^-: doping acceptor density [m^{-3}]

Inversion

$n_s >> N_a$

n_s electron density at surface

N_a doping acceptor density ? 578, 613

Inversion example

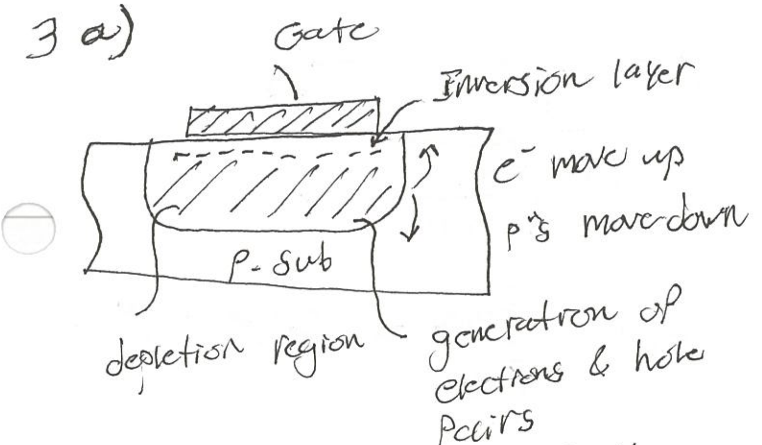
$$\frac{d^2\phi}{dx^2} = \frac{qn_i}{\epsilon_s} e^{\phi/V_T}$$

Accumulation example

$$\frac{d^2\phi}{dx^2} = \frac{-qn_i}{\epsilon_s} e^{-\phi/V_T}$$

trying to get ~~depletion~~ of $X_d = \sqrt{\frac{2\epsilon_s}{qN_a}}$

3 a)



a) Generation, The formation of the inversion layer occurs as follows:

1) Ramp up gate voltage
2) Rapidly deplete holes as they are pushed into the substrate
3) At this stage, in the depletion region

$p \approx 0$, $n = \frac{n_i^2}{N_a} \approx$ small

Consider indirect recombination

$$R = \frac{np - n_i^2}{\tau_p(n+n_i) + \tau_n(n+n_i)}$$

$$n, p \to 0 \Rightarrow R = \frac{-n_i}{\tau_p + \tau_n} < 0$$

$\tau_p : [s^{-1}]$: ptype time constant $= \frac{1}{c_p N_T}$

$\tau_n : \frac{1}{c_n N_T}$

- Because R is less than zero, electrons are being generated

b) strong inversion
inversion layer at $Si - SiO_2$ interface

$n_s\ [m^{-3}]$: electron density at ? (surface or semiconductor)

$N_A\ [m^{-3}]$: acceptor substrate density

$n_s >> N_A$

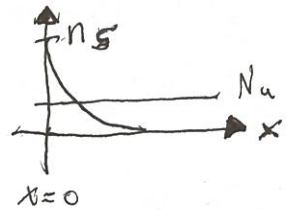
$$\frac{d^2\phi}{dx^2} = \frac{q}{\epsilon_s}(n - p - D)$$

Strong Inversion

$n_s > N_a$

$\phi_s > -\phi_p$

$\phi_s [V]$: electric potential at surface
$\phi_p [V]$: electric potential in bulk p-substrate



@ & near $x = 0$

$p \approx 0$

$n_s >> N_a$

e-field E: $[NC^{-1}]$ $[Vm^{-1}]$ $[kg\,m\,s^{-3}A^{-1}]$

$\int - dx$

electric potential : ϕ $[V]$ $[kg\,m^2 A^{-1} s^{-3}]$

$\times q$ $[C][As]$

energy E: $[J]$ $[kg\,m^2 s^{-2}]$

$$\frac{d^2\phi}{dx^2} = \frac{q}{\epsilon}(n - p - D)$$

$$\therefore \frac{d^2\phi}{dx^2} = \frac{q}{\epsilon} n$$

$n = n_i e^{(E_F - E_i)/kT}$

$n = n_i e^{\phi/V_T}$ $\because V_T = \frac{kT}{q}$

$$\therefore \frac{d^2\phi}{dx^2} = \frac{q n_i}{\epsilon_s} e^{\phi/V_T}$$

$V_T [V]$: thermal ~~voltage~~ voltage $= 0.02586\ V$ @ $T = 300K$

$\because \phi_s \equiv \phi(x = 0)$
Peak potential at $Si - SiO_2$ interface

$\phi'(x) = \phi(x) - \phi(0)$

& $\because e^{\phi'/V_T} = \frac{n}{n_s}$

replace n_i with n?

$$\therefore \frac{d^2\phi'}{dx^2} = \frac{q n_s}{\epsilon_s} e^{\phi'/V_T}$$

?

note $\frac{d^2\phi}{dx^2} = \frac{d}{dx}\left(\frac{d\phi'}{dx}\right) = -\frac{dE}{dx}$

$$-\frac{dE}{dx} = -\frac{dE}{d\phi'}\frac{d\phi'}{dx} = \frac{dE}{d\phi'} E = \frac{1}{2} d(E^2) \frac{1}{d\phi'}$$

S.O.V. I.B.P. chain ?

$$\frac{d^2\phi'}{dx^2} = \frac{1}{2}\frac{1}{d\phi'}\, d(E^2)$$

$$\int_{E(x)}^{E(\infty)=0} d(E^2) = \frac{2qn_s}{\epsilon}\int_{\phi'(x)}^{\phi'(\infty)} e^{\phi'/V_T}\cdot d\phi'$$

$$0 - E^2(x) = \frac{2qn_s}{\epsilon}\left(\frac{V_T}{1}\right)\left[e^{\phi'(\infty)/V_T} - e^{\frac{\phi(x)}{V_T}}\right]$$

$$e^{\phi'(\infty)/V_T} \approx 0$$

$$\therefore\; E(x) = \sqrt{\frac{2qn_sV_T}{\epsilon_s}}\, e^{\phi'/2V_T} = -\frac{d\phi'}{dx}$$

$$d\phi'\, e^{-\phi'/2V_T} = -\sqrt{\frac{2qn_sV_T}{\epsilon_s}}\, dx$$

?

$$\int_0^{x} e^{-\phi'/2V_T}\, d\phi' = \int_0^{x} -\sqrt{\frac{2qn_sV_T}{\epsilon_s}}\, dx$$

$$-2V_T\left[-e^{\frac{-\phi'(x)}{2V_T}} - e^{\frac{-\phi(0)}{2V_T}}\right] = \sqrt{\frac{2qn_sV_T}{\epsilon_s}}\, x$$

$$-2V_T\left[1 - e^{-\frac{\phi'(x)}{2V_T}}\right] = \sqrt{\frac{2qn_sV_T}{\epsilon_s}}\, x$$

$$e^{-\frac{\phi'(x)}{2V_T}} = \frac{\sqrt{\frac{2qn_sV_T}{\epsilon_s}}\, x}{2V_T} + 1$$

$$e^{\frac{\phi'(x)}{V_T}} = \frac{1}{\left(\frac{\sqrt{\frac{2qn_sV_T}{\epsilon_s}}\, x}{2V_T} + 1\right)^2}$$

definition of n

$$n = \frac{n_s}{\left(\frac{\sqrt{\ }\, x}{2V_T} + 1\right)^2}$$

$$n = \frac{n_s}{\left(\frac{x}{\sqrt{2}\sqrt{\frac{V_T\epsilon_s}{qn_s}}} + 1\right)^2}$$

$$L_D = \sqrt{\frac{V_T\epsilon_s}{qn_s}} = \text{Debye length}$$

$$n = \frac{n_s}{\left(\frac{x}{\sqrt{2}L_D} + 1\right)^2}$$

(?) how are we showing inversion layer charge density as a function of distance? We are just showing n as a function of distance

c) $x_d = \sqrt{\frac{2\epsilon_s}{qN_a}\phi_b} >>$

ϕ_b = built in potential

typical depletion length

→ how are you deriving x_d ?

Conduction band moving away from E_f, causing depletion.

accumulation - of majority carrier

Flat band

$\boxed{N\ MOS}$

p majority in bulk — ~~p type~~

--- E_i

----- E_f

~~N~~ majority in bulk — $\boxed{P\text{-}\cancel{type}\ MOS}$

--- E_f

--- ~~E_i~~

? depletion - of majority carrier

inversion of majority with minority carrier

P-mos, ~~N type~~ n bulk substrate, inversion layer holes

n-mos, p bulk substrate, inversion layer e

NMos

------- E_i

--- E_f

p-type bulk

form an inversion layer of electrons

PMOS

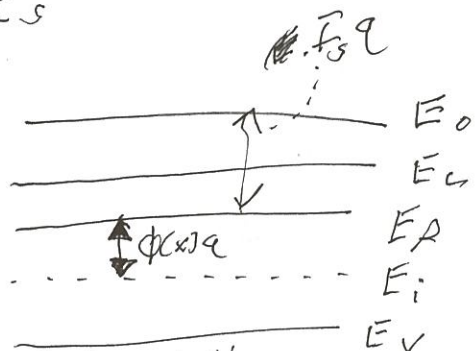
n-type bulk — Flat band

--- E_f

~~---~~ E_i

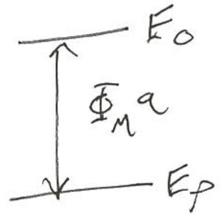
M | O | S

$\Phi_M > \Phi_s$



PMOS, n bulk

$\Phi_M > \Phi_s$

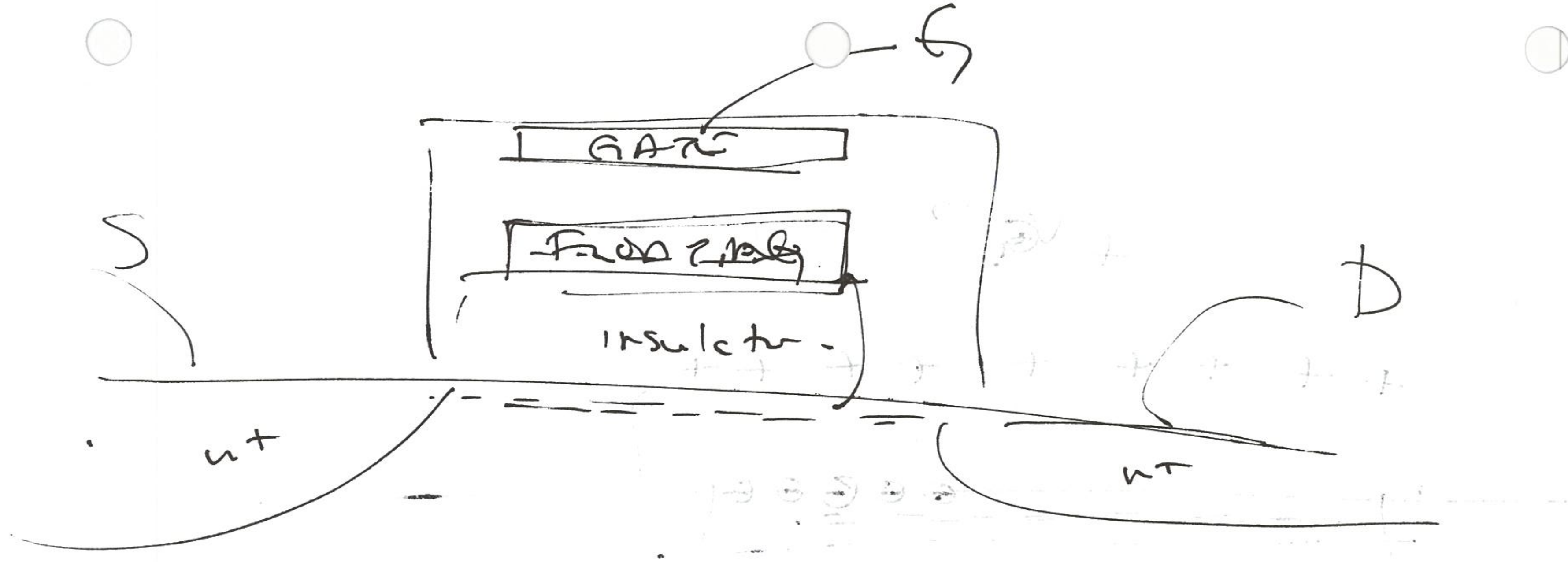


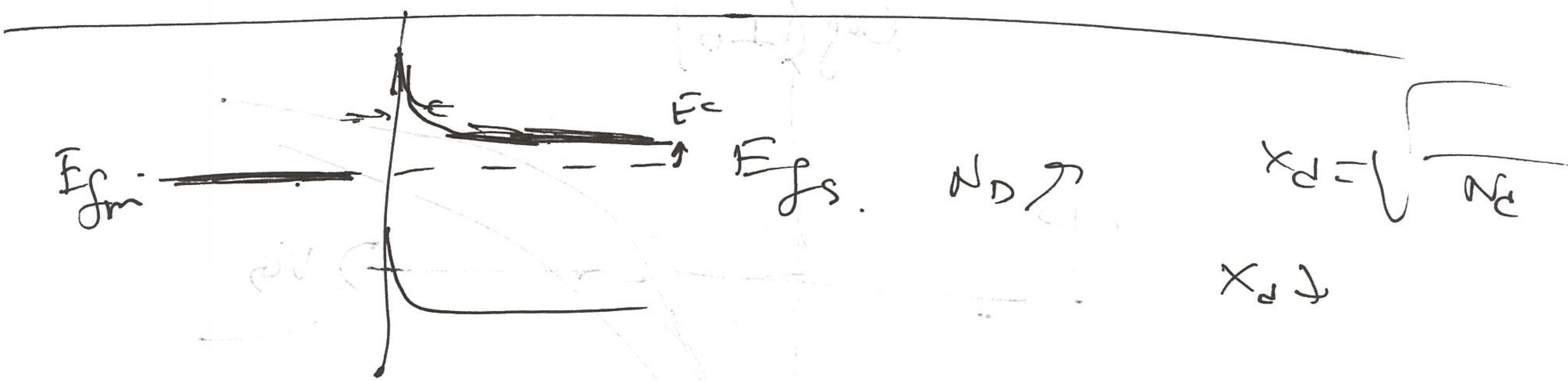
$[kg\, m^2 A^{-1} s^{-3}]$ $[C] = [A s]$

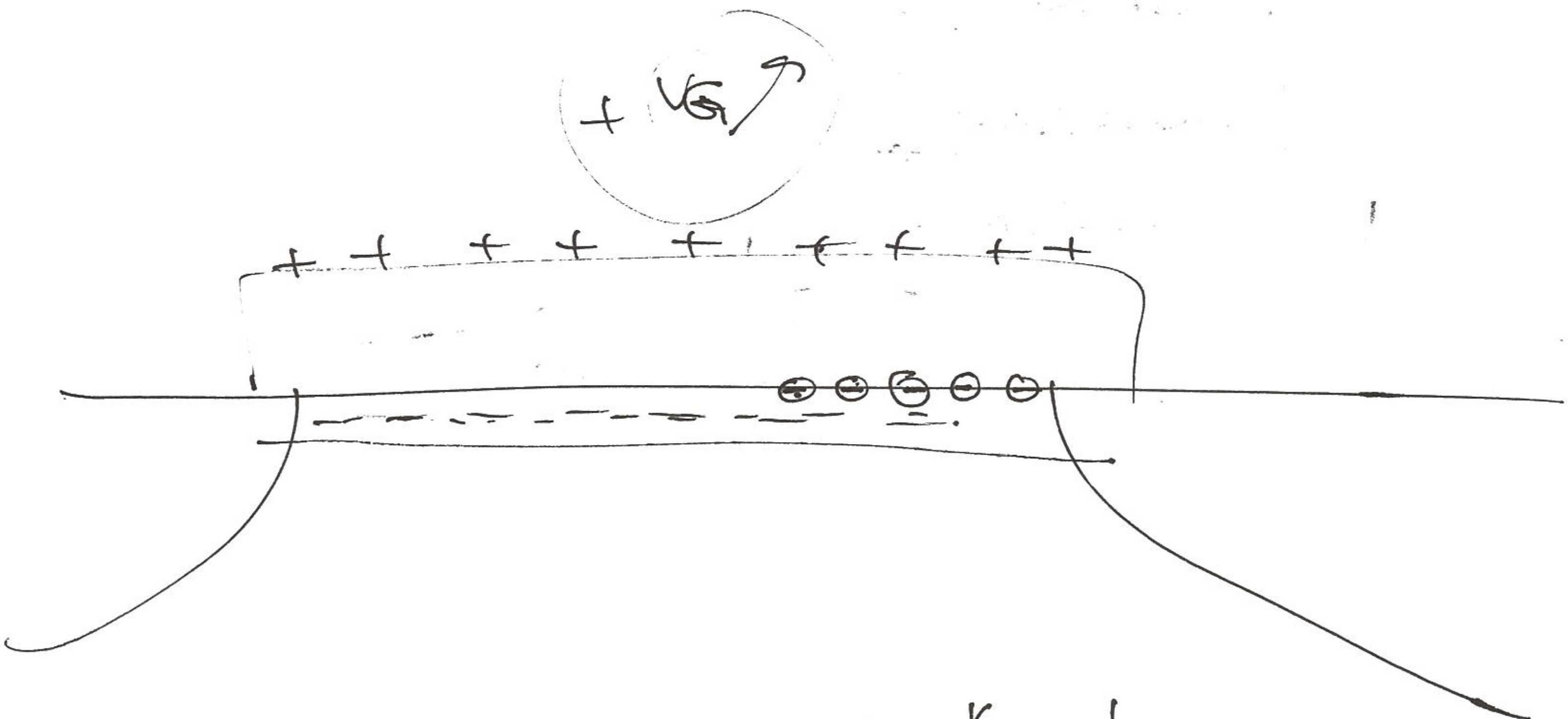
$[V][C] = [kg\, m^2 \cancel{A} s^{-2}]$

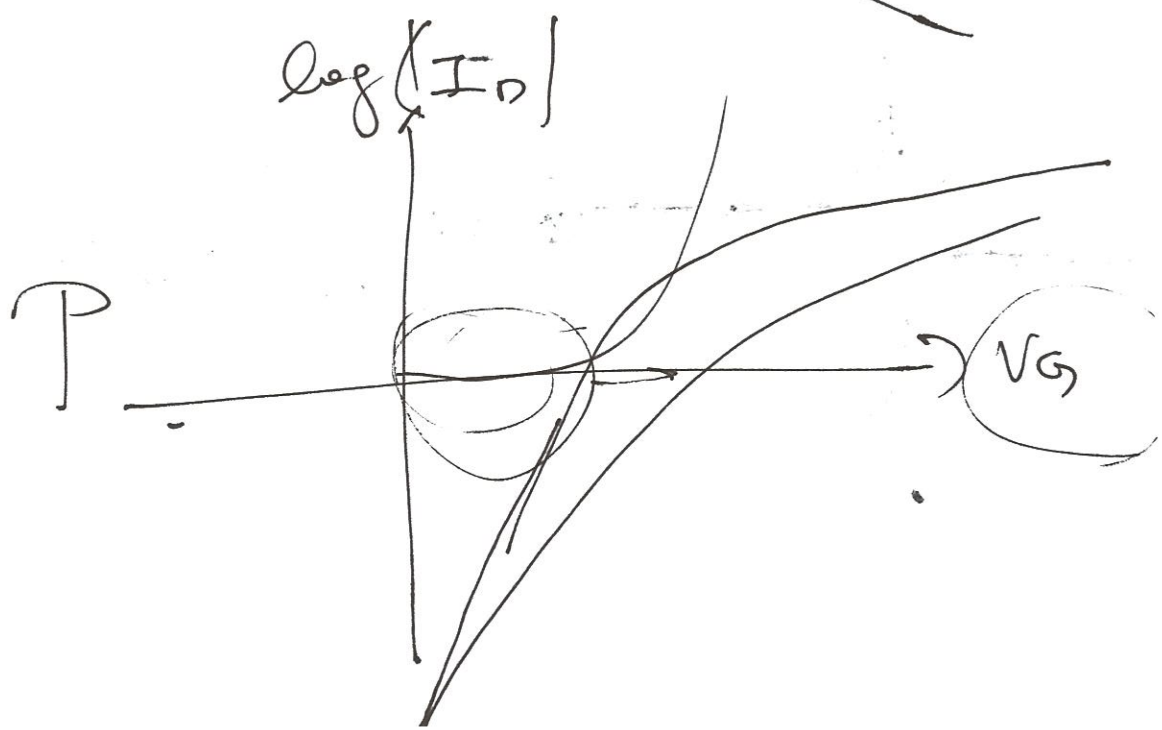
$[J]$

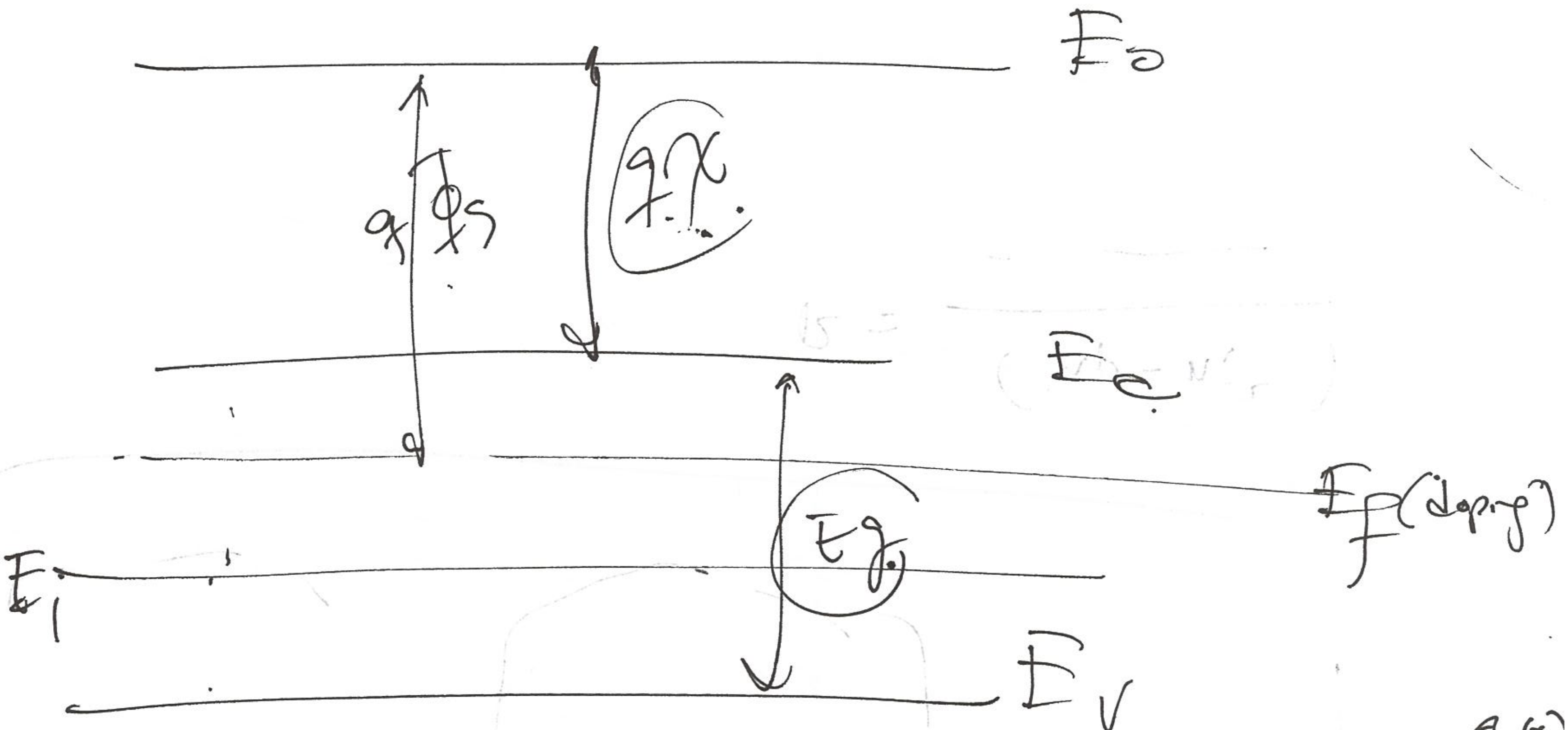
E


G
GATE
Floating
insulator
S
D
n+
n+

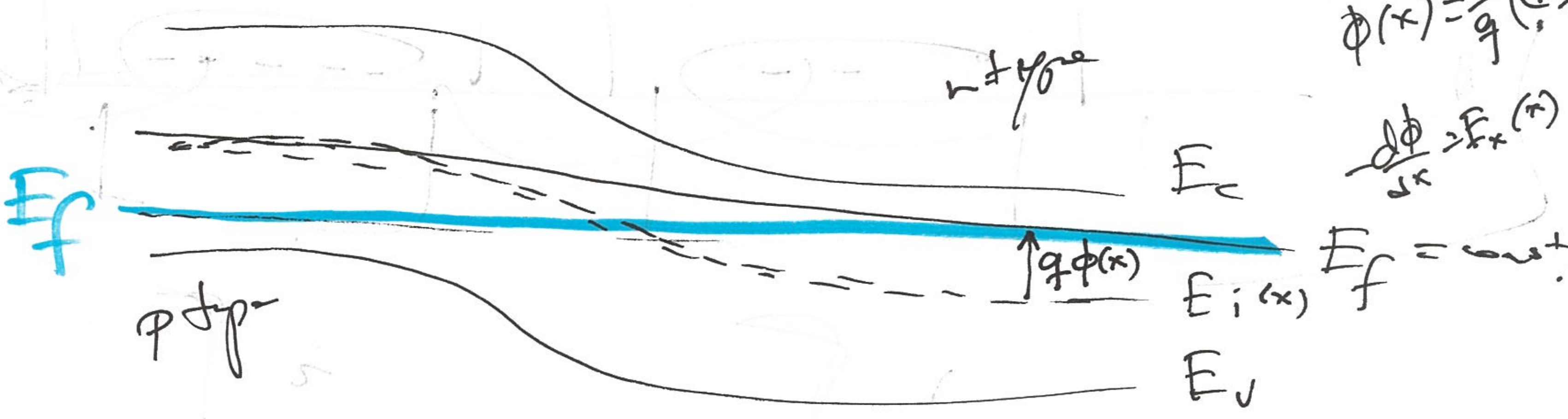

E_c
E_{fm}
E_{fs}
$N_D \nearrow$
$x_d = \sqrt{\frac{}{N_d}}$
$x_d \downarrow$


+ VG ↗


log |I_D|
P
VG



F.E.

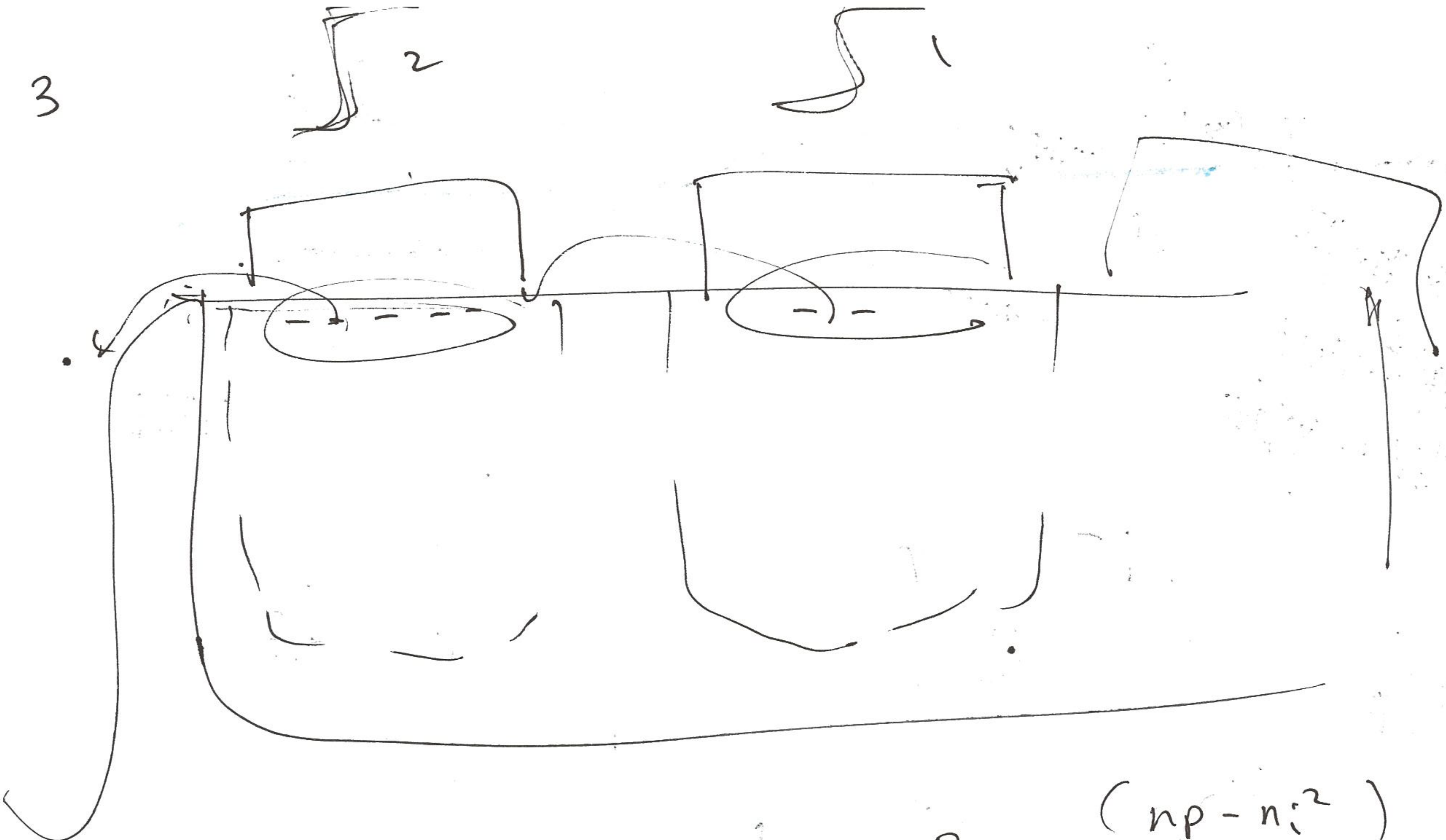


$$q\phi(x) = E_f - E_i(x)$$

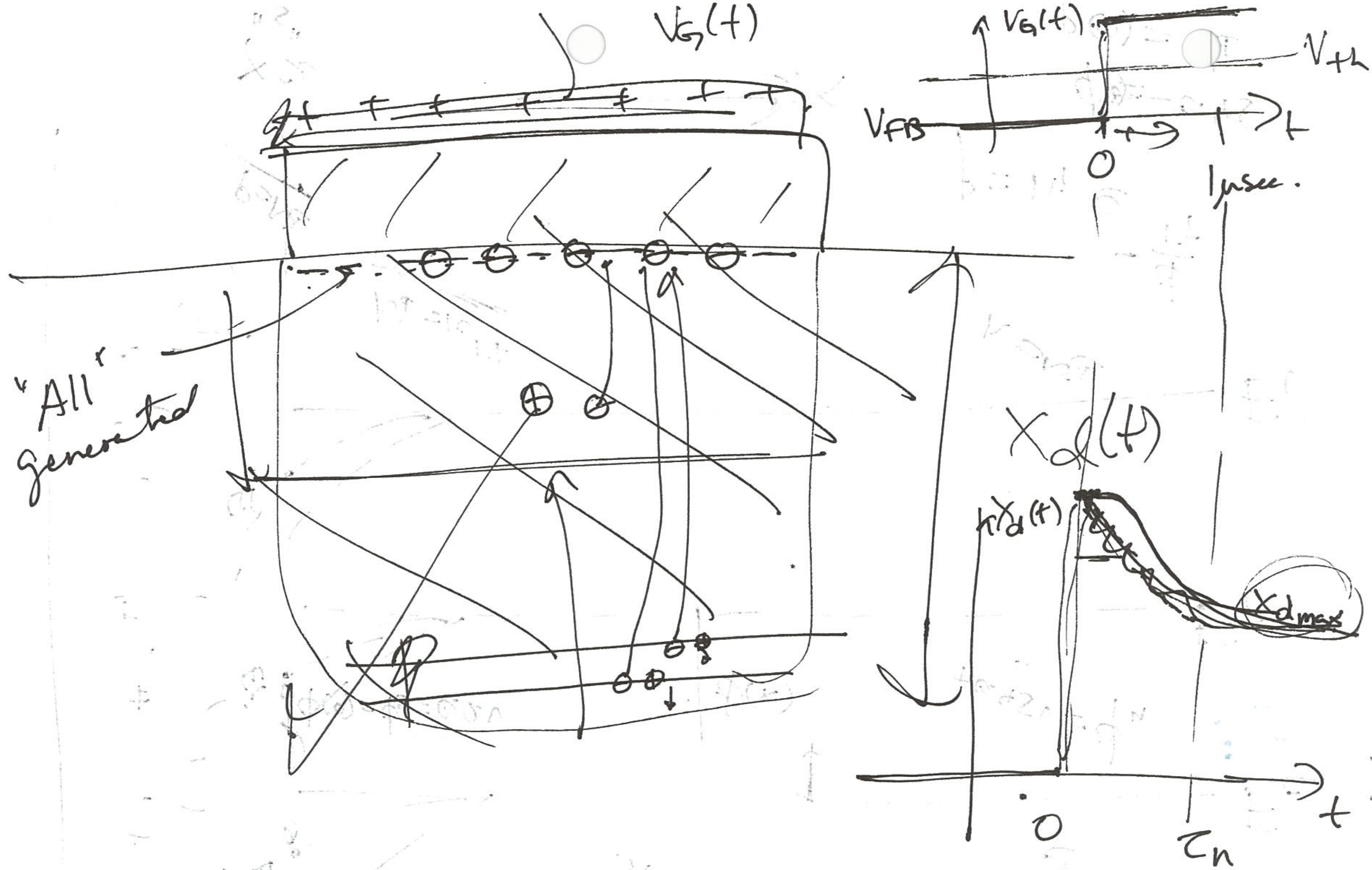
$$\phi(x) = \frac{1}{q}\left(E_f - E_i(x)\right)$$

$$-\frac{d\phi}{dx} = E_x(x) = -\frac{1}{q}\nabla E$$

3



$$R = \frac{(np - n_i^2)}{-\quad\_\_\_\_}$$



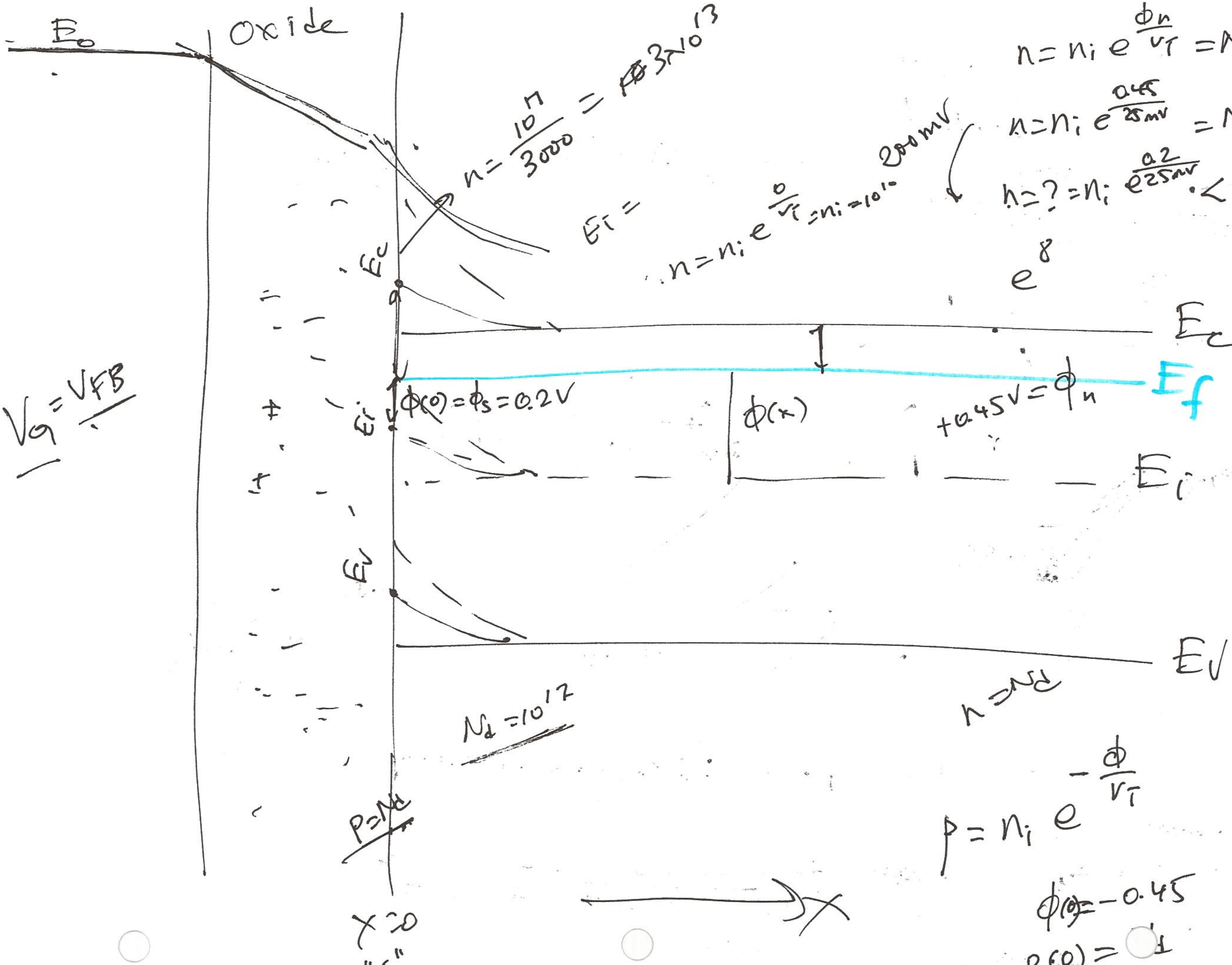
τ_n, τ_p.

$\tau_n = 10^{-6}$ sec.

$n = \frac{n_i^2}{N_A}$ = small

$p = 10^{17}$

$n = 10^3$



$n = \frac{10^{17}}{3000} = 3 \times 10^{13}$

$E_i =$

$n = n_i e^{\frac{0}{V_T}} = n_i = 10^{10}$

200mV

$n = n_i e^{\frac{\phi_n}{V_T}} = N_d$

$n = n_i e^{\frac{0.45}{25mV}} = N_d$

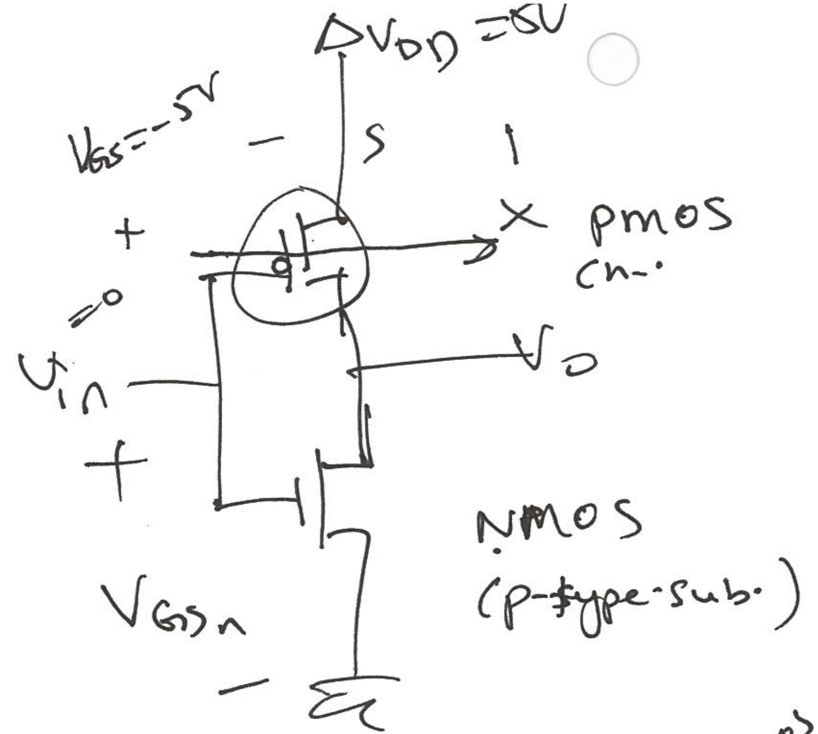
$n = ? = n_i e^{\frac{0.2}{25mV}} < N_d$

e^8

$p = n_i e^{-\frac{\phi}{V_T}}$

$\phi(0) = -0.45$

$p(0) = 1$



$E = -qV$ (energy)

$$\frac{dE}{dx} = \frac{\rho}{\epsilon}$$

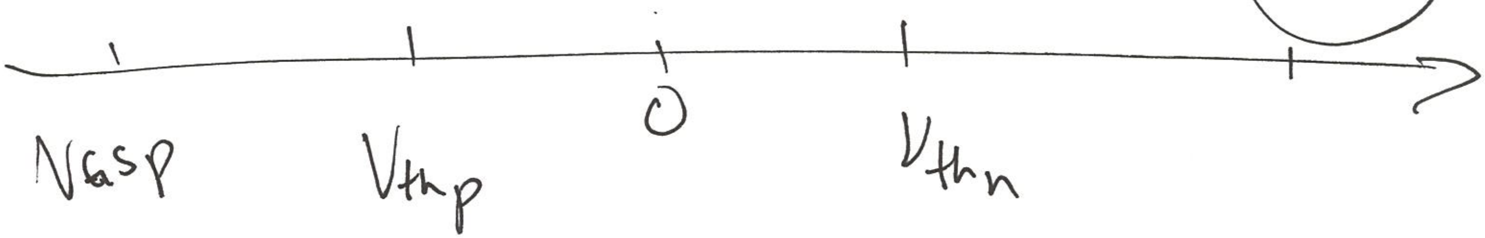
$$-E = -\frac{d\phi}{dx}$$ (E field)

$$\frac{d^2\phi}{dx^2} = -\frac{\rho}{\epsilon}$$

$V_{in} = +5V$ NMOS ON

$V_{GSp} = 0V \Rightarrow$ pmos OFF

V_{GSn}



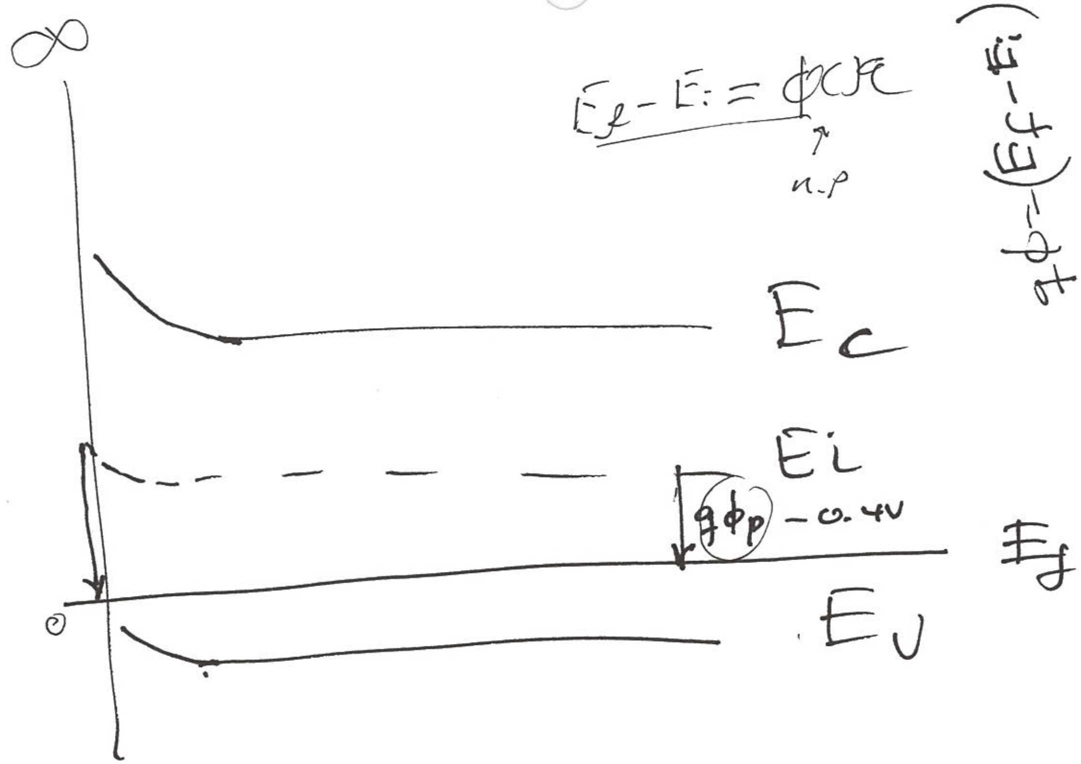
$V_{in} = 0$

pmos ON

$V_{GSn} = 0$

$V_{in} = 0$

NMOS OFF



$\underline{E_f - E_i = \phi q}$ n.p

$-\phi t = (E_f - E_i)$

$\phi_p = -V_T \ln\left(\frac{N_a}{n_i}\right)$

$\phi_p = -0.4V$

$\phi(x=\infty) = -0.4V$

$p = n_i e^{-\phi/V_T}$

$\phi \to -\infty$

$p \to \infty$

$\therefore$

$p >> N_a$

$\therefore$ accumulating

N-mos
ptype bulk $-\infty$ ——— E_i in Flat Band
——— E_f

T.E.

$p = N_a$

~~T.E. all time~~

$p = n_i e^{-\phi/V_T} = N_a$

$\phi_p = -V_T \ln\left(\frac{N_a}{n_i}\right)$ - only valid in bulk

ϕ_p
Substrate potential

T.E. no current
only band bending $p = n_i e^{-\phi/V_T}$

$I_M > I_S$

$E_i \uparrow$ then E_f

$\therefore$ ~~$E_i > E_f$~~

$\to E_f - E_i$

$\therefore$ negative value $E_i > E_f$

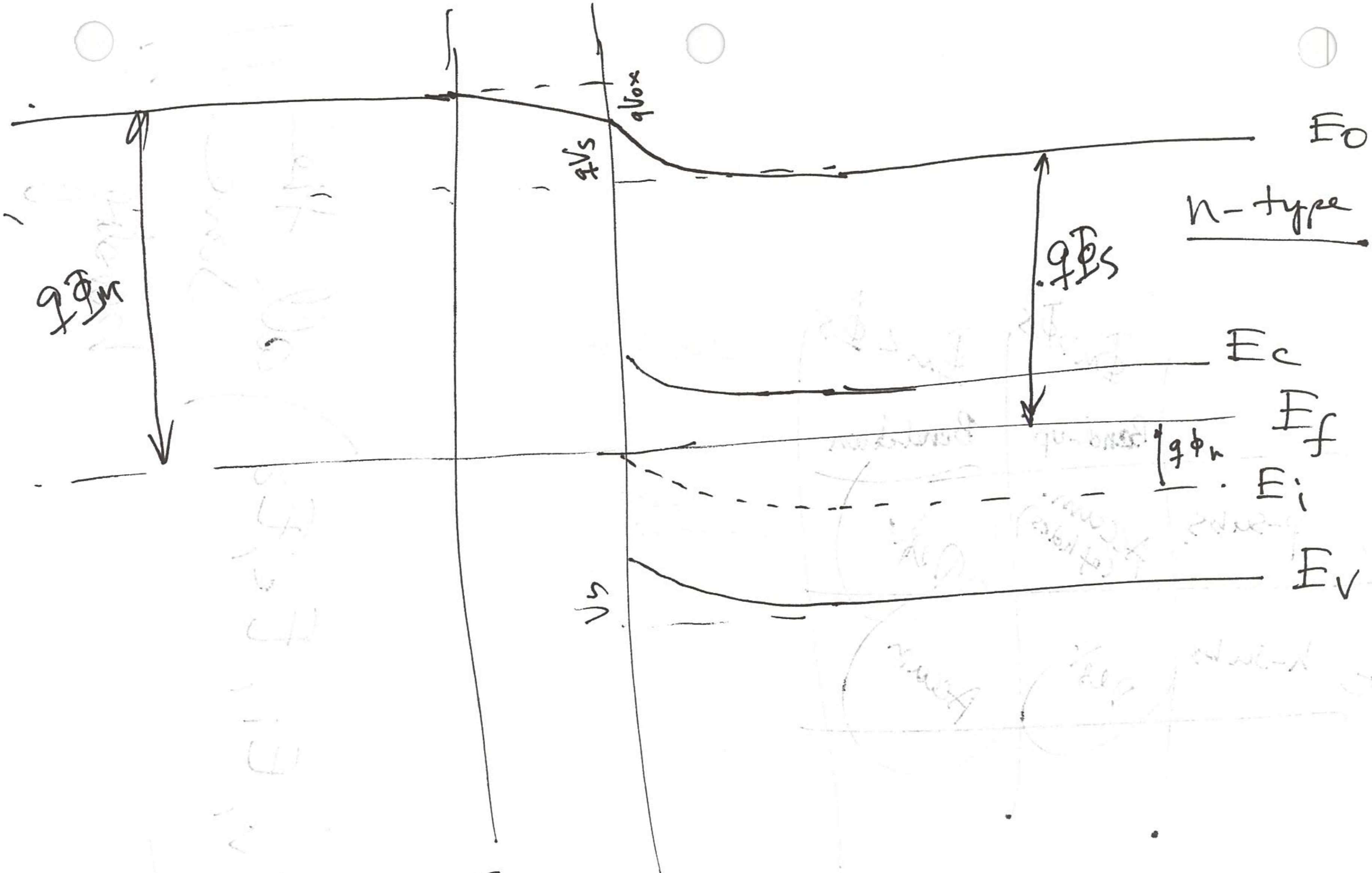
$p = N_a$

$$\boxed{p = n_i e^{-\phi/V_T}} = N_a$$

$E_i \to \infty$

$\phi \; |E_f - E_i| \to \infty$

ϕ becomes more negative



$$\Phi_m > \Phi_s$$

$$\Phi_m - \Phi_s = \Phi_{ms} = V_s + V_{ox}$$

		$\Phi_m > \Phi_s$ Bend-up	$\Phi_m < \Phi_s$ Bend-down
(nMOS)	p-subs.	Accum. (of holes)	Depl.
(pMOS)	n-subs	Depl.	Accum

~~Flat Band~~

—— E_F

- - - - - E_i

V_g when bands dont bend, Flat band

(E_V, E_i, E_c, E_0) all the same!! no bending

Device Electronics
2018-04-20
Week 14

May 4 final exam review
~~[illegible]~~
Final Exam on Monday May 14th
Final project due on May 11th
HW # 4 Due May 3rd

Accumulation:

P-Substrate: Accum, FB, Depletion

$\xrightarrow{V_G}$

Weak inversion, onset of inversion, Strong inversion

n-Substrate: Strong inversion, onset of inversion, Weak inversion

$\xrightarrow{V_G}$

Depletion, FB, Accumulation

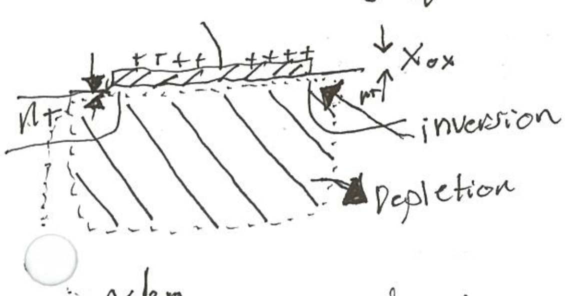
L_D = Debye length $\ll X_d$ Depletion

$$L_D = \sqrt{\frac{\epsilon_s V_T}{q n_s}}$$

$V_T \ln\left(\frac{\text{Doping}}{n_i}\right) = \phi_B$

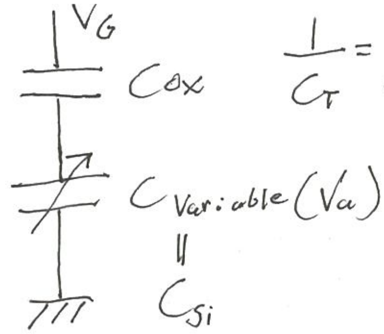
n_s = Doping density

$n_s \gg$ Doping
$\phi_B \gg V_T \sim 25\text{ mV}$



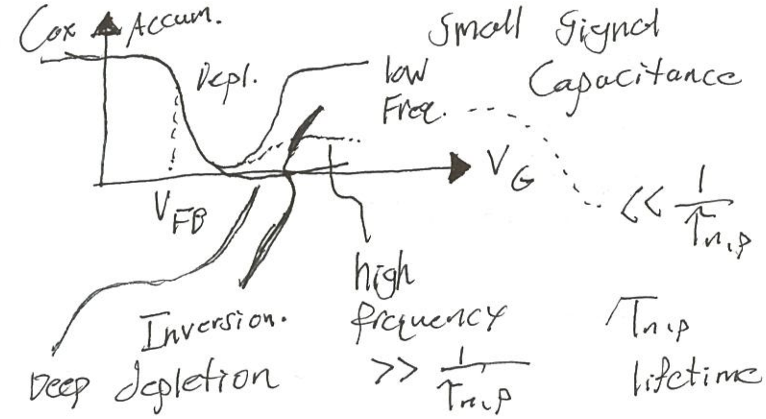
Deep depletion

$X_d > X_{d,max}$



$$\frac{1}{C_T} = \frac{1}{C_{ox}} + \frac{1}{C_{Si}}$$

$$C_{ox} = \frac{\epsilon_{ox}}{X_{ox}}, \quad C_{Si} = \frac{\epsilon_{Si}}{(X_d)}$$



Small Signal Capacitance

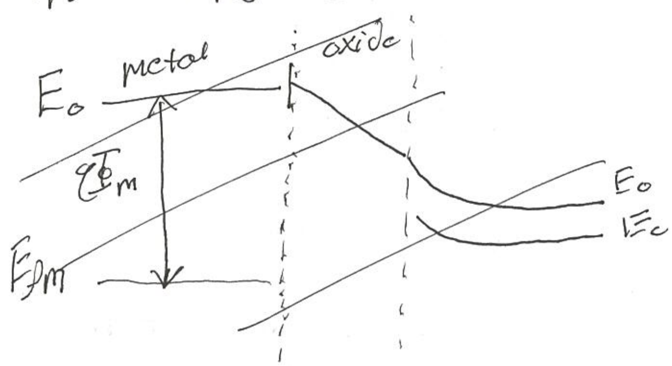
$\ll \frac{1}{\tau_{n,p}}$

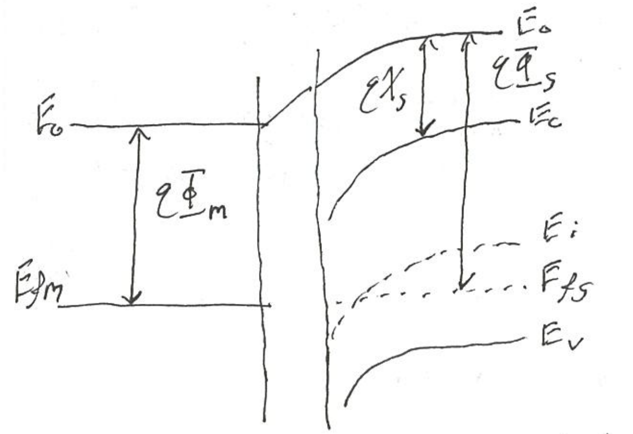
high frequency $\gg \frac{1}{\tau_{n,p}}$

$\tau_{n,p}$ lifetime

· Charge coupled devices

$V_{FB} = \Phi_{ms} + \ldots$

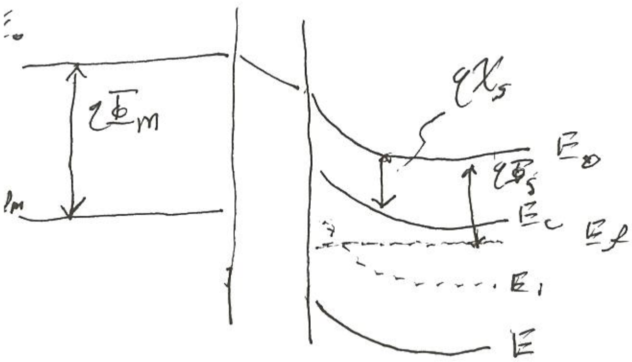




Metal   Oxide   Semiconductor

p-type

$\Phi_m < \Phi_s$



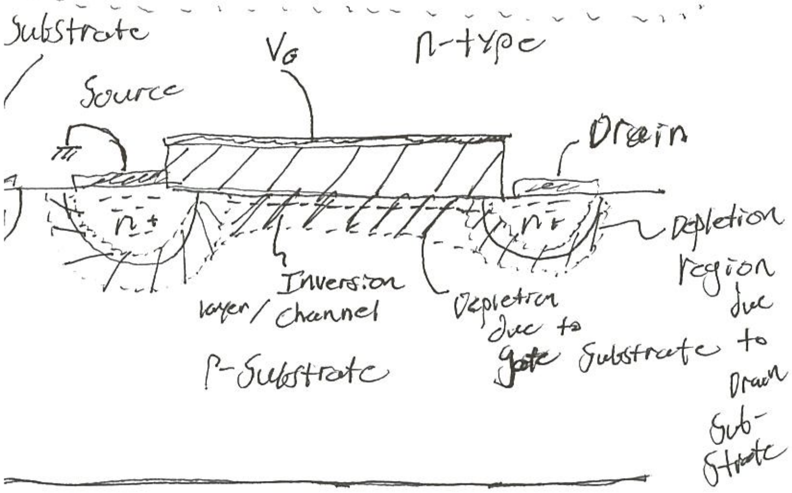
P-type
n bulk
$\Phi_m > \Phi_s$
accumulation @ $V_g = 0$

Accumulation ρ_s
↓
Strong inversion

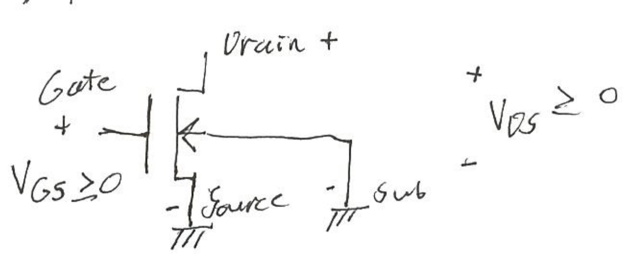
n-type
p-Substrate

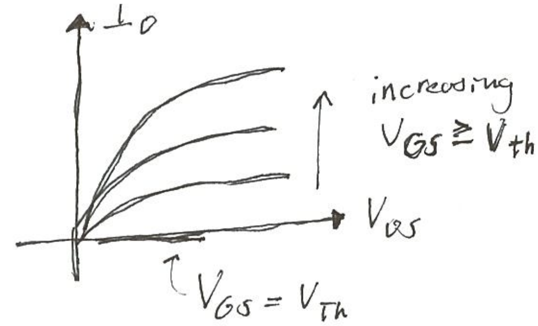
$$n(x) = \frac{q n_s}{\left(\frac{x}{\sqrt{2} L_D} + 1\right)^2}$$

in proof $n_s \gg$ Doping



N Mos transistor





We ~~just~~ have a depletion region

$V_{DS} = 0$

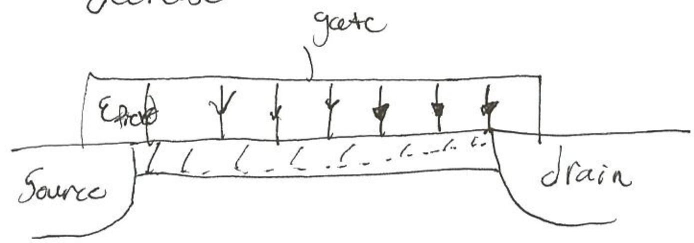
as V_G increases
- depletion region increases
- sufficient $V_G > V_{th}$
- create inversion layer
- more V_G improves conductivity of ~~gate voltage~~ inversion layer/channel

If Substrate grounded
- increasing drain voltage (+)
- keep getting a larger depletion region

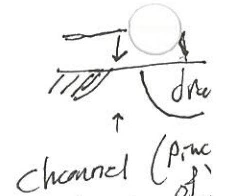
→ ~~st~~ decreasing voltage from gate to the channel

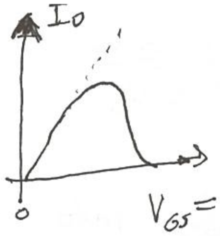
e field on gate oxide decreases

→ resulting in weaker channel current decrease in inversion



Efield gets smaller
∴ inversion layer gets smaller
resistivity increases
- eventually E field small enough

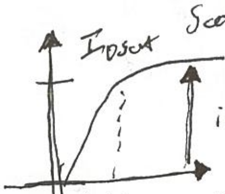




$V_{GS} = V_{th}$

Pinch off, does a transistor act like that no!

Saturation current



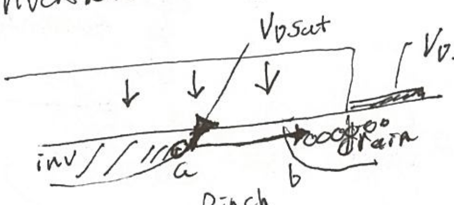
increasing $V_{GS} \geq V_{th}$ Pinch-off resistor

V_{Dsat}, pinch off ("resistor is cut")

Field Effect Transistor (FET)

pinched off inversion layer, but still conducting

- the charges are jumping ~~from~~ from inversion to drain



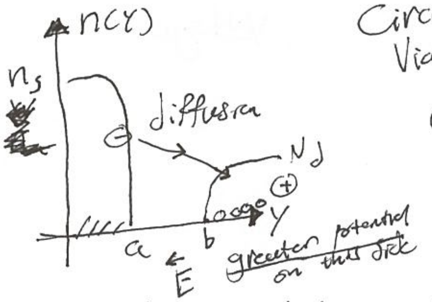
Pinch off

depletion in middle

Depletion region

- how does an electron jump
- current before pinch off, drift ~~cur~~ current
- pinch off, no drift current

diffusion current still remains

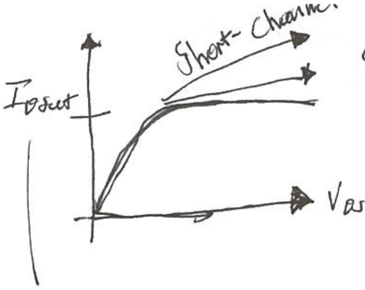


drift, diffusion, drift

Circuit complete via diffusion,

Current remains flat, as ~~the~~ pinch off increases

Resistor (channel) gets shorter, therefore less resistance

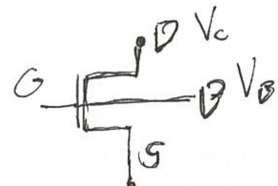


Current slightly increases as the pinchoff expands and the resistor (chan gets shorter

Saturation current

- diffusion current allows for the circuit to remain
- hardly any holes (mobile carriers) in the depletion region
- how to take into account substrate and source

derive threshold voltage



V_C / V_{source}

Same voltage between source & drain

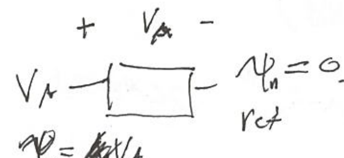
∴ V_B reference (Bulk)

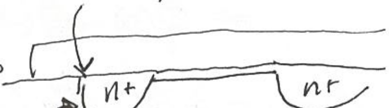
quasi fermi potentials $\psi_n = \psi_p = 0$

∴ V_C (at source & drain)

quasi-fermi-potential ⇒ voltage difference

$\psi_n = \psi_p = V_C - V_B$



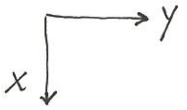


$\phi = V_T \ln\left(\frac{N_D}{n_i}\right) + (V_C - V$

Value of electrostatic potential

$n = n_i e^{\frac{\phi - \psi_n}{V_T}}$

What is happening between the Source & the drain

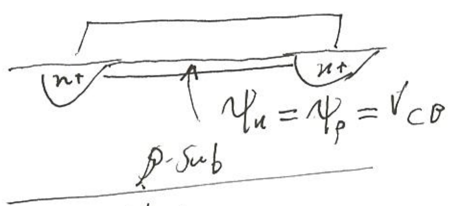


$$\vec{J}_n = -q\mu_n n \nabla \psi_n$$

$$J_{ny} = -q\mu_n n \frac{d\psi_n}{dy} = 0$$

$$\Rightarrow \psi_n(y) = \text{Constant} = V_{CB}$$

in Channel



N MOS

At onset of inversion

$$n_s = n(x=0) = N_a$$

$$n = n_i e^{(\phi - \psi_n)/V_T}$$

$$\Rightarrow \phi(x=0) = V_T \ln\left(\frac{N_a}{n_i}\right) + V_{CB} = \phi_B$$

Want this ↑!!

$x=0$ —— $\phi = V_T \ln\left(\frac{N_a}{n_i}\right) + V_{CB}$

threshold Condition $\Big\}\, V_s$

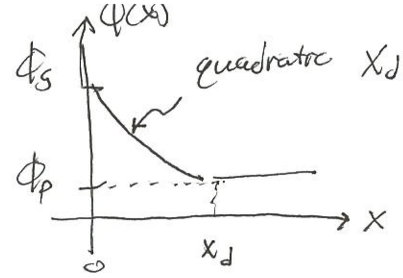
x —— $\phi = \phi_p = -V_T \ln\left(\frac{N_a}{n_i}\right)$

V_s = Voltage difference

$$= \phi(0) - \phi(x_d)$$

$$\phi(x) = \phi(0) + \frac{V_s}{x_j^2}(x - x_j)^2 \quad 0 \le x \le x_d$$

ϕ_s · potential of the Surface



$$x_d = \sqrt{\frac{2\epsilon_s V_s}{q N_a}}$$

deep into the Substrate

$$E_x(x=0) = E_s = \sqrt{\frac{2q N_a V_s}{\epsilon_s}}$$

repeat steps at home!

$$V_s\Big|_{@\,threshold} = 2|\phi_p| + V_{CB}$$

Source voltage

$$\Rightarrow V_G\Big|_{@\,threshold} = V_{th} = 2|\phi_p| + V_c + V_{FB} + \frac{1}{C_{ox}}\sqrt{2q N_a \epsilon_s (2|\phi_p| + V_{CB})}$$

Flat band

Source to Substrate Voltage

$$V_{GS}\Big|_{@\,thr} = 2|\phi_p| + V_{FB}^o + \frac{1}{C_{ox}}\sqrt{2q\, N_a \epsilon_s (2|\phi_p| + V_{CB})}$$

V_{CB}

Body effect

Variation of threshold Voltage & Substrate Voltage

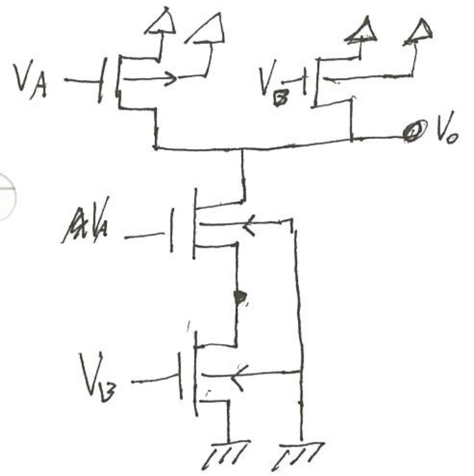
Basic CMOS logic design

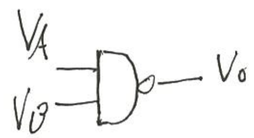
Nand Gate

Charges Inversion layer Charge

$Q_n \approx -C_{ox}(V_{GS} - V_{th})$

$V_{GS} \geq V_{th}$





~~Nan~~ NAND-2

Potent differen Source & the substrate

difference

Body effect

~~A~~ ∴ V_t ↓ more negative

$|V_{tp}|$ ↑

V_{tn} ↑

When Source a high potential →

→ gate even more positive to invert the channel

all depends on the voltage difference

threshold voltage ↑ content

depletion thickness

$X_{dmax} = \frac{2\epsilon_s}{qN_a}(2|\phi_p| + V_{CB})$

$|Q_d|$ = depletion layer charge

$V_{th} = 2|\phi_p| + V_{FB}^{\,0} + \cancel{X} + \frac{|Q_d|}{C_{ox}}$

V_{GS}

onset of inversion

plus charges get deposited ~~as~~ negative

Joseph Crandall
2018-04-23
HomeWork #4
Device Electronics
Read from Pierret
Chapters 15-19

Lecture HW: Derive the ~~Dy~~ Debye length inversion layer expression for a p-substrate MOS device, similar to the derivation we did in class, for the Debye length Accumulation layer.

Ch 15: P 3, 5
Ch 16: P 2, 7, 8, 13, 15
Ch 17: P 2, 4, 6, 10, 13
Ch 18: P 5, 7, 15
Ch 19: none

Silvaco Design an n-channel MOSFET of your own choosing width a gate width W = 5 um. Clearly state all of the design parameters of your device (oxide thickness, substrate doping, metal-semiconductor work-function, low-field mobility, etc.) Then vary gate length from L = W to L = 10 nm in sufficient number of gate length increments and compute the threshold voltage for each gate length value. Plot the threshold voltage calculated as a function of the gate length. Explain what if any variation in the threshold voltage as a function of decreasing gate length.

Ch 15 P. 3

a)

$$\to \int_0^y I_D\,dy' = I_D y$$

$$= 2qZ\mu_n N_D a \int_0^{V(y)} \left[1 - \frac{W(V)}{a}\right] dV$$

$$\to \left[\frac{W(V)}{a}\right] = \left(\frac{V_{bi} + V - V_G}{V_{bi} - V_P}\right)^{1/2}$$

$$\to y = \frac{2qZ\mu_n N_D a}{I_D}\int_0^V \left[1 - \frac{W}{a}\right] dV$$

$$= \frac{2qZ\mu_n N_D a}{I_D}\int_0^V \left[1 - \left(\frac{V_{bi} - V - V_G}{V_{bi} - V_P}\right)\right.$$

$$= \frac{2qZ\mu_n N_D a}{I_D}\left[V - \frac{2}{3}(V_{bi} - V_P)\left\{\left(\frac{V + V_{bi} - V_G}{V_{bi} - V_P}\right)^{3/2} - \cancel{\left(\frac{V_{bi} - V_G}{\ }\right)} - \left(\frac{V_{bi} - V_G}{V_{bi} - V_P}\right)^{3/2}\right\}\right]$$

$$\to L = \frac{2qZ\mu_n N_D a}{I_D}\int_0^{V_D}\left[1 - \frac{W}{a}\right] dV$$

$$= \frac{2qZ\mu_n N_D a}{I_D}\int_0^{V_D}\left[1 - \left(\frac{V_{bi} + V - V_G}{V_{bi} - V_P}\right)^{1/2}\right] dV$$

$$= V_D - \frac{2}{3}(V_{bi} - V_P)\left[\left(\frac{V_D + V_{bi} - V_G}{V_{bi} - V_P}\right)^{3/2} - \left(\frac{V_{bi} - V_G}{V_{bi} V_P}\right)^{3/2}\right]$$

$$\int x^n dx = \frac{x^{n+1}}{n+1}$$

$$\therefore \left(\frac{y}{L}\right) = \frac{V - \frac{2}{3}(V_{bi} - V_P)\left[\left(\frac{V + V_{bi} - V_G}{V_{bi} - V_P}\right)^{3/2} - \left(\frac{V_{bi} - V_G}{V_{bi} - V_P}\right)^{3/2}\right]}{V_D - \frac{2}{3}(V_{bi} - V_P)\left[\left(\frac{V_D + V_{bi} - V_G}{V_{bi} - V_P}\right)^{3/2} - \left(\frac{V_{bi} - V_G}{V_{bi} - V_P}\right)^{3/2}\right]}$$

b)
$V_G = 0$ [V]
$V_D = 5$ [V]
$V_{bi} =$ ~~10 [V]~~ 1 [V]
$V_P =$ ~~4/8~~ -8 [V]

$$\therefore \frac{y}{L} = \frac{V - \frac{2}{3}(1 - (-8))\left[\left(\frac{V + 1 - 0}{1 - (-8)}\right)^{3/2} - \left(\frac{1 - 0}{1 - (-8)}\right)^{3/2}\right]}{5 - \frac{2}{3}(1 - (-8))\left[\left(\frac{5 + 1 - 0}{1 - (-8)}\right)^{3/2} - \left(\frac{1 - 0}{1 - (-8)}\right)^{3/2}\right]}$$

$$= \frac{V-6[(V+1)^{3/2}-1]}{5-6(6^{3/2}-1)}$$

$\rightarrow V=1$

$$\frac{Y}{L} = \frac{V-6[(V+1)^{3/2}-1]}{5-6(6^{3/2}-1)}$$

$$= \frac{1-6[(1+1)^{3/2}-1]}{5-6(6^{3/2}-1)}$$

$$= \frac{1-6[1.83]}{5-6(14.7-1)} = \boxed{0.12927}$$

$\rightarrow V=2$

$$\frac{Y}{L} = \frac{2-6[(2+1)^{3/2}-1]}{5-6(6^{3/2}-1)}$$

$$= \frac{2-6[4.196]}{5-6(13.69)} = \boxed{0.3004}$$

$\rightarrow V=3$

$$\frac{Y}{L} = \frac{3-6[(3+1)^{3/2}-1]}{5-6(6^{3/2}-1)}$$

$$= \frac{3-6[7]}{5-6(13.69)} = \boxed{0.5655}$$

$\rightarrow V=4$

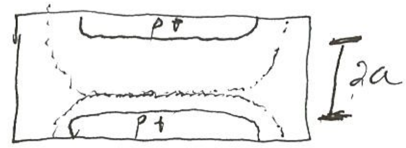
$$\frac{Y}{L} = \frac{4-6[(4+1)^{3/2}-1]}{5-6(6^{3/2}-1)}$$

$$= \frac{4-6[10.18]}{5-6(13.69)} = \boxed{0.7399}$$

the tabular representation are same as compared to the textbook

ch 15 p.5

a) $V_{GB}=0$



Pinch off channel with $V_D=0$

b) $V_p = -8[V]$

$V_{bi} = 1[V]$

$V_{GB}=0 \quad V_D=0$

$$2a = 2\left[\frac{2K_S\varepsilon_0}{qN_D}(V_{bi}-V_p)\right]^{1/2}$$

$$= \left[\frac{2K_S\varepsilon_0}{qN_D}(V_{bi}-V_{PT})\right]^{1/2} + \left[\frac{2K_S\varepsilon_0}{qN_D}V_{bi}\right]^{1/2}$$

Summation of top gate depletion width and the bottom gate depletion width

$$V_{bi} \Rightarrow \quad 2(V_{bi}-V_p)^{1/2} = (V_{bi}-V_{PT})^{1/2} + V_{bi}^{2}$$

$$V_{PT} = V_{bi} - \left[2(V_{bi}-V_p)^{1/2} - V_{bi}^{1/2}\right]^2$$

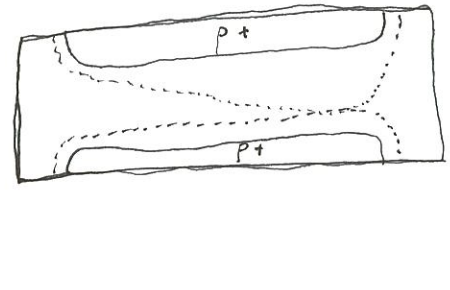
$V_{bi} = 1[V] \quad V_p = -8[V]$

$$V_{PT} = 1-[2\sqrt{9}-1]^2 = -24\ V$$

top depletion width needs to be wider than specified condition, when the two gates are tied together, which necessitate a larger valued of the applied voltage $|V_G|$

c) $V_{PT} < V_{GT} < 0 \quad V_{GB}=0$

the bottom depletion width still continues to the constriction of the channel



d) $V_D = V_{Dsat}$

$W_T + W_B \to 2a$ & $V(L) = V_{Dsat}$

$$W_T = \left[\frac{2K_S\varepsilon_0}{qN_D}(V_{bi}+V-V_{GT})\right]^{1/2}$$

$$W_B = \left[\frac{2K_S\varepsilon_0}{qN_D}(V_{bi}+V-V_{GB})\right]^{1/2}$$

$V_{GB} = 0$

$$\Rightarrow 2a = \left[\frac{2K_S\varepsilon_0}{qN_D}(V_{bi}+V_{Dsat}-V_{GT})\right]^{1/2} + \left[\frac{2K_S\varepsilon_0}{qN_D}(V_{bi}+V_{Dsat})\right]^{1/2}$$

$$2a = \left[\frac{2K_S\varepsilon_0}{qN_D}(V_{bi}-V_{PT})\right]^{1/2} + \left[\frac{2K_S\varepsilon_0}{qN_D}V_{bi}\right]^{1/2}$$

$$\therefore \boxed{(V_{bi}-V_{PT})^{1/2} + V_{bi}^{1/2} = (V_{bi}+V_{Dsat}-V_{GT})^{1/2}}$$

e) $V_{GB} = 0$ V_{Dsat} operation

is greater than ~~$V_{GB} = V_{GT}$~~

V_{Dsat} for $V_{GB} = V_{GT}$ operation

· top depletion width needs to be wider and allows more current flow and higher V_{Dsat} at pinch off.

→ $V_{bi} = 1[V]$ $V_P = -8[V]$

$V_{PT} = -24[V]$

$V_{Dsat} = V_G - V_P$

$= 6V$ $V_{GB} = V_{GT} = -2[V]$

When $V_{GT} = -2V$

$V_{Dsat}(V_{GB} = 0)$ is greater than

$V_{Dsat}(V_{GB} = V_{GT}$ operation)

Two expression are equal if $V_{GT} = 0$

f) $$I_D = -Z\int_{W_T(Y)}^{2a-W_B(Y)} J_{Ny}\,dx$$

$$= Z\int_{W_T(Y)}^{2a-W_B(Y)} \left(q\mu_n N_D \frac{dV}{dY}\right)dx$$

$$= qZ\mu_n N_D \frac{dV}{dy}\left[2a - W_B(Y) - W_T(Y)\right]$$

$$\rightarrow I_D = qZ\mu_n N_D \frac{dV}{dY}\left[1 - \frac{W_T+W_B}{2a}\right]$$

$$\frac{W_T+W_B}{2a} = \frac{(V_{bi}+V-V_{GT})^{1/2} + (V_{bi}+V)^{1/2}}{(V_{bi}-V_{PT})^{1/2} + V_{bi}^{1/2}}$$ for $V_{GB} = ($

$$I_D = \frac{2qZ\mu_n N_D a}{L}\left[V_D - \frac{2}{3}\frac{(V_{bi}+V_D-V_{GT})^{3/2} + }{(V_{bi}-V_{PT})^{1/2} + V_{bi}^{1/2}}\right.$$

$$\rightarrow (V_{bi}+V_D)^{3/2} - (V_{bi}-V_{GT})^{3/2}\ V_{bi}^{3/2}$$

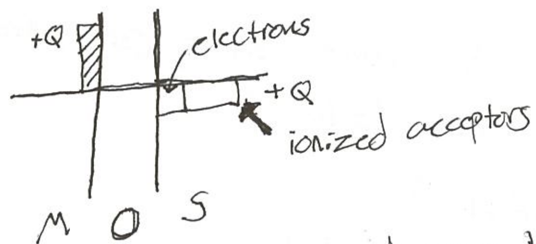
Ch 16 P.2

a) Figure 16.8 (c)

onset of inversion for ($\phi_S = 2\phi_F$)

$= 24kT/q$,

→ Where the semiconductor surface potential is ϕ_S and the reference voltage releted to the doping concentration is ϕ_F!



Depicts the block charge diagram which describes the charge situation inside the ideal p bulk MOS-C

b) obtain total charge in the semiconductor for each point, seperate block charges are added.

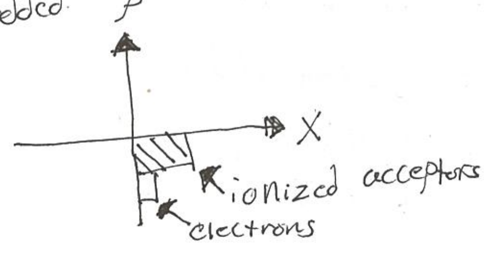


Figure 1 spike at the position $x'=0$, formation of an inversion layer of electrons at the surface.

$n_{surface} = N_A$

@ bias point for the terminal voltage V_T, the surface resistivity ρ_s is as follows. ⇒

$\rho_s = -q(n_{surface} + N_A)$

$= -2qN_A$

charge q

number of acceptors is N_A

$x=0$, ρ/qN_A

$\rho/qN_A = -2$

at $x=0$ at the onset of inversion the resistivity parameter for the acceptor concentration is -2

c)

$N_A = n_i e^{\phi_F(KT/q)}$

N_D donor atom concentration

$n_i = 1\times10^{10}$

$q = 1.6\times10^{-19}$ C

$K = 8.617\times10^{-5}$ eV/K

$T = 300$K

$\phi_F(KT/q)$ is 12

$N_A = n_i e^{\phi_F(KT/q)}$

$= 1\times10^{10} e^{12}$

$= 1.63\times10^{15}$ cm^{-3}

Mos depletion Width

$$W_T = \left[\frac{2K_s\varepsilon_0}{qN_A}(2\phi_F)\right]^{1/2}$$

Semiconductor dielectric constant $K_s = 11.8$

Permittivity of free space $\varepsilon_0 = 8.854\times10^{-14}$ F/cm

$$W_T = \left[\frac{2K_s\varepsilon_0}{qN_a}(2\phi_F)\right]^{1/2}$$

$$= \left[\frac{2(11.18)(24)(0.0259)}{(1.6\times10^{-19})(1.63\times10^{-15})}\right]^{1/2}$$

$= 0.706$ um

Mos depletion Width $\boxed{W_T = 0.706 \text{ um}}$

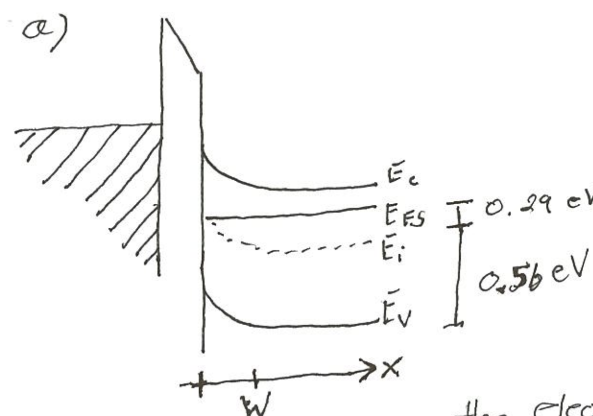
P7.

energy band diagram for an ideal $x_o = 0.2$ um MOS-C

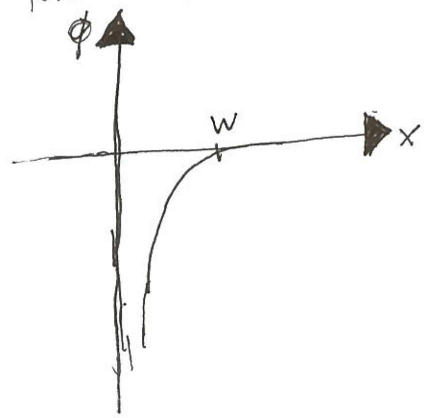
$T = 300$K

$E_F = E_i$ at the Si–SiO$_2$ interface

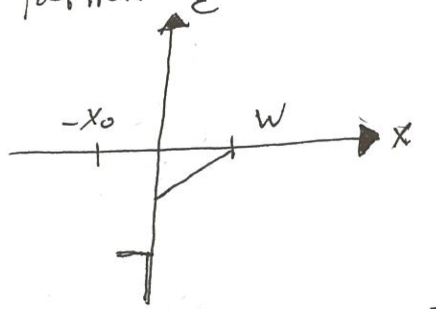
a)



ideal MOS-C depicting the electrostatic potential ϕ as a function of position x



b) electric field $\mathcal{E}$ inside the oxide and semiconductor as function of position

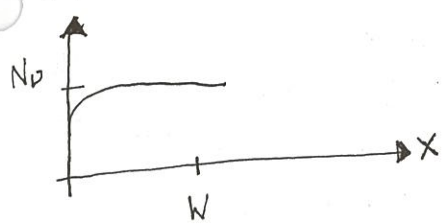


c) The Fermi energy level E_F is position independent; hence, the equilibrium conditions prevail inside the semiconductor

d) $n = n_i e^{(E_F - E_i)/KT}$

Here, intrinsic concentration is: n_i
intrinsic Fermi level is: E_i
Boltzmann's constant is: K
Temperature is: T

N_D versus the position W



e) $Si - SiO_2$

$E_F = E_i$

$(n|_{x=0} \cong n_i)\ 10^{10}\ cm^{-3}$

f) $N_D \cong n_{bulk}$

$= n_i e^{[E_{FS} - E_i(bulk)]/KT}$

$= 10^{10} e^{0.29/0.0259}$ $\left[\begin{matrix} E_{FS} = 0.29\,V \\ E_i(bulk) = 0\,V \end{matrix}\right]$

$\boxed{N_D = 7.29 \times 10^{14}\ cm^{-3}}$

g) Semiconductor Surface Potential

$\phi_s = \left(\frac{1}{q}\right)[E_i(bulk) - E_i(surface)]$

$= \left(\frac{1}{1.6\times10^{-19}C}\right)[E_i(bulk) - E_i(surface)$

$= -0.29\,V$

h) gate voltage V_G

$$V_G = \phi_s - \frac{K_s X_o}{K_o}\sqrt{\frac{2qN_D}{K_s \varepsilon_0}(-\phi_s)}$$

$$= 0.29 - \frac{11.8(2\times10^{-5})}{3.9}\sqrt{\frac{2(1.6\times10^{-19})(7.29\times10^{\ })}{(11.8)(8.85\times10^{-14})}}$$

$\rightarrow \times\sqrt{-(0.29)}$

$\boxed{= -0.78\,V}$

Charge of electron q is 1.6×10^{-19} C

Semiconductor dielectric constant: K_s is 11.8

Permittivity ε_0 is 8.85×10^{-14}

Oxide dielectric constant $K_o = 3.9$

i) Calculate voltage drop across the oxide layer: $\Delta\Phi_{ox}$

$\Delta\Phi_{ox} = V_G - \Phi_s$

$= 0.78\,V + 0.29\,V$

$\boxed{= -0.49\,[V]}$

j) Calculate the defined voltage V_δ

$$V_\delta = -(q/2)\frac{K_s X_o^2}{K_o^2 \varepsilon_0} N_D$$

$\Longrightarrow$

$$= -\left(\frac{1.6\times10^{-19}}{2}\right)\frac{11.8(2.5\times10^{-5})}{(3.9)^2(8.85\times10^{-14})}(7.29\times10^{14})$$

$$\boxed{\approx 0.20\,[V]}$$

The normalized small signal capacitance

~~[illegible]~~

$$\frac{C}{C_o} = \frac{1}{\sqrt{1+V_G/V_\delta}}$$

$$= \frac{1}{\sqrt{1+0.78/\cancel{[illegible]}\ 0.20}}$$

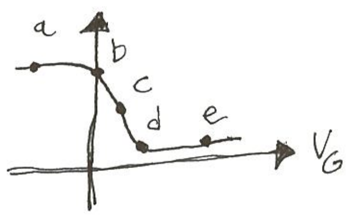
$$= \boxed{0.45}$$

Oxide Capacitance: C_o

Capacitance is: C

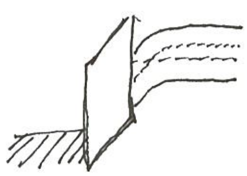
$$\frac{C}{C_o} = 0.45$$

P.8

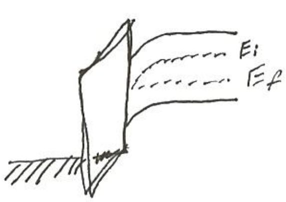


Bias condition Capacitive

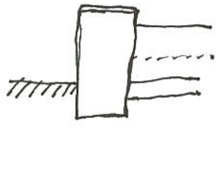
Inversion e



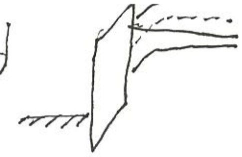
Depletion c



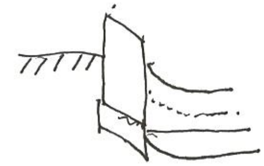
Flat Band b



$V_G = V_T$ d

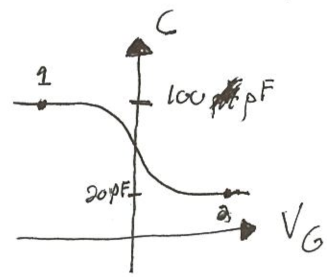


Accumulation a



P.13

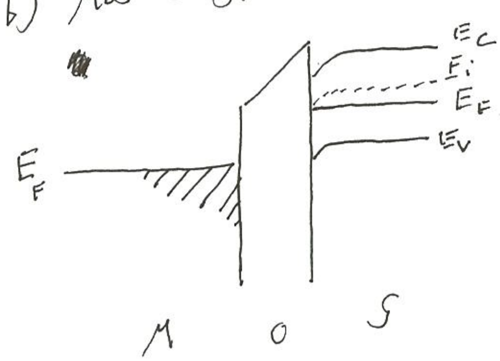
C-V Characteristic exhibited by an MOS-C is displayed



a) MOS-C has P-type doping

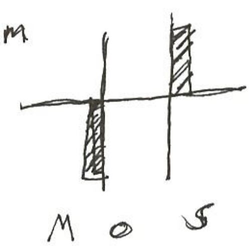
For P-type devices the maximum Capacitance C_{max} happens for a negative gate Voltage V_G, and the minimum capacitance happens for positive gate Voltage V_G

b) MOS energy band diagram at point 2



P-type MOS-C inversion

c) block diagram for Point 1



d) oxide thickness

$$x_o = \frac{K_o \varepsilon_o A_G}{C_{max}}$$

$K_o = 3.9$: oxide dielectric constant

$A_G = 3\times10^{-3}\ cm^2$: gate area

$C_{Max} = 100\ pF$: Maximum ~~Capacitan~~ Capacitance

$\varepsilon_o = 8.854\times10^{-15}$ Permittivity of free space

Calculate the oxide thickness x_o

$$x_o = \frac{(3.9)(8.854\times10^{-15})(3\times10^{-3})}{10^{-10}}$$

$$\boxed{= 0.104\ \mu m}$$

e) W_T : ~~width for the delta depletion~~ Width for the delta-depletion

$$W_T = \frac{K_S x_o}{K_o}\left(\frac{C_o}{C} - 1\right)$$

K_S : Semiconductor dielectric constant $= 11.8$

C : Capacitance $= 20pF$

Calculate width W_T for delta-depletion

$$W_T = \frac{K_S x_o}{K_o}\left(\frac{C_o}{C} - 1\right) = \frac{(11.8)(0.104\ \mu m)}{3.9}\left(\frac{100pF}{20pF} - 1\right)$$

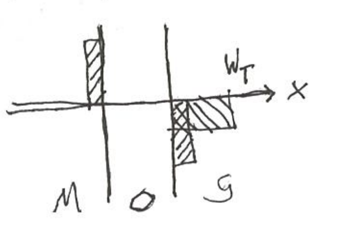
$$\boxed{= 1.26\ \mu m}$$

$$\boxed{N \cong 5\times10^{14}\ cm^{-3}}$$

Ch 16. P.15

~~Plot total dc depletion versus V_G characteristics~~

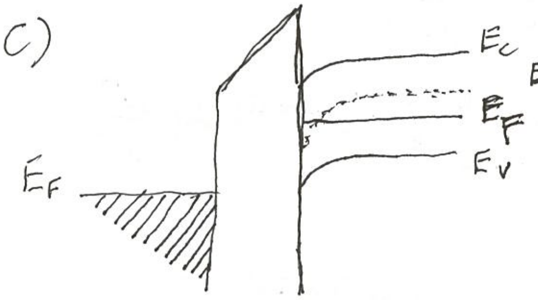
a) p type or n type semiconductor direct current state of an ideal MOS capacitor



Semiconductor is p-type because the inversion layer of negative charge of electrons is characteristic of p-type semiconductors.

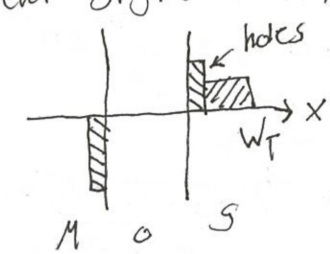
b) There is an inversion layer width $n_s > N_A$, this is the characteristic of the Inversion biased semiconductor

c)

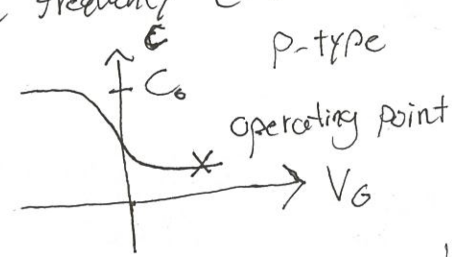


band diagram of the block diagram

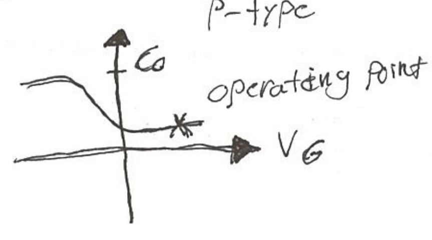
d) modified block charge diagram with the charge gate inside the MOS-C when a high frequency alternat current signal is applied



e) high frequency C-V characteristic



f) block diagram for the totally depleted MOS-C



Ch 17: P.2

ideal n-channel MOSFET

$T = 300K$
$Z = 50\ \mu m$
$L = 5\ \mu m$
$x_o = 0.05\ \mu m$
$N_A = 10^{15}\ cm^{-3}$
$\mu_n = 800\ cm^2 V^{-1} s^{-1}$

a) Voltage related to the doping concentration

$$\phi_F = \frac{KT}{q} \ln\left(\frac{N_a}{n_i}\right)$$

$$= \frac{8.617 \times 10^{-5}\ eV/K\ (300\ K)}{1.6 \times 10^{-19} C} \ln\left(\frac{10^{15}}{10^{10}}\right)$$

$$= 0.298\ V$$

$$V_T = 2\phi_F + \frac{K_S x_o}{K_o} \sqrt{\frac{4qN_A}{K_S \varepsilon_o} \phi_F}$$

Semiconductor dielectric constant

$K_s = 11.8$

oxide dielectric constant: $K_o = 3.9$
Permittivity $\varepsilon_o = 8.854 \times 10^{-14}$
Acceptor concentration: $N_A = 10^{15}\ cm^{-3}$

$$\cancel{V_T = 2(0.298) + \frac{(11.8)(5\times10^{-6})}{3.9}}$$

$$V_T = 2(0.298) + \frac{(11.8)(5\times10^{-6})}{3.9} \sqrt{\frac{4(1.6\times10^{-19})(10^{15})}{(11.8)(8.854\times10^{-14})} (0.298)}$$

$$\boxed{= 0.8\ V}$$

b) Capacitance C_o

$$C_o = \frac{K_o \varepsilon_o}{x_o} = \frac{(3.9)(8.854\times10^{-14})}{(5\times10^{-6})}$$

$$\boxed{= 6.90 \times 10^{-8}\ F\,cm^{-2}}$$

Saturation drain current

$$I_{Dsat} = \frac{Z \bar{\mu}_n C_o}{2L} (V_G - V_T)^2$$

Width of MOSFET channel $Z = 5\times10^{-3}$
Effective electron mobility $\mu_n = 800$
Length of MOSFET $L = 5\times10^{-4}$
Gate Voltage $V_G = 2V$

$$I_{Dsat} = \frac{(5\times10^{-3})(800)(6.9\times10^{-8})}{2(5\times10^{-4})} (2 - 0.8)^2$$

$$\boxed{= 0.397\ mA}$$

c) MOS depletion width W_T

$$W_T = \left[\frac{2K_S\varepsilon_o}{qN_A}(2\phi_F)\right]^{1/2}$$

$$= \left[\frac{2(11.8)(8.854\times10^{-14})}{(1.6\times10^{-19})(10^{15})}(2\times 0.298)\right]^{1/2}$$

$$= 0.882\ \mu m$$

Voltage V_W

$$V_W = \frac{qN_A W_T}{C_o}$$

$$= \frac{(1.6\times10^{-19})(10^{15})(0.882\times10^{-4})}{(6.90\times10^{-8})}$$

$$\boxed{= 0.205\ V}$$

$V_G - V_T = 1.20\ V$

$$V_{Dsat} = (V_G - V_T) - V_W\left\{\left(\frac{(V_G - V_T)}{2(\phi_F)} + \left(1 + \frac{V_W}{4(\phi_F)}\right)\right)^{1/2} - \left[1 + \frac{(V_G - V_T)}{2(\phi_F)}\right]\right\}$$

$$= 1.20 - 0.205\left\{\left(\frac{1.20}{2(0.298)} + \left(1 + \frac{0.205}{4(0.298)}\right)\right)^{1/2} - \left[1 + \frac{0.205}{4(0.298)}\right]\right\}$$

$$= 1.06\ V$$

Calculate the parameter

$$\frac{Z\bar{\mu}_n C_o}{L} = \frac{(5\times10^{-3})(800)(9.60\times10^{-8})}{5\times10^{-4}}$$

$$= 5.52\times10^{-4}\ A\,V^{-2}$$

Drain Current I_D

$$I_D = \frac{Z\mu_n C_o}{L}\left\{(V_G-V_T)V_D - \frac{V_D^2}{2} - \frac{4}{3}V_W\phi_F\left[\left(1+\frac{V_D}{2\phi_F}\right)^{3/2} - \left(1+\frac{3V_D}{4\phi_F}\right)\right]\right\}$$

$$= 5.52\times10^{-4}\left\{(1.20)1.06 - \frac{1.06^2}{2} - \frac{4}{3}(0.205)(0.298)\left[\left(1+\frac{1.06}{2(0.298)}\right)^{3/2} - \left(1+\frac{3\times1.06}{4(0.298)}\right)\right]\right\}$$

$$\boxed{= 0.349\times10^{-3}\ A}$$

d) Conductance g_d

$$g_d = \frac{Z\mu_n C_o}{L}(V_G - V_T)$$

$$= 5.52\times10^{-4}(2-0.8)$$

$$\boxed{= 0.662\ mS}$$

e) Calculate the transconductance

$$g_m = \frac{Z\bar{\mu}_n C_o}{L}(V_G - V_T)$$

$$= 5.52\times10^{-4}(2-0.8)$$

$$\boxed{g_m = 0.662\ mS}$$

f) Calculate the transconductance g_m When $V_G = 2V$ and $V_D = 2V$

$$g_m = \frac{Z\bar{\mu}_n C_o}{L}V_{Dsat}$$

$$= 5.52\times10^{-4}(1.06)$$

$$\boxed{= 0.585\ mS}$$

g) Calculate the maximum frequency f_{max}

$$f_{max} = \frac{\bar{\mu}_n V_D}{2\pi L^2}$$

$$= \frac{(800)(1)}{2\pi(5\times10^{-4})^2}$$

$$\boxed{= 509\ MHz}$$

Ch 17: P.4

V_T versus NA for ideal n-channel MOSFET (attached)

Ch 17: P.6

Saturation drain current I_{Dsat}

$$I_{Dsat} = \frac{Z\bar{\mu}_n C_o}{2L}\left[(V_G - V_T)V_D - \frac{V_D^2}{2}\right]$$

Z: ~~MOSFET~~ MOSFET channel

μ_n: Effective electron mobility is $\bar{\mu}_n$

L: Length of MOSFET

V_G: Gate Voltage

V_D: Drain Voltage

V_T: inversion depletion transition gate Voltage

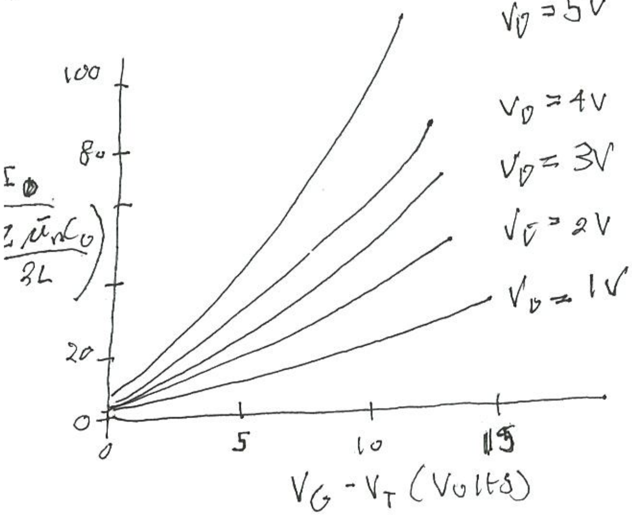
$$\left.\frac{dI_D}{dV_D}\right|_{V_G = constant} = \frac{Z\bar{\mu}_n C_o}{2L}\left[(V_G - V_T)V_D - \frac{V_D^2}{2}\right]$$

$$= 0$$

$$\boxed{V_{Dsat} = V_G - V_T}$$

Ch 17: p.10

Sketch current I_D versus $V_G - V_T$ characteristics from a n-channel MOSFET



I_D versus $V_G - V_T$ characteristics

→ Assume voltage $V_G - V_T$ raised proportionaly from drain voltage (V_D) is kept constant

$$I_D = \frac{Z\bar{\mu}_n C_o}{2L}(V_G - V_T)^2$$

→ drain fluctuates as the square of $V_G - V_T$ if $V_G - V_T < V_D$

When $V_G - V_T$ is equal to drain voltage V_D, the device transfers into the linear state operation

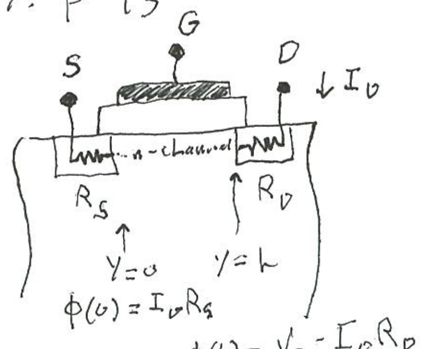
→ I_D in the linear region

$$I_D = \frac{Z\bar{\mu}_n C_o}{L}\left[(V_G - V_T)V_D - \frac{V_D^2}{2}\right]$$

fluctuates linearly with ~~Vo~~ $V_G - V_T$

→ As voltage V_D increases, one remains on the voltage squared part of the curve, when $V_G - V_T$ becomes greater than V_D, a ~~linear region~~ the function continues as a linear function.

Ch 17: P 13



The dimensions of MOSFET are reduced to achieve higher operational frequencies and higher packing densities, Rs and RD have become increasingly important. using the square law theory, the source & drain resistances are appropriately taken into account by replacing V_D with $V_D - I_D(R_S + R_D)$ and V_G with ~~$V_G - I_D R_S$~~ in $V_G - I_D R_S$ drain current: I_D

$$I_D = \frac{Z}{L}\bar{\mu}_n C_o\left[(V_G - V_T)V_D - \frac{V_D^2}{2}\right]$$

→ Saturated drain voltage V_{Dsat}

$$V_{Dsat} = V_G - V_T$$

→ Write expression for saturated drain current

$$I_{Dsat} = \frac{Z\bar{\mu}_n C_o}{2L}(V_G - V_T)^2$$

$$I_D = -\frac{Z}{L}\bar{\mu}_n \int_{I_D R_S}^{V_D - I_D R_D} Q_N \, d\phi$$

Source Resistance R_S

drain Resistance R_D

channel voltage @ $Y=0$ & $Y=L$

$V(0) = I_D R_S$

$V(L) = V_D - I_D R_D$

→ Square law theory

$Q_N = -C_o(V_G - V_T - \phi)$

$$I_D = -\frac{Z}{L}\bar{\mu}_n \int_{I_D R_S}^{V_D - I_D R_D} \left[-C_o(V_G - V_T - \phi)\right] d\phi$$

$$= \frac{Z}{L}\mu_n C_o \left\{ \left[V_G \phi \Big|_{I_D R_S}^{V_D - I_D R_D} \right] - V_T \phi \Big|_{I_D R_S}^{V_D - I_D R_D} - \frac{1}{2}\phi^2 \Big|_{I_D R_S}^{V_D - I_D R_D} \right\}$$

$$= \frac{Z}{L}\bar{\mu}_n C_o \left\{ \left[V_G[V_D - I_D R_D - I_D R_S]\right] - V_T[V_D - I_D R_D - I_D R_S] - \frac{1}{2}[V_D - I_D R_D]^2 + \frac{1}{2}[I_D R_S]^2 \right.$$

$$= \frac{Z}{L}\mu_n C_o \left[[V_G - V_T][V_D - I_D(R_D + R_S)] - \frac{1}{2}[V_D - I_D R_D]^2 + \frac{1}{2}[I_D R_S]^2 \right]$$

$\therefore V_D = V_D - I_D(R_D + R_S)$

$$= \frac{Z}{L}\bar{\mu}_n C_o \left\{ [V_G - V_T][V_D] - \frac{1}{2}\left\{ [V_D^2 + [I_D R_D]^2 - 2V_D I_D R_D] - [I_D R_S]^2 \right\} \right\}$$

$$= \frac{Z}{L}\bar{\mu}_n C_o [V_G - V_T][V_D] - \frac{1}{2}\left\{ V_D^2 + I_D^2(R_D^2 - R_S^2) - 2V_D I_D R_D \right\}$$

$$I_D = \frac{Z\bar{\mu}_n C_o}{L}\left\{ (V_G - V_T)[V_D] - \frac{[V_D]^2}{2} \right\}$$

$V_D = V_{Dsat}$

$Q_N(L) = 0$

$\phi(L) = V_{Dsat} - I_{Dsat} R_D$

$0 = -C_o[V_G - V_T - (V_{Dsat} - I_{Dsat} R_D)]$

$V_G - V_T - (V_{Dsat} - I_{Dsat} R_D) = 0$

$V_{Dsat} - I_{Dsat} R_D = V_G - V_T$

$\therefore$

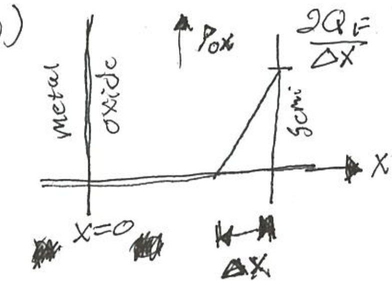
$$\boxed{I_{Dsat} = \frac{Z\bar{\mu}_n C_o}{2L}\left(V_G - I_{Dsat} R_{DS} - V_T\right)^2}$$

Ch:18 P. 5

Charge is distributed a short distance into the oxide from the Si-SiO$_2$ interface

a) $\Delta V_G = -\dfrac{Q_F}{C_o}$

b)



$$\rho_{ox} = \begin{cases} 0 & 0 \le x \le x_o - \Delta x \\ \dfrac{2Q_F}{\Delta x^2}x' & 0 \le x' \le \Delta x \qquad x' = x - x_o + \Delta x \end{cases}$$

$$\Delta V_G\binom{\text{oxide}}{\text{charges}} = \Delta V_G - \Delta V_G'$$

$$= \frac{1}{K_o \varepsilon_o}\int_0^{x_o} x \rho_{ox}(x)\, dx$$

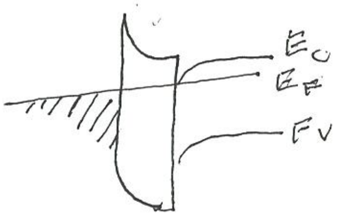
$$\Delta V_G = \frac{1}{K_o \varepsilon_o}\int_0^{x_o} x \rho_{ox}\, dx$$

$$= -\frac{1}{K_o\varepsilon_o}\left(\frac{2Q_F}{\Delta x^2}\right)\int_0^{\Delta x} x'(x'+x_o-\Delta x)\,dx$$

$$= \boxed{-\frac{Q_F}{C_o}\left(1-\frac{\Delta x}{3x_o}\right) = \Delta V_G}$$

Ch 18. P.7

a)



* interface trap density in structure is negligible

- assume $Q_M = 0$
  no charge in the oxide

  $\rho_{ox} = 0$

  $\mathcal{E}_{ox}$ = constant value

The oxide energy bands are a function of position.

If ρ_{ox} is not equal to zero
$\mathcal{E}_{ox}$ becomes a function of location and the oxide energy bands

b) Q_F is not equal to zero

- normal components of the D-field are continuous
- When no plane of charge, there is a discontinuity in the D field equal to the charge cm^{-2}

· Slope of bands zero ∴

$$\mathcal{E} = \left(\frac{1}{q}\right)\left(\frac{dE_c}{dx}\right) = 0$$

- Semiconductor side $\mathcal{E}$ is non-zero & positive

∴ there exists a plane of charge at or near the interface

Ch 18 P. 15

a) flat band conditions

$$V_{FB} = V_G = 0$$

$$= \phi_{MS} - \frac{Q_F}{C_o} - \frac{Q_M\gamma_m}{C_o} - \frac{Q_{IT}(2\phi_F)}{C_o}$$

Substitute corresponding values

$$V_{FB} = -0.46 - \frac{(1.6\times10^{-19})(5\times10^{-6})}{(3.9)(8.85\times10^{-14})}(2\times10^{11}+0+0-4\times10^{-11})$$

$$\boxed{V_{FB} \simeq 0\,V}$$

b) Threshold voltage is determined as followed:

$$V_T = 2\phi_F - \frac{K_S}{K_O}x_o\sqrt{\frac{4qN_D}{K_S\varepsilon_o}(-\phi_F)}$$

$$\phi_F = -\frac{kT}{q}\ln\left(\frac{N_D}{n_i}\right)$$

~~Substitute~~

$$\phi_F = -0.0259\ln\left(\frac{10^{15}}{10^{10}}\right)$$

$$\simeq -0.298\,V$$

Threshold voltage for perfect MOSFET

$$V_T = -(2)(0.298) - \left(\frac{11.8}{3.9}\right)(5\times10^{-6})\sqrt{\frac{(4)(1.6\times10^{-19})(10^{15})(0.298)}{(11.8)(8.85\times10^{-14})}}$$

$V_T = -0.80\ V$

$V_T = V_{Ti} + V_{FB}$

$= 0 - 0.80$

$\boxed{= -0.80\ V}$

c) MOSFET on enhancement mode device

→ P-channel device there is no inversion layer at zero bias

→ no drain current flow, when $V_G = 0$

→ MOSFET is in off state at zero bias which is an enhancement mode device

→ MOSFET device in <u>enhancement mode</u>

P. Derive Debye length inversion layer expression for p-substrate MOS device

Inversion $n >> N_a$

at or near $x=0$

$$= \frac{q n_i}{\epsilon_s} e^{\phi/V_T}$$

$$\frac{d^2\phi}{dx^2} = \frac{-\rho}{\epsilon_s} = \frac{q}{\epsilon_s}(n - p - 0) \quad \begin{matrix} \rho = 0 \\ p = 0 \\ n_s >> N_a \end{matrix}$$

$$\frac{d^2\phi}{dx^2} = \frac{q n_i}{\epsilon_s} e^{\phi/V_T} \qquad n = n_i e^{\phi/V_T}$$

$n_s = n_i e^{\phi_s/V_T}$

$\phi' = \phi_p - \phi_s$

ϕ_s: surface potential

ϕ_p: potential in the bulk substrate

$$\frac{d^2\phi}{dx^2} = \frac{q n_i}{\epsilon_s} \frac{N_s}{N_s} e^{\phi/V_T}$$

$$= \frac{q N_s}{\epsilon} e^{(\phi - \phi_s)/V_T}$$

$n_s = n_i e^{\phi_s/V_T}$

MOS $\longrightarrow x$

$$\frac{d^2\phi}{dx^2} = \frac{q N_s}{\epsilon_s} e^{\phi'/V_T}$$

$$E = -\frac{d\phi}{dx} = \frac{-d\phi'}{dx}$$

$$\frac{dE}{dx} = -\frac{q N_s}{\epsilon_s} e^{\phi'/V_T}$$

$$\frac{dE}{dx} = \frac{dE}{d\phi'}\frac{d\phi'}{dx} = -E\frac{dE}{d\phi'} \quad \text{chain rule}$$

$$E\frac{dE}{d\phi'} = \frac{q N_s}{\epsilon_s} e^{\phi'/V_T}$$

$$\frac{1}{2}E^2 \frac{d}{d\phi'} = K e^{\phi'/V_T}$$

$$\frac{1}{2}\frac{d}{d\phi}(E^2) = 2E\frac{dE'}{d\phi'}\cdot\frac{1}{2}$$

$$\frac{1}{2}\int_{E(x)}^{E(\infty)} d(E^2) = K\int_{\phi'(x)}^{\phi(\infty)} e^{\phi'/V_T}\, d\phi'$$

$$\frac{1}{2}\left[E^2(\infty) - E^2(x)\right] =$$

$$-E^2(x) = 2K V_T\left[e^{\phi'(\infty)/V_T} - e^{\phi'(x)/V_T}\right]$$

$$E^2 = 2K V_T e^{\phi'(x)/V_T}$$

$$E(x) = \sqrt{2KV_T}\; e^{\phi(x)/2V_T}$$

$$-\frac{d\phi'}{dx} = \sqrt{2KV_T}\;\ldots$$

$$\int_{\phi'(0)}^{\phi'(x)} e^{\phi'/2V_T}\, d\phi' = \sqrt{2KV_T}\int_0^x dx$$

$$2V_T\left[e^{\frac{\phi'(x)}{2V_T}} - 1\right] = \sqrt{2KV_T}\; x$$

$$e^{\phi'/2V_T} = \sqrt{2KV_T}\; x\; \frac{1}{2V_T} + 1$$

$$e^{\phi'/2V_T} = \frac{x}{\sqrt{2}L_D} + 1$$

$$L_D = \sqrt{\frac{\epsilon_s V_T}{qN_s}}$$

$$K = \frac{qN_s}{\epsilon_s}$$

$$\rho(x) = q\,p(x) = qN_s\, e^{\phi'/V_T}$$

$$\boxed{\rho(x) = \frac{qN_s}{\left(\frac{x}{\sqrt{2}L_D} + 1\right)^2}}$$

P. Silvaco attached

Device Electronics
2018-04-27
Week 15
Review on Friday @ 3:30

Threshold Voltage:

$$V_{th} = 2|\phi_p| + V_c + V_{FB}^0 + \frac{1}{C_{ox}}\underbrace{\sqrt{2qN_a \epsilon(2|\phi_p| + V_{CB})}}_{|Q_{depletion}|}$$

↑ Gate voltage and not V_{GS} (V_{th})

↑ Source voltage (V_c)

↑ Φ_{MS} (V_{FB}^0)

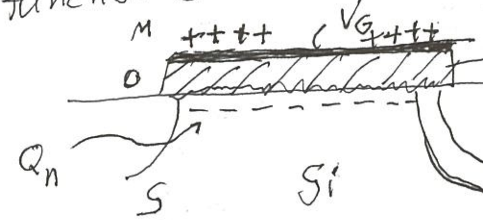
$Q_{depletion}$: depletion charge

$$X_{d,max} = \sqrt{\frac{2\epsilon_s}{qN_a}(2|\phi_p| + V_{CB})}$$

$$\boxed{Q_n = -C_{ox}(V_G - V_{th})}$$

Inversion layer charge

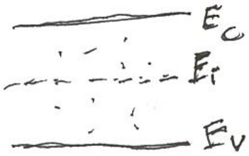
Φ_{ms}: metal semiconductor work function difference



Some traped charges in the oxide (SiO_2)

Some dangeling bonds

N-Mos example



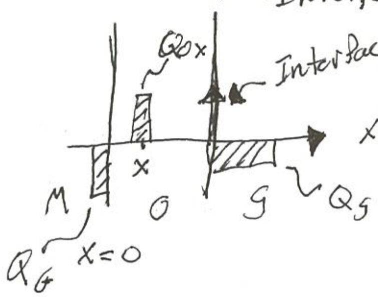
→ Some dangeling bonds may exist in the band gap

→ Can attract negative charges

how to take in account charges

extra charges, at interface and in the oxide

$$V_{FB} = V_{FB}^0 + \Delta V_{FB}$$ ← due to oxide or interface charges



Some depth in oxide

Q_G: gate charge

Q_{ox}: oxide charge

Q_S: charge silicon

$$Q_{ox} + (Q_G + Q_S) = 0 \quad \Big| \; Q_S = 0$$

New flat band condition

gate charge to compensate for oxide charge

$Q_{ox} = -Q_G$ (Want the following)

$$E_{ox} = \frac{-Q_{ox}}{\epsilon_{ox}}$$

$$\Delta V_{FB} = x_1 E_{ox} = \frac{-x_1 Q_{ox}}{\epsilon_{ox}}$$

generalized

$$\Delta V_{FB} = -\frac{1}{C_{ox}}\int_0^{X_{ox}} x_1 \rho_{ox}(x_1)\,dx_1$$

$$\Delta V_{FB} = -\frac{X_{ox}}{\epsilon_{ox}}\int_0^{x_{ox}} \frac{x_1}{x_{ox}} \rho_{ox}(x_1)\,dx$$

$$\Delta V_{FB} = \frac{-1}{C_{ox}}\underbrace{\int_0^{X_{ox}} \frac{x}{X_{ox}} \rho_{ox}(x)\,dx}$$

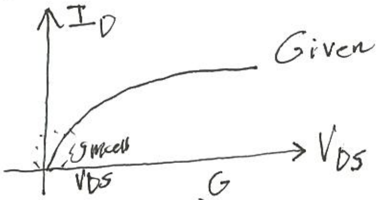
C_{ox}: Capacitance of oxide

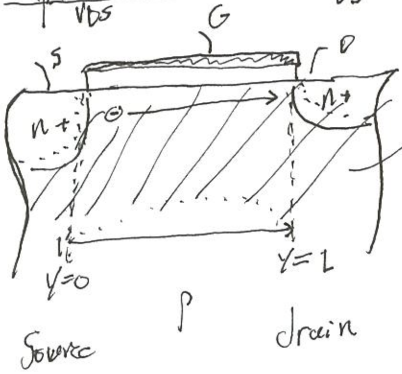
Change in → Mean of the oxide charge

$$\boxed{V_{FB} = \Phi_{MS} - \frac{Q_{int}}{C_{ox}} - \frac{1}{C_{ox}}\int_0^{X_{ox}} \frac{x}{X_{ox}} \rho_{ox}(x)\,dx}$$

Work function (dangeling) interface oxide (processing, tunneling)

Drain Current:





depletion of holes (only electrons)
Recombination ≈ 0

N-MOS

Electron current:

$$\frac{dn}{dt} = \frac{1}{q}\nabla\cdot\vec{J_n} - R \qquad S.S. \Rightarrow \frac{dn}{dt} = 0$$

small V_{DS} w.r.t. V_{GS} : Vertical electric field,

current dominated in y direction

I_{DS} : drain current

$$J_{ny} = \text{only}$$

$$\frac{1}{q}\frac{dJ_{ny}}{dy} - R$$

$$R = 0$$

$$\therefore$$

$$\frac{dJ_{ny}}{dy} = 0 \quad \therefore J_{ny}(y) = \text{Constant Value}$$

$$J_{ny} = -q\mu_n n \frac{d\Psi_{ny}}{dy} \Rightarrow -q\mu_n n \frac{V_{DS}}{L}$$

Ψ_{ny} … Quasi fermi potential

$$\frac{d\Psi_{ny}}{dy} = \text{constant} = \frac{V_{DS}}{L}$$

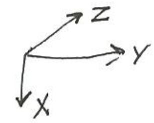
Ψ_{ny} = linear function of y.

$$\Psi_n(y) = \frac{V_{DS}}{L}\cdot y$$

→ Inversion layer thickness determined by debye length

$\sim L_D$

$$I_D = \int_0^W dz \int_0^{x_{inv}} dx\, J_{ny}(x)$$



$$I_D = W\int_0^{x_{in}} -q\mu_n n(x)\frac{V_{DS}}{L}\,dx$$

$$= -q\mu_n W\frac{V_{DS}}{L}\int_0^{x_{in}} n(x)\,dx$$

$$\underbrace{\left(-q\int_0^{x_{in}} n(x)\,dx\right)}_{\text{Inversion layer charge}}$$

drain current

· Inversion layer charge $\boxed{Q_{inv} = Q_n}$

$$\boxed{I_D = \mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS} - V_{th})\cdot V_{DS}}$$

Sign disappears because Q_n is a negative charge for electrons

Transit time:

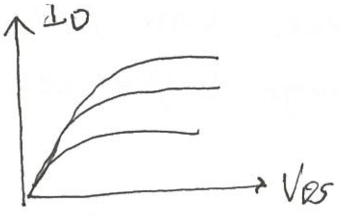
$$T_t = \frac{L}{V_{dr}} = \frac{L}{\mu_n E_y} \qquad E_y = \frac{V_{DS}}{L}$$

$$I_D = \frac{-Q_n}{T_t}$$

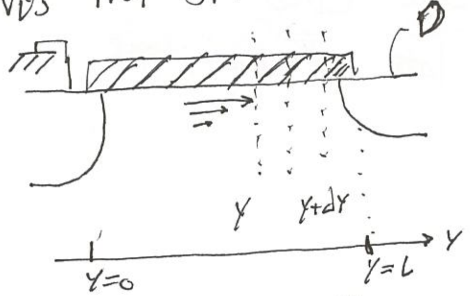
$$Q_n = WL\,Q_n$$

$Q_n = -C_{ox}(V_{gs} - V_{th})$

- Speed of device limited by minimum transit time
- What happens when V_{DS} is not small?



V_{DS} not small



$$\psi_n(y) = ?$$

- Current density flows through certain cross sections
  - going to encounter a voltage ~~drop~~ drop

$$d\psi_n = I_D \cdot dR = I_D \cdot \frac{1}{\mu_n n} \cdot \frac{dy}{X_{inv} W}$$

$$\downarrow$$

?

Conductivity $\mu_n n$

resistivity $= \frac{1}{cond\ldots}$

$$d\psi_n = I_D \cdot dR = I_D \cdot \frac{1}{\mu_n n} \cdot \frac{dy}{X_{inv} \cdot W}$$

$$d\psi_n = \frac{I_D \cdot dy}{\mu_n n W \cdot X_{inv}}$$

$$d\psi_n(y) = \frac{-I_D \cdot dy}{\mu_n W Q_n(y)}$$

$$\psi_n(0) = 0, \ \psi_n(L) = V_{DS}$$

$$I_D = -\mu_n\left(\frac{W}{L}\right)$$

$$\int_0^L I_D \cdot dy = -\mu_n W Q_n(y)\, d\psi_n$$

$I_D L$ ~~...~~

$$I_D = -\mu_n\left(\frac{W}{L}\right)\int_{\psi_n=0}^{\psi_n=V_{DS}} Q_n(y)\, d\psi_n$$

$$Q_n = -C_{ox}(V_{GS} - V_{th})$$

$$Q_n = -C_{ox}(V_G - V_{th})$$

← Without source reference

$$Q_n = -C_{ox}(V_G - 2|\phi_p| - V_{FB} - \underset{\substack{\uparrow \\ \text{"}V_c\text{"}}}{\psi_n}) + -$$

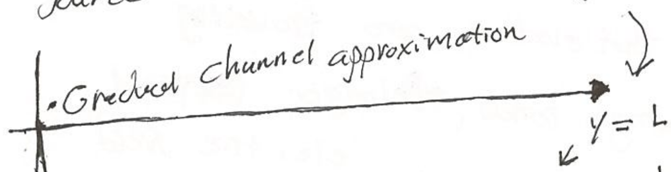
local quasi fermi potential

$$\sqrt{2\epsilon_s q N_a (2|\phi_p| + \underbrace{\psi_n(y) - V_B}_{\text{"}V_{CB}\text{"}})}$$

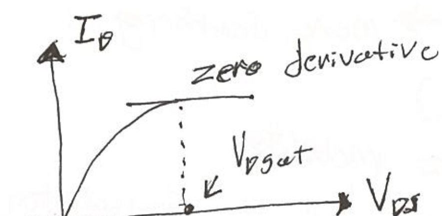
quasi fermi potential difference from some point y in the channel to the substrate

- Simplest derivation when taken from the source

$\psi_n(y) \to V_s$



$$I_D = \mu_n C_{ox}\left(\frac{W}{L}\right)\left[(V_G - V_{th})V_{DS} - \frac{V_{DD}^2}{2}\right]$$



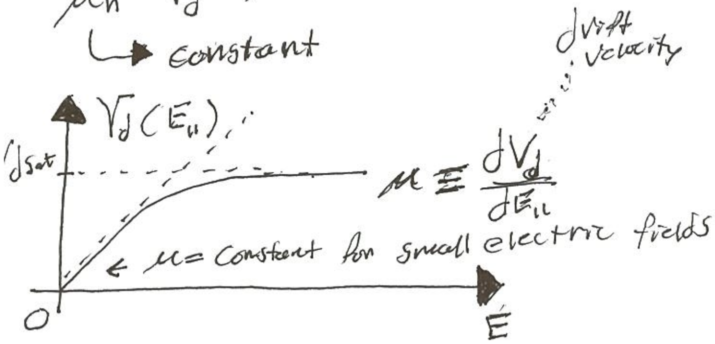
$$V_{Dsat} = V_G - V_{th} \Rightarrow I_{DSat} = \frac{\mu_n}{2}\left(\frac{W}{L}\right) \to$$

$$\to C_{ox}(V_G - V_{th})^2$$

· as channel pinches off we go into saturation.

Mobility

$\mu_n : V_d = \mu E$
↳ constant



→ V_d
→ $E_\parallel$

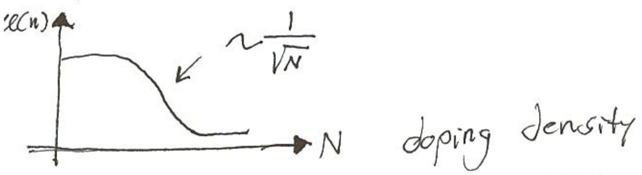
$E_\parallel$ Parallel electric field

$\vec{J}_n = -q\mu_n(\vec{E})\, n \nabla \psi_n$

$T_{th} = \frac{L}{V_{dsat}}$

V_{dsat} : saturation velocity

~~doping~~, conductivity $\sigma = \mu \cdot n$



→ Note that electrons are traveling

dangling bonds, → larger vertical electric field

→ closer electrons are to interface

→ more scattering

$\mu_n(E_\parallel, N, E_\perp)$
→ all decrease mobility

effects require more of a simulation

→ short channel effect

→ pinchoff effect

→ drift diff quasi fermi PN junction

capacitance, MS junction, IV charac,

Short, MOS device, work function

capacitance, charge coupled devices

gate voltage

→ Reading Period Class
3:00, lots of questions

Device part 1

2018-05-04 (filled in 2018-05-12)

T.E $J_n \neq 0$, $J_p \neq 0$

ψ_n, ψ_p : Quasi fermi potential

$$n = n_i e^{\frac{(\phi - \psi_n)}{V_T}}$$

$$p = n_i e^{\frac{-(\phi - \psi_p)}{V_T}}$$

$\psi_n(x) = ?$

$\psi_p(x) = ?$

Semic.

$$\vec{J_n} = -q\mu_n n \nabla \psi_n$$

$$\vec{J_p} = -q\mu_p p \nabla \psi_p$$

Conductor: (Diff eqn = 0)

$$\vec{J} = -q\mu n \nabla \phi$$

Basics

Poisson Eqn. $\nabla^2 \phi = \frac{-\rho}{\varepsilon}$

$$\rho = q(\underbrace{p - n}_{\text{mobile}} + \underset{\uparrow \text{ fixed doping}}{D})$$

$$D = N_D^+ - N_A^-$$

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon}$$

$$\vec{E} = -\nabla \phi$$

current continuity equations

elec. $\frac{dn}{dt} = \frac{1}{q} \nabla \cdot \vec{J_n} - R$

$R > 0$: recombination

$R < 0$: Generation

holes: $\frac{dp}{dt} = -\frac{1}{q} \nabla \cdot \vec{J_p} - R$

$$\vec{J_n} = q(D_n \nabla n - \mu_n n \nabla \phi)$$

$$\vec{J_p} = -q(D_p \nabla p + \mu_p p \nabla \phi)$$

Thermal Equilibrium : $\vec{J_n} = 0$, $\vec{J_p} \equiv 0$ or $R \equiv 0$

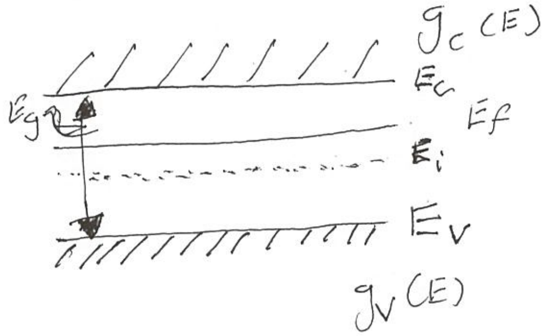
$\therefore n = n_i e^{\phi/V_T}$ T. E.

$p = n_i e^{-\phi/V_T}$

$V_T = \frac{KT}{q} = \frac{D_n}{\mu_n} = \frac{D_p}{\mu_p}$ Einstein Relation

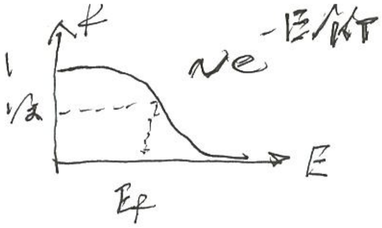
$$D_n \nabla n = \mu_n n \nabla \phi$$

$$\frac{\nabla n}{n} = \frac{\mu_n}{D_n} \nabla \phi$$



Fermi-Dirac statistics: (TE Concept)

$$f_{FD}(E) = \frac{1}{1 + e^{\frac{E - E_F}{KT}}}$$



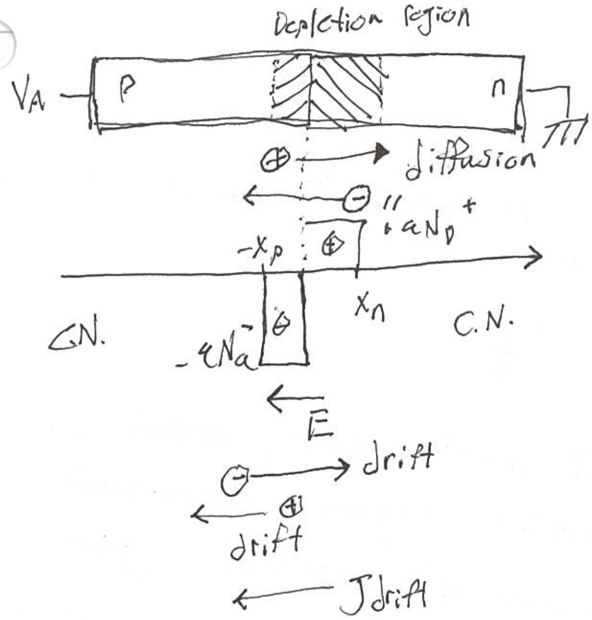
$$n = n_i e^{\frac{E_f - E_i}{kT}}$$

$$p = n_i e^{\frac{E_i - E_f}{kT}}$$

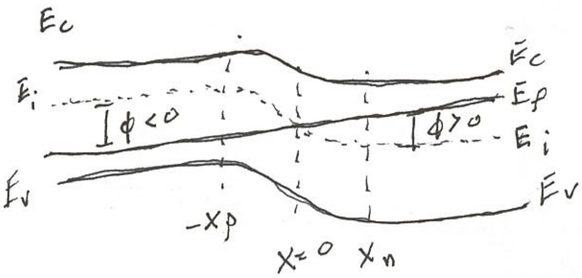
$$\phi = \frac{E_f - E_i}{q}$$

2018-03-04
Device review part ?

pn-junction:

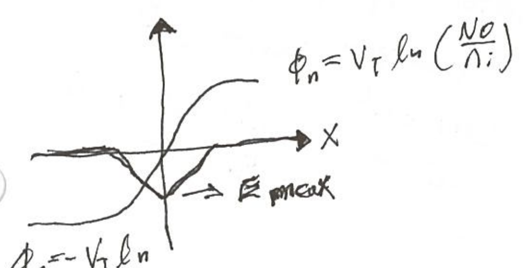


In T.E.
$$J^{drift} = J^{diff}$$
$$\therefore \vec{J} \equiv 0$$



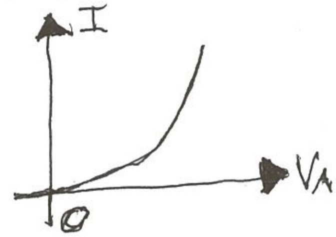
Depletion region approximation:

$0 \leq x \leq x_n$:
$$\frac{d^2\phi}{dx} = \frac{-qN_D^+}{\epsilon_s}$$



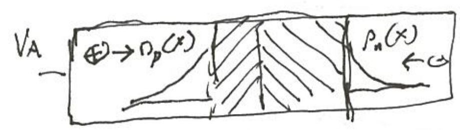
$$\phi_p = -V_T \ln\left(\frac{N_a}{n_i}\right) + V_A$$

understand the current flow



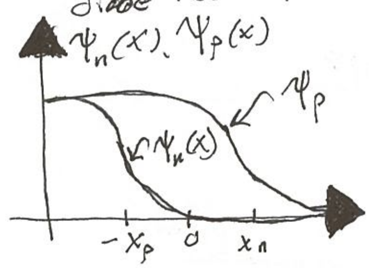
- transmit current from p to n

$$J = J_n(-x_p) + J_p(+x_n)$$



$I = J \cdot A$ ideal diode
$\Rightarrow$

ideal diode equation
$$I = I_0\left(e^{V_A/V_T} - 1\right)$$
additional effects:
$$I = I_0'\left(e^{V_A/nV_T} - 1\right)$$
$1 \leq n < 2$
diode ideality factor



$$np - n_i^2 = n_i^2\left(e^{\frac{\psi_p - \psi_n}{V_T}} - 1\right)$$

$$\vec{J}_n = -q\mu_n \nabla \psi_n$$
$$\vec{J}_p = -q\mu_p \nabla \psi_p$$

Shotty diodes

MOSFETS , BJT'S & JFETS
Yes   No

Metal Semiconductor Junction

ideal: $I = I_0 (e^{V_A/V_T} - 1)$

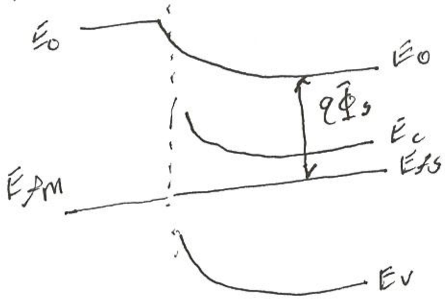
additional effects:

$I = I_0' (e^{V_A/nV_T} - 1)$

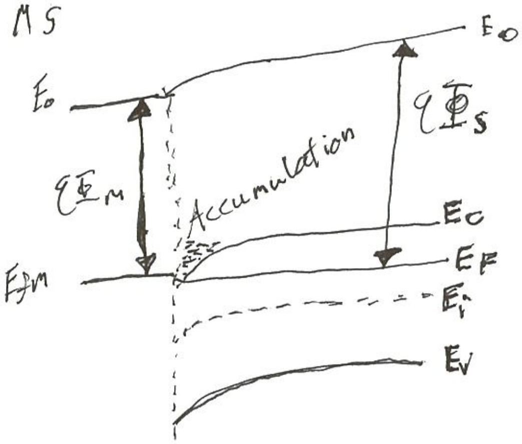
$K \leq n < 2$

diode ideality factor

M·S

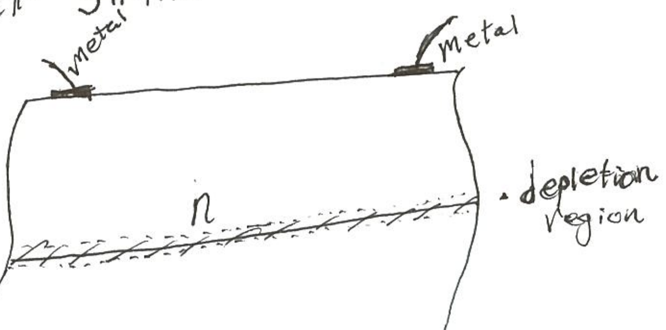


MS



n-type

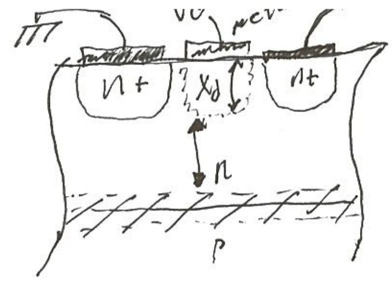
Jfet similar



~~Metal~~ workfunction greater than work function of n-type

Shotcky ~~junction~~ Junction

· dope ~~increas~~ workfunction of n-type ⇒ ohmic contact



· linear relationship, I can use to make resistor

$R = \rho \frac{L}{A}$

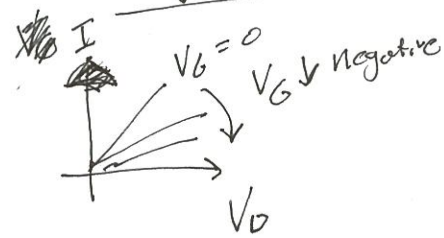
→ ~~Shot~~ Sholky diode, ~~ohmic~~ creats depletion region calculate thickness of depletion region.

Controlling → small current in a diode, negative bias

$V_G =$

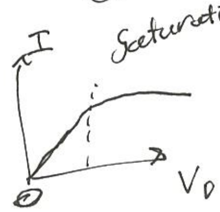
Want small I_G

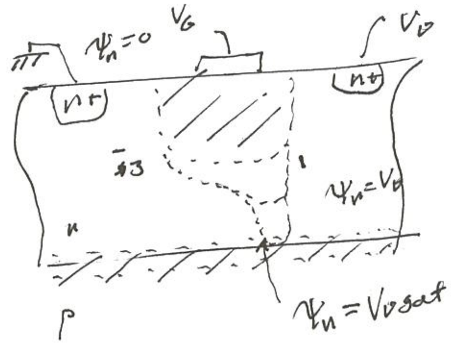
smaller A ∴ $R \to$ ~~dep~~ ∞



→ Calculate V_G (negative) when does ~~Capacitor~~ turn off current saturation why does it saturate

26

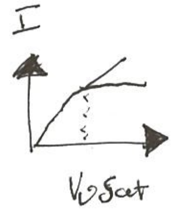




$V_G = -2V$ $V_D = 1V$

~~Increase~~
Increase V_D eventually a connected depletion region

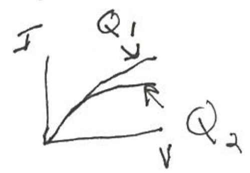
- V_{Dsat} for a given V_G
- V_D increased beyond V_{Dsat}



What does the depletion curve look like

flat or slope ?

1.6 JFET $L_1 < L_2$



Smaller length, decrease length of resistor

$V_D - V_{Dsat}$ voltage difference between depletion layer sides

MOS question

Given Metal & Semiconductor

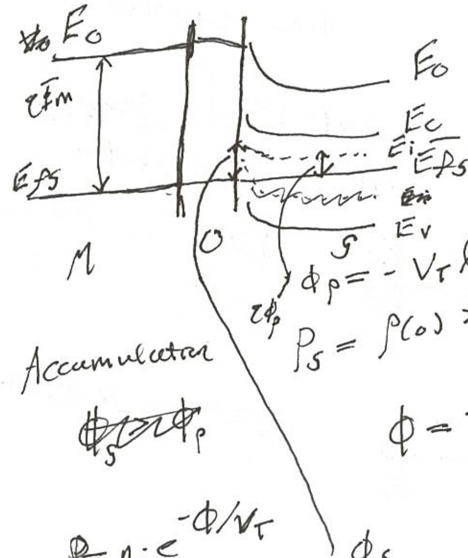
do we have depletion, accumulation or inversion ?

n-type or p-typ

3a.

p-type semiconductor

$\Phi_M > \Phi_S$



because p-type

$\phi_p = -V_T \ln\left(\frac{N_a}{n_i}\right)$

$\rho_s = \rho(0) \gg N_a$

$\phi = \frac{1}{q}(E_F - E_i)$

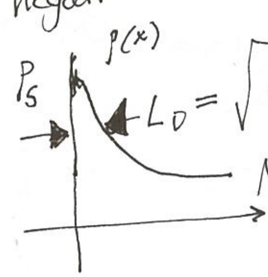
Accumulation

$\cancel{\phi_s \ge 2\phi_p}$

$\rho = n_i e^{-\phi/V_T}$

$\phi_s < \phi_p$

more negative   negative

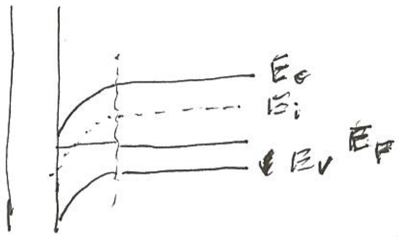


$L_D = \sqrt{\frac{\varepsilon_s V_T}{q \rho_s}}$

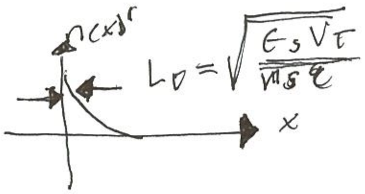
$X_d = \sqrt{\frac{\varepsilon_s V_s'}{N_a q}}$

$L_D \ll X_d$

In inversion



$n_s = n(x=0) >> N_a$

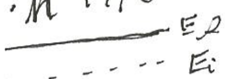


Band diagram

inversion
~~deplet~~ depletion
Flat band
Accumulation

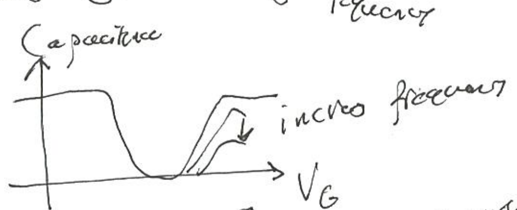
· p type Flat band ~~[illegible]~~
in Semiconductor
hole Majority

· n type electron majority



MOS Strong inversion
Capacitance = Oxide Capacitance

MOS device high f, lower Capacitance
frequency



~~from~~ Strong inversion

Capacitance decreases

generation time
inversion layer does not have time to
adjust, capacitor gets thicker