Joe Crandall's ModernPhysics Formula Sheet, covers special relativity and basic quantum mechanics, equations are repeated, Fields of Study Section Topic unknown Topic SubTopic Spacing for readability Special Relativity Principal of Relativity The Nature of Time $\Delta t > \Delta s > \Delta\tau$ The Metric Equation $\Delta s^2 = \Delta t^2 - \Delta d^2 = \Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2$ Proper Time $\Delta\tau_{AB} = \int_{t_A}^{t_B} (1-v^2)^{1/2} dt$ $\Delta\tau_{AB} = (1-v^2)^{1/2}\Delta t_{AB}$ $v = constant$ $v_0 = \frac{\sqrt{Gm}}{r}$ $G = 6.673x10^{-11}$ Binomial Approximation $\sqrt{1-v^2} \approx 1 - \frac{1}{2}v^2 for\, v \ll c$ $\Delta\tau_{AB} - \Delta t_{AB} = -\frac{1}{2}v^2\Delta t_{AB}$ Coordinate Transformations $\Delta t' = \gamma(\Delta t - \beta\Delta x)$ $\Delta x' = \gamma(-\beta\Delta t + \Delta x)$ $\Delta y' = \Delta y$ $\Delta z' = \Delta z$ $\Delta t = \gamma(\Delta t' + \beta\Delta x')$ $\Delta x = \gamma(\beta\Delta t' + \Delta x')$ $\Delta y = \Delta y'$ $\Delta z = \Delta z'$ $\gamma = 1/\sqrt{1-\beta^2}$ Lorentz Contraction $L = L_R\sqrt{1-v^2}$ $\theta = tan^{-1}(\frac{tan\theta'}{\sqrt{1-\beta^2}})$ The Cosmic-Speed Limit $v'_x = \frac{v_x - \beta}{1-\beta v_x}$ $v_x = \frac{v'_x + \beta}{1+\beta v'_x}$ $v'_y = \frac{v_y\sqrt{1-\beta^2}}{1-\beta v_x}$ $v_y = \frac{v'_y\sqrt{1-\beta^2}}{1+\beta v'_x}$ $v'_z = \frac{v_z\sqrt{1-\beta^2}}{1-\beta v_x}$ $v_z = \frac{v'_z\sqrt{1-\beta^2}}{1+\beta v'_x}$ Principal of Four-Momentum / Conservation of Four-Momentum

$$\begin{bmatrix} P_t \\ P_x \\ P_y \\ P_z \end{bmatrix} = \begin{bmatrix} m/\sqrt{1-v^2} \\ mv_x/\sqrt{1-v^2} \\ mv_y/\sqrt{1-v^2} \\ mv_z/\sqrt{1-v^2} \end{bmatrix} \qquad \begin{bmatrix} E_1 \\ P_1x \\ P_1y \\ P_1z \end{bmatrix} + \begin{bmatrix} E_2 \\ P_2x \\ P_2y \\ P_2z \end{bmatrix} = \begin{bmatrix} E_3 \\ P_3x \\ P_3y \\ P_3z \end{bmatrix} + \begin{bmatrix} E_4 \\ P_4x \\ P_4y \\ P_4z \end{bmatrix}$$

$E = \frac{m}{\sqrt{1-v^2}}$ $\rho = \frac{mv}{\sqrt{1-v^2}}$ $K = E - m$ $\frac{\rho}{E} = v$ Four Momemtum's Time and Space Components $\rho_t = \frac{m}{\sqrt{1-v^2}}$ $\rho = m + \frac{1}{2}mv^2\, when\, v \ll 1$ $K = m(\frac{1}{\sqrt{1-v^2}} - 1)$ $E_{rest} = mc^2$ Proporties of Four Momentum Conversion $P'_t = \gamma(P_t - \beta P_x)$ $P'_x = \gamma(-\beta P_t + P_x)$ $Four Momentum of Light$ $\frac{\rho}{E} = v = 1$ $\rho = E$ $m^2 = E^2 - \rho^2 = 0$

Quantum $E = \frac{1}{2}mv^2$

Binomial approximation, is x is a real number close to 0 and alpha is a real number then $(1+x)^\alpha \approx 1 + \alpha x$

The Quantum Theory of Light BlackBody Radiation Stefan's Law $e_{total} = a\sigma T^4$ Wien's Displacement law $\lambda_{max}T = 2.898x10^{-3}mK$ Plank Black Body Radiation Formula $u(f,T) = \frac{8\pi hf^3}{c^8}(\frac{1}{e^{\frac{hf}{k_bT}}-1})$ Photoelectric Effect Einstein's theory of the photoelectric effect $K_{max} = hf - \phi$ Photon Energy $E = \frac{hc}{\lambda} = hf \quad f = \frac{c}{\lambda}$ Bragg equation $n\lambda = 2dsin\theta \quad n = 1,2,3,...$ X-ray photon emission $\lambda_{min} = \frac{hc}{eV}$ If you increase the intensity of the light you increase the measured current Compton Effec $\lambda' - \lambda_0 = \frac{h}{m_ec}(1-cos\theta)$ Energy Conservation $E = m_ec^2 = E' + E_e$ Momentum Conservation $p = p'cos\theta + p_ecos\phi$ Energy and Momemtm $p_{photon} = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda}$ $p_{electron} = mv$

The Particle Nature of Matter $A = \frac{4}{3}\pi r^2$ $f = \frac{E_i - E_f}{h}$ $c = fh$ $\lambda = \frac{ch}{E_i - E_f}$ $K = \frac{1}{2}mv^2$ $mv = \frac{E_{photon}}{c}$ $E = E_{photon} + \frac{1}{2}mv^2$ $p = \frac{E_{photon}}{c}$ $\frac{1}{\lambda} = R(\frac{1}{n_f^2} - \frac{1}{n_i^2})$ $f = \frac{c}{\lambda}$ $E = hf$ $\frac{E}{c} = p$ $E = hcR(\frac{1}{n_f^2} - \frac{1}{n_i^2})$ if $n_i > n_f$ energy released $\frac{f}{c} = \frac{1}{\lambda}$ $f = \frac{E_i - E_f}{h}$ $\lambda = \frac{ch}{E_i - E_f}$ $\lambda = \frac{ch}{E_i - E_f}$ Millikan's stuff The Bohr Atom $\frac{1}{\lambda} = R_\infty(\frac{1}{n_f^2 - n_i^2})$ Plank-Enstein formula $E_i - E_f = hf$ size of allowed electron orbits $m_evr = n\hbar \quad \hbar = \frac{h}{2\pi}$ n = orbital levels Total energy of the atom $E = K + U = \frac{1}{2}m_ev^2 - k\frac{e^2}{r}$ permittitivity of free space $\epsilon_0 = 8.854x10^{-12}s^4A^2m^{-3}kg^{-1}$ Radii of Bohr orbits in hydrogen $r_n = \frac{n^2\hbar^2}{m_eke^2} \quad n = 1,2,3,..$ Bohr radius the smallest radius occurs for n = 1, is called the Bohr radius, denoted by a_0 $a_0 = \frac{4\pi\epsilon_0\hbar^2}{m_ee^2} = 5.292x10^{-11}m$ Energy levels of hydrogen and hydrogen like $E_n = -\frac{ke^2}{2a_0}(\frac{Z^2}{n^2}) = -13.6\frac{Z^2}{n^2}eV$ n = 1,2,3... z = 1,2,3... Emission wavelengths of hydrogen $\frac{1}{\lambda} = \frac{f}{c} = \frac{ke^2}{2a_0hc}(\frac{1}{n_f^2} - \frac{1}{n_i^2}))$ could the 1's be replaced by Z squared Corespondanced Principal

The Wave Nature of Matter $p = mv$ $\lambda = \frac{h}{p}$ $p = \gamma mv$ $\gamma = (1 - \frac{v^2}{c^2})^{-\frac{1}{2}}$ $\lambda = \frac{h\sqrt{1-(\frac{v}{c})^2}}{mv}$ $\lambda = \frac{h}{p}$ $p = \frac{h}{\lambda}$ $k = \frac{p^2}{2m}$ $k = \frac{3}{2}K_bT$ $\lambda p = \frac{hc}{E_{photon}}$ $E = fh$ $f = \frac{c}{\lambda}$ $E = \frac{ch}{\lambda}$ $\lambda = \frac{h}{p}$ $KE = \frac{1}{2}mv^2$ $p = mv$ $v = \frac{p}{m}$ $E = cp$ $KE = \frac{1}{2}\frac{p^2}{m}$ $KE = \frac{1}{2}\frac{E^2}{c^2m}$ $v = \frac{d}{t}$ $\lambda = \frac{h}{p}$ $p = mv$ $\lambda = \frac{h}{mv}$ $\lambda = \frac{h}{p}$ $p = mv$ $E = \frac{1}{2}mv^2$ $E = \frac{1}{2}pv$ $p = \sqrt{2E}$ $\lambda = \frac{h}{\sqrt{k2m}}$ $k = \frac{p^2}{2m}$ $k = (\frac{h}{\lambda})^2\frac{1}{2me}$ $KE \leftrightarrow E \leftrightarrow k \leftrightarrow K$ for most of these problems $\Delta p_x\Delta x \geq \frac{\hbar}{2}$ $p_x = \frac{\hbar}{2x} = \frac{h}{4\pi x}$ $E = \frac{1}{2}\frac{p^2}{m}$ $p = \sqrt{2Em}$ $\sqrt{2Em} = \frac{h}{4\pi x}$ $2Em = (\frac{h}{4\pi x})^2$ $E = \frac{(\frac{h}{4\pi x})^2}{2m}$ $E = cp$ $p_x = \frac{h}{4\pi x}$ $\frac{E}{c} = \frac{h}{4\pi x}$ $E = \frac{ch}{4\pi x}$ $\Delta E\Delta t \approx \frac{\hbar}{2}$ $\Delta p_x\Delta x \geq \frac{\hbar}{2}$ $\Delta E = \frac{h}{4\pi\Delta t}$ $E = cp$ $p = \frac{E}{c}$ $\lambda = \frac{hc}{E}$

Pilot Waves of De Broglie De Broglie Wavelengths $\lambda = \frac{h}{p} \quad f = \frac{E}{h} \quad p = \gamma mv \quad \gamma = (1 - \frac{v^2}{c^2})^{-\frac{1}{2}}$ $E^2 = p^2c^2 + m^2c^4 = \gamma^2m^2c^4$ Bohr orbits arise because the electron matter waves interfere constructively of a circular orbit $n\lambda = 2\pi r \quad \lambda = \frac{h}{m_ev}$ $m_evr = n\hbar$ Heisenberg Uncertainty Principle $\Delta x\Delta p_x \geq \frac{\hbar}{2}$ $\Delta y\Delta p_y \geq \frac{\hbar}{2}$ $\Delta z\Delta p_z \geq \frac{\hbar}{2}$ $\Delta t\Delta E \geq \frac{\hbar}{2}$

Quantum Mechanics in One Dimension $k = \frac{n\pi}{L}$ $E_n = n^2E_1$ $E_1 = \frac{\pi^2\hbar^2}{2mL^2}$ $E = \frac{hc}{\lambda}$ $n_i^2(E_1) - n_f^2E_1 = \Delta E$ $n = \frac{(\frac{\Delta E}{E_1}+1)}{2} = \frac{(\frac{\frac{hc}{\lambda}}{E_1}-1)}{2}$ $hf = E_i - E_f$ $\frac{h}{p}$ $E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$ $n = \frac{2Lmv}{h}$ $p = \frac{h}{\lambda}$ $\Delta E = \frac{hc}{\lambda} = hf$ $\frac{hc}{\lambda} = E_i = E_f$ $\Delta E = \frac{n^2\pi^2\hbar^2}{2mL^2}$ $\frac{hc}{\lambda} = \frac{1}{L^2}(\frac{n_i^2\pi^2h^2}{4\pi^22m} - \frac{n_f^2\pi^2h^2}{4\pi^22m})$ $L = \sqrt{\frac{\lambda}{hc}(\frac{n_i^2\pi^2h^2}{4\pi^22m} - \frac{n_f^2\pi^2h^2}{4\pi^22m})}$ $\lambda = \frac{hc}{E_i = E_f}$ $E = \frac{n^2h^2}{8mL^2}$ $\lambda = \frac{hc2mL^2}{(n_i^2h^2)-(n_f^2h^2)}$ $E = \frac{h^2}{8mL^2}$ If halfed $L_{new} = \frac{L_old}{2}$ Wavenumber = k angular freqeuncy $= \omega$ $kL = n\pi$ $k = \frac{2\pi}{\lambda}$ $\lambda = \frac{h}{p}$ $p = mv$ $n = \frac{L(\frac{2\pi}{\frac{h}{mv}})}{\pi}$ $n = \frac{2Lmv}{\pi}$ $k = \frac{p}{\hbar}$ $\omega = \frac{E}{\hbar}$ for nonrelativistic particle $\omega(k) = \frac{\hbar k^2}{2m}$ $E_n = \frac{n^2h^2}{8mL^2}$ $n = \frac{2Lmv}{h}$ Corespondance principle for electrons $p = \frac{x}{L} - \frac{1}{2\pi n}sin(\frac{2\pi nx}{L})$ classically left hand wall $\frac{x}{L}$ Probabilities of Infinite Square Well $p = \frac{x}{L} - \frac{1}{2\pi n}sin(\frac{2\pi nx}{L})$

Normalization of Wave Functions $\psi = Asin(X)$ $1 = \int_0^\pi |Asin(x)|^2 dx$ $A = \sqrt{\frac{2}{\pi}}$ Particle Probability Penetration Depth probability finding particle $x = L + \eta$ to $x = \eta$ $\eta =$ penetration distance L=width of wall $\langle p \rangle = \frac{1}{e^2}$ how many η outside well for outside/inside ratio = x $\frac{ln(x)}{2} = \eta$ Penetration at Top/Bottom of Well $E_n = \frac{n^2\pi\hbar^2}{2m(L+2\delta)^2}$ $\delta = \frac{\hbar}{\sqrt{2m(U-E)}} = \frac{1}{\alpha}$ U = depth of the well E = energy of the electron Energy levels in finite square well nucleaus as square well potential Transitions in a Harmonic Oscillator $\lambda = \frac{h}{p}$ $k = \frac{2\pi}{\lambda}$ $E = \frac{hc}{\lambda}$ $E_n = (n + frac12)\hbar\omega$ $\Delta E = \hbar\omega$ $E = \frac{hc}{\lambda}$ $f = \frac{c}{\lambda}$ $E_i - E_f = hf$ $\omega = \frac{hc}{\lambda\hbar((n_i+1/2)-(n_f+1/2))}$ $\omega = \frac{2\pi c}{\lambda((n_i+1/2)-(n_f+1/2))}$ $\Delta E = \frac{hc}{\lambda}$ $\Delta E = \frac{h\omega((n_i+1/2)-(n_f+1/2))}{2\pi}$ $\lambda = \frac{2\pi c}{\omega((n_i+1/2)-(n_f+1/2))}$ Energy Levels in harmonic oscillator spring constant = k harmonic oscillator zero point energy n = 0 $E_0 = \frac{1}{2}\frac{h}{2\pi}\sqrt{\frac{k}{m}}$ $\Delta E = \hbar\omega$ $hf = \hbar\omega = E$ $\omega = \sqrt{\frac{k}{m}}$ $E_n = (n+\frac{1}{2})(\frac{h}{2\pi})(\sqrt{\frac{k}{m}})$ $E = \frac{hc}{\lambda}$ $\frac{hc}{\lambda} = \hbar\omega((n_i + 1/2) - (n_f + 1/2))$ $\omega = \sqrt{\frac{k}{m}}$ $\lambda = \frac{hc\sqrt{m}2\pi}{h\sqrt{k}((n_i+1/2)-(n_f+1/2))}$ Expectation values in an Infinite Square well $n = 1\langle x \rangle = \frac{L}{2}$ $n = 1\langle x^2 \rangle = 0.283L^2$ $n = 1\langle p \rangle = 0$ Expectation Value and Most Probable Location $\psi(x) = Ae^{-x}(1 - e^{-x})forx \geq 0$ and $\psi(x) = 0forx \leq 0$ $\int_0^L (\psi_n(x))^2 dx = 1$ $\int_0^L (Ae^{-x}(1 - e^{-x}))^2 dx = 1$ $A^2\int_0^L e^{-4x} - 2e^{-3x} + e^{-2x}dx = 1$ $(\frac{-e^{-4L}}{4}) - (\frac{-1}{4}) + (\frac{2e^{-3L}}{3}) - (\frac{2}{3}) + (\frac{-e^{-2L}}{2}) - (\frac{-1}{2}) = \frac{1}{A^2}$ $lim \to \infty$ $A = \sqrt{12}$ $A = 3.464$ electron most likeley to be foun $(\psi_n)^2 dx = 0$ $\psi_n = 3.464e^{-x}(1 - e^{-x})$ $(3.464e^{-x}(1 - e^{-x}))^2 dx = 0$ ... $11.9993(\frac{2(1-e^{-x})}{e^{3x}} - \frac{2(1-e^{-x})^2}{e^{2x}}) = 0$ ... $x = 0.693147$ calculate expectation value of position x $\langle x \rangle = \int_0^\infty x|\psi(x)|^2 dx$ $\langle x \rangle = \int_0^\infty x|3.464e^{-x}(1 - e^{-x}))|^2 dx = 0$ $\langle x \rangle = 12\int_0^\infty x(e^{-x} - e^{-2x})^2 dx = 0$ $= \frac{13}{12} = 1.0833$

Quantum Mechanics in One Dimension, from test 2 WaveFunction Classic Probability Schrodiner Equation $E\psi(x) = U(x)\psi(x) - \frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}$ $E_n = \frac{\hbar^2 k^2}{2m} = \frac{n^2\pi^2\hbar^2}{2mL^2}$ Particle in Find the Wave Function $\psi(x) = Asinkx + Bcoskx$ Boundary conditions dictates that $\phi(x) = 0$ at x = 0 and x = L for x = 0, this means that B = 0 for x = L, this means that $kL = n\pi$ fro integer vaues of n $\psi_n(x) = Asin(\frac{n\pi x}{L})$ Find normalization constant for A $\int_\infty^\infty \psi_n(x)\psi_n(x)dx = 1 \to$ $A^2\int_0^L sin^2(\frac{n\pi x}{L}) \to A = \sqrt{\frac{2}{L}}$ Normalized wave function looks like th case of standing waves on a string with fixed ends: $\psi_n(x) = \sqrt{\frac{2}{L}}sin(\frac{n\pi x}{L})$ Find the Energy Levels From boundary conditions discrete wave number is: $k_n = \frac{n\pi}{L} = \sqrt{\frac{2mE_n}{\hbar^2}}$ This yields discrete energies $E_n = \frac{\hbar^2 k_n^2}{2m} = \frac{\hbar^2}{2m}(\frac{n^2\pi^2}{L^2}) = \frac{n^2h^2}{8mL^2}$ Particle in a box:Classical vs Quantum Classical Probability $P(x) = \frac{1}{L}$ Classical particle moving in a valley $P_{Cl}(x) = \frac{2}{T}\frac{1}{v(x)} = \frac{2}{T}\sqrt{\frac{m}{2(E-U(x))}}$ Quantum probability $P(x) = |\psi(x)|^2 = \frac{2}{L}sin^2(\frac{n\pi x}{L})$ KE of particle in Box $E_n = \frac{1}{2}mv^2 = \frac{n^2h^2}{8mL^2}$ $E_{min} = \frac{p^2}{2m} = \frac{\hbar^2}{8mL^2}$ Finite Square Well Schrodinger eqn outside finite well: $\alpha^2 = \frac{2m(V_0-E)}{\hbar^2}$ $k^2 = \frac{2mE}{\hbar^2}$ Penetration Depth $\eta = \frac{1}{\alpha} = \frac{\hbar}{\sqrt{2m(V_0-E)}}$ Harmonic Oscillator Potential energy $U(x) = \frac{1}{2}kx^2$ Energy Levels $E_n = (n+\frac{1}{2})\hbar\sqrt{\frac{k}{m}} = (n+\frac{1}{2})\hbar\omega$ $hf = \hbar\omega$ Zero point energy $E_0 = \frac{1}{2}\hbar\omega$

Tunneling Phenomena Tunneling Phenomena Potential Step Reflection coefficient $R = (\frac{k-\alpha}{k+\alpha})^2$ $\alpha = k_2$ $k = k_1$ $k = \frac{p}{\hbar}$ $\omega = \frac{E}{\hbar}$ $k = \frac{\omega p}{E}$ Transmission coefficient $T(E) = \{1 + \frac{1}{4}[\frac{U^2}{E(U-E)}]sinh^2\alpha L\}^{-1}$ $T = 1 - R = \frac{4k\alpha}{(k+\alpha)^2}$ Quantization o Angular Momentum $L = \sqrt{l(l+1)}$ Probability Distribution what is Bohr radius $r = \frac{n^2 a_0}{Z}$ $T = 1 - R$ $R = (\frac{k-\alpha}{k+\alpha})^2$ $T(E) = \{1 + \frac{1}{4}[\frac{U^2}{E(U-E)}]sinh^2\alpha L\}^{-1}$ $sinh(x) = \frac{e^x + e^{-x}}{2}$ $T \approx e^{-2\alpha L}$ Reflection and Transmision Coeficients Case1) $E < V_0$ and $\alpha^2 = \frac{2m(V_0-E)}{\hbar^2} > 0$ Case2) $E > V_0$ and $\alpha^2 = \frac{2m(E-V_0)}{\hbar^2} > 0$

Quantum Mechanics in Three Dimensions article in a 3D Box Normalized Wavefunction $\sqrt{\frac{8}{L^3}}sin(\frac{n_x\pi x}{L})sin(\frac{n_y\pi x}{L})sin(\frac{n_z\pi x}{L})$ Quantized Energy $E = (n_x^2 + n_y^2 + n_z^2)\frac{\pi^2\hbar^2}{2mL^2}$ Schrodinger equation in three dimensions $-\frac{\hbar^2}{2m}\nabla^2\psi + U(r)\psi = i\hbar\frac{\partial\psi}{\partial t}$ Laplacian $= \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ The time-independent Schrodinger equation $-\frac{\hbar^2}{2m}\nabla^2\psi + U(r)\psi = E\psi(r)$ allowed values of momentum components for particle in a box $|p_x| = \hbar k_1 = n_1\frac{\pi\hbar}{L}$ for $n_1 = 1, 2, ...$ $|p_y| = \hbar k_2 = n_2\frac{\pi\hbar}{L}$ for $n_2 = 1, 2, ...$ $|p_z| = \hbar k_3 = n_3\frac{\pi\hbar}{L}$ for $n_3 = 1, 2, ...$ Discrete energies allowed for a particle in a box $E = \frac{1}{2m}(|p_x|^2 + |p_y|^2 + |p_z|^2) = \frac{\pi^2\hbar^2}{2mL^2}\{n_1^2 + n_2^2 + n_3^2\}$ $E = \frac{1}{2m}(|p_x|^2 + |p_y|^2 + |p_z|^2) = \frac{\pi^2\hbar^2}{2m}\{(\frac{n_1}{L_1})^2 + (\frac{n_2}{L_2})^2 + (\frac{n_3}{L_3})^2\}$ Hydrogen Atom and Hydrogen Like $E_n = \frac{-13.6}{n^2}eV$ Degeneracy $= n^2$ $E_n = \frac{-Ke^2}{2a_0}\{\frac{Z^2}{n^2}\}$ Angular Momentum Vector $= |L| = \sqrt{l(l+1)}\hbar$ Z component $= L_z = m_l\hbar$ Orientations of L are quantized $= cos\theta = \frac{L_z}{|L|} = \frac{m_l}{\sqrt{l(l+1)}}$ Change in n and emission of Photon $13.6eV(\frac{1}{n_2^2} - \frac{1}{n_1^2}) = \frac{hc}{\lambda}(\frac{1}{1.6x10^{-19}})$ expectation values Example expectation values for a particle in a box in a general state labeled by n $\langle x \rangle = \frac{L}{2}$ Hydrogen ground state most probable radius(and hydrogen like) $P(r) = \frac{4Z^3}{a_0^3}r^2 e^{\frac{-2Zr}{a_0}}$ average radius of the electron $r = \frac{a_0}{Z}$ fixed n, electron with highest angular momentum found at distance $r = n^2 a_0$ the letters s,p,d,f used to designante $l = 0, 1, 2, 3$ Zeeman Effect magnetic moment $\mu = IA$ Potential energy due to external magnetic field in z-direction $U = -\vec{\mu}B = \mu_B m_l B$ magnet placed in B field has interaftion potential energy $U = -\mu B$ magnetic moment due to current loop $\vec{\mu} = -\frac{e}{2m}\vec{L}$ Zeeman Effect $E = E_n + \mu_B m_l B$ electron spin intrinsic $S = \frac{\sqrt{3}}{2}\hbar$ angle between electron's spin angular momentum and its +z component $Cos\theta = \frac{S_z}{S} = \frac{\frac{\hbar}{2}}{[\sqrt{3/2\hbar}]} = \frac{1}{\sqrt{3}}$ Electron transitions with spin $(2n+1)$ degeneracy with spin $2(n^2)$ electron spin

Atomic Structure J is total angular momentum with spin $J = L + S$ high l = circular orbit, low l - eleptical orbit due to pull from nucleaus $E_n = \frac{-13.6Z^2}{n^2}eV$ effective nuclear charge Z_{eff} $E_{nl} = -\frac{Z_{eff}^2}{n^2}(13.6eV)$ Mosely law for xray frequency $f = (2.47x10^{15}Hz)(Z-1)^2$