Module 1: Quantum Strangeness

1.1 What is quantum computing and why does it matter?

Currently, the excitement about quantum computing centers on two quite different applications:

- The promise of *faster* computation for some problems:
    - The prototypical example, the one we will cover, is Shor's algorithm for fast factorization, which threatens to un-do parts of conventional cryptography.
    - A less hyped but equally important example: fast computation of molecular properties which will enable *designing* molecules (drugs, materials etc).

- The promise of unbreakable security:
    - Strange quantum properties allow for two bits to be *entangled* so that observation of one bit affects the other.
    - Then, if Eve observes a passing bit sent by Alice to Bob, they'll know.
    - This kind of thing just can't be done with conventional hardware.

However, there's an even more important reason that garners little hype:

- There is a (quiet) revolution underway in the science and engineering of materials.

- It is now routinely possible in an average lab to manipulate individual atoms and even their constituent parts.

- And the laws of physics that operate at the atomic and sub-atomic level are *quantum mechanics*.

- Thus, it appears reasonable that *some form* of quantum phenomena will prevail in the future of computing.

- One could draw an analogy with the steam engine:
    - Almost no transportation today operates with steam engines.
    - This was not predictable in the early days of steam engines.
    - Yet, the steam engine certainly drove innovation in understanding how to make engines of all kinds, including today's engines.
   - Principles developed with steam engines apply broadly to many kinds of engines.

1.2 Prelude to quantum strangeness: some thought experiments

Gedanken or thought experiments have a long and rich history in science, especially so in physics. (Example: Einstein's wondering what it would be like to travel on a light beam led to a profound altering of physics.)

Let's start with something simpler: reasoning about a pellets fired at a screen:
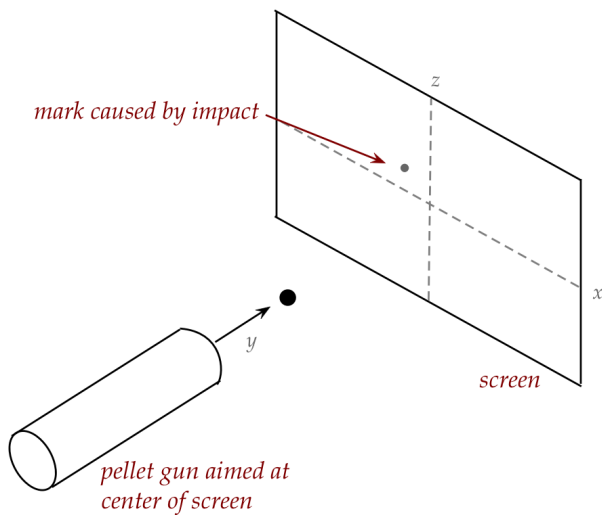
- As is typical in thought experiments, we'll *first* make simplifying assumptions and see if the assumptions need to be removed.

- Assume no wind, ignore gravity (because of short range), but assume that the pellet gun is imperfect in its ejection of the pellet.

- Each pellet hits the screen and leaves a visible mark.

- This leads to a "spread" of dots (marks) on the screen.

- Now suppose we imagine x-y-z axes with the origin at the center of where the gun is pointed (where a perfectly aimed pellet would land):
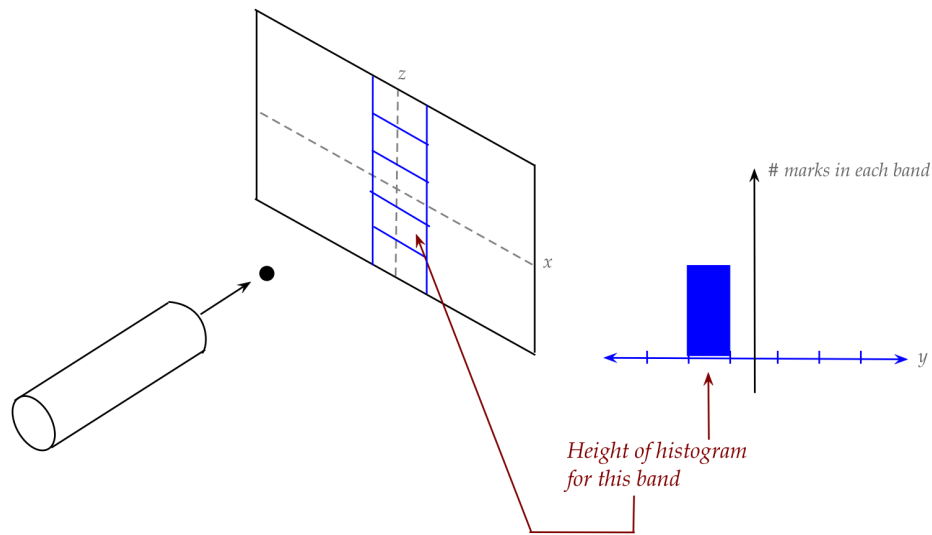
- Next, let's imagine horizontal bands in an area near the z-axis, as shown, and count how many pellets land in each band. This will give us a histogram along the z-direction.

In-Class Exercise 1: What is the shape of the histogram? Draw out a potential histogram by completing the histogram above. What would be expect to see with thinner and thinner bands (many of them), and lots of pellets? If we replaced horizontal bands with vertical bands near the x-axis to get an x-histogram, what would it look like?
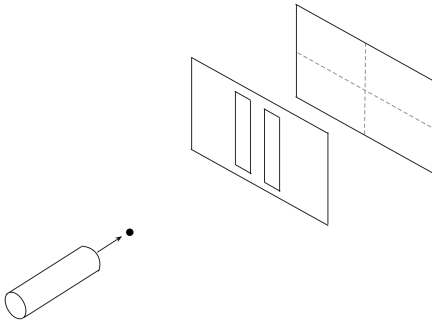
In-Class Exercise 2: How would your responses change if we placed a hole-in-a-wall between the gun and the screen?
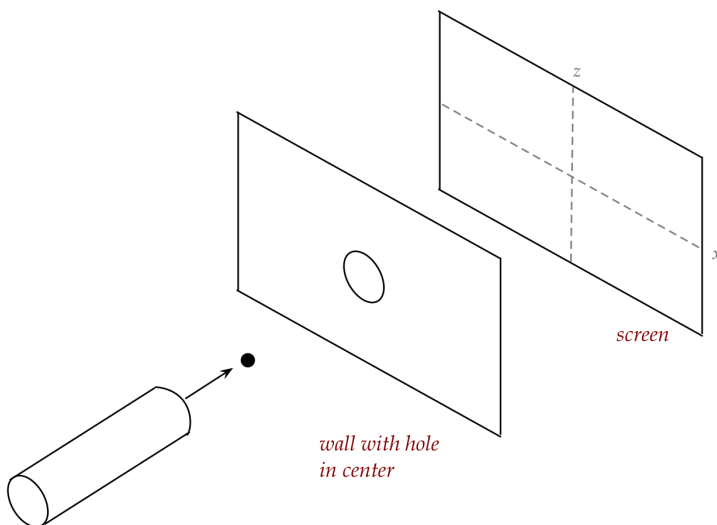
In-Class Exercise 3: Now consider moving the pellet gun further back so that the spread of trajectories is wide enough to encompass both holes shown here:
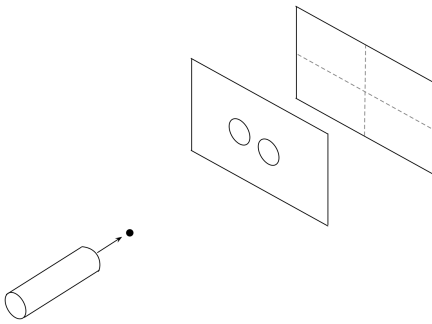
What would the x and y histograms look like? And how would those change if we changed holes to slits?

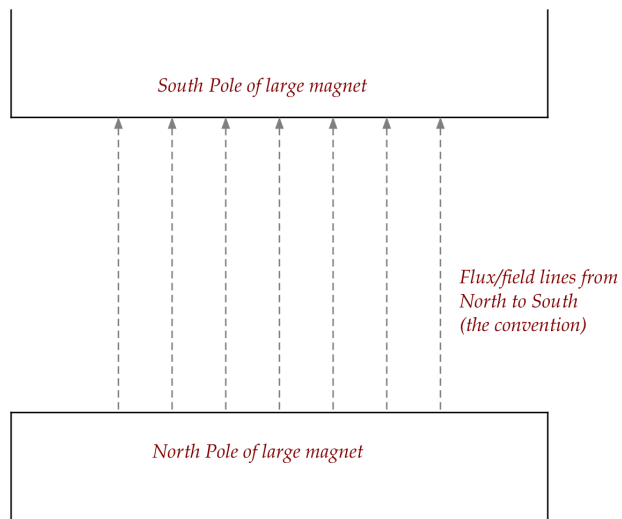The next thought experiment will be about *randomness*:

- Consider these two experiments:
    - Flip a coin and observe whether heads or tails faces up.
    - Shuffle a deck of cards and examine the top card.

- We often use probabilities to say
    - $\Pr[\text{heads}] = 0.5$ (coin flip)
    - $\Pr\left[\text{ace of spades}\right] = \frac{1}{52}$ (card example)

In-Class Exercise 4:

1. What is the source of randomness in each case?
2. For the coin-flip, could one in principle, knowing the launch velocity of the coin (and other variables) predict the outcome? What variables would be needed? Could a mechanical contraption be designed to flip a coin predictably?
3. Does this predictability extend to the card example?
4. What is an example of a physical measurement that is likely to have random errors, and for which an average over several measurements is needed?

For the next thought experiment, we need a bit of background in magnetism:

- Suppose we have the means to create a magnetic field that we can turn on or off at will:

South Pole of large magnet

Flux/field lines from North to South (the convention)

North Pole of large magnet

Some simplifying assumptions:

- The lines aren't straight the closer you get to the edge, so we'll focus on the area closer to the center.
- The density of field lines (field strength) is the same everywhere in the central region.
- The large magnet is placed on a frictionless table top.
  (So, this is a from-above view.)

- We'll use the term *homogeneous* to describe a magnetic field whose strength is the same everywhere (in the region of interest).

- Now consider placing a compass needle or a small bar magnet in this field that's slightly tilted.

South Pole of large magnet

N

S

North Pole of large magnet
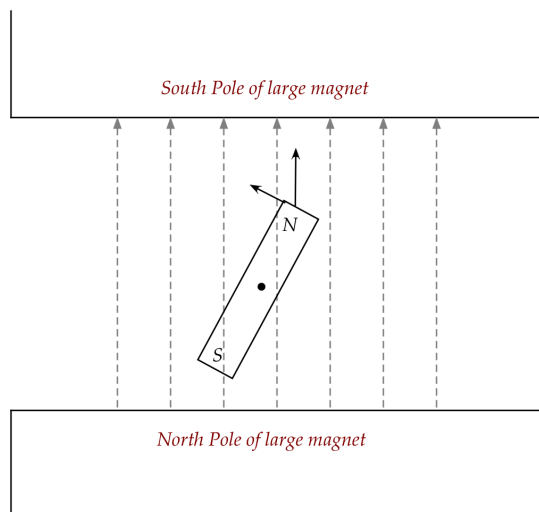
- Assume that the surface is frictionless (oiled, say) and that the bar magnet can move or rotate.

In-Class Exercise 5: How would you describe the motion of the bar magnet? What is the net force experienced by the center of the bar magnet?

Next, consider the same bar magnet placed in a magnetic field that's *inhomogenous* vertically:

South Pole of large magnet

Higher field strength

N

S

Lower field strength

North Pole of large magnet
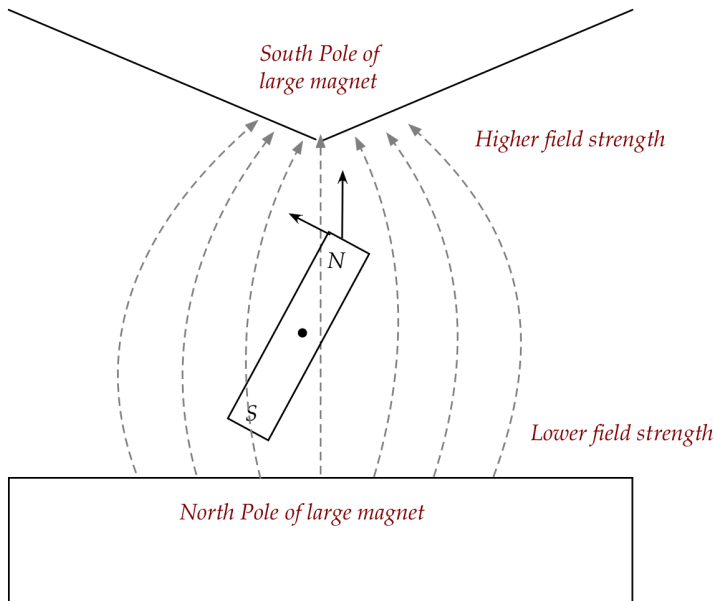
- By altering the shape of the larger magnet's South pole, one can arrange for a greater density of field lines closer to that pole.

In-Class Exercise 6: Now how would you describe the motion of the bar magnet? What is the net force experienced by the center of the bar magnet, and why?
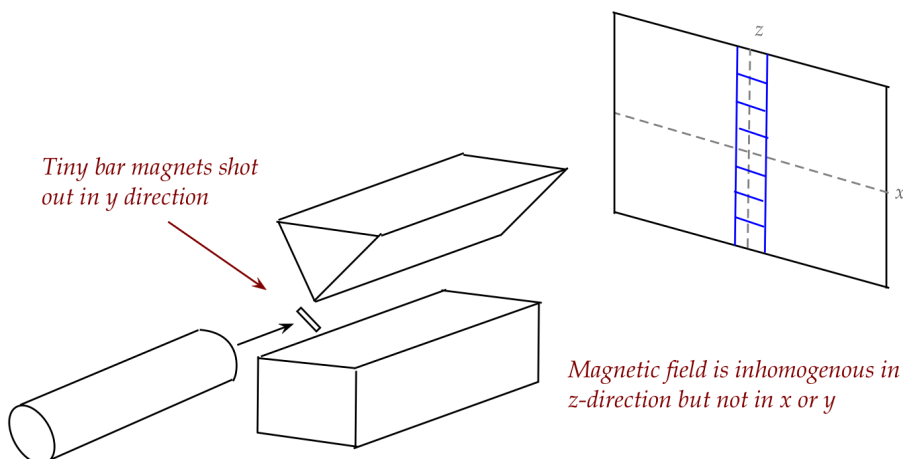
Next, we'll combine the two kinds of experiments by firing bar magnets through an inhomogenous field:

Tiny bar magnets shot out in y direction

Magnetic field is inhomogenous in z-direction but not in x or y

Some assumptions:

- We'll neglect the effects of gravity.

- One tiny bar magnet (a magnetized pellet) is shot out at a time, letting it hit the screen before the next one is sent.

- The pellet gun cannot control the initial orientation of a pellet, so assume that over time, pellets are launched at all kinds of angles, with no bias towards any particular angle.

In-Class Exercise 7: What does one expect for the z-histogram (vertical), and why? That is, what is the histogram going to look like if we count the "hits" in each little blue box? If one knows the landing spot for a particular bar magnet, how could one compute the vertical force experienced by the magnet?

The result and explanation above is what one would call the *classical* explanation based on what was known between the time of Newton and the very early 1900's.

1.3   Quantum strangeness: the Stern Gerlach experiment

In 1922, Otto Stern and Walther Gerlach performed their historic experiment in Frankfurt, Germany:

- Instead of firing small magnets, they launched silver atoms.
    - The stream of atoms was produced by heating silver in an oven, and placing successive barriers with small holes in the way to create the beam of atoms.
    - The detector was a coated glass plate.
    - The whole thing was placed in vacuum to prevent air molecules from interfering with the trajectories of the silver atoms.

- The fundamental assumption about silver atoms at the time:
    - They behave like mini magnets, and they emerge from the oven in a random orientation.
    - The thinking was: the outermost electron revolves around the nucleus, and it was already known that circularly moving electric charges (a current loop) exhibit magnetic behavior.

- Some interesting historical facts:
    - Part of the experiment's funding came from Goldman of Goldman-Sachs in NY, just based on a request letter.
    - A cigar made a difference (improved the visibility of the marks).
    - Sadly, Otto Stern had to leave Germany.

So, then, what did Stern-Gerlach discover?

Silver atoms
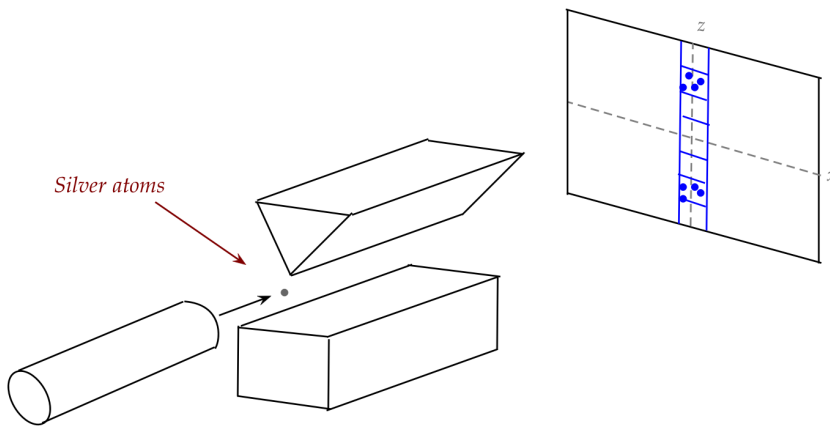
- There are two small vertically separated clusters.

- Based on the center of each cluster, one can calculate the numeric value of a magnetic property called *magnetic moment*. There are only two such values, one positive (the upper cluster), one negative (the other).

- If we give this a symbol, one could say:
   - Atoms in the upper cluster exhibited a magnetic moment of m_B.
   - Atoms in the lower cluster showed a magnetic moment of $-m_B$.

- The other big observation: the separation is up or down (along the magnetic field).

- These two observations can be stated more abstractly as:
   - The vectorial property called magnetic moment is *quantized* in both magnitude and direction.

- Here, quantization means: only particular values occur.
   - The magnitude takes on only two values: m_B or $-m_B$.
   - The direction (orientation) takes on only two values: up or down.

- Another key observation: if the magnetic field is turned off, the silver atoms behave just like pellets.

Many questions are now raised:

- Why are there two discrete deflections?
- Are there fundamentally two different kinds of silver atoms?
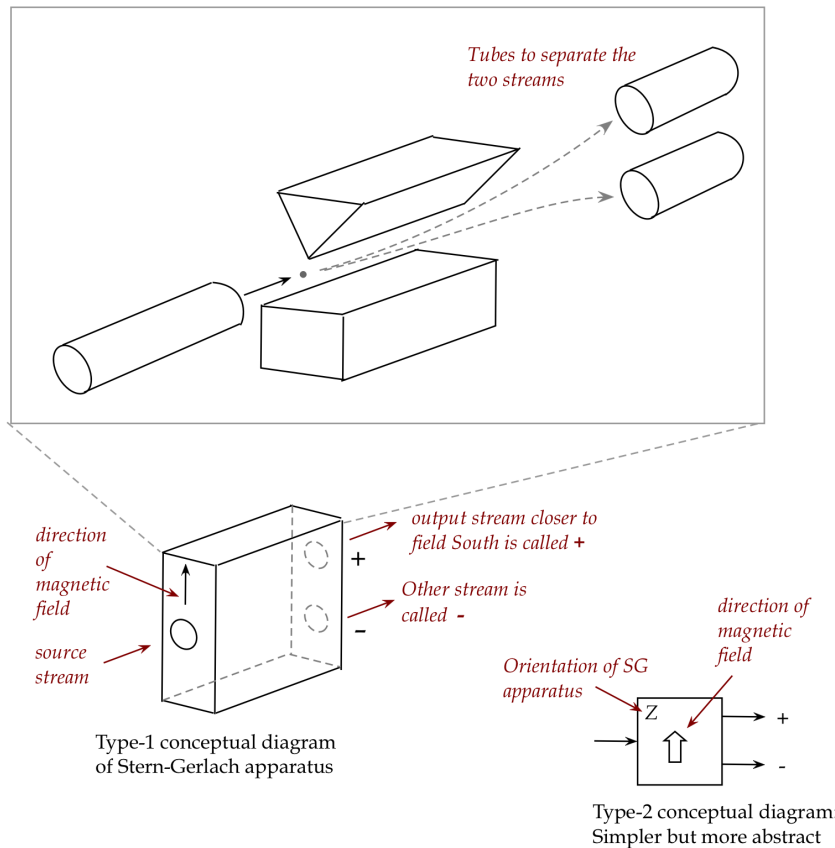- Does this happen with other atoms?

Let's answer the last one:

- Yes, it happens with other atoms but with different numbers of clusters vertically aligned.
- Example: nitrogen shows 4 clusters, sulfer shows 5. All are along the vertical direction.

In-Class Exercise 8: What other questions might one ask? What are you curious to know about this type of experiment?

This is where it starts to get interesting.

We'll use two diagrams to conceptually simplify *depicting* the Stern-Gerlach apparatus:

Tubes to separate the two streams

direction of magnetic field

output stream closer to field South is called +

Other stream is called −

source stream

+

−

Type-1 conceptual diagram
of Stern-Gerlach apparatus

Orientation of SG apparatus

direction of magnetic field

Z

+

−

Type-2 conceptual diagram:
Simpler but more abstract

- Assume that the two streams that earlier hit the screen in two distinct places are now fed through tubes to cleanly separate them into separate streams.

- The Type-1 diagram above is a conceptual diagram that simplifies the essence of the input and output streams:
    - There's one input port, and two output ports.
    - One of the output ports is labeled **+** (plus) to indicate that it's closer to the South end of the field.
    - The other is labeled **−**.
      (This is just the naming convention.)

- The diagram can be further conceptually simplifed into the Type-2 diagram shown that has all the needed info.
    - The Type-2 might be harder to use in visualizing, but it carries all the information about the experimental set up.

1.4 Multi-stage Stern Gerlach experiments

The thought experiment of examining the output and even subjecting the output to a further round of magnetic field traversal did not apparently occur to Stern-Gerlach.

It was in fact the undisputed champion of Gedanken experiments, Albert Einstein, who proposed it.

Consider the following set up:

Type-1 conceptual diagram

Type-2 conceptual diagram
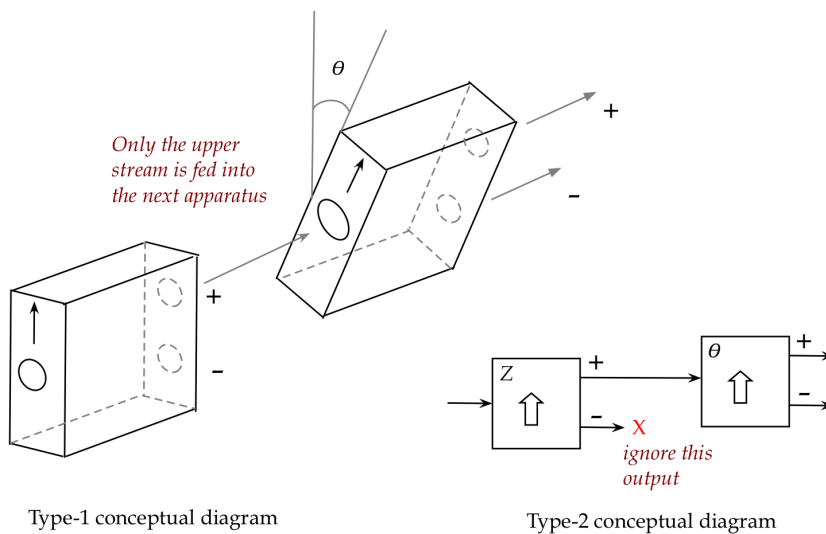
- Here, *one* of the output streams of the first SG apparatus is fed as input into a *second* SG apparatus.

- The other stream is merely blocked or ignored, but making sure it does not interfere with the first.

- The overall combination here has one input and three outputs
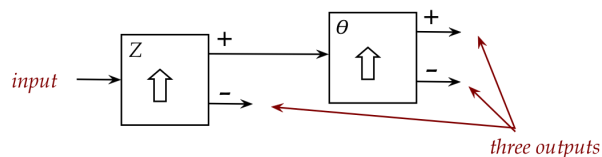
(Even if we block an output, we'll still be interested in the fraction of atoms that come out there.)

- The most common question is: what fraction of the input appears at each possible output?

- There's no reason to restrict the orientation of field to vertical: in this case the second apparatus is oriented at an arbitrary angle θ.

- Typically, in a multi-stage SG experiment, the first apparatus is oriented in the z-direction ($\theta = 0$) because we want a known starting point (either the upper or lower stream).

In-Class Exercise 9: Sketch, with modest neatness, the Type-1 and Type-2 diagrams for a two-stage SG experiment when the orientation of the second apparatus is

1. $\theta = 0$
2. $\theta = \pi$
3. $\theta = \frac{\pi}{2}$
4. $\theta = \frac{\pi}{4}$

In-Class Exercise 10: Sketch Type-1 and Type-2 diagrams for a *three*-stage SG experiment where:

1. The + output of the first stage is fed into the second.
2. The + output of the second stage is fed into a third apparatus.
3. The orientations of the three stages are: $\theta_1 = 0, \theta_2 = \frac{\pi}{2}, \theta_3 = 0$.


1.5 Stern Gerlach experiments, part I

We'll present results of modern versions of experiments that came after Stern and Gerlach's original experiment.

Let's start with a closer look at the single stage:
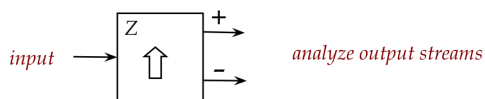
- Assume we can send large numbers of atoms through and carefully observe where each individual atom exits (+ or -).

- *What percentage of atoms exit in the + stream?*
 ▷ Answer: 50% (evenly distributed between + and -).
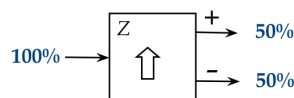
- *Is there a pattern to the exit streams, such as alternate + and -?*
 ▷ Answer: It is completely random.

- *Do some atoms go elsewhere, somewhere other than the + or - ports?*
  - ▷ Answer: No, all are fully accounted for in the +/- streams.

- *Do the observations above change with a different orientation θ?*
  - ▷ Answer: No

In-Class Exercise 11: How does one test that the splitting between two streams is random? Is it enough to merely establish that 50% exit through the + stream?

We'll next feed the + output into another SG apparatus oriented at $\theta = 0$:
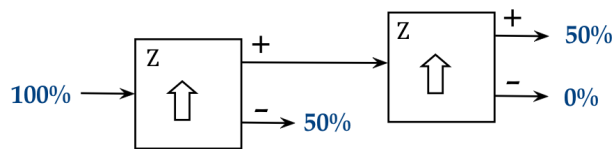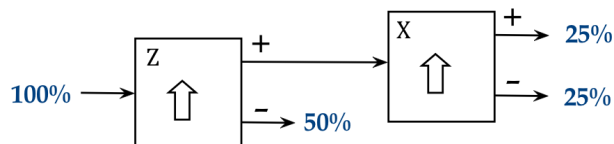

- The observed output: *all* the + stream atoms emerge at the + output of the second stage.

- A similar result obtains from sending the other stream:
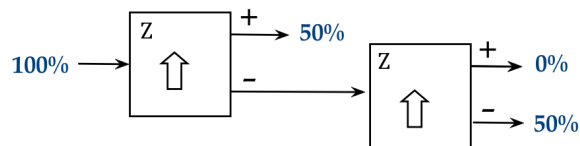
- At this point one might reason as follows:
  - ○ There are two kinds of atoms, + and -
  - ○ The first SG filters out one kind, which is confirmed by the second SG.

We'll now set the orientation of the second apparatus to align with the X direction, i.e., $\theta = \frac{\pi}{2}$:


- Now we see a random split of each + output atom between the horizontal + and - of the x-aligned second SG.

- A similar result is observed using the - output of the first SG:
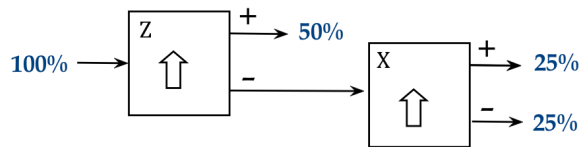
- What might one make of this?
  - One could argue that there's an "x" property that's randomly split across atoms, and so, all the "+z" atoms have equal numbers of "+x" and "-x".

We'll return to two-stage experiments shortly.

We also tweak our diagram to use *probabilities* instead of percentages:
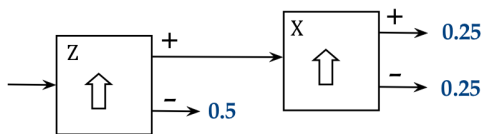
- If, over a large number of atoms, the percentage is 25% for a particular output port, we can say that the probability for a particular atom to emerge there is 0.25.

- One has to be careful about such statements because we'll need to assume *independence* between atoms.

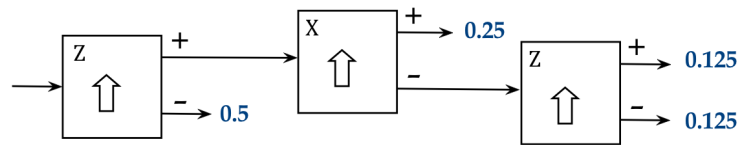**In-Class Exercise 12:** Explore the above notion of independence. Why does it matter?

1.6   Stern Gerlach experiments, part II

The obvious *three* stage experiments are to use Z-X-Z and Z-X-X:

- Here's the output of Z-X-Z the first with the + output of the middle SG:
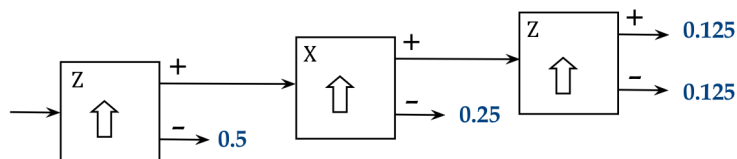
- And again with the other output of the X-aligned SG:

Z ↑ + → → X ↑ + → 0.25 → Z ↑ + → 0.125
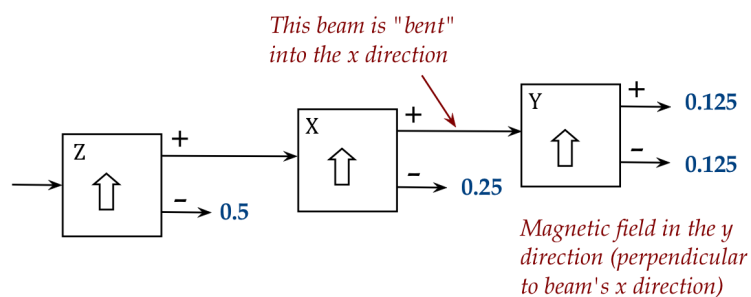   − → 0.5   −   − → 0.125

- The pattern of output from the third stage is observed to be random.

- What are we to make of this?
    - Why do "+z" atoms in the first experiment above, now filtered into "+x" split into "+z" and "-z" in the third stage?
    - What is causing the random output in the third stage?

- We can already see that the silver atoms do not carry a "+z" or "-z" property with them:
    - If it were true that each silver atom is either a "+z" kind or a "-z" kind (but not both), then it can't be that the purely "+z" stream after the first stage eventually produces "-z" in the third.

## 1.7   Stern Gerlach experiments, part III

Next, let's point out the observations from two additional variations of a three-stage set up:

- With some experimental difficulty, it turns out to be possible to change the direction of an atom stream by 90°.

- Then, we can apply a field in the Y direction and observe:

This beam is "bent" into the x direction

Z ↑ + → → X ↑ + → → Y ↑ + → 0.125
      − → 0.5   − → 0.25   − → 0.125

Magnetic field in the y direction (perpendicular to beam's x direction)

Now for something strange:

- It is possible, overcoming significant experimental challenges, to merge the two outputs of an SG apparatus:

Merge apparatus

The "tubes" emerging
from X are merged
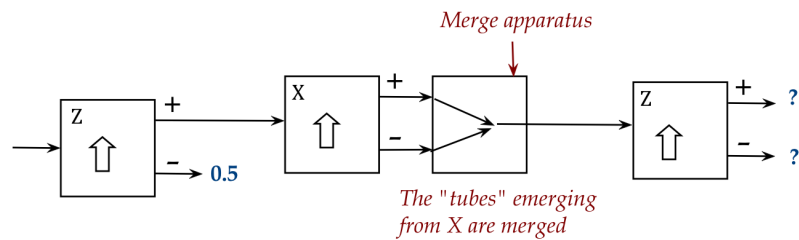
- The merge is such that every atom can go either way through the middle X apparatus but appear at the input of the third.

In-Class Exercise 13: What is a reasonable guess for the probabilities at two rightmost outputs? Start by asking: what is the probability that an atom arrives at the output of the merge apparatus? Then, further, what is the probability of such an atom to appear at the + output of the third stage? What is, therefore, the overall probability that an atom entering the first appears at the + of the third? [Answer provided in class]

If this wasn't strange enough, consider this variation:

Atom passage detector
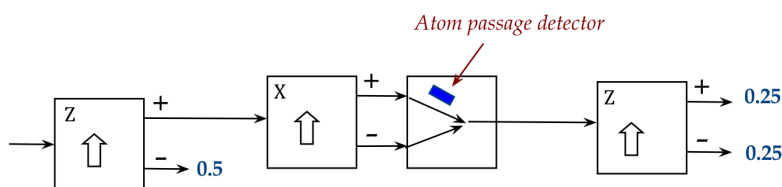
- One could use a faint source of light (photons) to illuminate which path an atom takes in the merge apparatus.

- Since we can send atoms slowly (one by one) through the whole apparatus, we can tell which path in the merge an atom takes.

- With this "which path" knowledge, the probabilities observed change drastically, as shown above.

Even stranger still:

- Suppose one can turn the *which-path detector* on or off at will for every atom that passes through the system.

- Then, consider only the subset of atoms when the which-path is ON:
    - For this subset of atoms, the output probabilities are equal (0.25 each).

- For the other atoms (when which-path is OFF), the probabilities are 0.5 (+ output) and 0 (for the - output)!

- Let's explain the situation a little further:

- Suppose you turn ON the which-path detector for atoms 1, 3, 4, 6, 10.
- And turn it OFF for atoms 2, 5, 7, 8, 9
- Then, the ON atoms (1, 3, 4, 6, 10) will split probabilistically evenly between the two outputs.
- This doesn't mean exactly evenly, but applying a coin-flip.
- But *every one of* atoms 2, 5, 7, 8, 9 will appear on the + output!

1.8 Stern Gerlach experiments, part IV

Now let's return to two-stage SG experiments that will ultimately be useful in pointing the way forward.

First, consider the second at $\theta = \pi$:

Downward field
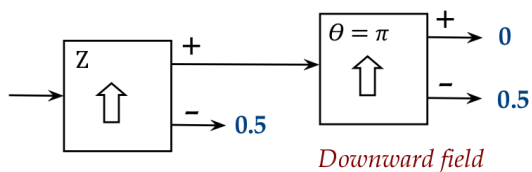
- Here, the second apparatus is turn through $180°$ so that it's upside down.

- The result is that all atoms entering the second come out of the - port.

In-Class Exercise 14: To clarify the orientation of the second SG apparatus, draw the Type-1 diagram for the above case.

Next, consider a general orientation θ such as $\theta = 30° = \frac{\pi}{6}$:

- We are interested in how the probability of observing an atom at the + output of the second SG varies with θ.

- Let $f(\theta)$ be this probability.

- If we were to experimentally observe and plot we'd get:
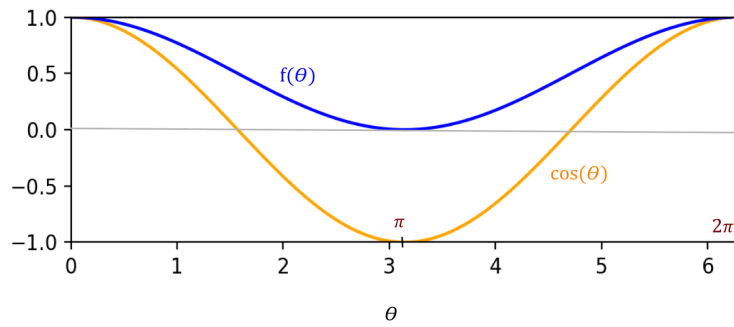
For comparison, we're also plotting $\cos(\theta)$, which will turn out to have a role to play.

- The function $f(\theta)$ above turns out to be $f(\theta) = \cos^2(\frac{\theta}{2})$, although that is quite hard to guess by looking at the figure.

Now let's start with some vague ideas:

- We appear to have properties that are *mutually exclusive*:
  - If one is observed (e.g., +), the other is not (e.g., -)

- If we sought to apply linear algebra, we'd suspect some kind of orthogonality:
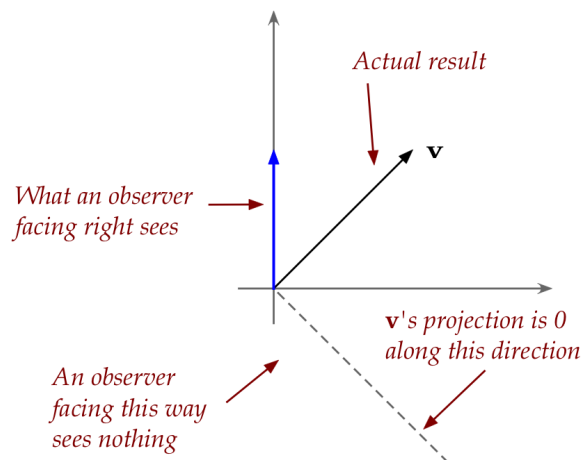
- If a vector **v** is the result of an experiment, then you can't observe it along a direction orthogonal to **v**.
  - This is because **v** has no *projection* along the orthogonal direction.

- Next, consider this observation from standard linear algebra:
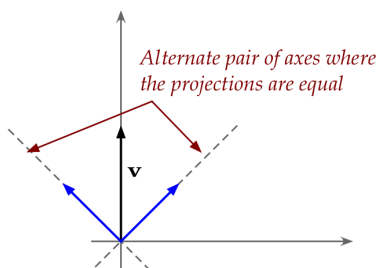
- o Recall that a "+z" stream split evenly between "+x" and "-x".
- o Could even projections suggest something similar?

- We know that the projection of $\mathbf{v}$ along the x-axis is $|\mathbf{v}|\cos(\theta)\mathbf{e}_1$.
  - o Yet, we have strange terms like $\cos^2(\frac{\theta}{2})$ from experimental observation.

At the moment, it's not clear that linear algebra can explain what we've observed.

Here are some of the key questions:

- Do atoms carry with them some internal information used to route them through various SG configurations?

- What accounts for the probabilistic nature of the observed outputs?

- When the orientation of the second SG is $\theta = 180°$ why does the output flip signs?

- How does one explain the unexpected results from the merge apparatus, including the which-path observations?
  (And why are we calling this *which-path*?)

In-Class Exercise 15: What are your own additional questions?

Before we answer these questions, we will examine another, very different experimental set up.
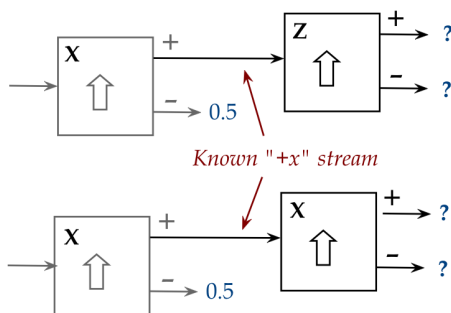
Lastly, let's revisit the basic 2-stage SG and point out something.

Recall these two scenarios with a first Z-stage:
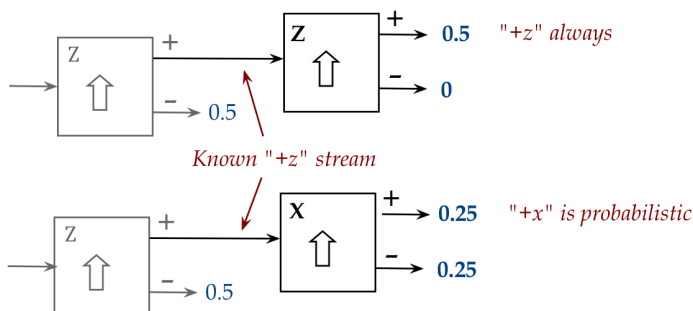
- Scenario 1: Second stage is Z-aligned.
- Scenario 2: Second stage is X-aligned.

In-Class Exercise 16: What is observed when the first stage is an X-aligned SG?
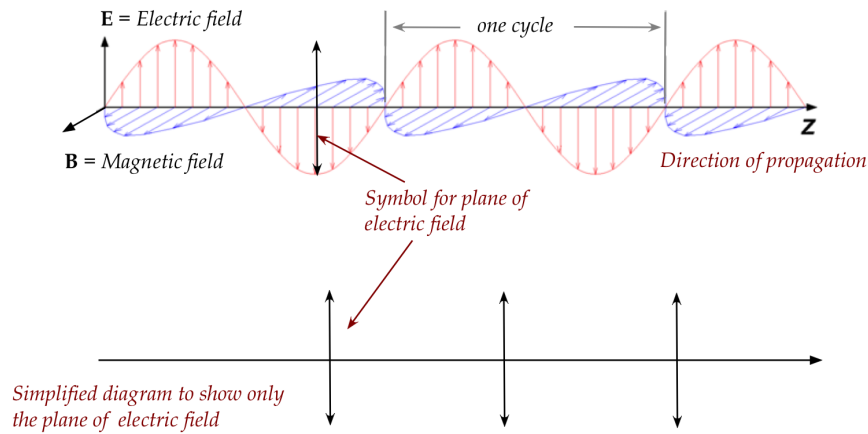
Known "+x" stream

The key observation:

- Think of the *second* stage as a *measuring* device.

- For example, in a Z followed by Z, the second Z says that the stream into the second is 100% (with certainty) a "+z" stream.

- Now consider a Z stage followed by an X stage:
    - The X stage output could be either "+x" or "-x".
    - There is no certainty involved.

- One could say "certainty in Z" goes with "uncertainty in X", and vice versa (as shown in the exercise).
    ▷ We can't have certainty in both Z and X

- This is called the *Heisenberg uncertainty principle*.

1.9 Experiments with light: background

Light is a complicated subject. We will greatly simplify to present a few relevant ideas:
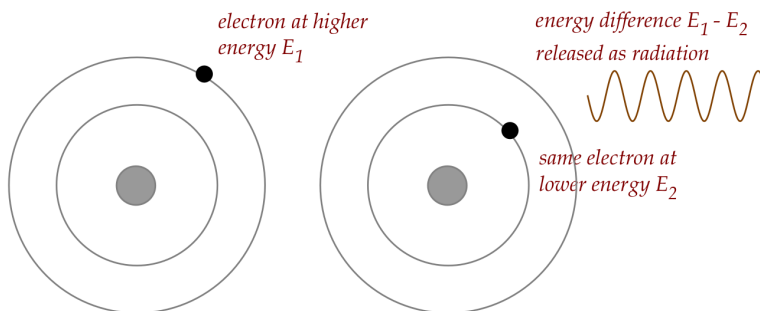
- There are two main conceptions of light:
    1. Light as a *wave*.
    2. Light as a "blob" or *photon*.

- Let's start with light as a wave:

E = *Electric field*

one cycle

B = *Magnetic field*

Direction of propagation

Symbol for plane of electric field

Simplified diagram to show only the plane of electric field

(Modified from: <u>Wikimedia Commons</u>)

Here,

- ○ Light consists of an electric-field wave, and a magnetic field wave.
- ○ In the above picture, the electric field wave is in one plane (vertical).

- What creates such a wave?
    - ○ We'll use an ancient and somewhat incorrect but simple "planetary" model of an atom to explain.
   - ○ An electron can release energy and change its orbit:

electron at higher energy E_1

energy difference $E_1 - E_2$ released as radiation

same electron at lower energy E_2

(Note: An electron can also *absorb* light and increase its energy)

- Light as a photon:
    - ○ The energy levels of an electron are *quantized*: they can take only one among a discrete set of values:

Allowed energy levels

These and others in between solid lines aren't possible
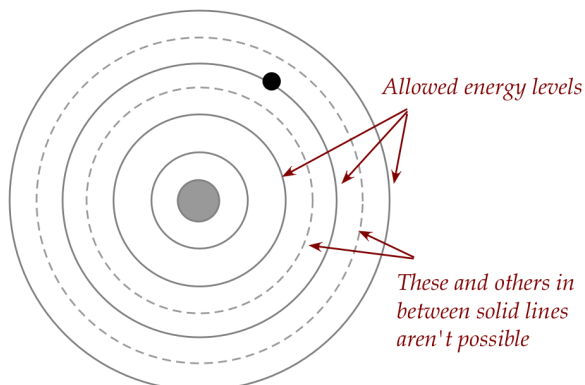
- o This means the differences (energy released as radiation) are also quantized.
- o When an electron transits "down" (to lower energy), the blob of radiation emitted is called a *photon*.
- o When an electron absorbs a photon, it transits up to higher energy.
- o A photon itself is wave-like (undulating electric/magnetic fields).
- o Photons are often called *wave packets*, a blob of a wave.

- Recall some wave terminology:
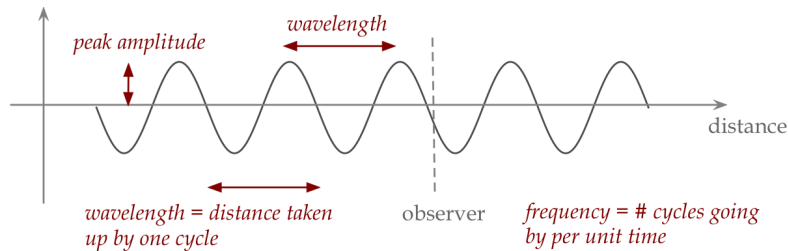
- o *Cycle*: One complete up and down (see figure above), or the wave shape from crest-to-crest. (This is not a number.)
- o ν = *frequency* = number of cycles going by per second.
- o λ = *wavelength* = actual physical distance occupied by one cycle.
- o v = *velocity* = distance per unit time covered by any cycle.

There are other properties (like wavenumber, phase, angular frequency) which we won't need.

- Relationship between photon energy and frequency:


$$E = h \nu$$

The takeaway:

- o Atoms can differ in how much energy is released/absorbed when electrons change orbits.
- o This difference is reflected in the *frequency* of the radiation emitted or absorbed.

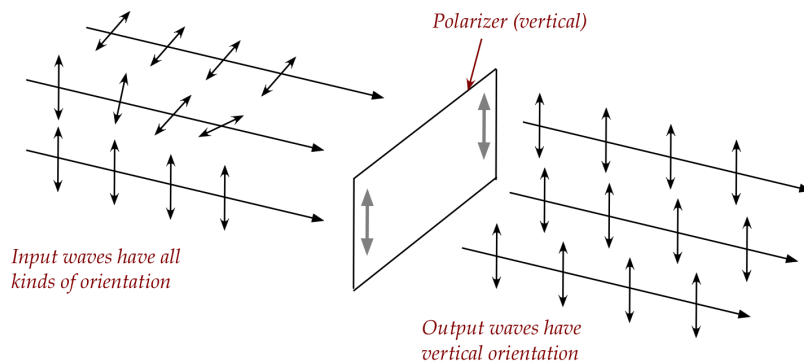The useful phenomenon of *polarization*:

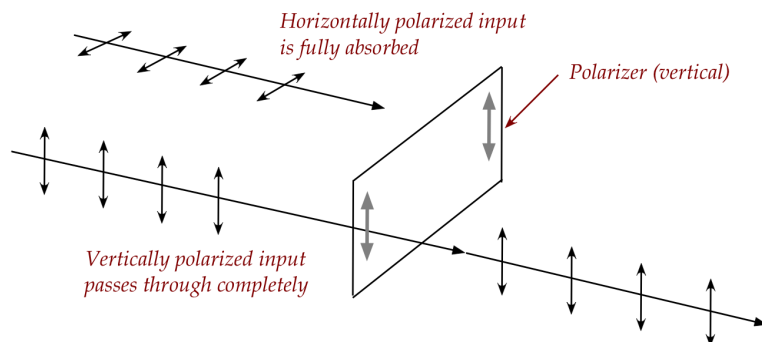- Recall our simplified diagram showing the plane of the electric field in a single wave:

Simplified diagram to show only the plane of electric field
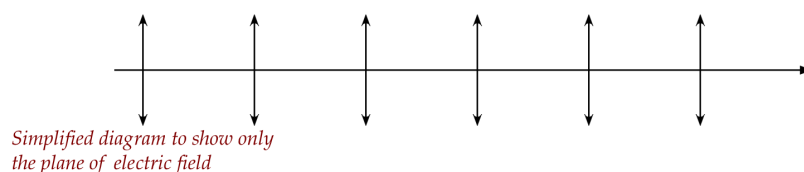
- It is possible for this plane to vary in a single wave, as in:

Continuously rotating electric field plane
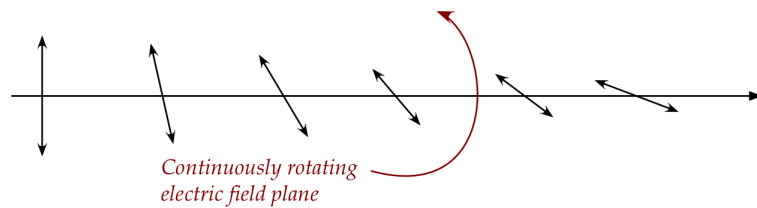
- Even if individual waves maintained a fixed orientation, the different waves will have different orientations.

- There are materials that will ensure that all waves that pass through have a fixed orientation as determined by the material:
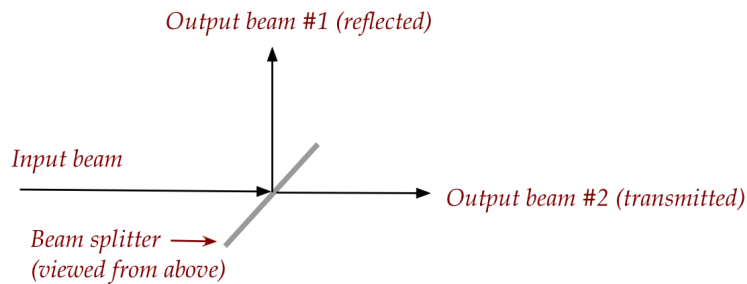

Polarizer (vertical)

Input waves have all kinds of orientation

Output waves have vertical orientation

- Note:

Horizontally polarized input is fully absorbed

Polarizer (vertical)

Vertically polarized input passes through completely

  - Light that comes out of a polarizer is called *polarized light*.
  - A vertically polarized wave will pass through entirely.
  - For waves of other orientation, part will pass through (with vertical output) and part will be absorbed.
  - A horizontally polarized wave will be fully absorbed and will not get through a vertical polarizer.

- Beam vs. single wave/photon:
  - A typical light source will produce many waves (from many atoms).
  - Alternatively: many photons from many atoms.
  - A *beam* is a combination of these.
  - Note: we'll later see that it's possible to send a single photon.

- An even more interesting and useful kind of polarizer is a *polarizing beam splitter (PBS)*:

Polarizing beam splitter

Unpolarized beam

Vertically polarized beam

Horizontally polarized beam
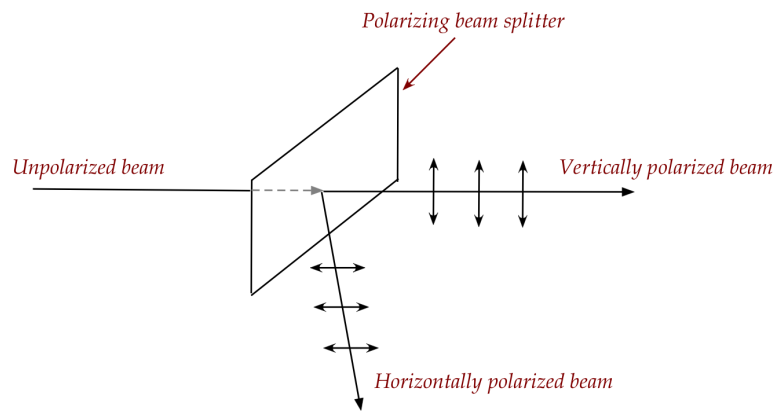
- When an unpolarized beam arrives, it splits (roughly 50% each) into two beams that travel in different directions.
- One beam is vertically polarized, the other horizontally polarized.

• A *beam splitter* merely splits the input beam into two beams (without polarization):

Output beam #1 (reflected)

Input beam

Output beam #2 (transmitted)

Beam splitter ⟶
(viewed from above)

- One common construction uses a half-silvered mirror.
- Which is why part of the light passes through, and part is reflected.

In-Class Exercise 17: Consider the following two polarizers in sequence:

Unpolarized beam

?

Vertical polarizer

Horizontal polarizer
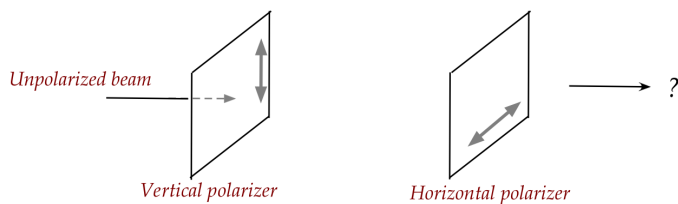
What fraction of the input light reaches the other (right) side of the second polarizer?

In-Class Exercise 18: Consider the inserting a 45° polarizer in the middle:

Unpolarized beam

45⁰ polarizer → *45⁰ polarizer*
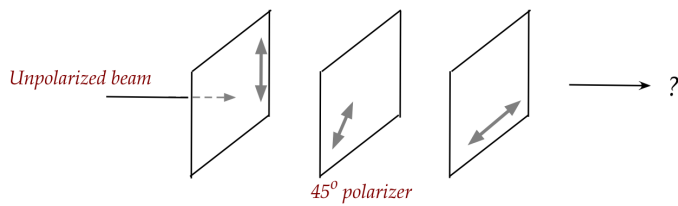
What fraction of the input light reaches the other (right) side of the third polarizer?

So now let's turn to the fundamental question about the nature of light: is it a wave or a photon, or both?
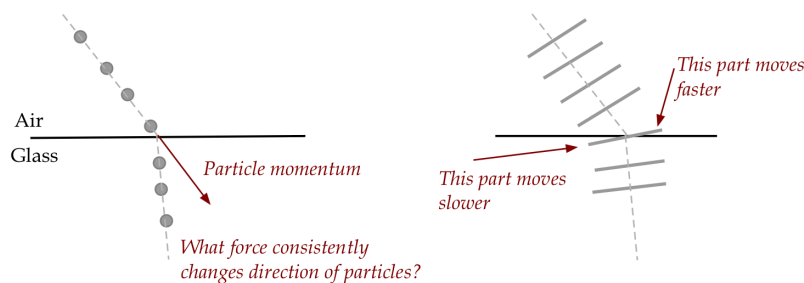
The answer, as it turns out, is surprising.

1.10 Experiments with light: some history

In the early days of physics, Newton thought that light consisted of small (invisible to the eye) *particles*:

- His reasoning: waves (such as water and sound) bend around corners but he did not observe that in light:


Water wave bends around a corner

Light is observed not to bend

- Unfortunately, as he knew, the particle theory did not explain *refraction*:

Air

Glass

Particle momentum

What force consistently changes direction of particles?

This part moves faster

This part moves slower

- His contemporary, Christian Huygens, proposed a wave theory to explain refraction (right side of figure).

Augustin Fresnel's experiment:

Thin fringe → *Dark shadow cast by obstacle*

Tiny light "Arago" spot

Opaque obstacle

Thin beam
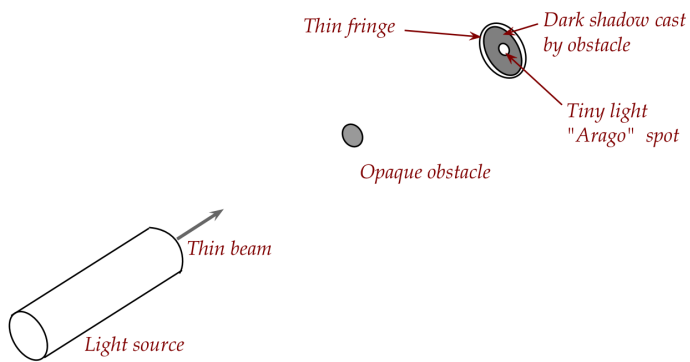
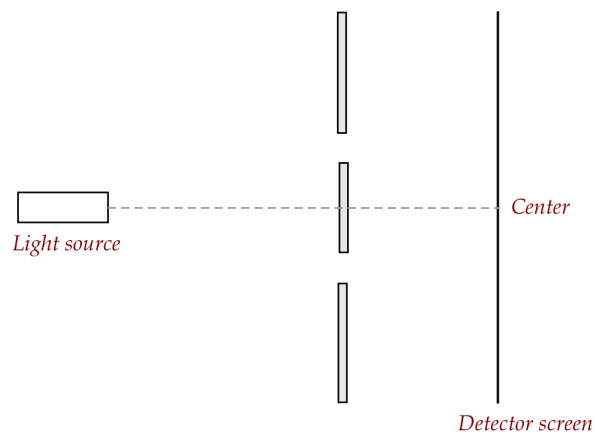*Light source*

- A wave theory would explain bending around an obstacle if the obstacle is small enough.

The famous double-slit of Thomas Young:

- A point light source is aimed at an opaque wall with two tiny slits:

Light source ------------------------------- *Center*

Detector screen

Think of the screen as a photographic plate that can, over time, show the accumulated intensity of light at each position along the plate.

- If one slit is covered, the distribution seen is like the one-hole pellet case:

z

Blocked
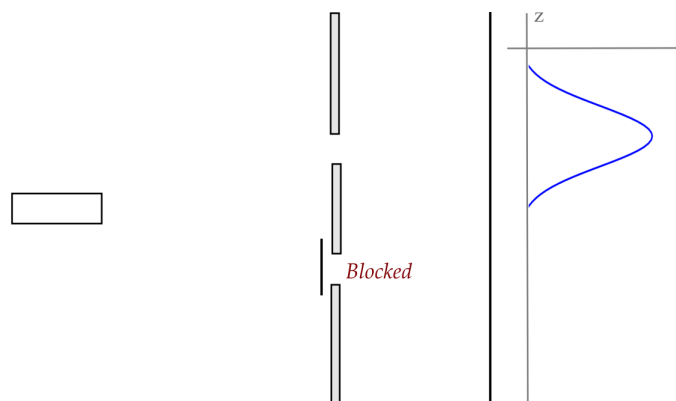
In-Class Exercise 19: What is the distribution (histogram) expected if both slits are open and if light is made of particles? What is the height of the distribution at the center?

The actual distribution with both slits open:

interference pattern
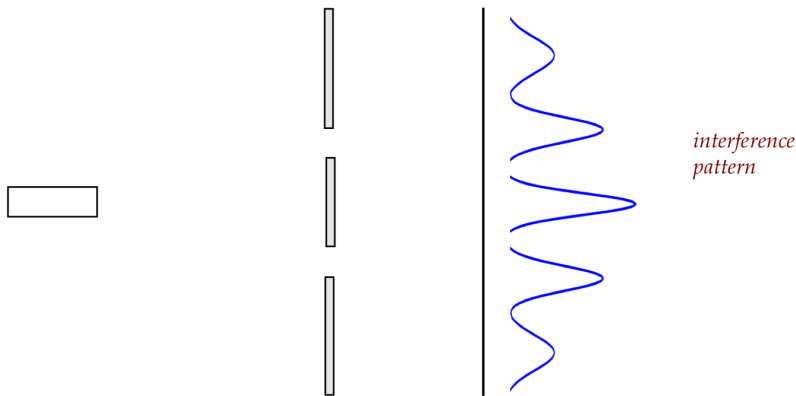
This seems to establish that light is a wave, with the *interference pattern* seen with water waves:

interference pattern along the screen will show added crests and troughs
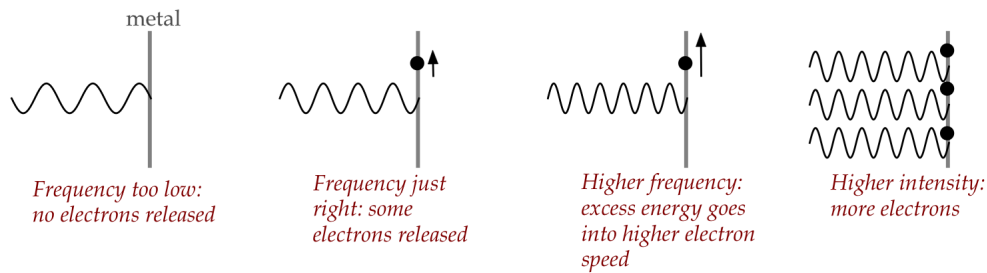
added crests in the center
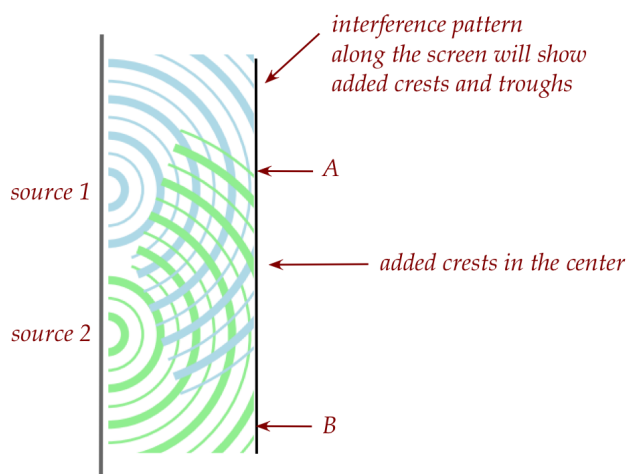
Note:

- Think of the screen as "virtual", because a real screen will involve the waves bouncing back.
- The water molecule at point A moves up and down, away from its equilibrium.
- The amount of motion (amplitude) depends on how the two wave shapes add up at point A. (Two crests will have the highest amplitude. A crest and a trough will have zero amplitude.)
- The water molecules and points A and B are different.

Enter Einstein:

- He proposed that *quantitative* observations on the photoelectric effect could only be explained if light were *quantized*, i.e., particles:

metal

Frequency too low: no electrons released

Frequency just right: some electrons released

Higher frequency: excess energy goes into higher electron speed

Higher intensity: more electrons

- The reasoning:
  - If light is all waves, then any frequency should release electrons, and electrons released should depend on intensity.
  - Actual observation: no amount of intensity for below-threshold frequency light releases any electrons!
  - Above the threshold, higher frequency correlates with current (electron velocity).
  - Thus, if $E = h\nu$ is energy needed by an electron, the threshold frequency is ν and for $\nu' > \nu$ the excess energy $h(\nu' - \nu)$ is converted into electron kinetic energy.

Many years later, an experiment with very low intensity light:

D2 = Detector 2

reflected

transmitted

D1 = Detector 1

Source

BS = Beam Splitter
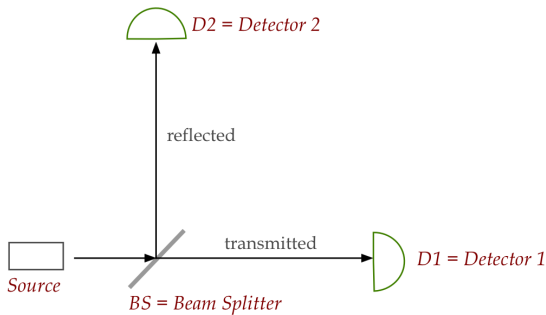
- A light source that can be made very low intensity.

- If light were a wave, the split should result in half-intensity in each direction.

- And ... both detectors above should activate, and simultaneously.

- What actually happens: only one detector goes off.

- Adding up energy used shows no loss anywhere: it's all accounted for.

- Even more troubling: the detector pattern is random.
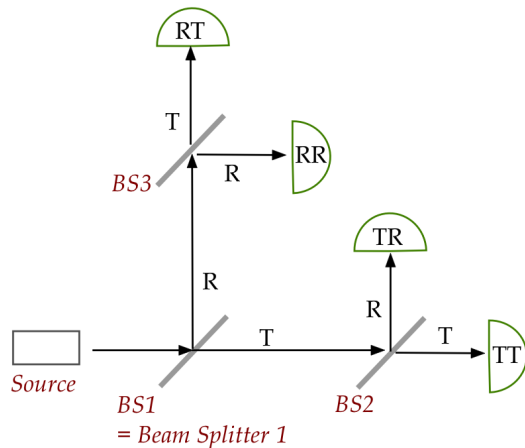
Thus, it now looks like light consists of photons.

But what about that double-slit interference pattern?

This is where things start to get strange.

1.11 Experiments with light: Mach-Zehnder interferometry

We'll now examine a series of light-based experiments using multiple beam splitters.

Let's start with three (non-polarizing) beam splitters in this configuration:

- Individual photons are released one by one so that each detection event corresponds to a single photon.

- The probability of transmission (T) is equal to the probability of reflection (R).

- Each of the four detectors are labeled according to how an arriving photon underwent transmission and reflection.

- Example: the top most detector RT detects photons that were reflected (R) at BS1 and then transmitted (T) at BS3.

- Note: once you know which detector went off, the path from source to that detector is unique.

In-Class Exercise 20: For a given photon, what is the probability that detector RT goes off? What are the probabilities for the other detectors?

Next, let's reconfigure the setup slightly:
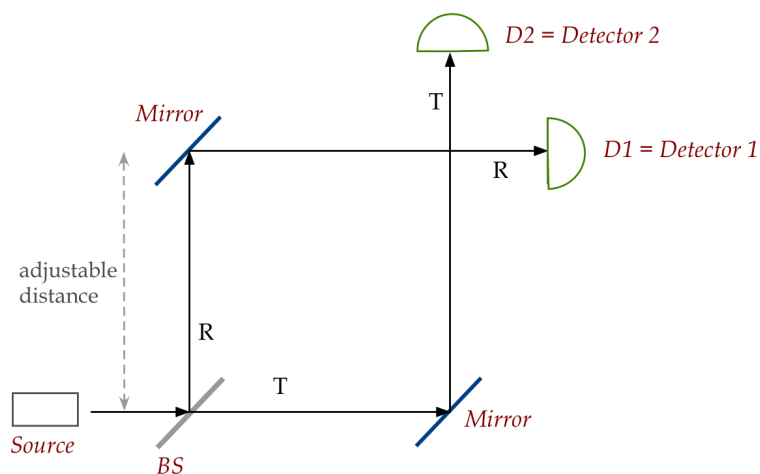
Note:

- The mirrors are 100% reflective.
- The distance to the top mirror is adjustable, and if increased, D1 is moved up by the same distance.

In-Class Exercise 21: For a given photon, what are the probabilities for detecting at D1 vs D2? Does the adjustable distance matter?

Next, we'll set up what is known at a *Mach-Zehnder* interferometer:
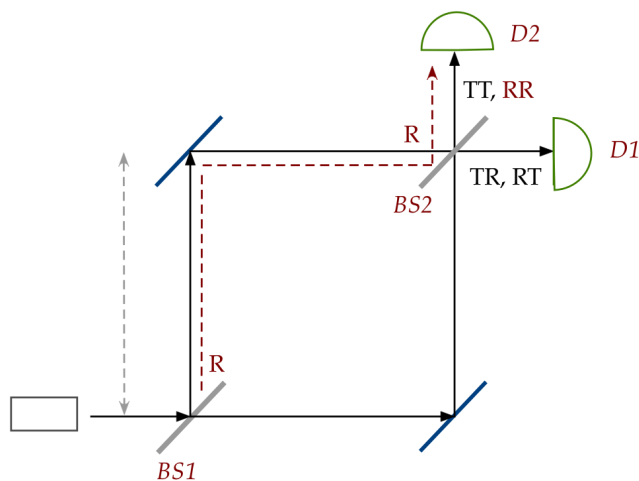
Note:

- There is now a beam splitter (BS2) where the two paths cross.

- Thus, a photon arriving at BS2 can be transmitted or reflected.

- Any photon thus undergoes one of four possibilities at the two beam splitters (in sequence):
  - TT: transmitted through both BS1 and BS2
    - ▷ detected at D2

- o RR: reflected through both BS1 and BS2
  - ▷ detected at D2
- o RT: reflected at BS1, transmitted at BS2
  - ▷ detected at D1
- o TR: transmitted at BS1, reflected at BS2
  - ▷ detected at D1

In-Class Exercise 22: Based *only* on the reasoning in the previous two exercises, what is the probability of detection at D1? At D2?

Then, presumably it should be surprising that:

1. $\Pr[\text{Detect at D2}] \;=\; 0$
2. $\Pr[\text{Detect at D1}] \;=\; 1$

That is, *all* photons are detected only at D1!

Note:

- We are assuming the setup where every photon is sent one at a time.

- The detection is complete: every detection event corresponds to a photon (the energies add up correctly).
  - ▷ A "half photon" is never detected

- The distance to the top mirror is adjusted carefully to make this happen.

- Further adjustments show that one can control the probabilities of detection at D1 (and therefore at D2).
  - ▷ At another distance, we get the reverse: $\Pr[\text{Detect at D2}] \;=\; 1$

If this were not strange enough, consider this variation:
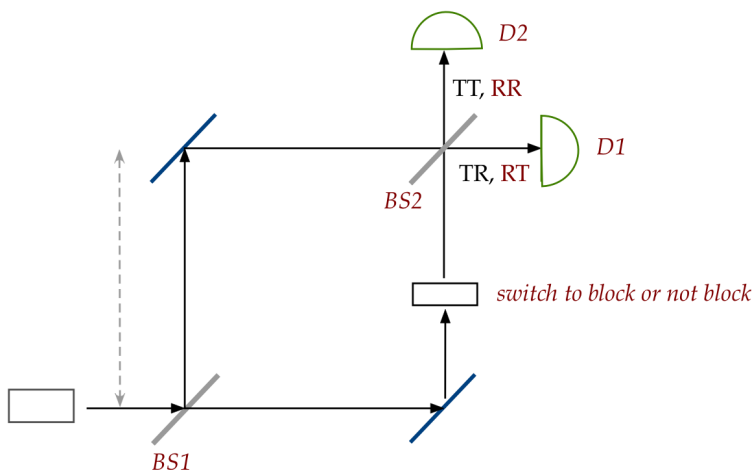
- Here, we insert a controllable device that can block a photon.

- This can be turned on and off at will.

- Suppose we now start sending photons and call these photons 1, 2, 3, ..., 10.

- Next, suppose we apply the *block* for photons 1, 3, 5, 6, 10 but not for photons 2, 4, 7, 8, 9.

- The photons 1, 3, 5, 6, 10 randomly distribute between D1 and D2.
   ▷ These are photons for which the path is known.

- The photons 2, 4, 7, 8, 9 *all* reach D1!
   ▷ These are photons for which the path is unknown.

- Another way of stating "unknown" is that the paths are *indistinguishable* at the second beam splitter.

As we will later see, the fact that paths are indistinguisable is what is called *quantum interference*:

- This will turn out to be fundamentally different from water-wave interference.

- And, even more surprisingly, will turn out to be the foundation for algorithm design!
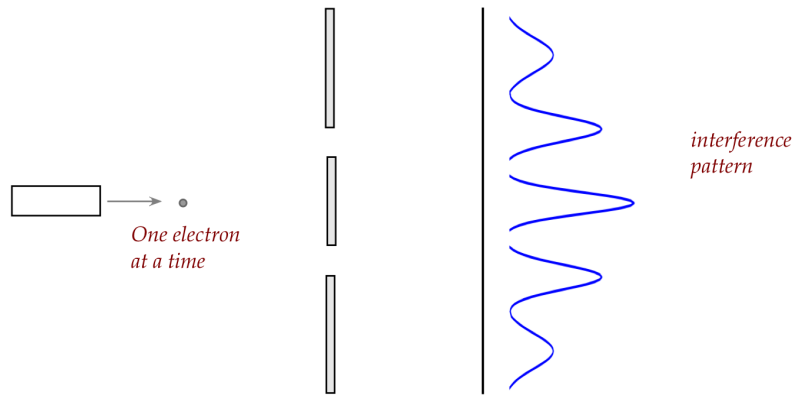

1.12   Double-slit revisited

Lastly, let's go back to the historic double-slit experiment and send in one photon at a time:
   ▷ An interference pattern is observed.

If one of the slits is closed, one gets the pellet pattern.

It is possible to send larger particles than photons:

- Sending in *electrons* one at a time results in the same interference pattern:

- With larger objects, one can detect which slit using photons:

Bounce a photon to detect which slit

Electron source

Equal probability for each slit
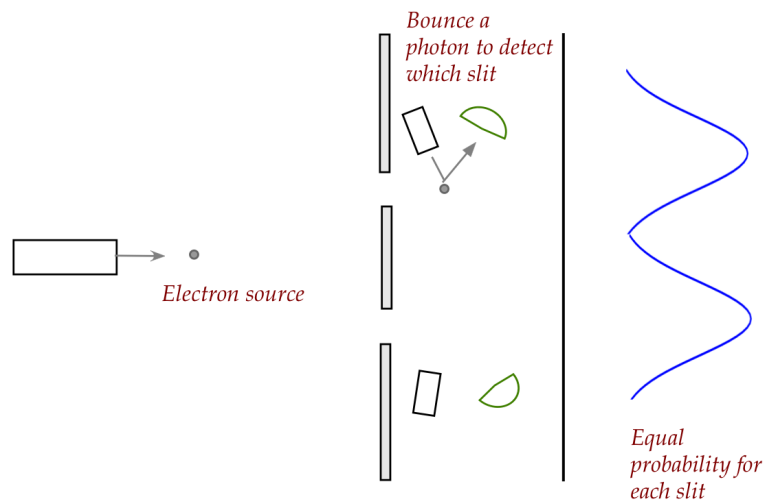
- But knowing which path now *removes* interference.

- The double-slit experiment has been performed for larger particles such as 60-atom molecules, with the same results.

- The fundamental question is: what exactly does a particle do when confronted with multiple paths?
    - Does it pick one randomly?
        - ▷ No, because that would create the two-hump distribution
    - Does it go through *both*?
        - ▷ This seems to defy any conventional notions we have
    - Something else?
        - ▷ Indeed. And we'll have more to say about this.

This concludes our first foray into the quantum world:

- As we've seen there are several counter-intuitive outcomes of experiments.

- Classical (Newtonian) explanations fail to explain these outcomes.

- None of these are easy to understand and have baffled scientists for over a century when the first such experiments (Stern-Gerlach) were conducted.

- This introduction was for fun and curiosity: we don't really need to understand these experiments to learn quantum computing.

- We will see a few more examples later that will be even more shocking:
  - The delayed-choice experiment.
  - The quantum eraser experiment.

- What we *will* examine and that will a key role in quantum computing are the underlying foundations that explain what we've seen:
  - *Complex-number linear algebra*: the mathematical elements
  - *Superposition*: This is the feature that, loosely, explains the photon taking "two paths" simultaneously.
  - *Interference*: the "wave like" addition/substraction.
  - *Entanglement*: we didn't really see this above, but it it is yet another unique and baffling quantum property.