THE GEORGE WASHINGTON UNIVERSITY
School of Engineering and Applied Science
Department of Electrical and Computer Engineering

Preliminary Examination

Fall 2005

General Instructions

Read carefully before starting.

i) As a transition year, you have the choice of (please choose one option):

☐ **NEW FORMAT**: Solve 5 problems in all; at most 2 questions may be selected from the same section.

☐ **OLD FORMAT**: Solve five (5) problems in accordance with the following selection rule: choose two (2) problems from any one of the seven sections, two (2) problems from one of the other sections, and the remaining problem from any section, including from either of the sections previously selected.

ii) It is required that you to clearly mark your choice of format on this cover sheet of the exam.

iii) Effective next year the new format as posted on the website will only go.

iv) Please write your name and student number below:

______________________________ ____________________
Student Name **Student Number**

v) Solve each problem in a <u>separate</u> blue book. Write the section number, problem number, and your student number on the front of each blue book. **DO NOT WRITE YOUR NAME ON THE BLUE BOOK.**

vi) Submit solutions to only five (5) problems. Use only **ONE** blue book per problem.

vii) For each problem, make a special effort to give the answers in a clear form.

viii) The exam will begin at 9:00 a.m. and end at 2:00 p.m.

!!GOOD LUCK!!

Section 1

1. Consider a 6-stage pipelined processor running at 5 GHz. Assuming that it takes one clock cycle per stage:

   (a) How long will it take to run a program that executes 500 instructions that has 40 conditional branches assuming that it will branch 50% of the time?

   (b) What is the average CPI of this machine running this program?

   (c) Is there an advantage to lengthening the pipeline to 7-stages if then the clock could be increased to 8GHz?

2. A 10 MB on board cache has a cycle time of 0.2 ns. It is used to access a 4GB, 10 ns RAM. Assuming that each line in the cache is 10 kB, if the hit rate is .95, what is the average cycle time of the cache for,

(a) a look-through memory when reading?

(b) a look-aside memory when reading?

(c) a write-through.

(d) What is the miss penalty?

3. Different codes are used to represent data in a digital computer. Explain the following:

   (a) What is a run-length-limited code and where is it used?

   (b) What is an excess three code and where is it used?

   (c) What is a Gray code and where is it used?

Section 2

4. Let M_n be the discrete-time process defined as

$$M_n = \frac{X_{n+1} - X_{n-1}}{2}$$

where $\{X_n\}_{n=-\infty}^{+\infty}$ is a sequence of zero-mean unit-variance independent identically distributed random variables.

(a) Find the mean, variance, and autocovariance of M_n. Sketch the autocovariance function.

(b) Is the process wide sense stationary? Explain your answer.

5. Let X be a positive continuous random variable with the cumulative distribution function

$$F_X(x) = \begin{cases} 0, & x < 0 \\ \dfrac{x^2}{1+x^2}, & x \geq 0. \end{cases}$$

The random variable Z is defined as

$$Z = F_X(X) = \frac{X^2}{1+X^2}.$$

Find and plot the cumulative distribution function $F_Z(z)$ and the probability distribution function $f_Z(z)$ for Z.

6. Let K be a discrete random variable with the probability mass function

$$p_K(k) = \begin{cases} \frac{1}{9}, & k \in \{0,1,2,3,4,5,6,7,8\} \\ 0, & \text{otherwize.} \end{cases}$$

Furthermore, let us define

$$X = (1-\delta_{K,8})\left(|\cos\frac{2\pi K}{8}| + |\sin\frac{2\pi K}{8}|\right)\cos\frac{2\pi K}{8}$$
$$Y = (1-\delta_{K,8})\left(|\cos\frac{2\pi K}{8}| + |\sin\frac{2\pi K}{8}|\right)\sin\frac{2\pi K}{8}$$

where

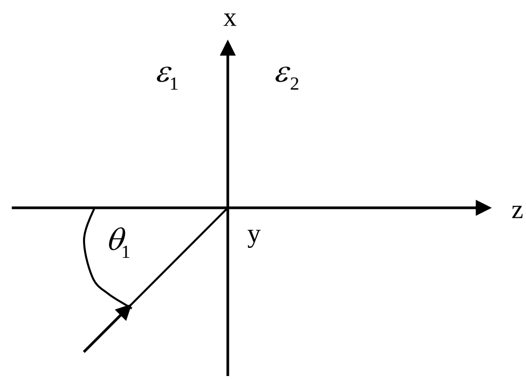
$$\delta_{m,n} = \begin{cases} 1, & m = n \\ 0, & m \neq n. \end{cases}$$

Prove whether X and Y are independent or not.

Section 3

7. Two electric dipoles are excited by prescribed equal currents alternating sinusoidally in time at frequency f. The dipoles are each located on and oriented parallel to the z axis of a Cartesian coordinate system. Their centers are a distance $\lambda = c/f$ apart. Find the radiation pattern of the two dipoles. Here c is the free space velocity of light.

8. A plane wave is incident in the interface between 2 dielectrics ε_1 and $\varepsilon_2, \varepsilon_1 > \varepsilon_2$.

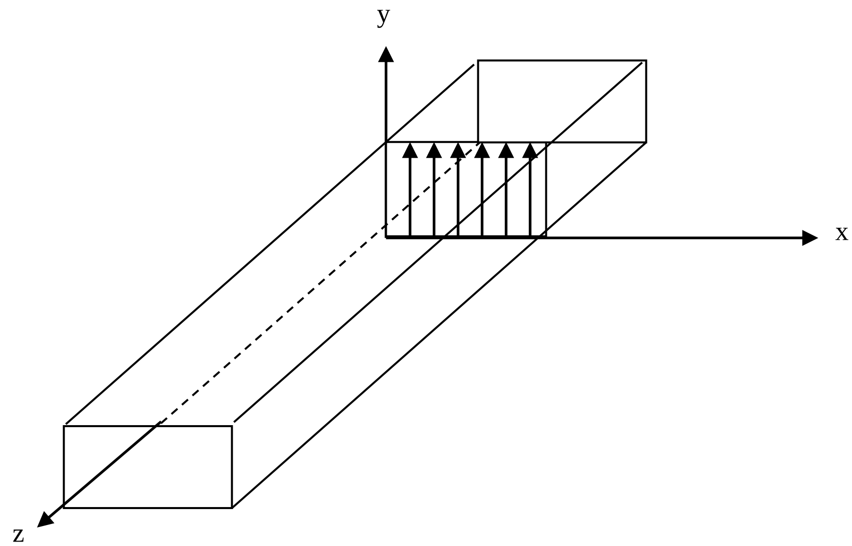


a) Find the angle θ_{1c} such that all waves incident with $\theta_1 > \theta_{1c}$ are "totally reflected".

b) For $\theta_1 > \theta_{1c}$, describe the field (if any) in the region z > 0 in the ε_2 dielectric.

c) If $\underline{E}$ is perpendicular to the plane of the incidence, $\underline{E} = E_y\ \underline{\hat{y}}$, find the phase of the reflection coefficient.

9. An infinitely long rectangular waveguide with perfectly conducting walls



is excited by the current sheet in the plane z = 0,

$$\underline{J}_s = -J_0 \sin\frac{\pi x}{a}\,\hat{y} \quad , \quad 0 \le x \le a, 0 \le y \le b$$

where J_0 is a constant phasor and $\hat{y}$ is a unit vector in the y direction. Assume harmonic time dependence $\varepsilon^{+j\omega t}$.

a) Find the electric field in the guide for all positive and negative values of z.

b) Determine the time average power flowing down the guide in the positive z direction.

Section 4

10. A linear system is defined by the differential equation

$$\frac{d^2 y}{dt^2} + 2\frac{dy}{dt} + 5y = x + 3\frac{dx}{dt}$$

with $x(t)$ the input and $y(t)$ the output.

a) Find the output when the input is $7\cos(4t)$. (Reduce the final result to a real function.)

b) Find the impulse response. Is the system stable? Explain.

c) Find the output for $t \geq 0$ with initial conditions $y(0) = 1,\ y'(0) = -1$ when the input is $7\cos(4t)U(t)$.

Identify the zero state, the zero input, the transient and the steady state responses.

11. The sequence $y[n]$ satisfies the following difference equation

$$y[n+2]-y[n+1]+\frac{1}{2}y[n]=u[n]$$

where

$$u[n]=\begin{cases}1\,;\,n\geq 0\\0\,;\,n<0\end{cases}$$

a) Assuming $y[0]=0$ and $y[1]=0,$ find the causal solution of the difference equation using Z transforms. (Reduce the final result to a real sequence.)

b) Suppose the $y(n\Delta t)=y[n]/\Delta t$ represent samples of a bandlimited signal with a bandwidth of 10 H z. Find the Fourier transform of the signal.

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12. The Laplace Transform $F(s)$,

$$F(s) = \frac{s+4}{\left(s^2+25\right)\left(s-3\right)},$$

can represent several functions of time.

a) Find the functions.

b) Identify the function that has a single-sided Laplace transform.

c) Identify the function that has a Fourier transform. Find the Fourier transform.

Section 5

13. **Time evolution of systems – Solve by linear algebra**

a) *Discrete time evolution*
There is a population of owls and mice. The population is changing each year, because owls eat mice, the more mice there are, the more food the owls have and next year there will be more owls. But the more owls are there, the more mice they eat, and there will be less mice the next year. So that the population of each species depends on its own population and on the other species' population.

Let the owl population in a year n be given by $x_1(n)$ hundreds, and the mice population in year n $x_2(n)$ in tens of thousands.

In yr 0 $x_1=2$ $x_2=3$.
Assume that the equations, governing the population rate are:

$$x_1(n) = 0.4\, x_1(n-1) + 0.6\, x_2(n-1)$$
$$x_2(n) = -0.3\, x_1(n-1) + 1.3\, x_2(n-1)$$

Find the population in year *n*.

b) *Continuous time evolution*
The change in the population of two species is modeled by the following system:

$$dx/dt = 5x - y$$

$$dy/dt = -2x + 4y$$

The initial populations are x(0), y(0)

1) What will happen in the long term?

2) Does the outcome depend on the initial population? If yes, how? (Will both species grow and survive, i.e. live in symbiosis; or one will be eliminated; or are they competitive, i.e. populations fluctuating?)

3) Investigate the cases of critical values of y(0)/x(0)

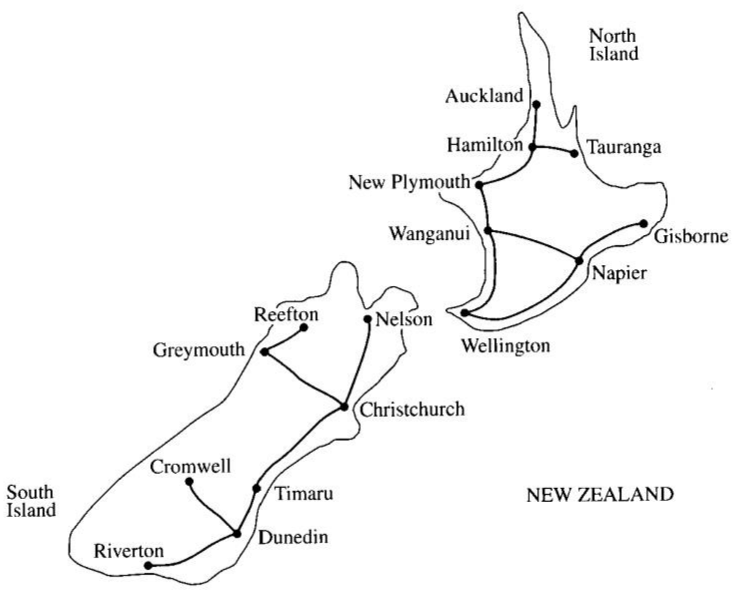
14. Assume that the figure shows the high-speed fiberoptic cable network of the North and South islands of New Zeeland.

Set up the connectivity matrix of the graphs.

Find the Gould index, i.e. the eigenvector belonging to the dominant eigenvector, normalized to 1. (*Use the power method to find λ_{max}.)*

a) Determine which city has the highest connectivity on each islands.

b) Compare the connectivities of both islands. Which one is the most connected?



15. Find the solution in the form of the least squares for A**x** = **b**, where

$$A = \begin{bmatrix} 1 & -2 & -1 \\ 2 & 0 & 1 \\ 2 & -4 & 2 \\ 4 & 0 & 0 \end{bmatrix} \quad and \quad \vec{b} = \begin{bmatrix} -1 \\ 1 \\ 1 \\ -2 \end{bmatrix}$$

by

a) computing the Gram-Schmidt factorization of A into A=QR,

where Q is an (mxn) orthogonal matrix, and

R is an (nxn) invertible upper triangular matrix with matrix elements as $r_{11}=||\mathbf{a}_1||$;

$r_{kk} =||\mathbf{a}_k-\mathbf{p}_{k-1}||$ for k=2,.., n.; $r_{ik}=\mathbf{q}_i\mathbf{a}_k^T$ (**a** are the column vectors of A, **p** are projection matrices and **q** are columns of Q)

b) Show in general that when A is factored into QR, the matrix equation becomes:

$$R\mathbf{x} = Q^T\mathbf{b}, \quad \text{and.} \quad \mathbf{x} = R^{-1}Q^T\,\mathbf{b}$$

c) solve for **x** by back substitution.

Section 6

16. Show that a *pn*-junction with uniform acceptor and donor densities $\boldsymbol{N_a}$ and $\boldsymbol{N_d}$, respectively, has a built-in potential of $\boldsymbol{V_T \ln(N_a N_d / n_i^2)}$**.** Here, $\boldsymbol{V_T}$ denotes the thermal voltage and $\boldsymbol{n_i}$ denotes the intrinsic carrier density. If the *p* and *n* regions can be accessed via Ω-contacts, can this potential be measured by a voltmeter? Justify your answer.

17. Derive an expression for the threshold voltage of a *p*-channel MOSFET. Derive and clearly state all necessary expressions. In this derivation you must include the following physical mechanisms and parameters:

(i) The metal-semiconductor work function difference Φ_{MS}.
(ii) The gate insulator thickness t_{OX} and dielectric constant ε_{OX}.
(iii) The silicon dielectric constant ε_{Si} and substrate doping N_d.
(iv) The fixed oxide charge Q_f and Si-insulator interface charge Q_{it}.
(v) The source to substrate potential V_{BS}.

In current IC fabrication the channel length commonly is in the order of the source and drain depletion region widths. Comment on any changes to the threshold voltage in this case.

18. Employing energy band diagrams, describe how to build a Schottky Ω-contact on *p*-type silicon. You must provide necessary calculations employing the Poisson equation in terms of the metal work function ϕ_m, electron affinity χ, built in potential ϕ_p, temperature T, etc. If or when necessary, state your assumptions with respect to the above parameters.

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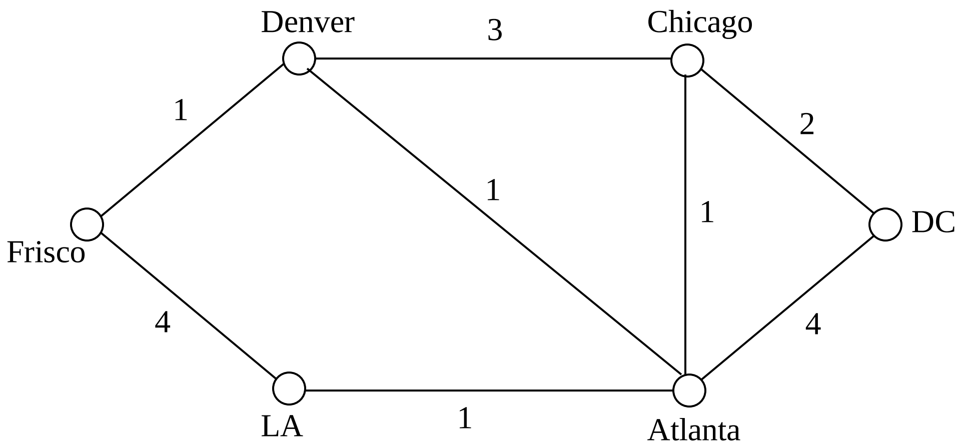
Section 7

19. Given a continental map of the United States below.



The numbers given in the links are the costs you have to pay when you use the corresponding link. Obtain:

a) the central map for this backbone network,

b) the minimum cost route from Frisco to DC,

c) what would you do if this network is a distributed network?

20. a) Compute the S/N in db needed to transmitting signal of 64 Kb/s capacity over a 3 Khz noisy phone line.

b) A voice signal of bandwidth 4 Khz is transmitting through a PCM processor. The accepted maximum quantization error is 0.5% of the peak message amplitude. Compute:

i. The minimum channel capacity required.

ii. The minimum bandwidth of the channel.

21. Given C(6, 3) and $g(x) = x^3 + x^2 + 1$.

a) Compute all the code words in C(6, 3).

b) How many error bits is this C(6, 3) able to detect?

c) How many error bits will this C(6, 3) be able to correct?