THE GEORGE WASHINGTON UNIVERSITY
School of Engineering and Applied Science
Department of Electrical and Computer Engineering

Preliminary Examination

Spring 2008

General Instructions

Read carefully before starting.

Solve 5 problems in all; at most 2 questions may be selected from the same section.

Please write your name and student number below:

______________________________	____________________
Student Name	**Student Number**

Solve each problem in a separate blue book. Write the section number, problem number, and your student number on the front of each blue book. **DO NOT WRITE YOUR NAME ON THE BLUE BOOK.**

Submit solutions to only five (5) problems. Use only **ONE** blue book per problem.

For each problem, make a special effort to give the answers in a clear form.

The exam will begin at 10:00 a.m. and end at 3:00 p.m.

Only Calculators provided by the department at the examination will be allowed.

Section 1

1. A floating point adder has been designed as a pipeline with four stages to perform operations such as align mantissa, add, .. etc.. If you are to use that pipeline to sum the elements of a floating point array of 100 elements, what is the minimum number of clocks needed for such computation? Show how you scheduled your computations. Also you can assume the availability of multiplexers and delay elements that can introduce delay time in multiples of pipeline clocks at the pipeline input or in the feedback path if needed. However, you should state your assumptions clearly and sketch how you modified the inputs of the pipeline.

2. On a microprocessor with a MIPS-like RISC instruction set and operating at 2 GHz, a program with the following mix is to be run: 15% stores, 25% loads, 15% branches, and 35% integer arithmetic, 5% integer shift, and 5% integer multiply. Given that stores require one cycle, load instructions require two cycles, branches require 2 cycles if they are taken and four cycles otherwise, integer ALU instructions require one cycle, and integer multiplies require ten cycles, and probability of a branch to be taken is 0.1:

   a. Compute the overall CPI and the throughput in MIPS.

   b. Consider a strength-reducing optimization that converts multiplies by a compile-time constant into a sequence of shifts and adds. For this instruction mix, 50% of the multiplies can be converted to shift-add sequences with an average length of three instructions. Assuming a fixed frequency, compute the change in instructions per program, cycles per instruction, and overall program speedup.

3. Consider the problem of scaling a matrix “a” of nxn elements by a constant “c” as shown in the following c program, where n is equal to 64 and each integer is represented in the underlying machine by four bytes.

```
#define c       5
int a[n][n] ;
main ()
{
        int i, j;
        for( i = 0 ; i < n ; i++)
        {
                for ( j= 0 ; j < n ; j++)
                        a[i][j] = a[i][j]*c;
        }
}
```

The cache memory size of the system is 4KB and the cache line size is sixteen bytes. Knowing that C distributes data in memory in row major order (i.e. row by row) and assuming that the cache was initially filled by as many elements of “a” as it can hold then:

a. Sketch the cache architecture and show how the elements of each array will be distributed in the cache system and compute the hit rate if a direct mapped cache is used

b. Repeat a if the outer and the inner loops are exchanged

c. Compute the hit rate if a two way set associative cache architecture is used instead in case a

d. Compute the hit rate if a two way set associative cache is architecture is used instead in case b

Section 2

4. A stochastic system has m components that are randomly turned on and off independently of each other. The on/off events of each component are governed by exponential processes with average rates $1/\lambda$ and $1/\mu$, respectively.

Determine the probability that exactly k components are on, for k=0, 1, 2, ..m

5. The impulse response of a linear time-invariant linear system is represented by

$$h(t) = \begin{cases} 1 \; ; & 0 \le t \le T \\ 0 \; ; & \text{otherwise.} \end{cases}$$

a) Find the autocorrelation function and power spectral density of the output $Y(t)$ when the input $X(t)$ is a stationary zero mean white Gaussian process with autocorrelation

$$R_X(\tau) = E[X(t+\tau)X(t)] = N_0/2 \; \delta(\tau) \; .$$

b) Find the cross correlation $R_{XY}(\tau) = E[X(t+\tau)Y(t)]$

c) Find the p.d.f $f_{Y_1 Y_2 Y_3}(y_1, y_2, y_3; t_1, t_2, t_3)$

6. Consider two independent, identically distributed, zero mean Gaussian random variables x and y, each with variance σ^2. Consider next the complex random variable z given by

$$z = x + jy = re^{j\theta}$$

Derive the probability density functions of r and θ.

Section 3

7. A center-fed dipole antenna of length ℓ has a current distribution which is given by

$$I(z) = \begin{cases} I_0 \sin k\left(\dfrac{\ell}{2} - z\right), & 0 < z < \dfrac{\ell}{2} \\ I_0 \sin k\left(\dfrac{\ell}{2} + z\right), & -\dfrac{\ell}{2} < z < 0 \end{cases}.$$

Here k is the free space wavenumber. The antenna configuration is shown in Figure 1.

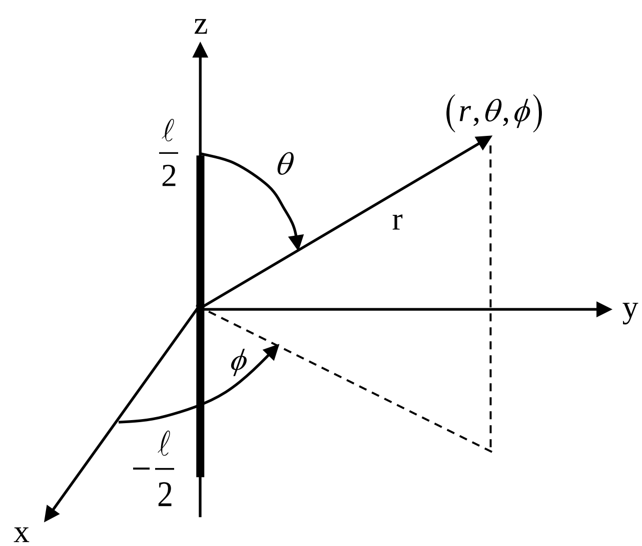

Figure 1

a) What are the conditions on r, ℓ, and k so that the observation point will be in the far field of the antenna?

b) Find the electric field in the radiation zone (far field).

c) Derive an analytic expression for the radiation pattern of the antenna. Sketch it.

8. A certain air filled parallel-plate waveguide has a plate separation of d = 0.5 cm.

a. List the five lowest frequencies modes (TE_m, TM_m) that can propagate on this line and compute their cut-off frequencies.

b. What are the phase and group (v_p and v_g) velocities for the TM_2 mode at a frequency equal to 1.6 times the TM_2 cut off frequency?

9. A uniform plane wave is obliquely incident at an angle θ_i at the interface between two nonmagnetic $\left(\mu_1 = \mu_2 = \mu_0\right)$ dielectric media as shown in Figure 2. The relative permittivity of the second medium is known to be $\varepsilon_{2r} = 3,$ and the electric field of the incident wave is given by

$$\mathbf{E}_i\left(x, z, t\right) = \hat{\mathbf{y}} E_0 \cos\left[12\times10^9 t - 40\sqrt{3}\left(x+z\right)\right]$$

a. Calculate the relative dielectric constant ε_{1r} and the angle of incidence θ_i.

b. Write the corresponding expression for the magnetic field of the incident wave (i.e., $\boldsymbol{H}_i(x, z, t)$).

c. Determine the percentage of the incident power that will be transmitted across the interface.

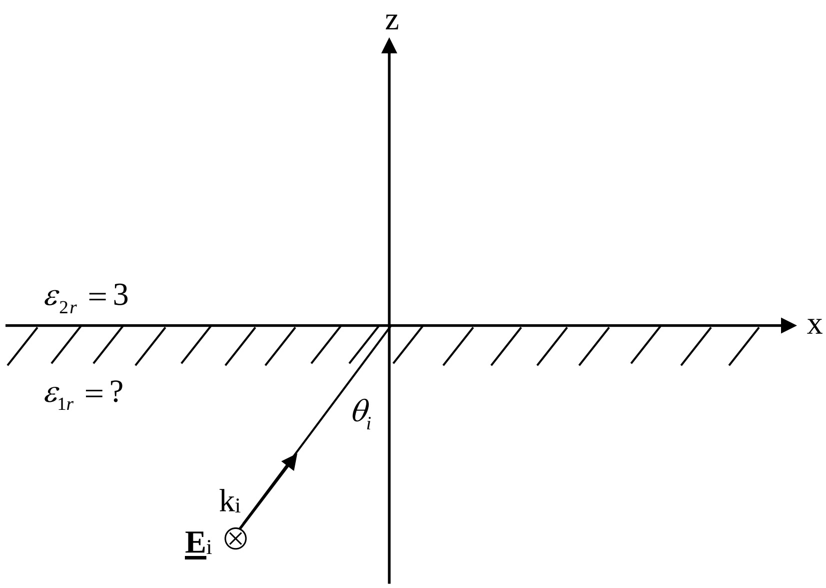

Figure 2

Section 4

10. A linear system is defined by the differential equation

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = x + \frac{dx}{dt}$$

with $x(t)$ the input and $y(t)$ the output.

Find

a) The output when the input is $2\cos(6t)$.

b) The impulse response.

c) The output for $t \geq 0$ with initial conditions $y(0) = 1, y'(0) = -1$ when the input is $2\cos(6t)U(t)$.

Identify the zero state, the zero input, the transient, and the steady state responses.

11. The sequence $y[n]$ satisfies the following difference equation

$$y[n+2]+\frac{1}{6}y[n+1]-\frac{1}{6}y[n]=u[n]$$

where

$$u[n]=\begin{cases}1\,;\,n\geq 0\\ 0\,;\,n<0\end{cases}$$

a) Assuming $y[0]=0$ and $y[1]=0,$ find the causal solution of the difference equation using Z transforms.

b) Suppose the $y(n\Delta t)=y[n]/\Delta t$ represent samples of a bandlimited signal with a bandwidth of 10Hz. Find the Fourier transform of the signal.

12. The Fourier transform $F(\omega)$ of a bandlimited signal $f(t)$ is given by

$$F(\omega) = \begin{cases} \cos^2\left(\dfrac{\pi\omega}{2\Omega}\right) & ; |\omega| \le \Omega, \\ 0 & ; |\omega| > \Omega. \end{cases}$$

a) The signal is sampled at intervals $\Delta t = \pi / \Omega$. Find the samples.

b) Find the Z transform of the sampled sequence $f[n] = f(n\Delta t)\Delta t$.

c) Find $f(t)$.

d) Suppose the signal is sampled at intervals $\Delta t' = 2\pi / \Omega$ and its Fourier transform approximated by

$$\hat{F}(\omega) = \Delta t' \sum_{n=-\infty}^{\infty} f(n\Delta t') e^{-i\omega n\Delta t'}.$$

Compute and sketch $\hat{F}(\omega)$.

Section 5

13. Find the straight line that minimizes the vertical rms error to the points:

(1.0, 2.5), (2.0, 3.7) (3.0, 4.6) (4.0, 5.8).

14. Find the eigenvalues and eigenvectors for the following matrix

$$\begin{bmatrix} 3 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

15. What is the solution to the following equation starting from the initial condition that $x = 1$ and $y = 2$

$$\frac{d}{dt}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Section 6

16. Consider the drift-diffusion model.

a) State the expressions for the drift and diffusion currents for electrons and holes.

b) Based on these expressions, show that the current density for electrons and holes can be expressed as follows in terms of quasi-Fermi potentials:

$$\vec{J}_n = -q\mu_n n \nabla \psi_n \text{ and } \vec{J}_p = -q\mu_p p \nabla \psi_p .$$

c) State the conditions for a metal-semiconductor junction to act like an Ohmic contact. Explain why the difference between the electrostatic and quasi-Fermi potential is independent of the applied bias at the Ohmic contacts.

17. An ideal long-base abrupt *pn*-junction silicon diode has an uniform cross section and constant doping on both sides of the junction. The diode has the following doping characteristics: $N_a = 2 \times 10^{16}$ cm^{-3} on the *p*-side and $N_d = 4 \times 10^{16}$ cm^{-3} on the *n*-side. The minority-carrier lifetimes are $\tau_n = 10^{-6}$ s and $\tau_p = 10^{-8}$ s, respectively. (Recombination within the space-charge region is negligible and minority carriers flow is only by diffusion in the charge neutral regions.)

a) Plot the built-in potential and label key values and coordinates?

b) Calculate the density of the minority carriers at the edge of the space-charge region when the applied voltage is 0.6 V.

c) Employing the drift-diffusion model show that the excess electron charge density in the "*p*-region" is given by the following expression:
$n_p'(x) = n_{p0}(e^{qVa/kT} - 1)\, e^{(x+xp)/Ln}$. Note that the usual variable definitions apply.

d) Determine the resulting electron current density.

18. Consider a MOS system:

a) In MOSFETs the threshold voltage is a measure of channel inversion. In this context, describe the influence of the substrate doping. How does the substrate doping influence what is commonly referred to as the “body effect”?

b) Describe the “long-channel effect” in MOS transistors. How does this effect change the threshold voltage of a MOSFET?

c) Describe at least one way to construct and operate a MOS device that can be employed as a voltage controlled capacitor.

Section 7

19. Consider sending a file of $F=ML$ bits over a path consisting of Q nodes interconnected by transmission links. Each link operates at a speed of R bits per second. If the file is transmitted in packet switching mode, it is segmented into packets of size L bits. For the following three cases, calculate the end-to-end delay from the beginning of file transmission to its complete reception by the receiver, assuming each link is k meters long. Also calculate the effective throughput as a fraction of R.

a) For virtual circuit packet switching operation, the setup time for the VC is t seconds and the amount of packet overhead is h bits.

b) For datagram operation, the packet over head is $2h$ bits.

c) For circuit switched operation, a setup time of t seconds and h bits of overhead for the complete file.

d) Comment on the relative merits of the three transmission modes.

20. Consider a slotted ALOHA system with a high and low power packets such that when a high power and a lower power packet collide, the high power packet is perfectly captured by the receiver. Otherwise, when the packets of the same power level collide we have a collision and packets need to be retransmitted. The arrival rates of high and low power packets (including retransmissions) are G_H and G_L. Find the throughput for high and low power packets, S_H and S_L. What value of G_L maximizes the total throughput? Find the maximum throughput of this slotted ALOHA system with capture?

21. Consider a statistical multiplexing system in which n sources independently generate packets with probability p per packet interval and sharing m packet slots on the transmission channel, where $n>m$. Derive a formula for the packet loss (average number of discarded packets divided by the average number of arriving packets). What is the packet loss for $n=2m$ and $p=1/2$.

Section 8

22. Assume the existence of a stimulus that begins to de polarize the cell membrane of an excitable cell. Ex plain/describe/discuss how an action potential is then generated. Your answer should include but not necessarily be limited to, the following points:

a) Voltage-gated sodium and potassium channels.

b) The relative timing of sodium- and potassium-channel gating.

c) Relative permeability changes of sodium and potassium channels.

d) Recovery of membrane potential to its resting level.

e) Positive afterpotential.

23. Explain/describe/discuss the local mechanisms (i.e., changes in local pH, pO_2 and pCO_2) that control ventilation/perfusion matching in the human lung.

24. The Figure (below) shows data from an experiment to measure the transport of two different substances (S_1, squares; S_2, triangles) into human red blood cells. Answer parts a) and b) based on the Figure and using your knowledge of cellular membrane transport mechanisms.

a. Which substance (S_1 or S_2) has the higher K_m? Explain.

b. Using the information provided, is it possible to determine if S_2 is being transported up its electrochemical gradient? Explain.