THE GEORGE WASHINGTON UNIVERSITY
School of Engineering and Applied Science
Department of Electrical and Computer Engineering

Preliminary Examination

Fall 2002

General Instructions

Read carefully before starting.

i) Solve five (5) problems in accordance with the following selection rule: choose two (2) problems from any one of the seven sections, two (2) problems from one of the other sections, and the remaining problem from any section, including from either of the sections previously selected.

ii) Solve each problem in a <u>separate</u> blue book. Write the section number, problem number, and your student number on the front of each blue book. **DO NOT WRITE YOUR NAME ON THE BLUE BOOK.**

iii) Submit solutions to only five (5) problems. Use only **ONE** blue book per problem.

iv) For each problem, make a special effort to give the answers in a clear form.

v) The exam will begin at 9:00 a.m. and end at 2:00 p.m.

!!GOOD LUCK!!

Section 1

1.

a) An NPN transistor has a base length that is 45 times the diffusion length, l_d, for minority carriers in it. If the doping of the emitter is 200 times the doping in the base, what is the α of the transistor?

b) What would the β of the transistor be?

c) If the emitter-base depletion layer is subject to a voltage V, it has a thickness t_d and a cross-sectional area of A_d. What is C_{eb} (the contribution to the input capacitance in a common emitter configuration due to that junction) at a voltage of $2V$?

d) What would the total input capacitance be if the gain of the transistor is G and the base-collector capacitance is C_{bc}?

2. According to the standard drift-diffusion transport model, the electron current density, $\mathbf{J}_n$, is the summation of the drift and diffusion currents:

$$\mathbf{J}_n = q\,(\mu_n\, n\, \nabla\, \phi - D_n\, \nabla n\,).$$

Here, q is the electronic charge, μ_n is the electron mobility, n is the electron charge density, ϕ is the electrostatic potential, and D_n is the electron diffusion constant.

a) When the current density is zero, show that the electron charge density can be expressed as follows:

$$n = n_i \exp(\phi\, /v_T)$$

where v_T is thermal voltage and n_i is the intrinsic charge density.

b) The electron quasi-Fermi potential, ψ_n, is defined as follows:

$$n = n_i \exp[(\phi - \psi_n\,)/v_T].$$

Derive an expression for the electron current density in terms of n and ψ_n. Based on this expression, what is the physical interpretation of the quasi-Fermi potential?

3. Provide clear and concise answers to the following questions on MOS devices:

a) Describe the "long-channel effect" in MOS transistors. How does this phenomenon vary the threshold voltage of a device?

b) In MOSFET devices the threshold voltage is a measure of channel inversion. In this context, describe the influence of the substrate doping. How does the substrate doping influence what is commonly referred to as the "body effect"?

c) Describe one way to construct and operate a MOS device that can be employed as a voltage controlled capacitor.

Section 2

4. Suppose you have a message of size L bits that needs to be sent over a transmission line. The probability of bit error is given to be p and assume bit errors to be statistically independent. The message is broken into packets and every packet must be of the same size X (assume the last packet is padded with bits to make its length equal to X). Each packet has a fixed header size ($H < X$) and carries $X - H$ bits of the payload. If a packet contains an error (including errors in padded bits), then the receiver requests the packet to be retransmitted.

a) Derive an expression for the optimum packet size, X^*, that maximizes the throughput over the transmission line (i.e., Message size in bits/Average # of bits transmitted). You may use $(1-p)^X \approx e^{-pX}$.

b) Find an approximate value for X^* if $L = 999{,}900$ bits, $H = 100$ bits, and $p = 10^{-6}$. Compare the throughputs for $X = 10^6$, 10^4, and 10^3 bits.

5. A network designer is going to choose between FDM and TDM over a link. His goal is to accommodate as many 10 kbps users as possible on the link (assume TDM is preferred in case of a tie). The design parameters are as follows: channel bandwidth = 100 kHz; spectral efficiency, i.e., bps/Hz = 2. For FDM, a guard band of 1 kHz has to be provided between adjacent channels, and for TDM each frame consists of 8 data bits per user and 10 framing bits.

   a) Which scheme must the designer choose?

   b) Assume everything else is fixed, what must the FDM guard band be for the designer to reverse his decision in (a).

   c) Assume a guard band of 1 kHz again. Suppose the goal now is to maximize the number of 20 kbps users. Will the choice in (a) be changed?

6. A network is supposed to be built in order to connect N stations.

a) What is the minimum number of bi-directional links we need to build a network such that there is a path between any two stations?

b) Assume N = 5. Sketch three network topologies with the minimum number of links and a path between any two stations.

c) How many bi-directional links are needed if any two stations are directly connected by a link?

d) Assume N = 64 and the stations are serviced by a token-ring LAN. For the following cases, calculate the time it takes to transfer a free token to the next station using the three free token reinsertion strategies: (i) after completion of transmission, (ii) after return of busy token, and (iii) after return of packet. Assume 1000-bit packet; 100 Mbps transmission speed; 5-bit latency/adapter; 100 meters between stations; and 2×10^8 m/s propagation speed.

Section 3

7. The Fourier transform F(ω) of a bandlimited signal f(t) is given by

$$F(\omega)=\begin{cases}1+\sin^4\left(\dfrac{\pi\omega}{\Omega}\right) & ; \quad |\omega|\le\Omega, \\ 0 & ; \quad |\omega|>\Omega.\end{cases}$$

Note: $F(\omega)=\int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt.$

a) The signal is sampled at intervals $\Delta t=\pi/\Omega.$ Find the samples.

b) Find the Z- transform of the sampled sequence $f[n]=f(n\Delta t)\Delta t.$

c) Suppose the signal is sampled at intervals $\Delta t'=2\pi/\Omega$ and its Fourier transform is approximated by

$$\hat{F}(\omega)=\Delta t'\sum_{n=-\infty}^{\infty} f\left(n\Delta t'\right)e^{-i\omega n\Delta t'}.$$

Compute and sketch $\hat{F}(\omega)$ within the band $|\omega|\le\Omega.$

8. The inverse of the Z- transform

$$F(z) = \frac{z}{\left(z^2 + 1/4\right)(z+5)^2}$$

can represent several sequences.

a) Find all the sequences.

b) One of the sequences represents the coefficients of the Fourier series expansion of the Fourier transform of a bandlimited function. Identify the sequence and find the Fourier transform of the corresponding bandlimited function.

9. A linear system is defined by the differential equation

$$\frac{dy(t)}{dt} + \alpha(1 - 0.2\cos 2t)\, y(t) = x(t)$$

where $\alpha > 0$, with x(t) the input and y(t) the output.

a) Assuming a causal system, find the impulse response.

b) With $x(t) = \exp(0.1\alpha \sin 2t)u(t + 3)$ and initial condition $y(-3) = 1$, find the output for $t \geq -3$. In your solution identify the zero state, the zero input, the transient and the steady state response.

Note: u(.) denotes the unit-step function.

Section 4

10. A time harmonic plane wave with electric field

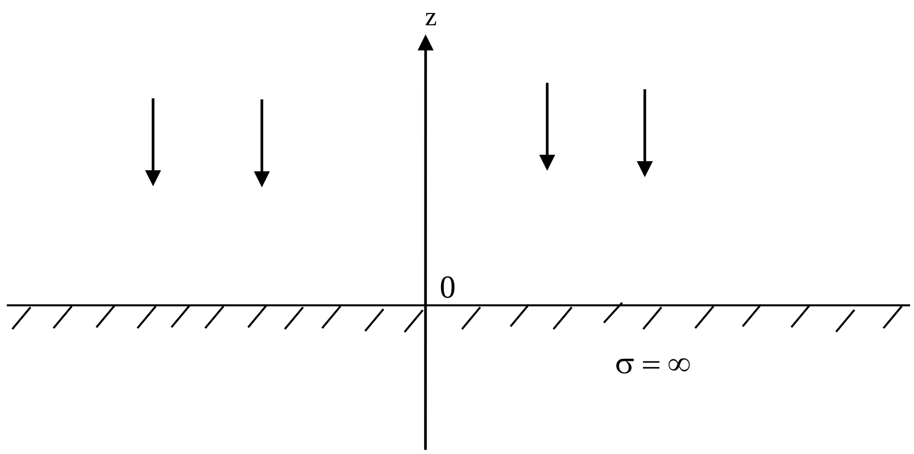
$$\underline{E}_i(z) = E_0\left(\hat{x} - j\hat{y}\right)e^{+jk_0 z}$$

is normally incident on a perfectly conducting half space. Here E_0 is a real constant and k_0 is the free space wavenumber

a) What is the polarization of the incident wave?

b) Find the reflected electric field.

c) What is the polarization of the reflected electric field?



11. The electric field of a time-harmonic plane wave in the free space region x > 0 is given by $\mathbf{E} = \left(\sqrt{3}\hat{x} + j\hat{y}\right)e^{-\alpha k_0 x - j\beta k_0 y}$, where α ($\alpha > 0$) and β are real, $k_0 = \omega/c_0$, and $\hat{x}$ and $\hat{y}$ are Cartesian unit vectors. Here c_0 is the velocity of light in free space

Find

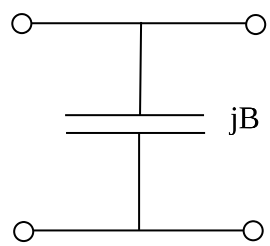
a) α and β.

b) If the given electric field is generated by a totally internally reflected wave in a dielectric occupying the half-space x < 0, what is the angle of incidence of the wave in the dielectric if the dielectric constant of the medium is 4? (Assume magnetic properties of free space). What is the critical angle?

c) Find the electric and magnetic field components within the dielectric.

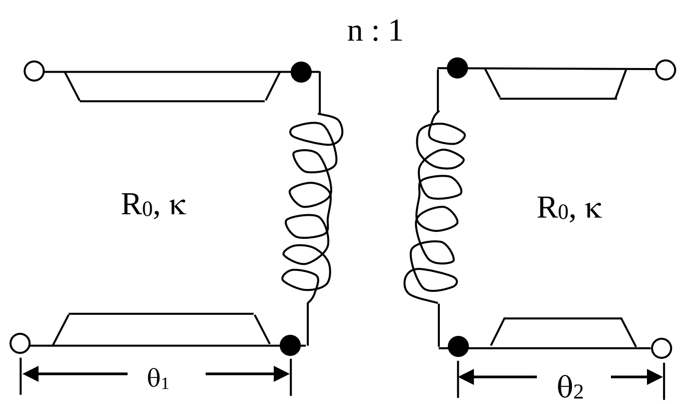
d) Compute the (complex) Poynting vector within the dielectric.

12. A capacitor is placed in shunt (in parallel) across a lossless transmission line of real characteristic impedance R_0. Using this value also as the normalization number at each of the two ports:



a) Compute the normalized voltage scattering matrix for this shunt capacitor in terms of its susceptance $B > 0$.

Every lossless reciprocal 2-port may be re presented by a transformer equivalent circuit as shown below.



$$\kappa = \frac{2\pi}{\lambda}$$

$\theta_1 = \kappa$ (line 1)

$\theta_2 = \kappa$ (line 2)

b) In terms of the susceptance B, compute values for the three parameters of the transformer equivalent circuit, n, θ_1, θ_2.

c) The shunt capacitor circuit is obviously symmetrical about a center line, while the transformer equivalent circuit b) is not symmetrical in form. Using additional transformer and/or transmission line elements, create a (transformer) equivalent circuit that is symmetrical in form.

Section 5

13. Given an unknown random variable Y and a known random variable X with the following statistics: $E\{X\} = 1$, $Var\{X\} = 4$, and $E(XY) = 7$; compute

a) αX such that $\alpha X + 1$ is the mean square estimate of Y,

b) the mean square error of this estimate,

c) show that $[Y - (\alpha X + 1)] \perp X$.

14. An ensemble member of a stationary random process $X(t)$ is sampled N times $(t_i,\ i = 1,\ \ldots,\ N)$. By treating the samples as random variables $X_i = X(t_i)$, an estimate, $\hat{X}$, of the mean of X is given by

$$\hat{X} = \frac{1}{N}\sum_{i=1}^{N} X_i.$$

a) Find $E\left[\hat{X}\right]$.

b) Find Var $\hat{X}$ under the assumption that the samples are sampled far enough apart in time so that they are independent.

c) What happens to Var $\hat{X}$ as $N \to \infty$? What does this mean?

Assume $E[X(t)] = m$ and Var $X(t) = \sigma^2$.

15. Suppose X is a random variable which is Poisson distributed with mean λ. Here λ itself is a random variable which is exponentially distributed with mean 1.

Find $P(X = n)$ for $n = 0, 1, 2, \ldots$, as a function of n alone.

Section 6

16. Let

$$A = \begin{bmatrix} 1 & 1 \\ 2 & -1 \\ -2 & 4 \end{bmatrix}.$$

a) Find orthonormal vectors q_1, q_2, and q_3 so that q_1 and q_2 form a basis for the column space of A.

b) Which of the four fundamental subspaces contains q_3?

c) Find the projection matrix P projecting onto the left nullspace (not the column space!) of A.

d) Find the least squares x solution to $Ax = (1, 2, 7)^t$.

17. Consider the system of first order linear ODEs

$$\frac{dx}{dt} = -7x + 2y \quad \frac{dy}{dt} = -6x.$$

Find two independent real-valued solutions $\begin{pmatrix} x^{(1)} \\ y^{(1)} \end{pmatrix}$ and $\begin{pmatrix} x^{(2)} \\ y^{(2)} \end{pmatrix}$ of this system and hence find the solution $x(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$ which satisfies the initial condition $x(0) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

18. a) Find the Singular Value Decomposition of the following matrix:

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = U\,\Sigma V^T.$$

b) Find all entries in the SVD:

$$A = \begin{bmatrix} u_1 & u_2 \end{bmatrix} \begin{bmatrix} \sigma_1 & \\ & \sigma_2 \end{bmatrix} \begin{bmatrix} v_1 & v_2 \end{bmatrix}^T$$

c) Find the pseudoinverse A^+ of matrix A, and using A^+ find the (minimal norm) east squares solution to A **x = b**, for

$$\mathbf{b} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Section 7

19. Consider a 32-bit processor with a 64-bit external (processor-to-memory) data bus. The frequency of the external clock that synchronizes the processor with memory (and clocks the synchronous bus between the two) is N MHz.

    (Please state clearly any assumptions you make for each question and show all work.)

    a) What is the **maximum** data transfer rate between the processor and a 64-bit memory when the processor performs a 4-2-2-2 burst transfer?

    b) What is the actual data transfer rate between processor and a 16-bit memory when the processor performs normal (i.e., non-burst) transfers?

    c) Assume the processor is a little-endian processor connected to a 32-bit memory, issues the proper number of byte-enables, and executes the instruction “Load a 16 bit operand from (hexadecimal) location 1234567A”. If the 16-bit operand is AABB, draw the timing diagram of the normal (non-burst) bus cycles that fetches this operand. (Show the values of the byte enables the hexadecimal values placed on the address and data bus byte lanes).

20. Consider a 32-bit big endian processor (with a 32-bit address and a 32-bit data bus). We want to build a system around it that has both some ROM memory and some RAM memory; ROM will be allocated to the lowest memory addresses and RAM immediately next to it.

    The ROM portion has 16-bit width, 1MB total capacity, and uses 4Kx4-bit **ROM** chips. The RAM portion has a 32-bit width, 8MB total capacity, and uses 16Kx2-bit **DRAM** chips.

    Design both the ROM and DRAM memory subsystems and interface them to the processor's address and data buses.

21. Assume that one 16-bit and two 8-bit processors are to be interfaced to a system bus. The following details are given:

a. All processors have the hardware features necessary for any type of data transfer: programmed I/O, interrupt-driven I/O, and DMA

b. All processors have a 16-bit address bus

c. Two memory boards, each of 64 Kbytes capacity, are interfaced with the system bus. The designer wishes to use a shared memory that is as large as possible.

d. The system bus supports a maximum of four interrupt lines and one DMA line.

Make any other assumptions necessary, and

(1) Give the system bus specifications in terms of number and types of lines

(2) Explain how the aforementioned devices are interfaced to the system bus.