THE GEORGE WASHINGTON UNIVERSITY
School of Engineering and Applied Science
Department of Electrical and Computer Engineering

Preliminary Examination

Spring 2006

General Instructions

Read carefully before starting.

NEW FORMAT: Solve 5 problems in all; at most 2 questions may be selected from the same section.

Please write your name and student number below:

______________________________	________________
Student Name	**Student Number**

Solve each problem in a <u>separate</u> blue book. Write the section number, problem number, and your student number on the front of each blue book. **DO NOT WRITE YOUR NAME ON THE BLUE BOOK.**

Submit solutions to only five (5) problems. Use only **ONE** blue book per problem.

For each problem, make a special effort to give the answers in a clear form.

The exam will begin at 10:00 a.m. and end at 3:00 p.m.

!!GOOD LUCK!!

Section 1

1. Let $\{X_i\}_{i=1}^{N}$ be a sequence of N independent identically distributed random variables. Each random variable X_i is uniformly distributed between -2 and +2. Also, let

$$Z_N = \frac{1}{N}\sum_{i=1}^{N} |X_i|.$$

(a) Find the probability density function of Z_1.

(b) Find the probability density function of Z_2.

(c) Find the approximate probability density function of Z_N, when $N \gg 1$.

2. The random process $Z(t)$ is defined by

$$Z(t) = Xt + Y$$

where X and Y are a pair of random variables with means $m_X = 1$ and $m_Y = 2$, variances $\sigma_X^2 = 1$, $\sigma_Y^2 = 4$, and correlation coefficient $\rho_{XY} = 0.5$.

(a) Find the mean and autocovariance of $Z(t)$.

(b) Find the probability density function of $Z(t)$ if X and Y are jointly Gaussian random variables.

(c) Is $Z(t)$ a wide-sense stationary random process? Explain your answer.

(Note: The general form of the Gaussian probability density function is

$$f_V(v) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(v-m)^2}{2\sigma^2}}$$

where m is the mean and σ^2 is the variance of a Gaussian random variable V.)

3. Let X and Y be two Gaussian random variables with variances equal to 1 and 4, respectively. These two random variables are transformed to the random variables U and V as

$$U = X + \mu Y$$
$$V = X - \mu Y \, .$$

(a) Find μ such that U and V are uncorrelated.

(b) For the values of μ found in (a), are U and V independent? Explain your answer.

(c) Find the joint probability density function for U and V when μ has the values found in (a), the mean values of X and Y are 1 and 2, respectively, and the correlation coefficient $\rho_{XY} = 0.5$.

Section 2

4. Consider a semiconductor device under thermal equilibrium, and that the Fermi energy E_f is within the bandgap throughout the device, i.e., non-degenerate system. Assume that the drift-diffusion model is employed to describe current transport. In this context, show that the electrostatic potential ϕ can be related to the Fermi energy as follows:

$$\phi = (1/q)\,(E_f - E_i),$$

where, q is the electronic charge and E_i his the mid-gap energy. State any necessary assumptions, define all symbols used and state all derivations employed.

5. Derive an expression for the threshold voltage of an n-channel MOSFET. The derivation must include the following physical mechanisms and parameters:

(i) The metal-semiconductor work function difference Φ_{MS}.

(ii) The gate insulator thickness t_{OX} and dielectric constant ε_{OX}.

(iii) The silicon dielectric constant ε_{Si} and substrate doping N_a.

(iv) The fixed oxide charge Q_f and Si-insulator interface charge Q_{it}.

(v) The source to substrate potential V_{BS}.

State all necessary assumptions and derive all necessary expressions. Comment on any changes to the threshold voltage when the channel length is in the order of the source and drain depletion region widths.

6. The following questions relate to the Debye length.

a) The Debye length is a measure of the variation of charge. Is it a measure for mobile or fixed charge? Explain.

b) Consider the *p*-side edge of a *pn*-diode depletion region with constant doping $\boldsymbol{N_a} = 5 \times 10^{16}\ \mathrm{cm}^{-3}$. Derive an expression for the Debye length in terms of relevant parameters. State any assumptions or approximations employed.

c) The Debye length in typical device applications is considered to be a small length in comparison to deletion lengths. State two operating conditions in a MOSFET device where the Debye length is an important parameter.

Section 3

7. A look through cache memory for a d-ram has a cycle time of 2 ns. The miss penalty is 8 ns.

(a) If the hit ratio is 0.9, what is the average cycle time of the memory combination?

(b) If the cache size is doubled, the hit ratio is increased to 0.95. What is the percent decrease in average cycle time?

(c) What would the advantage in cycle time be of using a look aside memory instead?

(d) Are there any disadvantages to using a look aside memory?

8. A pipelined microprocessor has depth of six. The cycle time of the six stages are 0.3, 0.5, 0.4, 0.4, 0.4 and 0.6 ns. respectively.

(a) How long does it take the microprocessor to perform a single instruction?

(b) What is the average time required to perform 200 instructions assuming there are no branches?

(c) What is the average time required to perform 200 instructions assuming 5% of the instructions are branches?

9. (a) Give three representations for signed integers.

   (b) Show the steps required to multiply two floating point numbers in IEEE single precision format.

   (c) Give three examples of exceptions that can be encountered in part (b) and explain how they would be detected.

Section 4

10. The following events occur between the primary station A and the two secondary stations B and C on a multidrop error-free half-duplex line using HDLC protocol: *Events* e_1, e_2, e_3, e_4, where

$e_1 =$ A, activates the link with B and C using a normal response mode.

$e_2 =$ A polls B for traffic, B responds by sending four I frames, then A only acknowledges B without granting B additional further transmission rights.

$e_3 =$ A polls C for traffic and C only acknowledges A.

$e_4 =$ A sends three frames to B and grants B the right to transmit. B responds by sending five additional frames and A acknowledges.

(a) Show the frames exchanged between the primary station A and the two secondary stat ions (B and C).

(b) Now assume t hat the noise corrupted the transmission of the first frame out of the five frames sent out by B to A in event e_4. Show two possible procedures for error recovery. Also, assume that the window size is seven.

Note: Use the abbreviations *A, YN(s)N(R), P /F* to describe a frame where *A* is the address field, *Y* is the type of frame, *P/F* is the poll/final bit, *N(s)* is the sending sequence number, and *N(R)* is the receiving sequence number.

11. When information bits of a certain message are grouped into blocks, overhead bits are needed in every block to provide functions such as synchronization, error control, and addressing. Assume each block contains N bits including H overhead bits. The line bit error rate is $\in$ and errored blocks must be retransmitted. A message with 1352 bits is to be transmitted.

Determine the optimum block length for this message N_{opt} such that the average number of total transmitted bits (including retransmissions) is minimum. Assume the overhead bits H are fixed and equal 168 bits regarding the block size $\in = 10^{-4}$. Repeat when $\in = 10^{-3}$.

12. The total number of stations N in an asynchronous CDMA packet radio network equals four. Users' communication with each other is equally likely. Packets arrive according to a Poisson process with rate λ packets/unit time. The packet length (and as a result, the service time) follows an exponential distribution with mean $1/\mu$ time units. A user may not transmit or receive simultaneously. Assume perfect acknowledgement, zero propagation time, and perfect capture, whereby correct reception is guaranteed once the packet is locked onto. If multiple users (packets) are destined to the same receiver, one packet is captured; the others are considered to be collided.

Let i represent the number of successfully communicating pairs, $i = 0, 1, 2$ and j represent the number of collided single terminals in the network, $j = 0, 1, \ldots, 4$. Then, the channel state can be rep resented by (i, j).

(a) Draw the steady-state transition diagram representing the system states. Show the transition probabilities.

(b) Obtain $P(i, j)$ for all I and j as a function of λ and μ.

(c) Obtain the average network throughput.

Section 5

13. An electromagnetic field in free space $\mu_0 = 4\pi \times 10^{-7}$ henry/meter, $\varepsilon_0 = 8.85 \times 10^{-12}$ farads/meter, is specified as by the vector phasor

$$\vec{E}(\vec{r}) = \vec{E}_0 \varepsilon^{-j\vec{k}\cdot\vec{r}}$$

where $\vec{E}_0 = \vec{x}^0$ the unit vector in the x direction of a rectangular coordinate system(x, y, z).

$$\vec{r} = x\vec{x}^0 + y\vec{y}^0 + z\vec{z}^{0^\circ}$$
$$\vec{k} = -j\vec{y}^0 + 2\vec{z}^0.$$

(a) What is the frequency f of the electromagnetic field (Hz)?

(b) Describe the equi-phase surfaces of the field. Write a general equation for the equi-phase surfaces.

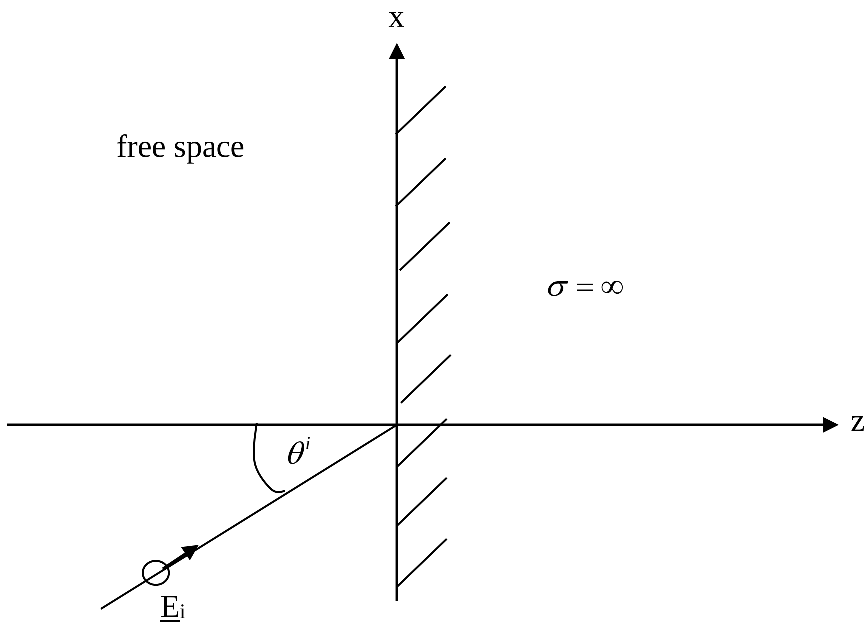
(c) Describe the constant magnitude-of-field surfaces. Write a general equation for these equal-magnitude surfaces.

(d) Evaluate the time average power density as a function of position.

14. A parallel polarized plane wave

$$\underline{E}^{(i)} = \underline{\hat{y}}_0 e^{-jk_0(x+z)/\sqrt{2}} \quad , \quad k_0 = \omega\sqrt{\mu_0 \varepsilon_0}$$

is incident on a perfectly conducting half space as is shown below.

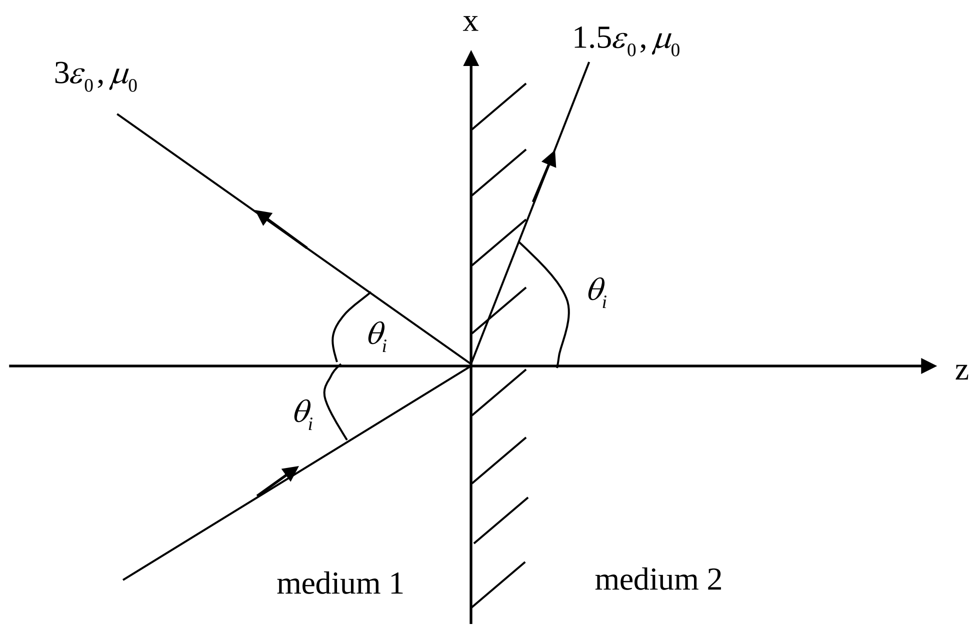


(a) Determine the angle of incidence;

(b) Find the reflected field;

(c) What is the surface current $\underline{J}_s$, on the surface of the conductor?

15. A parallel, polarized electromagnetic wave impinges from medium 1 $(\varepsilon_1 = 3\varepsilon_0, \mu_1 = \mu_0)$ to medium 2 $(\varepsilon_2 = 1.5\varepsilon_0, \mu_2 = \mu_0)$ as is shown below.



(a) What is the critical angle, θ_c?

(b) Given that the magnetic field in medium 1 is:

$$\boldsymbol{H}(x,z) = \hat{y}\left(e^{-jk_i \cos\theta_i t} - \rho e^{jk_i \cos\theta_i z}\right)e^{-jk_i \sin\theta_i x}, \; z < 0$$

where $k_i = \omega\sqrt{\varepsilon_1 \mu_1}$ and

$$\rho = \frac{\sqrt{\varepsilon_2}\cos\theta_i - \sqrt{\varepsilon_1}\cos\theta_i}{\sqrt{\varepsilon_2}\cos\theta_i + \sqrt{\varepsilon_1}\cos\theta_i} \quad ,$$

(c) what is the magnetic field in medium 2 when $\theta_i = 30^\circ$? (Assume $k_0 = \omega\sqrt{\mu_0 \varepsilon_0} = 1$ in parts (b) and (c).)

(d) Same as (b) but $\theta_i = 60^\circ$.

Section 6

16. Determine if the linear simultaneous algebraic equations have a solution. If so, find all solutions.

$$\begin{aligned} x_1 - x_2 - x_3 + x_4 &= 3 \\ x_1 + x_2 + 3x_3 + x_4 &= -1 \\ 2x_1 + x_2 + 4x_3 + x_4 &= 0 \\ x_2 + 2x_3 + x_4 &= -2 \end{aligned}$$

17. The following data have been collected; these are (x,y) data points.

$$\begin{array}{llllllll} X: & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ Y: & 2 & 3 & 11 & 20 & 22 & 30 & 50 \end{array}$$

Find the parabola line that best fits all the data points (in a least-squares sense).

18.

$$A = \begin{bmatrix} 1 & 0 & 3 & 1 \\ -1 & 1 & 7 & 2 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & -3 & -2 \end{bmatrix}$$

(a) What is the rank of the matrix A?

(b) How many eigenvalues does the matrix have? Find all of these.

(c) How many eigenvectors does the matrix have? Find all of these

(d) What is the dimension of the null space of A?

Section 7

19. (a) Find the SVD of the following $M \times 9$ matrix:

$$\mathbf{A} = [\mathbf{a_1} \quad \mathbf{a_1} \quad \mathbf{a_1} \quad \mathbf{a_1} \quad \mathbf{a}_2 \quad \mathbf{a}_2 \quad \mathbf{a}_2 \quad \mathbf{a}_3 \quad \mathbf{a}_3]$$

where $M > 9$ and $\mathbf{a_1}, \mathbf{a}_2$ and $\mathbf{a}_3$ are orthogonal column vectors with

$$\mathbf{a}_1^H \mathbf{a}_1 = 4, \ \mathbf{a}_2^H \mathbf{a}_2 = 9, \mathbf{a}_3^H \mathbf{a}_3 = 16$$

(b) Using the results of (a) solve the system

$$\mathbf{Ax} = \mathbf{y}$$

for $\mathbf{x}$ in the LMS sense.

(c) Compute the LMS error

20. The inverse of the double -sided Laplace transform

$$F_{II}(s) = \frac{s-1}{(s-2)^2(s^2+16)}$$

can represent several time functions, depending on the choice of the integration path in the inversion formula. Find and sketch the time functions. Identify the function for which $F_{II}(s)$ represents the single-sided Laplace transform. Find the corresponding Fourier transform.

21. A linear system is defined by the differential equation (DE)

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 25y = x(t)$$

(a) Find the output $y(t)$ when the input $x(t)$ is $cos(4t + \pi/4)$

(b) Find the impulse response

(c) Using Fourier Transforms solve the DE for $t \geq -5$ with initial conditions $y(-5) = 1, y'(-5) = 1$ and

$$x(t) = \begin{cases} 1; t > 1/2 \\ 0; t < 1/2 \end{cases}$$