THE GEORGE WASHINGTON UNIVERSITY
School of Engineering and Applied Science
Department of Electrical and Computer Engineering

Preliminary Examination - Fall 2014

Friday, October 17, 2014

General Instructions. Read carefully before starting.

Solve 5 problems in all; at most 2 questions may be selected from the same section.

Candidates registered in the following Focus Areas must answer at least two of the five questions from the relevant section as follows:

Biomedical Engineering:	Section 8
Computer Architecture & High Performance Computing:	Section 1
Communications & Networks:	Section 3
Electrical Power & Energy:	Section 9
Electromagnetics, Radiation Systems & Microwave Engineering:	Section 4
Electronics, Photonics & MEMS:	Section 6
Signal & Image Processing, Systems & Controls:	Section 3

Please write your name, student number and Registered Focus Area below:

________________________________ __________________

Student Name **Student Number**

____

Focus Area in which Registered

Solve each problem in a **separate** blue book. Write the section number, problem number, and your student number **on the front of each blue book**. **DO NOT WRITE YOUR NAME ON THE BLUE BOOK.**

Submit solutions to **only** five (5) problems. Use only **ONE** blue book per problem.

For each problem, make a special effort to give the answers in a clear form.

The exam will begin at 10:00 a.m. and end promptly at 3:00 p.m.

Only Calculators provided by the department at the examination will be allowed. Personal items including cell phones and other electrical devices must be relinquished prior to the start of the examination.

This is a CLOSED BOOK, CLOSED NOTES EXAMINATION

SECTION 1

1) An interleaved memory subsystem with 10 bit addresses uses the least significant 2 address bits to select the desired memory module via an appropriate decoder.

a) Sketch the address space layout in the modules

b) To accelerate the effective access time, the modules are accessed concurrently when applicable and output data are latched, where data is then accessed sequentially by the application from the fast latches. This can take place while a new set of words are being concurrently retrieved from the memory modules into the latches. The memory access time is T and the latch access time is t, where M=T/t to avoid data overrun. Derive an expression for accessing a 20-word vector from memory, if the memory access time is T and the latch access time is t. Consider the best and worst case scenarios.

c) Sketch the overall architecture of this memory subsystem, showing the decoder details and how the modules are controlled.

2) Assume a 5-stage pipelined processor with forwarding and a predict-taken branch predictor, for the following code

```
        LW   R2, 0(R1)
Label1: BEQ  R2, R0, Label2  ; not take once, then taken
        LW   R3, 0(R2)
        BEQ   R3, R0, Label1 ; taken
        ADD R1, R3, R1
Label2:  SW   R2, 0(R2)
```

a. Draw the pipeline execution diagram for this code, assuming no delay slots and braches execute in the EX stage

b. Draw the diagram, assuming delay slots are used

3) Let us consider that we have a five stage pipeline with Fetch (IF), Decode (ID), Execute (EX), Memory access (MEM) and Register Writeback (WB) stages. You may assume that four of the five pipe stages consume one cycle each EXCEPT MEM stage that consumes two clock cycles. The above said pipeline could forward the results of EX and MEM stages back to instructions that have already been decoded and waiting to execute. Draw a multi cycle pipeline diagram showing the progress of instructions through the pipeline at each clock cycle tick. Indicate the total number of cycles to complete the following set of instructions through the pipeline. Also, compute the speedup of this setup over a baseline architecture that does not have the forwarding capability.

```
LW R5, 10(R2)        #Load word

ADD R10, R5, R6   #R10=R5+R6

MUL R10, R10, R5 #R10=R10*R5

SW R10, 10(R2)     #Store word
```

SECTION 2

4) An $n \times n$ matrix A has eigenvalue λ_0 with multiplicity $m = n$. Prove that

$$f(\mathbf{J}) = \begin{bmatrix} e^{\lambda_0 t} & te^{\lambda_0 t} & \dfrac{t^2 e^{\lambda_0 t}}{2!} & \cdots & \dfrac{t^{m-2} e^{\lambda_0 t}}{(m-2)!} & \dfrac{t^{m-1} e^{\lambda_0 t}}{(m-1)!} \\ 0 & e^{\lambda_0 t} & te^{\lambda_0 t} & \cdots & \dfrac{t^{m-3} e^{\lambda_0 t}}{(m-3)!} & \dfrac{t^{m-2} e^{\lambda_0 t}}{(m-2)!} \\ 0 & 0 & e^{\lambda_i t} & \cdots & \cdots & \vdots \\ \vdots & \vdots & \vdots \quad \vdots & & te^{\lambda_0 t} & \dfrac{t^2 e^{\lambda_0 t}}{2!} \\ 0 & 0 & 0 \quad 0 & & e^{\lambda_0 t} & te^{\lambda_0 t} \\ 0 & 0 & 0 \quad 0 & & \cdots & e^{\lambda_0 t} \end{bmatrix}, \quad \mathbf{J} = \begin{bmatrix} \lambda_0 & 1 & \cdots & 0 & 0 \\ 0 & \lambda_0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \lambda_0 & 1 \\ 0 & 0 & \cdots & 0 & \lambda_0 \end{bmatrix}$$

HINT. Utilize the characteristic polynomial of **A**.

5) You are given the system

$$G(s) = \frac{1}{s^3}$$

Using state variable feedback, modify the given system so that its impulse response will be approximately equivalent to that of a system that has a natural frequency of $\sqrt{2}$ radians/s and a time constant of $\sqrt{2}$ s. Explain and justify your answer.

6) The system $S = \{\mathbf{A}, \mathbf{b}, \mathbf{c}\}$ is excited by an input $u(t) = e^{2t}[1(t) - 1(t-1)]$ where $1(t)$ is the unit step function and $\mathbf{x}(0)$ is the initial state.

a) Find the complete response using an equivalence transformation; (any other solution will be graded as zero)

b) Is it possible to find initial conditions such that, under zero-input conditions, the unstable modes of the system will not appear at the output? If yes, do it; if no, explain why not.

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 1 \\ -1 & 0 & 2 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{c} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$$

SECTION 3

7) Messages arrive at a central office switch according to a Poisson distribution, with an average arrival rate of 4 messages per second.

a) Determine the probability that in a 100 millisecond interval at least one message will arrive.

b) Determine the probability of exactly 4 messages arriving per second.

The messages are exponentially distributed in length with average length 8000 bits. The switch has an infinite buffer and is served by a 64 kilobit per second transmission circuit.

c) Determine the traffic intensity for the switch in Erlangs.

d) Determine the probability distribution of the buffer loading in messages.

e) Determine the average waiting time of a message in the buffer in seconds.

8) The random process $X(t) = \exp(at)$ consists of a family of exponential sample functions determined by the value of the continuous random variable a. In terms of the density function f(a) of a, determine the expected value of X(t), its autocorrelation function R(t1, t2) and its first order density function f(x, t).

9) Let $x1, x2, x3, \ldots$ be a sequence of independent and identically distributed binary random variables with values 1 and -1 and associated probabilities p and 1-p. Consider the partial sum $s(n) = x1 + x2 + x3 + \ldots\ x(n)$, with $s(0) = 0$.

a) Determine the range of values of r.

b) Find $P\{s(n) = r\}$

SECTION 4

10) The magnetic field of a plane electromagnetic wave propagating through a certain nonmagnetic material is given by

$$\mathbf{H} = \mathbf{z}30\cos(10^8 t - 0.5y + 1.2x) \qquad \text{(mA/m)}$$

Find the following:

a) The direction of wave propagation.

b) The phase velocity.

c) The wavelength in the material.

d) The relative permittivity of the material.

e) The electric field phasor.

11) A plane wave radiated by a distant antenna is incident in air upon a plane soil surface located at z = 0.

(Assume yz plane is the plane of incidence. z > 0 is soil with a relative permittivity of 4; and z < 0 is air) The frequency of the electromagnetic wave is 600 MHz and the angle of incidence is 30 degrees. Electric field is polarized along **x**-direction and its amplitude is 10 V/m.

a) Is this a TE or TM wave? Is this a perpendicular polarization or parallel polarization problem?

b) Apply the Snell's Law and find angle of transmission first and then calculate the transmission coefficient.

c) Derive phasor domain expressions for the transmitted electric and magnetic field vectors.

d) Calculate time average power density vector in soil.

Normal Incidence	TM Polarization	TE Perpendicular Polarization
$\Gamma = \frac{E_{r0}}{E_{i0}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$	$\Gamma_{TM} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$	$\Gamma_{TE} = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$
$\tau = \frac{E_{t0}}{E_{i0}} = \frac{2\eta_2}{\eta_2 + \eta_1}$	$\Gamma_{TM} = \frac{2\eta_2 \cos\theta_t}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$	$\Gamma_{TE} = \frac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$

12) There is a 30 cm long air filled transmission line with an intrinsic impedance of 40 Ω. If this tranmission line is connected to 80 Ω resistance at the load (at $z = 0.3$ m) and a DC generator of 8 V and internal resistance of 20 Ω (at $z = 0$ m), calculate and plot

a) Voltage, $V(z)$, along the whole transmission line for $t = 3.5$ ns.

b) Current, $I(t)$, at $z = 15$ cm for $0 < t < 5T$.

SECTION 5

13) A real-valued continuous-time signal $s(t)$ has Fourier transform $S(\omega)$ such that $S(\omega)=0$ for $|\omega| \geq B$ [rad/s].

a) Find the Fourier transform of

$$y(t) = s(t)\cos(\omega_0 t)$$

expressed using $S(\omega)$.

b) The signal $y(t)$ passes through a circuit with frequency response

$$H(\omega) = \alpha(\omega) e^{j\varphi(\omega)}$$

where $\alpha(\omega)$ is the amplitude response and $\varphi(\omega)$ is the phase response. Find the Fourier transform of the signal $z(t)$ at the output of the circuit.

c) Assume that $\omega_0 \geq B$ and that for $\omega \in [\omega_0 - B, \omega_0 + B]$ the circuit frequency response can be expressed as

$$H(\omega) = \alpha(\omega_0) e^{j\varphi(\omega_0) + j\varphi'(\omega_0)(\omega-\omega_0)}$$

where $\varphi'(\omega)$ is the first derivative of the phase response. Find the circuit frequency response $H(\omega)$ for $\omega \in [-\omega_0 - B, -\omega_0 + B]$.

d) Show that the output signal $z(t)$ can be expressed as

$$z(t) = \alpha(\omega_0) s(t - \tau_g(\omega_0)) \cos(\omega_0 (t - \tau_p(\omega_0)))$$

where $\tau_g(\omega) = -\varphi'(\omega)$ is the group delay and $\tau_p(\omega) = -\dfrac{\varphi(\omega)}{\omega}$ is the phase delay.

14) A real-valued discrete-time finite-energy pulse is generated as

$$p[n] = ap[n-1] + \delta[n]$$

where $p[n] = 0$ for $n < 0$, while $\delta[n] = 1$ for $n = 0$ and $\delta[n] = 0$ for $n \neq 0$.

a) Find z-transform $P(z)$, its region of convergence and the time-domain expression for the pulse. What is the range of a?

b) The time-autocorrelation function of the pulse $p(n)$ is defined as

$$r_p[n] = \sum_{k=-\infty}^{+\infty} p[k+n]p[k]$$

for n being any integer. Find z-transform $R_p(z)$, its region of convergence and the time-domain expression for the time-autocorrelation function.

c) The pulse $p[n]$ is the input of the real-valued discrete-time system with system function $H(z)$. Let the output of the system be denoted as $y[n]$. Show that the z-transform of the time-autocorrelation function of $y[n]$ can be expressed as

$$R_y(z) = H(z)H(z^{-1})R_p(z).$$

d) Let $H(z)$ in part (c) be

$$H(z) = 1 - z^{-M}.$$

Find the time-autocorrelation function of $y[n]$. What is the energy of $y[n]$?

15) Let us consider the train of pulses

$$s(t) = \sum_{n=-\infty}^{+\infty} g(nT_s) h(t - nT_s)$$

where $g(nT_s)$ are samples of the bandlimited continuous-time signal $g(t)$ with Fourier transform $G(\omega) = 0$ for $|\omega| \geq W$ [rad/s] and $h(t)$ is the rectangular pulse of width T

$$h(t) = \begin{cases} 1, & 0 \leq t < T \\ 0, & \text{otherwize.} \end{cases}$$

a) Find the Fourier transform $S(\omega)$ of $s(t)$ expressed using $G(w)$ and the Fourier transform $H(\omega)$ of $h(t)$.

b) What conditions must satisfy T_s and T such that the perfect reconstruction of $g(t)$ is possible by lowpass filtering of $s(t)$. Find the frequency response $R(\omega)$ of the lowpass filter that provides the perfect reconstruction.

c) Sketch the amplitude response of $R(\omega)$.

SECTION 6

16) Please provide concise answers to the following device physics related questions.

a) Explain the primary difference between direct and indirect band-gap semiconductors in terms of the electron energy-momentum relationship. Which type of semiconductor is more suitable for a photo-detector application and why?

b) In general, the Debye length in semiconductors is considered to be a small length compared to the depletion thicknesses. Give at least two examples in semiconductor devices where one would observe variations in properties over the Debye length.

17) The drift-diffusion transport model states that the electron current density, J_n, is the sum of the drift and diffusion currents: $J_n = -q\,(\mu_n\, n\ \nabla\phi - D_n\ \nabla n\,)$, where, q is the electronic charge, μ_n is the electron mobility, n is the electron charge density, ϕ is the electrostatic potential, and D_n is the electron diffusion constant.

a) Show that under thermal equilibrium conditions, the electron charge density can be expressed as follows: $n = n_i \exp(+\phi\ /v_T)$, where v_T is thermal voltage and n_i is the intrinsic charge density.

b) The electron quasi-Fermi potential, ψ_n, is a useful quantity to describe the non-equilibrium operation of semiconductor devices. It is defined as follows: $n = n_i \exp[(\phi - \psi_n\,)/v_T]$. Derive an expression for the electron current density in terms of n and ψ_n. Note that a very similar expression can be derived for the hole current density, as well. With this potential in mind, explain how a pn-junction can have an electric potential across it, yet exhibit zero terminal current and zero measured voltage drop across its terminals at thermal equilibrium.

18) A long-base abrupt *pn*-junction diode has a uniform cross section and constant doping on both sides of the junction. The device is doped as follows: N_a = $2.0 x 10^{16}$ cm^{-3} *p*-type and N_d = $1 x 10^{17}$ cm^{-3} *n*-type. The electron and hole carrier lifetimes in the material are $\tau_n = 3 x 10^{-6}$ s and $\tau_p = 5 x 10^{-6}$ s, respectively. Employing these parameters, answer the following questions:

a) What is the built-in potential of the *pn*-junction barrier?

b) If the device were to be modified as a *pin*-diode where an intrinsic layer of 0.2 μm is inserted between the *p* and the *n* layers, what will be the value of the built-in potential?

c) When the *pn*-diode (without the intrinsic layer) is forward biased with 0.5 V, calculate the density of the minority carriers at the edge of the space-charge regions.

d) Under the bias condition above, plot the minority and majority carrier currents as a function of distance from the junction.

SECTION 7

19) Consider a network link with a link rate of 100 Kb/s and a queue length of 500 packets. Suppose that an audio application sends audio data using RTP and UDP over the network link with an overhead of 50 bytes per packet. The audio is encoded at 32 Kb/s and we want audio packets to be large enough so that the overhead is at most 20% of the total.

a. How long does it take to acquire the audio data needed to “fill” a packet?

b. What is the queuing delay at the network link?

c. What is the delay if the link rate reduces to 10Kb/s?

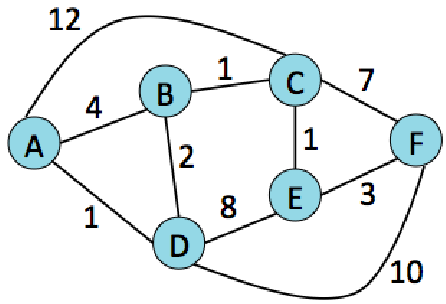
20) Consider a data link layer with the following parameters: Frame transmission time at the sender is t_f=20 microseconds. ACK or NAK transmission time at the receiver is t_{ack}=10 microseconds. Link propagation delay on both directions is t_{prop}=25 microseconds. Suppose frame processing time at both sender and receiver is negligible, i.e., t_{proc}=0. Finally, overall round-trip probability of frame error on the link is r=0.04.

a. Assume that for the Stop-and-Wait ARQ scheme, the TIMEOUT at the sender is chosen optimally. What is the resulting throughput (frames/second)?

b. In the Go-Back-N ARQ scheme, if the link is error free, what is the minimum window size *N* that is able to keep the link busy?

c. Choose the window size in (Part b) and now consider the link error probability r=0.04. What is the throughput (frames/second) of the Go-Back-N ARQ scheme?

21) Consider the following network with nodes A through F.



a. If Dijkstra's algorithm is employed for routing on this network, show how the algorithm finds the path that a packet travels from F to A. (Show your steps!)

b. Suppose we would like to modify the routing policy to find a path from F to A with the minimum number of hops. Can we use Dijkstra's algorithm for this purpose? Justify your answer.

SECTION 8

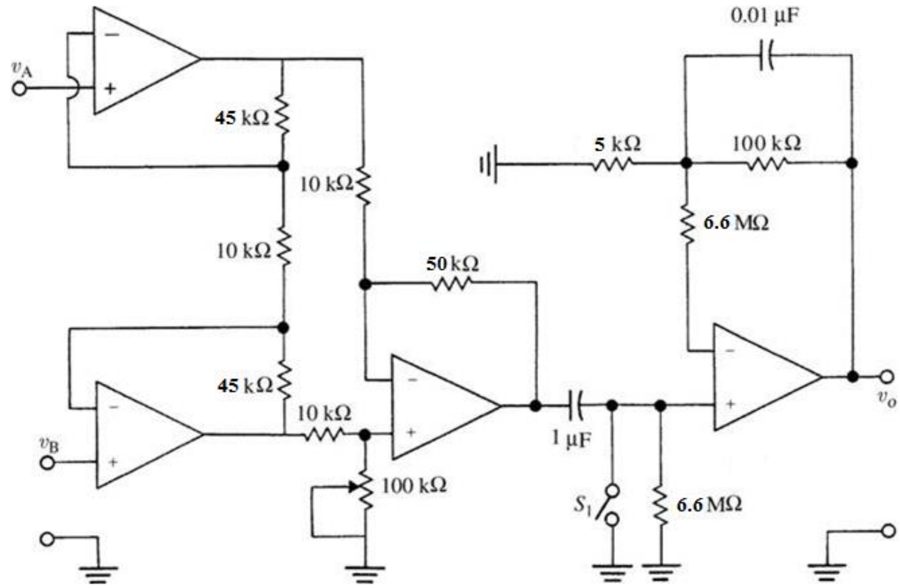
22) a) Define sensitivity and specificity of a diagnostic test. What false results will low sensitivity lead to? How about low specificity?

b) Suppose 100 people took the HIV-1 infection test, the test results *versus* the actual conditions of these people are given in the table below: *e.g.* among the 20 people who were actually infected, 19 were tested positive and 1 was tested negative. Calculate the sensitivity (as a percentage), specificity, negative predictive value (NPV) and positive predictive value (PPV) of this test.

		Actual Condition	
		Infected (20)	Not Infected (80)
Test Result	*Positive*: Test shows "infected"	19	3
	Negative: Test shows "not infected"	1	77

23) a) Draw a typical electrocardiogram (ECG) and label all waves and intervals. Explain what is happening electrically within the heart during each wave or interval.

b) An ECG preamplifier is shown below. Calculate the mid-band gain, low-frequency cutoff and high-frequency cutoff for this amplifier.



c) Explain the purpose of the driven-right-leg system used in ECG measurement systems.

24) a) Describe Beer's law for optical absorption.

b) If the transmission ($T = I/I_0$) of light (from a monochromatic light source) through a 1cm long clinical sample with DNA concentration of 1ng/mL is $T = 90\%$, what would the transmission be through the same length sample if the DNA concentration is increased to 2ng/mL? (Assume DNA is the only absorber at this wavelength in the sample).

SECTION 9

25) A three-phase 200MVA, 20kV, 60Hz salient pole synchronous machine has parameters Xd = 1.1 pu, Xq = 0.6 pu and Ra~0. The machine delivers 180MW at 0.9 lagging power factor to an infinite busbar.

Calculate the excitation voltage and the power angle. Draw the phasor diagram. (Hint: use per unit values and give your answers in pu)

26) A wind turbine is to be designed with an electrical power output of 2.0 MW. The rated upwind free wind speed is 13 m/s. Determine the length of the rotor blades in meters and the rotational speed of the rotor in rev/min if the tip-speed ratio whose value as 7.0 determines the maximum Power Coefficient of 0.45. Use the density of air as 1.225 kg/m^3

27) A 450MVA, 20kV, 60-Hz round-rotor synchronous generator has an Inertia constant H = 3s.

Displayed on the axes below are Torque/Angle characteristics for various faults occurring on a double circuit transmission line when connected between a synchronous generator and an infinite busbar. Using the Equal Area Criterion, determine the critical switching times for both a 3φ fault and a Double Line to Ground fault when the input torque from the turbine is 1.0 pu as shown in the diagram.