THE GEORGE WASHINGTON UNIVERSITY
School of Engineering and Applied Science
Department of Electrical and Computer Engineering

Preliminary Examination

Spring 2009

General Instructions

Read carefully before starting.

Solve 5 problems in all; at most 2 questions may be selected from the same section.

Please write your name and student number below:

______________________________	______________________
Student Name	**Student Number**

Solve each problem in a separate blue book. Write the section number, problem number, and your student number on the front of each blue book. **DO NOT WRITE YOUR NAME ON THE BLUE BOOK.**

Submit solutions to only five (5) problems.

For each problem, make a special effort to give the answers in a clear form.

The exam will begin at 10:00 a.m. and end at 3:00 p.m.

Only Calculators provided by the department at the examination will be allowed.

Section 1

1. In the design of a main memory subsystem of a small computer with 16Kbytes, both low order interleaving and high order interleaving are considered. Either way, 8 modules of 2 Kbytes each are used.

   a. Sketch the memory subsystem architecture in both cases showing how the bits of the full physical address will be used to enable the modules and access data within the modules.

   b. Sketch how the addresses space will be laid out in the modules showing at least the first and last address of each module.

   c. If each memory module has an access time of T, and data from each module can be read into a fast buffer with an access time of t, how long would it take to read 8 consecutive bytes (i.e. ones with consecutive addresses) in each of the cases?

   d. Repeat c for 12 bytes.

2. A 2 GHz processor has a linear instruction pipeline constructed with stages for fetch, decode, read-operand, execute, and write-result.

   a. What would be the exact throughput (in MIPS) and speed up over a non-pipelined version if 100 instructions were executed?

   b. Repeat part if the probability of an instruction to be a successful branch is 0.2

   c. What would be your answer for "b" if the pipeline was converted into a super-pipeline driven by a clock which is 4 times faster?

3. Consider the problem of computing the average pixel value in a 256x256 image where pixel values are represented as unsigned bytes. Assuming an architecture where the cache memory size is 16KB and the cache line size is sixteen bytes, and that the cache was initially filled by as many elements of the image as it can hold:

   a. Sketch the cache architecture and show how the pixels of each image will be distributed in the cache system and compute the hit rate if a direct mapped cache is used. Assume C-language row major order memory layout.

   b. Repeat part a if FORTRAN, which uses a column major order memory layout, is used.

   c. Compute the hit rate if a two way set associative cache architecture is used instead in case a

   d. Compute the hit rate if a two way set associative cache is architecture is used instead in case b

Section 2

4. Let $\underline{X}$ be a binomial random variable.

a) Obtain the characteristic function for $\underline{X}$, i.e., $\Phi_{\underline{X}}(s) = E\left(e^{s\underline{X}}\right)$.

b) Compute the mean and the variance for $\underline{X}$.

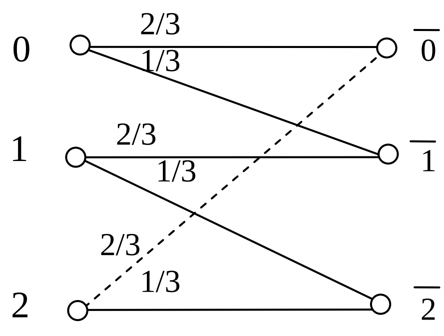
5. A ternary channel (3 symbols of input/output) is given below.

a priori probabilities

$P(0) = \frac{1}{2},$

$P(1) = \frac{3}{8},$

$P(2) = \frac{1}{8},$



Compute

a) $P(E) = average\ probability\ of\ error,$

b) a posteriori probabilities, $P(0/E), P(1/E), P(2/E).$

c) Determine which received symbol is most likely to be in error.

6. Let a random process be $\underline{X}(t) = A^2 \cos^2\left(\omega_C t + \underline{\theta}\right)$, where A and ω_C are constants and $\underline{\theta}$ is a random variable uniformly distributed in $\left[-\pi, \pi\right]$.

a. Is $\underline{X}(t)$ wide sense stationary?

b. If not, why not? If so, compute the mean and autocorrelation of $\underline{X}(t)$.

Section 3

7. A uniform length of lossless (perfect conductor) coaxial line, inner conductor radius $a = 1cm$ and outer conductor $b = 2cm$ is filled with uniform isotropic dielectrics. In the portion of coaxial line to the right of the plane (arbitrarily defined) $z = 0$ the dielectric permittivity is $\varepsilon_1 = 16\varepsilon_0$ whereas in a portion of coaxial line to the left of that plane the dielectric permittivity is $\varepsilon_1 = 4\varepsilon_0$.

a. Calculate the reflection coefficient in the left-hand portion of line at the plane $z = 0$.

b. Calculate the voltage standing wave ratio [VSWR] in that same left-hand portion of coaxial line (to the left of $z = 0$).

c. Find the distance from the junction plane $z = 0$ to the first maximum of in the voltage standing wave pattern expressed in terms of fractions of a wavelength in the left-hand portion of the coaxial line. Make a LARGE clear drawing of the reflection coefficient plane illustrating your calculation.

d. Filling a section of the coaxial line to the right of the plane $z = 0$ with a new dielectric with arbitrary permittivity to be chosen, design this section so that the input reflection coefficient in the left-hand dielectric at the plane $z = 0$ is equal to zero (impedance match). Specify the required permittivity of the dielectric and length of the new section of transmission line to be inserted (length in terms of fractions of the wavelength within the newly added section). Make a drawing of the reflection plane showing the effect of the new matching section.

8. The complex phasor (time dependence exp $(+j\omega t)$) representing the electric field of a plane wave in free space is

$$\boldsymbol{E} = E_0 e^{\frac{j\pi}{2}} \left(\boldsymbol{x} - \boldsymbol{y}\right) e^{-j\left(\frac{\omega}{c}\right)\frac{(x+y)}{\sqrt{2}}}$$

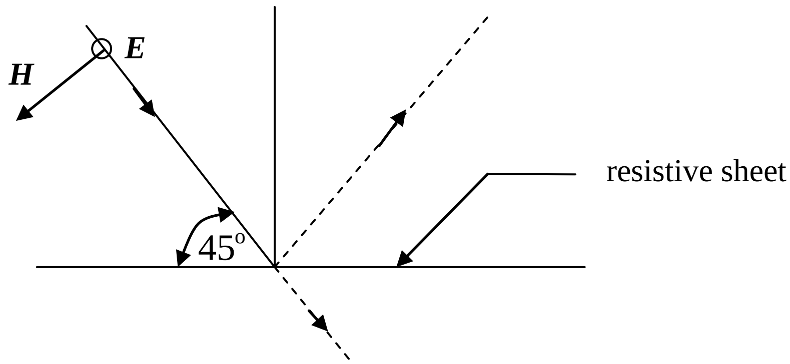
where ω is the angular frequency, c is the free space velocity of light, and E_0 is a real constant.

a. What is the direction of propagation of the wave?

b. What is the polarization of the wave?

c. Find $\boldsymbol{H}(x,y,z,t)$.

9. A plane wave, whose $\boldsymbol{E}$ field is parallel to a resistive sheet and has an amplitude of 1 volt/meter, is incident at 45° onto the resistive sheet.



The sheet is characterized by a linear relationship between the surface current $\boldsymbol{K}$ flowing in it and the tangential electric field of $\boldsymbol{E}_t$:

$$\boldsymbol{K} = G\,\boldsymbol{E}_t$$

Calculate the value of G for which the intensities of the transmitted and reflected waves are equal.

Section 4

10. The input $e^{i\omega t}U(t)$ to a linear system gives the output

$$-\frac{e^{-t^4/2}}{t^3+4i\omega}+\frac{e^{i\omega t}}{t^3+4i\omega}$$

for all real t and ω. (Note: $U(t)$ denotes the unit step function.)

a) Find the system impulse response

b) Find and sketch the output when the input is $f(t)=\begin{cases}1\,;|t|\le 1\\ 0\,;\,|t|>1\end{cases}$

11. The sequence $y[n]$ satisfies the following difference equation:

$$y[n+2]-\frac{8y[n+1]}{15}+\frac{1}{15}y[n]=u[n]$$

where

$$u[n]=\begin{cases}1\,; n\geq 0\\ 0\,; n<0\end{cases}$$

a) Assuming $y[0]=2$ and $y[1]=3$ find the causal solution.

b) Suppose the $y(n\Delta t)=y[n]/\Delta t$ (with Δt the sampling interval) represent samples of a band limited signal with a bandwidth of 30 Hz. Find the Fourier Transform of the signal.

12. A linear system is defined by the differential equation

$$\frac{d^2 y}{dt^2}+\frac{dy}{dt}+y=x-3\frac{dx}{dt}$$

with $y(t)$ the input and $x(t)$ the input.

Find

a) the output when the input is $\cos 5t$

b) the impulse response

c) the output for $t \geq 0$ with initial conditions $y(0)=1$, $y'(0)=-1$ when the input is $\cos 5tU(t)$ and identify the zero state, zero input, the transient and the steady state responses.

Section 5

13. Find the straight line that minimizes the vertical rms error to the points:

$$(1.0, 2.1), (2.0, 1.9), (3.0, 4.3), (4.0, 5.2)$$

14. For the following matrix

$$\begin{bmatrix} 3 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

a. What is the product of the eigenvalues?

b. What is the sum of the eigenvalues?

c. Are the eigenvectors orthogonal and why?

15. a) Write the following equation as a 2nd order ODE.

$$\frac{d}{dt}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 & -1 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

b) What is the solution to the homogeneous equation?

c) What is the solution that starts from x = 1 and y = 2?

Section 6

16. According to the drift-diffusion transport model the hole current density, $\boldsymbol{J}_p$, is the superposition of the drift current and the diffusion current given as follows:

$$\boldsymbol{J}_p = -q\,(\mu_p\, p\ \nabla\phi + D_p\ \nabla p\ ).$$

In the above expression, q is the electronic charge, μ_p is the hole mobility, p is the hole charge density, ϕ is the electrostatic potential, and D_p is the hole diffusion constant.

a) Under thermal equilibrium conditions show that the hole charge density can be expressed as follows:

$$p = n_i \exp(-\phi / v_T)\ ,$$

where v_T is thermal voltage and n_i is the intrinsic charge density.

b) The hole quasi-Fermi potential, ψ_p, can be defined as follows:

$$p = n_i \exp[-(\phi - \psi_p\ )/v_T].$$

Derive an expression for the hole current density in terms of p and ψ_p. Based on this expression, explain the physical meaning of the quasi-Fermi potential.

17. Provide brief and clear answers to the following questions:

a) Sketch the band diagram of a metal-semiconductor junction where the workfunction of the metal is larger than that of the semiconductor. Can such a system be employed to construct an Ohmic contact? If so, how can this be accomplished?

b) Keeping all other parameters constant, describe the impact of the shortening of channel length on the value of the threshold voltage of a MOS transistor.

c) Explain the relevance of the saturation velocity of the electron on the intrinsic limits of the speed of response of a MOSFET. What factors determine the speed of electrons in the channel of a MOSFET?

18. An ideal long-base abrupt *pn*-junction silicon diode has a uniform cross section and constant doping on both sides of the junction. The doping profile of the diode is as follows: $N_a = 6.0x10^{15}$ cm^{-3} *p*-type and $N_d = 2x10^{16}$ cm^{-3} *n*-type, where the minority-carrier lifetimes are $\tau_n = 1x10^{-6}$ s and $\tau_p = 2x10^{-6}$ s, respectively. Assume that the effects within the space-charge region are negligible and that minority carriers flow only by the diffusion mechanism in the charge neutral regions.

a) Determine the value of the built-in potential across the pn-junction barrier.

b) When the applied forward bias is 0.3 V, calculate the density of the minority carriers at the edge of the space-charge region.

c) Under the bias condition above, sketch the minority and majority carrier currents as a function of distance from the junction.

Section 7

19. Let A and B be two stations attempting to transmit on an Ethernet. Each has a steady queue of frames ready to send; A's frames will be numbered $A_1, A_2,$ and so on, and B's similarly. Let $T = 51.2 \mu s$ be the exponential backoff base unit.

Suppose A and B simultaneously attempt to send frame 1, collide, and happen to choose backoff times of $0 \times T$ and $1 \times T,$ respectively, meaning A wins the race and transmits A_1 while B waits. At the end of this transmission, B will attempt to retransmit B_1 while A will attempt to transmit $A_2.$ These first attempts will collide, but now A backs off for either $0 \times T$ or $1 \times T,$ while B backs off for time equal to one of $0 \times T, ..., 3 \times T.$

a) Give the probability that A wins this second backoff race immediately after this first collision; that is, A's first choice of backoff time $k \times 51.2$ is less than B's.

b) Suppose A wins this second backoff race. A transmits $A_3,$ and when it is finished, A and B collide again as A tries to transmit A_4 and B tries once more to transmit $B_1.$ Give the probability that A wins this third backoff race immediately after the first collision.

20. A group of 100 sources offers messages with exponentially distributed lengths to a 1200-bps line. The average message length is 200 bits, including overhead, and each source generates one message every 20 sec. Access to the line is controlled by message-switching concentration with an infinite queue. Determine the following:

a) Probability of entering the queue

b) Average queuing delay for all arrivals

c) Probability of being in the queue for more than 1 sec

d) Utilization of the transmission link.

21. A uniform PCM system is defined to encode signals in the range of -8159 to +8159 in quantization intervals of length 2. (The quantization interval at the origin extends from -1 to +1.) Signals are encoded in sign-magnitude format with a polarity bit = 1 denoting a negative signal.

a) How many bits are required to encode the full range of signals?

b) How many unused codes are there?

c) Determine the quantization noise, noise power, and signal-to-noise ratio (in decibels) of each of the following sample values: 30.2, 123.2, -2336.4, and 8080.9.

Section 8

22. a) How does the lipid solubility of solute molecules affect their ability to permeate biological membranes?

b) For molecules of similar lipid solubility, how does molecular weight influence permeability?

c) What is the approximate molecular weight above which the permeation of water-soluble substances by diffusion occurs at negligible rates?

d) How do water-soluble molecules that are much larger than this cut off enter and leave the cell?

23. a) List all the functions of skeletal muscle.

b) Briefly discuss and sketch the organization of a skeletal muscle and a skeletal muscle fiber.

c) What are the key elements that make force generation by cross-bridge cycling possible?

d) List in sequence the events involved in excitation-contraction coupling of skeletal muscle contraction and relaxation.

e) How is the force generated by a skeletal muscle varied?

24. a) Draw a diagram of the layers of the retina, and discuss briefly the main components of the retina

b) Describe the steps by which light energy is transduced to a receptor potential.