THE GEORGE WASHINGTON UNIVERSITY
School of Engineering and Applied Science
Department of Electrical and Computer Engineering

Preliminary Examination

Spring 2010

General Instructions

Read carefully before starting.

Solve 5 problems in all; at most 2 questions may be selected from the same section.

Please write your name and student number below:

Student Name	**Student Number**
______________________________	________________

Solve each problem in a separate blue book. Write the section number, problem number, and your student number on the front of each blue book. **DO NOT WRITE YOUR NAME ON THE BLUE BOOK.**

Submit solutions to only five (5) problems.

For each problem, make a special effort to give the answers in a clear form.

The exam will begin at 10:00 a.m. and end at 3:00 p.m.

Only Calculators provided by the department at the examination will be allowed.

SECTION 1

1. For the following question, assume a simple single-issue, five-stage pipeline for a processor with one cycle latency for each pipe stage. IF- fetches the instruction; ID- Decodes instruction and reads registers; EX- arithmetic computation or Memory Address Calculation; MEM- Memory Access for Load, store and WB- Writes back result into registers and completes operation.

   Consider the following code sequence:
   (Instruction format: Operation Destination, Src 1, Src 2)

   LD R1, 20(R2)
   ADD R3, R1, R4
   MUL R5, R2, R4

   a) With the help of a diagram, show how the instructions will advance through various stages of the pipeline. Indicate the total number of cycles needed to complete the execution of all three instructions.

   b) Assume that there is a forwarding circuitry that can feed the outputs of the EX and MEM stages into the input of EX stage. With this new feature, show how the instructions will advance through the pipeline stages. Indicate the total number of cycles needed to complete the execution of the code sequence.

   c) Assume that the processor has the capability to look through multiple instructions at once and figure out the data dependences between them. In order to prevent bubbles in the pipeline due to dependent instructions, the processor now acquires a new capability to dispatch instructions out of program order. Remember that there is still only one instruction that is dispatched for execution at any given cycle. With this capability of out-of-order execution along with data forwarding described in (b), show the instructions advancing through the pipeline. Calculate the number of cycles needed to complete the code sequence.

   d) In real world, load instructions take multiple cycles to fetch the data from memory. This increases the latency of execution for loads. Suggest two different ways that you could use to minimize stalling of the pipeline due to such exposed load latencies.

2. a) Assume a memory with 128 blocks whose block frame addresses are 0..127, a cache memory with 32 blocks numbers from 0..31, and the cache is cold.

For memory block frame address 65, in which block(s) of the cache may it be stored if the cache is:

i) Directly mapped?

ii) Fully associative?

iii) 2-way set associative?

iv) 4-way set associative?

b) Please indicates the possible effects of cache organizational parameters on each category of cache misses. Please briefly explain.

Cache parameter	**Cold Misses**	**Capacity Misses**	**Conflict Misses**
Reduce capacity			
Increase capacity			
Reduce block size			
Increase block size			
Reduce associativity			
Increase associativity			

3. a) In the following table, each of the rows corresponds to an I/O technique, while each of the columns represents a feature. For each of the three techniques indicate all applicable features by placing x in the corresponding cell(s).

	Processor is tied in I/O	Transfer rate is limited by CPU speed	Processor Periodically Checks whether I/O device ready	Processor Must relinquish control of the bus	When I/O is complete, processor receives an interrupt
Interrupt Driven I/O					
Programmed I/O					
DMA					

b) Consider a disk system with 960 512-byte sectors per track where the disk rotates at 3600 rpm. A processor reads one sector from the disk using interrupt-driven I/O with one interrupt per byte. If it takes 2.5 microseconds to process each interrupt, what percentage of time will the processor spend handling I/O, if you ignore seek time.

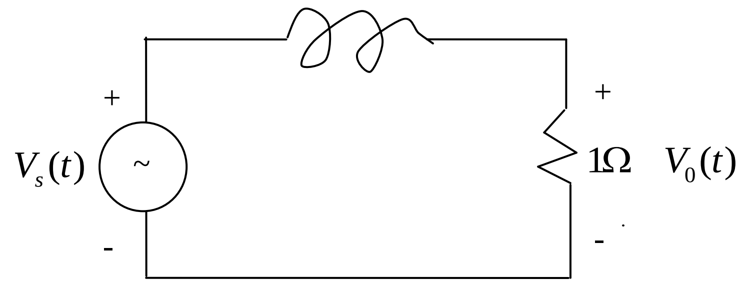
c) Repeat b with all of its parameters if a DMA controller is used, and each DMA operation transfers an entire sector.

SECTION 2

4. Let g(x) > 0 and g(x) nondecreasing function of x.

Prove $P\left\{|x| \geq \lambda\right\} \leq \dfrac{E\left\{g\left(|x|\right)\right\}}{g\left(\lambda\right)}$, *for* $\lambda > 0$.

5. Given a circuit below,



$V_s(t)$ is a w.s.s. process with 0 mean and $R_{V_s,V_s}(t) = e^{-|\tau|}$ autocorrelation. Compute

a) the mean of $V_0(t)$ and

b) autocorrelation of $V_0(t)$, $R_{V_0V_0}(\tau)$.

6. Let the density function of a geometric random variable be

$$f_{\underline{x}}(x) = \sum_{k=0}^{\infty} p\left(1-p\right)^{k}\ \delta(x-k).$$

Compute

a. the mean of $\underline{x}$, $\eta_{\underline{x}}$,

b. the variance of $\underline{x}$, $\sigma_{\underline{x}}^{2}$,

c. the third moment of $\underline{x}$, m_3.

SECTION 3

7. An electromagnetic field in free space, $\mu_0 = 4\pi \times 10^{-7}$ henry/meter, $\varepsilon_0 = 8.85 \times 10^{-12}$ farads/meter, is specified as by the vector phasor

$$\underline{E}(\underline{r}) = \underline{E}_0 \varepsilon^{-j\underline{k}g\underline{r}}$$

where $\underline{E}_0 = \hat{x}$ the unit vector in the x direction of a rectangular coordinate system (x, y, z).

$$\underline{r} = x\hat{x} + yy + z\hat{z}$$
$$\underline{k} = -j\hat{y} + 2\hat{z}.$$

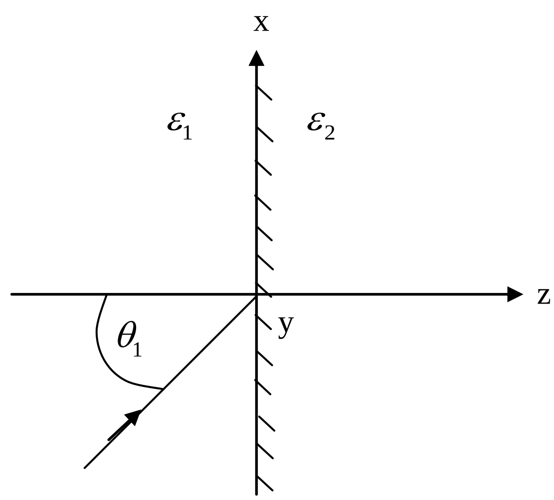
a) What is the frequency f of the electromagnetic field (Hz)?

b) Describe the equi-phase surfaces of the field. Write a general equation for the equi-phase surfaces.

c) Describe the constant magnitude-of-field surfaces. Write a general equation for these equal-magnitude surfaces.

d) Evaluate the average power as a function of position.

8. A plane wave is incident in the interface between two dielectrics ε_1 and $\varepsilon_2, \varepsilon_1 > \varepsilon_2$.



a) Find the angle θ_{1c} such that all waves incident with $\theta_1 > \theta_{1c}$ are "totally reflected".

b) For $\theta_1 > \theta_{1c}$, describe the field (if any) in the region z > 0 in the ε_2 dielectric.

c) If $\underline{E}$ is perpendicular to the plane of the incidence, $\underline{E} = E_y\ \hat{y}$, find the phase of the reflection coefficient.

9. The vector potential phasor, $A(\underline{r})$, produced by an electric current density $\underline{J}(\underline{r})$ is given by the integral

$$\underline{A}(\underline{r}) = \frac{\mu}{4\pi}\int \underline{J}(\underline{r}')\frac{e^{-jk|\underline{r}-\underline{r}'|}}{|\underline{r}-\underline{r}'|}d\underline{r}'$$

where $\underline{r} = x\hat{x} + y\hat{y} + z\hat{z}$ is the position vector to the field observation point and $\underline{r}'$ a similar position vector to the current source point.

a) Carefully introduce and justify the approximations to produce a formula for the approximate vector potential for the far field.

b) Derive the far-field electric field intensity (radiation field) produced by surface currents in the z = 0 plane

$$\begin{aligned} \underline{J}_s(x,y,0) &= \hat{y}\cos\left(\frac{\pi x}{2a}\right) && \begin{array}{l} -a < x < +a \\ -b < y < +b \end{array} \\ &= 0 && \quad otherwise; \end{aligned}$$

in terms of the given parameters and any other constants you may find it necessary to introduce and define.

SECTION 4

10. A discrete linear time-invariant (LTI) system yields $y[n]$ for an input $x[n]$,

$$x[n] = \left(\frac{1}{2}\right)^n u[n]$$

$$y[n] = \delta[n] + a\left(\frac{1}{4}\right)^n u[n],$$

where *a* is a parameter to be determined subsequently.

i) Compute the Z-transforms $X(z)$ and $Y(z)$ of $x[n]$ and $y[n]$.

ii) Evaluate the response of an LTI system with impulse response h[n] to an input $x[n] = (-2)^n$, for all n.

iii) Determine the value of the parameter *a* in the system function of LTI system, $H(z)$ so that an input $x[n] = (-2)^n$, for all *n*, produces a zero output $y[n] = 0,$ for all n.

iv) Now, by evaluating the complex inversion integral for $H(z)$ over a suitable contour in the complex z-plane, obtain the impulse response function $h[n]$ for this LTI system.

v) Compute the out put of this LTI system for the input $x[n] = 1,$ for all *n*.

11. a) Demonstrate the evaluation of the generalized limit

$$\lim_{\varepsilon \to \infty} \frac{1}{\varepsilon^2}\left[u(t) - 2u(t-\varepsilon) + u(t-2\varepsilon)\right]$$

b) Construct the analytic signal $\hat{f}(t)$ corresponding to the real signal

$$f(t) = \frac{\sin 2at}{\pi t}.$$

Since there are an infinite number of signals [$f(t) + jg(t)$ may be any real signal] the real part of which is the given real signal, you (your construction) must demonstrate that $\hat{f}(t)$ is indeed the analytic signal, i.e., that the Fourier transform of $\hat{f}(t)$ is zero for $\omega < 0$.

12.

The inverse of the double-sided Laplace transform

$$F_{II}(s) = \frac{s-2}{(s-3)^2(s^2+16)}$$

can represent several time functions, depending on the choice of the integration path in the inversion formula. Find the s ketch the time functions. Identify the function for which $F_{II}(s)$ represents the single-sided Laplace transform. Find the corresponding Fourier transform.

SECTION 5

13.

a) Compute the $LU = A$ factorization of the matrix

$$A = \begin{bmatrix} a & b & c \\ 2a & 5b & c \\ 4a & b & 3c \end{bmatrix}$$

b) Solve

$$A\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} \text{ for } \mathbf{x} = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^T$$

by first setting $L\mathbf{w} = \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}$, solving for $\mathbf{w}$,

and then solving $U\mathbf{x} = \mathbf{w}$ for $\mathbf{x}$

14.

a) Find the projection of $\mathbf{b} = \begin{bmatrix} e^{i\pi/4} & 0 & e^{-i\pi/4} & 1 \end{bmatrix}^T$ on the vector $\mathbf{a} = \begin{bmatrix} 1 & 1 & 0 & 1 \end{bmatrix}^T$

b) Find the projection of $\mathbf{b}$ on the hyperplane defined by the columns of the matrix

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$$

15. Determine the Singular Value Decomposition of the matrix

$$A = \begin{bmatrix} 1+i & 0 & 0 \\ 0 & 1 & 2 \\ 1-i & 0 & 0 \\ 0 & 1 & 2 \end{bmatrix}$$

SECTION 6

16. A semiconductor device is under thermal equilibrium conditions, and the Fermi energy E_f is within the bandgap throughout the device, i.e., it is a non-degenerate semiconductor. Consider that the drift-diffusion model is employed to describe current transport. In this framework, show that the electrostatic potential ϕ is related to the Fermi energy by the following expression:

$$\phi = (1/q)\,(E_f - E_i),$$

Here, q is the electronic charge and E_i is the mid-gap energy. State any necessary assumptions, define all symbols used and show all derivations employed.

17. The hole quasi-Fermi potential, ψ_p, can be defined as follows:

$$p = n_i \, exp[-(\phi - \psi_p)/v_T].$$

(a) Derive an expression for the hole current density in terms of p and ψ_p. You may start from the following expression for the hole current density, $\boldsymbol{J}_p$:

$$\boldsymbol{J}_p = -q\,(\mu_p\, p\, \nabla \phi + D_p\, \nabla p\,).$$

where, q is the electronic charge, μ_p is the hole mobility, p is the hole charge density, ϕ is the electrostatic potential, and D_p is the hole diffusion constant.

(b) Based on this expression, explain the physical meaning of the quasi-Fermi potential.

(c) Now, consider an NMOS transistor with source, drain and substrate Ohmic contacts. If the quasi-Fermi potential through out the entire device is 2V, what is the voltage difference between the source and the drain? What is the resulting drain current?

18. Consider a long-base abrupt *pn*-junction diode with a uniform cross section and constant doping on both sides of the junction. The doping density of the device is: $N_a = 8.0 \times 10^{15}$ cm^{-3} *p*-type and $N_d = 3 \times 10^{16}$ cm^{-3} *n*-type, where the minority-carrier lifetimes are $\tau_n = 2.5 \times 10^{-6}$ s and $\tau_p = 4 \times 10^{-6}$ s, respectively. Assume that the effects within the space-charge region are negligible and that minority carriers flow only by the diffusion mechanism in the charge neutral regions.

    a) Calculate the built-in potential of the pn-junction barrier?

    b) When the device is forward biased with 0.4 V, calculate the density of the minority carriers at the edge of the space-charge regions.

    c) Under the bias condition above, plot the minority and majority carrier currents as a function of distance from the junction.

SECTION 7

19. A series of information characters consisting of 7 binary data digits and a single even parity check digit is transmitted over a channel with bit error rate p. At the receiver each character is checked for errors and is discarded if the parity check fails. Otherwise it is passed on the end user.

a) Determine the probability distribution of the number of errors in the received character

b) Determine the expected number of errors in the received character

c) Determine the expected number of characters received in error

d) Determine the probability of receiving a character with an error pattern that goes undetected by the parity digit

e) Determine the effective throughput of the system, measured as the average number of correct data digits per character that are passed on the end user

20. A series of 10 information frames is to be transmitted from a primary to a secondary in half duplex mode across an established data link, using the ARM HDLC protocol. The primary transmits one frame at a time and waits for an acknowledgement before transmitting the next frame. Describe in diagram form how the following problems are solved:

a) An error in the third information frame.

b) An error in the frame acknowledging receipt of the sixth frame.

c) The absence of a response from the secondary after the eighth frame.

Next, assume that the 10 information frames sent by the primary carry 100 bits of information each. The velocity of propagation on the link is $2x10^8$ meters per second, the link is 10 km long and has a capacity of 9600 bits per second. Determine

d) The link throughput in bits per second

e) The efficiency of utilization as a percentage of channel capacity

Ignore the processing and disconnect times, assume the link is error free in both directions, and no protocol errors occur.

SECTION 8

21. a) Draw a functional overview diagram showing all components of the nervous system.

b) Explain the process of propagation of an action potential over myelinated and unmyelinated axons. Show diagrams of the movement of ions over the cell membrane in both cases.

22. a) Discuss the structure and function of a cardiac muscle cell.

b) Describe the process of a normal cardiac contraction.

c) List hormones involved in cardiovascular regulation, and describe their function.

23. a) Describe the structure and function of a hair cell.

b) Describe all steps in the reception of sound and the process of hearing.

c) Show a complete drawing of the auditory system.