THE GEORGE WASHINGTON UNIVERSITY
School of Engineering and Applied Science
Department of Electrical and Computer Engineering

Preliminary Examination - Spring 2015

Friday, February 20, 2015

General Instructions. Read carefully before starting.

Solve 5 problems in all; at most 2 questions may be selected from the same section.

Candidates registered in the following focus areas must answer two of their five questions from the relevant section as follows:

Biomedical Engineering	Section 8
Computer Architecture and High-Performance Computing:	Section 1
Communications & Networks:	Section 3
Electrical Power & Energy:	Section 9
Electromagnetics, Radiation Systems & Microwave Engineering:	Section 4
Electronics, Photonics & MEMS:	Section 6
Signal & Image Processing, Systems & Controls:	Section 3

Please write your name and student number below:

______________________________ ____________________

Student Name **Student Number**

Solve each problem in a **separate** blue book. Write the section number, problem number, and your student number **on the front of each blue book**. **DO NOT WRITE YOUR NAME ON THE BLUE BOOK.**

Submit solutions to **only** five (5) problems. Use only **ONE** blue book per problem.

For each problem, make a special effort to give the answers in a clear form.

The exam will begin at 10:00 a.m. and end promptly at 3:00 p.m.

Only Calculators provided by the department at the examination will be allowed. Personal items including cell phones and other electrical devices must be relinquished prior to the start of the examination.

This is a CLOSED BOOK, CLOSED NOTES EXAMINATION

SECTION 1

1) Assume a five-stage single-pipeline micro architecture with Fetch, Decode, Execute, Memory Access and Register Writeback. All operations take one clock cycle with the exception of Memory Access that takes 2 cycles. Using a multicycle pipeline diagram, demonstrate the progress of each instruction on every clock cycle until the last instruction of the second iteration of the following loop.

```
Loop:  LW    R1, 0(R5)
       ADDI  R1, R1, #1
       SUB   R4, R3, R2
       SW    R1, 0(R3)
       BNZ   R4, Loop
```

2) Given a two-core processor with a shared last level cache. Assume that words A and B are in the same cache block. For the following sequence of events, identify each miss as a hit, a true sharing miss, or a false sharing miss, and explain what happens in CPU, cache and memory.

Time	Core1	Core2
1	Write A	
2		Read B
3	Write A	
4		Write B
5	Read B	
6		Read A

3) A microprocessor core has an instruction pipeline with k stages. It is desired to execute a program that has n dynamic instructions.

   a. How many pipeline clocks would it take to execute the program on a single core? What is the throughput of the pipeline in instructions per clock?

   b. What is the efficiency of the pipeline and the speed up over a non-pipelined version?

   c. Repeat 1 and 2 if the probability of an instruction to be a branch is p and the probability that a branch is taken is q?

   d. If the chip has m cores but due to the network and other parallelization overheads the efficiency of the parallel implementation drops to 80%, what will be your answers to 1, 2 and 3 considering the whole chip?

   e. In no more than one 2 lines discuss how Amdahl's law would change your answer to 4 if the sequential fraction is equal to 20% for very large values of m?

SECTION 2

4) For the system $S = \{\mathbf{A}, \mathbf{b}, \mathbf{c}\}$

$$\mathbf{A} = \begin{bmatrix} -2 & 0 & 0 \\ -1 & -2 & 1 \\ 1 & 0 & 0 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{c} = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}$$

Using a *similarity transformation*, find the complete response for an initial state $x(t_0) = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ and input $u(t) = \{12\cos \pi t\}1(t)$ where $1(t)$ is the unit step function.

NOTE. No other method of solution will be accepted.

5) For the system $\mathbf{A} = \begin{bmatrix} -1 & -8 \\ 0.5 & -1 \end{bmatrix}$, $\mathbf{b} = \begin{pmatrix} 1 \\ 0.5 \end{pmatrix}$, $\mathbf{c} = \begin{bmatrix} -1 & 1 \end{bmatrix}$

a. Design a state observer;

b. Using the state estimates from part a), find an appropriate state feedback such that the system will have a purely oscillatory response with a natural frequency of oscillation $\omega_n = 2\ radians\ /\ second$.

6) The following are examples of simultaneous linear algebraic equations $\mathbf{y} = \mathbf{A}\mathbf{x}$ with unknown $\mathbf{x}$.

$$1)\quad \begin{bmatrix} 1 & -2 & 4 \\ 2 & -1 & 3 \\ 0 & 2 & -2 \\ -2 & 3 & -7 \\ 1 & 1 & 1 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -8 \\ -7 \\ 6 \\ 13 \\ 1 \end{pmatrix};\quad 2)\quad \begin{bmatrix} 1 & 1 & 2 & 0 \\ -1 & 2 & 1 & -3 \\ 2 & 0 & 2 & 6 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

For each, find

a. The dimension of the range space of **A** (rank);

b. The dimension of the null space of **A** (nullity);

c. All solutions $\mathbf{x}$. for the equations.

SECTION 3

7) (3 parts)

Part 1: You toss a coin N times. The coin shows head with probability p and tail with probability 1-p. Compute the following probabilities:

a. All the tosses result in tails

b. The coin shows head at least once

c. The coin shows tail exactly k times, for $0 \leq k \leq N$

Part 2: You simultaneously roll two dice. Find the probability that the first die lands on a higher value than the second die.

Part 3: You play a game by simultaneously rolling two dice. If the sum of the two numbers is at most 5, you win and the game ends. If the sum is 7 or 10 you lose and the game also ends. In any other case you have to roll again.

a. Find the probability that the game ends after exactly four rolls.

b. Find the probability that you win.

8) A nonlinear device has an input X which is uniformly distributed over the interval -2 to +3 and a transfer function $Y = X^2$.

a. Find the probability density function and cumulative probability function of Y.

b. Find the expected value and variance of Y.

9) Consider a two-state Markov chain with transition probabilities $p_{12} = p$ and $p_{21} = q$. Assume q is fixed but the value of p can be selected from the interval (0,1). Also assume a payoff of r units every time we visit state 2 and a cost of p every time we visit state 1.

   a) Compute the long-term payoff per time step as a function of p and q.

   b) Find the value of p that maximizes the payoff.

SECTION 4

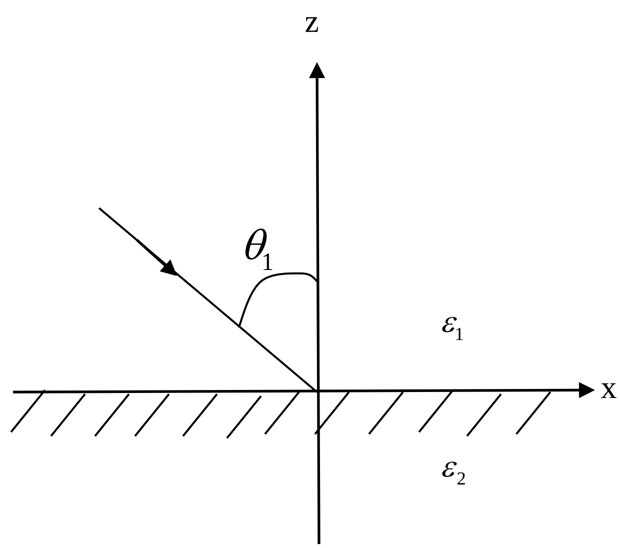
10) A plane wave having a magnetic field vector-phasor

$$\mathbf{H}(\mathbf{r}) = \hat{\mathbf{y}} H_{0y} \exp\left[-j\left(x k_1 \sin\theta_1 - z k_1 \cos\theta_1\right)\right]$$

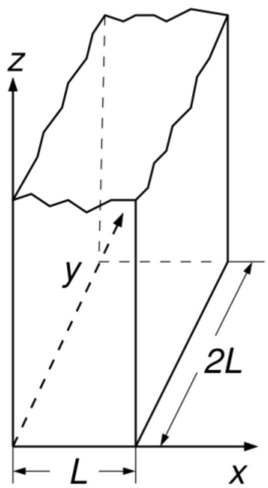
is incident on a dielectric interface at z = 0. Assume that the dielectric half space for z > 0 has permittivity ε_1 and the half space for z < 0 has permittivity ε_2, with $\varepsilon_1 = 2\varepsilon_2$. Here ω is the angular frequency and $k_1 = \omega\sqrt{\varepsilon_1 \mu_0}$.

a. Find the range of θ_1, such that there is no propagating wave in the region z < 0 (total reflection).

b. Find the value of θ_1 for which the incident wave is completely transmitted into the region z < 0, (no reflection). This is Brewster's angle.



11)



Consider a rectangular waveguide, infinitely long in the z direction, with transverse dimensions $L_x = L$ and $L_y = 2L$ as illustrated in the figure. The walls are a perfect conductor.

(a) *What are the boundary conditions for the electric field intensity (**E**) at the walls?*

(b) *Derive a general expression for electric field propagating in the waveguide and write the exact expression for the lowest order mode. (Hint: The lowest mode has the electric field in the x direction only.)*

(c) *For the lowest mode that can propagate, find the phase velocity and the group velocity.*

(d) *The possible modes of propagation in such waveguides separate naturally into two classes. What are these two classes and how do they differ physically?*

12) The radial component of the radiated power density of an infinitesimal linear dipole of length $\ell \ll \lambda$ is given by $\mathbf{W_{av}} = \hat{\mathbf{r}}(A_0 \sin^2\theta)/r^2$, where A_0 is constant, θ is the usual spherical coordinate, and $\hat{\mathbf{r}}$ is the radial unit vector. Determine

a. Radiation intensity

b. The total radiated power

c. The maximum directivity of the antenna, and

d. Directivity as a function of θ.

Note that: $\sin^3(x) = (3/4)\sin(x) - (1/4)\sin(3x)$

SECTION 5

13) A real-valued discrete-time finite-energy pulse is generated as

$$p[n] = ap[n-1] + \delta[n]$$

where $p[n] = 0$ for $n < 0$, while $\delta[n] = 1$ for $n = 0$ and $\delta[n] = 0$ for $n \neq 0$.

a. Find z-transform $P(z)$, its region of convergence and the time-domain expression for the pulse. What is the range of a?

b. The time-autocorrelation function of the pulse $p(n)$ is defined as

$$r_p[n] = \sum_{k=-\infty}^{+\infty} p[k+n]p[k]$$

for n being any integer. Find z-transform $R_p(z)$, its region of convergence and the time-domain expression for the time-autocorrelation function.

c. The pulse $p[n]$ is the input of the real-valued discrete-time system with system function $H(z)$. Let the output of the system be denoted as $y[n]$. Show that the z-transform of the time-autocorrelation function of $y[n]$ can be expressed as

$$R_y(z) = H(z)H(z^{-1})R_p(z) .$$

d. Let $H(z)$ in part (c) be

$$H(z) = 1 - z^{-M} .$$

Find the time-autocorrelation function of $y[n]$. What is the energy of $y[n]$?

14) The signal

$$s(t) = t - \frac{1}{2}$$

should be approximated on interval $[0,1)$ by a sum of three functions $g(t)$, $\sqrt{2}g(2t)$ and $\sqrt{2}g(2t-1)$ where

$$g(t) = \begin{cases} +1, & 0 \le t < \frac{1}{2} \\ -1, & \frac{1}{2} \le t < 1. \end{cases}$$

a. Find the coefficients multiplying the functions in the sum such that they provide the least mean square error approximation $\hat{s}(t)$ of $s(t)$.

b. Write the expression for the approximation $\hat{s}(t)$ obtained in part (a). Sketch the approximation.

15) Let the signal $m(t)$ be bandlimited, i.e. its Fourier transform $M(f) = 0$ for $|f| \geq B$, and the signal $c(t)$ be highpass, i.e. its Fourier transform $C(f) = 0$ for $|f| \leq B$.

a. Show that the Hilbert transform of the product $m(t)c(t)$ is the product $m(t)\hat{c}(t)$, where $\hat{c}(t)$ is the Hilbert transform of $c(t)$.

b. Find the Hilbert transform of

$$\frac{\sin(\pi 2Bt)}{\pi 2Bt}\cos(2\pi 2Bt).$$

Note: Hilbert transform of a signal $x(t)$ is defined as

$$\hat{x}(t) = \frac{1}{\pi}\int_{-\infty}^{+\infty}\frac{x(\tau)}{t-\tau}\mathrm{d}\tau.$$

This corresponds in the Fourier transform domain

$$\hat{X}(f) = -j\operatorname{sign}(f)X(f)$$

where $X(f)$ and $\hat{X}(f)$ are the Fourier transforms of $x(t)$ and $\hat{x}(t)$, respectively, and

$$\operatorname{sign}(f) = \begin{cases} +1, & f > 0 \\ 0, & f = 0 \\ -1, & f < 0 \end{cases}.$$

SECTION 6

16) Provide clear and concise answers to the following device physics questions.

a. For common device length scales, the Debye length is considered to be small in comparison to the depletion thicknesses. State two conditions in a MOSFET where the Debye length is relevant and explain why.

b. Based on the energy-momentum relationship for electrons, explain the difference between direct and indirect band-gap semiconductors. Employing this description, explain how basic Generation-Recombination (G-R) processes take place in such materials. Which type of material is a good choice for a photo-detector and why?

17) Consider a long *p*-channel MOSFET. Derive an expression for the threshold voltage and all necessary expressions. Carefully explain your basic assumptions and considerations. The following physical mechanisms and parameters need to be incorporated into the model:

(i) Insulator thickness t_{OX} and its dielectric constant ε_{OX}.

(ii) The metal-semiconductor work function difference Φ_{MS}.

(iii) Silicon dielectric constant ε_S and substrate doping N_D.

(iv) Substrate to source potential V_{SB}.

(v) Fixed insulator charge Q_f and Si-Insulator interface charge Q_{it}.

What is the applicability of the above derivation to a short-channel device? Carefully state how the threshold voltage may change as the length of the device decreases.

18) The drift-diffusion model states that the hole current density, J_p, is the sum of the drift and diffusion currents. Namely, $J_p = -q\,(\mu_p\, p\ \nabla\phi + D_p\ \nabla p\,)$, where, q is the electronic charge, μ_p is the hole mobility, p is the hole charge density, ϕ is the electrostatic potential, and D_p diffusion constant.

a. State the condition for Thermal Equilibrium and show that under thermal equilibrium the hole charge density can be expressed as follows: $p = n_i\, exp(-\phi\,/v_T)$, where v_T is thermal voltage and n_i is the intrinsic charge density.

b. The hole quasi-Fermi potential, ψ_p, is a useful parameter and it is defined as follows: $p = n_i\, exp[-(\phi - \psi_p\,)/v_T]$. Derive an expression for the hole current density in terms of p and ψ_p. Based on this derivation explain how a *pn*-junction can have a built-in electric potential across it, yet exhibit zero terminal current and zero measured voltage drop across its terminals at thermal equilibrium.

SECTION 7

19) Consider a slotted ALOHA network with a total arrival rate of 60 frames per second. Suppose the length of each time slot is 50 milliseconds.

a. What is the probability of a successful frame transmission in this network?

b. On average, how many frames are successfully transmitted per second?

c. Repeat Parts (a) and (b) for a Pure ALOHA network with the same parameters. Is the throughput higher or lower than the slotted ALOHA? Why?

20) Suppose three hosts are connected by two links as shown in the figure. Host A sends packets to Host C and Host B serves merely as a relay. However, as indicated in the figure, they use different ARQ protocols for reliable communication (i.e., Go-Back-N for A→B and Selective Repeat for B→C). Notice that B is a regular host running both as a receiver (to receive packets from A) and sender (to forward A's packets to C). B's receiver immediately relays incoming packets to B's sender. Assume that round-trip-time (RTT) for a single packet is slightly longer than the 3 packet transmission times. A packet timeout interval for Go-Back-N protocol is slightly longer than RTT, while send and receiver window sizes of Selective Repeat are both four packets.

a. Draw **side-by-side** the timing diagrams for A→B and B→C transmission up to the time where the first 7 packets from A show up on C. Assume that the 2nd and 5th packets arrive in error to B on their first transmission, and the 5th packet arrives in error to C on its first transmission.

b. Should ACKs be sent from C back to A? Why?

21) A token ring LAN interconnects M=20 stations using a star topology in the following way. All the input and output lines of the token-ring stations interfaces are connected to a cabinet where the actual ring is placed. Suppose that the distance from each station to the cabinet is 100 meters and that the ring latency per station is 8 bits. Assume that frames are 1250 bytes and that the ring speed is 25Mbps.

a. What is the maximum possible arrival rate that can be supported if stations are allowed to transmit an unlimited number of frames/tokens?

b. What is the maximum possible arrival rate that can be supported if stations are allowed to transmit 1 frame/token using single-frame operations?

SECTION 8

22) Explain different mechanisms of regulation of blood pressure and blood flow.

23) Discuss hormonal regulation of calcium.

24) Describe (including relevant drawings) the structure and function of the vestibular complex.

SECTION 9

25) A three-phase 50MVA, 20kV, 60Hz salient pole synchronous machine has parameters Xd = 1.0 pu, Xq = 0.7 pu and Ra~0. The machine delivers 40MW at 0.85 lagging power factor to an infinite busbar.

Calculate the excitation voltage and the power angle. Draw the phasor diagram. (Hint: use per unit values and give your answers in pu)

26) A wind turbine is to be designed with an electrical power output of 3.5 MW. The rated upwind free wind speed is 14 m/s. Determine the length of the rotor blades in meters and the rotational speed of the rotor in rev/min if the tip-speed ratio is determined as 0.45 at the maximum Power Coefficient value of 7.5

Use $\rho_{air} = 1.225 \frac{k_g}{m^3}$

27) A 200MVA, 20kV, 60-Hz round-rotor synchronous generator has an Inertia constant H = 5s.

Displayed on the axes below are Torque/Angle characteristics for various faults occurring on a double circuit transmission line when connected between a synchronous generator and an infinite busbar. Using the Equal Area Criterion, determine the critical switching times for both a 3φ fault and a L-L fault when the input torque from the turbine is 1.0 pu as shown in the diagram.