THE GEORGE WASHINGTON UNIVERSITY
School of Engineering and Applied Science
Department of Electrical and Computer Engineering

Preliminary Examination

Fall 2001

General Instructions

Read carefully before starting.

i) Solve five (5) problems in accordance with the following selection rule: choose two (2) problems from any one of the seven sections, two (2) problems from one of the other sections, and the remaining problem from any section, including from either of the sections previously selected.

ii) Solve each problem in a separate blue book. Write the section number, problem number, and your student number on the front of each blue book. **DO NOT WRITE YOUR NAME ON THE BLUE BOOK.**

iii) Submit solutions to only five (5) problems. Use only **ONE** blue book per problem.

iv) For each problem, make a special effort to give the answers in a clear form.

v) The exam will begin at 9:00 a.m. and end at 2:00 p.m.

!!GOOD LUCK!!

Section 1

1. Consider the matrix

$$A = \begin{pmatrix} 4 & -1 & 1 \\ -1 & 4 & -1 \\ 1 & -1 & 4 \end{pmatrix}.$$

a) Given that one eigenvalue of A is $\lambda = 6$, find the remaining eigenvalues.

b) Find three linearly independent eigenvectors of A.

c) Find an *orthogonal* matrix Q and a diagonal matrix Λ, so that $A = Q\Lambda Q^T$.

2. Consider this sequence: $G_0 = 0, G_1 = 1$ and $G_{k+2} = {(G_k + G_{k+1})}/{2}$. (So G_{k+2} is the average of the previous two numbers G_k and G_{k+1}.) This problem will find the limit of G_k as $k \to \infty$.

a) Find a matrix A which satisfies

$$\begin{bmatrix} G_{k+2} \\ G_{k+1} \end{bmatrix} = A \begin{bmatrix} G_{k+1} \\ G_k \end{bmatrix}.$$

b) Find the eigenvalues and eigenvectors of A.

c) Write $A^k = S\Lambda^k S^{-1}$, where Λ is a diagonal matrix. You do **not** need to multiply this out to get a single matrix.

d) Find the limit as $k \to \infty$ of the numbers G_k.

3. Consider the differential equation (ODE)

$$\frac{du}{dt} = Au \quad \text{with} \quad A = \begin{bmatrix} -2 & 3 \\ 2 & -3 \end{bmatrix} \quad \text{and} \quad u(0) = \begin{bmatrix} 5 \\ 0 \end{bmatrix}.$$

a) Find the eigenvalues and eigenvectors and diagonalize to $A = S\Lambda S^{-1}$.

b) Solve for $u(t)$ starting from the given $u(0)$.

c) Compute the matrix e^{At} using S and Λ.

d) As t approaches infinity, find the limits of $u(t)$ and e^{At}.

Section 2

1. Consider a hypothetical little-endian processor with the following specs:

- 32-bit external address bus, 32-bit external data bus, non-multiplexed
- no on-chip cache
- 32-bit internal address registers, 8-bit internal data registers
- synchronous external bus, requiring 3 clocks per bus cycle.

This processor is connected to an 8-bit memory, whose data lines are connected to the lower byte-lane of the processor's external data bus.

Assume that the 32-bit stack pointer points to location 12345678 (which is the next free location in the stack to receive data), and that the "return address" saved on top of the stack was ABCDEF12.

Show the timing diagram when the program executes the already fetched instruction RET (return from subroutine). Show the most important control signals (such as read/write, memory/IO, byte-enables, operand size indicators, external port size indicator) placed on the control bus and the actual hexadecimal values transferred on each byte lane of the external address and data buses.

State all your assumptions.

2. A virtual memory system has a virtual address space of 8K words, a main memory of 4K words, and page and block (or page frame) sizes of 1K words. The following page references occur by a program during a given time interval. (Only page changes are listed. If the same page is referenced again, since it does not cause a page fault it is not listed twice.)

4 2 0 1 2 6 1 4 0 1 0 2 3 5 7

Determine the four pages that are resident in main memory after each page reference if the replacement algorithm is FIFO.

3. Consider a 32-bit processor (32-bit addresses) and a cache with the following specs:

- Block or cache line size = 128 bits.
- Validity is done on a doubleword (32-bit) basis.
- Cache fills can be either on a doubleword basis or on a whole cache line basis.
- It distinguishes among 4 different access spaces.
- It is 32-set 4-way set-associative cache.
- It supports only the write-back policy.

Give the block diagram of the cache's structure and indicate how the physical address fields are to be interpreted by the cache controller.

Section 3

1. When answering the following questions on MOS devices please provide clear and concise answers.

   a) In a MOSFET device the threshold voltage is a measure of channel inversion. In this context, describe the influence of the substrate bias and what is commonly referred to as the “body effect”.

   b) Describe the “short-channel effect” in MOS transistors. How does this phenomenon vary the threshold voltage of a device?

   c) The scaling down of MOSFET device dimensions naturally involves decreasing the thickness of the gate dielectric. One aspect of this process is to identify low dielectric materials to replace the gate SiO_2. Why is it beneficial to reduce the dielectric constant of the insulating layer between the gate and the active region of the MOS device?

   d) Describe one way to construct and operate a MOS device that can be employed as a voltage controlled capacitor.

2. Consider an ideal long-base abrupt *pn*-junction silicon diode with uniform cross section and constant doping on both sides of the junction. The diode is made from $N_a = 1.5x10^{16}$ cm^{-3} *p*-type and $N_d = 3x10^{16}$ cm^{-3} *n*-type materials in which the minority-carrier lifetimes are $\tau_n = 10^{-6}$ s and $\tau_p = 10^{-8}$ s, respectively. (Effects within the space-charge region are negligible and that minority carriers flow only by diffusion mechanism in the charge neutral regions.)

a) What is the value of the built-in potential?

b) Calculate the density of the minority carriers at the edge of the space-charge region when the applied voltage is 0.6 V.

c) Sketch the majority and minority carrier currents as functions of distance from the junction, under the bias in part (b).

3. Formulate the Poisson and the steady-state drift-diffusion current continuity equations in terms of the following state variables: Electrostatic potential, electron and hole quasi-Fermi potentials: (ϕ, φ_n, φ_p). State any advantages and/or disadvantages of employing the quasi-Fermi potentials.

Section 4

1. A Hertzian dipole situated at the origin and oriented along the x-axis carries a current of $I_1 = I_0 \cos \omega t$. A second Hertzian dipole having current $I_2 = I_0 \sin \omega t$ is also situated at the origin but oriented along the z-axis. Find the polarization of the radiated electric field (far field) at

a) a point on the x-axis,

b) a point on the z-axis,

c) a point on the y-axis, and

d) a point on the line $x = y$, $z = 0$.

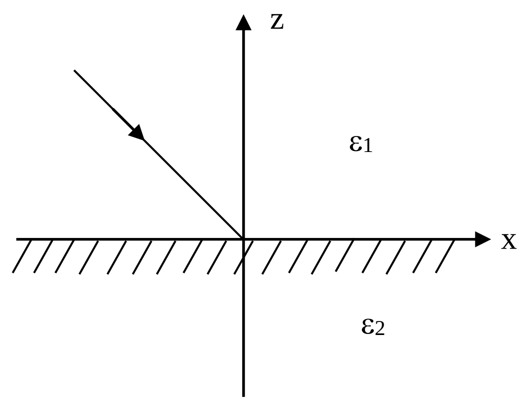
2. A plane wave having a magnetic field vector-phasor

$$\boldsymbol{H}(\boldsymbol{r}) = \hat{\boldsymbol{y}}\, H_{0y} \exp\left[-j\left(xk_1 \sin\theta_1 + zk_1 \cos\theta_1\right)\right]$$

is incident on a dielectric interface at z = 0. Assume that the dielectric half space for z > 0 has permittivity ε_1 and the half space for z < 0 has permittivity ε_2, with $\varepsilon_1 = 2\varepsilon_2$.

a) Find the range of θ, such that there is no propagating wave in the region z < 0 (total reflection).

b) Find the value of θ for which the incident wave is completely transmitted into the region z < 0, (no reflection). This is Brewster's angle. Here ω is the angular frequency and $k_1 = \omega\sqrt{\varepsilon_1 \mu_0}$.



3. The complex phasor (time dependence exp (+jωt)) representing the electric field of a plane wave in free space is

$$\boldsymbol{E} = E_0 e^{\frac{j\pi}{2}} \left(\hat{\boldsymbol{x}} - \hat{\boldsymbol{y}}\right) e^{-j\left(\frac{\omega}{c}\right)\frac{(x+y)}{\sqrt{2}}}$$

where ω is the angular frequency, c is the free space velocity of light, and E_0 is a real constant.

a) What is the direction of propagation of the wave?

b) What is the polarization of the wave?

c) Find $\boldsymbol{H}(x, y, z, t)$.

Section 5

1. A transmitter sends one out of two signals: a +1V DC signal with probability p_+ or a –1V DC signal with probability p_-. When the +1V DC signal is transmitted, a receiver receives one out of three signals: a +1V DC signal with probability α, a 0V DC signal with probability 2ε, or –1V DC signal with probability ε. When the –1V DC signal is transmitted, the receiver receives one out of these three signals: a –1V DC signal with probability β, a 0V DC signal with probability 3ε, or a +1V DC signal with probability 2ε.

   a) Find the probability that the +1 DC signal is transmitted given that the +1V DC signal or the 0V signal is received.

   b) Find the probability that the –1 DC signal is transmitted given that the –1V DC signal or the 0V signal is received.

   c) Assuming $p_+ = p_-$, find the range of ε for which the probability found in (c) is larger than the probability found in (b).

2. A meeting of N persons starts when all of them arrive at the meeting place. Each person arrives randomly and independently of others with the probability density function of arrival time $f_T(t), t \geq 0$.

a) Find the probability density function that the meeting will start at t, $t \geq 0$ as a function of $f_T(\cdot)$.

b) Assume that the arrival times are uniformly distributed between 0 and 1 hour. Find the probability that the meeting will start within the first half hour.

c) Assume that the arrival times are exponentially distributed, i.e. $f_T(t) = 2e^{-2t}$. Find the probability that the meeting will start within the first half hour.

d) Compare the probabilities found in (b) and (c).

3. In a square wave the n-th transient occurs at instant $T(n) = nT_0 + W(n)$, where $W(n)$ are independent random variables identically distributed with zero mean, variance σ^2, and $-\frac{T_0}{2} < W(n) < \frac{T_0}{2}$. (This is a model for the jitter. If $W(n) = 0$, the square wave is periodic with period T_0.)

a) Is $T(n)$ a wide sense stationary random process? Justify your answer.

b) The intertransient times are measured as $I(n) = T(n) - T(n-1)$. Is $I(n)$ a wide sense stationary random process? Justify your answer.

c) Find the power spectral density of $I(n)$. (You may need the discrete-time Fourier transform: $F(\omega) = \sum_{n=-\infty}^{+\infty} f(n)e^{-j\omega n}$.)

Section 6

1. The signal

$$f(t) = \cos\left(\frac{2\pi t}{T}\right)$$

is to be approximated in the interval $0 \le t \le T$ in the LMS sense by N functions defined by

$$\phi_n(t) = p_\delta(t - n\delta);\ n = 0,1,2,...N-1,$$

where

$$p_\delta(t) = \begin{cases} \frac{1}{\sqrt{\delta}}\ ;\ 0 < t \le \delta, \\ 0\ ;\ otherwise, \end{cases}$$

and T/N.

a) Denoting the partial sum by

$$f(t) = \sum_{n=0}^{N-1} \hat{f}_n \phi_n(t),$$

find the coefficients $\hat{f}_n$.

b) Compute the LMS error as a function of T and N.

c) Prove that the LMS error approaches 0 as $N \to \infty$.

2. The input to a linear system gives rise to the output $\dfrac{1}{2+i\omega}e^{i8\omega t^2}$,
for all real .

a) Is the system time invariant? Justify your answer.

b) Find the impulse response.

c) Is the system causal?

d) Find the output when the input is the pulse

$$p(t)=\begin{cases}1; |t|<2,\\ 0; |t|\geq 2.\end{cases}$$

3. The Fourier transform of a bandlimited function $f(t)$ is given by

$$F(\omega) = \begin{cases} \dfrac{1}{5/4 - \cos(\pi\omega/\Omega)} \; ; \; |\omega| \le \Omega, \\ 0 \; ; \; otherwise \end{cases}$$

The function is sampled at intervals $\Delta t = \pi/\Omega$.

a) Find the Z transform of the sampled sequence.

b) Find the samples $f(n\Delta t)$.

c) Suppose the function is sampled at intervals $\Delta t' = 2\pi/\Omega$ and its Fourier transform is reconstructed using the formula

$$\hat{F}(\omega) = \Delta t' \sum_{n=-\infty}^{\infty} f(n\Delta t') e^{-i\omega n\Delta t'}$$

Compute and sketch $\hat{F}(\omega)$ in the interval $|\omega| \le \Omega$.

Section 7

1. An NRZ baseband code with voltage pulses of constant amplitude +/- 5 volts and duration 2 milliseconds is used to transmit a binary data sequence.

   a) What is the rate of transmitted data in bits per second?

   b) What is the amount of power in the transmitted signal?

   c) What is the amount of energy in the signal per transmitted binary digit?

   d) How much of the total energy in a single transmitted binary digit is contained in the frequency range from 0 to 500 Hertz?

   e) For the NRZ code, find the energy spectral density of a single pulse by use of the Fourier Transform.

   f) Repeat part (e) for a Bipolar code pulse of similar amplitude and time duration.

2. Let S be the expected transmission time for sending a data frame and let F be the feedback transmission time for sending an ACK or NAK frame. Let P be the expected one–way propagation and processing delay.

   a) Find the expected time T between successive frame transmission in a stop-and-wait system.

   b) Let x be the probability of frame error in a data frame and y be the probability of frame error in an ACK or NAK frame. Find the probability z that a data frame is correctly received and correctly acknowledged on a given transmission.

   c) Show that 1/z is the expected number of times a data frame must be transmitted for a stop-and-wait system.

   d) Combining parts (a), (b), and (c), find the expected time required per data frame. Evaluate for $S = 1$, $F = P = 0.1$, and $x = y = 10^{-3}$.

e) Four signals m1(t), m2(t), m3(t), and m4(t) are bandlimited to 3.6 kilohertz, 1.2 kilohertz, 1.2 kilohertz, and 1.2 kilohertz, respectively. These signals are to be transmitted in a time division multiplex arrangement.

a. Design a multiplexor that will accommodate all four signals, assuming that each signal is quantized to 1024 levels.

b. Determine the frame length of the multiplexor output.

c. Determine the required bandwidth of the transmission channel.