

**THE GEORGE WASHINGTON UNIVERSITY**  
**School of Engineering and Applied Science**  
**Department of Electrical and Computer Engineering**

**Preliminary Examination - Fall 2016**

**Thursday, October 13, 2016**

**General Instructions. Read carefully before starting.**

**Solve 5 problems in all; at most 2 questions may be selected from the same section.**

Candidates registered in the following Focus Areas must answer at least two of the five questions from the relevant section as follows:

Computer Architecture & High Performance Computing:	Section 1
Communications & Networks:	Section 3
Electrical Power & Energy:	Section 8
Electromagnetics, Radiation Systems & Microwave Engineering:	Section 4
Electronics, Photonics & MEMS:	Section 6
Signal & Image Processing, Systems & Controls:	Section 3

Please write your name, student number and Registered Focus Area below:

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**Student Name**

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**Student Number**

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**Focus Area in which Registered**

Solve each problem in a separate blue book. **Write the section number, problem number, and your student number on the front of each blue book.**

**DO NOT WRITE YOUR NAME ON THE BLUE BOOK.**

Submit solutions to **only** five (5) problems. Use only **ONE** blue book per problem.

For each problem, make a special effort to give the answers in a clear form.

The exam will begin at 10:00 a.m. and end promptly at 3:00 p.m.

**Only Calculators provided by the department at the examination will be allowed. Personal items including cell phones and other electrical devices must be relinquished prior to the start of the examination.**

**This is a CLOSED BOOK, CLOSED NOTES EXAMINATION**

## Section 1

1.

A. Precisely define each of the following terms in no more than two lines

- i. I/O operation: \_\_\_\_\_
- ii. Memory mapped I/O: \_\_\_\_\_
- iii. I/O Mapped I/O: \_\_\_\_\_
- iv. DMA: \_\_\_\_\_

B. Design a 16x4 memory subsystem with low-order interleaving using 8x2 memory chips for a computer system with an 8-bit address bus. You can use any logic gates, decoders or

C. multiplexers as necessary.

2. Assume a five-stage single-pipeline micro architecture with Fetch, Decode/Register Read, Execute, Memory Access and Register Writeback and Result forwarding support between the pipeline stages. The operand value should be available to the instruction before the Execute stage. Using a multicycle pipeline diagram, demonstrate the progress of each instruction on every clock cycle until the last instruction of the first iteration of the following loop. Calculate the total number of cycles

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Loop: LH  R2, 6(R1)
      SUB R6, R6, R2
      SH  R6, 6(R1)
      BEZ R6, Loop
```

3. Machine X has 64 bit virtual address and 40 bit physical address. The page size is 16KB. The L1 cache is directed mapped 16KB, and L2 is four-way set associative 4MB. Virtually indexed and physically tagged cache. 2-way associative TLB with 256 entries. Please draw a diagram to clearly explain the translation from a virtual address to a physical address, and the interactions to the TLB and L1/L2 Cache. Indicates how many bits are used for each component.

## Section 2

4. Do the following systems of equations have a solution? If so, find all solutions.

$$1 = x_1 + 2x_3 + 2x_4$$

a)  $0 = 2x_1 + x_2 - 3x_3 - 2x_4$

$$0 = 3x_1 + 2x_2 + 3x_3 + 2x_4$$

$$? \quad 1 = x_1 - x_2 - x_3 + x_4$$

b)  $-2 = -2x_1 + 5x_2 - x_3 - x_4$

$$? \quad 3 = x_1 + 3x_2 + 6x_3 - 9x_4$$

5. Given the system  $S=\{\mathbf{A},\mathbf{b},\mathbf{c}\}$  , where

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix}; \quad \mathbf{b} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}; \quad \mathbf{c} = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}$$

- a. Is the system internally stable?
- b. Is the system BIBO stable?
- c. It is desired that the response of the system have a time constant  $\tau = 1 \text{ s}$  and a natural frequency of oscillation  $\omega_n = 1 \text{ radian} / \text{s}$ . Can you do it using state feedback? If, yes, do it and show all your steps. If no, explain in detail why not.



6. Solve the linear time-invariant nonhomogeneous vector differential equation for  $x(t)$

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 1 & 1 \\ -1 & 0 & 2 \end{bmatrix} \mathbf{x} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} t$$

## Section 3

7. Assume  $X(t)$  is a stationary white noise process with autocorrelation function

$$R_X(\tau) = q\delta(\tau)$$

$X(t)$  is applied to a linear system with impulse response function

$$h(t) = e^{-ct} \text{ for } t \geq 0 \text{ and } c > 0$$

Find the autocorrelation function of the output  $Y(t)$  of the system.

8. Let  $N$  be the total number of messages in two identical M/M/1 queues, each operating independently with input rate  $\lambda$  and service rate  $\mu$ .
- a) Determine the steady-state probability of  $n$  messages in the system as a whole.
  - b) Determine the average number of messages in the system as a whole.

9. The random process  $X(t) = \exp(at)$  consists of a family of exponential sample functions determined by the value of the continuous random variable  $a$ . In terms of the density function  $f(a)$  of  $a$ , determine the expected value of  $X(t)$ , its autocorrelation function  $R(t_1, t_2)$  and its first order density function  $f(x, t)$ .

## Section 4

10. A uniform sinusoidal plane wave in free space with the following phasor expression for the electric intensity

$$\mathbf{E}_i(x, z) = \hat{\mathbf{y}}10e^{-j(6x+8z)}$$

is incident on a perfectly conducting plane located at  $z=0$ .

- a) Find the frequency and wavelength of the wave.
- b) Write the time dependent expression for the incident electric and magnetic fields.
- c) Determine the angle of incidence.
- d) Find the time harmonic electric and magnetic fields of the reflected wave.

11. A 3GHz y-polarized uniform plane wave propagates in the  $+x$  direction in a non-magnetic medium having a relative dielectric constant 2.5 and a loss tangent  $10^{-2}$ .

a) Determine the distance over which the amplitude of the propagating wave will be cut in half.

b) Determine the characteristic impedance, the wavelength, the phase velocity, and the group velocity of the wave in the medium,

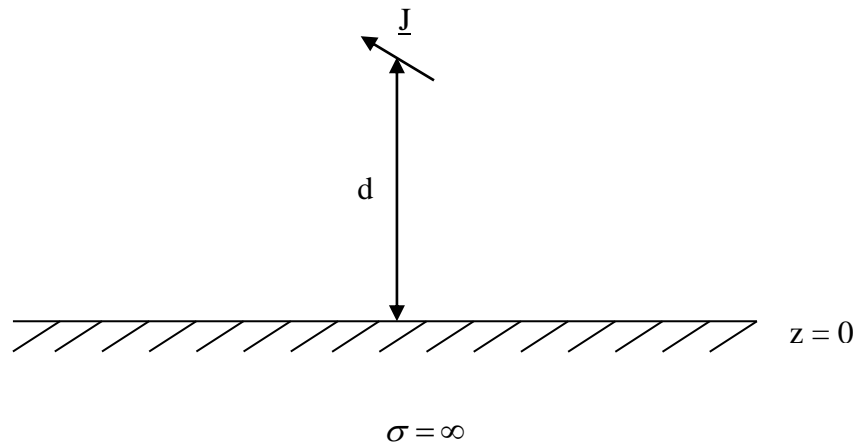
c) Assuming  $\mathbf{E} = \hat{\mathbf{y}}50\sin(6\pi 10^9 t + \pi/3)$  at  $x=0$ , write the time varying expression for  $\mathbf{H}$  for all  $t$  and  $x$ .



12. A short dipole of strength  $\mathbf{J}$  is located above a perfectly conducting plane located at  $z = 0$ . The dipole strength is

$$\mathbf{J} = J_0 (\cancel{\hat{x}} + \hat{z})$$

where  $\hat{x}$  and  $\hat{z}$  are unit vectors in the  $x$  and  $z$  directions and  $J_0$  is a constant.. The distance of the dipole from the conductor is “ $d$ ” as shown in the figure below.



- a) Find the location, direction and magnitude of the image.
- b) Find the radiation pattern.

## Section 5

13. The Wigner time-frequency distribution for a signal  $s(t)$  is defined as

$$W_s(t, f) = \int_{-\infty}^{+\infty} s^*\left(t - \frac{\tau}{2}\right) s\left(t + \frac{\tau}{2}\right) e^{-j2\pi f\tau} d\tau$$

where  $s^*(t)$  is complex conjugated  $s(t)$  and  $j = \sqrt{-1}$ .

(a) Show that

$$\int_{-\infty}^{+\infty} W_s(t, f) df = |s(t)|^2.$$

(b) Show that

$$\int_{-\infty}^{+\infty} W_s(t, f) dt = |S(f)|^2$$

where  $S(f)$  is the Fourier transform of  $s(t)$ .

(c) Show that

$$\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} W_s(t, f) dt df = E_s$$

where  $E_s$  is the signal energy, i.e.,  $E_s = \int_{-\infty}^{+\infty} |s(t)|^2 dt$ .

(d) Let  $g(t) = s(t - t_0)$ , show that

$$W_g(t, f) = W_s(t - t_0, f).$$

(e) Let  $h(t) = s(t) e^{j2\pi f_0 t}$ , show that

$$W_h(t, f) = W_s(t, f - f_0).$$

14. The signal

$$s(t) = t - \frac{1}{2}$$

should be approximated on interval  $[0,1)$  by a sum of three functions  $g(t)$ ,  $\sqrt{2}g(2t)$  and  $\sqrt{2}g(2t-1)$  where

$$g(t) = \begin{cases} +1, & 0 \leq t < \frac{1}{2} \\ -1, & \frac{1}{2} \leq t < 1. \end{cases}$$

- (a) Find the coefficients multiplying the functions in the sum such that they provide the least mean square error approximation  $\hat{s}(t)$  of  $s(t)$ .
- (b) Write the expression for the approximation  $\hat{s}(t)$  obtained in part (a). Sketch the approximation.

15. Assume that  $x[n]$  is a real-valued discrete-time signal and  $h[n]$  is a real-valued impulse response of linear time-invariant discrete-time system. Let  $y_1[n] = x[n] \star h[n]$  represent filtering the signal in the forward direction, where  $\star$  stands for convolution. Now filter  $y_1[n]$  backward to obtain  $y_2[n] = y_1[-n] \star h[n]$ . The output is then given by reversing  $y_2[n]$  to obtain  $y[n] = y_2[-n]$ .
- (a) Show that this set of operation is equivalently represented by a filter with impulse response  $h_o[n]$  as  $y[n] = x[n] \star h_o[n]$  and express  $h_o[n]$  in terms of  $h[n]$ .
- (b) Show that  $h_o[n]$  is an even signal and find the phase response of a system having impulse response  $h_o[n]$ . Is the system causal?
- (c) Let  $H(z)$  and  $H_o(z)$  be  $z$ -transforms of  $h[n]$  and  $h_o[n]$ , respectively, and that  $h[n]$  is causal. If  $H(z) = 1/(1 - 0.9z^{-1})$  find  $H_o(z)$ , the region of convergence of  $H_o(z)$ , and  $h_o[n]$ .
- (d) Repeat (c) if  $H(z) = 1 - 0.9z^{-1}$ .

## Section 6

**16. Provide brief explanations and justifications to the following statements:**

a) Derive the Einstein Equations. State your assumptions.

b) Assume Silicon at room temperature. Where is  $E_F$  for  $n = 5 \times 10^{19} \text{ cm}^{-3}$ ?

c) Explain the difference of a direct vs. indirect bandgap semiconductor. What implications does do these have for photo-absorption (carrier generation) and emission processes (carrier recombination)?

**17. Mobility**

- a) How is Mobility defined?
- b) What units does it have?
- c) Briefly describe two microscopic mechanisms that define the properties in mobility (include temperature).
- d) Describe the interrelationship between mobility, effective mass, and band structure. How can you improve the mobility?
- e) Why is it important to increase mobility for many (opto)electronic devices?



### 18. PN Junction Electrostatics

Consider a pn-junction with uniform doping densities in the p- and n- regions of  $N_a = 1 \times 10^{17} \text{ cm}^{-3}$  and  $N_d = 5 \times 10^{16} \text{ cm}^{-3}$ , respectively. The depletion region approximation can be employed to describe the charge profile.

- a) Under thermal equilibrium conditions, compute and plot the net charge density,  $\rho(x)$ , the electric field and the electrostatic potential. Clearly indicate the length of the depletion region on each side of the junction.
- b) Plot the energy-band diagram based on the above calculations. Label all
- c) How can you increase the current in a PN junction?
- d) How does the IV curve of a PN junction differ from that of the MS junction? What device implications does such difference have?

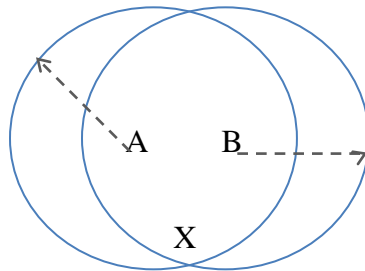
## Section 7

19. Suppose that computer A is sending a file to computer B using a private Ethernet with no other computers using it. They are connected by 100m of wire. Bits travel at the rate of  $2 \times 10^8$  m/s in this wire. Suppose the Ethernet has a bandwidth of  $10^9$  bits per second.
- What is the propagation delay of the connection?
  - How long would it take for  $10^6$  bits to finish traveling from computer A to computer B?
  - Suppose you measured the time it takes to transmit a  $10^6$  bit file from computer A to computer B. In practice, the time you measure is longer than time you calculated above. What factors could have resulted in this?

20. Consider a data link layer with the following parameters: Frame transmission time at the sender is  $t_f=20$  microseconds. ACK or NAK transmission time at the receiver is  $t_{ack}=10$  microseconds. Link propagation delay on both directions is  $t_{prop}=25$  microseconds. Suppose frame processing time at both sender and receiver is negligible, i.e.,  $t_{proc}=0$ . Finally, overall round-trip probability of frame error on the link is  $r=0.04$ .

- a. Assume that for the Stop-and-wait ARQ scheme, the TIMEOUT at the sender is chosen optimally. What is the resulting throughput (frames/second)?
- b. In the Go-Back-N ARQ scheme, if the link is error free, what is the minimum window size  $N$  that is able to keep the link busy?
- c. Choose window size in Part b and now consider the link error probability  $r=0.04$ . What is the throughput (frames/second) of the Go-Back-N ARQ scheme?

21. The figure below shows a WIFI network with an access point, X and two hosts, A and B. The large circles indicate the coverage areas of the two hosts. The coverage area for X is not shown, but you may assume that it includes both hosts. Assume RTS/CTS are not used.



Suppose X is transmitting a packet at time 0 and finishes sending it at time  $100 \mu\text{s}$ . Also, A gets a packet to send at time 50 that takes  $100 \mu\text{s}$  to send and is assigned a backoff timer of 100. B gets a packet at time 90 that takes  $150 \mu\text{s}$  and is assigned a backoff timer of 150.

- a) For each of the three hosts, what time do they successfully deliver their packets? You may ignore the inter-frame spacing and the time required for ACKs.
- b) Now, suppose RTS/CTS is enabled. In this case, what time do they successfully deliver their packets? You may assume that the time needed to send RTS, CTS and A

## Section 8

22. A three-phase 250MVA, 20kV, 60Hz salient pole synchronous machine has parameters  $X_d = 1.1$  pu,  $X_q = 0.6$  pu and  $R_a \approx 0$ . The machine delivers 230MW at 0.9 lagging power factor to an infinite busbar.

Calculate the excitation voltage and the power angle. Draw the phasor diagram.  
(Hint: use per unit values and give your answers in pu)

23. A wind turbine is to be designed with an electrical power output of 7.0 MW. The rated upwind free wind speed is 13 m/s. Determine the length of the rotor blades and the height of the supporting tower in meters and the rotational speed of the rotor in rev/min if the tip-speed ratio whose value as 7.0 determines the maximum Power Coefficient of 0.45. Use the density of air as  $1.225 \text{ kg/m}^3$



24. A 450MVA, 20kV, 60-Hz round-rotor synchronous generator has an Inertia constant  $H = 5\text{s}$ .

Displayed on the axes below are Torque/Angle characteristics for various faults occurring on a double circuit transmission line when connected between a synchronous generator and an infinite busbar. Using the Equal Area Criterion, determine the critical switching times for both a 3 $\phi$  fault and a Line to Line fault when the input torque from the turbine is 1.0 pu as shown in the diagram.

