THE GEORGE WASHINGTON UNIVERSITY
School of Engineering and Applied Science
Department of Electrical and Computer Engineering

Preliminary Examination

Fall 2004

General Instructions

Read carefully before starting.

i) Solve five (5) problems in accordance with the following selection rule: choose two (2) problems from any one of the seven sections, two (2) problems from one of the other sections, and the remaining problem from any section, including from either of the sections previously selected.

ii) Solve each problem in a separate blue book. Write the section number, problem number, and your student number on the front of each blue book. **DO NOT WRITE YOUR NAME ON THE BLUE BOOK.**

iii) Submit solutions to only five (5) problems. Use only **ONE** blue book per problem.

iv) For each problem, make a special effort to give the answers in a clear form.

v) The exam will begin at 9:00 a.m. and end at 2:00 p.m.

!!GOOD LUCK!!

Section 1

1. Consider the typical road network of the figure below. It represents an area of downtown Jacksonville, Florida. The streets are all one-way with the arrows indicating the direction of traffic flow. The flow of traffic in and out of the network is measured in vehicles per hour (vph). The figures given here are based on midweek peak traffic hours, 7 a.m. to 9 a.m. and 4 p.m. to 6 p.m. An increase of 2 percent in the overall flow should be allowed for during the Friday evening traffic flow. Construct a mathematical model that can be used to analyze this network.

   Assume that the following traffic law applies.

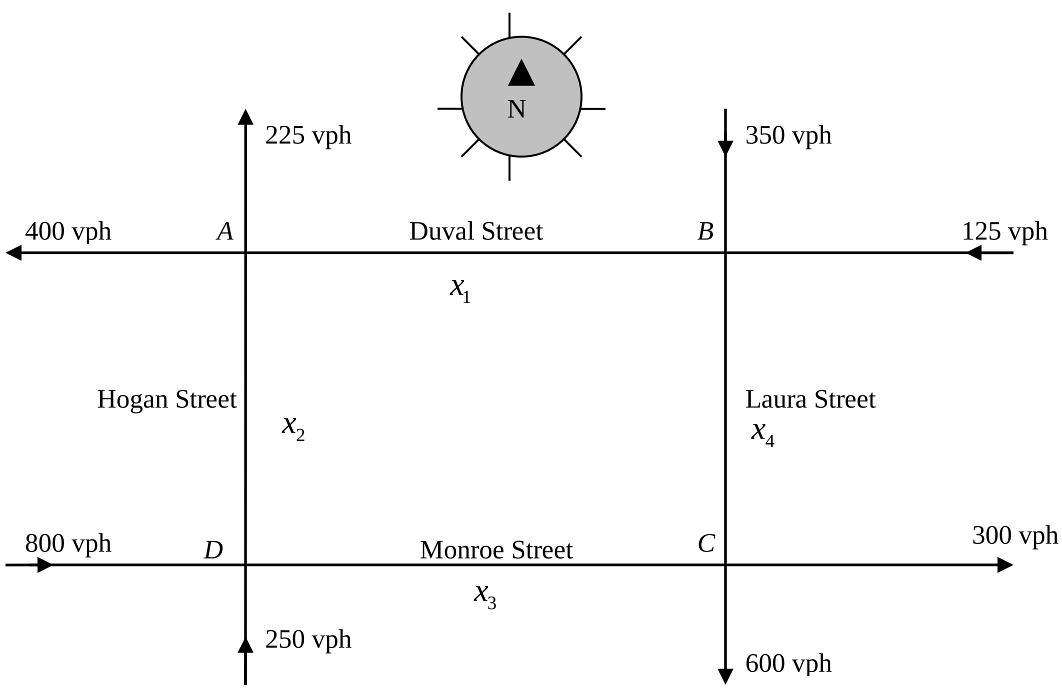
   All traffic entering an intersection must leave that intersection.

   This conservation of flow constraint (compare it to the first of Kirchhoff's laws for electrical networks) leads to a system of linear equations.

   a) Set up and solve the resulting system.

   b) Describe your conclusions about the solution.

   c) For a roadwork on Monroe Street, x_3 should be minimized. Estimate how much should be x_4 without causing "negative flow" along other roads.



Downtown Jacksonville, FL

2. Let

$$A = \begin{bmatrix} 1 & -1 & 4 \\ 1 & 4 & -2 \\ 1 & 4 & 2 \\ 1 & -1 & 0 \end{bmatrix}.$$

a) Describe the 4 subspaces associated with this matrix (rank, dimension, basis vectors.

b) Find an orthonormal basis for the column space of A.

3. The city of Mawtookit maintains a constant population of 300,000 people from year to year. A political science study estimated that there were 150,000 Independents, 90,000 Democrats, and 60,000 Republicans in the town. It was also estimated that each year 20 percent of the Independents become Democrats and 10 percent become Republicans. Similarly, 20 percent of the Democrats become Independents and 10 percent become Republicans, while 10 percent of the Republicans defect to the Democrats and 10 percent become Independents each year. Let

$$\mathbf{x} = \begin{bmatrix} 150,000 \\ 90,000 \\ 60,000 \end{bmatrix}$$

and let $\mathbf{x}^{(1)}$ be a vector representing the number of people in each group after 1 year.

a) Find a matrix A such that $A\mathbf{x} = \mathbf{x}^{(1)}$.

b) Show that $\lambda_1 = 1.0, \lambda_2 = 0.5, \textit{and } \lambda_3 = 0.7$ are the eigenvalues of A, and factor A into a product SDS^{-1}, where D is diagonal.

c) Which group will dominate in the long run? Justify your answer by computing $\lim_{n\to\infty} A^n\mathbf{x}$.

Section 2

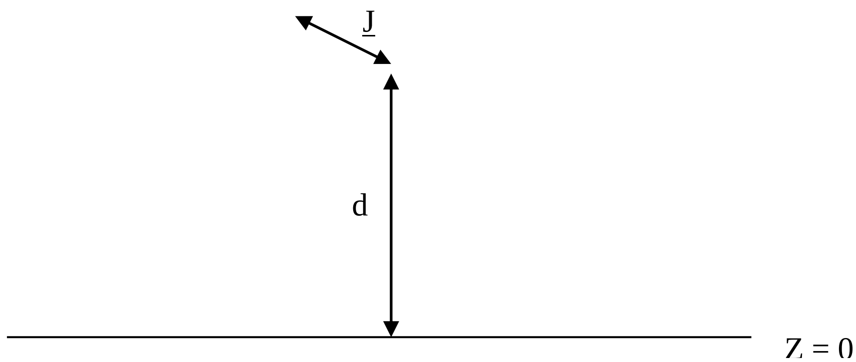
4. When two transmission lines are to be connected in cascade, a reflection of the wave to be transmitted from one to the other side will occur if they do not have the same characteristic impedances. Show that a new quarter-wave length of line, inserted as a transformer between the two cascaded lines, can cause the first line to see its characteristic impedance Z_{01} as a termination, thus eliminating the reflection.

   As part of your demonstration, find the characteristic impedance of the new quarter wavelength section in terms of the different characteristic impedances Z_{01} and Z_{02} of the original two lines.

5. A short dipole of strength J is located above a perfectly conducting plane z = 0. The dipole strength is

$$\underline{J} = \underline{J}\left(\hat{x} + \hat{z}\right)$$

where $\hat{x}$ and $\hat{z}$ are unit vectors in the x and z directions. The distance of the dipole from the conductor is "d".



a) Find the location, direction and magnitude of the image.

b) Find the radiation pattern.

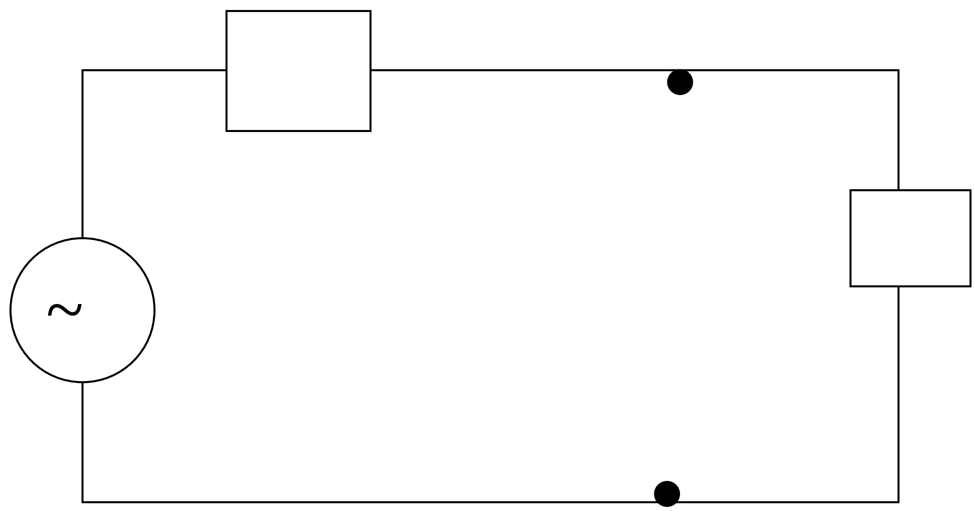
6. The circuit diagram shown below represents an antenna connected to a receiver load.

Label each circuit element in the diagram.

a) Indicate unambiguously which box(es) represent (parts of) the antenna and which do not.

b) Indicate unambiguously which box contains the radiation resistance of the antenna.

c) What is the physical significance (if any) of the power dissipated in this radiation resistance?



Section 3

7. A digital computer has a memory of 64K x 16 bits and a cache memory of 1K 16-bit words. The cache uses direct mapping with a block size of 16-bit words.

   a) How many bits are there in the tag, index (or group), and byte fields of the address format?

   b) How many bits are there in each line of the cache, and how are they divided into different fields? Include a valid bit.

   c) How many blocks can the cache accommodate?

   Explain your work and state all your assumptions.

8. A microprocessor based system (wordlength and data bus width of 8 bits) employs RAM chips of 256x4-bits and ROM chips of 1024x4-bits.

The microprocessor system needs 8K of contiguous RAM and 8K of contiguous ROM space. The one highest-order bit of the 16-bit address bus is assigned to 0 for RAM and to 1 for ROM. Assume all unused (don't care) address bits are zero.

a) How many RAM and ROM chips are needed?

b) Draw the interface of these chips to the microprocessor buses.

c) Give the address ranges in hexadecimal for RAM and ROM modules of 8-bit widths.

9. Consider a little-endian 16-bit CPU connected to a memory subsystem via a 16-bit data bus D0-D15, a 16-bit address bus A0-A15, and a control bus over which the CPU sends control signals to memory.

Memory system is organized as identical arrays of 8-bit bytes. The 16-bit memory subsystem is divided into two 8-bit modules, an upper module (UMODULE) and a lower module (LMODULE), in order to implement low-order memory interleaving. The rules for accessing operands in memory are as follows:

- 16-bit operands aligned at even locations are fetched in one cycle. (The least significant byte over data lines D0-D7.)
- Misaligned 16-bit operands are fetched in two cycles

The CPU issues to memory the following control signals:

- MREAD=H: to perform a read from memory
- MWRITE=H: to perform a write to memory
- ENUMOD=H: to enable the 8-bit memory module connected to data bus lines D8-D15
- ENLMOD=H: to enable the 8-bit memory module connected to data bus lines D0-D7

a) Draw the two 8-bit memory modules and their interconnection to the data bus lines. Show the numbering of the lowest five hexadecimal addresses in each of the two 8-bit memory modules.

b) Indicate what control signals are used to select each memory module.

c) Consider the 16-bit word operand: ABCD where the leftmost byte is the most significant byte. If this operand is stored starting at (byte) memory location 1234,

1. How many bus cycles will it need to fetch the operand from memory?

2. What information (in hex) will travel over the data bus lines?

3. What address or addresses (in hexadecimal) will the CPU send to memory?

4. What control signals does the CPU issue to memory to select the proper module per bus cycle?

d) If the above operand is stored starting at memory location 1235, answer the above questions 1 through 4.

Section 4

10. Let the random process $Z(t), -\infty < t < +\infty,$ be defined as

$$Z(t) = Y\cos(t) + X\sin(t)$$

where X and Y are independent random variables such that

$$P(X = -1) = P(Y = -1) = 2/3$$
$$P(X = 2) = P(Y = 2) = 1/3.$$

a) Is $Z(t)$ a wide sense stationary process? Explain.

b) Is $Z(t)$ a strict sense stationary process? Explain. (Hint: Consider $E\left\{Z^3(t)\right\}$.)

11. Let X_1, X_2 and X_3 be independent zero-mean unit-variance random variables. Also, let

$$
\begin{aligned}
Y_1 &= X_1 \\
Y_2 &= X_1 + X_2 \\
Y_3 &= X_1 + X_2 + X_3 \\
\hat{Y}_3 &= a_1 Y_1 + a_2 Y_2
\end{aligned}
$$

where a_1 and a_2 are constants. Find a_1 and a_2 such that

$$E\left\{\left(Y_3 - \hat{Y}_3\right)^2\right\}$$

is minimum. (In this case $\hat{Y}_3$ is called the minimum mean square error linear estimate of Y_3.)

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12. Let the random variable Y be given by

$$Y = X_1 + X_2 + X_3$$

where X_1, X_2 and X_3 are independent random variables such that

$$P(X_i = 1) = p$$
$$P(X_i = 0) = 1 - p$$

for $i = 1, 2, 3$.

a) Determine the characteristic function of X_i.

b) Determine the characteristic function of Y.

c) Determine $E\{Y^2\}$.

d) Determine the probability mass function of Y.

Section 5

13. a) Find the SVD of the following $M \times 9$ matrix:

$$\mathbf{A} = \begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_1 & \mathbf{a}_1 & \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_2 & \mathbf{a}_2 & \mathbf{a}_3 & \mathbf{a}_3 \end{bmatrix}$$

where $M > 9$ and $\mathbf{a}_1$, $\mathbf{a}_2$ and $\mathbf{a}_3$ are orthonormal column vectors.

b) Using the results of a) solve the system

$$\mathbf{Ax} = \mathbf{y}$$

for $\mathbf{x}$ in the LMS sense.

c) Show the columns of $\mathbf{A}$ constitute a frame in the three-dimensional subspace spanned by $\mathbf{a}_1$, $\mathbf{a}_2$ and $\mathbf{a}_3$. What are the frame bounds?

14. The inverse of the double-sided Laplace transform

$$F_{II}(s) = \frac{s}{(s+2)^2 (S^2+4)}$$

can represent several time functions, depending on the choice of the integration path in the inverse formula. Find and sketch the time functions. Identify the function for which $F_{II}(s)$ represents the single-sided Laplace transform. Find the corresponding Fourier transform.

15. A linear system is defined by the differential equation (DE)

$$\frac{d^2y}{dt^2}+2\frac{dy}{dt}+5y=x(t).$$

a) Find the output $y(t)$ when the input $x(t)$ is $\sin(2t+\pi/8)$.
b) Find the impulse response.
c) Using Fourier Transforms solve the DE for $t\geq -5$ with the initial conditions $y(-5)=1,\ y'(-5)=1$ and

$$x(t)=\begin{cases}1\,;\, t>0\\ 0\,;\, t<0\end{cases}.$$

Section 6

16. Answer the following questions relating to the Debye length.

a) The Debye length is a measure of the variation of charge. Is it relevant for mobile or fixed charge? Explain.

b) Consider the n-side edge of a *pn*-diode depletion region with doping $N_d = 4\text{x}10^{16}\ \text{cm}^{-3}$. Derive an expression for the Debye length in terms of relevant parameters. State any assumptions or approximations employed.

c) The Debye length in typical device applications is considered to be a small length. State two regions or conditions in a MOSFET device where the Debye length is relevant.

17. State the conditions (and/or assumptions) when the drift-diffusion model is suitable to describe current transport in semiconductors.

a) Derive the expressions for the electron and hole drift-diffusion currents in terms of the electrostatic potential ϕ and the electron hole charge densities ($\boldsymbol{n}$, $\boldsymbol{p}$).

b) Formulate the Poisson and the steady-state drift-diffusion current continuity equations in terms of the electrostatic potential, and the electron and hole quasi-Fermi potentials: **(ϕ, φ_n, φ_p).**

c) Based on the response above, how can one interpret the quasi-Fermi potentials?

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18. Derive an expression for the threshold voltage of an *n*-channel MOSFET. Clearly explain and derive all necessary expressions. Account for the following physical mechanisms and parameters:

(i) Metal-semiconductor work function difference $\boldsymbol{\Phi_{MS}}$.

(ii) Insulator thickness $\boldsymbol{t_{OX}}$ and dielectric constant $\boldsymbol{\varepsilon_{OX}}$.

(iii) Silicon dielectric constant $\boldsymbol{\varepsilon_{Si}}$ and substrate doping $\boldsymbol{N_a}$.

(iv) Fixed oxide charge $\boldsymbol{Q_f}$ and Si-SiO_2 interface charge $\boldsymbol{Q_{it}}$.

(v) Substrate to source potential $\boldsymbol{V_{SB}}$.

State, if any, the change in the above threshold voltage if the channel length is in the order of the source and drain depletion region widths. What is such an effect typically referred to in the literature?

Section 7

19. A data source produces messages at random moments in time, at an average rate of x messages per second and with the time between successive messages exponentially distributed.

a) Find the probability distribution of the number of messages produced per second.

b) Determine the average number of messages produced in 5x seconds.

c) Determine the probability that the source produces exactly 1 message in x seconds.

20. A data link between stations A and B operates under the control of the Asynchronous Response Mode of the HDLC protocol.

a) Show the structure of all three types of frames used in the protocol.

b) Show the action of the protocol in response to an error occurring in the transmission of an information frame from A to B.

21. A bus local area network uses the CSMA/CD access procedure to allocate the channel among its 3 stations. Derive the probability of a collision, assuming all 3 stations follow the same message generation and message duration model, but that the messages of one of the stations occur only half as often but are twice as long as those of the other 2 stations.