Dr. Helgert's Fall 2007 Preliminary Exam Questions for ECE 203

1. A radio receiver filters, amplifies and rectifies the voltage across the terminals of an antenna. The voltage may be due to thermal noise alone or it may be due to the sum of a signal and the thermal noise. The final output of the receiver, sampled at a certain time t, is a random variable X. It is desired to choose, based on the value of the sample X, between the hypothesis H_1 that a signal is present and the hypothesis H_0 that no signal is present. The a priori probability of a signal is $P(H_1) = 0.5$. When no signal but only background noise is present at the input to the receiver, the probability density function for X is

$$f_X(x|H_0) = c_0 \exp(-c_0 x) u(x) \quad .$$

When a signal is also present, the probability density for X is

$$f_X(x|H_1) = c_1 \exp(-c_1 x) u(x) \quad ,$$

where $u(x)$ is the unit step function and $c_0 > c_1$.

a) Given that a particular value x is observed at the receiver output, what is the probability that a signal is present? That is, determine $P(H_1|X = x)$.

b) If we wish to minimize the probability P_e of error in deciding whether a signal is present or not, over what range of values of x should we decide "signal present" and over what range of values should we decide "signal absent"?

c) Determine the minimum attainable P_e in terms of c_0 and c_1.

2. The input to a quantizer is a random positive voltage v whose pdf is

$$f_V(v) = a \exp(-av) u(v)$$

where $u(v)$ is the unit step function. The output of the quantizer is equal to the integer k when

$$k < V \le k+1, \quad 0 \le k < \infty.$$

Find the expected value of the output of the quantizer.