THE GEORGE WASHINGTON UNIVERSITY
School of Engineering and Applied Science
Department of Electrical and Computer Engineering

Preliminary Examination

Fall 2006

General Instructions

Read carefully before starting.

Solve 5 problems in all; at most 2 questions may be selected from the same section.

Please write your name and student number below:

______________________________	______________________
Student Name	**Student Number**

Solve each problem in a separate blue book. Write the section number, problem number, and your student number on the front of each blue book. **DO NOT WRITE YOUR NAME ON THE BLUE BOOK.**

Submit solutions to only five (5) problems. Use only **ONE** blue book per problem.

For each problem, make a special effort to give the answers in a clear form.

The exam will begin at 10:00 a.m. and end at 3:00 p.m.

Computers and/or calculators are not to be used

Section 1

1. An on board cache takes 0.2 ns to determine if it is hit or a miss, and 0.15 ns more to deliver the content. In a certain application, when connected to a 3 ns D-ram, it has a hit ratio of 0.93.
    a. What is the average access time for a look-through configuration?
    b. What is the average access time for a look-aside configuration?
    c. Discuss the pros and cons of the two configurations.

2. A pipeline microprocessor has "k" stages. If the pipeline is to execute "n" instructions, then

    a. Derive an expression for the execution time in clock cycles
    b. Derive an expression for the Speed up of this microprocessor over a similar one which is not pipelined
    c. Derive an expression for the throughput in instructions/cycle
    d. What will be the execution time if the probability of an instruction to be a branch is "p" and the probability that a branch is taken is "q"?

3. Answer the following questions
    a. A computer system is using 11 bits to represent signed integers. How many negative and how many positive numbers are represented if the system is encoding these numbers using: i. the sign and magnitude notation? ii. the one’s complement notation? iii. the two’s complement notation?
    b. What are the largest and the smallest possible numbers that can be represented in the IEEE single precision format?
    c. Show the steps required to add 2 floating point numbers in the IEEE single precision format? Can pipelining help such operations? How?

Section 2

1. The random variables X has a cumulative distribution function $F_X(x)$. Define a new random variable Y by

$$Y = g(X) = F_X(x).$$

Show that Y is uniformly distributed over [0, 1]. Conversely, if we want a random variable with some given probability density $f_X(x)$, and we know that Y is uniformly distributed over [0, 1], we can take $X = g^{-1}(Y)$, where $g^{-1}(Y)$ is the inverse function to that defined above, with

$$F_X(x) = \int_{-\infty}^{x} f_X(u)du$$

in terms of the desired probability density function. Thus a sample value from a uniform [0, 1] distribution, which is readily available from a standard computer random number generation subroutine, may be simply converted to a sample from some arbitrary known distribution.

What function $g^{-1}(Y)$ yields a sample of a Cauchy random variable by transforming a sample from a uniform [0, 1] distribution?

2. A communication channel accepts an arbitrary voltage input v and outputs a voltage Y = v + N, where N is a Gaussian random variable with mean 0 and variance σ^2. Suppose that when a 1 is transmitted, v = V and when a 0 is transmitted, v = -V. The receiver decides a 0 was sent if Y is negative and a 1 otherwise. Find the probability of the receiver making an error if a 0 was sent; if a 1 was sent. Evaluate for V = 2 and $\sigma^2 = 4$.

3. The sample X of a speech signal is a Laplacian random variable with parameter $\alpha = 1$. Suppose that X is quantized by a uniform quantizer consisting of four intervals: $(-\infty, -a]$, $(-a, 0]$, $(0, a]$, and $(a, \infty]$.

a) Find the value of a so that X is equally likely to fall in each of the four intervals. (If a is an irrational number, do not at this point convert to a decimal with roundoff error.)

b) Find the representation point x_1 for X in $(0, a]$ that minimizes the mean-squared error (conditioned on $0 < X \le a$), that is find x_1 which minimizes

$$\int_0^a (x - x_1)^2 f_X(x)dx.$$

Hint: Find x_1 in terms of the value of a found above but do not convert to a decimal.)

Find the representation points x_2 for interval $(-a, 0]$, x_3 for interval $(a, \infty]$, and x_4 for interval $(-\infty, -a]$.

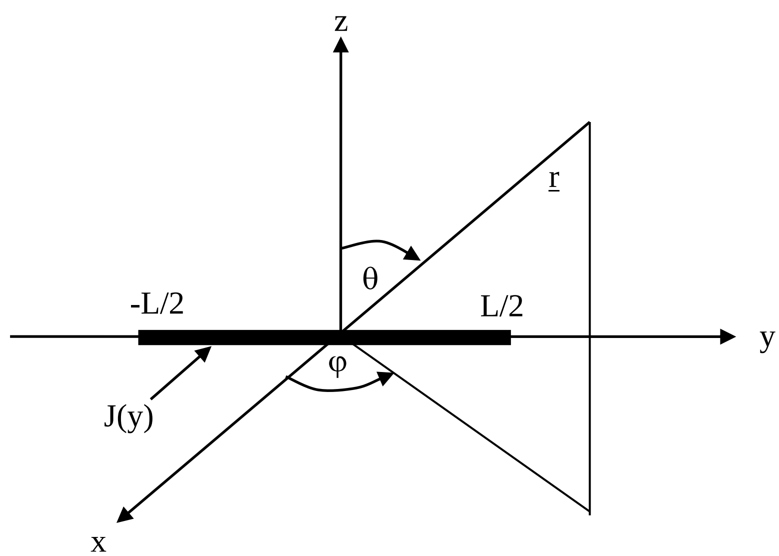
c) Evaluate the mean-squared error of the quantizer.

Section 3

1. A time harmonic linear distribution of electric current along the y-axis is of the form

$$J(y) = \begin{cases} 1, & |y| \le L/2 \quad , \\ 0, & |y| > L/2 \quad . \end{cases} .$$

Find the electric and magnetic fields in the far zone. Use the coordinate system shown.



2. An electric field is given as the complex vector phasor

$$\boldsymbol{E} = \hat{\boldsymbol{x}} + (1 + j)\hat{\boldsymbol{y}}.$$

(a) Write the real time dependent electric field corresponding to the above phasor if the radian frequency of the harmonic oscillation is given as ω_0..

(b) Sketch the path the electric field vector tip traces out in time (the polarization ellipse). The sketch must by LARGE, clear and labeled to indicate how the given phasor $\boldsymbol{E}$ is related to your sketch. SHOW THE DIRECTION IN WHICH THE ELLIPSE IS TRACED..

(c) Calculate the ellipticity of the ellipse (ratio of major to minor axis lengths).

3. In examining plane wave reflection from a dielectric half space, a certain angle of incidence is called the “Brewster” angle.

(a) What distinguishes the Brewster angle from other angles of incidence?

(b) Derive an expression for the Brewster angle.

Section 4

1. The inverse of the double -sided Laplace transform

$$F_{II}(s) = \frac{s+3}{(s-4)(s^2+16)}$$

can represent several functions of time, depending on the choice of the integration path in the inversion formula.

a) Find and sketch the functions.
b) Identify the function for which $F_{II}(s)$ represents the single-sided Laplace transform.
c) Identify the function that has a Fourier transform and find the Fourier transform.

2. A linear system is defined by the differential equation

$$\frac{d^2 y(t)}{dt^2} + 2\frac{dy(t)}{dt} + 5y(t) = x(t) + 2\frac{dx(t)}{dt}$$

with $x(t)$ the input and $y(t)$ the output

a) Find $y(t)$ when the input is $\cos(4t)$ (Express the final result in terms of real quantities only)

b) Find the impulse response

c) Find the output for $t \geq 0$ with initial conditions $y(0) = 1,\ y'(0) = -1$ when the input is $\cos(4t)U(t)$

Identify the zero state , the zero input ,the transient and the steady state responses.

3. The Fourier transform $F(\omega)$ of a bandlimited signal $f(t)$ is given by

$$F(\omega) = \begin{cases} \cos^4\left(\dfrac{\pi\omega}{2\Omega}\right) e^{-i\frac{2\pi\omega}{\Omega}} & ; |\omega| \le \Omega, \\ 0 ; |\omega| > 0 \end{cases}$$

a) The signal is sampled at intervals $\Delta t = \pi / \Omega$. Find the samples.
b) Find the Z transform of the sampled sequence $f[n] = f(n\Delta t)\Delta t$
c) Find $f(t)$
d) Suppose the signal is sampled at intervals $\Delta t' = 2\pi / \Omega$ and its Fourier transform is approximated by

$$\hat{F}(\omega) = \Delta t' \sum_{n=-\infty}^{\infty} f(n\Delta t') e^{-i\omega n\Delta t'}$$

Compute and sketch $\hat{F}(\omega)$

Section 5

1. Consider the matrix

$$A = \begin{bmatrix} 4 & -1 & 1 \\ -1 & 4 & -1 \\ 1 & -1 & 4 \end{bmatrix}.$$

a) Given that one eigenvalue of A is $\lambda = 6,$ find the remaining eigenvalues.

b) Find the **linearly independent** eigenvectors of A.

c) Check your results.

d) Diagonalize matrix A so that $A = Q\Lambda Q^T$.

2. Solve the ordinary differential equation **algebraically**:

$$\frac{du}{dt} = Au \quad \text{with } A = \begin{bmatrix} -2 & 3 \\ 2 & -3 \end{bmatrix} \text{ and } u(0) = \begin{bmatrix} 5 \\ 0 \end{bmatrix}.$$

a) Find the eigenvalues and eigenvectors; and diagonalize $A = S\Lambda S^{-1}$. Check your results.

b) Solve for $u(t)$ starting from the given $u(0)$.

c) Compute the matrix e^{At} using S and Λ.

d) As t approaches infinity, find the long term limits of $u(t)$ and e^{At}.

3. Matrix A is defined as

$$A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & 2 \\ 2 & 2 & 3 \end{bmatrix}.$$

a) Decide if A is positive definite.

b) What is the norm of this matrix?

c) What is the condition number for A?

d) Generate matrix B = A-1. From its **rank** find all the eignevalues of B and then the eigenvalues of A, without actually calculating them.

Section 6

Problem 1

The standard drift-diffusion transport model states that the hole current density, $\boldsymbol{J}_p$, is the summation of the drift and diffusion currents:

$$\boldsymbol{J}_p = -q\,(\mu_p\, p\; \nabla\phi + D_p\; \nabla p\,).$$

In this expression, q is the electronic charge, μ_p is the hole mobility, p is the hole charge density, ϕ is the electrostatic potential, and D_p is the hole diffusion constant.

a) Show that the hole charge density can be expressed as follows:

$$p = n_i \exp(-\phi\,/v_T)$$

when the semiconductor is in thermal equilibrium. Note: v_T is thermal voltage and n_i is the intrinsic charge density.

b) The hole quasi-Fermi potential, ψ_p, can be defined as follows:

$$p = n_i \exp[-(\phi - \psi_p\,)/v_T].$$

Derive an expression for the hole current density in terms of p and ψ_p. Based on this expression, what is the physical interpretation of a constant level quasi-Fermi potential?

Problem 2

Based on the physical operation of semiconductor devices, provide brief and clear answers to the following questions:

a) Draw the band diagram of a metal-semiconductor barrier where the workfunction of the metal is larger than that of the semiconductor. Can such a system be employed to construct an Ohmic contact? If so, what key parameters need to be controlled?

b) With all other parameters remaining unchanged, describe the effect of the shortening of channel length on the value of the threshold voltage of a MOS device.

c) Is the saturation velocity of an electron important in determining the intrinsic limits on the speed of response of a MOSFET? If so, how? What factors determine the speed of electrons in the channel of a MOSFET?

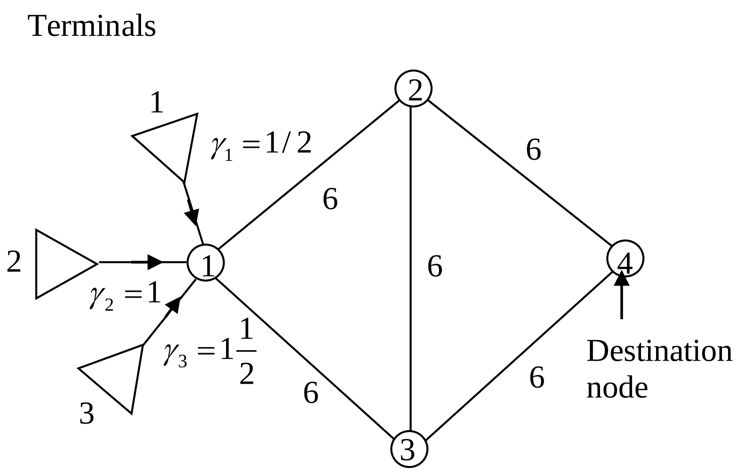
Problem 3

Consider an ideal long-base abrupt *pn*-junction silicon diode with uniform cross section and constant doping on both sides of the junction. The doping of the diode is as follows: N_a = 5.0x10^{15} cm^{-3} *p*-type and N_d = 1x10^{16} cm^{-3} *n*-type, where the minority-carrier lifetimes are τ_n =1.5x10^{-6} s and τ_p =2x10^{-7} s, respectively. In this problem, assume that the effects within the space-charge region are negligible and that minority carriers flow only by the diffusion mechanism in the charge neutral regions.

a) Determine the value of the so-called built-in potential across the barrier?
b) When the applied reverse bias is 0.4 V, calculate the density of the minority carriers at the edge of the space-charge region.
c) Under the bias condition above, sketch the minority and majority carrier currents as a function of distance from the junction.

Section 7

1. Consider the packet-switched network shown below. Terminals 1, 2, and 3 generate Poisson traffic at the rate of $\gamma_1 = 1/2$, $\gamma_2 = 1$, and $\gamma_3 = 11/2$ packets/sec, respectively, as shown. All packets are $1/\mu = 1/6$ sec long, on the average, exponentially distributed. All are destined for node 4, as shown. All line capacities are $\mu = 6$ packets/sec, as shown.



2. A stop-and-wait protocol uses positive acknowledgements (acks) and timeouts only. (No negative acknowledgements are used.) The ack arrives at the transmitter at time t_{ack} after transmission of a frame. Take $t_{out} > t_{ack}$. Frames are all length t_t and are always available for transmission. A frame is received in error with probability *p*. Show that the maximum frame throughput is

$$\lambda_{\max} = (1 - p)/t_T\left[1 + (b - 1)p\right]$$

where $b = (t_T + t_{out})/(t_T + t_{ack}) > 1$.

3. A coaxial cable bus 20-km long has 1000 stations connected to it: Electromagnetic energy propagates in both directions at a speed of 200,000 km/sec. The line capacity is 10 Mbps. Packets transmitted at each station are all 10 kbits long.

a) A token (permission to transmit) is passed sequentially from station 1 to 2 to … to 1000, and then back to 1 again, starting the cycle over. (This is then called a token bus.) There are 8 bits of latency at each station. Each station generates 8 packets per 10 sec on the average. Calculate the average scan (cycle) time. What is the maximum rate of transmission of packets at each station?

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b) Pure random access (pure Aloha) is used instead, with stations transmitting at will. Calculate the maximum possible rate of transmission of packets at each station.