THE GEORGE WASHINGTON UNIVERSITY
School of Engineering and Applied Science
Department of Electrical and Computer Engineering

Preliminary Examination

Spring 2007

General Instructions

Read carefully before starting.

NEW FORMAT: Solve 5 problems in all; at most 2 questions may be selected from the same section.

Please write your name and student number below:

______________________________	______________________
Student Name	**Student Number**

Solve each problem in a <u>separate</u> blue book. Write the section number, problem number, and your student number on the front of each blue book. **DO NOT WRITE YOUR NAME ON THE BLUE BOOK.**

Submit solutions to only five (5) problems. Use only **ONE** blue book per problem.

For each problem, make a special effort to give the answers in a clear form.

The exam will begin at 10:00 a.m. and end at 3:00 p.m.

!!GOOD LUCK!!

Section 1

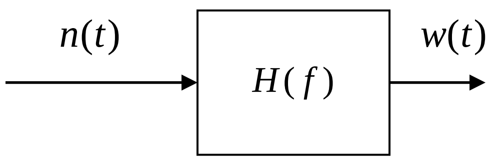
1. Consider linear system shown in Figure 1 with input noise process n(t) defined by the following autocorrelation function: $R_n(\tau) = \frac{N_0}{2}\delta(\tau)$.

   a) Find the power spectral density of noise n(t).

   b) Find the power spectral density of output noise w(t).

   c) What is the noise power at the output of the filter.

   d) Find the autocorrelation function of output noise w(t).



$$H(f) = \begin{cases} 2, & -W \le f \le W \\ 0, & \text{elsewhere} \end{cases}$$

2. The packet transmission error probability is p. The packet is transmitted until it is received correctly at the receiver and each transmission is independent of previous transmissions.

a) Derive the probability mass distribution of required number of packet transmissions.

b) What is the average number of transmissions?

3. Consider zero-mean Gaussian random variables x and y, each with variance σ^2.

a) Derive the probability density function of $z = \sqrt{x^2 + y^2}$.

b) Calculate $E(z)$ and $E(z^2)$.

c) What is the probability that z is smaller than $\sqrt{E(z^2)}$?

Section 2

4. Consider an air-filled parallel plate guide having perfectly conducting plates that are separated by a distance "a". A TE_1 mode is excited in the waveguide. Its electric field is:

$$\boldsymbol{E}(x,z) = \hat{\boldsymbol{y}} A \sin\frac{\pi x}{a} e^{-j\beta z} \quad , \quad A = \boldsymbol{constant}$$

a) Determine β.

b) Determine the magnetic field in the guide.

c) What is the surface current induced in the perfectly conducting plates?

5. A certain plane wave in glass $\left(\varepsilon = 2\varepsilon_0\right)$ is characterized by $\boldsymbol{E} = \left(\sqrt{2}\hat{\boldsymbol{x}} + \hat{\boldsymbol{y}} - \hat{\boldsymbol{z}}\right)e^{-j2\pi\cdot 10^6 (y+z)} (V/m)$ where y and z are given in meters.

a) In what direction is this wave propagating?

b) What is its frequency (Hz)?

c) What polarization does the wave have?

d) Evaluate the complex Poynting vector $\underline{S}$ at the origin.

6. A time harmonic plane wave with electric field

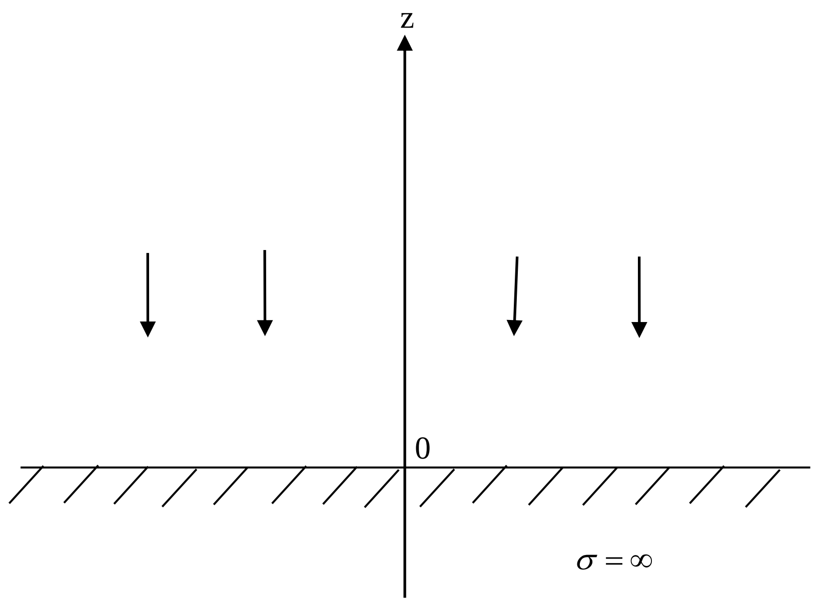
$$\boldsymbol{E}_i(z) = E_0(\hat{\boldsymbol{x}} - j\hat{\boldsymbol{y}})e^{+jk_0 z}$$

is normally incident on a perfectly conducting half space. Here E_0 is a real constant and k_0 is the free space wavenumber.

a) What is the polarization of the incident wave?

b) Find the reflected electric field.

c) What is the polarization of the reflected electric field?



Section 3

7. Consider using a 2-way set-associative cache memory of 2 kilobytes, where each cache line is 16 bytes, in a system that has 1 Mbyte of main memory.

a) Sketch the memory system organization showing how memory locations are mapped into the cache.

b) Sketch the physical memory address layout for the purpose of accessing the cache.

c) Under which conditions would a set-associative cache become equivalent to a fully associative cache?

d) Under which conditions would a set-associative cache become equivalent to a direct mapped cache?

e) Discuss the cost and performance tradeoffs associated with the three common cache organizations: direct mapped, fully associative, and set associative.

8. A 2-way superscalar microprocessor has k stages per pipeline and uses a pipeline clock cycle of T seconds. If n instructions are to be executed:

a) Derive an expression for the execution time.

b) Derive an expression for the Speed up of this microprocessor over a similar one which is neither a superscalar nor pipelined.

c) Derive an expression for the throughput.

d) What will be the execution time if the probability of an instruction to be a branch is "p" and the probability that a branch is taken is "q"?

9. Answer the following questions.

a) A computer system is using 11 bits to represent signed integers. How many negative and how many positive numbers are represented if the system is encoding these numbers using: i. the sign and magnitude notation? ii. the one's complement notation? iii. the two's complement notation?

b) What are the largest and the smallest possible numbers that can be represented in the IEEE single precision format?

c) Show the steps required to add 2 floating point numbers in the IEEE single precision format? Can pipelining help such operations? How?

Section 4

10. What is the distance, in 4-dimensional space (w, x, y, z), from the point (1, 3, 2, 5) to the hyperplane $3x + 4y + z = 20$?

11. Show that for a 3 by 3 matrix, the determinant of the product is the product of the determinants of the individual matrices.

12. Given the matrix:

$$\begin{vmatrix} 3 & 3 & 2 & 1 \\ 6 & 6 & 3 & 2 \end{vmatrix}$$

Define the row space, the column space, the null space, and the left null space.

Section 5

13. The inverse of the Z-transform

$$F(z) = \frac{z}{(z^2 - z + 1)(z - 2)^2}$$

can represent several possible sequences.

a) Find all the sequences.

b) One of the sequences represents the coefficients of the Fourier series expansion of the Fourier Transform of a band-limited signal. Identify the sequence and find the Fourier Transform of the corresponding band-limited signal.

14. The input $e^{i\omega t}U\left(t\right)$ to a linear system gives the output

$$-\frac{e^{-t^5}}{t^4+i\omega}+\frac{e^{i\omega t}}{t^4+i\omega}$$

for all real t and ω. (Note: $U\left(t\right)$ denotes the unit step function)

a) Find the system impulse response.

b) Find and sketch the output when the input is $f(t)=\begin{cases}1\,;|t|\le 1\\ 0\,;\,|t|>1\end{cases}$.

15. A linear system is defined by the differential equation

$$\frac{d^2y}{dt^2} + \frac{dy}{dt} + y = 2 + 3\frac{dx}{dt}$$

with $y(t)$ the output and $x(t)$ the input.

Find

a) the output when the input is $\sin(5t - \pi/4)$,

b) the impulse response,

c) the output for $t \geq -8$ with initial conditions $y(-8) = 1$, $y'(-8) = -1$ when the input is $\sin(5t - \pi/4)U(t+8)$ and identify the zero state, zero input, the transient and the steady state responses.

Section 6

16. A series of 10 information frames is to be transmitted from a primary to a secondary in half duplex mode across an established data link, using the ARM HDLC protocol. The primary transmits one frame at a time and waits for an acknowledgement before transmitting the next frame. Describe in diagram form how the following problems are solved:

a) An error in the third information frame.

b) An error in the frame acknowledging receipt of the sixth frame.

c) The absence of a response from the secondary after the eighth frame.

Next, assume that the 10 information frames sent by the primary carry 100 bits of information each. The velocity of propagation on the link is 2×10^8 meters per second, the link is 10 km long and has a capacity of 9600 bits per second. Determine

d) The link throughput in bits per second

e) The efficiency of utilization as a percentage of channel capacity

Ignore the processing and disconnect times, assume the link is error free in both directions, and no protocol errors occur.

17. You have the choice between using a digital transmission system or an analog transmission system for the output of a microphone with a frequency response from 0 to 2000 hertz. In the digital case you use 64 levels of quantization for the signal and a biphase baseband code. In the analog case you decide to use phase modulation of the carrier.

How many signals could be carried by a channel of bandwidth 200 kilohertz in the analog and digital modes? State all your assumptions

18. Two DTE's are communicating via modems and RS-232D interfaces over a half duplex link. Message transmission is one-directional, with each message consisting of M bits, of which N bits are overhead and M-N bits are user data. After a message is transmitted, the sender waits for an acknowledgement consisting of K bits, before sending the next message. All transmissions are error-free.

a) As a function of the modem transmission rate R, the propagation delay T, and the RTS-to-CTS delay D, determine the effective user data throughput in bits per second.

b) What is the user data throughput for the following parameter values:

R = 9600 bps
M = 1000 bits
N = 24 bits
K = 24 bits
T = 1 milliseconds
D = 10 milliseconds

Section 7

19. The following questions relate to the Debye length in semiconductor devices:

a) The Debye length is a measure of the variation of charge. Is it relevant for mobile or fixed charge? Explain.

b) Generally, the Debye length in typical device applications is considered to be a small length. State two regions or conditions in a MOSFET device where the Debye length is relevant.

c) Consider the *p*-side edge of a *pn*-diode depletion region with doping $N_a = 4 \times 10^{16}\ \text{cm}^{-3}$. Derive an expression for the Debye length in terms of relevant parameters. State any assumptions or approximations employed.

20. The drift-diffusion transport model states that the hole current density, J_p, is the summation of the drift and diffusion currents:

$$J_p = -q\ (\mu_p\ p\ \nabla\ \phi + D_p\ \nabla p\ ).$$

In this expression, q is the electronic charge, μ_p is the hole mobility, p is the hole charge density, ϕ is the electrostatic potential, and D_p is the hole diffusion constant.

a) When the semiconductor is in thermal equilibrium, show that the hole charge density can be expressed as follows:

$$p = n_i \exp(-\phi\ /v_T)$$

where v_T is thermal voltage and n_i is the intrinsic charge density.

b) The hole quasi-Fermi potential, ψ_p, is defined as follows:

$$p = n_i \exp[-(\phi - \psi_p\ )/v_T].$$

Derive an expression for the hole current density in terms of p and ψ_p. Based on this expression, state the physical interpretation of the quasi-Fermi potential.

21. In this problem you are to derive an expression for the threshold voltage of an *n*-channel MOSFET. In this derivation, clearly explain and derive all necessary expressions. Account for the following physical mechanisms and parameters:

i. Metal-semiconductor work function difference $\boldsymbol{\Phi_{MS}}$.

ii. Insulator thickness $\boldsymbol{t_{OX}}$ and dielectric constant $\boldsymbol{\varepsilon_{OX}}$.

iii. Silicon dielectric constant $\boldsymbol{\varepsilon_{Si}}$ and substrate doping $\boldsymbol{N_a}$.

iv. Fixed oxide charge $\boldsymbol{Q_f}$ and Si-SiO_2 interface charge $\boldsymbol{Q_{it}}$.

v. Substrate to source potential $\boldsymbol{V_{SB}}$.

Where applicable, state the change in the threshold voltage if the channel length is in the order of the source and drain depletion region widths. What is this effect referred to in the literature?

Section 8

In your answers to any or all of these questions, be sure to provide quantitative as well as descriptive information. In particular, address these issues: What rates, volumes, masses, temperatures, etc. are involved? What equations govern the underlying behaviors and processes?

22. Assume the existence of a stimulus that begins to depolarize the cell membrane of an excitable cell. Explain/describe/discuss how an action potential is then generated. Your answer should include, but not necessarily be limited to, the following points …

A. Voltage gated sodium and potassium channels.
B. The relative timing of sodium and potassium channel gating.
C. Relative permeability changes of sodium and potassium channels.
D. Recovery of membrane potential back to its resting level.
E. Positive afterpotential.

23. Describe/explain/discuss Ca^{2+} handling (e.g., Ca^{2+} entry to and exit from the muscle cells, Ca^{2+} storage in and release from organelles within the muscle cells) in human cardiac muscle. In addition, compare (discuss similarities and differences) in Ca^{2+} handling among human cardiac, skeletal, and smooth muscle.

24. Explain/describe/discuss the local mechanisms (i.e., changes in local pH, pO_2 and pCO_2) that control ventilation/perfusion matching in the human lung.