THE GEORGE WASHINGTON UNIVERSITY
School of Engineering and Applied Science
Department of Electrical and Computer Engineering

Preliminary Examination – Spring 2020

Friday, February 21, 2020

General Instructions. Read carefully before starting.

Solve 5 problems in all; at most 2 questions may be selected from the same section.

Candidates registered in the following focus areas must answer two of their five questions from the relevant section as follows:

Computer Architecture and High-Performance Computing:	Section 1
Communications & Networks:	Section 3
Electrical Power & Energy:	Section 8
Electromagnetics, Radiation Systems & Microwave Engineering:	Section 4
Electronics, Photonics & MEMS:	Section 6
Signal & Image Processing, Systems & Controls:	Section 3

Please write your name and student number below:

_______________________________________ G____________________

Student Name **Student Number**

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Focus Area in which Registered

Solve each problem in a **separate** exam book. Write the section number, problem number, and your student number **on the front of each blue book**. **DO NOT WRITE YOUR NAME ON THE EXAM BOOK.**

Submit solutions to **only** five (5) problems. Use only **ONE** exam book per problem.

For each problem, make a special effort to give the answers in a clear form.

The exam will begin at 10:00 a.m. and end promptly at 3:00 p.m.

Only Calculators provided by the department at the examination will be allowed. Personal items including cell phones and other electrical devices must be relinquished prior to the start of the examination.

This is a CLOSED BOOK, CLOSED NOTES EXAMINATION

Section 1

1. In the following problem do not write any final answer without setting up a detailed equation that reflects the complete basis of your answer. Such equations will be treated as part of your answers and failing to do so will incur penalties even if the final answer is correct.

   A microprocessor X has 16 processor cores running at a clock speed of 2.2 GHz. Each core can complete 4 floating point operations per clock cycle.

   a) What is the peak throughput of the microprocessor in MFLOPS?

   b) A program A was run on Microprocessor X. If 15% of program A is inherently sequential, what would be the throughput of the microprocessor in MFLOPS?

   c) What would be the speed up of the microprocessor while running program A as compared to the sequential case?

   d) What would be the efficiency of the microprocessor while running program A?

2.

Using a multi-cycle clock diagram, show the clock-by-clock execution of the following set of instructions. You may assume that forwarding logic is available. Indicate the total number of cycles needed to complete all of the instructions.

LW \$t1, 0(\$s2)
Add \$t2, \$t1, \$t2
LW \$t3, 0(\$t2)
Sub \$t3, \$t3, \$t2

3.

Assume there are half as many D-cache accesses as I-cache accesses, and that the I-cache and D-cache are responsible for 25% and 15% of the processor's power consumption in a normal four-way set associative implementation. The average power consumption for normal four-way set associative cache is 0.28 for I-cache and 0.35 for D-cache. Using the way selection approach with a four-way set associative cache increases the average access time for I-cache by 1.04 and for D-cache by 1.13. Determine if way selection improves performance per watt.

Section 2

4.

For $\mathbf{A} = \begin{bmatrix} 8 & -2 & -1 \\ 4 & -2 & -3 \\ 2 & -3 & -3 \end{bmatrix}$, $y_1 = \begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix}$ and $y_2 = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix}$

a) Find the dimension of the range space of **A** (rank);
b) Find the dimension of the null space of **A** (nullity);
c) Find all solutions **x**, for simultaneous linear algebraic equations $y_1 = \mathbf{Ax}$ and $y_2 = \mathbf{Ax}$
d) Find the best solution, in the least square sense, to the following system of equations:

$$\begin{aligned} -2 &= x_1 - 2x_2 \\ 5 &= x_1 - 2x_2 \\ 1 &= -2x_1 + x_2 \\ -3 &= x_1 - 3x_2 \end{aligned}$$

e) Determine if each of the following sets of vectors are linearly independent or not:

$$\left\{ \begin{bmatrix} -1 \\ -9 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} \right\} \quad \text{and} \quad \left\{ \begin{bmatrix} 2-i \\ -i \end{bmatrix}, \begin{bmatrix} 1+2i \\ -i \end{bmatrix}, \begin{bmatrix} -i \\ 3+4i \end{bmatrix} \right\}$$

Note: i represents the complex imaginary number with $i^2=-1$ and take complex number field.

5.

Consider a controllable and observable system with the following transfer function:

$$G(s) = \frac{(s+1)(s+2)}{(s-2)(s+3)(s-1)}$$

Is it possible, by using <u>state feedback</u> to change it to $G(s) = \dfrac{(s+1)}{(s-2)(s+3)}$? If yes, do it.

Is the resulting system controllable? observable? If no, explain why not?

6. Consider the following linear system and answer the following questions

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -3 & -3 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 2 & 1 \end{bmatrix} x(t)$$

a) Determine the time domain response y(t) if the input u(t)=5 for t >0 and initial value of the state is $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ at t=0.

b) Determine the transfer function G(s) for this system.
c) Is this system internally stable?
d) Is this system controllable? observable?

Section 3

7.
(3 parts)
Part 1: You toss a coin N times. The coin shows head with probability p and tail with probability 1-p. Compute the following probabilities:

a) All the tosses result in tails
b) The coin shows head at least once
c) The coin shows tail exactly k times, for $0 \le k \le N$

Part 2: You simultaneously roll two dice. Find the probability that the first die lands on a higher value than the second die.

Part 3: You play a game by simultaneously rolling two dice. If the sum of the two numbers is at most 5, you win and the game ends. If the sum is 7 or 10 you lose and the game also ends. In any other case you have to roll again.

a) Find the probability that the game ends after exactly four rolls.
b) Find the probability that you win.

8.

A nonlinear device has an input X which is uniformly distributed over the interval -2 to +3 and a transfer function $Y = X^2$.

a) Find the probability density function and cumulative probability function of Y.
b) Find the expected value and variance of Y.

9.

Consider a two-state Markov chain with transition probabilities $p_{12} = p$ and $p_{21} = q$. Assume q is fixed but the value of p can be selected from the interval (0,1). Also assume a payoff of r units every time we visit state 2 and a cost of p every time we visit state 1.

a) Compute the long-term payoff per time step as a function of p and q.
b) Find the value of p that maximizes the payoff.

Section 4

10. The electric field of a plane wave propagating in a medium with a relative dielectric constant of $\varepsilon_r = 8$ and free space permeability is given by

$$\boldsymbol{E} = (\hat{\boldsymbol{x}} + \alpha \hat{\boldsymbol{y}}) e^{-j(2k_0 x + k_0 \xi y)} \quad , k_0 = \omega\sqrt{\varepsilon_0 \mu_0}$$

a) Find the constant ξ such that $\xi > 0$;

b) Determine α;

c) Find the magnetic field;

d) What is the average power density of the wave?

11. The magnetic field of a mode in a parallel-plate air waveguide with a plate separation of 2.5 cm is given by

$$H_z(x, y) = Ce^{-j640\pi x/3}\cos(160\pi y)$$

where x and y are both in meters.

a) Is this a TE_n or TM_n mode? What is n? Is it a propagating or non-propagating mode?

b) What is the operating frequency?

c) Find the corresponding electric field.

d) Find the lowest order mode of the same type (TE or TM) that does not propagate at this operating frequency.

12.

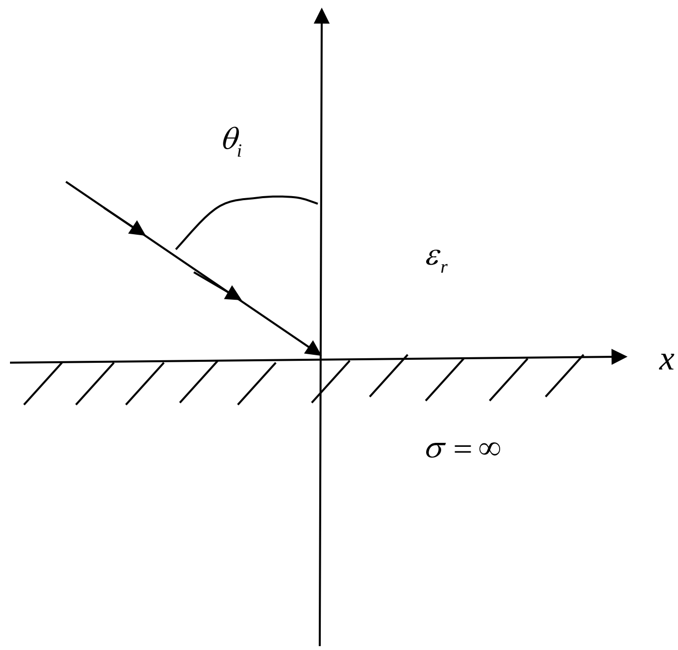

Figure 1

A dielectric medium $(\mu = \mu_0)$ lies above a perfectly conducting plane as shown in Figure 1. A plane wave having the electric field

$$\mathbf{E}(x,z) = \left(3E_0\hat{\mathbf{x}} + \frac{9E_0}{4}\hat{\mathbf{z}}\right)e^{-j3k_0x+j4k_0z}$$

is incident upon a perfectly conducting plane. Here E_0 is a constant and k_0 is the free space wavenumber $\left(k_0 = \sqrt{\mu_0\varepsilon_0}\right)$.

a) What is the angle of incidence θ_i?

b) What is the dielectric constant ε_r of the medium?

c) Find the z component of the reflected electric field.

Section 5

13.

(a) Find the Fourier transform of

$$x(t) = \frac{\sin(\pi 2Bt)}{\pi 2Bt} \cos(2\pi f_c t)$$

where $f_c > 2B > 0$.

(b) Find the Hilbert transform $\hat{x}(t)$ of $x(t)$.

(c) Find the analytic signal $\psi(t)$ of $x(t)$.

(d) Find the complex envelope $\gamma(t)$ of $x(t)$.

14.

The relationship between the output $y[n]$ and the input $x[n]$ of a discrete-time product modulator can be written as:

$$y[n] = \sum_{m=-\infty}^{+\infty} x[m]e^{j\omega_c m}h[n-m]$$

where $h[n]$ is the impulse response of a linear time-invariant (LTI) system.

a) Is the product modulator a linear system? Explain your answer.
b) Is the product modulator time-invariant? Explain your answer.
c) Assume that $h[n]$ corresponds to a causal system. What condition must $h[n]$ satisfy in this case? Is then the product modulator a causal system as well? Explain your answer.
d) Assume that $h[n]$ provides bounded-input bounded-output (BIBO) stability. What condition on $h[n]$ is necessary and sufficient for BIBO stability? Is then the product modulator BIBO stable as well? Explain your answer.
e) Let $X(z)$, $Y(z)$, and $H(z)$ be z-transforms of $x[n]$, $y[n]$, and $h[n]$, respectively. Express the above input-output relationship of a discrete-time modulator in z-transform domain.

15.

(a) Using the Gram-Schmidt orthogonalization procedure find set of orthonormal basis functions to represent the three signals $s_1(t)$, $s_2(t)$, and $s_3(t)$ shown in Figure 1. Sketch the orthonormal basis functions.

(b) Express each of these signals in terms of the set of basis functions found in part (a).

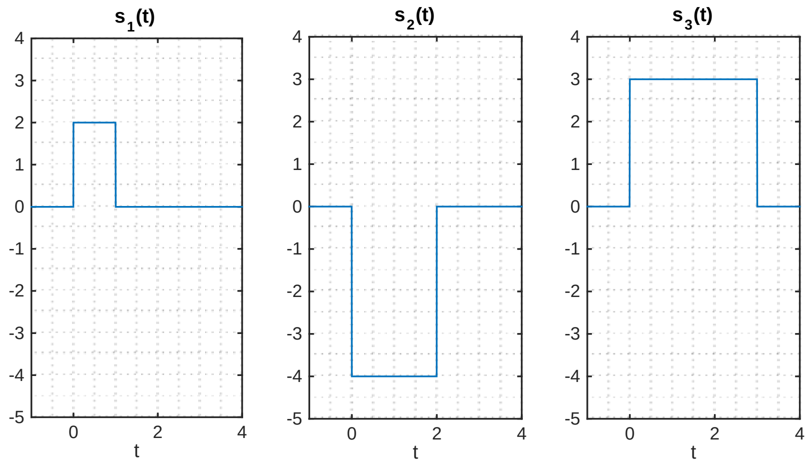

Figure 1: Three signals

Section 6

16. **Provide brief explanations and justifications to the following statements:**

a) Using band-theory and energy-related arguments, explain why a metal conducts, an insulator blocks current, and a semiconductor conducts current only under certain situations.

b) A PN junction is used as a photodetector. Assume that light is shining on all parts of the diode equally. Which part of the photodiode is most critical for photo detection and why? In this application, should the devices be under forward or reverse bias?

c) After repeated operations of a PMOS MOSFET, hole-type interface traps are formed in the Si-SiO2 interface, does this process increase or decrease the threshold voltage? Draw the band-diagram and the sub-threshold IV curve to illustrate your answer.

17. **MOS Band Diagram**

a) Draw the band diagram of a MOS system where the "metal" work function Φ_M is smaller than the silicon work function Φ_S. Assume that there are no applied voltages at the n-type substrate (doping N_D) and the gate. ON the diagram clearly label the following parameters and functions: electron affinity in the semiconductor x, the Fermi level E_f, the conduction and valance band edges E_c and E_v, the band-gap E_g, the mid-gap E_i, the thickness of the oxide d, the potential drop in the oxide ϕ_{ox}, and the potential drop in the semiconductor $\phi(x)$.

b) For this device, what is the most likely outcome when no voltages are applied: inversion or accumulation?

c) If the substrate is grounded, what voltage must be applied to the gate to reach flat-band conditions?

18. **Mobility and effective Mass**

a) State the defining equation for the material mobility of a semiconductor. Name two physical mechanisms that effect the mobility. State their temperature dependency. Sketch (qualitatively) the mobility for silicon as a function of doping level for both the acceptor and donor doped silicon, and briefly explain the trend curves.

b) Derive an equation for the effective mass of a semiconductor. Comment on its relationship to energy band structure. Thus explain, what band structure shape would lead to a high mobility.

Section 7

19.

A series of 10 information frames is to be transmitted from a primary to a secondary in half duplex mode across an established data link. The primary transmits one frame at a time and waits for an acknowledgement before transmitting the next frame. Describe in diagram form how the following problems are solved:

a) An error in the third information frame.

b) An error in the frame acknowledging receipt of the sixth frame.

c) The absence of a response from the secondary after the eighth frame.

Next, assume that the 10 information frames sent by the primary carry 100 bits of information each and the acknowledgment from the secondary contains 40 bits of information. The velocity of propagation on the link is $2x10^8$ meters per second, the link is 10 km long and has a capacity of 9600 bits per second. Determine

d) The link throughput in bits per second

e) The efficiency of utilization as a percentage of channel capacity

Ignore the processing and disconnect times, assume the link is error free in both directions, and no protocol errors occur.

20.

a)
You have the choice between using a digital transmission system or an analog transmission system for the output of a microphone with a frequency response from 0 to 2000 hertz. In the digital case you use 64 levels of quantization for the signal and a biphase baseband code. In the analog case you decide to use phase modulation of the carrier.

How many signals could be carried by a channel of bandwidth 200 kilohertz in the analog and digital modes? State all your assumptions

b)
Find the permissible dynamic range of an analog signal digitized according to the pulse code modulation method with 10 bits per sample and a required signal-to-quantization ratio of 20 dB.

21.

A frame of N bits, including an even parity bit, is transmitted over a line that has a bit error rate of p and operates at a speed of 1 kilobit per second. Errors are generated independently of each other.

a) Write an expression for the probability that the received frame has k bits in error in terms of N and p.

b) Find the probability that the first error occurs after 5 milliseconds of transmission time.

Section 8

22.

A three-phase 200MVA, 20kV, 60Hz salient pole synchronous machine has parameters Xd = 1.1 pu, Xq = 0.7 pu and Ra~0. The machine delivers 200MW at 0.9 lagging power factor to an infinite busbar.

Calculate the excitation voltage and the power angle. Draw the phasor diagram. (Hint: use per unit values and give your answers in pu)

23.

A wind turbine is to be designed with an electrical power output of 4.0 MW.
The rated upwind free wind speed is 10 m/s. Determine the length of the rotor blades and the height of the supporting tower in meters and the rotational speed of the rotor in rev/min if the tip-speed ratio whose value as 7.0 determines the maximum Power Coefficient of 0.40. Use the density of air as 1.225 kg/m^3

24.

A 600MVA, 20kV, 60-Hz round-rotor synchronous generator has an Inertia constant H = 3s.

Displayed on the axes below are Torque/Angle characteristics for various faults occurring on a double circuit transmission line when connected between a synchronous generator and an infinite busbar. Using the Equal Area Criterion, determine the critical switching times for both a 3φ fault and a Single Line to Ground fault when the input torque from the turbine is 1.0 pu as shown in the diagram.