THE GEORGE WASHINGTON UNIVERSITY
School of Engineering and Applied Science
Department of Electrical and Computer Engineering

Preliminary Examination

Fall 2007

General Instructions

Read carefully before starting.

Solve 5 problems in all; at most 2 questions may be selected from the same section.

Please write your name and student number below:

____________________________________ __________________

Student Name **Student Number**

Solve each problem in a <u>separate</u> blue book. Write the section number, problem number, and your student number on the front of each blue book. **DO NOT WRITE YOUR NAME ON THE BLUE BOOK.**

Submit solutions to only five (5) problems. Use only **ONE** blue book per problem.

For each problem, make a special effort to give the answers in a clear, legible form.

The exam will begin at 10:00 a.m. and end at 3:00 p.m.

Computers and calculators may not be brought into the examination room.

CLOSED BOOK EXAMINATION

SECTION 1

1. Consider using a 4-way set-associative cache memory of 4 kilobytes, where each cache line is 16 bytes, in a system that has 2 Mbyte of main memory.
    a. Sketch the memory system organization showing how memory locations are mapped into the cache
    b. Sketch the physical memory address layout for the purpose of accessing the cache
    c. Under which conditions would a set-associative cache become equivalent to a fully associative cache?
    d. Under which conditions would a set-associative cache become equivalent to a direct mapped cache?
    e. Discuss the cost and performance tradeoffs associated with the three common cache organizations: direct mapped, fully associative, and set associative.

2. A 4-way superscalar microprocessor has k stages per pipeline and uses a pipeline clock cycle of T seconds. If n instructions are to be executed:

    a. Derive an expression for the execution time
    b. Derive an expression for the Speed up of this microprocessor over a similar one which is neither a superscalar nor pipelined
    c. Derive an expression for the throughput
    d. What will be the execution time if the probability of an instruction to be a branch is “p” and the probability that a branch is taken is “q”?

3. Answer the following questions
   a. A computer system is using 16 bits to represent signed integers. How many negative and how many positive numbers are represented if the system is encoding these numbers using: i. the sign and magnitude notation? ii. the one’s complement notation? iii. the two’s complement notation?
   b. What are the largest and the smallest possible numbers that can be represented in the IEEE single precision format?
   c. Show the steps required to add 2 floating point numbers in the IEEE single precision format?
   d. Using your answer in c, sketch the basic structure of pipeline arithmetic process that can perform this operation

SECTION 2

4. Let $\{X_i\}_{i=-\infty}^{+\infty}$ be a sequence of independent identically distributed Laplacian random variables whose probability density function is given as

$$f_{X_i}(x) = \frac{\lambda}{2} e^{-\lambda|x-m|},\ x \in (-\infty, +\infty),\ \lambda > 0,\ m \in (-\infty, +\infty).$$

Moreover, let $\{Y_i\}_{i=-\infty}^{+\infty}$ be a sequence of independent identically distributed discrete random variables such that $P[Y_i = -1] = P[Y_i = +1] = \frac{1}{2}$. Also, assume that $\{X_i\}_{i=-\infty}^{+\infty}$ and $\{Y_i\}_{i=-\infty}^{+\infty}$ are jointly independent. The sequence $\{Z_i\}_{i=-\infty}^{+\infty}$ is defined as

$$Z_i = \begin{cases} X_{\frac{i}{2}}, & i \text{ even} \\ Y_{\frac{i-1}{2}}, & i \text{ odd.} \end{cases}$$

(a) Find m and λ such that the sequence $\{Z_i\}_{i=-\infty}^{+\infty}$ is wide sense stationary.

(b) For the values of m and λ found in (a), is the sequence $\{Z_i\}_{i=-\infty}^{+\infty}$ strict sense stationary? Explain your answer.

5. The random variable X has mean m_X and variance σ_X^2 while the random variable Y has mean m_Y and variance σ_Y^2. The random variables X and Y are jointly Gaussian with the correlation coefficient ρ_{XY}.

(a) Find the probability density function of $Z = aX + bY$.
(b) Find the correlation coefficient ρ_{ZX}.
(c) Calculate (a) and (b) for $m_X = m_Y = 0$, $\sigma_X = \sigma_Y = a = b = 1$, and $\rho_{XY} = 0.5$.

(Note: The general form of the Gaussian probability density function is

$$f_V(v) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(v-m)^2}{2\sigma^2}}$$

where m is the mean and σ^2 is the variance of a Gaussian random variable V.)

6. The input to a quantizer is a random positive voltage v whose pdf is

$$f_V(v) = a\exp(-av)u(v)$$

where $u(v)$ is the unit step function. The output of the quantizer is equal to the integer k when

$$k < V \le k+1, \quad 0 \le k < \infty.$$

Find the expected value of the output of the quantizer.

SECTION 3

7. The electric field of a plane wave propagating in a medium with a relative dielectric constant of $\varepsilon_r = 8$ and free space permeability is given by

$$\boldsymbol{E} = \left(\hat{\boldsymbol{x}} + \alpha \hat{\boldsymbol{y}}\right) e^{-j(2k_0 x + k_0 \xi y)} \quad , k_0 = \sqrt{\varepsilon_0 \mu_0}\,\omega$$

a) Find the constant ξ such that $\xi > 0$;

b) Determine α;

c) Find the magnetic field;

d) What is the average power density of the wave?

8. The vector potential $\mathbf{A}(\mathbf{r})$ produced by time-harmonic $e^{+j\omega t}$ currents $\mathbf{J}(\mathbf{r}')$ in free space is given by

$$\mathbf{A}(\mathbf{r}) = \frac{\mu}{4\pi} \iiint_{V'} \frac{\mathbf{J}(\mathbf{r}') e^{-jk|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|}.$$

i) Systematically show how to introduce approximations to obtain a direct expression for this vector potential in the far-field $|\mathbf{r}| \gg |\mathbf{r}'|$.

ii) Assume that the far field vector potential $\mathbf{A}_1(\mathbf{r})$ produced by a current distribution $\mathbf{J}(\mathbf{r}'-\mathbf{r}_1')$ is given. Show that the far field vector potential $\mathbf{A}_2(\mathbf{r})$ produced by a translated current distribution $\mathbf{J}(\mathbf{r}'-\mathbf{r}_2')$, identical in magnitude and direction with the first can be written in terms of $\mathbf{A}_1(\mathbf{r})$ and a function of the vector displacement $\mathbf{r}_2 - \mathbf{r}_1$.

iii) Show analytically that the same kind of relation holds for the electric and magnetic far-fields $\mathbf{E}_1(\mathbf{r})$, $\mathbf{H}_1(\mathbf{r})$ and $\mathbf{E}_2(\mathbf{r})$, $\mathbf{H}_2(\mathbf{r})$.

9. The magnetic field of a particular mode in a parallel-plate air waveguide with a plate separation of 2.5 cm is given by

$$H_z(x, y) = Ce^{-j640\pi x/3}\cos(160\pi y)$$

where x and y are both in meters.

a) Is this a TE_n or TM_n mode? What is n? Is it a propagating or non-propagating mode?
b) What is the operating frequency?
c) Find the corresponding electric field.
d) Find the lowest order mode of the same type (TE or TM) that does not propagate at this operating frequency.

SECTION 4

10. A linear time-invariant (LTI) system with input x[n] and output y[n] is specified by the difference equation

$$y[n-1] - \frac{5}{2} y[n] + y[n+1] = x[n] .$$

The system may or may not be stable or causal.

i) Find the Z transform of the systems function (unit impulse response) H(z).

ii) On consideration of the singularities of H(z) in the complex z-plane, mark CLEARLY the different possible regions of convergence (ROC) for H(z).

iii) Solve for the different unit input response functions h[n] corresponding to each of the regions of convergence.

11. The periodic signal $f(t)$ with period T is represented by the Fourier series

$$f(t) = \sum_{n=-N}^{n=N} c_n e^{i2\pi n/T}$$

The signal is sampled at regular intervals $\Delta t + T$. Show that the signal $\hat{f}(t)$ reconstructed from these samples may be represented by

$$\hat{f}(t) = f\left(t \frac{\Delta t}{T + \Delta t}\right)$$

provided the time increment Δt is chosen such that

$$\Delta t < T/2N$$

12. An input $e^{i\omega t}$ to a linear system results in the output

$$\frac{e^{i\omega\left(1-\frac{2v}{c}\right)t}}{(a+i\omega)\left(b+v^2t^2\right)}$$

where a, b, v and c are real positive constants and $2v/c<1$.

a) Is the system time invariant? Explain.
b) Find the impulse response
c) Is the system causal?
d) Find the output when the input is the pulse

$$p(t)=\begin{cases}1; -T\le t\le T\\ 0; |t|>T\end{cases}$$

SECTION 5

13. a) What is the iteration matrix using the Jacobi method for the following set of equations

$$4I_1 + 2I_2 = 6$$
$$2I_1 + 5I_2 = 7$$

b) What is the iteration matrix using the Gauss-Seidel method for the above set of equations?

c) What is the condition number for these equations?

d) Starting from [0 0], what is the value of I after one iteration for the Jacobi method?

e) Starting from [0 0], what is the value of I after one iteration for the Gauss-Seidel method?

14. Find the solution that minimizes the mean squared error for the following set of equations:

$$3x+5y=6$$
$$2x+4y=5$$
$$x+3y=4$$
$$2x+y=4$$

15. If $\mathbf{A} = \begin{bmatrix} 3.0000 & 1.0000 & 1.0000 \\ 1.0000 & 3.0000 & 0.5000 \\ 1.0000 & 0.5000 & 3.0000 \end{bmatrix}$.

a) Show that 2.5 is one of the eigenvalues.
b) What are the other eigenvalues of **A**?
c) Construct a set of orthonormal eigenvectors.
d) Find a similarity transformation that diagonalizes **A**.

SECTION 6

16. Derive an expression for the threshold voltage of a long channel *p*-channel MOSFET. Derive all necessary expressions and explain your basic assumptions and considerations. Account for the following physical mechanisms and parameters:

(i) Metal-semiconductor work function difference Φ_{MS}.
(ii) Insulator thickness t_{OX} and dielectric constant ε_{OX}.
(iii) Silicon dielectric constant ε_{Si} and substrate doping N_d.
(iv) Fixed oxide charge Q_f and Si-SiO_2 interface charge Q_{it}.
(v) Substrate to source potential V_{SB}.

Next, consider a device where the channel length is only 130 nm, i.e., a short channel device. Discuss the applicability of the above derivation for such a device. What are the primary short channel device parameters that effect the change in the threshold voltage from the above long channel derivation?

17. The following are questions related to basic device physics concepts. Please be concise in your responses.

a) Explain the difference between direct and indirect Generation-Recombination (G-R). In your explanation state what specific solid state material properties are critical for a material to exhibit direct vs. indirect G-R. State at least one practical material choice consideration in designing a photo-detector.
b) For typical device parameters, the Debye length is considered to be a small length compared to the depletion thicknesses. State two regions or conditions in a MOSFET device where the Debye length is relevant and explain why this is so.

18. According to the drift-diffusion transport model the hole current density, J_p, is the sum of the drift and diffusion currents: $J_p = -q\ (\mu_p\ p\ \nabla\phi + D_p\ \nabla p\ )$, where, q is the electronic charge, μ_p is the hole mobility, p is the hole charge density, ϕ is the electrostatic potential, and D_p is the hole diffusion constant.

a) Show that when the semiconductor is in thermal equilibrium, the hole charge density can be expressed as follows: $p = n_i\, exp(-\phi\,/v_T)$, where v_T is thermal voltage and n_i is the intrinsic charge density.

b) The hole quasi-Fermi potential, ψ_p, is defined as follows: $p = n_i\, exp[-(\phi - \psi_p\ )/v_T]$. Derive an expression for the hole current density in terms of p and ψ_p. Based on this (a very similar expression can be derived for the electron current density, as well), explain how a *pn*-junction can have an electric potential across it yet exhibit zero terminal current and zero measured voltage drop across its terminals at thermal equilibrium.

SECTION 7

19. Consider a Selective Reject ARQ (SR-ARQ) protocol over a communication link with a packet error probability equal to P. Explain the operation of SR-ARQ and derive a formula for its average throughput.
HINT: First derive an expression for a probability mass function of an average number of transmissions.

20. Telephone calls arrive at a local exchange according to a Poisson process at the rate of 10 calls per hour.

a) What is the probability that more than 3 calls arrive in a 10 minute interval?

b) Given that 5 calls arrived between 2 P.M. and 2:30 P.M., what is the probability that all of the calls arrived between 2 and 2:15 P.M.

21. The hypercube network is a regular network topology used for computer interconnection networks. A hypercube of order n (integer) has 2^n nodes, and each node has a distinct n-bit binary address. Nodes whose binary addresses differ in exactly one bit are connected by an (undirected) link.

a) What is the number of links in the network?

b) The diameter of a network is defined to be the maximum (over all node pairs) of the shortest paths between nodes. What is the diameter of the hypercube? (Assume that the length of a path is the number of links on the path.).

c) Suppose a node transmits packets to every other node in the network. The destination node of the packet is chosen with equal probability. Find the average number of links traversed by an arbitrary packet.

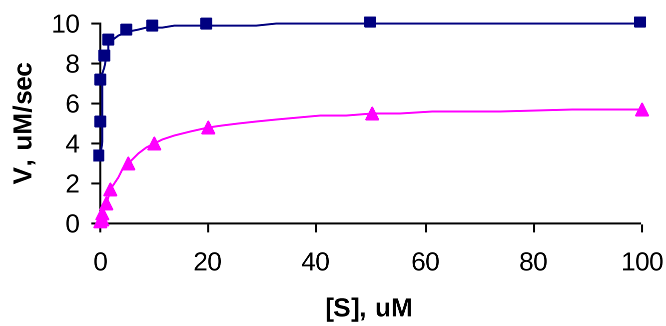
SECTION 8

22. Assume the existence of a stimulus that begins to depolarize the cell membrane of an excitable cell. Explain/describe/discuss how an action potential is then generated. Your answer should include, but not necessarily be limited to, the following points …

A. Voltage gated sodium and potassium channels.
B. The relative timing of sodium and potassium channel gating.
C. Relative permeability changes of sodium and potassium channels.
D. Recovery of membrane potential back to its resting level.
E. Positive afterpotential.

23. Describe/explain/discuss the mechanism of the generator potential of the photoreceptor cells in the human eye.

24. The Figure (below) shows data from an experiment to measure the transport of two different substances (S_1, squares; S_2, triangles) into human red blood cells. Answer parts (A) and (B) based on the Figure and using your knowledge of cellular membrane transport mechanisms.



A. Which substance (S_1 or S_2) has the higher K_m? Explain.

B. Using the information provided is it possible to determine if S_2 is being transported up its electrochemical gradient? Explain.