THE GEORGE WASHINGTON UNIVERSITY
School of Engineering and Applied Science
Department of Electrical and Computer Engineering

Preliminary Examination - Spring 2014

Thursday, February 13, 2014

General Instructions. Read carefully before starting.

Solve 5 problems in all; at most 2 questions may be selected from the same section.

Candidates registered in the following focus areas must answer two of their five questions from the relevant section as follows:

Biomedical Engineering:	Section 8
Computer Engineering:	Section 1
Communications & Networks	Section 3
Electrical Power & Energy	Section 9
Electromagnetics, Radiation Systems & Microwave Engineering	Section 4
Microelectronics & VLSI	Section 6
Signal & Image Processing, Systems & Controls	Section 3

Please write your name and student number below:

________________________________ ____________________

Student Name **Student Number**

Solve each problem in a **separate** blue book. Write the section number, problem number, and your student number **on the front of each blue book**. **DO NOT WRITE YOUR NAME ON THE BLUE BOOK.**

Submit solutions to **only** five (5) problems. Use only **ONE** blue book per problem.

For each problem, make a special effort to give the answers in a clear form.

The exam will begin at 10:00 a.m. and end promptly at 3:00 p.m.

Only Calculators provided by the department at the examination will be allowed. Personal items including cell phones and other electrical devices must be relinquished prior to the start of the examination.

This is a CLOSED BOOK, CLOSED NOTES EXAMINATION

Section 1

1) Assume that a microarchitecture has a five stage pipeline-- Fetch, Decode, Execute, Memory and WriteBack, each stage taking one cycle latency to finish its intended operation. The pipeline is single issue, i.e., it can take only one instruction up for execution at a given time cycle. Assume that there are two forwarding paths-- (1) forward results of EX stage to the next immediate instruction entering the EX stage, and (2) forward the results of MEM stage to the incoming instruction entering the EX stage. You should assume that all ALU related operations including load/store address computation are done in the EX stage of the pipeline, the actual reading or writing of the data values happen in the MEM stage, and that all load/store operations are cache hits needing only one cycle to finish their memory-related operations.

On this machine, show stage-by-stage progress of instruction execution and compute the total number of cycles needed to finish the execution using the following set of instructions:

Load R1, 8(R10)

Add R2, R3, 5

Mul R4, R2, R1

Store R4, 8(R10)

2) For a direct mapped cache design with 32 bit address, the bits of the address are used as follows: bits 32-10 for Tag, bits 9-5 for Index, and bits 4-0 for offset.

a) What is the cache block size? How many entries does the cache have?

b) Assuming the cache starts empty, for the following sequence byte-addressed cache reference: 0, 4, 16, 132, 232,160,1024, 30,140, 3100, 180, 2180.

i. What is the hit ratio? How many blocks are replaced?

ii. List the final state of the cache for each valid entry.

3) a) Draw a diagram to illustrate the internal organization of a magnetic hard drive.

b) What is the average time to read or write a 4096-byte sector for a typical disk rotating at 10,000 RPM? The average seek time is 5 ms, the transfer rate is 80 MB/sec, and the controller overhead is 0.2 ms. Assume that the disk is idle so that there is no waiting time.

Section 2

4) Consider the dual systems

$$S = \{\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}\} \text{ and } S_d = \{\mathbf{A}_d, \mathbf{B}_d, \mathbf{C}_d, \mathbf{D}_d\}$$

a) Prove that if S is controllable, then S_d is observable and if S is observable, then S_d is controllable;

b) It is claimed that

i. If S stable in the sense of Lyapunov, it does not necessarily imply that S_d is also stable in the sense of Lyapunov. Prove or disprove the claim.

ii. If S is BIBO stable it does not necessarily imply that S_d is also BIBO stable. Prove or disprove the claim.

5) The transfer function of a system is

$$H(s) = \frac{s - 2}{s^5 + 5s^4 + 9s^3 + 9s^2 + 8s + 4}$$

a) Is the system asymptotically stable? Marginally stable?

b) If it is asymptotically stable, is the time constant, $t > 1$ or $t < 1$?

c) If it is not asymptotically stable, could you modify H(s) to make it so with a time constant $t\ {}^{3}\ 1$? If yes, indicate how you would do it and show the modified system.

6) Given an m x k matrix $\mathbf{A}$ and an k x n matrix $\mathbf{B}$, prove that

$\rho(\mathbf{AB}) = \min(\rho(\mathbf{A}),\ \rho(\mathbf{B}))$ where ρ designates rank.

Section 3

7) The input to a digital filter is an iid random sequence $\ldots, X_{-1}, X_0, X_1, \ldots$ with $E[X_i] = 0$ and $Var[X_i] = 1$. The output $\ldots, Y_{-1}, Y_0, Y_1, \ldots$ is related to the input by the formula

$$Y_n = X_n + X_{n-1} \quad \text{for all integers } n.$$

Find the expected value $E[Y_n]$ and the autocovariance function $C_Y[m, k]$.

8) a) Suppose the continuous-time random process $X(t)$ has autocorrelation function

$$R_X(\tau) = 1 + e^{-\alpha|\tau|}, \quad -\infty < \tau < \infty$$

where $\alpha > 0$. Find the power spectral density function for $X(t)$.

b) Suppose the random process $X(t)$ is the voltage across a series RLC circuit. Let $Y(t)$ be the voltage across the capacitor. Find the power spectral density function of the process $Y(t)$.

9) Show that if $X(t)$ is a wide sense stationary Gaussian process, then $X(t)$ is a stationary Gaussian process.

Section 4

10) A uniform plane wave having a wavelength of 1 cm is propagating in an infinite medium with $\varepsilon = 2\varepsilon_0$ and $\mu = 2\mu_0$. the wave is polarized in the $\hat{\mathbf{x}}$ direction and is propagating in the +z direction with an average power density of 1 watt/m^2. In addition, the time varying electric field has a maximum at z=t=0.

a) Find the propagation constant, β, and the frequency of the wave.

b) Determine the phasor electric and magnetic fields.

c) Find the time varying electric field.

11.

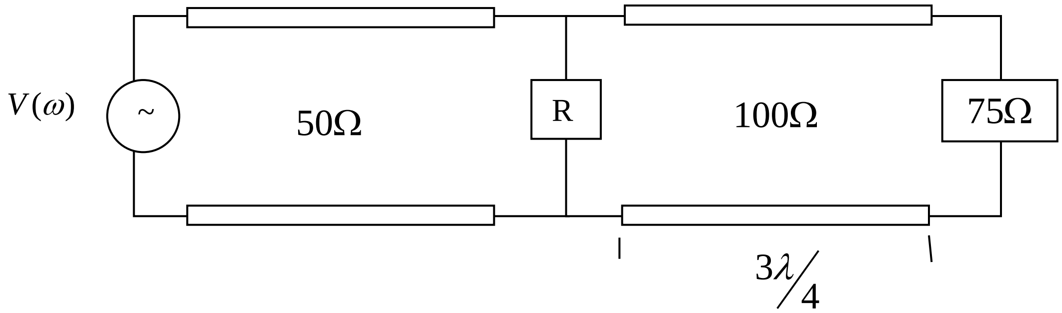


Figure 1

a) Determine the value of R so that no power is reflected down the 50Ω line shown in Fig. 1.

b) Given that the source (time harmonic) V = 10 volts, what is the power dissipated in the resistance R and the 75Ω load?

12)

a) Show that in the far zone, the electric field radiated by a Hertzian dipole (placed along the z-axis, have a length of δ and constant current I_0) can be approximated by

$$E_\theta(r,\theta) \cong j\eta \frac{kI_0\delta e^{-jkr}}{4\pi r}\sin\theta \qquad (1)$$

where r is the distance between the observation point and the dipole, θ is the angle between the z-axis and r-vector as shown in Fig. 2 and η is the impedance of the background medium.
Hint: $\hat{z} = \hat{R}\cos\theta - \hat{\theta}\sin\theta$

b) Use Eq. (1) and obtain an approximate expression for the electric field radiated by a Hertzian dipole (vertical electric dipole) that is placed a distance h above an infinite, flat, perfect electric conductor (PEC) as shown below. Again, the infinitesimal dipole is assumed to have a length of δ and constant current I_0. The arrow indicates the polarity of the source. The observation point P is in the far-zone, i.e. $r \gg \lambda$.
Note: Partial credit might be given for an appropriate sketch explaining the implementation of image theory that could be used to solve this problem.

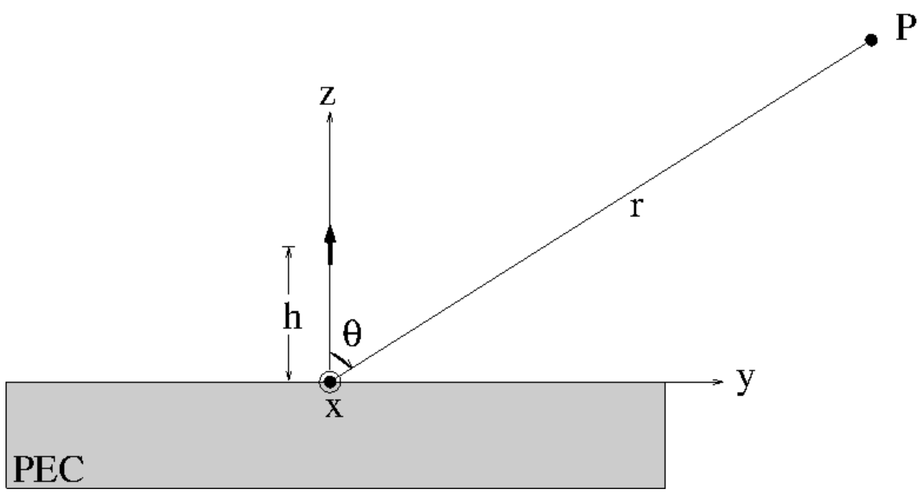


Fig. 2 Dipole above ground plane

Section 5

13) a) Find the Fourier series of the Dirac delta impulse train

$$v(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT) .$$

b) The ideal uniform samples of a continuous-time signal $g(t)$ are obtained as

$$x(t) = g(t)v(t) .$$

Find the spectrum (Fourier transform) of $x(t)$ assuming that the spectrum of $g(t)$ is $G(f)$.

c) Under what conditions $g(t)$ can be reconstructed without loss from $x(t)$? Explain the reconstruction procedure.

d) Another train of uniform samples is obtained as

$$y(t) = g(t)w(t)$$

where

$$w(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT - \frac{T}{2})$$

i.e., $w(t)$ is the Dirac delta impulse train $v(t)$ shifted to the right by $\frac{T}{2}$.

Find the spectrum of $w(t)$. What is the condition for errorless reconstruction of $g(t)$ from $w(t)$?

e) The two ideal sample trains are merged, i.e.,

$$z(t) = x(t) + y(t) .$$

What is the spectrum of $z(t)$? Is the errorless reconstruction of $g(t)$ from $z(t)$ possible and under what condition?

14) The Wigner time-frequency distribution for a signal $s(t)$ is defined as

$$\mathrm{W}_s(t, f) = \int_{-\infty}^{+\infty} s^*(t - \frac{\tau}{2}) s(t + \frac{\tau}{2}) \mathrm{e}^{-j2\pi f \tau} \, \mathrm{d}\tau$$

where $s^*(t)$ is complex conjugated $s(t)$ and $j = \sqrt{-1}$.

a) Show that

$$\int_{-\infty}^{+\infty} \mathrm{W}_s(t, f) \, \mathrm{d} f = | s(t) |^2 .$$

b) Show that

$$\int_{-\infty}^{+\infty} \mathrm{W}_s(t, f) \, \mathrm{d} t = | S(f) |^2$$

where $S(f)$ is the Fourier transform of $s(t)$.

c) Show that

$$\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \mathrm{W}_s(t, f) \, \mathrm{d} t \, \mathrm{d} f = E_s$$

where E_s is the signal energy, i.e., $E_s = \int_{-\infty}^{+\infty} | s(t) |^2 \, \mathrm{d} t$.

d) Let $g(t) = s(t - t_0)$, show that

$$\mathrm{W}_g(t, f) = \mathrm{W}_s(t - t_0, f) .$$

e) Let $h(t) = s(t) \mathrm{e}^{j2\pi f_0 t}$, show that

$$\mathrm{W}_h(t, f) = \mathrm{W}_s(t, f - f_0) .$$

15) a) Find all impulse responses of a linear time-invariant system that can have Laplace transform

$$H(s) = \frac{-3}{(s+2)^2(s-1)}.$$

b) Which of these impulse responses correspond to a bounded-input/ bounded-output system? Find its frequency response.

c) The input of the system described by the impulse response in (b) is

$$x(t) = \cos\left(t + 2\tan^{-1}\left(\frac{1}{2}\right)\right).$$

Find the output $y(t)$ of the system.

Section 6

16) Provide clear explanations to the following questions:

a) Draw the energy band diagram of a metal-semiconductor junction under the condition that the work-function of the metal layer is larger than the work-function of the semiconductor layer. Does such a material system allow the construction of an Ohmic contact? If so, describe how this can be accomplished employing the band-diagram. If not, why not?

b) Describe how the depletion region of a *pn*-junction acts as a "current source" under reverse bias. State at least two physical mechanisms that allow this to take place.

c) Why is the saturation velocity of the electron important to determine the intrinsic limit of the response speed of a MOSFET? List any relevant physical factors that determine the speed of electrons in the channel of a MOSFET?

17) The drift-diffusion transport model states that the hole current density, $\boldsymbol{J}_\mathrm{p}$, is the superposition of the drift and the diffusion currents, and is expressed as follows:

$$\boldsymbol{J}_p = -q\,(\mu_p\, p\ \nabla\phi + D_p\ \nabla p\,).$$

Here,* $\mu_\mathbf{p}$ *denotes the mobility,* p *is the hole charge density,* ϕ *is the electrostatic potential,* $\mathbf{D_p}$ *is the hole diffusivity and* q *is the electronic charge.

a) ***State the condition for thermal equilibrium. Then, show that the hole charge density can be expressed as follows under thermal equilibrium conditions:***

$$p = n_i\, exp(-\phi\,/v_T)\ ,$$

Note that v_T is thermal voltage and n_i is the intrinsic charge density.

b) ***The definition of the hole quasi-Fermi potential,* $\psi_\mathbf{p}$*, is as follows:***

$$\mathbf{p = n_i\, exp[-(\phi\ -\ \psi_p\ )/v_T].}$$

Based on this definition, derive an expression for the hole current density in terms of* p *and* $\psi_\mathbf{p}$*. In the context of the expression you derive, clearly articulate the physical interpretation of the quasi-Fermi potential.

18) Basic *pn*-junction operation: Consider the ideal so-called "long-base" abrupt *pn*-junction silicon diode that has a uniform cross section and constant doping on both sides of the *pn*-junction. The diode is doped as follows: $N_a = 5.0 \times 10^{16}$ cm^{-3} *p*-type and $N_d = 3 \times 10^{16}$ cm^{-3} *n*-type. For this material, the minority-carrier lifetimes are: $\tau_n = 2 \times 10^{-6}$ s and $\tau_p = 1 \times 10^{-6}$ s, respectively. You may assume that the effects within the space-charge region are negligible and that the minority carriers flow only by diffusion in the charge neutral regions.

a) Draw/sketch the band-diagram for this system. Also, plot the electrostatic potential, the net charge density and the corresponding electric field.

b) Determine the value of the built-in potential across the pn-junction.

c) Calculate the density of the minority carriers at the edge of the space-charge region for a forward bias of 0.25V.

d) Under the above bias condition, calculate and plot the minority and majority carrier currents as a function of distance from the junction.

Section 7

19) A packet multiplexer is fed by 4 data sources and can transfer at most two packets to its output in one slot. If more than 2 packets are generated, the excess packets are dropped. (Assume dropped packets are randomly selected.) Each source generates a packet in each time slot with probability 0.5.

a) What is the expected throughput of the multiplexer, i.e., what is the average number of packets transferred to the output per slot?

b) What is the average number of dropped packets per slot?

c) What is the probability that a given input packet would be dropped?

20) A 100 km link of rate 1 Mbps uses the Stop-and-Wait ARQ protocol to transfer frames of length 1000 bits. Assume that signals propagate on the link at a speed of 200,000 kilometers per second, and ignore the size of ACKs.

a) What is the effective data rate of the link if there are no errors?

b) Derive an expression for the link length L as a function of the effective data rate R, assuming no errors.

c) If the bit error rate is 10^{-4}, what is the effective data rate?

21) Suppose that a group of 32 stations is serviced by a token-ring LAN. Calculate the frame transfer times for each of the two cases below for each of these two token reinsertion strategies: (i) token is released as soon as the frame is transmitted, and (ii) token is released after the entire frame recirculates back to the source.

a) 1000-bit frame; 10 Mbps speed; 2.5-bit latency/adapter; 50 meters between stations

b) Same as (a) except 100 Mbps speed and 8-bit latency/adapter.

Section 8

You need to choose up to 2 questions from **either** Sub-section 8A **OR** Sub-section 8B. You are not allowed to answer questions from **BOTH** Sub-sections.

Sub-section 8A

22) a) Draw the anatomy of a typical cell; list various organelles and other components of the cell and explain the function of these components.

b) Draw the cell membrane, and explain its structure and function.

23) a) List and explain primary functions of the skeletal system.

d) Draw the structure of a typical bone and explain the main components.

e) Discuss different types of bone cells and their function.

24) a) Describe 3 mechanisms of hypothalamic control over endocrine system.

b) What hormones are produced by the adrenal gland and what is their function?

Sub-section 8B

25) Define sensitivity and specificity of a diagnostic test. What false results will arise more often if a test has low sensitivity but high specificity? How about low specificity but high sensitivity? Suppose 100 people take the H1N1 virus infection test, 5 people are actually infected. Fill in the following table for the following cases (a): Sensitivity = 100%, Specificity = 0%; (b): Sensitivity = 0%, Specificity = 100% .

<table>
<tr><td colspan="2" rowspan="2"></td><td colspan="2">Actual Condition</td></tr>
<tr><td>Infected (5)</td><td>Not Infected (95)</td></tr>
<tr><td rowspan="2">Test Result</td><td>Positive: Test shows "infected"</td><td>TP</td><td>FP</td></tr>
<tr><td>Negative: Test shows "not infected"</td><td>FN</td><td>TN</td></tr>
</table>

26) Draw a typical electrocardiogram and label all waves and intervals. Explain what is happening electrically within the heart during each wave or interval.

27) a) Define and sketch Einthoven’s triangle.

b) Define Wilson’s central terminal and sketch the corresponding electrode connections.

Section 9

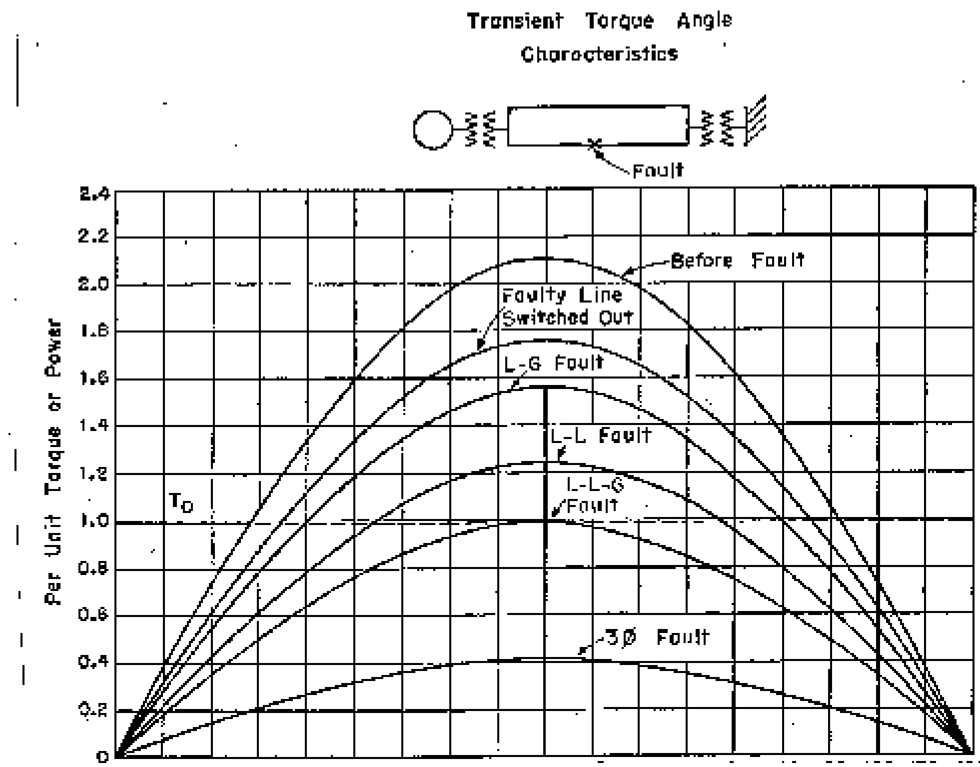
28) A three-phase 200MVA, 20kV, 60Hz salient pole synchronous machine has parameters Xd = 1.0 pu, Xq = 0.7 pu and Ra~0. The machine delivers 160MW at 0.9 lagging power factor to an infinite busbar.

Calculate the excitation voltage and the power angle. Draw the phasor diagram. (Hint: use per unit values and give your answers in pu)

29) A wind turbine is to be designed with an electrical power output of 3.0 MW. The rated upwind free wind speed is 13 m/s. Determine the length of the rotor blades in meters and the rotational speed of the rotor in rev/min if the tip-speed ratio is determined at the maximum Power Coefficient value as 7.0 and the density of air is 1.225 kg/m^3

30) A 250MVA, 20kV, 60-Hz round-rotor synchronous generator has an Inertia constant H = 3s.

Displayed on the axes below are Torque/Angle characteristics for various faults occurring on a double circuit transmission line when connected between a synchronous generator and an infinite busbar. Using the Equal Area Criterion, determine the critical switching times for both a 3φ fault and a Line-Line fault when the input torque from the turbine is 1.0 pu as shown in the diagram.



28)