THE GEORGE WASHINGTON UNIVERSITY
School of Engineering and Applied Science
Department of Electrical and Computer Engineering

Preliminary Examination

Fall 2003

General Instructions

Read carefully before starting.

i) Solve five (5) problems in accordance with the following selection rule: choose two (2) problems from any one of the seven sections, two (2) problems from one of the other sections, and the remaining problem from any section, including from either of the sections previously selected.

ii) Solve each problem in a separate blue book. Write the section number, problem number, and your student number on the front of each blue book. **DO NOT WRITE YOUR NAME ON THE BLUE BOOK.**

iii) Submit solutions to only five (5) problems. Use only **ONE** blue book per problem.

iv) For each problem, make a special effort to give the answers in a clear form.

v) The exam will begin at 9:00 a.m. and end at 2:00 p.m.

!!GOOD LUCK!!

Section 1

1a.

Set up a linear system of equations and solve it.
In a certain town 30% of the married women get divorced each year and 20% of the single women get married each year.

a) Assuming that the total population of women remains constant, how many married women (M(t)) and how many single women (S(t)) will be there after one year? Write down a matrix that describes the transition of women between the two groups each year.

b) After 3 years? After 10 year? Sketch your data for M(t) and S(t). Find the steady state vector for the matrix in a).

c) Write out explicitly the spectral decomposition of the solution, based on the eigenvectors of A.

1b.

A strain of bacteria has a growth rate that is proportional to the size of the population. Initially, there are 100 bacteria; after 3 hours there are 1600.

a) If $p(t)$ denotes the number of bacteria after t hours, find the formula for $p(t)$.

b) How long does it take the population to double?

c) When will the population reach 1 million?

2a. A(n x n) ; Λ = $\mathbf{S}^{-1}$ **A S**

Diagonalize matrix A:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Check your solution by back substitution.

2b. a) Find the singular value decomposition for the following matrix:

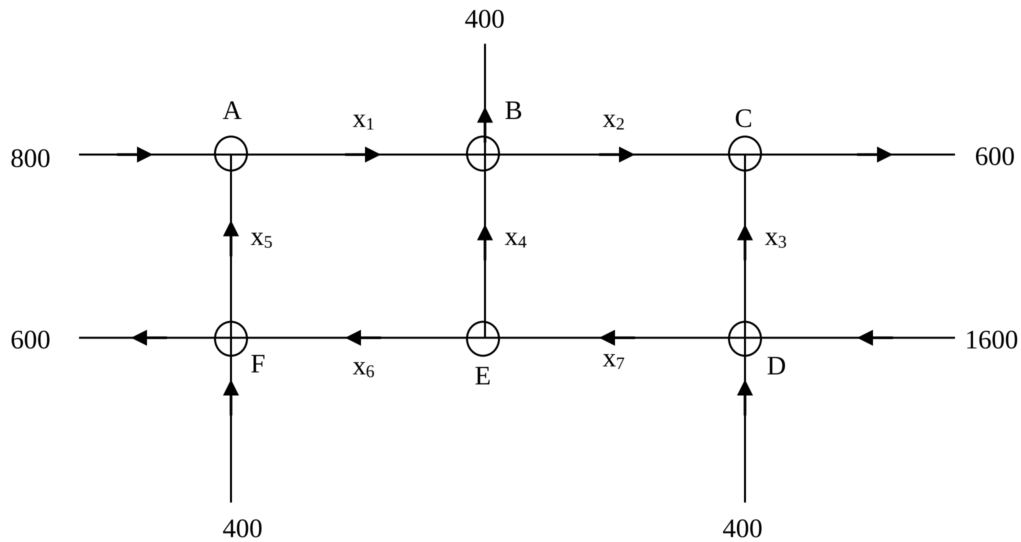
$$A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

b) Extend $\{\vec{u}_1, \vec{u}_2\}$ to an orthonormal basis for $\mathbf{R}^3$.

c) Find the pseudoinverse of A.

d) Check your solution by back substitution.

3. **Flow of traffic** – network of one-way streets.



Set up a system of equations that represent the traffic flow for the network shown in the figure. The numbers give the average flow into and out of the network at peak traffic hours.

a) Solve the system of equations, i.e. determine the traffic flow in the network.

b) What is the traffic flow if $x_6 = 300$ and $x_7 = 1300$ vehicles per hour?

c) The street segments AB and BC should be closed for road repair, i.e. $x_1 = x_2 = 0$. What is the traffic flow in this case?

d) To avoid non-physical negative flow, you might want to change the direction of traffic for some one-way streets, which flow direction should be changed?

Section 2

4. Consider a 5-stage pipelined RISC scalar processor, with one clock cycle per stage.

   a) List the microoperations needed to execute a “register-relative” LOAD instruction.

   b) List the microoperations needed to execute an “ALU-type instruction.”

   c) Draw the execution timing diagram of the pipeline for the 3 instruction code LOAD-ALU-ALU (i.e., a Load instruction in the program is followed by two ALU-type instructions).

   d) What is the throughput rate of the pipeline? How long will it take to execute these 3 instructions?

   e) Indicate whether there are any possible problems/hazards and present some possible solutions.

Make sure you state all your assumptions.

5. Design a 16-bit memory of total capacity 8192 bits using SRAM chips of size 64x4 bits. Arrange these chips in a rectangular array configuration showing all logic required for interfacing it to the data and address bus lines of a little-endian processor. Assume this designed target memory is to be assigned to the highest address space. The design should allow for both byte and 16-bit word transfers.

   Make sure you state all your assumptions.

6. Consider a 32-bit big-endian processor, with a 32-bit external data bus, a 29-line external address bus, and 4 “byte-enables”. The processor executes a store operation to store the operand AABBCCDD to memory location 12345676.

   a) Show the timing diagram (with the hexadecimal values on the bus lines and the most important signals) when the memory subsystem is 32 bits wide.

   b) Show the timing diagram (with the hexadecimal values on the bus lines and the most important signals) when the memory subsystem is 16 bits wide.

   Make sure you state all your assumptions.

Section 3

7. An electromagnetic field in free space, $\mu_0 = 4\pi \times 10^{-7}$ henry/meter, $\varepsilon_0 = 8.85\times 10^{-12}$ farads/meter, is specified as by the vector phasor

$$\vec{E}(\vec{r}) = \vec{E}_0 \varepsilon^{-j\vec{k}\cdot\vec{r}}$$

where $\vec{E}_0 = \vec{x}^0$ the unit vector in the x direction of a rectangular coordinate system (x, y, z).

$$\vec{r} = x\vec{x}^0 + y\vec{y}^0 + z\vec{z}^0$$
$$\vec{k} = -j\vec{y}^0 + 2\vec{z}^0.$$

a) What is the frequency f of the electromagnetic field (Hz)?

b) Describe the equi-phase surfaces of the field. Write a general equation for the equi-phase surfaces.

c) Describe the constant magnitude-of-field surfaces. Write a general equation for these equal-magnitude surfaces.

d) Evaluate the time average power density as a function of position.

8. A 50 ohm characteristic impedance transmission line is terminated in a load impedance $Z_L = 50 - j50$ (ohms). The transmission line is excited by a wave incident on the load with a wavelength of 10 meters.

a) Calculate the reflection coefficient at the load.

b) Calculate the standing-wave-ratio on the transmission line.

c) Calculate the physical distance from the load to the voltage maximum nearest the load*.

d) Calculate the physical distances from the load at which a shunt capacitor can be placed across the transmission line to match the load to the characteristic impedance of the transmission line*.

e) *Make two suitable sketches the reflection coefficient plane (Smith Chart) which illustrate the calculations done in connection with c) and d).

9. The vector potential, vector phasor $A(\vec{r})$, produced by an electric current density $\vec{J}(\vec{r})$ is given by the integral

$$\vec{A}(\vec{r}) = \frac{\mu}{4\pi}\int \vec{J}(\vec{r}')\frac{e^{-jk|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|}d\tau'$$

where $\vec{r} = x\vec{x}^0 + y\vec{y}^0 + z\vec{z}^0$ is the position vector to the field observation point and $\vec{r}'$ a similar position vector to the current source point.

a) Carefully introduce and justify the approximations to produce a formula for the approximate vector potential for the far field.

b) Derive the far-field electric field intensity (radiation field) produced by surface currents in the z = 0 plane

$$\begin{aligned} \vec{J}_s(x,y,0) &= \vec{y}^0\cos\left(\frac{\pi x}{2a}\right) \quad && \begin{matrix} -a < x < +a \\ -b < y < +b \end{matrix} \\ &= 0 && \textit{otherwise}; \end{aligned}$$

in terms of the given parameters and any other constants you may find it necessary to introduce and define.

Section 4

10. a) Compare between the operation of the stop-and-wait ARQ, the Goback N ARQ (GBN), and the selective repeat ARQ. For comparison purposes use window size of N = 7, normalized propagation delay a = 4 (N < 2a + 1). In each case explain contingencies taken into account for damaged frames and damaged ACKs.

Prove the utilization in stop-and-wait ARQ is given by $U = \frac{(1-p)}{(1+2a)}$ where p is the probability that a single frame is in error.

b) For a satellite channel (one way end-to-end propagation delay is 0.25 sec) calculate the link utilization if p = 0.1 and a stop-and-wait ARQ is used to transmit 2000 bit frames with rate 1 Mbps.

11. Two packets P_1 and P_2 simultaneously arrive at a host to be transmitted over the network. The host has been idle prior to these arrivals and needs to decide which packet to transmit first. The goal is to minimize the average delay that the two packets experience prior to completely leaving the node.

Assume that the packet lengths are T_1 and T_2 respectively, with $T_1 > T_2$. The host has three alternatives:

i) It transmits P_1 first.

ii) It transmits P_2 first.

iii) It flips a fair coin to determine which packet to transmit first.

Calculate the average delay for the three strategies and compare their performances. Explain the results clearly.

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12. Consider a data communication system using the send and wait protocol. The number of bits in the messages to be transmitted by the system are uniformly distributed between 500 and 1500 bits. Acknowledgement messages are 20 bits in duration. The transmission rates in both the forward and feedback channels are 9600 bps. Assume that the bit error rates in the forward and feedback channel are 10^{-4}, independent from bit to bit.

    a) What is the probability that both the message and its ACK are received correctly?

    The timeout interval after transmission is T = 20 msec. Assume that the round trip delay has the following probability distribution function:

$$
\begin{aligned}
B(t) &= 0, && t \leq 10\ m\mathrm{sec} \\
&= 1 - e^{-200(t-0.01)}, && t > 10\ m\mathrm{sec}
\end{aligned}
$$

    b) What is the total time required to transmit a message on the average?

Section 5

13. A device has two inputs x_1 and x_2 and one output y defined as

$$y = \begin{cases} x_1 + x_2 & \text{if} \quad x_1 \geq x_2, \\ 0 & \text{if} \quad x_1 < x_2. \end{cases}$$

Two random signals $X_1(t)$ and $X_2(t)$ are applied to the inputs x_1 and x_2 respectively. The corresponding output is $Y(t)$. Find the average value of

$$D(t) = |Y(t) - X_1(t)|$$

assuming that $X_1(t)$ is uniformly distributed between 0 and 2, $X_2(t)$ is uniformly distributed between 0 and 2ρ, where $\rho \leq 1$, and $X_1(t)$ and $X_2(t)$ are independent.

14. Let X be a Laplace random variable with the probability density function $f_X(x) = \frac{\alpha}{2} e^{-\alpha|x-1|}$. In order to build a nonuniform quantizer, the range of X should be partitioned in six non-overlapping intervals: $I_1 = (-\infty, x_{-2}], I_2 = (x_{-2}, x_{-1}]$, $I_3 = (x_{-1}, 1], I_4 = (1, x_1], I_5 = (x_1, x_2]$, and $I_6 = (x_2, +\infty)$. Find the partition points x_{-2}, x_{-1}, x_1, x_2 such that $P(X \in I_k) = P(X \in I_l)$, $k = 1, \ldots, 6$, $l = 1, \ldots, 6$, i.e. such that X is equally probable to be in any of these intervals.

15. For the sequence of independent identically distributed continuous random variables $\{X_n\}_{n=-\infty}^{+\infty}$, find

a) the probability that X_4 is a local maximum (i.e., $X_4 > X_3$ and $X_4 > X_5$),

b) the probability that X_4 is a local maximum given that X_2 is a local maximum.

(Note that you may need to use the following equality:

$$\int_a^b f(x)F^n(x)dx = \frac{1}{n+1}\left(F^{n+1}(b) - F^{n+1}(a)\right)$$

where $f(x)$ is the probability density function, $F(x)$ is the corresponding cumulative distribution function, and $n = 0, 1, 2, \ldots$).

Section 6

16. According to the standard drift-diffusion transport model, the hole current density, J_p, is the summation of the drift and diffusion currents:

$$J_p = -q\,(\mu_p\, p\, \nabla \phi + D_p\, \nabla p\,).$$

Here, q is the electronic charge, μ_p is the hole mobility, p is the hole charge density, ϕ is the electrostatic potential, and D_p is the hole diffusion constant.

a) When the semiconductor is in thermal equilibrium, show that the hole charge density can be expressed as follows:

$$p = n_i \exp(-\phi\,/v_T)$$

where v_T is thermal voltage and n_i is the intrinsic charge density.

b) The hole quasi-Fermi potential, ψ_p, is defined as follows:

$$p = n_i \exp[-(\phi - \psi_p\,)/v_T].$$

Derive an expression for the hole current density in terms of p and ψ_p. Based on this expression, what is the physical interpretation of the quasi-Fermi potential?

17. Provide clear and concise answers to the following questions on the physical operation of semiconductor devices:

a) With all other parameters remaining unchanged, describe the effect of the narrowing of channel width (not length) on the value of the threshold voltage of a MOS device.

b) Draw the band diagram of a metal-semiconductor barrier where the workfunction of the metal is larger than that of the semiconductor. Can such a system be employed to construct an Ohmic contact? If so, how?

c) State at least two intrinsic limits on the speed of response of a MOSFET.

18. In this problem we consider an ideal long-base abrupt *pn*-junction silicon diode with uniform cross section and constant doping on both sides of the junction. The doping of the diode is as follows: $N_a = 1.0x10^{16}$ cm^{-3} *p*-type and $N_d = 4x10^{16}$ cm^{-3} *n*-type, where the minority-carrier lifetimes are $\tau_n = 2x10^{-6}$ s and $\tau_p = 2x10^{-8}$ s, respectively. (Assume that the effects within the space-charge region are negligible and that minority carriers flow only by the diffusion mechanism in the charge neutral regions.)

i) Calculate the value of the built-in potential across the barrier?

ii) Calculate the density of the minority carriers at the edge of the space-charge region when the applied reverse bias is 0.5 V.

iii) Sketch the minority and majority carrier currents as a function of distance from the junction under the bias condition stated above.

Section 7

19. The inverse of the Z-transform

$$F(z) = \frac{z}{(z^2 - \sqrt{2}z + 1)(z - 4)}$$

can represent several possible sequences.

a) Find all the sequences

b) One of the sequences represents the coefficients of the Fourier series expansion of the Fourier Transform of a band-limited signal. Identify the sequence and find the Fourier Transform of the corresponding band-limited signal.

20. The input $e^{i\omega t}U(t)$ to a linear system gives the output

$$-\frac{e^{-t^4/2}}{t^3+2i\omega}+\frac{e^{i\omega t}}{t^3+2i\omega}$$

for all real t and ω. (Note: $U(t)$ denotes the unit step function)

a) Find the system impulse response

b) Find and sketch the output when the input is $f(t)=\begin{cases}1 & ; \quad |t|\le 1\\ 0 & ; \quad |t|>1\end{cases}$

20. The sequence $y[n]$ satisfies the following difference equation:

$$y[n+2]-\frac{8y[n+1]}{15}+\frac{1}{15}y[n]=u[n]$$

where

$$u[n]=\begin{cases}1\,;n\geq 0\\0\,;n<0\end{cases}$$

a) Assuming $y[0]=1$ and $y[1]=2$ find the causal solution.

b) Suppose the $y(n\Delta t)=y[n]/\Delta t$ (with Δt the sampling interval) represent samples of a band limited signal with a bandwidth of 20 Hz. Find the Fourier Transform of the signal.