THE GEORGE WASHINGTON UNIVERSITY
School of Engineering and Applied Science
Department of Electrical and Computer Engineering

Preliminary Examination

Fall 2000

General Instructions

Read carefully before starting.

i) Solve five (5) problems in accordance with the following selection rule: choose two (2) problems from any one of the seven sections, two (2) problems from one of the other sections, and the remaining problem from any section, including from either of the sections previously selected.

ii) Solve each problem in a separate blue book. Write the section number, problem number, and your student number on the front of each blue book. **DO NOT WRITE YOUR NAME ON THE BLUE BOOK.**

iii) Only five problems will be graded. The proctor will check off the 5 problems that are to be graded for your exam. Use only **ONE** blue book per question.

iv) For each problem, make a special effort to give the answers in a clear, reduced form. Present any accompanying discussion using complete sentences. Make **LARGE** well-labeled diagrams.

v) The exam will begin at 10:00 a.m. and end at 3:00 p.m.

!!GOOD LUCK!!

Section 1

1. Consider a stop-and-wait ARQ. Assume:

   p = probability of bit error and independent bit errors over a packet
   n = packet length
   Δn = packet overhead (preamble)
   = length of ACK

   Assuming that propagation and turnaround (processing) delays are equal to 0,

   a) Derive transmission efficiency (effective throughput) of stop-and-wait ARQ.

   b) Find the optimum packet size.

2. The slotted ALOHA throughput is given by $S = Ge^{-G}$. Now, suppose that there are two traffic classes, high power and low power, characterized by G_H and G_L, such that a high power packet is always correctly received when a low power packet contends with it for the slot (perfect capture).

   a) What is the throughput for high power packets? Explain.

   b) What is the throughput for low power packets? Explain.

   c) What is the total throughput for high and low power packets? For $G_L e^{-G_L} = e^{-1}$, find the maximum throughput.

3. Consider M users sharing the same channel of total capacity equal to 1. Each user is characterized with Poisson traffic of average rate $\lambda' = \lambda/M$. Packets are of fixed lengths such that one time unit is required for one packet transmission if the total bandwidth (capacity) is allocated to this single packet.

   a) Describe the queueing model if M users share the channel using FDMA. Calculate the total average time per packet in the system if the average queueing delay is given by Polachek-Khinchin formula

$$w = \frac{\lambda' \overline{x^2}}{2\left(1 - \frac{x'}{\mu'}\right)}$$

   where x is the packet duration, $\overline{x^2}$ is the second moment of packet duration, and μ' is the average packet service time.

b) Describe the queueing model for TDMA if M users share the same channel. Calculate the total average time per packet in the system. Compare with FDMA. Explain.

Section 2

1. Which of the following are legitimate autocorrelation functions of a real stationary random process?

a) $$R_1(\tau)=\begin{cases}2 & ; \quad |\tau|<1\\ 0 & ; \quad otherwise\end{cases}$$

b) $$R_2(\tau)=\begin{cases}\tau & ; \quad 0\le\tau\le 1\\ -\tau & ; \quad -1<\tau\le 0\end{cases}$$

c) $$R_3(\tau)=2\frac{\sin(2\pi\tau)}{2\pi\tau} \quad ; \quad |\tau|<\infty$$

d) $$R_4(\tau)=|\tau| \quad ; \quad |\tau|\le 1$$

e) $$R_5(\tau)=2\cos(\omega\tau) \quad ; \quad |\tau|<\infty$$

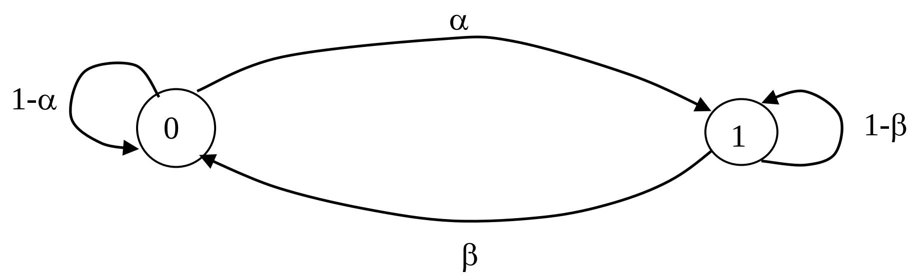
You must give <u>specific</u>, <u>quantitative</u> reasons for each answer. A simple yes or no (guess) is not an acceptable answer.;

2. Packets arrive at a Router such that the <u>intervals</u>, T_i, between arrivals are random, identically distributed, independent and one-sided exponential, α, i.e.

$$f_{T_i}(t)=\alpha e^{-\alpha t} \quad , \quad t>0$$
$$f_{T_i}(t)=0 \quad , \quad otherwise$$

Find an expression for the probability that two (2) consecutive packets arrive in an interval exceeding t_0 seconds, in terms of α and t_0.

3. A sequence of binary symbols {0, 1} evolves according to a Markov chain such that the next bit depends on the current bit according to the transition probabilities shown.



a) Assume we start such that the first bit is a zero, what is the probability that the 10^{th} bit is also a zero if:

(1) $\alpha = 0.1$, $\beta = 0.8$;
(2) $\alpha = 0.8$, $\beta = 0.1$;
(3) $\alpha = 0.1$, $\beta = 0.1$.

c) For the n-th bit, what is the condition on α and β so that as $n \to \infty$, $P_n(0)$ and $P_n(1)$ are independent of n? Give examples, i.e., choose α, β numerically and find

$$\lim_{n \to \infty} P_n(0) \; \textit{and} \; \lim_{n \to \infty} P_n(1).$$

Do <u>NOT</u> choose $\alpha = \beta$.

Section 3

1. An antenna located at the origin of a rectangular coordinate system radiates a time-harmonic electromagnetic field of radian frequency ω_0 in all directions. The electric field associated with radiation in the direction of the positive x-axis, for large values of x, is specified by the peak value phasor

$$\vec{E}(x,0,0;\omega_0) \sim \left[(1+j)\hat{y} + j\sqrt{3}\hat{z}\right]\frac{e^{-jkx}}{x};$$

$\hat{x}, \hat{y}, \hat{z} \equiv$ *unit vectors along the respective cartesian axes*

$j = \sqrt{-1},$

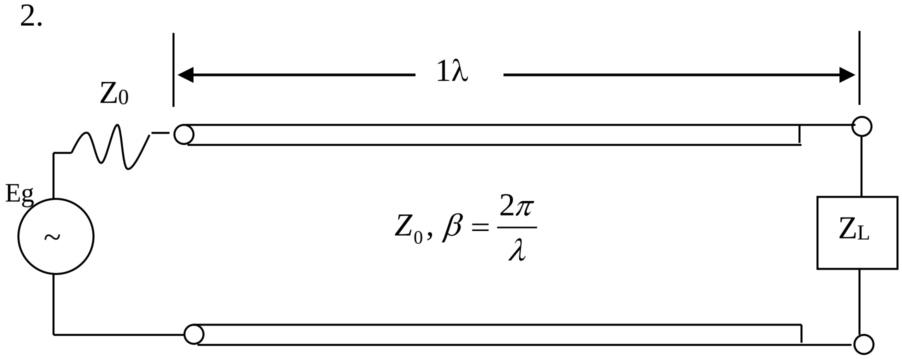
$k = \omega_0\sqrt{\mu_0\varepsilon_0}.$

a) Write the time dependent electric field vector $\vec{E}(x,0,0;t)$ which corresponds to the given phasor.

b) Calculate the ellipticity (ratio of the major axis to minor axis) of the polarization ellipse and the angle the major axis makes with the positive y-axis.

c) From Maxwell's equations, calculate the magnetic field phasor $\vec{H}$ associated with the given electric field.

2.



A lossless transmission line with real characteristic impedance Z_0 and real propagation constant β is terminated in a load $Z_L = (1.5 – j0.4)\ Z_0$.

a) What is the voltage standing wave ratio (VSWR) on the transmission line?

b) What is the distance from the load (towards the generator) to the nearest maximum of the VSWR pattern expressed in fractions of a wavelength?

c) What is the distance from the load (expressed in fractional wavelengths) to the point nearest the load at which a pure susceptance jB could be placed (in shunt) across the transmission line to match the resulting total load to the transmission line, i.e., reduce the magnitude of the reflection coefficient for the combined load plus susceptance to zero?

d) What is the value of the required susceptance (magnitude and algebraic sign?)

e) In terms of Eg, Z_L, Z_0, $2\pi/\lambda$, jB and any/or additional assumed parameters you find necessary, calculate the power delivered by the generator to the load after the correct matching susceptance jB has been attached in place.

4. A region of uniform density of charge of value ρ_0 (Coulombs/meter^3) is in the shape of a sphere of radius "a" (meters). The sphere is suspended above an infinite conducting half-space the surface of which is coincident with the x, y plane of a Cartesian coordinate system so that the center of the spherical charge distribution is located at (0, 0, h), $h \geq a$.

a) Calculate the surface density of charge at all points of the x, y plane.

b) Calculate the total charge on the x, y plane.

c) Calculate the value of the electrostatic potential function $V(\bar{r})$ at all points <u>on the z axis</u> above and below the surface of the conducting half-space. $V(\bar{r}) \to 0$ at infinite distances from the spherical charge distribution.

d) Calculate the total electrostatic vector force (if any) on the spherical charge distribution (magnitude and direction).

Section 4

1. Please be brief and to the point when answering the following questions.

   a) The threshold voltage of a MOSFET depends on the ability of the gate bias to invert the channel. In this context, explain the concept of "strong inversion" in a MOS system in terms of the band structure and mobile charge densities.

   b) Consider a moderately doped semiconductor. It is well known that the mobility of electrons increases with decreasing temperature. Based on this, can we automatically assume that the conductivity increases, as well? For instance, if one is attempting to improve device and circuit performance (such as, lower noise, parasitic resistance, etc.) can one expect further improvements regardless of how much the temperature is lowered?

   c) In integrated circuits Ohmic contacts are formed by making a metal-semiconductor contact. Explain how such a system is designed, such that it does not display Schottky diode behavior.

   d) Describe one way to construct and operate a device that can be employed as a voltage-controlled capacitor. State at least one application where such a device is of practical interest.

2. Consider the NMOS transistor shown below with substrate doping N_a, channel length L, channel width W, oxide thickness X_{ox}, channel mobility μ_n and flat-band voltage V_{FB}. Derive an expression for the threshold voltage V_{th} when there is no channel current, (set $V_S = V_D$).

3. Consider the following pin-diode where all dimensions are given in microns. The net doping density is N_a in the p-region, N_d in the n-region and zero in the intrinsic region. Assume that the doping in the p and n regions is uniform. Compute and plot the electrostatic potential in the pin-diode under reverse bias V_a in terms of the given parameters. State one reason why the “i” in the pin-diode is useful.

Section 5

1. The Fourier transform F(ω) of a bandlimited signal f(t) is given by

$$F(\omega) = \begin{cases} \cos^4\left(\dfrac{\pi\omega}{\Omega}\right) & ; \quad |\omega| \le \Omega, \\ 0 & ; \quad |\omega| > \Omega. \end{cases}$$

a) The signal is sampled at intervals $\Delta t = \pi/\Omega$. Find the samples.

b) Find the Z-transform of the sampled sequence $f[n] = f(n\Delta t)\,\Delta t$.

c) Find f(t).

d) Suppose the signal is sampled at intervals $\Delta t' = 2\pi/\Omega$ and its Fourier transform is approximated by

$$\hat{F}(\omega) = \Delta t' \sum_{n=-\infty}^{\infty} f(n\Delta t')\, e^{-i\omega n\Delta t'}.$$

Compute and sketch $\hat{F}(\omega)$ within the band $|\omega| \le \Omega$.

2. The inverse transform of the Z- transform, given by

$$F(z) = \frac{z}{\left(z^2 + 1/16\right)\left(z+2\right)^2},$$

can represent several sequences, depending on the region of convergence in the z-plane.

a) Find all the sequences.

b) One of the sequences represents the coefficients of the Fourier series expansion of the Fourier transform of a bandlimited function. Identify the sequence and find the Fourier transform of the corresponding bandlimited function.

3. The input $e^{i\omega t}$ to a linear system results in the output $\dfrac{e^{i2\omega t}}{t^6 + i2\omega}$ for all real ω and t.

a) Is the system time-invariant? Justify your answer.

b) Find the system impulse response.

c) Is the system causal? Justify your answer.

d) Find the output when the input is

$$f(t) = \begin{cases} 1 & ; \quad -1 \le t \le 1, \\ 0 & ; \quad otherwise. \end{cases}$$

Section 6

1. Consider two processors A and B, each having an on-chip cache. You are asked to figure out which one fills (or updates) its cache line from the processor's external main memory faster. Explain how you would do it. (State all assumptions you are making in your thought process.)

2. Consider a recent scalar pipelined processor with a 5-stage pipeline. The processor has internal Harvard architecture with on-chip ICACHE and DCACHE.

   a) Indicate the internal CPU hardware component assigned to each of these 5 stages. (Assume there exists separate ALU and separate "effective-address calculation" hardware.)

   b) If the processor is a RISC-type with 32-bit fixed length instructions, has "load-store architecture", and its memory accesses require "register-based" effective address calculation (i.e., the specified register's content is added to the displacement contained within the instruction), then:

      1) Indicate which hardware component is used during each clock cycle for the LOAD instruction.
      2) Indicate which hardware component is used during each clock cycle for an ALU-type instruction.
      3) Show the cycle by cycle pipeline flow for the following program sequence:

         LOAD instruction
         ALU-type instruction
         STORE instruction

   State all your assumptions.

3. Consider a 16-bit big-endian processor with a 24-bit memory address.

   a) Design a 4 Mbyte mail memory using 1024 x 4 SRAM chips and arrange these chips on a two-dimensional PC board.

   b) Interface this memory to the processor through its address and data buses. Show
      - how the address bits placed on the address bus are decoded by the memory subsystem, and
      - how each 8-bit byte-section of the memory is connected to an 8-bit byte-lane of the data bus.

   State all your assumptions.

Section 7

1. Solving a system of linear equations, give an estimate of the population P(t) of the US in the year 2010, based on the following data (in millions). Base your estimate both on a linear and a parabolic fit. Determine:

   a) the population of the USA in 2010 for both cases;

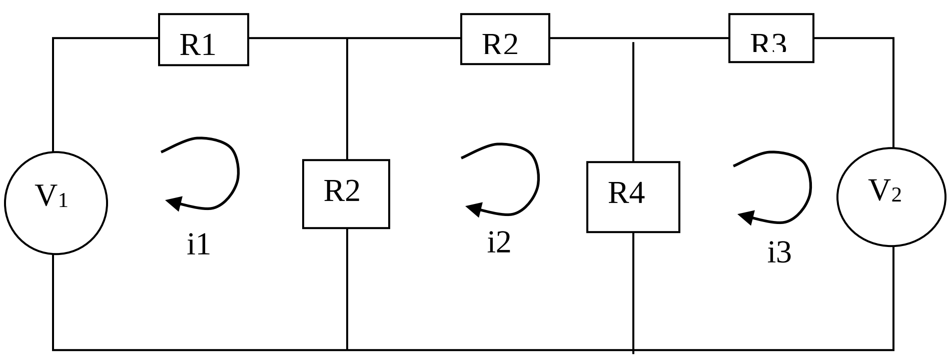
   b) The error of the fits, as defined by err = sum (A**x**-**b**).

t	P(t)
1930	76.0
1950	150.7
1970	203.2
1990	247.1

2. An electric circuit is analyzed by solving a system of linear equations A**x** = **b**. Consider the circuit shown in the figure. Given the values for the resistors, (R1 = R2 = R3 = R4 = R5 =10Ω) and the voltages V1 = V2 = 5V),

   a) find the mesh currents i1, i2, and i3;

   b) calculate the error function err = sum (A**x**-**b**).



3. Given the following matrix:

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 8 \end{bmatrix},$$

a) Compute A^TA and its eigenvalues $\sigma_1^2, 0$ and unit eigenvectors $\mathbf{v}_1$ and $\mathbf{v}_2$;

b) Compute AA^T and its eigenvalues $\sigma_2^2, 0$ and unit eigenvectors $\mathbf{u}_1$ and $\mathbf{u}_2$;

c) Verify that $A\mathbf{v}_1 = \sigma_1\mathbf{u}_1$;

d) Find all entries in the Singular Value Decomposition:

$$A = \begin{bmatrix} \boldsymbol{u}_1 & \boldsymbol{u}_2 \end{bmatrix} \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix} \begin{bmatrix} \boldsymbol{v}_1 & \boldsymbol{v}_2 \end{bmatrix}^T$$

e) Write down orthonormal bases for the four fundamental subspaces of this A.