November 13, 2019
CSC-208-01, Discrete Structures I
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Ex 2.2 Q8, 78

In Exercises 3–26, use mathematical induction to prove that the statements are true for every positive integer *n*. [*Hint*: In the algebra part of the proof, if the final expression you want has factors and you can pull those factors out early, do that instead of multiplying everything out and getting some humongous expression.]

8. $1^3 + 2^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$

Let the base case be n = 1:

$\sum_{i=1}^{1} 1^3 = 1$ and $\frac{1^2(1+1)^2}{4} = \frac{(2)^2}{4} = 1$

So the statement is true for the base case.
Now assume the statement is true starting at a random n, such that (sigma) i = k:

$$\sum_{i=1}^{k} i^3 = \frac{k^2(k+1)^2}{4}$$

Now try to move one more number:

$$\sum_{i=1}^{k} i^3 = \frac{k^2(k+1)^2}{4} + (k+1)^3$$

Factoring $(k+1)^2$ out:

$$\sum_{i=1}^{k} i^3 = (k+1)^2 \left(\frac{1}{4}k^2 + (k+1)\right)$$

Rearranging the right-hand expression:

$$\sum_{i=1}^{k} i^3 = (k+1)^2 \left(\frac{k^2 + 4k + 4}{4}\right)$$

Simplify $k^2 + 4k + 4$:

$$\sum_{i=1}^{k} i^3 = (k+1)^2 \left(\frac{(k+2)^2}{4}\right)$$

Therefore, by moving $(k+1)^2$ to the top of the fraction,

$$\sum_{i=1}^{k} i^3 = \frac{(k+1)^2(k+2)^2}{4}$$

We have proven that from any number, we can successfully move one more number, thus proving this by induction.

78. Prove that any amount of postage greater than or equal to 12 cents can be built using only 4-cent and 5-cent stamps.

Let the base cases be n = 12, n = 13, n = 14, and n = 15:

$3 * 4¢ + 0 * 5¢ = 12¢$

$2 * 4¢ + 1 * 5¢ = 13¢$

$1 * 4¢ + 2 * 5¢ = 14¢$

$0 * 4¢ + 3 * 5¢ = 15¢$

So the statement is true for all the base cases.

Now, assume that P(k) is true for $12 \leq r \leq k$ is true, where $r, k \in N$ and $k \geq 15$, or

12, 13, 14, 15, … (k - 3), (k - 2), (k - 1), k

Take postage for k + 1: $k + 1 = (k - 3) + 4$, that is you can get k + 1 adding a 4¢ stamp to a $k - 3$ postage. If you have $P(k - 3)$ [which is definitely within $12 \leq r \leq k$] , you also get $P(k + 1)$ by adding an additional 4¢ stamp.