Math 551
Practice Problems for Final Exam

- ***Attention: Textbook, notes, calculators and other electronic devices are NOT allowed during exams.***

1) Find a basis for the column space col(A) of the matrix

$$A = \begin{bmatrix} 1 & -2 & 3 & 2 & 0 \\ 2 & -4 & 5 & 0 & 1 \\ -3 & 6 & 0 & 1 & 1 \end{bmatrix}.$$

Based on your finding, determine the rank of A.
2) Find a basis for the null space ker(A) of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 1 & 1 & 1 & 1 \end{bmatrix}.$$

Based on your finding, determine the rank of A.
3) Find the eigenvalues of the matrix

$$A = \begin{bmatrix} 2 & 0 & 0 \\ -1 & 3 & 2 \\ 1 & 4 & 5 \end{bmatrix}.$$

4)The eigenvalues of the matrix

$$A = \begin{bmatrix} -3 & 3 & 1 \\ -10 & 8 & 2 \\ 20 & -12 & -2 \end{bmatrix}$$

are $\lambda_1 = -1$ and $\lambda_2 = 2$. Find the corresponding eigenspaces $E_{\lambda_1}(A)$ and $E_{\lambda_2}(A)$ and their dimensions. Based on your findings, determine whether A is diagonalizable.
5) Consider the following subspace in $\mathbb{R}^4$:

$$W = \text{span}\left\{ \begin{bmatrix} 1 \\ 2 \\ -1 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \end{bmatrix} \right\}.$$

Find an orthonormal basis for W.
6) Consider the following set of vectors

$$\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

Determine whether this set of vectors forms an orthogonal basis of $\mathbb{R}^3$. If it does, determine whether it also forms an orthonormal basis.
7) Determine a, b, c and d such that the following matrix is an orthogonal matrix

$$A = \begin{bmatrix} 2/3 & \sqrt{2}/2 & \sqrt{2}/6 \\ 2/3 & a & b \\ c & 0 & d \end{bmatrix}.$$

8) Is the matrix

$$A = \begin{bmatrix} 2 & 1 & 0 \\ -1 & -6 & 2 \\ -1 & -3 & 1 \end{bmatrix}$$

invertible? If yes find its inverse A^{-1}. If no explain why.

9) In $\mathbb{R}^4$, consider the vectors

$$\vec{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 1 \\ -2 \\ 3 \\ -4 \end{bmatrix}, \quad \vec{v}_3 = \begin{bmatrix} 1 \\ 2 \\ -3 \\ -4 \end{bmatrix}, \quad \vec{w} = \begin{bmatrix} 4 \\ -28 \\ 30 \\ 0 \end{bmatrix}.$$

Determine whether $\vec{w}$ belongs to the subspace $V = \operatorname{span}\{\vec{v}_1, \vec{v}_2, \vec{v}_2\}$.

10) Find *all* solutions of the linear system

$$\begin{cases} x_1 + 2x_2 + x_3 + 12x_5 = -2 \\ x_1 + 2x_2 + 2x_3 - 2x_4 + 4x_5 = 1 \\ x_1 + 2x_2 + 5x_3 - 7x_4 - 18x_5 = 4 \end{cases}$$