Homework #3 Joseph Webster & Michael Yangoni

1) $n\ln(n) \equiv n\lg(n) < n\sqrt{\sqrt{n}} < n^{1.3} < n\sqrt{n} < n^2 < n^{\lg(n)} < (1.001)^n < 2^n \equiv 2^{n+7} < n! < n^n$

2) ① $\sum_{k=1}^{n} \lg(k) + \sum_{k=1}^{n} \lg(k) = T(n)$; where n = number of elements.

② $\sum_{i=1}^{g(n)} X(f(i)) \subseteq X[g(n) \cdot f(g(n))] \rightarrow 3.28$

$\rightarrow 2\left(\sum_{k=1}^{n} \lg(k)\right) \rightarrow 2 \cdot n \cdot \lg(n)$

$\rightarrow T(n) \in \Theta(2n\lg(n))$, but, we can disgard the constant (2) because at large values of (n), we can see that it doesn't matter

$\boxed{\therefore f(n) = n \cdot \lg(n)}$

3) ⓐ

```
z ← 0
for k ← 1 to n*n
    q ← 1
    while q ≤ k
        q ← q + 1
        z ← z + 1
```

$\sum_{k=1}^{n^2} \Theta(n)$

$\sum_{q=1}^{K} \Theta(n)$

Howell's: $\Theta(\text{outer}(\text{inner}(\text{outer})))$

$\Theta(n^2(n(n)'))$

$n^2 \cdot n^2$

$\boxed{\Theta(n^4)}$

ⓑ

```
z ← 0
for k ← 1 to n*n
    q ← 1
    s ← k*k
    while q ≤ s
        q ← q * 2
        z ← z + 1
```

$\lg(k^2)$

$\sum_{q=1} \Theta(k)$

$\sum_{k=1}^{n^2} \Theta(n)$

$\sum_{k=1}^{n^2} \lg(k^2)$

$\boxed{\Theta(n^2 \cdot \lg(n))}$

$n^2, \lg((n^2)^2)$

(3) (c)

```
z ← 0
k ← 1
while k < n
    for q ← 1 to k
        z ← z + 1
    k ← k * 2
```

$$\sum_{q=1}^{k} 2^{n}$$

$$\sum_{k=1}^{\lg(n)} \Theta(2^{k-1})$$

will choose to use the upperbound of lg(n)

$$\sum_{k=1}^{\lg(n)} \Theta(n)$$

$$\sum_{k=1}^{\lg(n)} 2^{k-1} \to \Theta(2^{\lg(n) \cancel{-1}})$$

$$\boxed{\Theta(n)}$$