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1.1 Find a reccurence for $T(n)$

$$T(n) = T(\lfloor \tfrac{n}{2} \rfloor) + 1$$

1.2

$$T(n) = T(\lfloor \tfrac{n}{2} \rfloor) + 1 \quad n^0 \to q = 0$$

$$r = \log_2(1) \to 0 \; ; \; r = q$$

$$T(n) \in \Theta(n^0 \lg(n)) \to T(n) \in \Theta(\lg(n))$$

2.1 $T(n) = 3T(\tfrac{n}{3}) + n^2$

$$r = \log_{(3)}(3) = 1 \; ; \; q = 2 \; ; \; r < q$$

$$\therefore T(n) \in \Theta(n^2)$$

2.2 $T(n) = 9T(\tfrac{n}{3}) + n^2$

$$r = \log_3(9) = 2 \; ; \; q = 2 \; ; \; r = q$$

$$\therefore T(n) \in \Theta(n^2 \lg(n))$$

2.3 $T(n) = T(n-1) + n\sqrt{n} \to T(n) = T(n-1) + n^{1.5}$

Recursion on Slightly Larger Problems

- $g(n) = h(n) + g(n-1) \quad \to g(n) = \sum_{i=1}^{n} h(i)$
- $T(n) = n\sqrt{n} + T(n-1) \quad \to T(n) = \sum_{i=1}^{n} n^{1.5}$ or "$\Theta(n^{2.5})$"

  but, $n\sqrt{n} = n^{1.5}$

$$\therefore T(n) = \sum_{i=1}^{n} n^{1.5} \to \Theta((n)\cdot(n)^{1.5}) \to \Theta(n^{2.5})$$

because "$g(n)$" is non-decreasing.

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$\boxed{3.1}$ $u = s = .6$

n	1	2	3	4	5	6	7	8	9	10
$T(n)$	2	4	6	8	13	17	27	27	35	35

$u = .8,\ s = .6$

n	1	2	3	4	5	6	7	8	9	10
$T(n)$	2	4	6	8	15	24	40	56	81	81

$\boxed{3.2}$

$T(n) \geq cn^2$ $\forall n$ where $n \geq 0, c > 0$

$\underline{n \geq 5}$

$$T(n) = T(\lceil s \cdot n \rceil) + T(\lceil u \cdot n \rceil) + 1$$
$$\geq c(\lceil s \cdot n \rceil)^2 + c(\lceil u \cdot n \rceil)^2 + 1$$
$$\geq c(sn) + c(un)$$
$$\geq cn^2(s^2 + u^2) \geq cn^2 \cdot 1$$

$\therefore s^2 + u^2 \geq 1$ and $c > 0$

$\underline{1 \leq n \leq 4}$

$$T(n) \geq cn^2$$
$$T(n) = 2n$$
$$\geq 2cn^2 \rightarrow T(n) \geq \cancel{2}cn^2 \rightarrow \frac{T(n)}{n^2} \geq c$$

n	1	2	3	4
$T(n)$	2	4	6	8
n^2	1	4	9	16
c	2	1	2/3	1/2

$\boxed{\therefore \text{The largest "c" that will work is } .5}$

$\boxed{3.3}$

- $\underline{s = u = .6}$

→ check the constraint for s & u: $(.6)^2 + (.6)^2 \geq 1$

↳ $.72 \geq 1$ ✗, Does not hold.

- $\underline{s = .6,\ u = .8}$

→ check the constraint for s & u: $(.6)^2 + (.8)^2 \geq 1$?

↳ $.36 + .64 \geq 1$ ✓, Constraints hold

n	1	2	3	4	5	10
$T(n)$	2	4	6	8	15	81
cn^2	.5	2	4.5	8	12.5	50

$c = .5$ ✓

→ $T(n) \geq cn^2$, holds when $1 \leq n \leq 10$ and $c = .5$

→ Because, when $n = 10$, $T(n) \geq .5(10)^2$ which is: $81 \geq 50$, and because $\forall n$ from 1 to 9 are < 10, it must follow that $T(n) \geq cn^2$ holds for $\forall n : 1 \leq n \leq 10$