Michael Yangmi & Joseph Webster

1.1) - $U(4) = 2$ because in the worst case, where the elements of 'A[]' are non-decreasing with no repetition, and 'x' is either the first or last element of A.

- For instance: $A = [\underset{lo}{1}, \underset{q}{2}, 3, \underset{hi}{4}]$

$lo = 1$
$hi = 4$
$q = (1+4) \text{ div } 2 = 2$

- Pre-pass on the while loop body

- Assume $A[q] < x$:

$A = [1, 2, \underset{lo}{3}, \underset{hi}{4}]$

2 elements in Array $\begin{cases} lo' = q + 1 = 3 \\ hi' = 4 \end{cases}$

$q' = (3+4) \text{ div } 2 = 3$

- First pass on the while loop body

- Assume $A[q] < x$:

$A = [1, 2, 3, \underset{hi}{\underset{lo}{4}}]$

1 element in array $\begin{cases} lo'' = q' + 1 = 4 \\ hi'' = n = 4 \end{cases}$

$q = (4+4) \text{ div } 2 = 4$

- Second pass on the while loop body

∴ There are two full passes in a worst-case scenario, so $\underline{U(4) = 2}$.

1.2)

m	5	6	7	8	9
U(m)	2	2	2	3	3

1.3) show that (for $m \geq 2$) expresses $U(m)$ in terms of $U(\lfloor \frac{m}{2} \rfloor)$. Show that when $m = 3, 8$ coincides with 1.2.

- $U(\lfloor \frac{m}{2} \rfloor) + 1$
- $m = 3$; $U(\lfloor 3/2 \rfloor) + 1 = U(\lfloor 1.5 \rfloor) + 1 = 0 + 1 = \underline{1}$
- $m = 8$; $U(\lfloor 8/2 \rfloor) + 1 = U(\lfloor 4 \rfloor) + 1 = 2 + 1 = \underline{3}$

1.4) $\log_2(m)$ or $\lg(m)$ should satisfy $U(m)$.

- $m = 1$, $\log_2(1) = 0$ ✓
- $m = 3$, $\log_2(3) \approx 1.5$, which rounded down is 1 ✓
- $m = 8$, $\log_2(8) = 3$ ✓

Michael Yangni & Joe Webster

2)

$f(n)$	$n \geq 10{,}000$	$n \geq 100{,}000{,}000$
n	10,000	100,000,000
$n \cdot \lg(n)$	1,003	4,523,072
$n \cdot \sqrt[3]{n}$	1,000	1,000,000
$n \cdot \sqrt{n}$	465	215,444
n^2	100	10,000
n^4	10	100
$(1.001)^n$	9215	18430
$(1.1)^n$	97	194
2^n	14	27
100^n	2.0	4.0
$n!$	8.0	12
n^n	6.0	9.0

3.1) $3n^4 + 7n^3 + 6n^2 + 9n + 5 \in O(n^4)$

Def if there exists $c > 0$ and $n_0 \geq 0$ such that for all "n" with $n \geq n_0$, $f(n) \leq c\,g(n)$

Proof ① $3n^4 + 7n^3 + 6n^2 + 9n + 5 \leq 3n^4 + 7n^4 + 6n^4 + 9n^4 + 5n^4$

② $\boxed{3n^4 + 7n^3 + 6n^2 + 9n + 5 \leq 30n^4 \text{ where } n_0 \geq 1 \;\&\; c = 30.}$

3.2) $n^5 - 7n^4 + 2n^2 - n \notin O(n^4)$

- Assume $n^5 - 7n^4 + 2n^2 - n \leq c \cdot n^4$, where "c" > 0 and $n_0 \geq 1$ is true $\forall n$ where $n \geq n_0$.
- By Algebra, we can show: $n^5 \leq c \cdot n^4 + 7n^4 - 2n^4 + n^4$.
- By applying algebra again, we divide both sides by "n^4"
  ↳ $n \leq c + 7 - 2 + 1 \Rightarrow \underline{n \leq c + 6}$
- $n \leq c + 6$ cannot hold true $\forall n$, as "c" will be set & "n" will be growing.
- ∴ We've proved a contradiction with our assumption and deduction. So, $n^5 - 7n^4 + 2n^2 - n \notin O(n^4)$