CIS 575. Introduction to Algorithm Analysis
Assignment #4, Spring 2019
Due Thursday, February 21, 11:59pm

You may if you so prefer work in groups of two in which case each name should be listed on your answer but only one of you should submit.

1. (10p). Consider the following program (which may be what you constructed in Assignment 1) whose running time $T(n)$ we want to estimate, as a function of $n = hi - lo$:

Find(x, A, lo, hi)
    $q \leftarrow (lo + hi)$ div 2
    if $A[q] = x$
        return q
    else if $A[q] < x$
        return Find$(x, A, q + 1, hi)$
    else
        return Find$(x, A, lo, q - 1)$

1. (5p) Write a recurrence for $T(n)$. (You may assume arithmetic operations take time in $\Theta(1)$.)
2. (5p) Solve that recurrence, by using the "Master Theorem". (You should indicate which version you use, and what are the given values of a, b, etc)

2. (12p). Solve each of the recurrences

$$T(n) = 3\,T(\frac{n}{3}) + n^2 \tag{1}$$

$$T(n) = 9\,T(\frac{n}{3}) + n^2 \tag{2}$$

$$T(n) = T(n-1) + n\sqrt{n} \tag{3}$$

Your answers should be of the form $T(n) \in \Theta(f(n))$, with f as simple as possible. You should justify your answers, for example by appealing to the "Master Theorem" (if applicable).

3. (18p). Given real numbers s, u with $0 < s \leq u \leq 0.8$, consider the function T given by

$$\begin{aligned} T(n) &= 2n && \text{for } n \leq 4 \\ T(n) &= T(\lceil sn \rceil) + T(\lceil un \rceil) + 1 && \text{for } n \geq 5 \end{aligned}$$

This is well-defined, since when $n \geq 5$ then $n - un = (1-u)n \geq 0.2n \geq 1$ and thus $\lceil sn \rceil \leq \lceil un \rceil < n$.

1. (5p) Tabulate $T(n)$, for n from 1 to 10, with $s = u = 0.6$, and also with $s = 0.6$ and $u = 0.8$.
2. (8p) Use the *substitution method* to find a constraint on the values of s and u such that you can prove by induction in n that
$$T(n) \geq cn^2 \text{ for all } n \geq 0$$
for some positive real number c; list the *largest* c that will work.
3. (5p) Compare your "experimental" results from part 1 to your "theoretical" results from part 2. That is, for $s = u = 0.6$ and also for $s = 0.6, u = 0.8$, tell whether the constraint from part 2 is satisfied; if so, with c the constant you found in part 2, tell whether $T(n) \geq cn^2$ does indeed hold for all $n \leq 10$.