CIS 575. Introduction to Algorithm Analysis
Assignment #2, Spring 2019
Due Thursday, February 7, 11:59pm

You may if you so prefer work in groups of two in which case each name should be listed on your answer but only one of you should submit.

1. (18p). Assuming that $A[1..n]$ is non-decreasing and that x occurs in $A[1..n]$, consider the below program (which may be what you constructed in Assignment 1):

$lo, hi, q \leftarrow 1, n, (1+n)$ div 2
while $A[q] \neq x$
  if $A[q] < x$
    $lo \leftarrow q + 1$
  else
    $hi \leftarrow q - 1$
  $q \leftarrow (lo + hi)$ div 2
return q

If $n = 1$, the body of the `while` loop will not be executed; if $n = 2$, it will be executed once (if $A[1] < x$) or not at all (if $A[1] = x$); if $n = 3$, it will be executed once (if $A[2] \neq x$) or not at all.

Let $U(m)$ denote the maximum number of iterations of the `while` loop body when $A[lo..hi]$ has m elements (that is, $m = hi - lo + 1$). We can thus tabulate the first entries of U as follows:

m	1	2	3	4	5	6	7	8	9
$U(m)$	0	1	1						

1. (5p): Find $U(4)$. You must argue for your answers.
2. (5p): Provide the remaining missing entries in the above table, that is, compute $U(m)$ for $m = 5..9$. You do not need to argue for your answers.
3. (4p): Use your table to write a *recurrence* (a recursive equation) that (for $m \geq 2$) expresses $U(m)$ in terms of $U(\lfloor \frac{m}{2} \rfloor)$. You do not need to argue for your answer, but show that for $m = 3, 8$ it coincides with your answer to (2).
4. (4p): Find a general formula (not recursively defined) for $U(m)$. You do not need to argue for your answer, but show that for $m = 1, 3, 8$ it coincides with your answer to (2).

2. (12p). For each of the following functions f of n, indicate how big the natural number n must be in order for $f(n)$ to be at least 10,000, and for $f(n)$ to be at least 100,000,000 (that is, 10^8). You may give approximate answers but they should have at least two significant figures.

n	$n \lg(n)$	$n \sqrt[3]{n}$	$n \sqrt{n}$	n^2	n^4	$(1.001)^n$	$(1.1)^n$	2^n	100^n	$n!$	n^n

3. (10p). Prove, using *only* the definition of "big-O" and *not* any auxiliary results stated on the slides or in the textbook(s), that

1. (5p): $3n^4 + 7n^3 + 6n^2 + 9n + 5 \in O(n^4)$
2. (5p): $n^5 - 7n^4 + 2n^2 - n \notin O(n^4)$