ENEE469O Final Project
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Rebuilding the Transportation Cost Minimization Model

Abstract:

Efficient transportation systems are critical to reducing operational costs and improving service delivery across industries. Inspired by research leveraging linear programming (LP) to address transportation challenges, this project develops and implements an LP-based model aimed at minimizing total transportation costs within a given network. The model considers multiple constraints including supply capacities, demand fulfillment, and route specific transportation costs. After constructing the optimization model, we applied the simplex method to identify the optimal routing strategy. The results demonstrate the practical utility of LP in optimizing real-world transportation systems and provide insights into how small adjustments in constraints or cost coefficients can lead to significant operational improvements. The final project code can be found on github at: https://github.com/jwei132/ENEE469O_Final_Project

Problem Formulation:

Our project focuses on the implementation of a transportation cost minimization model to determine the optimal way to transport goods between n suppliers to m destinations. The model takes into account the following parameters:

- Supply capacity (s): a list of n values describing the amount of a product that each supply node can provide.
- Demand (d): a list of m values that describes the amount of goods demanded by each destination node.
- Cost matrix (C): An nxm matrix describing the cost of sending goods from a specific supplier to a specific destination. The entry in row i, column j represents the cost of sending one product from supplier node i to destination node j.

The solution to the program is an nxm transport matrix (X), where entry $x_{i,j}$ describes the number of products transported from supplier i to node j.

The objective function of our model is to minimize the value of:

$$\sum_{i=1}^{n}\sum_{j=1}^{m} c_{i,j}x_{i,j}$$

Where:

- $c_{i,j}$ is the entry in row i, column j of the cost matrix C
- $x_{i,j}$ is the entry in row i, column j of the transport matrix X

Given the objective function, the model is subject to 3 different constraints. First are supply constraints, which ensure that the amount of products shipped out from each supplier doesn't exceed the supply capacity of that supplier. Secondly we have demand constraints that ensure that the amount received by each destination node is equal to the amount demanded. These two constraints are modeled by the following equations.

$$\sum_{j=1}^{n} x_{i,j} \le s_i \qquad\qquad \sum_{i=1}^{m} x_{i,j} = d_j$$

Where:

- s_i is the i^{th} entry of supply vector s.
- d_j is the j^{th} entry of demand vector d.

For the supply constraints, we can see that we will have n supply constraints, one for each supply node. Similarly, we will have m demand constraints, one for each destination node.

Lastly, we also apply a non-negativity constraint, since it does not make sense to send a negative amount of products to a destination. This constraint is modeled by the following equation:

$$x_{i,j} \ge 0,\ 1 \le i \le n,\ \ 1 \le j \le m$$

This constraint set will have n*m values since each element of the matrix X must be non-negative.

Solution Methods:

To solve this problem, we must represent our problem in the general form of a linear program. To do this, we flatten the nxm matrices X and C into vectors $\bar{x}$ and $\bar{c}$, each with length n*m by appending the rows of the matrix into a vector. Doing this converts our optimization problem into the form:

$$min_{x \in \Re^{n \times m}} \left(\bar{c}^T \bar{x} \right)$$

$$A_{ub} \bar{x} \leq b_{ub}$$

$$A_{eq} \bar{x} = b_{eq}$$

Where:

- A_{ub} is matrix with dimensions p x (n*m), where p is the number of inequality constraints. In this case, we have n supply constraints and n*m non-negative constraints on the. So p = n+n*m
- A_{eq} is matrix with dimensions q x (n*m), where q is the number of equality constraints. Since the only equality constraints of our model are the demand constraints, q is equal to m.

After setting up our constraints based on the input values, we solve the linear program using the simplex algorithm. After the program is solved and we obtain an optimal solution $\bar{x}$ *. We can then convert $\bar{x}$ * back into an nxm matrix (by dividing the vector into vectors of length n and putting each vector in order as the rows of the X matrix) and then calculate the final cost of transportation with the original objective function.

Implementation

The final implementation of this problem was done in python using numpy arrays and matrices to do the model setup. Once we had the variables representing the objective function and constraint sets, we were able to use the linear program solver from scipy to solve the linear program using the simplex algorithm.

To make the code cleaner, we decided to make our solver a python class which stores private variables, the matrices and vectors representing our linear program. To use the class, the user would create a new class object and provide the source capacity vector, the destination demand vector, and the cost matrix associated with their system. Then the solve function is called on the object which performs the flattening step outline in the previous section while applying the scipy solver on the result. This generates a solution in the form of a general linear program which the python class then reconstructs into the transportation matrix we expect. Lastly, the solve function also calculates the objective function and returns the total cost of using the solved transportation matrix with the given cost matrix.

In our example, we set the supply capacity and destination demand vectors as constants and made a simple loop to populate the cost matrix with random values within a certain range.

Conclusion:

Through this project, we successfully developed and implemented a linear programming model to address the transportation cost minimization problem. By translating real-world supply, demand, and cost constraints into a solvable mathematical framework, we were able to apply the simplex method to find optimal transportation strategies. The results confirmed that even modest changes in cost coefficients or constraint values can significantly influence the overall solution, offering valuable insight into the sensitivity of logistical systems. Our model demonstrates the practical power of linear programming in optimizing transportation networks and provides a strong foundation for further enhancements, such as incorporating time windows, dynamic demand, or multi-modal transportation. This project not only reinforced our understanding of LP theory and optimization algorithms but also emphasized their direct applicability to complex engineering and operations problems while giving us an opportunity to learn how to use python to solve optimization problems.