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# 1 sm Theory

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**Parent Theories:** indexedLists, patternMatches

## 1.1 Datatypes

*configuration* = CFG ('input list) 'state ('output list)

## 1.2 Definitions

[TR\_def]

```
⊤ TR =
(λ a₀ a₁ a₂ .
  ∀ TR'.
    (∀ a₀ a₁ a₂ .
      (exists NS Out s ins outs .
        (a₁ = CFG (a₀::ins) s outs) ∧
        (a₂ = CFG ins (NS s a₀) (Out s a₀::outs))) ⇒
      TR' a₀ a₁ a₂) ⇒
    TR' a₀ a₁ a₂)
```

[Trans\_def]

```
⊤ Trans =
(λ a₀ a₁ a₂ .
  ∀ Trans'.
    (∀ a₀ a₁ a₂ . (exists NS . a₂ = NS a₁ a₀) ⇒ Trans' a₀ a₁ a₂) ⇒
    Trans' a₀ a₁ a₂)
```

## 1.3 Theorems

[configuration\_one\_one]

```
⊤ ∀ a₀ a₁ a₂ a₀' a₁' a₂' .
  (CFG a₀ a₁ a₂ = CFG a₀' a₁' a₂') ⇔
  (a₀ = a₀') ∧ (a₁ = a₁') ∧ (a₂ = a₂')
```

[TR\_cases]

```
⊤ ∀ a₀ a₁ a₂ .
  TR a₀ a₁ a₂ ⇔
  ∃ NS Out s ins outs .
    (a₁ = CFG (a₀::ins) s outs) ∧
    (a₂ = CFG ins (NS s a₀) (Out s a₀::outs))
```

## [TR\_clauses]

$$\vdash (\forall x \ x1s \ s1 \ out1s \ x2s \ out2s \ s2 . \\ \text{TR } x \ (\text{CFG } x1s \ s1 \ out1s) \ (\text{CFG } x2s \ s2 \ out2s) \iff \\ \exists NS \ Out \ ins . \\ (x1s = x::ins) \wedge (x2s = ins) \wedge (s2 = NS \ s1 \ x) \wedge \\ (out2s = Out \ s1 \ x::out1s)) \wedge \\ \forall NS \ Out \ x \ x1s \ s1 \ out1s \ x2s \ out2s . \\ \text{TR } x \ (\text{CFG } x1s \ s1 \ out1s) \\ (\text{CFG } x2s \ (NS \ s1 \ x) \ (Out \ s1 \ x::out2s)) \iff \\ \exists ins . (x1s = x::ins) \wedge (x2s = ins) \wedge (out2s = out1s)$$

## [TR\_complete]

$$\vdash \forall s \ x \ ins \ outs . \\ \exists s' \ out . \\ \text{TR } x \ (\text{CFG } (x::ins) \ s \ outs) \ (\text{CFG } ins \ s' \ (out::outs))$$

## [TR\_deterministic]

$$\vdash \forall NS \ Out \ x1 \ ins1 \ s1 \ outs1 \ ins2 \ outs2 \ ins'_2 \ outs'_2 . \\ \text{TR } x1 \ (\text{CFG } (x1::ins1) \ s1 \ outs1) \\ (\text{CFG } ins2 \ (NS \ s1 \ x1) \ (Out \ s1 \ x1::outs2)) \wedge \\ \text{TR } x1 \ (\text{CFG } (x1::ins1) \ s1 \ outs1) \\ (\text{CFG } ins'_2 \ (NS \ s1 \ x1) \ (Out \ s1 \ x1::outs'_2)) \iff \\ (\text{CFG } ins2 \ (NS \ s1 \ x1) \ (Out \ s1 \ x1::outs2)) = \\ (\text{CFG } ins'_2 \ (NS \ s1 \ x1) \ (Out \ s1 \ x1::outs'_2)) \wedge \\ \text{TR } x1 \ (\text{CFG } (x1::ins1) \ s1 \ outs1) \\ (\text{CFG } ins2 \ (NS \ s1 \ x1) \ (Out \ s1 \ x1::outs2))$$

## [TR\_ind]

$$\vdash \forall TR' . \\ (\forall NS \ Out \ s \ x \ ins \ outs . \\ \text{TR}' \ x \ (\text{CFG } (x::ins) \ s \ outs) \\ (\text{CFG } ins \ (NS \ s \ x) \ (Out \ s \ x::outs))) \Rightarrow \\ \forall a0 \ a1 \ a2 . \text{TR } a0 \ a1 \ a2 \Rightarrow \text{TR}' \ a0 \ a1 \ a2$$

## [TR\_rules]

$$\vdash \forall NS \ Out \ s \ x \ ins \ outs . \\ \text{TR } x \ (\text{CFG } (x::ins) \ s \ outs) \\ (\text{CFG } ins \ (NS \ s \ x) \ (Out \ s \ x::outs))$$

## [TR\_strongind]

$$\vdash \forall TR' . \\ (\forall NS \ Out \ s \ x \ ins \ outs . \\ \text{TR}' \ x \ (\text{CFG } (x::ins) \ s \ outs) \\ (\text{CFG } ins \ (NS \ s \ x) \ (Out \ s \ x::outs))) \Rightarrow \\ \forall a0 \ a1 \ a2 . \text{TR } a0 \ a1 \ a2 \Rightarrow \text{TR}' \ a0 \ a1 \ a2$$

## [TR\_Trans\_lemma]

$$\vdash \text{TR } x \ (\text{CFG } (x::\text{ins}) \ s \ \text{outs}) \\ (\text{CFG } \text{ins } (\text{NS } s \ x) \ (\text{Out } s \ x::\text{outs})) \Rightarrow \\ \text{Trans } x \ s \ (\text{NS } s \ x)$$

## [Trans\_cases]

$$\vdash \forall a_0 \ a_1 \ a_2. \ \text{Trans } a_0 \ a_1 \ a_2 \iff \exists \text{NS}. \ a_2 = \text{NS} \ a_1 \ a_0$$

## [Trans\_Equiv\_TR]

$$\vdash \text{TR } x \ (\text{CFG } (x::\text{ins}) \ s \ \text{outs}) \\ (\text{CFG } \text{ins } (\text{NS } s \ x) \ (\text{Out } s \ x::\text{outs})) \iff \text{Trans } x \ s \ (\text{NS } s \ x)$$

## [Trans\_ind]

$$\vdash \forall \text{Trans}' . \\ (\forall \text{NS } s \ x. \ \text{Trans}' x \ s \ (\text{NS } s \ x)) \Rightarrow \\ \forall a_0 \ a_1 \ a_2. \ \text{Trans } a_0 \ a_1 \ a_2 \Rightarrow \text{Trans}' a_0 \ a_1 \ a_2$$

## [Trans\_rules]

$$\vdash \forall \text{NS } s \ x. \ \text{Trans } x \ s \ (\text{NS } s \ x)$$

## [Trans\_strongind]

$$\vdash \forall \text{Trans}' . \\ (\forall \text{NS } s \ x. \ \text{Trans}' x \ s \ (\text{NS } s \ x)) \Rightarrow \\ \forall a_0 \ a_1 \ a_2. \ \text{Trans } a_0 \ a_1 \ a_2 \Rightarrow \text{Trans}' a_0 \ a_1 \ a_2$$

## [Trans\_TR\_lemma]

$$\vdash \text{Trans } x \ s \ (\text{NS } s \ x) \Rightarrow \\ \text{TR } x \ (\text{CFG } (x::\text{ins}) \ s \ \text{outs}) \ (\text{CFG } \text{ins } (\text{NS } s \ x) \ (\text{Out } s \ x::\text{outs}))$$



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