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# 1 sm Theory

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**Parent Theories:** indexedLists, patternMatches

## 1.1 Datatypes

*configuration* = CFG ('input list) 'state ('output list)

## 1.2 Definitions

[TR\_def]

$$\begin{aligned} \vdash \text{TR} = & \\ & (\lambda a_0 a_1 a_2. \\ & \quad \forall TR'. \\ & \quad (\forall a_0 a_1 a_2. \\ & \quad \quad (\exists NS \text{ Out } s \text{ ins } \text{ outs}. \\ & \quad \quad \quad (a_1 = \text{CFG } (a_0 :: \text{ins}) \text{ } s \text{ } \text{outs}) \wedge \\ & \quad \quad \quad (a_2 = \text{CFG } \text{ins } (NS \text{ } s \text{ } a_0) (\text{Out } s \text{ } a_0 :: \text{outs}))) \Rightarrow \\ & \quad \quad TR' a_0 a_1 a_2) \Rightarrow \\ & \quad TR' a_0 a_1 a_2) \end{aligned}$$

[Trans\_def]

$$\begin{aligned} \vdash \text{Trans} = & \\ & (\lambda a_0 a_1 a_2. \\ & \quad \forall Trans'. \\ & \quad (\forall a_0 a_1 a_2. (\exists NS. a_2 = NS a_1 a_0) \Rightarrow Trans' a_0 a_1 a_2) \Rightarrow \\ & \quad Trans' a_0 a_1 a_2) \end{aligned}$$

## 1.3 Theorems

[configuration\_one\_one]

$$\begin{aligned} \vdash \forall a_0 a_1 a_2 a'_0 a'_1 a'_2. \\ & (\text{CFG } a_0 a_1 a_2 = \text{CFG } a'_0 a'_1 a'_2) \iff \\ & (a_0 = a'_0) \wedge (a_1 = a'_1) \wedge (a_2 = a'_2) \end{aligned}$$

[TR\_cases]

$$\begin{aligned} \vdash \forall a_0 a_1 a_2. \\ & \text{TR } a_0 a_1 a_2 \iff \\ & \quad \exists NS \text{ Out } s \text{ ins } \text{ outs}. \\ & \quad (a_1 = \text{CFG } (a_0 :: \text{ins}) \text{ } s \text{ } \text{outs}) \wedge \\ & \quad (a_2 = \text{CFG } \text{ins } (NS \text{ } s \text{ } a_0) (\text{Out } s \text{ } a_0 :: \text{outs})) \end{aligned}$$

**[TR\_clauses]**

$$\begin{aligned}
& \vdash (\forall x \ x1s \ s_1 \ out1s \ x2s \ out2s \ s_2. \\
& \quad \text{TR } x \ (\text{CFG } x1s \ s_1 \ out1s) \ (\text{CFG } x2s \ s_2 \ out2s) \iff \\
& \quad \exists NS \ Out \ ins. \\
& \quad \quad (x1s = x::ins) \wedge (x2s = ins) \wedge (s_2 = NS \ s_1 \ x) \wedge \\
& \quad \quad (out2s = Out \ s_1 \ x::out1s)) \wedge \\
& \quad \forall NS \ Out \ x \ x1s \ s_1 \ out1s \ x2s \ out2s. \\
& \quad \text{TR } x \ (\text{CFG } x1s \ s_1 \ out1s) \\
& \quad \quad (\text{CFG } x2s \ (NS \ s_1 \ x) \ (Out \ s_1 \ x::out2s)) \iff \\
& \quad \exists ins. (x1s = x::ins) \wedge (x2s = ins) \wedge (out2s = out1s)
\end{aligned}$$
**[TR\_complete]**

$$\begin{aligned}
& \vdash \forall s \ x \ ins \ outs. \\
& \quad \exists s' \ out. \\
& \quad \text{TR } x \ (\text{CFG } (x::ins) \ s \ outs) \ (\text{CFG } ins \ s' \ (out::outs))
\end{aligned}$$
**[TR\_deterministic]**

$$\begin{aligned}
& \vdash \forall NS \ Out \ x_1 \ ins_1 \ s_1 \ outs_1 \ ins_2 \ ins'_2 \ outs_2 \ outs'_2. \\
& \quad \text{TR } x_1 \ (\text{CFG } (x_1::ins_1) \ s_1 \ outs_1) \\
& \quad \quad (\text{CFG } ins_2 \ (NS \ s_1 \ x_1) \ (Out \ s_1 \ x_1::outs_2)) \wedge \\
& \quad \text{TR } x_1 \ (\text{CFG } (x_1::ins_1) \ s_1 \ outs_1) \\
& \quad \quad (\text{CFG } ins'_2 \ (NS \ s_1 \ x_1) \ (Out \ s_1 \ x_1::outs'_2)) \iff \\
& \quad (\text{CFG } ins_2 \ (NS \ s_1 \ x_1) \ (Out \ s_1 \ x_1::outs_2)) = \\
& \quad \text{CFG } ins'_2 \ (NS \ s_1 \ x_1) \ (Out \ s_1 \ x_1::outs'_2)) \wedge \\
& \quad \text{TR } x_1 \ (\text{CFG } (x_1::ins_1) \ s_1 \ outs_1) \\
& \quad \quad (\text{CFG } ins_2 \ (NS \ s_1 \ x_1) \ (Out \ s_1 \ x_1::outs_2))
\end{aligned}$$
**[TR\_ind]**

$$\begin{aligned}
& \vdash \forall TR'. \\
& \quad (\forall NS \ Out \ s \ x \ ins \ outs. \\
& \quad \quad \text{TR}' \ x \ (\text{CFG } (x::ins) \ s \ outs) \\
& \quad \quad \quad (\text{CFG } ins \ (NS \ s \ x) \ (Out \ s \ x::outs))) \Rightarrow \\
& \quad \forall a_0 \ a_1 \ a_2. \text{TR } a_0 \ a_1 \ a_2 \Rightarrow \text{TR}' \ a_0 \ a_1 \ a_2
\end{aligned}$$
**[TR\_rules]**

$$\begin{aligned}
& \vdash \forall NS \ Out \ s \ x \ ins \ outs. \\
& \quad \text{TR } x \ (\text{CFG } (x::ins) \ s \ outs) \\
& \quad \quad (\text{CFG } ins \ (NS \ s \ x) \ (Out \ s \ x::outs))
\end{aligned}$$
**[TR\_strongind]**

$$\begin{aligned}
& \vdash \forall TR'. \\
& \quad (\forall NS \ Out \ s \ x \ ins \ outs. \\
& \quad \quad \text{TR}' \ x \ (\text{CFG } (x::ins) \ s \ outs) \\
& \quad \quad \quad (\text{CFG } ins \ (NS \ s \ x) \ (Out \ s \ x::outs))) \Rightarrow \\
& \quad \forall a_0 \ a_1 \ a_2. \text{TR } a_0 \ a_1 \ a_2 \Rightarrow \text{TR}' \ a_0 \ a_1 \ a_2
\end{aligned}$$

**[TR\_Trans\_lemma]**

$$\vdash \text{TR } x \text{ (CFG } (x::\text{ins}) \text{ } s \text{ outs)} \\ \text{(CFG ins (NS } s \text{ } x) \text{ (Out } s \text{ } x::\text{outs}))} \Rightarrow \\ \text{Trans } x \text{ } s \text{ (NS } s \text{ } x)$$
**[Trans\_cases]**

$$\vdash \forall a_0 \ a_1 \ a_2. \text{Trans } a_0 \ a_1 \ a_2 \iff \exists NS. \ a_2 = NS \ a_1 \ a_0$$
**[Trans\_Equiv\_TR]**

$$\vdash \text{TR } x \text{ (CFG } (x::\text{ins}) \text{ } s \text{ outs)} \\ \text{(CFG ins (NS } s \text{ } x) \text{ (Out } s \text{ } x::\text{outs}))} \iff \text{Trans } x \text{ } s \text{ (NS } s \text{ } x)$$
**[Trans\_ind]**

$$\vdash \forall Trans'. \\ (\forall NS \ s \ x. \text{Trans}' \ x \ s \text{ (NS } s \text{ } x)) \Rightarrow \\ \forall a_0 \ a_1 \ a_2. \text{Trans } a_0 \ a_1 \ a_2 \Rightarrow \text{Trans}' \ a_0 \ a_1 \ a_2$$
**[Trans\_rules]**

$$\vdash \forall NS \ s \ x. \text{Trans } x \ s \text{ (NS } s \text{ } x)$$
**[Trans\_strongind]**

$$\vdash \forall Trans'. \\ (\forall NS \ s \ x. \text{Trans}' \ x \ s \text{ (NS } s \text{ } x)) \Rightarrow \\ \forall a_0 \ a_1 \ a_2. \text{Trans } a_0 \ a_1 \ a_2 \Rightarrow \text{Trans}' \ a_0 \ a_1 \ a_2$$
**[Trans\_TR\_lemma]**

$$\vdash \text{Trans } x \ s \text{ (NS } s \text{ } x) \Rightarrow \\ \text{TR } x \text{ (CFG } (x::\text{ins}) \text{ } s \text{ outs)} \text{ (CFG ins (NS } s \text{ } x) \text{ (Out } s \text{ } x::\text{outs}))}$$



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