Fragmentation, mathematical ignorance, and the metalinguistic reply*

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Abstract

On the *coarse-grained view*, any distinctions between mental contents can be characterized using possible worlds. The coarse-grained view is sometimes thought to be refuted by straightforward counterexamples involving failures of logical omniscience. An independently motivated theory of mental fragmentation enables one avoid the counterexamples while preserving the coarse-grained view.

1 Introduction

How fine-grained are mental contents?

On the *coarse-grained* view, any distinctions between mental contents can be characterized using possible worlds. This view has been famously (infamously?) endorsed in Stalnaker (1984), Lewis (1970, sec. V), Lewis (1975), Lewis (1994), and Braddon-Mitchell and Jackson (2007). According to the simplest version of the coarse-grained view, a mental content is just a set of possible worlds.

On the *fine-grained* view, there are mental contents that differ in ways that cannot be captured using possible worlds. On one version of the fine-grained view, a mental content is a Fregean sense (Frege 1956). On another version, a mental content is a set of possible or impossible worlds Fagin et al. (1995), Hintikka (1975).

The coarse-grained view is sometimes thought to be refuted by straightforward counterexamples. In this paper we will argue that this is not so. We will show that a friend of coarse-grained contents has the resources to address the counterexamples. Of course, defusing counterexamples is not the same as showing that the coarse-grained view is correct. An overall assessment of the strength of the view is beyond the scope of this paper.

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2 *The coarse-grained view*

In this section we will describe a simple example of a coarse-grained view of mental content. It is based on a coarse-grained notion of information according to which a batch of information is just a set of possible worlds—the worlds at which that information obtains.

To motivate the view, it is useful to consider some examples:

Successes A soldier walks an intricate path to successfully evade the mines littering a mine-field (Braddon-Mitchell and Jackson 2007). A crossword puzzlist fills in just the letters that complete a difficult puzzle.

What helps explain each success is that the subject has access to appropriate information.

Failures A second soldier steps on a mine. A second crossword puzzlist scratches his head and leaves the puzzle blank.

What helps explain each failure is that the subject lacks access to appropriate information.

How does the possession of a given batch of information relate to states of belief, and to belief attributions? And what, exactly, is the connection between information possession and rational action? Here is a SIMPLE PLAN for answering these questions:

SIMPLE STATE A subject's belief state is the information she possesses. (Since a batch of information is taken to be a set of possible worlds, this means that the subject's belief state is a set of worlds.)

SIMPLE ATTRIBUTION A subject believes that p if and only if every world in her belief state is such that p.

SIMPLE ACTION An action is predicted for a subject if and only if it tends to bring about her desires at worlds compatible with the information she possesses.

Suppose, for example, that the soldier walking the minefield possesses the information that there are no mines by the river. Then according to SIMPLE STATE the soldier's belief state is a set of worlds at which there are no mines by the river, and according to SIMPLE ATTRIBUTION, the soldier believes that there are no mines by the river. Finally, suppose that the soldier's options are traveling by the river or traveling on high ground. Then SIMPLE ACTION

predicts that she will travel by the river (assuming that she desires not to step on a mine).

A more sophisticated version of SIMPLE PLAN would replace each of the plan's three theses with probabilistic analogues. For instance:

SIMPLE STATE* A subject's belief state is her credence function—her assignment of subjective probabilities to propositions, understood as sets of possible worlds.

SIMPLE ATTRIBUTION* A subject is confident that p if and only if she assigns high enough credence to the set of worlds in which p.

SIMPLE ACTION* An action is predicted for the subject if and only if no rival action is assigned greater expected value by subject's credence function (given the subject's values).

We will mostly focus on non-probabilistic versions of SIMPLE PLAN. But our arguments can be adapted to fit the probabilistic case.

Some philosophers have argued that a theory of mind based on either version of SIMPLE PLAN would be hopeless (Field 1986, Speaks 2006, Williamson 2016). We agree. SIMPLE PLAN faces decisive objections, and we will review some of them below. However, we believe that there is independent reason to adopt a *fragmented* view of mental states. We will motivate such a view and use it to develop FRAGMENTED PLAN, a replacement for SIMPLE PLAN that is able to overcome the objections we consider, while continuing to presuppose a coarse-grained conception of information. In so doing, we aim to remove one significant obstacle to giving a coarse-grained account of mental contents.

3 *Objection 1: The problem of metaphysical omniscience*

SIMPLE PLAN faces two decisive objections.[1]

It is an immediate consequence of SIMPLE STATE and SIMPLE ATTRIBUTION that belief ascriptions are closed under metaphysical entailment: if every world at which ϕ is true is also a world at which ψ is true, then $\ulcorner S$ believes that $\phi\urcorner$ can only be true if $\ulcorner S$ believes that $\psi\urcorner$ is.

SIMPLE STATE and SIMPLE ATTRIBUTION also entail that every subject believes every necessary truth. For if ϕ is a necessary truth, SIMPLE ATTRIBUTION entails that $\ulcorner S$ believes that $\psi\urcorner$ is true, regardless of what S's belief state happens to be.

[1]Here we will focus on the non-probabilistic version of SIMPLE PLAN. The probabilistic version faces analogous problems.

A particularly pressing consequence of this result is that SIMPLE PLAN entails that all subjects believe all logical and mathematical truths, and have beliefs that respect all logical entailments. That is wildly implausible. This difficulty—the *problem of logical omniscience*—is our present concern.

4 *A simple metalinguistic reply*

The coarse-grained theorist might follow Stalnaker (1984) and Braddon-Mitchell and Jackson (2007, 200–201), and reply that mathematical and logical uncertainty can be partly metalinguistic.[2] For example, she might posit that uncertainty about whether $12 \times 18 = 216$ can derive in part from uncertainty about the following metalinguistic claim:

C The sentence "$12 \times 18 = 216$" is true.

Since C is contingent, the coarse-grained theorist might hope to avoid the implication that everyone believes C. And she might hope to analyze mathematical uncertainty in terms of uncertainty about claims such as C.

Here is a problem for the metalinguistic reply.[3] In representing a subject's mental state, it is not enough to do justice to what they are ignorant about. We must also do justice to the information that they possess. Even if an ordinary subject is ignorant of the fact that $18 \times 5 = 90$, she can be expected

[2]Stalnaker thinks that "Under certain conditions, the content of an assertion is not the proposition determined by the ordinary semantic rules, but instead the diagonal proposition of the propositional concept determined." (Stalnaker 1987, 124) and argues that something similar happens in the case of belief attributions. Braddon-Mitchell and Jackson (2007, 201) endorses a similar idea: "When Jones discovers that a triangle she already knew to be equiangular is also and necessarily equilateral, she does indeed discover something, but arguably not about how things are but about how it is that two different sentences constructed out of interestingly different materials represents the very same way things are."

[3]Objections of this kind have been raised by Field (1978, 34–35), Field (1986), Soames (2009), Speaks (2006, 448–450), Williamson (2016) and Bacon (2017). Here is how Williamson presses the point:

> One problem for his view is that for any formula A in standard mathematical notation, the biconditional True($\ulcorner A \urcorner$) $\leftrightarrow$ A will follow logically from known axioms of a standard compositional theory of truth for the mathematical language. Since on Stalnaker's view our knowledge is closed under such logical consequence, it implies that we already know the biconditional, so the metalinguistic claim is equivalent for us to the original mathematical claim. Thus semantic ascent to the metalinguistic level only postpones the problem. Williamson (2016, footnote 1)

to know the meanings of mathematical vocabulary and the compositional rules of mathematical language. So she will know—and hence believe—basic metalinguistic claims such as that "18" refers to 18. But these basic metalinguistic claims together entail that

"$12 \times 18 = 216$" is true if and only if $12 \times 18 = 216$,

and hence entail that C is true.

Now, according to SIMPLE STATE and SIMPLE ATTRIBUTION, one believes whatever is entailed by one's beliefs. So SIMPLE STATE and SIMPLE BELIEF entail that everyone believes true metalinguistic claims such as C. So it appears futile for the coarse-grained theorist to analyze mathematical uncertainty in terms of uncertainty about claims such as C.

5 Objection 2: actions arising from mathematical ignorance

An additional objection to SIMPLE PLAN focuses not on belief attributions, but rather on the relationship between information possession and dispositions to act.

The objection is best explained with an example.[4] Suppose that Alice and Bob have the same desires but that they each have a false belief about 12×18: Alice thinks that it equals 214, Bob thinks that it equals 218. They are remodeling their bathroom, and the plan calls for a grid of 12 tiles by 18 tiles. When shopping for the required tiles, Alice will be disposed to buy a different number of tiles than Bob.

Such subjects are clearly possible, but it is difficult to see how a friend of SIMPLE PLAN can capture Alice and Bob's mathematical ignorance in a way that predicts the correct tile-shopping behavior.

To see why, notice that *no* possible worlds are compatible with either Alice's or Bob's beliefs, since each has beliefs incompatible with the necessary

[4]We take this example to capture the key point of a nice example introduced in (Field 1986):

> if I offer someone who doesn't know much mathematics $1000 for an example of a plane map that requires more than four colors to color (according to the usual coloring conventions), he will behave very differently than he would if I had offered $1000 for a trisection of a Euclidean 60 degree angle by straight edge and compass; to explain this, I need to attribute different beliefs and desires to him in the different cases, and it is prima facie difficult to see how I can do this in a relevant way if the desire to do one impossible task is identified with the desire to do any other impossible task.

truth that $12 \times 18 = 216$. So in the absence of a special story, the coarse grained theorist must represent each of their belief states by the very same set: the empty set. But according to SIMPLE ACTION, differences in predicted behavior between subjects only derive from differences in the possible worlds compatible with each agent's beliefs (when desires are held fixed). So it seems that the coarse grained theorist cannot account for the difference between Alice and Bob's tile-shopping behavior.

A friend of SIMPLE PLAN might insist that a suitable difference between Alice and Bob's belief states are ready to hand: Alice believes, for example, that 214 tiles are required to for the bathroom, while Bob believes that 218 are required. There is no problem representing *that* difference in terms of sets of possible worlds, since the set of worlds in which 214 tiles are required is distinct from the set of worlds in which 218 are required. Furthermore, that difference seems to produce just the right predicted actions according to SIMPLE ACTION.

This proposal is vulnerable to a difficulty analogous to the one raised above for the metalinguistic reply. For as before, it is not enough for the coarse grained theorist to do justice to Alice and Bob's ignorance. He must also do justice to their knowldege. In particular, Alice and Bob each believe that the bathroom requires 12×18 tiles. And any world compatible this fact is a word at which the bathroom requires 216 tiles. These considerations put pressure on the coarse-grained theorist to represent Alice and Bob's belief states by sets containing only worlds in which the bathroom requires 216 tiles. But doing so will not account for either Alice or Bob's tile-shopping behavior.

So SIMPLE PLAN does not supply the coarse-grained theorist with an adequate way to represent a failure of logical omniscience.

6 The state of play

The above objections should not be understood as the complaint that coarse-grained theorists have failed to produce a general theory of ordinary-language belief attributions or a general theory of predicted action. That demand would be unreasonable, since it is so difficult to give such theories in any framework whatsoever. Rather, the challenge is that certain deep structural features of the coarse-grained framework seem to immediately rule out any such theory.

We will answer this challenge by giving up on SIMPLE PLAN but retaining a coarse-grained view of mental contents.

Our motivating idea is that the across-the-board notion of information

possession should be replaced with a notion of information accessible *relative to a condition*. We will motivate this picture and then use it to reply to the above objections.

7 *Information access*

Why replace the across-the-board notion of information possession with a relative notion of accessible information? Because information can be represented in a way that makes it accessible for some purposes, but inaccessible for others (Stalnaker 1991, 437–438).

For example, recall the two crossword puzzlists from §2. One of them confidently fills in the correct answer; the other is stumped. Let us fill in some details. Suppose that the relevant portion of the puzzle looks like this:

_ _ _ _ M T

The goal is to fill in the blanks to complete a word of English. Further suppose that each puzzlist knows that "dreamt" is a word of English, and knows how to spell it. Indeed, each puzzlist realizes from the start that "dreamt" is a word of English that ends in MT.

So why is one puzzlist disposed to fill in the blanks with DREA, while the other is disposed to gnash his teeth, curse, and fill in nothing?

We suggest that both puzzlists possess the information they need to fill in the blanks, but that the conditions relative to which they have access to this information are different. Let D be the set of worlds in which *dreamt* is a word of English spelled D-R-E-A-M-T. Both puzzlists have access to D for the purpose of using the word "dreamt" in a written essay. And they both have access to D for the purpose of answering the question "Is 'dreamt' a word of English ending in MT?".

But for the purpose of filling in the blanks in "_ _ _ _ M T", only the first puzzlist has access to D.

So if we'd like to represent the difference between the two puzzlists, our representation of each of them should not attempt to specify what information he possesses, period. It should instead say what information he has access to for what purposes.[5]

[5] In an illuminating paper on the role of the organization of memory in human reasoning, Cherniak (1983, 166) notes that creatures with mental organizations anything like ours constantly face a hard computational problem: quickly accessing memories relevant to their current situation. Cherniak convincingly argues that there is therefore a practical need for a small short-term memory store that supports fast—but not exhaustive—searching and consistency checking. It is to be expected that the heuristics underlying such searches will

For example, we might represent the informational state of the struggling puzzlist with the following sort of table, which we shall call an *access table*:

Choice condition	*Accessible information*
dreamt salient	$I_!$
dreamt not salient	$I_?$
[more conditions]	[information accessible relative to those conditions]

Here $I_!$ and $I_?$ are sets of possible worlds representing batches of (coarse-grained) information. $I_!$ contains only worlds in which *dreamt* is a word of English spelled D-R-E-A-M-T. $I_?$ contains other worlds as well.

The table reflects that the puzzlist's dispositions "factor" into two natural components: one component associated with situations in which the word "dreamt" has been made salient, and another in which it has not.[6]

8 *The fragmentationist plan*

Once a relativised notion of information possession is in place, it is straightforward to amend SIMPLE STATE and SIMPLE ACTION appropriately:

FRAGMENTED STATE A subject's belief state is ~~the information she possesses~~ **her access table**.

operate differently in different circumstances, and hence that different information will be accessible in different circumstances. Indeed, the necessity for heuristics that narrow memory searches was already recognized by Hume: "as the production of all the ideas to which [a] name may be applied, is in most cases impossible, we abridge that work by a more partial consideration, and find but few inconveniences to arise in our reasoning from that abridgement" (Hume 1738, 21, as cited in Cherniak 1983, 176).

[6]The suggestion in Stalnaker (1984) that logical omniscience failures can be understood in terms of fragmented belief states was the core motivation for the present model. Braddon-Mitchell and Jackson (2007, 199–200) also uses fragmented coarse-grained belief states to accommodate failures of logical omniscience. Yalcin (2008, Ch. 3), Yalcin (2015), and Yalcin (2016) develop that same suggestion, proposing an elegant model on which all-or-nothing belief is relative to questions, understood as partitions of logical space. The treatment of logical omniscience failures in those works uses privileged partitions to represent which propositions are accessible to an agent, and so differs from the present treatment. (See especially Yalcin (2016, n. 26).) Egan (2008) endorses a treatment of fragmented credences similar to the present one and interestingly suggests that mental fragmentation might be practically indispensable for agents with perceptual belief forming mechanisms anything like human ones—mechanisms that are less than perfectly reliable but which nevertheless produce immediate belief in certain circumstances.

FRAGMENTED ACTION An action is predicted for a subject if and only if it tends to bring about her desires at worlds compatible with the information she possesses **in her current choice condition**.

It is less straightforward to appropriately amend SIMPLE ATTRIBUTION to take into account a relativised notion of information possession. What is needed is a replacement for SIMPLE ATTRIBUTION, reproduced here for convenience:

SIMPLE ATTRIBUTION A subject believes that p if and only if every world in her belief state is such that p.

Our proposal will be a version of the dispositional analysis of belief set forth in Schwitzgebel (2002): "To believe that P [...] is nothing more than to match to an appropriate degree and in appropriate respects the dispositional stereotype for believing that P." What Schwitzgebel means by a dispositional stereotype is a profile of dispositions to react in specified ways in a specified set of conditions. We will modify that proposal by suggesting a particular specification for what a dispositional stereotype consists in. We will suggest that each belief attribution is associated with a set of "diagnostic tests", and that a belief attribution is counted as true if the subject has "passing information" for a suitable subset of its associated tests.

What are the diagnostic tests associated with a given belief attribution?

To answer this question, recall the crossword puzzlist who is unable to fill in "_ _ _ _ M T", even though he knows how to spell *dreamt*. Despite the puzzlist's present difficulty, he does know, and hence believe, that *dreamt* is a word of English spelled D-R-E-A-M-T.

Now: think about how you might check whether your friend believes that *dreamt* is a word of English spelled D-R-E-A-M-T. You might assign him the task of writing an essay about his dreams on the previous night, check if he used *dreamt*, and if so, whether he spelled it correctly. You might show him DREAMT, and ask him whether it is a properly spelled word of English. And so on. These are the diagnostic tests associated with the belief ascription "My friend believes that *dreamt* is a word of English spelled D-R-E-A-M-T."

Our proposal is therefore that when your friend has the ability to pass a sufficient set of these tests, the belief ascription "My friend believes that *dreamt* is a word of English spelled D-R-E-A-M-T" is true.

Notice that on this proposal, the practice of belief attribution "plays favorites" by counting only certain test conditions as diagnostic. For example in order for the above belief attribution to be true, your friend needn't have

access to the information that *dreamt* is a word of English spelled D-R-E-A-M-T relative to all conditions. In particular, he need not be able to immediately fill in the blanks when faced with the crossword clue "_ _ _ _ M T".

With this as our background, we can state our proposal. Say that a row of the subject's access table counts as "diagnostic" for ϕ if it corresponds to one of the diagnostic tests associated with the attribution $\ulcorner S$ believes that $\phi\urcorner$. We propose:[7]

FRAGMENTED ATTRIBUTION A subject believes that ϕ if and only if each row of her access table which is diagnostic for ϕ contains passing information for ϕ (or at least a sufficient set of such rows does).

Here ends our explanation of FRAGMENTED PLAN, which we claimed above would help reply to Objections 1 and 2 from §§3–5.[8] It is time to reply to those objections.

[7]The present proposal is far from a full analysis of belief attributions. Our goal here is less ambitious: a first-approximation account of belief ascriptions that makes room for failures of logical omniscience. Though our account will not be adequate in general (Among other limitations, the account does not even hope to handle difficulties raised in Soames (2009)) we hope it will show that there is no essential tension between coarse-grained accounts of content and failures of logical omniscience.

[8]It is worth noting that one could also formulate a *probabilistic* version of FRAGMENTED PLAN, according to which an access table assigns to each elicitation condition a *probability function* rather than a set of worlds.

An important advantage of using probability functions is that they allow us to take advantage of Bayesian decision theory, and replace FRAGMENTED ACTION with a fragmented variant of decision theory, which allows for agents who have imperfect access to their information:

FRAGMENTED DECISION THEORY An option is predicted for a subject in a given elicitation condition if and only if no rival option is assigned greater expected value by the probability function in the row associated with that elicitation condition in the subject's access table.

What about FRAGMENTED ATTRIBUTION? In a probabilistic setting, it is best to speak about confidence rather than belief:

FRAGMENTED CONFIDENCE A subject is confident that ϕ if and only if each row of her access table which is diagnostic for ϕ has a probability function that assigns high credence to passing information for ϕ (or at least a sufficient set of such rows does).

In what follows will focus on the non-probabilistic version of FRAGMENTED ATTRIBUTION. But, as before, our arguments can be adapted to fit the probabilistic version.

9 *Reply to Objection 1*

Objection 1 is based on the observation that SIMPLE STATE and SIMPLE BELIEF entail that every subject believes every necessary truth, and that belief ascriptions are closed under metaphysical entailment. FRAGMENTED PLAN can avoid this problem because FRAGMENTED STATE and FRAGMENTED ATTRIBUTION do not entail that every subject believes every necessary truth, and do not entail that belief is closed under metaphysical entailment.

To illustrate the fact that FRAGMENTED STATE and FRAGMENTED ATTRIBUTION do not entail that belief is closed under metaphysical entailment, recall the access table associated with the crossword puzzlist (§7). Consider the following belief attributions:

1. The puzzlist believes that *dreamt* is an English word spelled D-R-E-A-M-T.

2. The puzzlist believes that to solve the puzzle, what is needed is an English word ending in MT.

3. The puzzlist believes that *dreamt* solves the puzzle.

The contents attributed in (1) and (2) together metaphysically entail the content attributed in (3). But according to FRAGMENTED ATTRIBUTION (1) and (2) are true but (3) is false.[9]

[9]For (1), the diagnostic tests are, for example, being asked "How do you spell *dreamt*?", or writing an essay about one's dreams on the previous night. And the passing information is mainly the information that *dreamt* is an English word spelled D-R-E-A-M-T. Since $I_!$ contains such information, and since the diagnostic tests all involve situations in which "dreamt" is salient to the subject, the puzzlist's access table contains passing information relative to elicitation conditions corresponding to the relevant diagnostic tests.

For (2), the diagnostic tests are, for example, being asked "Would an English word ending in MT suffice to solve the puzzle?", or "What would it take to solve this puzzle?". And the passing information is mainly the information that to solve the puzzle, what is needed is an English word ending in MT. But either $I_!$ or $I_?$ can be assumed to contain such information. So that the puzzlist's access table can be assumed to contain passing information relative to elicitation conditions corresponding to the relevant diagnostic tests.

For (3), the diagnostic tests are, for example, being faced with the puzzle, or being asked "Is there a word of English ending in MT?". And the passing information is that *dreamt* solves the puzzle, or answers the question. But $I_?$ does not contain such information. Since the diagnostic tests are all such that "dreamt" is not salient to the subject, this means that the puzzlist's access table does not contain passing information relative to elicitation conditions corresponding to the relevant diagnostic tests.

10 Reply to objection 2

Recall Alice and Bob remodeling a bathroom floor. They each realize that the floor requires a grid of 12 by 18 tiles. But the two of them have different false beliefs about the value of 12×18. As a result they are disposed to buy different numbers of tiles.

And recall objection 2: that a coarse-grained picture based on SIMPLE STATE and SIMPLE ACTION cannot satisfactorily model the mental states of Alice and Bob. The basic problem is that there is a tension in Alice and Bob's beliefs. Alice believes both that the bathroom requires 12×18 tiles and that it requires 214 tiles. Bob believes both that the bathroom requires 12×18 tiles and that it requires 218 tiles. As a result there are *no* possible worlds are compatible with every aspect of Alice's beliefs, or every aspect of Bob's beliefs. So SIMPLE PLAN requires that their belief states are both represented by the empty set. But in that case, SIMPLE ACTION won't predict the important behavioral differences between Alice and Bob.

Adopting FRAGMENTED PLAN avoids this difficulty because it removes the pressure to represent Alice and Bob's belief states by the empty set. For example, we might represent Alice's belief state using the following sort of access table:

Elicitation condition	*Accessible information*
Asked questions that test basic understanding of arithmetic	I_{und}
Asked "Is it the case that $12 \times 18 = 214$?"	I_{lim}
[more conditions]	[information accessible relative to those conditions]

Here I_{und} is a set of worlds in which all mathematical vocabulary items have their standard meanings (the numeral "12" refers to the number 12, "$\times$" has multiplication as its semantic value, and so on). At every such world, the sentence "$12 \times 18 = 214$" is false.

In contrast, I_{lim} is a set of worlds in which the sentence "$12 \times 18 = 214$" is true. The worlds in I_{lim} involve various oddities: in some of them, the numeral "12" has a nonstandard meaning; in others the symbol "$\times$" does; in others the language has nonstandard compositional rules.

This table does justice to two facts about Alice. Its first row represents that Alice *understands arithmetical vocabulary*. For example, if she were to be tested on whether "$\times$" expresses multiplication, or whether $\ulcorner 1 \times n = n \urcorner$ is true (for n a small enough numeral), she would pass the test.

Its second row represents that Alice *has a cognitive limitation*: she wrongly believes that $12 \times 18 = 214$. For example, she would answer affirmatively if asked "Is it the case that $12 \times 18 = 214$?".

We might represent Bob's belief state with a table whose first row is the same as Alice's but whose second row contains a set of worlds in which the sentence "$12 \times 18 = \mathbf{218}$" is true. The resulting tables predict the desired tile-buying behavior for Alice and Bob, but appeal only to course grained content (since the batch of information at each row consists of a set of possible worlds). This answers objection 2.

11 Conclusion

The problem of logical omniscience—as developed in objections 1 and 2—is often thought fatal to the coarse-grained view of mental content. But it is not: an appeal to mental fragmentation saves the day. Of course, other attacks based on the problem of logical omniscience are surely waiting in the wings.

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