Market Concentration, Growth, and Acquisitions*

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Abstract

I study an oligopolistic growth model in which firms innovate by creatively destroying their competitors' goods, improving their own goods, and developing new ones. A large firm is equivalent to a mass of small firms that coordinate their innovation to maximize joint profits. Larger firms adapt their innovation mix to avoid cannibalization, and as a result impose a high rate of creative destruction risk on their competitors without generating much growth. A tax on large firm acquisitions of their smaller competitors' goods may backfire by encouraging large firms to creatively destroy those goods instead. In a special case of the model with only creative destruction, I prove conditions so that a rise in large firm profitability leads to a fall in growth, and so that a fall in taxes on acquisitions leads to an *increase* in growth. In the full quantitative model, a fall in large firm fixed costs, calibrated to match the recent rise in concentration in the US, explains 41% of the fall in growth from the 1990s to the 2010s, as well as the burst in growth during the late 1990s, the positive across-industry correlation between growth and changes in concentration, and the fall in R&D efficiency. The size-dependent nature of innovation implies it is advantageous to be large, and yields a novel theory of the inverted-U relationship between concentration and growth.

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1 Introduction

Many authors have documented a rising share of revenue going to the top firms in industries at the national level in the US since the 1990s.[1] This trend has spurred research into its connection to the recent decline in growth, as well as the policy implications.[2] Over a similar time period, there was a dramatic shift for venture capital backed startups from going public to being acquired by public firms.[3] A particular concern is that large firms innovate less in favor of acquiring technologies instead.[4] The following questions emerge: How do large firms innovate? Is an increase in large firm innovation behind the fall in growth? If so, should policymakers limit acquisitions to reduce concentration, or allow acquisitions to discourage large firm innovation?

To answer these questions, I study an oligopolistic growth model in which firms can improve on and replace old goods via creative destruction[5], and develop new ones by expanding the set of varieties[6].[7] I take as the key feature of a large firm that they control a significant portion of their industry's innovation and as a result earn a significant portion of its sales. A large firm can be thought of as a super innovative firm or as a group of small firms that coordinate their innovation to maximize joint profits. I find that if firms can creatively destroy each others' goods and innovation rates are sufficiently responsive to discount rates, then a shock that increases large firm innovation incentives leads to lower growth. In that case, acquisitions that increase concentration are costly, but policies to limit them may backfire by encouraging large firms to innovate more instead. On the other hand, a tax on large firms' sales reduces their innovation and increases growth. In the quantitative model, a rise in large firm profitability calibrated to generate the observed rise in concentration in the US from the mid 1990s to the late 2010s explains 41% of the observed fall in the long-run growth rate, as well as the short-run growth burst in the late 1990s, the positive correlation across industries between changes in concentration and growth, the fall in the entry rate, and the fall in R&D efficiency.

[1]See Autor, Dorn, Katz, Patterson, and Van Reenen (2020), Barkai (2020).

[2]See Cavenaile, Celik, and Tian (2021), Aghion, Bergeaud, Boppart, Klenow, and Li (2022), Akcigit and Ates (2021), De Ridder (2021), Liu, Mian, and Sufi (2022), Olmstead-Rumsey (2022), and Garcia-Macia, Hsieh, and Klenow (2019).

[3]See Pellegrino (2021).

[4]See Letina, Schmutzler, and Seibel (2021) and Fons-Rosen, Roldan-Blanco, and Schmitz (2022).

[5]See Grossman and Helpman (1991b), Aghion and Howitt (1992), and Klette and Kortum (2004).

[6]See Romer (1990) and Grossman and Helpman (1991a).

[7]See Atkeson and Burstein (2019) for a model with creative destruction and expanding varieties.

The key mechanism is that large firms impose a high rate of creative destruction risk on their competitors relative to the growth they generate. Thus, as large firms' innovation incentives rise, they do not generate much more growth, but increase the discount rate on small firm innovation. If small firms are sufficiently responsive, overall growth falls.

The mechanism follows from two implications of the Arrow (1962) "replacement effect" that incumbents avoid cannibalizing their sales from past innovations. First, within an industry, larger firms face less creative destruction risk because incumbents direct creative destruction toward their competitors' goods rather than their own, and a larger firm controls more of the creative destruction in its industry. Second, larger firms have a stronger preference for creatively destroying their competitors' goods over improving on their own goods or developing new ones. Whereas creatively destroying a good mostly takes sales from the producer of that good, the latter types of innovation generate sales only through growth, which takes sales from the rest of the industry, and thus from large firms in proportion to their revenue shares.

How large firms use creative destruction provides a rationale for their existence—consolidating innovation technology in one firm—and for them to acquire goods, even without production synergies or increasing returns. Large firms shield their own goods, which raises their value, but target their competitors', which deters competition.

Without creative destruction, large firms just innovate relatively little, and so are relatively efficient. A rise in large firm innovation incentives increases growth. The return to consolidation is low or negative because competitors respond by innovating more.

A low creative destruction rate for large firm goods explains the finding in Akcigit and Kerr (2018) that larger firms' patents have lower external citation rates. Moreover, it generates the high sales-to-R&D ratios of industry leaders Olmstead-Rumsey (2022) documents[8], and the higher interest rate sensitivity of industry leaders' valuations Liu, Mian, and Sufi (2022) estimate[9].[10] The reasoning behind the size-dependent preference for creative destruction and the deterrence role of creative destruction fit the finding in Garcia-Macia, Hsieh, and Klenow (2019) that creative destruction generates a lot of

[8]To illustrate, if a firm's goods' sales do not depreciate, then its innovation problem is still well-defined with a positive interest rate, but in the long-run, its stock of sales from past innovations is infinite.

[9]A lower creative destruction rate implies a higher share of the total discount rate is the interest rate.

[10]Argente, Lee, and Moreira (2021) provide empirical evidence for *directed* creative destruction: the revenues of high sales products depreciate more quickly than the revenues of low sales products. Akcigit, Alp, and Peters (2021) argue that relatively fast creative destruction of goods produced by firm types that innovate less and tend to be small can explain high employment at old firms in US and Indian data.

excess reallocation across firms unrelated to growth, and so is responsible for a minority of growth in the US, but a majority of the rate at which firms discount innovations.

In Section 3, I study a single industry in a special case of the model in which firms innovate only by creatively destroying their competitors' goods. There is a continuum of small firms and a single large firm that is equivalent to a mass of small firms that collude to spare each others' goods. The large firm searches for acquisition opportunities in which it makes a take-it-or-leave-it offer for a small firm's good subject to a tax.

I derive sufficient statistics for the signs of the effects of large firm innovation incentives (profit or cost per innovation) and the acquisition tax on long-run growth. An increase in large firm innovation incentives reduces growth if small firm innovation is sufficiently elastic with respect to the rate at which small firms discount innovations. Further increases are more likely to decrease growth, so long-run changes in large firm innovation incentives imply an inverted-U relationship between concentration and growth.

A *decrease* in the acquisition tax *increases* growth only if an increase in large firm innovation incentives reduces growth, and in that case, if the elasticity of innovation with respect to the value of innovating is sufficiently high relative to the elasticity of the acquisition rate with respect to the acquisition surplus. Moreover, long-run changes in large firm innovation incentives imply that decreasing the tax increases growth if industry concentration is sufficiently high. These results depend on two opposing effects of reducing the tax and thereby increasing the surplus of each acquisition: 1) growth falls because acquisitions increase, so goods shift to the large firm where they stagnate; and 2) growth rises because the large firm innovates less in favor of acquiring instead. An econometrician studying the impact of exogenous acquisitions would estimate only the first effect.

I develop the quantitative macroeconomic model in Section 4, in which firms also innovate by improving on their own goods and developing new ones. There is a continuum of industries, each of which contains a single large firm and a measure of small firms. In Section 5, I calibrate the model to US data in 1983-1993, and conduct the key experiment: a rise in large firm profitability due to a fall in large firm per-good fixed costs, calibrated to match the rise in industry concentration until 2018. The shock explains 41% of the fall in the growth rate from 1983-1993 to 2003-2013. The model generates all of the observed short-run increase in the growth rate, and 20% of the positive correlation across industries between growth and the change in concentration along the transition path because small firm innovation is less responsive in the short-run and to industry-specific shocks. Long-run R&D/GDP relative to growth falls by 32% of the drop in the data because large firms creatively destroy their competitors' goods relatively quickly, so their innovation is relatively inefficient given convex costs, and becomes more so as it increases.

The strength of the main mechanism depends in particular on the productivity improvement required to creatively destroy a good, which determines the share of sales from creative destruction that come from business stealing rather than growth, and on the relative cost of creative destruction, which determines the share of the small firm discount rate due to creative destruction risk. I use patent data from Akcigit and Kerr (2018) to calibrate the former, and large job destruction flows from Garcia-Macia, Hsieh, and Klenow (2019), which in the model can only be explained by creative destruction, for the latter. I show in Section 5.4 that the main results are *stronger* if I set the former, and so the creative destruction share of *growth*, to zero because the relevant role of creative destruction is discounting small firm innovations, and because larger firms shift innovation more toward creatively destroying competitors' goods if a larger share of sales from doing so come from business stealing. Moreover, I show in Section 5.5.4 that if instead the model matches an empirical across-industry inverted-U relationship between concentration and growth, then the results are similar and robust to a wide range of parameter values.

The results are also quantitatively significant with either Compustat or Census concentration data (Section 5.5.3), and whether labor is flexible or fixed across production, innovation, and entry (Section 5.5.2): in the flexible case, small firms are infinitely responsive to their discount rate, and in the fixed case because innovation efficiency falls.

Finally, I show in Section 6 that in the calibrated model, the effects of a tax on large firm acquisitions of goods are qualitatively as in the special case. It is more beneficial to lower the tax after the rise in large firm profitability and concentration, and if the tax is already low. The elasticity of the acquisition rate with respect to the surplus is crucial: if it is above 1, then lowering the tax on the margin may improve growth, but it is optimal to ban acquisitions altogether. Acquisitions of *innovation technologies* are strictly bad.

These results highlight important features of optimal competition and innovation policy. First, in theories that focus only on *how much* large firms produce or innovate, high markups or the Arrow (1962) "replacement effect" imply that optimal policy encourages *more* large firm production or innovation.[11] Taking into account the multidimensional nature of innovation, such policies end up encouraging innovation that adds little social value but deters competition. The same force would likely apply in a static model of pro-

[11]See Edmond, Midrigan, and Xu (2021) on production, and Fons-Rosen, Roldan-Blanco, and Schmitz (2022) on innovation in the context of acquisitions.

duction in an industry with varying degrees of substitutability between different goods. Second, a rise in large firm activity may be associated with lower growth because of how large firms *build* their market share. Policies such as subsidizing acquisitions can increase concentration *and* growth as long as they reduce large firm innovation. More generally, optimal policy is difficult to implement because it entails a size-dependent tax/subsidy on creative destruction relative to other types of innovation.

Large Firms and Innovation:

Previous work on oligopolistic competition and innovation mostly focuses on the impact of a large firm's market share on its *magnitude* of innovation rather than the composition of its innovation, which drives this paper's results.[12] An exception is the theory in Argente, Baslandze, Hanley, and Moreira (2021) and mentioned in Akcigit and Ates (2021) that large firms use patents to deter competition. The mechanism is different, but the implied effects of large firm innovation are similar. The theory is complementary to the one I propose because creative destruction is most relevant where patent protection is weak.

In the context of the recent rise in concentration and fall in growth, the most similar papers are Aghion, Bergeaud, Boppart, Klenow, and Li (2022), De Ridder (2021), and Liu, Mian, and Sufi (2022). In the first two, all firms have infinitesimal market shares, and a rise in innovation by high process productivity firms results in less growth because they are difficult to compete with, so their presence reduces the return to creative destruction. As such, they find similar results from the rise of high productivity firms that I find from the rise of large, but equally productive firms. In Liu, Mian, and Sufi (2022), two large firms compete in each industry, and if one becomes sufficiently dominant, then growth slows because it optimally responds dramatically to innovation by its competitor. By contrast, my mechanism does not rely on a large firm responding directly to the actions of a single competitor. Thus, it may be more relevant when thinking about an industry with both large and small firms, especially in light of the estimates in Amiti, Itskhoki, and Konings (2019) on the responsiveness of a firm's price to its competitors' prices.

Although none of the papers mentioned so far study acquisitions, in each case acquisitions by large or productive firms would be bad for growth if they raise concentration[13] because no matter how those firms gain market share, they deter innovation by reducing

[12] See Shapiro (2012) for a discussion, and Cavenaile, Celik, and Tian (2021) for a recent example.

[13] This will generally be the case in models with infinitesimal firms whose innovation does not affect their acquisition opportunities.

the *flow profits* their competitors can earn. By contrast, I find that acquisitions can lead to both higher concentration and growth because it is the way large firms innovate that leads to lower growth by adding to the *discount rate* their competitors use for innovations.

The model generates an alternative explanation of the inverted-U relationship between concentration and growth across industries, estimated empirically in Aghion, Bloom, Blundell, Griffith, and Howitt (2005) and Cavenaile, Celik, and Tian (2021). If variation in large firm profitability or innovation capabilities drives variation across industries, then the inverted-U emerges because an increase in large firm innovation decreases growth only if concentration is sufficiently high so that large firm innovation is sufficiently distorted. Previous theories are based instead on two effects of competition: some is necessary to encourage dominant firms to innovate, but too much discourages any innovation. A crucial difference is that in this paper, there is an inverted-U across industries even if a rise in large firm innovation incentives always decreases growth at the aggregate level because the response of small firms is vital, and small firms respond more to aggregate shocks.

Large Firm Acquisitions of Small Competitors' Goods:
"Entry for buyout", described in Rasmusen (1988) and recently in Letina, Schmutzler, and Seibel (2021) and Fons-Rosen, Roldan-Blanco, and Schmitz (2022), is another theory in which the expectation of future valuable acquisitions increases growth: entrants innovate to be acquired. Yet, if large firm innovation is one-dimensional, then this effect needs to be weighed against the *negative* effect on large firm innovation. Indeed, in Fons-Rosen, Roldan-Blanco, and Schmitz (2022), reducing acquisitions improves growth because it increases large firm innovation. In my model, acquisitions are made with take-it-or-leave-it offers, so the entry for buyout effect is absent. Instead, acquisitions may lead to higher growth precisely *because* they reduce large firm innovation. A distinct implication is that acquisitions are more likely to increase growth if concentration is higher.

2 Industry Model

There is a unit measure of industries, indexed by $n \in [0, 1]$. In this section, I focus on a single industry n. Time is continuous and indexed by $t \in [0, \infty)$. At each t, there is a measure N_t of small firms, indexed by $i \in [0, N_t]$ and denoted by S, and a single large firm denoted by $i = L$. They produce a measure $M_{n,t}$ of intermediate goods, indexed by

$j \in [0, M_{n,t}]$, which they sell to a representative final good producer with R_t to spend. Firms hire labor at wage W_t to produce and innovate, and purchase a fixed cost input at price normalized to 1. They maximize expected discounted profits with discount rate r_t.

Although I focus on one industry in this section, I include a subscript n for industry-specific variables so that it is clear which variables are determined at the macroeconomic level, i.e., are exogenous to the industry: N_t, R_t, W_t, and r_t.

2.1 Static Block

2.1.1 Demand

At each time t, the final good producer takes as given a price offered by each firm for each intermediate good, $\{p_{n,t}(i,j)\}$. All versions of a good j are perfect substitutes, so it purchases only the cheapest ones, with price $p_{n,t}(j) = \min_{i \in [0,N_t] \cup \{L\}} \{p_{n,t}(i,j)\}$. It chooses demand of each good $\{y_{n,t}(j)\}$ to maximize a CES aggregate subject to spending R_t:

$$\max\{Y_{n,t}\} \qquad \text{s.t.} \qquad Y_{n,t}^{\frac{\gamma-1}{\gamma}} = \int_0^{M_{n,t}} y_{n,t}(j)^{\frac{\gamma-1}{\gamma}} dj \qquad \text{and} \qquad \int_0^{M_{n,t}} p_{n,t}(j) y_{n,t}(j) dj = R_t, \tag{1}$$

where $\gamma > 1$ is the constant elasticity of substitution across goods.

The First Order Condition for good j and the constraints imply the demand curve[14]

$$y_{n,t}(j) = p_{n,t}(j)^{-\gamma} P_{n,t}^{\gamma-1} R_t,$$

where $P_{n,t}$ is the industry price index so that $R_t = P_{n,t} Y_{n,t}$: $P_{n,t}^{1-\gamma} = \int_0^{M_{n,t}} p_{n,t}(j)^{1-\gamma} dj$.

2.1.2 Production and Competition

At each time t, production occurs in two stages. Each firm can potentially produce a version of each good in its industry with a version specific productivity $z_{n,t}(i,j)$. Let $z_{n,t}(j) \equiv \max_{i \in [0,N_t] \cup \{L\}} \{z_{n,t}(i,j)\}$ be the highest productivity available for good j at time t. We will see later that for each j, t, there is a unique firm with $z_{n,t}(i,j) = z_{n,t}(j)$.

Let $Z_{n,t}$ be an industry aggregate of productivity:

$$Z_{n,t}^{\gamma-1} = \int_0^{M_{n,t}} z_{n,t}(j)^{\gamma-1} dj,$$

[14] The First Order Condition yields $Y_{n,t}^{\frac{1}{\gamma}} y_{n,t}(j)^{\frac{-1}{\gamma}} = \zeta p_{n,t}(j)$, where ζ is the Lagrange multiplier on the budget constraint. Aggregating across all j and satisfying the budget constraint yields $\zeta = Y_{n,t}^{\frac{1}{\gamma}} P_{n,t}^{\frac{1-\gamma}{\gamma}} R_t^{\frac{-1}{\gamma}}$.

and define the relative productivity of a version of a good:

$$\tilde{z}_{n,t}(i,j) \equiv (z_{n,t}(i,j)/Z_{n,t})^{\gamma-1} \qquad\qquad \tilde{z}_{n,t}(j) \equiv (z_{n,t}(j)/Z_{n,t})^{\gamma-1}.$$

In the first stage of production, firms simultaneously choose for which goods they will pay a fixed cost. If firm i pays the fixed cost for good j, then they can produce their version in the second stage, i.e., using labor with production function

$$q_{n,t}(i,j) = z_{n,t}(i,j)l_{n,t}(i,j).$$

Otherwise, they can produce good j with productivity $z_{n,t}(j)/\sigma$, where $\sigma > 1$ captures the ability to imitate other firms' versions. The fixed cost for a version scales with its relative productivity, and is $\tilde{z}_{n,t}(i,j)f_S$ and $\tilde{z}_{n,t}(i,j)f_{L,n}$ for small firms and the large firm, respectively. In the second stage of production, fixed cost payments are common knowledge, firms simultaneously choose prices for each good, and sell the quantity demanded.

Finally, the large firm's revenue is subject to a tax τ_R; it is multiplied by $1-\tau_R$.

2.1.3 Static Optimization and Equilibrium

At each time t, firms choose fixed cost payments and prices to maximize static profits in a subgame perfect Nash equilibrium of the two stage game. I focus on the equilibrium in which for each good j, the sole producer is the unique firm with the most productive version; it pays the fixed cost, sets a markup of σ, and thus earns flow profits

$$\tilde{z}_{n,t}(j)\left(\left(1-\sigma^{-1}\right)R_t - f_S\right) \qquad\qquad \tilde{z}_{n,t}(j)\left(\left(1-\sigma^{-1}-\tau_R\right)R_t - f_{L,n}\right)$$

if small or large, respectively. Profits scale with relative productivity $\tilde{z}_{n,t}(j)$: variable profits scale given CES demand and a constant markup, and fixed costs scale by definition.

The equilibrium strategies are for each good: 1) only the firm with the most productive version pays the fixed cost; 2) for any first stage actions, the most productive producer in the second stage sets its price equal to the second-most productive producer's marginal cost, and other producers set their prices equal to their own marginal costs; 3) if multiple firms set the same price, demand is split evenly among those with the lowest marginal cost. Assumptions 1 and 2 hold for all t, and ensure that these strategies are best responses:

Assumption 1. $\sigma \leq \gamma/(\gamma-1)$.

Assumption 2. $\left(1-\sigma^{-1}\right)R_t \geq f_S$ *and* $\left(1-\left(1-\sigma^{1-\gamma}\right)\mathcal{L}_{n,t}\right)\left(1-\sigma^{-1}-\tau_R\right)R_t \geq f_{L,n}$.

I solve the static optimization problem formally in Appendix A.1. Since only the most productive producer of a good pays the fixed cost, absent imitation they want to set a markup of at least $\gamma/(\gamma-1)$. Assumption 1 thus implies that imitation binds and pins down their markup. Assumption 2 ensures that their variable profits exceed the fixed cost, and for the large firm takes into account that paying the fixed cost for one good lowers that good's price and thus the sales of the large firm's other goods.

2.1.4 Aggregation

Since all goods are sold with a markup σ, the industry price is $P_{n,t} = \sigma W_t / Z_{n,t}$, and industry output is $Y_{n,t} = Z_{n,t} L_t^p$, where production labor is $L_t^p = R_t/(\sigma W_t)$.[15] In that sense, $Z_{n,t}$ is the "correct" measure of industry productivity.

2.2 Dynamic Block

2.2.1 Innovation

Each firm contains entrepreneurs it uses to innovate. At each time t, an entrepreneur's firm chooses a new good development rate δ and for each good j in the industry, a creative destruction rate $\kappa(j)$. At Poisson arrival rate δ, the firm develops a new good; the productivity z of its version is drawn so that the expected value of $(z/Z_{n,t})^{\gamma-1}$ is 1, and all other firms' versions have productivity 0. At Poisson arrival rate $\int_0^{M_{n,t}} \kappa(j)dj$, the firm creatively destroys an old good, and the relative probability it creatively destroys good j is proportional to $\kappa(j)$. Upon creatively destroying good j, its version's productivity jumps to z, which is drawn so that $z > z_{n,t}(j)$, and the expected value of $z^{\gamma-1}$ is $(\lambda z_{n,t}(j))^{\gamma-1}$, where $\lambda > 1$ is the innovation step size. Whether a firm develops a new good or creatively destroys an old one, it becomes the (temporary) sole producer of that good.

The flow labor cost of an entrepreneur's innovation is

$$\alpha \int_0^{M_{n,t}} \tilde{z}_{n,t}(j) X_S(\kappa(j))dj + X_S(\delta) \qquad\qquad \alpha \int_0^{M_{n,t}} \tilde{z}_{n,t}(j) \beta_n X_L(\kappa(j))dj + \beta_n X_L(\delta)$$

if it is controlled by a small firm or the large firm, respectively, where $\alpha > 0$ is the relative cost of creative destruction and $\beta_n > 0$ is an industry-specific large firm innovation cost shifter. Each innovative activity has an independent cost that scales with the expected

[15] Industry revenue, $R_t = P_{n,t} Y_{n,t}$, over expenditures on production labor, $W_t L_t^p$, is the markup σ.

relative productivity of a successful innovation. For $I \in \{S, L\}$, the innovation cost function $X_I(\cdot)$ is twice continuously differentiable, strictly increasing, and weakly convex, with $X_I(0) = 0$ and a continuous marginal innovation cost elasticity for all $\kappa \geq 0$:

$$\epsilon_I(\kappa) \equiv \kappa X_I''(\kappa)/X_I'(\kappa).$$

The marginal innovation cost is increasing in the total rate, and it is costly to focus on one type of innovation or on creatively destroying a subset of goods.

2.2.2 Small vs. Large Firms

A small firm contains a single entrepreneur, so at a Poisson arrival rate it develops a new good or creatively destroys an old one. Thus, in equilibrium, it produces a finite number of goods. The large firm contains a unit measure of entrepreneurs, so at a *continuous rate* it develops new goods and creatively destroys old ones; it creatively destroys *each good* at a Poisson arrival rate. Thus, in equilibrium, it produces a *measure* of goods. This distinction is the only substantive difference between small and large firms.

2.2.3 Acquisitions

At each time t, the large firm chooses an acquisition opportunity rate $A_t(j)$ for each good j in the industry. At Poisson arrival rate $A_t(j)$, it encounters the firm with the most productive version of good j, and makes a take-it-or-leave-it offer subject to an acquisition tax rate $\tau_A > -1$. If the firm accepts, then its version is transferred to the large firm, the large firm pays the offered price, and pays τ_A times the price to the tax authority.

The flow labor cost of searching for potential acquisitions is $\int_0^{M_{n,t}} \tilde{z}_{n,t}(j) X_A(A_t(j)) dj$, where $X_A(\cdot)$ is twice continuously differentiable, strictly increasing, and weakly convex, with $X_A(0) = 0$ and a continuous marginal search cost elasticity for all $A \geq 0$:

$$\epsilon_A(A) \equiv A X_A''(A)/X_A'(A).$$

2.2.4 Dynamic Equilibrium

At each time t, the large firm chooses innovation rates, acquisition search rates, and acquisition price offers, and small firms choose innovation rates and acquisition price acceptance strategies to maximize expected present discounted profits using interest rate

r_t. I focus on Markov perfect equilibria in which the industry state is the large firm's share of industry relative productivity (or equivalently revenue):

$$\mathcal{L}_{n,t} \equiv \int_{j \in J_{n,t}} \tilde{z}_{n,t}(j)dj,$$

where $J_{n,t}$ is the set of goods of which the large firm has the most productive version, and the aggregate state is the measure of small firms N_t, expenditures R_t, the wage W_t, and the interest rate r_t. Moreover, each small firm develops new goods at the rate $\delta_{S,n,t}(\mathcal{L}_{n,t})$, creatively destroys each of its competitors' goods at the single rate $\kappa_{S,n,t}(\mathcal{L}_{n,t})$, and accepts any acquisition offer weakly above its value of a good; the large firm develops new goods at rate $\delta_{L,n,t}(\mathcal{L}_{n,t})$, creatively destroys each of its competitors' goods and its own goods at the single rates $\kappa_{L,n,t}(S;\mathcal{L}_{n,t})$ and $\kappa_{L,n,t}(L;\mathcal{L}_{n,t})$, respectively, searches for an acquisition opportunity for each of its competitors' goods at the single rate $A_{n,t}(\mathcal{L}_{n,t})$, and in an acquisition opportunity, always offers the small firm's value of the good. In each case, the subscript t captures the dependence on the aggregate state. A small firm chooses a different creative destruction rate for the goods it currently produces, but this rate is irrelevant because it produces a measure 0 of goods.

I solve the dynamic optimization problem in Sections 2.2.7 and 2.2.8.

2.2.5 Evolution of the Measure of Small Firms

I do not yet specify the entry/exit process that drives the evolution of the measure of small firms N_t. Exit will not affect a small firm's optimal innovation (Section 4.3).

2.2.6 Evolution of the Industry State and Growth

In equilibrium, the large firm revenue share evolves over time according to

$$\begin{aligned}\dot{\mathcal{L}}_{n,t} =& (\kappa_{L,n,t}(S;\mathcal{L}_{n,t}) + A_{n,t}(\mathcal{L}_{n,t}) + (\gamma-1)g_{L,n,t}(\mathcal{L}_{n,t}))(1-\mathcal{L}_{n,t}) \\ & - N_t(\kappa_{S,n,t}(\mathcal{L}_{n,t}) + (\gamma-1)g_{S,n,t}(\mathcal{L}_{n,t}))\mathcal{L}_{n,t}, \end{aligned} \tag{2}$$

where a dot over a variable indicates its derivative with respect to time t, and where $g_{n,t}(\mathcal{L}_{n,t}) \equiv \dot{Z}_{n,t}/Z_{n,t}$ is the growth rate of industry productivity, which is the sum of growth due to large firm innovation, $g_{L,n,t}(\mathcal{L}_{n,t})$, and to small firm innovation, $N_t g_{S,n,t}(\mathcal{L}_{n,t})$:

$$\begin{aligned}(\gamma-1)g_{L,n,t}(\mathcal{L}_{n,t}) &= \left(\lambda^{\gamma-1}-1\right)\left(\kappa_{L,n,t}(S;\mathcal{L}_{n,t})(1-\mathcal{L}_{n,t}) + \kappa_{L,n,t}(L;\mathcal{L}_{n,t})\mathcal{L}_{n,t}\right) + \delta_{L,n,t}(\mathcal{L}_{n,t}) \\ (\gamma-1)g_{S,n,t}(\mathcal{L}_{n,t}) &= \left(\lambda^{\gamma-1}-1\right)\kappa_{S,n,t}(\mathcal{L}_{n,t}) + \delta_{S,n,t}(\mathcal{L}_{n,t}). \end{aligned} \tag{3}$$

When a firm develops a new good or creatively destroys one of its own, it gains sales only through adding to industry productivity, i.e., generating growth, which takes sales from all other goods by depreciating their relative productivities. Each good loses the same fraction, so the large firm bears a share $\mathcal{L}_{n,t}$ of the cost. In the other extreme, in an acquisition, the large firm takes sales from the previous producer without generating any growth, and thus without any effect on other goods. Finally, creatively destroying a competitor's good yields a combination of the two: the innovator improves on the good, which generates growth and multiplies sales by $\lambda^{\gamma-1} - 1$ on average, but also takes the sales the good had before it was improved on.

There are therefore two reasons for the $1 - \mathcal{L}_{n,t}$ on the first line of equation (2): the first two terms in parentheses are only targeted at small firms that control a share $1-\mathcal{L}_{n,t}$ of sales, whereas the third term—growth—is discounted because a fraction $\mathcal{L}_{n,t}$ of the sales it generates is cannibalization. A similar logic applies to the $\mathcal{L}_{n,t}$ on the second line.

2.2.7 Small Firm Dynamic Optimization

A small firm's innovation optimization problem is the same regardless of the goods it produces because it takes industry aggregates as given. At each time t, a small firm chooses innovation rates $\delta, \{\kappa(j)\}$ to maximize

$$\left(\int_0^{M_{n,t}} \kappa(j)\lambda^{\gamma-1}\tilde{z}_{n,t}(j)dj + \delta \right) \Pi_{n,t}(\mathcal{L}_{n,t}) - W_t \left(\alpha \int_0^{M_{n,t}} \tilde{z}_{n,t}(j) X_S(\kappa(j))dj + X_S(\delta) \right),$$

where $\tilde{z}_{n,t}(j)\Pi_{n,t}(\mathcal{L}_{n,t})$ is the expected present discounted profits from being the most productive producer of good j at time t, and the value per unit of relative productivity, $\Pi_{n,t}(\cdot)$, is given by the HJB equation:

$$\begin{aligned} r_t\Pi_{n,t}(\mathcal{L}_{n,t}) = & \underbrace{\left(1-\sigma^{-1}\right)R_t - f_S}_{\text{flow profits}} - (\underbrace{N_t\kappa_{S,n,t}(\mathcal{L}_{n,t}) + \kappa_{L,n,t}(S;\mathcal{L}_{n,t})}_{\text{creative destruction risk}} + \underbrace{(\gamma-1)g_{n,t}}_{\text{growth}})\Pi_{n,t}(\mathcal{L}_{n,t}) \\ & + \dot{\mathcal{L}}_{n,t}\Pi'_{n,t}(\mathcal{L}_{n,t}) + \dot{\Pi}_{n,t}(\mathcal{L}_{n,t}). \end{aligned} \tag{4}$$

The right-hand side of the first line is flow profits and the rate at which a small firm's good's value depreciates: at a Poisson arrival rate, the good is transferred in its entirety to another firm (creative destruction risk), and at a continuous rate, industry growth erodes the good's relative productivity. The second line is the change in the value over time due to changes in the industry and aggregate state.

The First Order Conditions yield the optimal new good development rate and single rate at which a small firm creatively destroys each of its competitors' goods:

$$W_t X_S'(\delta_{S,n,t}(\mathcal{L}_{n,t})) \geq \Pi_{n,t}(\mathcal{L}_{n,t}) \qquad \alpha W_t X_S'(\kappa_{S,n,t}(\mathcal{L}_{n,t})) \geq \lambda^{\gamma-1}\Pi_{n,t}(\mathcal{L}_{n,t}). \tag{5}$$

Each holds with equality if the rate is strictly positive. There is a single creative destruction rate because costs are convex, and costs and benefits scale with relative productivity.

2.2.8 Large Firm Dynamic Optimization

I split the large firm optimization problem into two steps. First, subject to a given evolution of its revenue share, $\dot{\mathcal{L}}_{n,t}$, the large firm chooses δ, $\{\kappa(j)\}$, and $\{A(j)\}$ to minimize cost. It chooses one creative destruction rate for its own goods, and one creative destruction rate and one acquisition rate for its competitors' goods because costs are convex, and costs and benefits scale with relative productivity. Let ζ be the Lagrange multiplier on the $\dot{\mathcal{L}}_{n,t}$ constraint. The First Order Conditions yield the optimal rates:

$$\begin{aligned}
\beta_n W_t X_L'(\delta_{L,n,t}(\mathcal{L}_{n,t})) &\geq (1-\mathcal{L}_{n,t})\zeta \\
\beta_n \alpha W_t X_L'(\kappa_{L,n,t}(L;\mathcal{L}_{n,t})) &\geq \left(\lambda^{\gamma-1}-1\right)(1-\mathcal{L}_{n,t})\zeta \\
\beta_n \alpha W_t X_L'(\kappa_{L,n,t}(S;\mathcal{L}_{n,t})) &\geq \left(\left(\lambda^{\gamma-1}-1\right)(1-\mathcal{L}_{n,t})+1\right)\zeta \\
W_t X_A'(A_{n,t}(\mathcal{L}_{n,t})) &\geq \zeta - (1+\tau_A)\Pi_{n,t}(\mathcal{L}_{n,t}),
\end{aligned} \tag{6}$$

where each inequality holds with equality if the rate is strictly positive, and where I impose that the large firm always acquires a good (at minimum price) conditional on getting the opportunity because otherwise it would not search.

The marginal benefits on the right-hand sides reflect that the large firm discounts the value of an innovation based on the share that comes from growth because that share partly cannibalizes its sales of its other goods (discussed in Section 2.2.6). Thus, it fully discounts developing a new good and creatively destroying one of its own by $1-\mathcal{L}_{n,t}$, does not discount an acquisition at all, and only discounts the portion of the value of creatively destroying a competitor's good that comes from improving on the good.

Expressions (6) imply a unique solution given $\dot{\mathcal{L}}_{n,t}$ because each rate is increasing in ζ. Let $\bar{X}_{n,t}\left(\dot{\mathcal{L}}_{n,t};\mathcal{L}_{n,t}\right)$ be the implied cost of innovation and acquisition search/payments.[16]

[16] $\bar{X}_{n,t} = \beta_n[\alpha((1-\mathcal{L}_{n,t})X_L(\kappa(S))+\mathcal{L}_{n,t}X_L(\kappa(L)))+X_L(\delta)]+(1-\mathcal{L}_{n,t})(X_A(A)+A(1+\tau_A)\Pi_{n,t}(\mathcal{L}_{n,t}))$.

Second, the large firm chooses $\dot{\mathcal{L}}_{n,t}$, taking as given the cost function $\bar{X}_{n,t}(\cdot,\cdot)$. The HJB equation for the large firm's expected discounted profits, $V_{n,t}(\mathcal{L}_{n,t})$, is

$$\begin{aligned} r_t V_{n,t}(\mathcal{L}_{n,t}) =& \mathcal{L}_{n,t}\left(\left(1-\sigma^{-1}-\tau_R\right)R_t - f_{L,n}\right) + \max_{\dot{\mathcal{L}}}\left\{\dot{\mathcal{L}} V'_{n,t}(\mathcal{L}_{n,t}) - \bar{X}_{n,t}\left(\dot{\mathcal{L}};\mathcal{L}_{n,t}\right)\right\} \\ &+ \dot{V}_{n,t}(\mathcal{L}_{n,t}), \end{aligned} \tag{7}$$

where $\dot{V}_{n,t}(\cdot)$ is the effect of changes in the aggregate state over time. If any innovation or acquisition rate is strictly positive, then $\zeta = V'_{n,t}(\mathcal{L}_{n,t})$.

3 Special Case: Only Creative Destruction

To derive analytical results and isolate a key mechanism, I first study a special case of the industry model with only creative destruction. Firms cannot develop new goods, and I set the elasticity of substitution across goods, γ, to 1. When a firm creatively destroys good j, their new version has productivity $\lambda z_{n,t}(j)$. Total expenditures, the wage, and the interest rate are fixed over time at R, W, and r, and I set the relative cost of creative destruction α, the measure of small firms N_t, and the measure of goods $M_{n,t}$ to 1. I drop the industry subscript n to eliminate unnecessary notation.

3.1 Firm Optimization and Equilibrium

The key implication of $\gamma = 1$ is that each good receives revenue R, regardless of its relative productivity. It follows that growth has no direct effect on sales: 1) if a firm creatively destroys its own good, it does not gain sales; 2) if a firm creatively destroys a competitor's good, it only gains sales by taking the good, and the improvement is irrelevant; 3) the value of a firm's good depreciates over time only due to creative destruction risk.

There is now a Markov perfect equilibrium without an industry state. Small firms creatively destroy their competitors' goods at rate κ_S, and the large firm creatively destroys and acquires its competitors' goods at rates κ_L and A, respectively. Firms do not creatively destroy their own goods. The large firm revenue share $\mathcal{L}_t$ affects the growth rate, but is no longer relevant for innovation decisions because there is no cannibalization.

Adapting equations (2) and (3) yields

$$\dot{\mathcal{L}}_t = (\kappa_L + A)(1-\mathcal{L}_t) - \kappa_S \mathcal{L}_t \qquad\qquad g_t = \ln(\lambda)((1-\mathcal{L}_t)\kappa_L + \kappa_S).$$

The growth rate reflects that large firm innovation targets only a fraction of the goods in the industry, whereas each small firm targets all goods but a set of measure zero. I focus on balanced growth path equilibria in which $\mathcal{L}_t$, and so g_t, are constant over time:

$$\mathcal{L} = (\kappa_L + A)/(\kappa_L + A + \kappa_S) \qquad\qquad g = \ln(\lambda)\kappa_S(2\kappa_L + A + \kappa_S)/(\kappa_L + A + \kappa_S). \quad (8)$$

3.2 Large Firm Innovation and Growth

I first set acquisitions to 0 and study the long-run effects of shocks and policy changes that only directly affect large firm innovation incentives: changes to the large firm innovation cost shifter β, fixed cost f_L, and revenue tax rate τ_R. To do so, I vary β, f_L, and τ_R, and characterize the implied relationship between equilibrium large firm innovation κ_L and the balanced growth path growth rate g. To map the results into the effects of β, f_L, and τ_R more directly, Proposition 1 in Section 3.3 provides a condition under which equilibrium κ_L is a continuous increasing function of $((1 - \sigma^{-1} - \tau_R)R - f_L)/\beta$.

Small firm optimization yields

$$WX_S'(\kappa_S) \geq \Pi = \frac{(1 - \sigma^{-1})R - f_S}{r + \kappa_L + \kappa_S}, \quad (9)$$

which holds with equality if $\kappa_S > 0$, and implies that equilibrium small firm innovation κ_S is a decreasing function of equilibrium κ_L. Equations (8) thus yield balanced growth path g as a function of equilibrium κ_L. Neither function depends directly on β, f_L, or τ_R.

Throughout, I suppose $\kappa_L < \kappa_L^*$, where $\kappa_L^* \in [0, \infty]$ is the large firm innovation rate above which in equilibrium, $\kappa_S = 0$, and so $g = 0$. As such, equilibrium κ_S and balanced growth path g are continuously differentiable functions of equilibrium κ_L. Decompose g and its derivative with respect to equilibrium κ_L:

$$g = \underbrace{\bar{\kappa}}_{\text{Discount}} \underbrace{\frac{g}{\bar{\kappa}}}_{\text{Composition}} \qquad\qquad \frac{\partial g}{\partial \kappa_L} = \underbrace{\frac{\partial \bar{\kappa}}{\partial \kappa_L}\frac{g}{\bar{\kappa}}}_{\text{Discount Effect}} + \underbrace{\bar{\kappa}\frac{\partial (g/\bar{\kappa})}{\partial \kappa_L}}_{\text{Composition Effect}},$$

where $\bar{\kappa} \equiv \kappa_L + \kappa_S$ is the equilibrium creative destruction rate a small firm faces, the discount rate it applies to a good (the denominator on the right-hand side of inequality (9)) minus the interest rate, and $g/\bar{\kappa} = \ln(\lambda)(1 - \mathcal{L}^2)$ is a function of κ_L/κ_S, the composition of creative destruction risk between large and small firm innovation.

The "Discount Effect" is positive. More large firm innovation implies less small firm innovation, but still a higher equilibrium small firm discount rate because innovation costs are convex. Holding fixed its composition, more creative destruction implies more growth.

The "Composition Effect" is negative. Holding fixed small firm creative destruction risk, more large firm innovation shifts its source in equilibrium to the large firm. Immediately, large firm goods face a lower rate of creative destruction, and in the long-run, they are a larger share of the industry (thus, the squared $\mathcal{L}$), each of which implies less growth.

Combining the two effects, $\partial g/\partial \kappa_L$ is the product of a strictly positive function and a sufficient statistic for whether following a change in large firm innovation incentives, more large firm innovation is associated with a higher balanced growth path growth rate:[17]

$$\operatorname{sign}\left(\frac{\partial g}{\partial \kappa_L}\right) = \epsilon_S(\kappa_S) - \frac{2\mathcal{L}}{1-\mathcal{L}} \frac{\bar{\kappa}}{r+\bar{\kappa}}, \tag{10}$$

which depends on three measurable equilibrium outcomes. In particular, $\epsilon_S(\kappa_S)$ is the *inverse elasticity of small firm innovation with respect to the value of innovating.* Thus, large firm innovation reduces growth if small firms are sufficiently responsive to the discount rate, if the large firm earns a sufficient share of industry revenue, and if creative destruction risk is a sufficiently significant component of the small firm discount rate.

The following theorem characterizes the effects of non-marginal changes in large firm innovation incentives. If those incentives drive variation across industries or over time, then it characterizes the long-run relationship between industry concentration and growth because concentration is increasing in large firm innovation.

Theorem 1. *Vary β, f_L, and τ_R. If in equilibrium, $\kappa_L < \kappa_L^*$ (so $\kappa_S > 0$ and $g > 0$), then balanced growth path g is a continuously differentiable function of κ_L such that:*

1. *At $\kappa_L = 0$, $\partial g/\partial \kappa_L \geq 0$, and strictly so if and only if $\epsilon_S(\kappa_S) > 0$.*
2. *There exists a $\kappa_L' < \kappa_L^*$ such that for all $\kappa_L \in (\kappa_L', \kappa_L^*)$, $\partial g/\partial \kappa_L < 0$.*
3. *If $\epsilon_S(\cdot)$ is constant, then g is single-peaked, and so increasing then decreasing.*

Proof. See Appendix A.2. ■

If the large firm has an insignificant revenue share, then more large firm innovation implies faster growth because the large firm innovates like a small firm and targets all goods, so the composition effect is zero. In the other limit, the growth rate is decreasing in large firm innovation and goes to zero because the large firm deters small firm innovation with a fast creative destruction rate, but in the long-run, there are no small firm goods to

[17]The strictly positive function is $\ln(\lambda)(1-\mathcal{L})^2 \frac{r+\bar{\kappa}}{\epsilon_S(\kappa_S)(r+\bar{\kappa})+\kappa_S}$.

creatively destroy. The final property of g states that if the elasticity of small firm innovation with respect to the discount rate is constant, then this pattern fully characterizes the relationship between large firm innovation and growth: g exhibits an inverse-U shape.

3.3 Equilibrium Uniqueness

The following proposition provides a condition for the equilibrium to be unique given parameters, and characterizes properties of the equilibrium as a function of parameters.

Proposition 1. *Suppose $X_L(\cdot)$ and $X_A(\cdot)$ are strictly convex and for all κ_L, $\epsilon_L(\kappa_L)(1+\epsilon_S(\kappa_S)) \geq 1$, where κ_S is given by κ_L and small firm optimization (inequality (9)). There is a unique equilibrium. If there are no acquisitions ($A = 0$), then equilibrium κ_L is a continuous increasing function of $\left(\left(1-\sigma^{-1}-\tau_R\right)R - f_L\right)/\beta$. Moreover,*

1. *Equilibrium κ_L is increasing in τ_A, and strictly so if κ_L and A are strictly positive.*
2. *Equilibrium A is decreasing in τ_A, and strictly so if A is strictly positive.*
3. *If equilibrium κ_L, κ_S, and A are strictly positive, then κ_L, κ_S, A, and balanced growth path g are continuously differentiable with respect to τ_A.*

Proof. See Appendix A.3. ■

3.4 Acquisition Policy

I study the long-run effects of the acquisition tax rate τ_A, focusing on the role of large firm innovation. To have a unique equilibrium, I impose the conditions in Proposition 1. For τ_A to affect the balanced growth path g, suppose κ_L, κ_S, and A are strictly positive.

Decompose τ_A's effect on the balanced growth path growth rate:

$$\frac{\partial g}{\partial \tau_A} = \underbrace{\frac{\partial A}{\partial \tau_A}\frac{\partial g}{\partial A}}_{\text{Acquisition Effect}} + \underbrace{\frac{\partial \kappa_L}{\partial \tau_A}\frac{\partial g}{\partial \kappa_L}}_{\text{Innovation Effect}},$$

where $\partial g/\partial A$ is taken holding innovation rates fixed, and $\partial g/\partial \kappa_L$ is taken holding acquisitions fixed, but not small firm innovation, (the derivative from Section 3.2 with $A > 0$).

The "Acquisition Effect", which is positive, is the effect an econometrician would estimate following an exogenous acquisition. An increase in τ_A decreases the surplus from each acquisition, which decreases the equilibrium acquisition rate, and so reduces the large firm's revenue share and increases the balanced growth path growth rate. Put simply, actual acquisitions increase concentration, and so reduce growth.

The "Innovation Effect" has the same sign as $\partial g/\partial \kappa_L$, studied for $A = 0$ in Section 3.2. For the large firm, the outside option to creatively destroying a competitor's good is acquiring it. An increase in τ_A increases the equilibrium incentive to innovate because it reduces the surplus of an acquisition. Therefore, cutting the acquisition tax can be good for growth only if large firm innovation reduces growth. Put simply, the expectation of future high surplus acquisitions reduces large firm innovation, which may increase growth.

If $\epsilon_S(\kappa_S) = 0$, then $\partial g/\partial \tau_A$ is the product of a strictly positive function and a sufficient statistic for whether an increase in τ_A leads to faster growth on the balanced growth path:[18]

$$\operatorname{sign}\left(\frac{\partial g}{\partial \tau_A}\right) = \frac{1}{\epsilon_A(A)} - \frac{1}{1-\mathcal{L}} \frac{2\kappa_L + A}{\epsilon_L(\kappa_L)\,(r+\bar{\kappa}) - \kappa_L} \left(1 + \frac{(1+\tau_A)\Pi}{\mathcal{S}}\right)^{-1}, \tag{11}$$

which depends on measurable equilibrium outcomes. $\mathcal{S}$ is the surplus of an acquisition, which accrues to the large firm, $(1+\tau_A)\Pi$ is the tax-inclusive acquisition price, and $\epsilon_L(\kappa_L)$ and $\epsilon_A(A)$ are the *inverse elasticities of large firm innovation and the acquisition rate with respect to the value of innovating and of acquiring*, respectively. Thus, taxing acquisitions reduces growth if large firm innovation is sufficiently responsive, if the acquisition rate is sufficiently unresponsive, and if the acquisition surplus is sufficiently high.

The more complicated analog to expression (11) for $\epsilon_S(\kappa_S) > 0$ is in Appendix A.4. The additional result is that for an increase in τ_A to reduce growth, small firm innovation must be sufficiently responsive, as expected given the results in Section 3.2.

The following theorem supposes that variation in large firm innovation incentives drives variation across industries or over time, and characterizes the relationship between industry concentration and the effect of a change in the acquisition tax rate.

Theorem 2. *Suppose $\epsilon_L(\cdot)$ and $\epsilon_A(\cdot)$ are constants, and $\epsilon_S(\cdot) = 0$. Vary f_L and τ_R. There is an $\mathcal{L}^*$ such that $\partial g/\partial \tau_A < 0$ if and only if on the balanced growth path, $\mathcal{L} > \mathcal{L}^*$ and $\Pi > W X_S'(0)$.*

Proof. See Appendix A.5. ■

Following an increase in large firm innovation incentives and the resulting rise in industry concentration, taxing acquisitions is more likely to reduce growth. To see why,

[18] See Appendix A.4 for the derivation.

the second of equations (8) implies that if $\epsilon_S(\cdot) = 0$, then

$$\partial g/\partial A = -(1-\mathcal{L})\kappa_L/(\kappa_L + A + \kappa_S) \qquad \partial g/\partial \kappa_L = -(2\kappa_L + A)/(\kappa_L + A + \kappa_S), \quad (12)$$

where the latter includes the implied equilibrium change in κ_S. As $\mathcal{L}$ goes to 1, $\partial g/\partial A$ becomes arbitrarily small relative to $\partial g/\partial \kappa_L$ because mechanically, the effect of A and κ_L on $\mathcal{L}$ goes to 0, but the effect of κ_S does not, which drives $\partial g/\partial \kappa_L$.

4 Quantitative Macroeconomic Model

I now embed the industry model from Section 2 into a macroeconomic model. There is a unit measure of industries, indexed by $n \in [0, 1]$, each of which is modeled as in Section 2. A representative final good producer aggregates the goods in all industries into the final good to sell to a representative household for consumption and to firms to cover fixed costs. The household consumes, inelastically supplies $\bar{L}$ units of labor, owns all the firms, and has access to a risk-free bond in zero supply. Exogenous exit and undirected endogenous entry determine the measure of small firms, which is the same in each industry.

4.1 Representative Household and the Interest Rate

The household chooses a path of consumption $\{C_t\}$ and bond holdings $\{B_t\}$ to maximize present discounted utility subject to the budget constraint at all t:

$$\max \int_0^\infty e^{-\rho t} \ln(C_t) dt \qquad \text{s.t.} \quad P_t C_t + \dot{B}_t = W_t \bar{L} + D_t + r_t B_t \quad \text{for all } t,$$

where $\rho > 0$ is the time discount rate, and the household takes as given paths for the final good price $\{P_t\}$, wage $\{W_t\}$, flow profits from firms $\{D_t\}$, and net interest rate $\{r_t\}$.

Bond market clearing implies that for all t, the net interest rate must compensate for ρ and the negative rate of change of the value of the numeraire: $r_t = \rho + \dot{C}_t/C_t + \dot{P}_t/P_t$.

4.2 Representative Final Good Producer

At each time t, the final good producer can aggregate industry composite goods $\{Y_{n,t}\}$ (equation (1) in Section 2.1.1) into final good output Y_t via a Cobb-Douglas aggregator: $\ln(Y_t) = \int_0^1 \ln(Y_{n,t}) dn$. It maximizes profits taking as given the final good price P_t, and industry composite good prices $\{P_{n,t}\}$ (determined by cost minimization in Section 2.1.1).

The First Order Condition for $Y_{n,t}$ implies constant expenditures across industries: $R_t = P_t Y_t$. Perfect competition implies zero profits, which yields $\ln(P_t) = \int_0^1 \ln(P_{n,t})dn$.

4.3 Entry and Exit

At each time t, each of an infinite mass of potential entrants can receive value 0 or pay $\xi > 0$ units of labor to draw an industry from the uniform distribution and enter as a small firm with an entrepreneur and a 0 productivity version of each good. Potential entrants maximize expected discounted profits using the interest rate to discount payoffs. For entry to be finite, the value of entry net the cost must be weakly negative.

Each small firm entrepreneur exits exogenously at Poisson arrival rate $\eta > 0$, after which its firm can produce, but not innovate. Thus, the measure of small firm *entrepreneurs*, which is relevant, does not equal the measure of producing small firms, which is irrelevant. The former, N_t, is the same in each industry because entry is *undirected*, and evolves over time due to entry e_t and exit according to $\dot{N}_t = e_t - \eta N_t$.

4.4 Aggregation and Normalization

At each time t, since expenditures are the same in each industry, it follows from industry aggregation in Section 2.1.4 that production labor L_t^p is the same in each industry. Aggregate output is $Y_t = Z_t L_t^p$, and the final good price is $P_t = \sigma W_t / Z_t$, where Z_t is aggregate productivity: $\ln(Z_t) = \int_0^1 \ln(Z_{n,t})dn$.

I normalize $P_t Z_t = 1$ for all t so that $W_t = \sigma^{-1}$, and expenditures are $R_t = L_t^p$. Aggregate productivity growth is $g_t \equiv \dot{Z}_t / Z_t = \int_0^1 g_{n,t}dn$.

4.5 Fixed Costs and Innovation Cost Functions

A unit of fixed cost requires Z_t units of the final good. With the normalization, fixed costs are as in Section 2.1.2: $\tilde{z}_{n,t}(i,j)f_S$ and $\tilde{z}_{n,t}(i,j)f_{L,n}$ for small and large firms, respectively.

I set the innovation cost function so the marginal cost has a constant elasticity $\epsilon > 0$:

$$X_S(x) = X_L(x) = (\epsilon + 1)^{-1} x^{\epsilon+1}.$$

We can interpret the cost shifter β_n as the large firm has a measure $\beta_n^{-1/\epsilon}$ of entrepreneurs.

4.6 Equilibrium

Firms play Markov perfect equilibria (Sections 2.1.3 and 2.2.4), and potential entrants, the household, and the final good producer optimize, taking as given paths of the aggregate state $\{N_t\}$ and aggregates $\{R_t, W_t, r_t\}$. At all t, final good producers earn zero profits, the value of entry is weakly less than the cost, the tax authority uses lump sum taxes/transfers to balance its budget, and markets clear for bonds, labor (production, innovation, and entry equal supply), and the final good (consumption and fixed costs equal supply).

I study balanced growth paths and convergence to them following unanticipated shocks. A balanced growth path is an equilibrium in which each industry state and the aggregate state are constant over time. Thus, aggregate quantities and prices, and industry and aggregate growth rates are constant, and the interest rate is $r = \rho$.

4.7 Welfare

The measure of welfare is that of the household, which depends on current productivity and weighted averages of future growth, production labor, and fixed costs:

$$\int_0^\infty e^{-\rho t}\ln(C_t)dt = \int_0^\infty e^{-\rho t}\left(\ln(Z_t) + \ln\left(L_t^p - \int_0^1((1-\mathcal{L}_{n,t})f_S + \mathcal{L}_{n,t}f_{L,n})dn\right)\right)dt$$
$$= \frac{\ln(Z_0)}{\rho} + \frac{\int_0^\infty \rho e^{-\rho t}g_t dt}{\rho^2} + \frac{\int_0^\infty \rho e^{-\rho t}\ln\left(L_t^p - \int_0^1((1-\mathcal{L}_{n,t})f_S + \mathcal{L}_{n,t}f_{L,n})dn\right)dt}{\rho},$$

Growth at t is discounted by ρ^2 because it raises consumption in all $t' > t$. On a balanced growth path, welfare is $\ln(Z_0)/\rho + g/\rho^2 + \ln\left(L^p - \int_0^1((1-\mathcal{L}_n)f_S + \mathcal{L}_n f_{L,n})dn\right)/\rho$.

4.8 Entrant Optimization

A small firm's value of innovating does not depend on the goods it produces. Thus, the value of entering industry n at time t is $E_{n,t}(\mathcal{L}_{n,t})$, which is given by the HJB equation:

$$\begin{aligned} r_t E_{n,t}(\mathcal{L}_{n,t}) = {} & \left(\kappa_{S,n,t}(\mathcal{L}_{n,t})\lambda^{\gamma-1} + \delta_{S,n,t}(\mathcal{L}_{n,t})\right)\Pi_{n,t}(\mathcal{L}_{n,t}) + \dot{\mathcal{L}}_{n,t}E'_{n,t}(\mathcal{L}_{n,t}) + \dot{E}_{n,t}(\mathcal{L}_{n,t}) \\ & - W_t(\epsilon+1)^{-1}\left(\alpha\kappa_{S,n,t}(\mathcal{L}_{n,t})^{\epsilon+1} + \delta_{S,n,t}(\mathcal{L}_{n,t})^{\epsilon+1}\right) - \eta E_{n,t}(\mathcal{L}_{n,t}). \end{aligned} \tag{13}$$

The right-hand side is the benefit (first line) and cost (second line) from innovation (Section 2.2.7), the effects of changes over time in the industry and aggregate state, and the risk of exit. The value of entry net the cost is $\int_0^1 E_{n,t}(\mathcal{L}_{n,t})dn - W_t\xi$.

4.9 Model Discussion

Before proceeding to the results, I discuss some of the main modeling choices.

The presence of imitation and Assumption 1 imply a constant markup across firms and goods. One effect is the number of industry states is 1 rather than 3, and new good development is equivalent to own good improvement. More deeply, this choice abstracts from the relationship between static prices and growth. If large firms set higher markups, then an increase in their revenue share increases prices relative to productivity, which implies *higher* profits and small firm innovation incentives. However, large firms also may produce with low marginal costs relative to productivity,[19] which has the opposite effect.

A consequence of making all entrants permanently small is that large firm profits do not affect entry. One justification is that if large firms exit more slowly or if firms take time to become large, then due to discounting, large firms are over represented in the cross section relative to their salience for a potential entrant. Second, this is the correct approach for the main experiment in Section 5.3 if the interpretation of the shock is that large firms pay higher firm-level fixed costs to achieve lower per-good fixed costs.

5 Results: The Effects of Large Firm Innovation

I characterize the effect of changes to large firm innovation incentives (the large firm fixed cost, revenue tax, and innovation cost) on industry concentration, growth, and welfare. I present long-run qualitative results that also provide intuition, then calibrate the model and compare quantitative results to US data. Throughout, the acquisition rate is 0.

5.1 Concentration and Growth in the Long-Run

I use small firm optimization and large firm *relative* innovation rates to characterize the long-run relationship between large firm revenue shares and growth. The following theorem, analogous to Theorem 1 and displayed graphically in Figure 1, shows that if innovation costs are quadratic, then *across industries*, the growth rate is a function of the large firm industry revenue share that exhibits an **inverted-U shape**. On the other hand, an *aggregate* increase in large firm innovation incentives always decreases the long-run growth rate, leaving aside quantitatively small general equilibrium labor market effects.

[19]Due to a higher process productivity as in Aghion, Bergeaud, Boppart, Klenow, and Li (2022) or more intangibles as in De Ridder (2021).

Theorem 3. *There are continuously differentiable functions $g_I(\cdot)$ and $g_A(\cdot)$ such that 1) on a balanced growth path, the industry n growth rate is $g_I(\mathcal{L}_n)$; and 2) if all industries are the same, the large firm fixed cost f_L, revenue tax rate τ_R, and innovation cost shifter β vary, and labor supply $\bar{L}$ adjusts so that balanced growth path production labor L^p is constant, then the long-run aggregate growth rate is $g_A(\mathcal{L})$. The following hold:*

1. $g_I(0) > 0$, $g_I'(0) > 0$, $g_A(0) > 0$, *and* $g_A'(0) = 0$.

2. *If* $\epsilon = 1$, *then* $\lim_{\mathcal{L}\to 1}(g_I(\mathcal{L})) = 0$ *and there exists a threshold* $\mathcal{L}^* \in (0,1)$ *such that* $g_I'(\mathcal{L}) > 0$ *for* $\mathcal{L} < \mathcal{L}^*$ *and* $g_I'(\mathcal{L}) < 0$ *for* $\mathcal{L} > \mathcal{L}^*$.

3. *For all* $\mathcal{L} > 0$, $g_A'(\mathcal{L}) < 0$, *and* $\lim_{\mathcal{L}\to 1}(g_A(\mathcal{L})) = 0$.

Proof. See Appendix A.6. ∎

As in Section 3.2, decompose the effect of large firm innovation incentives on the long-run growth rate into the discount and composition effects. On a balanced growth path, using equation (4), a small firm's value of an innovation is

$$\Pi_n(\mathcal{L}_n) = \frac{(1-\sigma^{-1})\,L^p - f_S}{\rho + N(\kappa_{S,n}(\mathcal{L}_n) + (\gamma-1)g_{S,n}(\mathcal{L}_n)) + \kappa_{L,n}(S;\mathcal{L}_n) + (\gamma-1)g_{L,n}(\mathcal{L}_n)}.$$

The denominator is the small firm discount rate, which includes depreciation from growth and creative destruction risk. The "discount effect" is the effect on the discount rate, holding fixed depreciation from growth relative to creative destruction risk; the "composition effect" is the effect on that ratio, holding fixed the discount rate.

Across industries, the discount effect is strictly positive because N is fixed and innovation costs are convex: more large firm innovation implies a higher discount rate and so more growth. Across balanced growth paths, the discount effect is zero due to free entry.

The composition effect is strictly negative; the more large firms innovate, the more the small firm discount rate is achieved through creative destruction risk rather than depreciation from growth. First, the sole force in Section 3.2: the reluctance of firms to creatively destroy their own goods is only relevant for large firms that are responsible for non-negligible shares of innovation and sales in their industries. Large firm goods stagnate, which reduces growth but not small firm creative destruction risk. Second, large firms have a size-dependent relative preference for creatively destroying competitors' goods:

$$\frac{\kappa_{L,n}(L;\mathcal{L}_n)}{\kappa_{L,n}(S;\mathcal{L}_n)} = \mathcal{D}(\mathcal{L}_n)\left(\frac{\lambda^{\gamma-1}-1}{\lambda^{\gamma-1}}\right)^{1/\epsilon} \qquad \frac{\delta_{L,n}(\mathcal{L}_n)}{\kappa_{L,n}(S;\mathcal{L}_n)} = \mathcal{D}(\mathcal{L}_n)\left(\frac{\alpha}{\lambda^{\gamma-1}}\right)^{1/\epsilon},$$

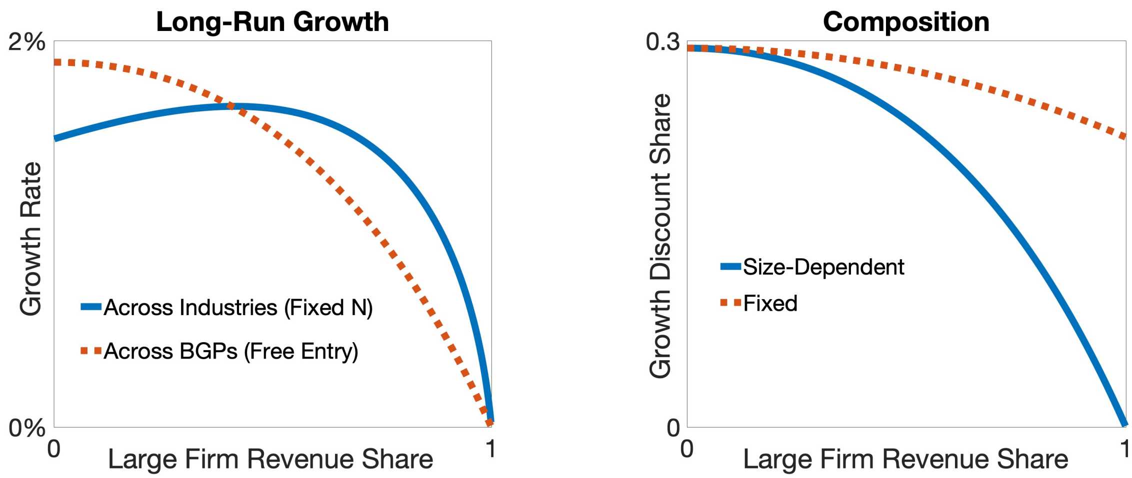

Figure 1: Both figures use the calibration in Section 5.2. Left panel: the solid blue and dotted orange lines display $g_I(\cdot)$ and $g_A(\cdot)$, respectively, from Theorem 3. Right panel: the solid blue and dotted orange lines display growth relative to the small firm discount rate (minus ρ) in equilibrium, and with large firm relative innovation rates set as if $\mathcal{L}_n = 0$, respectively.

where each is the product of $\mathcal{D}(\mathcal{L}_n) \equiv \left(\frac{1-\mathcal{L}_n}{(1-\lambda^{1-\gamma})(1-\mathcal{L}_n)+\lambda^{1-\gamma}}\right)^{1/\epsilon}$ and the analogous small firm relative innovation rate. $\mathcal{D}(\cdot)$, decreasing from $\mathcal{D}(0) = 1$ to $\mathcal{D}(1) = 0$, is the discount a large firm applies to generating growth rather than creatively destroying a competitor's good; a fraction $\lambda^{1-\gamma}$ of the sales gained through creative destruction is taken from the competitor, and thus does not cannibalize the sales of the large firm's other goods. I show the relative importance of the two mechanisms in the right panel of Figure 1.

5.1.1 Labor Market Effects and Approximating Long-Run Effects of Shocks

In the quantitative experiments, shocks that shift innovation to large firms result in an increase in aggregate expenditures to increase production labor and clear the labor market. Omitted from Theorem 3, this pushes up the return to innovating. Put another way, large firm innovation saves costs relative to small firm entry and innovation. Nonetheless, the effect on the long-run growth rate is not significant; the composition effect dominates.

It follows that given the effect of a shock to large firm innovation incentives (f_L, τ_R, or β) on the long-run large firm revenue share, $g_A(\cdot)$ from Theorem 3 provides a good approximation of the effect on long-run growth. Computing $g_A(\cdot)$ only requires the marginal cost elasticity ϵ, the relative creative destruction cost α, the innovation step size λ, the elasticity of substitution γ, and the initial large firm revenue share $\mathcal{L}$ and growth rate g. Thus, the size of the fall in g in the main experiment in Section 5.3 depends almost entirely on ϵ, α, λ, and γ because I calibrate the shock to match a given increase in $\mathcal{L}$.

5.2 Calibration

I calibrate a balanced growth path with identical industries. I set some parameters externally (Table 1) and internally calibrate the rest to jointly match empirical moments (Table 2). I set labor supply $\bar{L}$ so aggregate expenditure $R = L^p$ is 1. Small and large firms have the same fixed cost. The revenue tax rate is 0. The units of time are years.

5.2.1 Externally Calibrated Parameters

The exit rate η is the annual employment-weighted average firm exit rate from Boar and Midrigan (2022).[20] The demand elasticity γ is the median estimate from Broda and Weinstein (2006) at the most disaggregated level in the earliest time period, which is apt because I use the most disaggregated industry definition available for the large firm revenue share measure. The marginal innovation cost elasticity ϵ captures two elasticities: of a firm's total innovation rate with respect to its value or cost, and of a firm's *relative* innovation with respect to the value or cost of one type of innovation *relative* to another. Acemoglu, Akcigit, Alp, Bloom, and Kerr (2018) discuss a variety of studies that estimate the former elasticity to be 1. The proof of Theorem 3 shows that 1 is also a good target for the latter:[21] to match the inverted-U relationship between growth and concentration documented in Cavenaile, Celik, and Tian (2021), the elasticity of relative innovation rates cannot be much greater than 1 (≈ 1.62); otherwise, an industry's growth rate diverges to infinity as its large firm's revenue share goes to 1 because the large firm uses a high rate of growth as well as creative destruction to maintain its dominance.

5.2.2 Internally Calibrated Parameters

I match the BLS measure of aggregate productivity growth from Garcia-Macia, Hsieh, and Klenow (2019), the 1-year real interest rate from FRED, the cost-weighted average markup estimated in Compustat data by De Loecker, Eeckhout, and Unger (2020), and

[20] η should be the rate at which an entrepreneur stops innovating in their industry; closing one firm to start another should not count as an exit. Nonetheless, its effects on the long-run results are small because it only impacts the shift in labor from entry to production.

[21] See Proposition 2 in Appendix A.6.

Table 1: Externally Calibrated Parameters

Parameter	Description	Value
η	Exit Rate	0.04
γ	Demand Elasticity	3.1
ϵ	Innovation Cost Elasticity	1

Business Enterprise Expenditures on R&D/GDP from the OECD MSTI database, all averages from 1983-1993. The first two determine the time discount rate ρ: the real interest rate is $\rho + g$.[22] The markup is the imitation discount σ. R&D/GDP (innovation costs over nominal output) determines profits over sales, and so the fixed cost given σ. The distinction between the fixed cost and σ is only relevant so I can lower f_L in experiments.

I set λ so the average growth contribution of a creative destruction innovation, $\frac{\lambda^{\gamma-1}-1}{\gamma-1}$, matches the Akcigit and Kerr (2018) estimate of the average growth contribution of an external patent (that mostly cites other firms' patents).[23] On average, 13% of a good's sales immediately after creative destruction are novel, so creative destruction risk is 74% of the small firm discount (minus ρ), but only 34% of growth is due to creative destruction.

I match $\mathcal{L}$ to the sales-weighted average industry revenue share of the largest firm in 4-digit industries in Compustat in 1983-1993 from Olmstead-Rumsey (2022). As a robustness check in Section 5.5.3, I use Census industry concentration data, which include all firms rather than just public ones. Compustat is less complete but may better capture the relevant moment for cannibalization, which is a large firm's share of innovations rather than sales. $\mathcal{L}$ and g pin down the large firm innovation cost shifter β and entry cost ξ.

To calibrate α, I use large job destruction flows—the share of aggregate employment lost over a 5 year period at firms whose employment shrank by at least two-thirds—computed in Census data by Garcia-Macia, Hsieh, and Klenow (2019) to back out the rate at which a small firm's good is creatively destroyed. The model only implies a maximum value for these flows,[24] so they identify a maximum α: I suppose all creative destruction

[22] A real bond in the model must compensate for ρ and declining marginal utility due to growth in C_t.

[23] Their analogous estimate for internal innovations is lower at 0.051, and so implies a lower λ and a stronger composition effect, whereas Garcia-Macia, Hsieh, and Klenow (2019) estimate a slightly higher 0.081 using labor flows data through the lens of a growth model different from the one in this paper.

[24] The number of goods per firm depends on the productivity distribution of innovations and whether firms develop new goods or improve on their old goods, neither of which is specified. Each small firm

Table 2: Internally Calibrated Parameters and Data Moments

Parameter	Value	Moment	Value
ρ – Time Discount Rate	0.0194	Real Interest Rate	3.6%
σ – Imitation Discount	1.3	Markup	1.3
f_S, f_L – Fixed Cost	0.183	R&D/GDP	1.81%
λ – Innovation Step Size	1.067	External Innovation Step	0.069
ξ – Entry Cost	4.233	TFP Growth Rate	1.66%
β – Large Firm Innovation Cost	28.36	Large Firm Revenue Share	40.7%
α – Creative Destruction Cost	0.3114	Large Job Destruction Rate	25.57%

of small firm goods leads to large flows, which are thus $(1-\mathcal{L})(1-e^{-5(\kappa_S+\kappa_L(S))})$.[25] For consistency, I use Census concentration data (see Section 5.5.3) for $\mathcal{L}$, which yields a higher α than does the Compustat measure. These choices that yield a maximum α are conservative in the sense that lower values imply larger falls in long-run growth. I use an alternative calibration strategy in Section 5.5.4 as a robustness check.

The calibrated values for ϵ, α, λ, and γ, and so $g_A(\cdot)$ (Theorem 3), do not depend on other parameters because ϵ, α, λ, γ, $\mathcal{L}$, and g imply innovation compositions, which are sufficient. I use $g_A(\cdot)$ to confirm that lower α's yield larger falls in long-run growth.

5.3 Quantitative Experiment: A Rise in Large Firm Innovation

I ask whether a rise in concentration driven by a rise in large firm profitability can explain changes in US data since the mid-1990s. I interpret the shock as a shift from per-good to firm wide fixed costs—a rise in span of control—due to the rise in information technology.[26] I show in Section 5.5.1 that a fall in large firm innovation costs yields similar results.

The economy begins on the balanced growth path from Section 5.2. There is an unanticipated permanent fall in f_L to 0.17 in all industries, calibrated so the new balanced

may have many small goods and face a zero probability of shrinking by more than two-thirds in 5 years.

[25] I suppose depreciation due to growth, which on its own takes a share $1-e^{-5(\gamma-1)g}=0.16<2/3$ of sales, do not lead to large flows. For growth depreciation to affect large flows, a small firm must have multiple goods, which then reduces the large flows generated by creative destruction. I exclude innovation labor, which is small, so I can calibrate α without computing a dynamic equilibrium.

[26] See Aghion, Bergeaud, Boppart, Klenow, and Li (2022) for a discussion.

growth path large firm revenue share is 0.51, the 2018 sales-weighted average revenue share of the largest firm across 4-digit industries in Compustat from Olmstead-Rumsey (2022).

5.3.1 Industry Concentration and Aggregate Growth

I compare the effects on growth to data from Garcia-Macia, Hsieh, and Klenow (2019) in Table 3. The shock explains 41% of the long-run fall in the growth rate, and all of the short-run increase, mostly due to a temporary shift in labor from entry to production. The cumulative growth burst is smaller than in the data because it peaks in the model after 3 years, whereas in the data is an average over 10 years. The large firm revenue share and the growth rate (Figure 2) converge over a similar time interval as the gap between the initial calibration years, 1983-1993, and the target year for the shock, 2018.

Table 3: Growth After a Fall in f_L

Moment	Data	Model
Growth Burst	+0.64 ppt (39%) (1993-2003)	GDP: +0.77 ppt (46%) (first year)
		TFP: +0.12 ppt (7%) (first year)
Cumulative Burst	+6.4 ppt (39%) (1993-2003)	GDP: +0.91 ppt (18%) (3 years)
		TFP: +0.21 ppt (4%) (3 years)
Growth Fall	-0.34 ppt (-20%) (2003-2013)	-0.14 ppt (-8%) (New BGP)

ppt is percentage point rise, and in parentheses is the percent rise. Growth burst is the peak change in the growth rate. Cumulative burst is the peak difference in accumulated growth. GDP uses output over $\bar{L}$ to measure productivity. Growth fall is the long-run change.

I decompose the time path of the productivity growth rate in Figure 3. The negative composition effect is present immediately because innovation shifts to large firms, and becomes more so as large firm revenue shares increase. The small firm discount rate increases dramatically in the short-run—flow profits rise because aggregate expenditures increase to clear the labor market, and the net value of entry falls below zero because there are excess small firms—but by less in the long-run because the permanent increase in expenditures is small. For long-run growth to rise, the savings on entry labor would need to be huge: production labor needs to rise by 10% to cancel the composition effect.

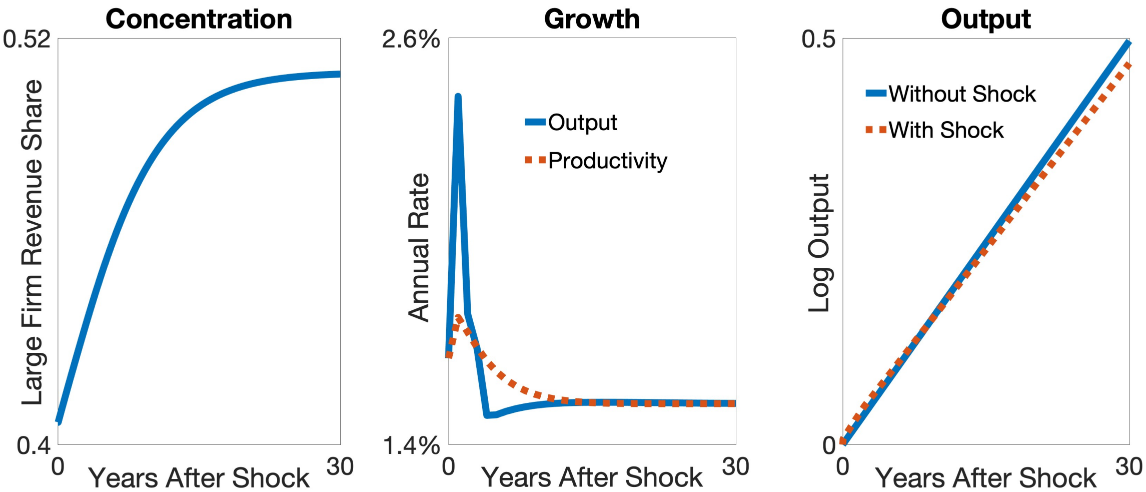

Figure 2: Transition paths after the shock to f_L. Left panel: $\mathcal{L}_t$. Middle panel: the dotted orange and solid blue lines are annual g_t and the growth rate of $Y_t/\bar{L}$. Right panel: the dotted orange and solid blue lines are $\ln(Y_t)$ in equilibrium and in a counterfactual without the shock.

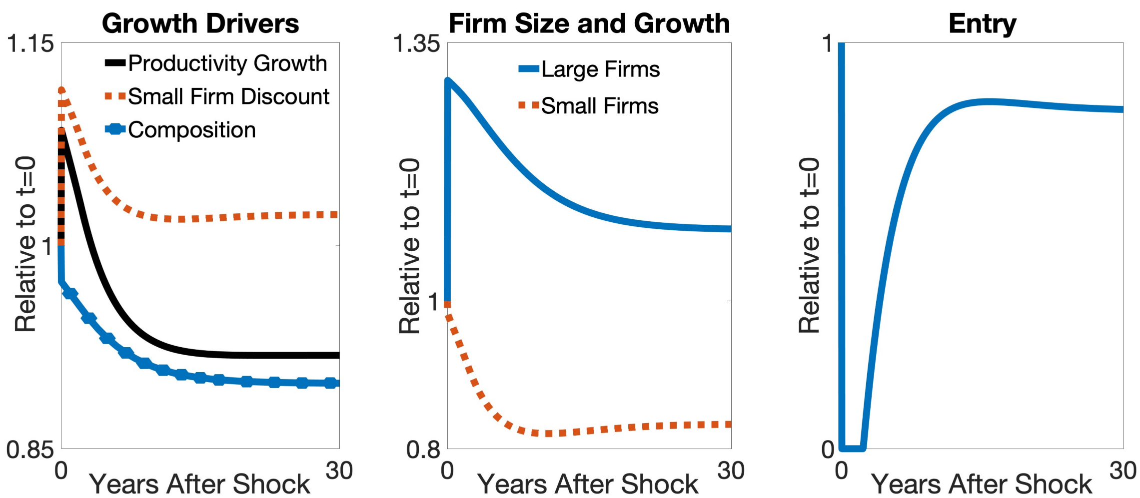

Figure 3: Variables relative to their values in the initial balanced growth path. Left panel: the solid black, dotted orange, and textured blue lines are g_t/g and its decomposition into the small firm discount rate (minus ρ) and growth over the discount rate (minus ρ). Middle panel: the solid blue and dotted orange lines are $g_{L,t}/g_L$ and $g_{S,t}/g_S$. Right panel: e_t/e.

5.3.2 Welfare and Size-Dependent Taxes

Including the transition path, the present discounted values of consumption (welfare) and output fall by the equivalent of a permanent 4.8% and 5.7% drop, respectively; output shifts from fixed costs to consumption. On its own, the decline in the long-run growth rate implies the equivalent of a 6.9% permanent fall in consumption. To see the importance of short-run vs. long-run effects, I compute the first time T so that the discounted value of output or consumption between 0 and T is lower after the shock than it would have been without the shock. For output and consumption, T is 19 and 34 years, respectively.

On the margin, decreasing the large firm revenue tax τ_R decreases welfare because it has the same effect as a fall in f_L, but without increasing consumption relative to output.

5.3.3 Labor Flows

The model matches the dynamics of large job destruction flows documented in Garcia-Macia, Hsieh, and Klenow (2019), using the model measure described in Section 5.2, $(1-\mathcal{L})\left(1-e^{-5(\kappa_S+\kappa_L(S))}\right)$, and equilibrium $\mathcal{L}$. The comparison is in Table 4. Large flows increase in the short-run because innovation shifts to creative destruction, but fall in the long-run because employment shifts to large firms that do not experience large flows.

Table 4: Labor Flows

Moment	Initial	Short-Run Change	Long-Run Change
Large Destruction Flows	0.26 (0.26)	+9% (+4%)	-15% (-13%)
Creative Growth Share	0.25 (0.27)	+3% (+5%)	-7% (-17%)
Creative Discount Share	0.74 (0.55)	+1% (-16%)	+3% (-3%)

Model (black): initial balanced growth path, change immediately after the shock, and change across balanced growth paths. Data (blue in parentheses): 1983-1993, change from 1983-1993 to 1993-2003, and change from 1983-1993 to 2003-2013.

The model matches their estimated dynamics of the creative destruction share of growth, using creative destruction *of small firm goods* for the model, which generates large flows. Including small firm creative destruction of large firm goods, the growth share still falls in the long-run—though by less than 1%—because large firms experience less creative destruction. On the other hand, the model does less well at matching their estimated dynamics of the creative destruction risk share of the small firm discount rate, not including the interest rate. The difference in levels is because Garcia-Macia, Hsieh, and Klenow (2019) assume creative destruction targets large and small firm goods equally, and use $\gamma = 4$ instead of 3.1, which increases depreciation due to growth. Moreover, they estimate a rise and then fall in the innovation step size, which increases and then decreases the creative destruction growth share relative to the creative destruction rate.

The model does not identify other statistics used in Garcia-Macia, Hsieh, and Klenow (2019) that depend on the productivity distribution of innovations, such as large job *creation* flows. Moreover, it says nothing about new good development vs. own good improvement because they have identical effects on sales and growth.

5.3.4 R&D Efficiency

The model matches the simultaneous rise in R&D/GDP and fall in growth in US data, which yield a fall in the growth rate relative to R&D/GDP from 0.91 in 1983-1993 to 0.69 in 2003-2013. In the model, across balanced growth paths, R&D/GDP rises from 1.81% to 1.82%, and the growth rate relative to R&D/GDP falls from 0.91 to 0.84. After the shock, large firms innovate more, and convex costs imply a fall in innovation efficiency.

Creative destruction is crucial to prevent a counteracting effect. Large firms are relatively efficient at developing new goods because to avoid cannibalization, they do relatively little; their marginal cost is 0.67 and 0.74 compared to that of small firms in the initial and new balanced growth paths, respectively. The opposite holds for creatively destroying competitors' goods because large firms face lower creative destruction rates and thus innovate more; their marginal cost is 1.08 and then 1.41 compared to that of small firms.

5.3.5 Industry Concentration and Industry Growth Rates

The model matches the finding in Ganapati (2021) that across industries in the US, rising concentration is associated with *faster* growth. With sector and time fixed effects, a 1 percent rise in the revenue share of the largest 4 firms in a 6-digit NAICS industry over a 5 year period is associated with a 0.1 percent rise in real output and a 0.2 percent rise in real output relative to employment. I run the same regression during the transition path by creating a measure 0 control group of industries with a constant large firm fixed cost. A 1 percent rise in an industry's large firm revenue share over a 5 year period is associated with a 0.04 percent rise in real output and in real output relative to employment.

The theory thus generates a parsimonious explanation for the short-run burst in growth and the positive relationship between rising concentration and growth across industries: small firm innovation is less responsive in the short-run and to industry-specific shocks.

5.3.6 Entry

The fall in entry (Figure 3) matches empirical results in Decker, Haltiwanger, Jarmin, and Miranda (2016):[27] the employment share of firms less than 5 years old fell by 20% during the early 1990s, and then 5% in the early 2000s; in the model, the same share (including production and innovation labor) falls by 16% across balanced growth paths.

5.4 The Role of Creative Destruction – Risk vs. Growth

The presence of creative destruction is crucial for the results thus far. Moreover, I confirmed at the end of Section 5.2 that a higher relative creative destruction cost α implies a lesser fall in long-run growth in the main experiment in Section 5.3. Now, I show that the relevant measure of the importance of creative destruction is the share of the small firm discount rate due to creative destruction risk rather than the share of growth due to creative destruction. I set the innovation step size λ to 1 so the latter is zero. I use two methods for calibrating the creative destruction cost α: 1) to match large job destruction flows as in the original calibration in Section 5.2; and 2) so $\alpha\lambda^{\gamma-1}$, and thus small firm creative destruction relative to new good development, is the same as in the original calibration. For other parameters, I use the same moments as in Section 5.2.

Growth falls by more than in Section 5.3 (Table 5). The increase is due to a stronger composition effect.[28] The discussion in Section 5.1 provides intuition: a lower λ shifts large firm innovation toward creatively destroying competitors' goods (lowers $\mathcal{D}(\cdot)$) because it decreases the share of sales from doing so that generate growth and thus cannibalization. Using labor flows to calibrate α instead of fixing $\alpha\lambda^{\gamma-1}$ yields a smaller change because to compensate for the shift in large firm innovation toward creative destruction, α must rise to get an opposite shift among small firms.

5.5 Different Shocks and Calibrations

I redo the main experiment in Section 5.3, but shock a different variable, use a different version of the model, and use different calibrations. The results are in Table 5.

[27] Garcia-Macia, Hsieh, and Klenow (2019) find similar results, but their third time period overlaps with the Great Recession.

[28] The positive discount effect is stronger because the initial growth rate is fixed, so more creative destruction implies a higher small firm discount rate, and thus a bigger increase for a given percent increase.

5.5.1 A Fall in Large Firm Innovation Costs

I change the large firm innovation cost shifter β rather than the fixed cost f_L. Innovation costs relative to output fall (unlike in the data) because innovation *rates* are similar to in Section 5.3, but innovation is cheaper. Long-run growth falls by less than in Section 5.3 because goods expenditures must rise by more to compensate for lower innovation labor.

Table 5: Alternative Shocks and Calibrations

Experiment	Growth	R&D/GDP	ξ	β	α	f_S, f_L	f_L^*
Original Experiment	1.52%	1.82%	4.23	28.4	0.311	0.183	0.17
$\lambda = 1$ Calibration 1	1.51%	1.82%	2.67	19.0	0.421	0.183	0.171
$\lambda = 1$ Calibration 2	1.50%	1.81%	1.85	14.2	0.272	0.185	0.175
Innovation Cost (β)	1.54%	1.67%	4.23	28.4	0.311	0.183	22 (β^*)
Fixed Labor	1.59%	1.98%	4.23	28.4	0.311	0.183	0.158
Census Calibration	1.61%	1.85%	4.19	40.2	0.311	0.186	0.178
Inverted-U Calibration	1.60%	1.86%	9.7	44.6	4.16	0.17	0.143

Each row is for a different experiment; it shows the internally calibrated parameters that sometimes vary, and long-run growth and R&D/GDP following the shock. The new value of the shocked parameter, which is β in the fourth case and f_L in the others, is in the last column.

5.5.2 Fixed Innovation Labor

I fix the labor used for each of entry, innovation, and production to their values in the initial balanced growth path. Each has a different wage, and a labor market that clears. If only entry is fixed, a rise in large firm innovation incentives increases growth because small firms are insufficiently responsive. They are more responsive if innovation labor is fixed as well because then large firm innovation pushes up the innovation labor wage.

The long-run growth rate falls, although by less than in Section 5.3 because convex costs imply that small firm innovation becomes more efficient as it decreases. Thus, a rise in large firm innovation incentives leads to lower growth if labor is fully flexible across uses, or fully fixed. The model's main mechanism is crucial in the fixed case as well because, as discussed in Section 5.3.4, it drives the fall in average innovation efficiency.

5.5.3 Census Concentration Measure

I match the large firm revenue share in the initial and new balanced growth paths to the sales-weighted average revenue share of the largest 4 firms in 6-digit NAICS industries in the US in 1997 (30.6%) and 2012 (35.9%), respectively, computed by Barkai (2020) in Census data using consistently defined industries over time; I use the same 2-digit sectors as Garcia-Macia, Hsieh, and Klenow (2019). I begin in 1997 because industry definitions change between 1992 and 1997, but the concentration increase would likely be larger if I started in 1983-1993 because Autor, Dorn, Katz, Patterson, and Van Reenen (2020) find increasing concentration in Census data since the 1980s. Growth falls by less than in Section 5.3 because a lower large firm revenue share implies a weaker composition effect.

5.5.4 Calibrating to Match the Inverted-U Relationship

Instead of large job destruction flows, I use the inverted-U relationship between growth and concentration across industries from Theorem 3, $g_I(\cdot)$, to calibrate the relative creative destruction cost α. Cavenaile, Celik, and Tian (2021) estimate inverted-U relationships between innovation measures and HHI (sum of squared revenue shares) in 4-digit industries in Compustat data, with sector and year fixed effects. I match the location of the peak of $g_I(\cdot)$ to the patent-maximizing large firm revenue share implied by their regression of industry patents on HHI and HHI squared: $\mathcal{L} = 0.83$.[29] The fall in growth is smaller than in Section 5.3 because α is higher, which implies less creative destruction.

Under this calibration strategy, the size of the fall in growth is robust. Recall from the ends of Sections 5.1 and 5.2 that given values for parameters ϵ, α, λ, and γ, for equilibrium outcomes $\mathcal{L}$ and g on the initial balanced growth path, and for $\mathcal{L}$ on the new balanced growth path, we can use $g_A(\cdot)$ from Theorem 3 to approximate g on the new balanced growth path. Moreover, the same values from the initial balanced growth path are sufficient to compute $g_I(\cdot)$ as well. Thus, I take ϵ and the initial values for $\mathcal{L}$ and g from the calibration in Section 5.2, and for any values of γ and λ, I calibrate α to match the inverted-U from Cavenaile, Celik, and Tian (2021). I then compute $g_A(0.51) - g_A(0.407)$ to approximate the change in long-run growth following the shock. I vary γ between 2 and 20, and for each γ, vary λ over its feasible range from 1 to the maximum above which the desired α does not exist (for example, to $\lambda = 1.05$ if $\gamma = 2$, and to $\lambda = 2.27$ if $\gamma = 20$).

[29] I match the maximizing HHI rather than the regression coefficients because $g_I(\cdot)$ is not a second-order polynomial, so to do the latter would require specifying an HHI distribution across industries.

The growth change, $g_A(0.51) - g_A(0.407)$, ranges from -0.055 to -0.07 percentage points, and variation is almost entirely due to γ: if $\gamma = 2$, then it ranges from -0.0554 to -0.0557, and if $\gamma = 20$, from -0.0662 to -0.0701.

6 Antitrust Policy: Acquisitions

6.1 Acquisitions of Goods

I analyze the effect of a change in the acquisition tax rate on growth, and the importance of concentration and the responsiveness of the acquisition rate for those effects. I parameterize the search function (Section 2.2.3) $X_A(A) = \omega_1 A^{\omega_2^{-1}+1}$ with a cost shifter $\omega_1 > 0$, and the *elasticity of the acquisition rate with respect to the acquisition surplus*, $\omega_2 > 0$.

6.1.1 The Effects of the Tax Rate on Growth

I compute balanced growth path growth for various acquisition tax rates in Figure 4, using the calibration after the fall in f_L from Section 5.3, and $\omega_2 \in \{1, 1/3, 0\}$. For $\omega_2 = 0$, $X_A(A)$ is 0 if $A \leq 0.05$ and $+\infty$ otherwise. For $\omega_2 \in \{1, 1/3\}$, I set ω_1 so the balanced growth path acquisition rate is 0.05 when the tax rate is at its minimum, -1.[30]

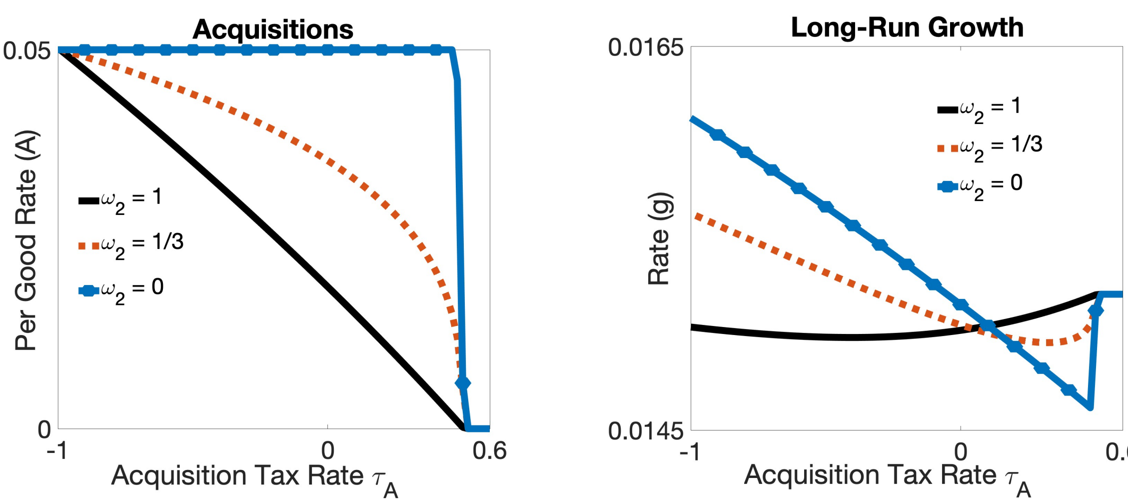

Figure 4: Variables on balanced growth paths as a function of τ_A, using the calibration following the drop in f_L in Section 5.3, and cost functions with different elasticities ω_2, and cost shifters ω_1 so that $A = 0.05$ at $\tau_A = -1$. Left panel: acquisition rate A. Right panel: growth rate g.

[30] ω_1 is 3.8 when ω_2 is 1, and 694 when ω_2 is 1/3.

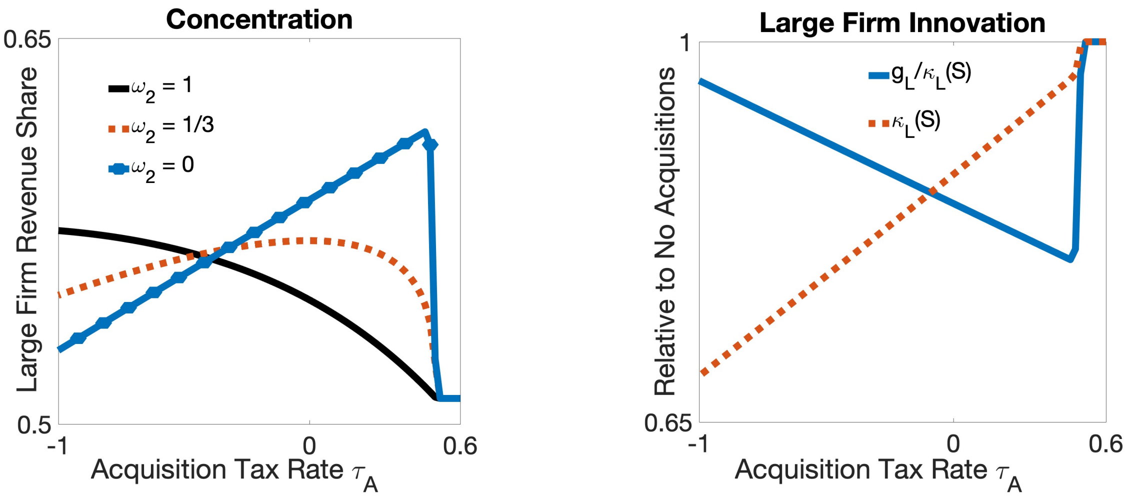

Figure 5: Left panel: see the caption in Figure 4; the large firm revenue share $\mathcal{L}$. Right panel: using the cost function that is 0 for $A \leq 0.05$ and infinite otherwise, the balanced growth path $g_L/\kappa_L(S)$ and $\kappa_L(S)$ as a function of τ_A relative to their values at $\tau_A = 0.6$.

If the tax rate is just below 0.6, then acquisitions occur but generate little surplus, so only the acquisition effect (Section 3.4), which lowers growth, is present: large firms have higher revenue shares, which has little effect on the rate at which they creatively destroy their competitors' goods, but reduces the rate at which they generate growth (Figure 5). As the tax falls, the acquisition surplus rises, so the innovation effect gains strength, which increases growth: large firms reduce all innovation to preserve acquisition opportunities.

Even if growth is lower with acquisitions than without, a tax cut on the margin can still increase growth, which is relevant if eliminating all acquisitions is not desirable.

6.1.2 The Acquisition Rate Elasticity and Concentration

To understand when it is more beneficial to lower the acquisition tax rate, I show in Figure 6 the break even tax rate and acquisition surplus at which the long-run growth rate is equal with acquisitions and without, for various values of ω_2, and using the calibration both before the main experiment in Section 5.3 and after.[31] The break even tax rate is higher and the break even surplus is lower—acquisitions more easily increase growth—if the acquisition rate is less elastic with respect to the surplus, and after the rise in concentration, as in Section 3.4. If the acquisition rate elasticity ω_2 is smaller, then so are total search costs relative to the marginal search cost, so it is cheaper to find acquisition

[31]In all cases, I set ω_1 so the acquisition rate is low to eliminate non-linear effects.

opportunities, and a high surplus is a powerful incentive to reduce innovation. After the rise in concentration, a given surplus is more beneficial for growth because the innovation effect is mechanically stronger relative to the acquisition effect, as we saw in Section 3.4.

The lower break even surplus after the rise in concentration demonstrates that the results do not simply reflect the higher surplus inherent in lower large firm fixed costs.

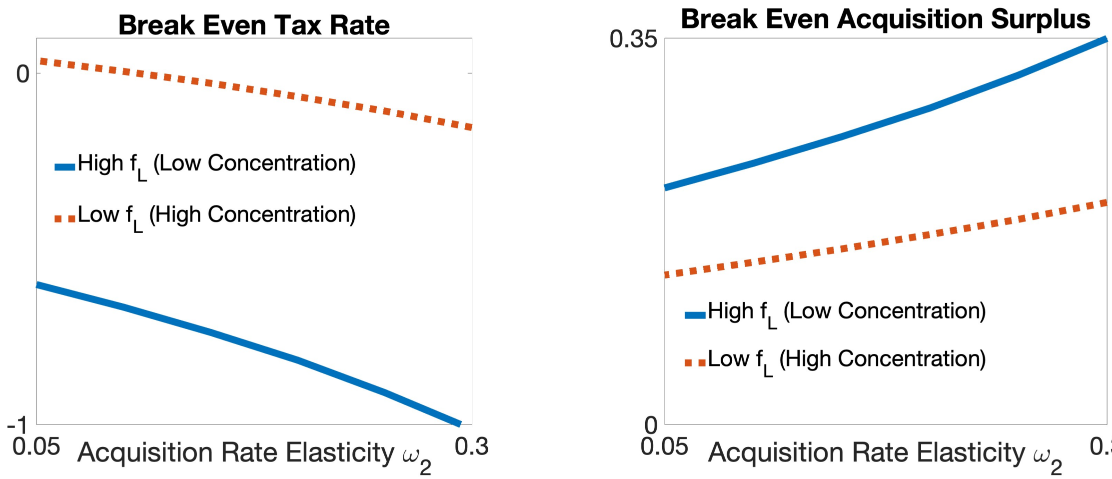

Figure 6: In each panel, the solid blue line uses the calibration from Section 5.2, and the dotted orange line uses the same but after the main experiment in Section 5.3. Left panel: the value of τ_A at which the economy with acquisitions has the same long-run growth rate as the one without, as a function of ω_2. Right panel: the surplus of an acquisition at the break even τ_A.

6.2 Acquisitions of Innovation Technology: The Value of Size

In the presence of creative destruction, it is advantageous to be large. Thus, the theory provides an explanation for large firm acquisitions of their competitors' goods and innovation technologies, even without production synergies or increasing returns. Equivalently, it explains the existence of large firms, and why they do not break themselves up.

A large firm has a one-time opportunity to acquire either the new good development or creative destruction technologies of a small share of its industry's small firm entrepreneurs. I compute the large firm value relative to the small firm value, first holding innovation rates fixed, and then allowing for re-optimization. I use the initial calibration from Section 5.2 so large and small firms differ only in size. The entrepreneurs still exit at rate η.

First, for either innovation technology, holding innovation rates fixed, the large firm is willing to pay 127% of the small firm's value. The large firm values goods more because it shields them from its creative destruction, which is 28% of the small firm discount rate.

Next, if the large firm can choose innovation rates, it is willing to pay 129% and 164% of the small firm's value for the new good development and creative destruction technologies, respectively. In the former case, the value of choosing innovation rates is low (129% vs. 127%) and can even be negative if small firms are more responsive, which reflects that large firms would like to commit to faster innovation, but cannot. They cut the entrepreneurs' new good development rates by 33% to avoid cannibalization, and small firms innovate more in response. In the latter case, the value of choosing innovation rates is high (164% vs. 127%), which reflects how large firms target creative destruction, and that a firm mostly cares about the rate at which its goods are creatively destroyed, rather than its competitors'. The large firm cuts the entrepreneurs' creative destruction of large firm goods by 91%, but *increases* creative destruction of small firm goods by 8% because it has a higher value of a good. In response, small firms innovate *less*.

7 Conclusion

I study the effects of the concentration of innovation. I use a model in which each industry consists of a single large firm and a continuum of small firms that can improve on and replace old goods (creative destruction), and develop new ones (expanding varieties). A large firm is equivalent to a mass of small firms that coordinate their innovation to maximize joint profits. I find that to avoid cannibalization, large firms improve on their goods and develop new ones relatively slowly, but creatively destroy their competitors' goods relatively quickly. Thus, they deter competition but generate relatively little growth.

I demonstrate three implications of how large firms innovate. If small firm innovation is sufficiently responsive to the discount rate, then a rise in large firm innovation incentives leads to lower long-run growth. In that case, large firm acquisitions of their competitors' goods have two effects with opposite implications for concentration, growth, and welfare: 1) goods shift to large firms, where they stagnate and make large firm innovation more focused on creatively destroying competitors' goods; and 2) large firms innovate less to preserve valuable acquisition opportunities. Finally, large firms value goods more, and there is a return to being large, even without increasing returns to scale in production; large firms shield their goods from their own creative destruction, which has little effect on small firms, and so does not result in a counteracting rise in small firm innovation.

In the quantitative model, a rise in large firm profitability calibrated to generate the recent rise in US industry concentration can explain significant shares of the observed fall in long-run growth, burst in short-run growth, positive correlation across industries between growth and changes in concentration, and fall in R&D efficiency. The share of the small firm discount rate due to creative destruction risk is a relevant measure of creative destruction's role, whereas the share of growth due to creative destruction is not.

The paper has novel implications for optimal competition and innovation policy. First, large firms are only harmful in how they achieve their size. Thus, policies may be beneficial even if they increase concentration as long as they reduce large firm innovation. Second, the Arrow (1962) replacement effect pushes large firms to reduce some types of innovation, but not others. Thus, subsidizing large firm innovation to undo the replacement effect ends up mostly encouraging creative destruction of small firm goods, and reducing growth.

The latter implication is relevant in other settings, and suggests avenues for future research. For example, suppose in a static model of oligopolistic competition, a firm can produce different types of goods, with varying degrees of substitutability with its competitors' goods. The same force that leads larger firms to set higher markups implies that they have a stronger preference for producing goods that are close substitutes with their competitors'. Thus, subsidizing large high markup firms to produce more may be costly, unlike in models in which all goods within an industry are equally substitutable.[32]

A Proofs and Derivations

A.1 Proof of Static Equilibrium (Section 2.1.3)

In the second stage, a firm without the most productive active version of good j cannot earn positive profits, so it is optimal for them to set price equal to marginal cost. A firm with the most productive active version would set a markup of at least $\gamma/(\gamma-1)$ if unconstrained by other producers of good j, so by Assumption 1, pricing below other producers' marginal costs is a binding constraint.[33]

In the first stage, a firm without the most productive version of good j earns zero second stage profits if it pays the fixed cost, so it is optimal not to. If the small firm with the most productive version pays the fixed cost, it earns profits $\tilde{z}_{n,t}(j)\left(1-\sigma^{-1}\right)R_t$, so it

[32]See Edmond, Midrigan, and Xu (2021).

[33]See Edmond, Midrigan, and Xu (2021) for the optimal markup with oligopoly, nested CES demand, and Bertrand competition.

is optimal to do so by Assumption 2. A large firm takes into account that paying the fixed cost for good j increases good j's productivity, which reduces the relative productivity of other goods. If the large firm pays fixed costs for a fraction x of its relative productivity, then its versions are replaced by imitations, and its industry revenue share is

$$\tilde{\mathcal{L}}_{n,t}(x) \equiv x\mathcal{L}_{n,t} / \left(1 - (1-x)\left(1-\sigma^{1-\gamma}\right)\mathcal{L}_{n,t}\right)$$

Total profit across both stages is $\tilde{\mathcal{L}}_{n,t}(x)\left(1-\sigma^{-1}-\tau_R\right)R_t - x\mathcal{L}_{n,t}f_{L,n}$, which is strictly concave in x. It is thus optimal for the large firm to pay the fixed cost for all goods for which it has the most productive version if the first derivative of total profits at $x=1$ is positive, which is the case if Assumption 2 holds.

A.2 Proof of Theorem 1

I use (10), which has the same sign as $\partial g/\partial \kappa_L$, to prove the theorem. First, the theorem is vacuously true if $\kappa_L^* = 0$, i.e., if there is an equilibrium without any innovation.

Suppose $\kappa_L^* > 0$ in which case in equilibrium, $\bar{\kappa} > 0$. For the first property, if $\kappa_L = 0$, then since $\kappa_L^* > 0$, it follows that $\kappa_S > 0$, and so $\mathcal{L} = 0$. Thus, the second term in (10) is 0, and the result follows. For the second and third properties, it is sufficient to show that as equilibrium κ_L increases due to changes in β, f_L, and τ_R, the balanced growth path $\frac{2\mathcal{L}}{1-\mathcal{L}}\frac{\bar{\kappa}}{r+\bar{\kappa}}$ increases, and diverges to infinity as κ_L converges to κ_L^*. As κ_L increases, κ_S must decrease (so $\mathcal{L}$ must increase) and $\bar{\kappa}$ must increase because if $\bar{\kappa}$ decreases, then κ_S must increase, which contradicts the fall in $\bar{\kappa}$. As κ_L converges to κ_L^*, κ_S goes to 0, so $\mathcal{L}/(1-\mathcal{L})$ goes to infinity, and $\bar{\kappa}$ goes to κ_L^*, so $\bar{\kappa}/(r+\bar{\kappa})$ does not go to 0.

A.3 Proof of Proposition 1

Use HJB equation (7) to solve for the equilibrium large firm value of a good:

$$\bar{V} \equiv V'(\mathcal{L}) = \frac{(1-\sigma^{-1}-\tau_R)R - f_L + W\beta X_L(\kappa_L) + WX_A(A) + A(1+\tau_A)\Pi}{r+\kappa_L+A+\kappa_S}. \tag{14}$$

Holding fixed parameters, use the large firm FOCs (6) with $\zeta = \bar{V}$ (and the strict convexity of $X_L(\cdot)$ and $X_A(\cdot)$), and small firm optimization (9) to write equilibrium large firm innovation κ_L, small firm innovation κ_S, the acquisition surplus $\mathcal{S} \equiv \bar{V} - (1+\tau_A)\Pi$, the small firm value of a good Π, the acquisition rate A, and so the right-hand side of (14) as continuous functions of $\bar{V}$ that are differentiable everywhere except at finitely many points. Let $T(\bar{V})$ be the function on the right-hand side. To prove that there is a unique equilibrium, it is thus sufficient to show that there is a unique $\bar{V}$ such that $T(\bar{V}) = \bar{V}$.

First, $\bar{V} > 0$ in equilibrium because $(1 - \sigma^{-1} - \tau_R)R - f_L > 0$ by Assumption 2, and the remaining terms on the right-hand side of (14) are weakly positive. It follows that $T(\bar{V})$ is bounded below by a strictly positive number, so that as $\bar{V}$ goes to 0, $T(\bar{V}) > \bar{V}$. To prove existence and uniqueness, it is thus sufficient to show that the derivative of $T(\bar{V})$ is bounded above by a number strictly below 1.

Canceling terms by using that if $\partial\kappa_L/\partial\bar{V} \neq 0$, then the FOC for κ_L holds with equality, and the same for A, we have that

$$T'(\bar{V}) = (A(1 + \tau_A)\partial\Pi/\partial\bar{V} - \bar{V}\partial\kappa_S/\partial\bar{V})/(r + \kappa_L + A + \kappa_S). \tag{15}$$

Differentiating Π and both sides of the FOCs for κ_L and κ_S with respect to $\bar{V}$ yields

$$\frac{\partial\Pi}{\partial\bar{V}} = \frac{-\Pi}{r + \bar{\kappa}}\frac{\partial\bar{\kappa}}{\partial\bar{V}} \qquad \frac{\partial\kappa_L}{\partial\bar{V}} = \frac{1}{\epsilon_L(\kappa_L)}\frac{\kappa_L}{\bar{V}} \qquad \frac{\partial\kappa_S}{\partial\bar{V}} = -\left(\epsilon_S(\kappa_S)\frac{r + \bar{\kappa}}{\kappa_S} + 1\right)^{-1}\frac{\partial\kappa_L}{\partial\bar{V}}. \tag{16}$$

We saw in Appendix A.2 that $\bar{\kappa}$ is increasing in κ_L, so $\partial\bar{\kappa}/\partial\bar{V} \geq 0$, and $T'(\bar{V})$ is less than

$$\frac{1}{\epsilon_S(\kappa_S)(r + \bar{\kappa})/\kappa_S + 1}\frac{1}{\epsilon_L(\kappa_L)}\frac{\kappa_L}{r + \kappa_L + A + \kappa_S},$$

which is strictly less than 1 because $r > 0$ and $(\epsilon_S(\kappa_S) + 1)\epsilon_L(\kappa_L) \geq 1$ by assumption.

For the next claim in the proposition, suppose $A = 0$ and define $\hat{V} \equiv \bar{V}/\beta$ and $\Upsilon \equiv ((1 - \sigma^{-1} - \tau_R)R - f_L)/\beta$. Dividing both sides of FOC (6) for κ_L by β shows that the equilibrium κ_L is a continuous increasing function of the equilibrium $\hat{V}$ that does not otherwise depend on Υ. Thus, to show that the equilibrium κ_L is a continuous increasing function of Υ, it is sufficient to show that the same holds for the equilibrium $\hat{V}$.

From small firm optimization (9), the equilibrium κ_S is also a continuous function of the equilibrium $\hat{V}$ that does not otherwise depend on Υ. Let $\hat{T}(\hat{V}, \Upsilon)$ be the right-hand side of (14) divided by β, where I make the dependence on Υ not through $\hat{V}$ explicit. From the first part of the proof and since $\hat{T}(\cdot,\cdot)$ is linear in Υ, $\hat{T}(\cdot,\cdot)$ is continuous in each of its arguments and differentiable everywhere except at finitely many points. Moreover, for each Υ, there is a unique equilibrium $\hat{V}$ such that $\hat{T}(\hat{V}, \Upsilon) = \hat{V}$. Totally differentiating with respect to Υ yields in equilibrium,

$$\partial\hat{V}/\partial\Upsilon = (1 - \partial\hat{T}(\hat{V}, \Upsilon)/\partial\hat{V})^{-1}\partial\hat{T}(\hat{V}, \Upsilon)/\partial\Upsilon.$$

The argument that $T'(\bar{V})$ is bounded above by a number strictly below 1 shows that the same is true for $\partial\hat{T}(\hat{V}, \Upsilon)/\partial\hat{V}$. It follows that $\partial\hat{V}/\partial\Upsilon > 0$ because $\partial\hat{T}(\hat{V}, \Upsilon)/\partial\Upsilon > 0$.

For the remaining claims, using the large firm FOCs and small firm optimization, it is sufficient to show that equilibrium $\bar{V}$ and $\mathcal{S}$ are strictly increasing and decreasing in τ_A, respectively (acquisitions depend on $\mathcal{S}$). The same argument as for $\partial\hat{V}/\partial\Upsilon$ shows that

$$\frac{\partial\bar{V}}{\partial\tau_A} = \frac{(1-T'(\bar{V}))^{-1}A\Pi}{r+\kappa_L+A+\kappa_S} = \frac{A\Pi}{r+\kappa_L+\kappa_S+A(1-(1+\tau_A)\partial\Pi/\partial\bar{V})+\bar{V}\partial\kappa_S/\partial\bar{V}},$$

where the second equality uses (15). Thus,

$$\frac{\partial\mathcal{S}}{\partial\tau_A} = \left(1-(1+\tau_A)\frac{\partial\Pi}{\partial\bar{V}}\right)\frac{\partial\bar{V}}{\partial\tau_A} - \Pi = \frac{-(r+\kappa_L+\kappa_S+\bar{V}\partial\kappa_S/\partial\bar{V})\Pi}{r+\kappa_L+\kappa_S+A(1-(1+\tau_A)\partial\Pi/\partial\bar{V})+\bar{V}\partial\kappa_S/\partial\bar{V}}.$$

The result follows because (16) and $(\epsilon_S(\kappa_S)+1)\epsilon_L(\kappa_L)\geq 1$ imply $\bar{V}\partial\kappa_S/\partial\bar{V}\geq -\kappa_L$.

A.4 Derivation of $\partial g/\partial\tau_A$

By assumption, the large firm FOCs (6) hold with equality (with $\zeta = \bar{V} \equiv V'(\mathcal{L})$). Differentiating each side of the FOC for κ_L with respect to $\bar{V}$ and for A with respect to $\mathcal{S} \equiv \bar{V} - (1+\tau_A)\Pi$ yield that $1/\epsilon_L(\kappa_L)$ and $1/\epsilon_A(A)$ are the elasticities of equilibrium κ_L and A with respect to equilibrium $\bar{V}$ and $\mathcal{S}$, respectively. Then, $\partial\bar{V}/\partial\tau_A$ and $\partial\mathcal{S}/\partial\tau_A$ from the end of Appendix A.3, and $\partial g/\partial A$ and $\partial g/\partial\kappa_L$ (the former holds innovation fixed and the latter includes the derivative of equilibrium κ_S with respect to equilibrium κ_L) from (12) in Section 3.4 imply that the derivative of balanced growth path g with respect to τ_A is the product of

$$\mathcal{L}(1-\mathcal{L})\frac{\kappa_L}{\kappa_L+A}\frac{1}{\mathcal{S}}\left(r+\kappa_L+\kappa_S-\frac{1}{\epsilon_S(\kappa_S)(r+\bar{\kappa})/\kappa_S+1}\frac{1}{\epsilon_L(\kappa_L)}\kappa_L\right)\frac{\partial\bar{V}}{\partial\tau_A},$$

which is strictly positive because $\epsilon_L(\kappa_L)(\epsilon_S(\kappa_S)+1)\geq 1$, and

$$\frac{(1-\mathcal{L})((1-\mathcal{L})\kappa_L+A)(\epsilon_S(\kappa_S)(r+\bar{\kappa})/\kappa_S+1)-((1+\mathcal{L}^2)\,\kappa_L+A)}{\mathcal{L}(1-\mathcal{L})[(r+\kappa_L+\kappa_S)\epsilon_L(\kappa_L)(\epsilon_S(\kappa_S)(r+\bar{\kappa})/\kappa_S+1)-\kappa_L]}\frac{\mathcal{S}}{\bar{V}}+\frac{1}{\epsilon_A(A)}, \tag{17}$$

where algebra shows that $\mathcal{S}/\bar{V} = (1+(1+\tau_A)\Pi/\mathcal{S})^{-1}$.

For $\partial g/\partial\tau_A$ to be strictly negative, the numerator of the first fraction in (17) must be strictly negative; the denominator is strictly positive because $\epsilon_L(\kappa_L)(\epsilon_S(\kappa_S)+1)\geq 1$. If the numerator is strictly negative, then $\partial g/\partial\tau_A<0$ only if the product of the first two fractions is sufficiently negative to outweigh $1/\epsilon_A(A)$. The numerator is strictly negative if and only if $\epsilon_S(\kappa_S)$ is sufficiently small. If that is the case, then the product of the first two fractions is strictly increasing in $\epsilon_S(\kappa_S)$ (an increase in $\epsilon_S(\kappa_S)$ makes the numerator less negative and increases the denominator), is strictly increasing in $\epsilon_L(\kappa_L)$ (an increase in $\epsilon_L(\kappa_L)$ increases the denominator), and is strictly decreasing in $\mathcal{S}/((1+\tau_A)\Pi)$. Finally, expression (11) follows from setting $\epsilon_S(\kappa_S)=0$.

A.5 Proof of Theorem 2

It is sufficient to show that $\mathcal{L}$, κ_L, A, and the second term in (11) (not including the minus) are increasing in $\aleph \equiv (1-\sigma^{-1}-\tau_R)R - f_L$, and do not otherwise depend on f_L and τ_R. In that case, if $\mathcal{L}$ is sufficiently high, then (11) is strictly negative, and κ_L and A are strictly positive (and $\kappa_S > 0$ by assumption), which implies that (11) is a sufficient statistic for whether $\partial g/\partial \tau_A < 0$.

The same argument as for $\partial \hat{V}/\partial \Upsilon$ in Appendix A.3 shows that $\bar{V}$ only depends on f_L and τ_R through $\aleph$, and that in equilibrium,

$$\partial \bar{V}/\partial \aleph = (1 - T'(\bar{V})^{-1}/(r + \kappa_L + A + \kappa_S) > 0.$$

Since $\epsilon_S(\cdot) = 0$, it follows from (16) in Appendix A.3 that $\bar{\kappa}$ and Π are constant in $\bar{V}$, which implies that $\mathcal{S}$ only depends on f_L and τ_R through $\aleph$, and that $\partial \mathcal{S}/\partial \aleph = \partial \bar{V}/\partial \aleph$. It follows from the large firm FOCs (6) and from (16) in Appendix A.3 that κ_L and A are increasing in $\aleph$, and that κ_S is decreasing in $\aleph$. Thus, $\mathcal{L}$ is increasing in $\aleph$. Finally, it follows that the second term in (11) (not including the minus) is increasing in $\aleph$ because $\epsilon_S(\cdot) = 0$ implies that $\epsilon_L(\cdot) \geq 1$, and so $\epsilon_L(\kappa_L)(r + \bar{\kappa}) > \kappa_L$.

A.6 Proof of Theorem 3

Throughout the proof, I focus on balanced growth paths, and drop time t subscripts. I begin by proving the results concerning $g_I(\cdot)$. To show there is a continuously differentiable function $g_I(\cdot)$ such that the industry n growth rate is $g_n = g_I(\mathcal{L}_n)$, I show that there are two continuously differentiable functions of the large firm industry revenue share, a strictly increasing $D(\cdot)$ and strictly decreasing $\tilde{g}(\cdot)$, so that for all n, $(\gamma - 1)g_n = D(\mathcal{L}_n)\tilde{g}(\mathcal{L}_n)$.

I show there is a strictly increasing continuously differentiable function $D(\cdot)$ so that for all industries n,

$$D(\mathcal{L}_n) = N(\kappa_{S,n}(\mathcal{L}_n) + (\gamma - 1)g_{S,n}(\mathcal{L}_n)) + \kappa_{L,n}(S; \mathcal{L}_n) + (\gamma - 1)g_{L,n}(\mathcal{L}_n). \tag{18}$$

Suppose (18) holds. From HJB equation (4) and the normalization in Section 4.4, a small firm's expected present discounted profit per unit of relative productivity is

$$\Pi_n(\mathcal{L}_n) = ((\sigma - 1)L^p - f_S)/(\rho + D(\mathcal{L}_n)). \tag{19}$$

From the evolution of the industry state over time (2) and growth due to small firms (3),

$$D(\mathcal{L}_n) = N(\lambda^{\gamma-1}\kappa_{S,n}(\mathcal{L}_n) + \delta_{S,n}(\mathcal{L}_n))/(1 - \mathcal{L}_n).$$

Thus, using (5) to write optimal small firm innovation as a function of $\Pi_n(\mathcal{L}_n)$ yields

$$(1-\mathcal{L}_n)D(\mathcal{L}_n)(\rho+D(\mathcal{L}_n))^{1/\epsilon} = N((\lambda^{\gamma-1})^{1+1/\epsilon}\alpha^{-1/\epsilon}+1)((\sigma-1)L^p - f_S)^{1/\epsilon}, \qquad (20)$$

where the right-hand side is a strictly positive number. Dropping the n subscripts on the left-hand side, (20) implicitly defines the function $D(\cdot)$ with the desired properties.

I now show that we can define a strictly decreasing continuously differentiable function $\tilde{g}(\cdot)$ so that for all industries n, $(\gamma-1)g_n = D(\mathcal{L}_n)\tilde{g}(\mathcal{L}_n)$. From the evolution of the industry state over time (2),

$$\frac{(\gamma-1)g_n}{D(\mathcal{L}_n)} = (1-\mathcal{L}_n)\frac{(\gamma-1)g_{S,n}(\mathcal{L}_n)}{\kappa_{S,n}(\mathcal{L}_n)+(\gamma-1)g_{S,n}(\mathcal{L}_n)} + \mathcal{L}_n\frac{(\gamma-1)g_{L,n}(\mathcal{L}_n)}{\kappa_{L,n}(S;\mathcal{L}_n)+(\gamma-1)g_{L,n}(\mathcal{L}_n)}. \qquad (21)$$

It follows from growth due to small firms (3) and small firm FOCs (5) that the first term on the right-hand side of (21) is $(1-\mathcal{L}_n)B/(B+1)$, where B is the rate at which a small firm generates growth relative to the rate at which it creatively destroys each good:

$$B \equiv \lambda^{\gamma-1} - 1 + (\alpha\lambda^{1-\gamma})^{1/\epsilon}.$$

It follows from growth due to large firms (3) and large firm FOCs (6) that the second term on the right-hand side of (21) is the product of $\mathcal{L}_n$ and

$$1-\left(1+(1-\mathcal{L}_n)(\lambda^{\gamma-1}-1)+\left(\frac{(1-\mathcal{L}_n)}{(1-\mathcal{L}_n)\lambda^{\gamma-1}+\mathcal{L}_n}\right)^{1/\epsilon}\left(\mathcal{L}_n(\lambda^{\gamma-1}-1)^{1+1/\epsilon}+\alpha^{1/\epsilon}\right)\right)^{-1}, \qquad (22)$$

which, dropping n subscripts, is a strictly decreasing continuously differentiable function of $\mathcal{L}$ that goes from $B/(B+1)$ at 0 to 0 at 1. Thus, we can define the desired $\tilde{g}(\cdot)$.

Next, $g_I(0) > 0$ because $D(0) > 0$ by (20), and the first term on the right-hand side of (21) is strictly greater than 0. Moreover, $g_I'(0) > 0$ because $D'(0) > 0$ by (20), and $\tilde{g}'(0) = 0$ since the first term on the right-hand side of (21) not including $1-\mathcal{L}_n$ is a constant and equal to the second term on the right-hand side not including $\mathcal{L}_n$.

The following proposition implies part of the second item in Theorem 3.

Proposition 2. *If $\epsilon < (1+\sqrt{5})/2$, then $\lim_{\mathcal{L}\to 1}(g_I(\mathcal{L})) = 0$. Otherwise, $\lim_{\mathcal{L}\to 1}(g_I(\mathcal{L})) = \infty$.*

Proof. First, (20) shows that as $\mathcal{L}$ goes to 1, $D(\mathcal{L})$ diverges to positive infinity, which implies that $\rho + D(\mathcal{L})$ converges to $D(\mathcal{L})$. It then follows from dividing each side of (20) by $1-\mathcal{L}$ and raising each side to $\epsilon/(\epsilon+1)$ that as $\mathcal{L}$ goes to 1, $D(\mathcal{L})$ converges to the product of a positive constant and $(1-\mathcal{L})^{-\epsilon/(\epsilon+1)}$.

Next, (21) and (22) show that as $\mathcal{L}$ goes to 1, $\tilde{g}(\mathcal{L})$ converges to 0, which implies that $\tilde{g}(\mathcal{L})$ converges to the product of a positive constant and $(1-\mathcal{L})^{\min\{1,1/\epsilon\}}$ because that is the lowest power of $1-\mathcal{L}$ contained in any term.

Thus, as $\mathcal{L}$ goes to 1, $g_I(\mathcal{L})$ converges to the product of a positive constant and $(1-\mathcal{L})^{\min\left\{\frac{1}{\epsilon+1},\frac{\epsilon+1-\epsilon^2}{\epsilon(\epsilon+1)}\right\}}$. If $\epsilon > 1$, then the minimum is the second option, which in that case is strictly decreasing in ϵ and negative if $\epsilon > (1+\sqrt{5})/2$. The proposition follows. ■

To complete the proof of the second item in Theorem 3, set $\epsilon = 1$. Since $g_I(0) > 0$, $g_I'(0) > 0$, $\lim_{\mathcal{L}\to 1}(g_I(\mathcal{L})) = 0$, and $g_I(\cdot)$ is continuously differentiable, it is sufficient to show that $g_I(\cdot)$ has at most one critical point. (20) shows that

$$D'(\mathcal{L}) = \frac{D(\mathcal{L})(\rho + D(\mathcal{L}))}{(\rho + 2D(\mathcal{L}))(1-\mathcal{L})},$$

and (21) along with the associated derivations shows that

$$\begin{aligned}
\tilde{g}(\mathcal{L}) &= \frac{B + \lambda^{1-\gamma}\mathcal{L}B/(B+1)}{B+1+\lambda^{1-\gamma}\mathcal{L}/(1-\mathcal{L})} \\
\tilde{g}'(\mathcal{L}) &= \frac{\tilde{g}(\mathcal{L})}{1-\mathcal{L}}\left(\frac{\lambda^{1-\gamma}(1-\mathcal{L})}{B+1+\lambda^{1-\gamma}\mathcal{L}} - \frac{\lambda^{1-\gamma}/(1-\mathcal{L})}{B+1+\lambda^{1-\gamma}\mathcal{L}/(1-\mathcal{L})}\right).
\end{aligned}$$

It follows from multiplying $(\gamma-1)g_I'(\mathcal{L}) = D'(\mathcal{L})\tilde{g}(\mathcal{L}) + D(\mathcal{L})\tilde{g}'(\mathcal{L})$ by $(1-\mathcal{L})/((\gamma-1)g_I(\mathcal{L}))$ and adding and subtracting 1 to $D'(\mathcal{L})\tilde{g}(\mathcal{L})$ that at a critical point of $g_I(\cdot)$,

$$-\frac{D(\mathcal{L})}{\rho + 2D(\mathcal{L})} + \frac{\lambda^{1-\gamma}(1-\mathcal{L})}{B+1+\lambda^{1-\gamma}\mathcal{L}} + \frac{B+1-\lambda^{1-\gamma}}{B+1+\lambda^{1-\gamma}\mathcal{L}/(1-\mathcal{L})} = 0.$$

The first term is strictly decreasing in $\mathcal{L}$ because $D(\cdot)$ is strictly increasing, and the second and third terms are strictly decreasing because each is strictly positive, their numerators are decreasing, and their denominators are increasing. Thus, there is at most one solution.

Finally, I prove the results concerning $g_A(\cdot)$. The arguments are the same as for $g_I(\cdot)$, except that $(\gamma-1)g_A(\mathcal{L}) = \tilde{D}\tilde{g}(\mathcal{L})$, where $\tilde{g}(\cdot)$ is as before, but $\tilde{D}$ is constant. Thus, the results follow because $\tilde{g}(\cdot)$ is continuously differentiable, $\tilde{g}(\mathcal{L})$ converges to 0 as $\mathcal{L}$ goes to 1, $\tilde{g}'(0) = 0$, and $\tilde{g}'(\mathcal{L}) < 0$ for all $\mathcal{L} > 0$.

I use the free entry condition to show that the desired constant $\tilde{D}$ exists. From HJB equation (13) and small firm FOCs (5), the value of entering industry n is

$$E_n(\mathcal{L}_n) = (\epsilon/(\epsilon+1))\left((\lambda^{\gamma-1})^{1+1/\epsilon}\alpha^{-1/\epsilon}+1\right)\Pi_n(\mathcal{L}_n)^{1+1/\epsilon}/(\rho+\eta).$$

The entry rate must be strictly positive on a balanced growth path, and all industries are identical, so the free entry condition fixes the value of entry: $E_n(\mathcal{L}_n) = W\xi$. It follows that for any variation in the large firm fixed cost f_L, revenue tax rate τ_R, and innovation cost shifter β, the balanced growth path present discounted profits a small firm earns per unit of relative productivity, $\Pi_n(\mathcal{L}_n)$, is the same. Thus, given constant labor used for production L^p, the result follows from (19).

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