Market Concentration, Growth, and Acquisitions

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Abstract

I study an oligopolistic growth model in which firms can innovate by creatively destroying their competitors' goods, innovating on their own varieties, and developing new varieties. To avoid cannibalization, larger firms innovate disproportionately through creative destruction, which generates little growth but deters other firms from innovating. A fall in large firm innovation costs, calibrated to match the recent rise in industry concentration in the US, explains almost half the fall in growth from the 1990s to the 2010s, as well as the burst in growth during the late 1990s and the positive across-industry correlation between changes in concentration and growth. Despite this link between the rise of dominant firms and the fall in growth, a substantial reduction in taxes on large firm acquisitions of their competitors' goods *increases* growth and welfare: to preserve valuable acquisition opportunities, large firms engage in less creative destruction. Dispersion in large firm innovation costs across industries yields a novel theory of the inverted-U relationship between concentration and growth.

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1 Introduction

Many authors have documented a rising share of revenue going to the top firms in industries at the national level in the US since the 1990s.[1] This trend has spurred research into its connection to the recent decline in growth, as well as the policy implications.[2] Over a similar time period, there was a dramatic rise in the rate at which venture capital backed startups are acquired relative to the rate at which they go public.[3] Two important questions emerge: Is large firm behavior behind the fall in growth? If so, should antitrust authorities limit acquisitions to reduce industry concentration and promote growth? These questions highlight the importance of understanding how large firms' market shares shape their, and their competitors', innovation incentives.

I argue that how large firms' market shares affect the *way in which they innovate* rather than just *how much they innovate*, and the resulting impact on their competitors' incentives to innovate, can help explain recent patterns in the data and has stark implications for optimal acquisition and innovation policy. The Arrow (1962) "replacement effect" states that for fear of cannibalization, large firms are reluctant to innovate. I show that this argument only applies to types of innovation that generate growth, and that large firms do not face the same disincentive when engaging in innovation that takes sales directly from their competitors. It follows that 1) a fall in large firm innovation costs, and therefore a rise in large firm innovation, *reduces* growth if their competitors' innovation rates are sufficiently responsive to the expected present discounted value of an innovation; and 2) in that case, policymakers face a novel trade-off when addressing large firm acquisitions of their smaller competitors' goods: a higher acquisition rate increases concentration and decreases growth, but a higher expected value of acquisitions to large firms reduces large firm innovation, leading to *lower* concentration and *faster* growth. A reduction in a tax on acquisitions increases growth if it has a sufficiently small effect on the acquisition rate. Moreover, since reducing large firm innovation is a benefit of acquisitions, not a cost, a fall in an acquisition tax has a stronger positive effect on growth when industries are more concentrated.

I formalize the theory in an endogenous growth model in which there is a continuum of industries, each of which consists of a single large firm and a continuum of small firms. Each firm can innovate by creatively destroying other firms' goods, developing new goods, or improving their own goods. The measure of small firms is determined by a free entry condition at the aggregate level, which implies that the value of entering is the expected value of entering into a random industry, and

[1] See Autor, Dorn, Katz, Patterson, and Van Reenen (2020), Barkai (2020).

[2] See Cavenaile, Celik, and Tian (2021), Aghion, Bergeaud, Boppart, Klenow, and Li (2022), Akcigit and Ates (2021), De Ridder (2021), Liu, Mian, and Sufi (2022), Olmstead-Rumsey (2022), and Garcia-Macia, Hsieh, and Klenow (2019).

[3] See Pellegrino (2021).

that the measure of small firms is the same in each industry. Dispersion in large firm innovation costs across industries drives dispersion in concentration and growth. I show that *across industries*, growth as a function of concentration displays an inverted-U shape, as observed in the data.[4] On the other hand, an *economy-wide* fall in large firm innovation costs has a much stronger tendency to reduce growth.

I calibrate the model to US data in the early 1990s and find that a fall in large firm innovation costs that generates the rise in concentration observed in the data can explain 41% of the eventual fall in the average growth rate. The model generates the observed temporary burst in aggregate growth in the late 1990s, as well as the positive across-industry correlation between changes in concentration and growth.

I introduce into the calibrated model an exogenous rate at which large firms can make take-it-or-leave-it offers to their smaller competitors to acquire their goods. I evaluate the effects of a tax on these acquisitions and find that a sufficiently large subsidy leads to an increase in welfare and a higher long-run growth rate relative to the economy without acquisitions. The required subsidy to increase welfare and surpass the no-acquisition growth rate is substantially lower when industries are more concentrated following the drop in large firm innovation costs.

The key mechanism in the model is that compared to small firms, large firms generate growth at a low rate relative to the rate at which they creatively destroy small firms' goods. When a firm improves their own goods or develops new ones, all the revenue they gain comes from generating growth, which takes revenue proportionally from all goods in the industry. When a firm creatively destroys a competitor's good, it improves the good's quality, generating revenue through growth, but mostly takes the revenue its competitor used to receive from the good. The larger a firm's revenue share, the more they discount generating revenue through growth because the more it cannibalizes their sales.

The free entry condition at the aggregate level implies that on a balanced growth path, the sum of growth and the rate of creative destruction faced by small firms is pinned down by the cost of entry since each serves as a discount rate on small firm profits. If large firm innovation costs fall across the economy, then innovation shifts toward large firms, whose revenue shares grow even bigger, and the composition of the discount rate on small firm profits shifts away from growth and toward creative destruction.

[4] See Cavenaile, Celik, and Tian (2021) for a direct measure of growth as a function of industry concentration, and Aghion, Bloom, Blundell, Griffith, and Howitt (2005) for an indirect measure that uses markups as a proxy for concentration.

Holding fixed the measure of small firms, which captures the effects of an industry-specific shock to the large firm's innovation cost or of the short-run response to an aggregate shock to large firm innovation costs, the sum of growth and creative destruction of small firms' goods is no longer fixed, and instead is increasing in large firm innovation. To understand why growth is highest in industries with intermediate levels of concentration, consider the two extremes. If the large firm has a very high innovation cost, then its revenue share is negligible and it innovates like a small firm. A marginal decrease in its innovation cost simply increases the total innovation rate and leads to faster growth. But after a sufficient decrease in its innovation cost, the large firm is nearly a monopolist, and has little incentive to develop new goods or improve on its old ones. The large firm maintains its dominance by creatively destroying its competitors' goods at such a fast rate that they see little benefit from innovating. Nonetheless, small firms produce so few goods that this creative destruction generates little growth. In the calibrated model, large firm revenue shares are sufficiently small to generate the short-run burst in growth and the positive across-industry correlation between changes in concentration and growth, following a fall in aggregate large firm innovation costs.

Large firm acquisitions of their smaller competitors' goods increase concentration and shift large firms further toward destructive types of innovation, pushing down growth and welfare. However, if acquisitions are valuable to large firms, they discourage large firm innovation because the more a large firm innovates, the less revenue remains for it to acquire. Thus, given an acquisition rate, reducing taxes on acquisitions shifts innovation to small firms who innovate in a more socially optimal way, increasing growth and welfare. This is especially true when industries are more concentrated because in that case, large firm innovation is particularly high and particularly geared toward creative destruction of small firms' goods.

Testable empirical predictions of the model's main mechanism are that firms can direct creative destruction efforts toward their competitors, and that smaller firms face higher discount rates on the profits from their innovations. The theory implies that this disparity increased as market concentration rose and growth fell in the US since the 1990s. Argente, Lee, and Moreira (2021) show that the revenues of high sales products depreciate more quickly than the revenues of low sales products, in line with the prediction that such products would be creatively destroyed more quickly if firms can direct their creative destruction efforts. Akcigit, Alp, and Peters (2021) show that a relatively high creative destruction rate for goods produced by firm types that innovate less, and tend to be small, can explain the high employment shares of old firms in US and Indian data. My theory provides an explanation for this disparity in creative destruction rates: firms focus their creative destruction toward their competitors' goods, so more innovative firms avoid a larger fraction of total creative destruction.

The theory also has implications for innovation policy. Research and development subsidies that target large firms may backfire and reduce growth by encouraging those firms to creatively destroy their competitors' goods, reducing smaller firms' incentives to innovate. Such subsidies are more likely to increase growth if they provide incentives to enter by targeting future large firms. It is even better to instead subsidize small firm innovation. A policy geared toward startups may accomplish these goals.

These results highlight an important subtlety in optimal competition and innovation policy. In theories that focus only on *how much* large firms produce or innovate, high markups or the Arrow (1962) "replacement effect" imply that it is optimal to subsidize high market shares and encourage *more* large firm production or innovation.[5] In these models, reducing large firm innovation is a cost of acquisitions, rather than a benefit.[6] In this paper, taking into account *the way in which* large firms innovate, policies that encourage large firm production or innovation backfire. Policies that offer large firms a tempting alternative to innovation increase growth by shifting activity to smaller firms that innovate in a more socially optimal way.

Large Firm Market Shares and Innovation:
Previous work on industries with oligopolistic competition and innovation mostly focuses on the impact of a large firm's market share on its *magnitude* of innovation.[7] A notable exception is the theory put forward in Argente, Baslandze, Hanley, and Moreira (2021) and mentioned in Akcigit and Ates (2021) that large firms use patents to deter competition. Although the mechanism is different, the result is similar in that large firm innovation generates little growth relative to the rate at which it deters small firm innovation. This theory is complementary to the one proposed here because creative destruction is most relevant in industries with weak patent protection.

Previous theories of the inverted-U relationship between market concentration and growth across industries, such as in Aghion, Bloom, Blundell, Griffith, and Howitt (2005) and Cavenaile, Celik, and Tian (2021), are based on the relative sizes of the "escape competition" and the Schumpeterian effects of competition. Smaller market shares increase pressure for firms to innovate to escape their competitors, but the expectation of the competition that pushes down market shares discourages innovation. In this paper, the magnitude of large firm innovation is driven by both effects, but the magnitude of small firm innovation depends only on the Schumpeterian effect: as the expected rate of growth and creative destruction increases, small firms innovate less. The inverted-U relationship depends on the response of small firm innovation to an increase in large firm innovation, and growth is decreasing in concentration when this response is sufficiently high. An important implication is

[5] See Edmond, Midrigan, and Xu (2021).
[6] See Fons-Rosen, Roldan-Blanco, and Schmitz (2022).
[7] See Shapiro (2012) for a discussion, and Cavenaile, Celik, and Tian (2021) for a recent example.

that if the entry rate and innovation are more responsive to economy-wide changes, then widespread higher concentration tends to be associated with lower growth at the aggregate level even if across industries or within an industry over time, higher concentration is associated with higher growth.

Large Firm Acquisitions of Small Competitors' Goods:
"Entry for buyout", described in Rasmusen (1988) and more recently, Letina, Schmutzler, and Seibel (2021) and Fons-Rosen, Roldan-Blanco, and Schmitz (2022), is another theory in which a high expected value of future acquisitions increases growth even though the distributional consequences of actual acquisitions do not. If an acquired firm receives a fraction of the surplus from the acquisition, then the expectation of being acquired increases the value of entry and innovation. Yet, if large firm innovation is one-dimensional, the entry for buyout effect needs to be weighed against the negative effect on large firm innovation; large firms have less reason to innovate if it erodes potential acquisition opportunities. In this paper, acquisitions are made with take-it-or-leave-it offers, so small firms face no incentive to enter or innovate to be acquired, and the entry for buyout effect is not present. Instead, acquisitions may be useful *because* they reduce large firm innovation, not in spite of it. A distinct implication of this theory is that expected acquisition opportunities are more beneficial when concentration is higher.

"Killer acquisitions", described in Cunningham, Ederer, and Ma (2021) and then in Letina, Schmutzler, and Seibel (2021) and Fons-Rosen, Roldan-Blanco, and Schmitz (2022), are acquisitions in which the acquiring firm does not produce the acquired good. The possibility of killer acquisitions is an additional potential cost of acquisitions because consumers cannot benefit directly from innovations that are not put into practice. Nonetheless, it only strengthens the positive effects of acquisitions on innovation discussed in this paper. The positive effect on growth of the expected value of future acquisitions is the same because if killer acquisitions are valuable to a large firm, then the large firm still reduces innovation to maintain acquisition opportunities. Moreover, the negative effect on growth of the actual acquisitions is lower because killer acquisitions are not produced and therefore have a smaller effect on large firms' revenue shares.

The Rise in Concentration and The Fall in Growth:
This paper is related to recent papers that study the effect of high productivity or superstar firms on growth, such as Aghion, Bergeaud, Boppart, Klenow, and Li (2022), De Ridder (2021), Cavenaile, Celik, and Tian (2021), and Liu, Mian, and Sufi (2022). A distinguishing feature of this paper is that it generates a parsimonious theory of the short-run burst in growth, the positive across-industry correlation between changes in concentration and growth, and the longer-run aggregate rise in concentration and fall in growth. The number of firms responds slowly and only to aggregate shocks. Without a fall in the number of firms, the dominant effect of a rise in large firm innovation

is simply a rise in total innovation and therefore growth.

Previous work has mostly considered models with only small firms, and so they focus on the effect of productivity dispersion across small firms, whereas I abstract from productivity dispersion and focus on the effect of large firms' market power. In particular, Aghion, Bergeaud, Boppart, Klenow, and Li (2022) and De Ridder (2021) focus on the channel that increased competition from high productivity competitors reduces less productive firms' markups, and therefore their incentive to grow. The channel I study in this paper is complementary in the sense that they focus on the flow profits a small firm receives from innovating, whereas I focus on the effective discount rate on small firm profits.

Liu, Mian, and Sufi (2022) study a growth model with two large firms in each industry, and find that a large firm can reduce growth by building a substantial productivity advantage over its competitor. The mechanism is that a bigger gap implies that the large firm will optimally cut its price by more in response to innovation by its competitor. On the other hand, as discussed, I focus on how a large firm's innovation decisions affect the rate at which its competitors discount their profits. An important difference is that my mechanism does not rely on a large firm responding directly to the actions of a single competitor. In that sense, my mechanism may be more relevant when thinking about the effect of a large firm on the innovation decisions of small firms. Finally, Cavenaile, Celik, and Tian (2021) study a growth model with large firms, but the pressure those large firms place on small firms has no effect on growth because small firms always have zero profits.

Akcigit and Ates (2021) and Olmstead-Rumsey (2022) propose theories in which exogenous changes in the economy's innovation technology cause a decline in growth, as well as a rise in market concentration and markups. The theory I propose reverses the causality and suggests that changes in industry structure—a rise in concentration—drive a decline in growth. Moreover, it provides an alternative explanation for the fall in the effect of a patent on a firm's market value documented in Olmstead-Rumsey (2022): the innovation is more quickly creatively destroyed by a large firm.

Model Building Blocks:

The model builds on two different strands of the growth literature, one focused on models of creative destruction[8], and one on expanding varieties models[9]. Recent work combines the two, but without large firms with positive market shares.[10]

[8]See Grossman and Helpman (1991b), Aghion and Howitt (1992), Klette and Kortum (2004), and Aghion, Bergeaud, Boppart, Klenow, and Li (2022).

[9]See Romer (1990) and Grossman and Helpman (1991a).

[10]See Atkeson and Burstein (2019).

The paper proceeds as follows. In section 2, I describe a simple illustrative model. In section 3, I describe the full quantitative model. In section 4, I discuss optimal firm behavior. In section 5, I characterize analytical results, calibrate the model, and show quantitative results on the effects of an aggregate fall in large firms' innovation costs. In section 6, I analyze the effects of acquisition policies. In section 7, I confirm the robustness of the model's results to allowing large firm entry and exit. In section 8, I conclude.

2 A Simple Industry Model

I first describe a simple industry model of creative destruction with a large firm that does not creatively destroy its own goods. The model illustrates the key mechanism of the theory. If firms can target creative destruction toward their competitors' goods rather than their own, then large firms will generate less growth for a given rate at which they creatively destroy their competitors' goods. If small firm innovation is sufficiently responsive to the rate at which small firm goods are creatively destroyed, then an increase in innovation by large firms reduces overall growth. The simple model also demonstrates important empirical predictions of this mechanism. Firms can direct creative destruction efforts toward their competitors, and smaller firms face higher discount rates on the profits from their innovations.

Time is continuous and indexed by $t \in [0, \infty)$. There is a unit measure of goods, indexed by $j \in [0, 1]$, each of which always receives the same revenue. At each time t, there is a measure N_t of small firms, and a single large firm. Each good is produced by a single firm at each time t, with productivity $z_t(j)$. The producer of a good receives flow profits π. Firms innovate by creatively destroying each others' goods. When a firm creatively destroys a good, it becomes the sole producer of that good, and the good's productivity, $z_t(j)$, is multiplied by $\lambda > 1$.

Industry productivity is $Z_t = \int_0^1 z_t(j)dj$, and the industry growth rate is $g_t = \frac{1}{Z_t}\frac{\partial Z_t}{\partial t}$. Denote by $S_t \in [0, 1]$ the measure of goods the large firm produces at time t, which is also the large firm's share of industry revenue.

Each small firm chooses a distribution of creative destruction rates, $\kappa_t(j)$, so that it creatively destroys a good at Poisson arrival rate $\int_0^1 \kappa_t(j)dj$, and the relative probability it creatively destroys good j is proportional to $\kappa_t(j)$. The flow cost of innovation is $\int_0^1 \kappa_t(j)^2 dj$. Small firms choose innovation rates to maximize the expected present discounted value of profits, where they discount future payouts by the real interest rate r.

In this simple model, I take the large firm's innovation decisions as given. The large firm creatively destroys goods produced by small firms at a flow rate rather than a Poisson arrival rate: in a finite interval of time, each small firm creatively destroys a finite *number* of goods, and the large firm creatively destroys a finite *measure* of goods. *The large firm does not creatively destroy its own goods.*

2.1 Long-Run Industry Concentration and Growth

I study balanced growth path Markov Perfect equilibria as a function of the large firm's exogenous innovation rate. All small firms creatively destroy all goods at the same rate, κ_S, since each good yields the same flow profits. The large firm creatively destroys small firms' goods at the exogenously given rate κ_L. The large firm's industry revenue share and the industry growth rate are constant over time at S and g, respectively. I consider two cases for determining the constant measure of small firms, N. In the first case, N is given by a free entry condition that fixes the value of being a small firm producing zero goods. In the second case, N is exogenously given.

I analyze the effects of changes in the rate at which the large firm creatively destroys small firms' goods, κ_L, on the large firm's industry revenue share, S, and growth, g, in the long-run. Since S is strictly increasing in κ_L in equilibrium, we can write g as a function of S. On a balanced growth path, the large firm's industry revenue share and growth are

$$S = \frac{\kappa_L}{N\kappa_S + \kappa_L}; \qquad g = (\lambda - 1)(N\kappa_S + (1 - S)\kappa_L). \tag{1}$$

Small firms creatively destroy goods and generate growth at rate $N\kappa_S$, and the large firm creatively destroys goods and generates growth at rate $(1 - S)\kappa_L$ since it does not creatively destroy its own goods. Small firm creative destruction is given by the First Order Condition:

$$\kappa_S = \frac{\pi/2}{r + N\kappa_S + \kappa_L},$$

where $N\kappa_S + \kappa_L$ is the Poisson arrival rate at which each small firm's good is creatively destroyed.

If N is determined by the free entry condition, then since the value of being a small firm producing zero goods is pinned down by the optimal small firm creative destruction rate, it follows that for any exogenous large firm creative destruction rate, κ_L, the total rate at which a small firm's good is creatively destroyed, $N\kappa_S + \kappa_L$, must be the same. From equation (1), the derivative of growth with respect to the large firm's industry revenue share is then

$$g'(S) = \frac{-2S}{1 + S}\frac{g}{1 - S},$$

which is always negative. If N is exogenously given, then

$$g'(S) = \left(\frac{-2S}{1+S} + \frac{1+r_0}{2+r_0} \right) \frac{g}{1-S},$$

where r_0 is the interest rate relative to the total rate of creative destruction: $r_0 = r/(N\kappa_S + \kappa_L)$. Since r_0 is strictly decreasing in S, in this case the derivative of growth with respect to the large firm's industry revenue share is negative if and only if $S > S^*$ for some $S^* \in (0, 1)$.

The derivative in the free entry case, and the first term of the derivative in the exogenous N case, is the *composition effect*. Holding fixed the total rate at which small firm goods are creatively destroyed, total small firm creative destruction falls one-for-one with the rise in large firm creative destruction. Small firm creative destruction contributes to growth at rate $\lambda - 1$ and large firm creative destruction contributes to growth at rate $(1 - S)(\lambda - 1)$, so the effect of the fall in the former always outweighs the effect of the rise in the latter, leading to a decreasing relationship between growth and the large firm's industry revenue share. The second term of the derivative in the exogenous N case is the *total innovation effect*. Holding fixed the ratio of small firm creative destruction to large firm creative destruction, $N\kappa_S/\kappa_L$, the total rate at which small firm goods are creatively destroyed, and thus growth, are increasing in large firm creative destruction. If the measure of small firms is exogenously given, then both the composition and total innovation effects are present because each small firm faces a convex innovation cost. If the large firm's revenue share is sufficiently small, then it makes little difference that the large firm does not creatively destroy its own goods, so the total innovation effect dominates the composition effect, which implies an increasing relationship between growth and the large firm's revenue share.

In the quantitative model I study in future sections, potential entrants pay entry costs not knowing into which industry they will enter. Thus, I interpret the exogenous N case as describing the relationship between concentration and growth *across industries* due to dispersion in large firm creative destruction, κ_L, as well as the short-run aggregate relationship between average concentration and growth due to widespread changes in κ_L. The free entry case describes the long-run aggregate relationship between average concentration and growth *over time* due to widespread changes in κ_L. Across industries, growth as a function of the large firm's revenue share exhibits an inverted-U shape. Following an aggregate increase in large firm innovation, in the short-run, concentration and growth may both increase, but in the long-run, concentration rises while growth falls.

3 Quantitative Model

Time is continuous and indexed by $t \in [0, \infty)$. There is a unit measure of industries, each of which consists of a measure of differentiated intermediate goods. There is a representative household who consumes the numeraire final good and inelastically supplies $\bar{L}$ units of labor. There is a representative final good producer that earns zero profits and purchases differentiated goods produced within each industry to convert into the final good for sale to the household. In each industry, a single large firm and a continuum of small firms use labor to produce, develop new goods, and creatively destroy old goods.

3.1 Representative Household

The household chooses a path of final good consumption to maximize the present discounted value of its utility:

$$\int_0^\infty e^{-\rho t} \ln(C_t) dt$$

subject to the budget constraint at each time t, $C_t = W_t \bar{L}_t + \Pi_t$, where C_t is final good consumption, the final good price is normalized to 1, W_t is the wage, and Π_t is flow profits from firms. The household owns all firms in the economy and takes the wage and profits as given. The household stochastic discount factor is $e^{-\rho t}/C_t$, and I denote its negative rate of change over time by

$$r_t = \rho + \dot{C}_t / C_t, \tag{2}$$

where a dot over a variable indicates its derivative with respect to time.

3.2 Representative Final Good Producer and Demand

At each time t, the representative final good producer chooses purchases of each good in each industry and sales of the final good to maximize profits:

$$C_t - \int_0^1 \int_0^{M_{n,t}} p_{n,t}(j) c_{n,t}(j) dj dn$$

subject to the production/aggregation functions

$$ln(C_t) = \int_0^1 ln(C_{n,t}) dn \qquad C_{n,t}^{\frac{\gamma-1}{\gamma}} = \int_0^{M_{n,t}} c_{n,t}(j)^{\frac{\gamma-1}{\gamma}} dj \quad \text{for all } n \in [0, 1],$$

where $M_{n,t}$ is the measure of goods available at time t in industry n, $p_{n,t}(j)$ and $c_{n,t}(j)$ are the price and real purchases of good j in industry n, respectively, and $\gamma > 1$ is the within-industry elasticity of substitution. The final good producer takes prices as given and earns zero profits. The First Order Condition for good j in industry n, along with the zero profit condition, implies the demand curve

$$c_{n,t}(j) = p_{n,t}(j)^{-\gamma} P_{n,t}^{\gamma-1} C_t, \tag{3}$$

where the industry price index is given by

$$P_{n,t}^{1-\gamma} \equiv \int_0^{M_{n,t}} p_{n,t}(j)^{1-\gamma} dj. \tag{4}$$

3.3 Intermediate Goods Producers

Each industry consists of a measure N_t of small firms, indexed by $i \in [0, N_t]$, and a single large firm, denoted by $i = L$. Since entry is undirected across industries, the measure of small firms can vary over time, but not across industries.

3.3.1 Production and Competition

At each time t, each firm can produce each good in its industry with a production function linear in labor and a firm-good-specific productivity:

$$q_{n,i,t}(j) = z_{n,i,t}(j) l_{n,i,t}(j). \tag{5}$$

All varieties of good j in industry n are perfect substitutes. At each time t, firms simultaneously choose prices to maximize static profits.

Let $z_{n,t}(j) \equiv \max\{z_{n,i,t}(j)\}_{i \in [0,N_t] \cup \{L\}}$ be the highest productivity with which any firm in industry n can produce good j at time t. In equilibrium, a firm i has strictly positive sales of good j only if it is the most productive producer, i.e., $z_{n,i,t}(j) = z_{n,t}(j)$, and its price is constrained to be less than the marginal cost of the second-most productive producer of good j.

Let $Z_{n,t}$ be an aggregate of productivity in industry n:

$$Z_{n,t}^{\gamma-1} \equiv \int_0^{M_t} z_{n,t}(j)^{\gamma-1} dj,$$

and define the relative productivity of good j: $\tilde{z}_{n,t}(j) \equiv (z_{n,t}(j)/Z_{n,t})^{\gamma-1}$. Going forward, I will usually characterize a good by its relative productivity.

3.3.2 Innovation

At each time t, each firm chooses two types of innovation rates to maximize the expected present discounted value of profits using the household stochastic discount factor: a rate of creative destruction for each good in its industry, and a rate of new good development. Conditional on creatively destroying good j, a firm's productivity for that good becomes $\lambda z_{n,t}(j)$, where $\lambda > 1$. Conditional on developing a new good, a firm's productivity for that good, λz, is drawn so that the expected value of $z^{\gamma-1}$ is equal to $Z_{n,t}^{\gamma-1}$. Moreover, whenever a firm creatively destroys a good or develops a new good, so that the new productivity for that good is $z_{n,t}(j)$, all other firms are able to produce that good with productivity at least equal to $z_{n,t}(j)/\sigma$, where $\sigma > 1$. Thus σ is the maximum possible gap between the productivities of the most productive and the second-most productive producers of a good.

Small Firms:
A small firm's innovation technology consists of a single *entrepreneur*. It chooses a distribution of creative destruction rates for goods in its industry, $\kappa_{n,i,t}(j)$, so that it creatively destroys a good at Poisson arrival rate $\int_0^{M_{n,t}} \kappa_{n,i,t}(j)dj$, and the relative probability it creatively destroys good j is proportional to $\kappa_{n,i,t}(j)$. The flow labor cost of creative destruction is

$$\alpha^{-1}\chi_C \int_0^{M_{n,t}} \tilde{z}_{n,t}(j)\kappa_{n,i,t}(j)^{\alpha}dj,$$

where $\alpha > 1$. A small firm chooses a Poisson arrival rate at which it develops a new good, $\delta_{n,i,t}$, subject to flow labor cost $\alpha^{-1}\delta_{n,i,t}^{\alpha}$. The costs of creatively destroying each good and of developing a new good are independent, and each scales with the expected relative productivity of the innovation. Finally, χ_C is the cost of creative destruction relative to new good development.

Large Firms:
In industry n, the large firm's innovation technology consists of a *measure* $\chi_{L,n}^{-1/(\alpha-1)}$ of entrepreneurs. It thus chooses a Poisson arrival rate at which it creatively destroys *each* good in its industry, $\kappa_{n,L,t}(j)$, and a *continuous rate* at which it develops a new good, $\delta_{n,L,t}$, subject to total flow labor cost

$$\alpha^{-1}\chi_{L,n}\left(\chi_C \int_0^{M_{n,t}} \tilde{z}_{n,t}(j)\kappa_{n,L,t}(j)^{\alpha}dj + \delta_{n,L,t}^{\alpha}\right),$$

where I take as given that the large firm optimally distributes its innovation across its entrepreneurs to minimize cost.

Small vs. Large Firms:
A small firm creatively destroys a single good and develops a new good at Poisson arrival rates, and thus becomes the most-productive producer of a finite number of goods in finite time. A large

firm creatively destroys a single good and develops a new good at a continuous rate, and thus becomes the most-productive producer of a finite measure of goods in finite time.

To be clear, a firm can creatively destroy a good that it already produces. For a small firm, this possibility is irrelevant since each small firm produces finitely many goods, and creatively destroys each good at an infinitesimal rate. For a large firm, this possibility is meaningful, and implies that the innovative capacity of the economy is not mechanically reduced as the large firm's share of old innovations grows.

Parameter Assumption and Discussion: To simplify analysis of the model, I make the following assumption:

Assumption 3.1. *The maximum productivity gap between the most productive and second-most productive producers of a good is weakly less than the creative destruction step size and the markup any firm would set if unconstrained, i.e.,* $\sigma \leq min\{\lambda, \gamma/(\gamma-1)\}$.

It follows that regardless of how a firm became the most productive producer of a good, the gap between that firm's productivity and the productivity of the second-most productive producer of the good is σ. I interpret this assumption as suggesting that firms can imitate each others' goods sufficiently well so that regardless of the gap between a firm's new innovation (creative destruction or new good development) and whatever came before it, the firm feels the same competitive pressure when pricing that good.

Assumption 3.1 implies that we can interpret new good development as firms innovating on their own goods. In either case, any gains enjoyed by a firm come from adding productivity to the industry, not from taking productivity from another firm, and in either case, the firm sets the same markup.

Since Assumption 3.1 implies that all firms set the same markup on all goods, I thus abstract from the effects of firms setting different markups on newly developed goods and on creatively destroyed goods, as well as the effects of large firms setting higher markups than small firms. Allowing for different markups substantially complicates the analysis. As large firms gain market share, they set higher markups, which reduces competition and encourages growth from small firms. This effect is mitigated or reversed if large firms are sufficiently more productive than small firms: they may set higher markups than small firms, but lower prices. Then, as large firms gain market share the industry price index falls, which decreases small firms' incentive to innovate.

3.3.3 Entry and Exit

Entry is undirected, so an entering firm draws an industry from the uniform distribution. At each moment in time, there is an infinite mass of potential entrants. If a potential entrant pays the cost of entry, then they draw an industry and enter as a single small firm that is not the most productive producer of any goods. Otherwise, the potential entrant receives value 0. The total cost of entry is increasing in the entry rate and is $\chi_E E_t^\epsilon$ units of labor, where E_t is the entry rate, χ_E is a cost-shifter, and $\epsilon \geq 1$ is the elasticity of total entry costs with respect to the entry rate. Thus, the marginal entrant faces an entry cost of $\chi_E \epsilon E_t^{\epsilon-1}$. At the lower bound for the elasticity, $\epsilon = 1$, the marginal entry cost is constant and there is a free entry condition. At the upper bound for the elasticity, $\epsilon = \infty$, the marginal entry cost is 0 if $E_t < 1$ and infinite if $E_t > 1$. In that case, the entry rate is always 1.

Each small firm exits exogenously at Poisson arrival rate $\eta > 0$. When a firm exits, it sells each good for which it is the most productive producer to another small firm (not the small firm that is the second-most productive producer of that good).

3.4 Equilibrium

At each moment in time, the goods market must clear, i.e., the amount each firm supplies of each good is equal to the representative household's demand for that good, and the labor market must clear, i.e., the labor used in production, for entry costs, and for innovation costs, must equal the labor inelastically supplied by the representative household.

Given the parameter restriction made in Assumption 3.1, I characterize each good by its type $f \in \{S, L\}$, which denotes whether the good's current producer is a small firm (S) or the large firm (L). Let $T_{n,t}(j)$ be good j's type in industry n at time t. For each type $f \in \{S, L\}$, define $\tilde{Z}_{f,n,t}$ to be the industry n measure of relative productivity of goods of type f at time t:

$$\tilde{Z}_{f,n,t} = \int\limits_{j:T_{n,t}(j)=f} \tilde{z}_{n,t}(j)dj,$$

where $\tilde{z}_{n,t}(j)$ is defined in Section 3.3.1. It follows that $\tilde{Z}_{S,n,t} + \tilde{Z}_{L,n,t} = 1$.

The industry state is the industry measure of relative productivity of goods of type L, $\tilde{Z}_{L,n,t}$. The aggregate state is the measure of small firms in each industry, N_t, and the distribution of industry states across industries.

I study Markov Perfect Equilibria in which firms' markups in industry n are given by static optimization of profits and are a function only of $\tilde{Z}_{L,n,t}$. In particular, markups are not a function of the industry aggregate of productivity, $Z_{n,t}$, the measure of small firms, N_t, or of time t. Firms' innovation decisions are given by dynamic optimization of expected discounted profits and are functions only of the industry state, $\tilde{Z}_{L,n,t}$, and the aggregate state when converging to a balanced growth path. Innovation decisions do not depend on the level of productivity, $Z_{n,t}$. Moreover, each firm creatively destroys all goods of each type f at the same rate. The entry rate is given by potential entrant dynamic optimization of expected discounted profits net of the marginal entry cost and is a function only of the aggregate state.

To be clear, firms can always observe all features of the economy when optimizing, but they suppose that other firms' actions depend only on the variables mentioned above. I show that it is then optimal for each firm also to condition their own actions only on the variables mentioned above.

I focus on balanced growth path equilibria and the convergence to a balanced growth path following unanticipated shocks. A balanced growth path is an equilibrium in which $\tilde{Z}_{L,n,t}$ is constant over time in each industry, the measure of small firms N_t is constant over time, and each firm's innovation decisions are functions only of $\tilde{Z}_{L,n,t}$ in their industry. It follows that aggregate productivity Z_t given by $\ln(Z_t) \equiv \int_0^1 \ln(Z_{n,t})dn$ grows at a constant rate.

3.5 Evolution of the Industry State and Growth

We will see in Section 4.2 that all small firms in industry n choose the same innovation rates. Moreover, each small firm creatively destroy all goods at the same rate. Thus, let $\kappa_{S,n,t}$ denote this rate of creative destruction, and let $\delta_{S,n,t}$ denote the rate at which small firms develop new goods. It follows that as a group, small firms in industry n creatively destroy each good at rate $N_t\kappa_{S,n,t}$, and develop a new good at rate $N_t\delta_{S,n,t}$. Let $\kappa_{L,n,t}(f)$ denote the rate at which the large firm in industry n creatively destroys a good that is currently produced by a type $f \in \{S, L\}$ firm, and let $\delta_{L,n,t}$ denote the rate at which the large firm develops new goods.

The industry state $\tilde{Z}_{L,n,t}$ evolves over time according to

$$\begin{aligned}\dot{\tilde{Z}}_{L,n,t} =& \lambda^{\gamma-1}\left(\delta_{L,n,t} + \left(1 - \tilde{Z}_{L,n,t}\right)\kappa_{L,n,t}(S) + \tilde{Z}_{L,n,t}\kappa_{L,n,t}(L)\right) \\ &- \tilde{Z}_{L,n,t}(N_t\kappa_{S,n,t} + \kappa_{L,n,t}(L)) - \tilde{Z}_{L,n,t}(\gamma - 1)g_{n,t},\end{aligned} \tag{6}$$

where $g_{n,t}$ is the growth rate of industry productivity $Z_{n,t}$:

$$(\gamma-1)g_{n,t} \equiv \frac{\partial Z_{n,t}^{\gamma-1}/\partial t}{Z_{n,t}^{\gamma-1}} = (\lambda^{\gamma-1}-1)\left(N_t\kappa_{S,n,t} + \left(1-\tilde{Z}_{L,n,t}\right)\kappa_{L,n,t}(S) + \tilde{Z}_{L,n,t}\kappa_{L,n,t}(L)\right)$$
$$+ \lambda^{\gamma-1}(N_t\delta_{S,n,t} + \delta_{L,n,t}).$$

In (6) for the evolution of the industry state over time, the first line is the inflow due to new good development, creative destruction of small firms' goods, and creative destruction of the large firm's own goods. The first term on the second line is the outflow due to creative destruction of the large firm's goods, and the last term is the outflow due to growth in $Z_{n,t}^{\gamma-1}$, which reduces relative productivity. In the expression for the industry growth rate, the first line is growth from creative destruction: the -1 in $\lambda^{\gamma-1}-1$ reflects the destroyed productivity of the old good; and the second line is growth from new good development in which all the productivity of new goods is novel.

3.6 Aggregation and Welfare

We will see in Section 4.1 that all firms set a markup σ on all goods. It follows that at time t, each industry uses the same quantity of labor in production, L_t^p, which implies that aggregate final good consumption is $C_t = Z_t L_t^p$, industry n consumption is $C_{n,t} = Z_{n,t}L_t^p$, and recalling that the final good price is normalized to 1, the wage is $W_t = Z_t/\sigma$. Household welfare is

$$\int_0^\infty e^{-\rho t}\left(\ln(Z_t) + \ln\left(L_t^p\right)\right)dt = \frac{\ln(Z_0)}{\rho} + \frac{\int_0^\infty \rho e^{-\rho t} g_t dt}{\rho^2} + \frac{\int_0^\infty \rho e^{-\rho t}\ln\left(L_t^p\right)dt}{\rho},$$

where g_t is the growth rate of aggregate productivity Z_t given by $g_t = \int_0^1 g_{n,t}dn$. Welfare depends on current productivity and weighted averages of future growth and labor used in production. Since growth in one period raises consumption in all future periods, it is discounted by ρ^2 rather than ρ.

Along a balanced growth path, the labor used in production is constant at L^p, and final good consumption and the wage grow at the aggregate productivity growth rate g. Welfare is

$$\ln(Z_0)/\rho + g/(\rho^2) + \ln(L^p)/\rho.$$

4 Firm Optimization

Before describing the firm problem, note that since small firms take industry aggregates as given, we can split their static profit maximization problem into a separate problem for each good. Moreover, when innovating, a small firm's problem is the same regardless of the goods it produces.

4.1 Static Profit Maximization: Prices

At each moment in time, firms choose prices simultaneously to maximize static profits. A small firm that is the most productive producer of a good with relative productivity $\tilde{z}_{n,t}(j)$ takes as given the industry price index, the wage, and industry revenue, and chooses a price to maximize static profits subject to the demand curve (3), the production function (5), and competition from the second-best producer, which Assumption 3.1 implies we can write as $p_{n,t}(j) \leq W_t\sigma/\tilde{z}_{n,t}(j)$. A large firm takes as given small firms' prices, the wage, and industry revenue, and chooses prices for its goods to maximize static profits subject to the demand curve (3), the production function (5), competition from the second-best producers of each of its goods, and aggregation (4), which determines the industry price index as a function of goods prices.

All firms would set a markup weakly greater than $\gamma/(\gamma-1)$ if unconstrained by the second-most productive producer.[11] Thus, by Assumption 3.1, all firms set a markup of σ on all goods. The static profits for a small firm producing a good with relative productivity $\tilde{z}_{n,t}(j)$ is thus $\tilde{z}_{n,t}(j)(1-\sigma^{-1})C_t$. The static profits of a large firm with relative productivity $\tilde{Z}_{L,n,t}$ is $\tilde{Z}_{L,n,t}(1-\sigma^{-1})C_t$. The industry revenue share of a large firm is $\tilde{Z}_{L,n,t}$.

4.2 Dynamic Profit Maximization: Innovation

At each moment in time, firms simultaneously choose innovation rates: a creative destruction rate for each good, and a new good development rate. In the dynamic problem, a firm takes as given its profit function from static optimization.

4.2.1 Small Firms

For a small firm to choose their optimal innovation rate, they must know the expected present discounted value of being the most productive producer of a good. The expected present discounted value of producing good j in industry n at time t is $\tilde{z}_{n,t}(j)\pi_{S,n,t}(\tilde{Z}_{L,n,t})$, given by the HJB equation:

$$\begin{aligned} r_t\pi_{S,n,t}(\tilde{Z}_{L,n,t}) =& (1-\sigma^{-1})C_t - (N_t\kappa_{S,n,t} + \kappa_{L,n,t}(S) + (\gamma-1)g_{n,t})\pi_{S,n,t}(\tilde{Z}_{L,n,t}) \\ &+ \dot{\tilde{Z}}_{L,n,t}\pi'_{S,n,t}(\tilde{Z}_{L,n,t}) + \dot{\pi}_{S,n,t}(\tilde{Z}_{L,n,t}), \end{aligned}$$

[11] See Edmond, Midrigan, and Xu (2021) for a derivation of the optimal markup with oligopoly, nested CES demand, and Bertrand competition.

where r_t is the discount rate implied by the household stochastic discount factor from equation (2). The first term on the right-hand side of the first line is flow profits, and the second term reflects the rate at which the good is creatively destroyed and the rate at which the good's relative productivity is depreciated by growth in industry productivity (recall that relative productivity is $\tilde{z}_{n,t}(j) \equiv (z_{n,t}(j)/Z_{n,t})^{\gamma-1}$ with $z_{n,t}(j)$ fixed over time). The second line captures the effects of changes over time in the industry state or in the aggregate state when the economy is converging to a balanced growth path.

A small firm chooses innovation rates to maximize the expected present discounted value of profits using the household stochastic discount factor, taking as given other firms' innovation rates, the industry and aggregate state, and the evolution of the states over time. As mentioned above, a small firm's innovation optimization problem is the same regardless of the goods they produce. The value function of a small firm with zero goods is given by the HJB equation:

$$\begin{aligned} r_t V_{S,n,t}(\tilde{Z}_{L,n,t}) = &\max_{\{\kappa(j)\}} \left\{ \int_0^{M_{n,t}} \kappa(j)\lambda^{\gamma-1}\tilde{z}_{n,t}(j)\pi_{S,n,t}(\tilde{Z}_{L,n,t})dj - W_t\alpha^{-1}\chi_C \int_0^{M_{n,t}} \tilde{z}_{n,t}(j)\kappa(j)^{\alpha}dj \right\} \\ &+ \max_{\delta} \left\{ \delta\lambda^{\gamma-1}\pi_{S,n,t}(\tilde{Z}_{L,n,t}) - W_t\alpha^{-1}\delta^{\alpha} \right\} - \eta V_{S,n,t}(\tilde{Z}_{L,n,t}) + \dot{\tilde{Z}}_{L,n,t}V'_{S,n,t}(\tilde{Z}_{L,n,t}) + \dot{V}_{S,n,t}(\tilde{Z}_{L,n,t}). \end{aligned}$$

The right-hand side of the first line is the optimization problem for the rate at which the small firm creatively destroys each good, the first term on the second line is the optimization problem for the rate at which the firm develops a new good, which uses the fact that the new good's expected relative productivity is $\lambda^{\gamma-1}$, and the remaining terms reflect the firm's exit rate and changes over time in the industry state and in the aggregate state when the economy is converging to a balanced growth path.

The First Order Conditions give the optimal new good development rate and the single rate at which a small firm creatively destroys each good in its industry:

$$\begin{aligned} \delta_{S,n,t} &= W_t^{\frac{-1}{\alpha-1}} \left(\lambda^{\gamma-1}\pi_{S,n,t}(\tilde{Z}_{L,n,t}) \right)^{\frac{1}{\alpha-1}} \\ \kappa_{S,n,t} &= (W_t\chi_C)^{\frac{-1}{\alpha-1}} \left(\lambda^{\gamma-1}\pi_{S,n,t}(\tilde{Z}_{L,n,t}) \right)^{\frac{1}{\alpha-1}}. \end{aligned} \tag{7}$$

A small firm values equally—condition on relative productivity—a good gained through new good development, through creatively destroying a small competitor's good, and through creatively destroying a large competitor's good because it does not internalize the different effects these innovations have on the industry state or growth. A small firm creatively destroys all goods at the same rate because the cost and benefit of an innovation both scale with its relative productivity.

4.2.2 Large Firms

A large firm chooses innovation rates to maximize the expected present discounted value of profits using the household stochastic discount factor, taking as given the aggregate state and its evolution over time, the current industry state, and small firm innovation rates as a function of the industry and aggregate state. The value function is given by the HJB equation:

$$\begin{aligned} r_t V_{L,n,t}(\tilde{Z}_{L,n,t}) &= \tilde{Z}_{L,n,t}(1-\sigma^{-1})C_t + \dot{V}_{L,n,t}(\tilde{Z}_{L,n,t}) \\ &+ \max_{\{\kappa(j)\},\delta} \left\{ \dot{\tilde{Z}}_{L,n,t}(\{\kappa(j)\},\delta;\tilde{Z}_{L,n,t})V'_{L,n,t}(\tilde{Z}_{L,n,t}) - W_t\alpha^{-1}\chi_{L,n}\left(\chi_C\int_0^{M_{n,t}}\tilde{z}_{n,t}(j)\kappa(j)^\alpha dj + \delta^\alpha\right)\right\}. \end{aligned}$$

The first term on the right-hand side of the first line is flow profits, and the second term reflects the changes over time in the aggregate state when the economy is converging to a balanced growth path. The second line is the optimization problem of the large firm choosing innovation rates; the benefit is through changes in the industry state, which depends on the large firm's innovation rates and the current industry state through the innovation rates of small firms.

The First Order Conditions give the optimal innovation rates:

$$\begin{aligned} \delta_{L,n,t} &= (W_t\chi_{L,n})^{\frac{-1}{\alpha-1}}\left(\lambda^{\gamma-1}(1-\tilde{Z}_{L,n,t})V'_{L,n,t}(\tilde{Z}_{L,n,t})\right)^{\frac{1}{\alpha-1}} \\ \kappa_{L,n,t}(L) &= (W_t\chi_{L,n}\chi_C)^{\frac{-1}{\alpha-1}}\left((\lambda^{\gamma-1}-1)(1-\tilde{Z}_{L,n,t})V'_{L,n,t}(\tilde{Z}_{L,n,t})\right)^{\frac{1}{\alpha-1}} \\ \kappa_{L,n,t}(S) &= (W_t\chi_{L,n}\chi_C)^{\frac{-1}{\alpha-1}}\left([1+(\lambda^{\gamma-1}-1)(1-\tilde{Z}_{L,n,t})]V'_{L,n,t}(\tilde{Z}_{L,n,t})\right)^{\frac{1}{\alpha-1}}. \end{aligned} \tag{8}$$

The benefit to the large firm of developing a new good or creatively destroying one of its own goods is that the innovation generates growth, which the large firm discounts by $1-\tilde{Z}_{L,n,t}$ because it adds to industry productivity and depreciates other goods' relative productivities. The benefit of creatively destroying a small competitor's good is in part that the innovation generates growth, but also that the innovation transfers the good's pre-innovation productivity from small firms to the large firm, which the large firm does not discount because it does not add to industry productivity.

4.2.3 Small vs. Large Firms:

We will see throughout the rest of the paper that a key determinant of the equilibrium growth rate is the ratio of the rate at which firms generate growth to the rate at which they creatively destroy each of their competitors' goods, i.e., the *composition* of innovation. From a firm's perspective, creative destruction and growth both depreciate the relative productivity of its goods, and the composition of innovation is irrelevant. The household disagrees because only growth generates a permanent reduction in prices.

The rate at which a small firm depreciates a competitor's relative productivity through growth relative to through creative destruction is

$$\frac{(\gamma-1)g_{S,n,t}}{\kappa_{S,n,t}} = \lambda^{\gamma-1}\chi_C^{\frac{1}{\alpha-1}} + \lambda^{\gamma-1} - 1, \tag{9}$$

where $(\gamma-1)g_{S,n,t} = \lambda^{\gamma-1}\delta_{S,n,t} + (\lambda^{\gamma-1}-1)\kappa_{S,n,t}$ is the rate at which a small firm generates growth in $Z_{n,t}^{\gamma-1}$. The same ratio for a large firm is

$$\begin{aligned}\frac{(\gamma-1)g_{L,n,t}}{\kappa_{L,n,t}(S)} =& \lambda^{\gamma-1}\chi_C^{\frac{1}{\alpha-1}}\left(\frac{\lambda^{\gamma-1}(1-\tilde{Z}_{L,n,t})}{\lambda^{\gamma-1}(1-\tilde{Z}_{L,n,t})+\tilde{Z}_{L,n,t}}\right)^{\frac{1}{\alpha-1}} \\ &+ (\lambda^{\gamma-1}-1)\left(1-\tilde{Z}_{L,n,t}+\tilde{Z}_{L,n,t}\left(\frac{(\lambda^{\gamma-1}-1)(1-\tilde{Z}_{L,n,t})}{(\lambda^{\gamma-1}-1)(1-\tilde{Z}_{L,n,t})+1}\right)^{\frac{1}{\alpha-1}}\right), \end{aligned} \tag{10}$$

which is strictly greater than $(\gamma-1)g_{S,n,t}/\kappa_{S,n,t}$ if $\tilde{Z}_{L,n,t} > 0$. The two terms in each of (9) and (10) are the ratio only including growth due to new good development and the ratio only including growth due to creative destruction, in that order. In each case, the term for large firms is lower than for small firms because the bigger a large firm's industry revenue share, the more they discount the value of growth, and the lower their incentive to develop new goods or creatively destroy their own goods relative to their incentive to creatively destroy their competitors' goods.

5 Results: Growth and Concentration

In this section, I show results that characterize the effect of changes in large firm innovation on industry concentration, growth, and welfare. I begin with qualitative results concerning long-run effects. I then calibrate the model, and analyze the effects of a fall in large firm innovation costs, which I compare to the data.

5.1 Qualitative Results: Concentration and Growth in the Long-Run

I focus first on the distribution of growth across industries and across economies along a balanced growth path. Without solving for a Markov Perfect Equilibrium of the dynamic game, I can characterize the relationship between large firm industry revenue shares and growth. These results also provide intuition for the quantitative exercises that follow. Throughout the section, I omit time t subscripts for variables that are constant over time.

The following theorem, displayed graphically by the solid blue line in Figure 1, shows that across industries in a single balanced growth path, growth as a function of the large firm's market share exhibits an inverted-U shape.

Theorem 5.1. *Suppose $\alpha \geq 2$ and the economy is on a balanced growth path. The long-run growth rate in an industry is a function of the long-run industry revenue share of the large firm, $h(\tilde{Z}_{L,n})$. There exists a threshold revenue share Z^* such that $h(\tilde{Z}_{L,n})$ is strictly increasing if $\tilde{Z}_{L,n} < Z^*$ and strictly decreasing if $\tilde{Z}_{L,n} > Z^*$.*

To gain intuition for the theorem note that along a balanced growth path, in industry n, the expected present discounted value of profits for a small firm from a good with relative productivity 1, over the wage, is

$$\frac{\pi_{S,n,t}}{W_t} = \frac{(\sigma-1)L^p}{\rho + N(\kappa_{S,n} + (\gamma-1)g_{S,n}) + \kappa_{L,n}(S) + (\gamma-1)g_{L,n}}, \tag{11}$$

and the large firm's revenue share is

$$\tilde{Z}_{L,n} = \frac{\kappa_{L,n}(S) + (\gamma-1)g_{L,n}}{N(\kappa_{S,n} + (\gamma-1)g_{S,n}) + \kappa_{L,n}(S) + (\gamma-1)g_{L,n}}. \tag{12}$$

The relationship across industries between the large firm's revenue share and the industry growth rate operates through the effective discount rate on small firm profits, the denominator on the right-hand side of (11). There are two effects analogous to the composition and total innovation effects from Section 2, the first of which implies a decreasing relationship between concentration and growth, and the second of which implies an increasing relationship.

We can see the composition effect, displayed graphically in the left panel of Figure 2, by holding fixed total innovation—the denominator of the right-hand side of (11) or (12). A higher large firm revenue share has two effects, each of which lowers the industry growth rate based on the analysis in Section 4.2.3. First, the large firm's innovation shifts away from growth and toward creative destruction of its competitors' goods. Second, the large firm's share of total innovation increases, shifting total innovation away from growth and toward creative destruction of small firms' goods.

We can see the total innovation effect, displayed graphically in the right panel of Figure 2, by holding fixed the composition of innovation—the fraction of the denominator of the right-hand side of (12) that is growth. A higher large firm revenue share implies higher total innovation and therefore a higher industry growth rate because small firm innovation is decreasing in $\pi_{S,n,t}/W_t$; if small firm innovation fell by enough to imply lower total innovation, then $\pi_{S,n,t}/W_t$ would increase and, by (7), small firms would optimally innovate more.

If the large firm's revenue share is sufficiently low, then the composition effect is small and the total innovation effect dominates; the large firm's innovation is not so shifted toward creative destruction of its competitors' goods, and the composition of its innovation does not change much in its revenue share. If the large firm's revenue share is sufficiently high, then the composition effect dominates because the large firm innovates almost entirely through creative destruction of its competitors' goods, so an increase in large firm innovation does not lead to higher growth.

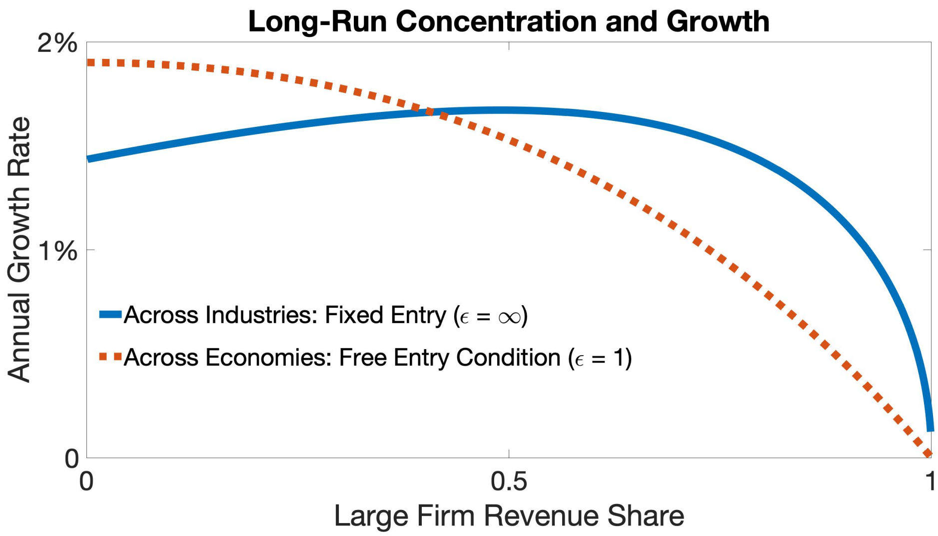

Figure 1: The lines depict growth on a balanced growth path at various levels of the large firm's industry revenue share. The solid blue line shows the growth rate across industries on a single balanced growth path, and the dotted orange line shows the growth rate across balanced growth paths in different economies, each with constant large firm revenue shares across industries. The figure is based on the calibration described in Section 5.2.

From this intuition, we also have the following theorem, displayed graphically by the dotted orange line in Figure 1, that compares balanced growth paths across economies.

Theorem 5.2. *Suppose $\alpha \geq 2$, a free entry condition holds, i.e., $\epsilon = 1$, and restrict attention to economies in which all industries are identical in equilibrium. Consider the balanced growth paths of two economies that differ in the large firm innovation cost, χ_L, and let labor supply $\bar{L}$ adjust so that the labor used in production, L^p, in the two balanced growth paths is the same. If one balanced growth path has a strictly higher large firm industry revenue share, then it has a strictly lower growth rate.*

Intuitively, the total innovation effect in this case is zero, and all that remains is the composition

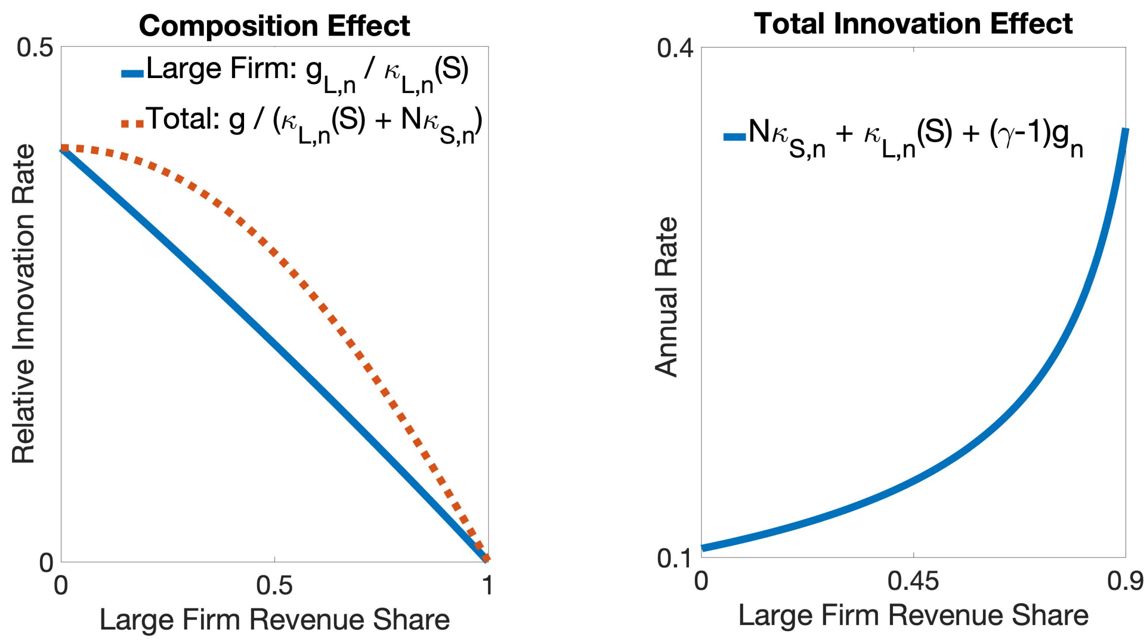

Figure 2: The left panel depicts growth relative to creative destruction of small firms' goods in an industry on a balanced growth path as a function of the large firm's revenue share, both for the large firm (the solid blue line) and for all firms (the dotted orange line). The right panel depicts the total innovation rate in an industry on a balanced growth path as a function of the large firm's revenue share. The figure is based on the calibration described in Section 5.2.

effect and the resulting negative relationship between concentration and growth. The free entry condition at the aggregate level, as well as holding fixed the labor used in production, implies that the effective discount rate on small firm profits is the same in each balanced growth path. In the quantitative exercise in Section 5.3, I allow the labor used in production to adjust, but the effect on long-run growth is small relative to the composition effect of the rise in concentration.

5.2 Calibration

I calibrate the model and solve it computationally to yield more results. I first calibrate the model to an initial balanced growth path in which all industries are identical. I set some parameters externally, and internally calibrate the rest to jointly match a set of moments in the data. The externally calibrated parameters as well as their sources are listed in Table 1. The internally calibrated parameters are listed in Table 2. The data moments used to calibrate the internally calibrated parameters as well as their sources are listed in Table 3. I set the minimum productivity gap, σ, equal to the innovation step size, λ, which is the largest possible value given Assumption 3.1. I set the household's labor supply, $\bar{L}$, so that in the initial balanced growth path, output relative to productivity, $C_t/Z_t = L^p$, is 1. The units of time are years.

The innovation cost elasticity, α, which I calibrate externally to 2, is particularly important because it determines how a large firm's innovation composition responds to its revenue share. I assume that creative destruction and new good development costs are independent, and that each innovation rate responds to the expenditures on that type of innovation as in the studies described in Acemoglu, Akcigit, Alp, Bloom, and Kerr (2018).

The innovation step size, λ, which I calibrate internally to 1.05, is important because it determines the fraction of a creative destruction innovation that improves on the replaced good and is novel, $(\lambda^{\gamma-1} - 1)/\lambda^{\gamma-1} = 0.1$. As λ increases and the fraction that is novel goes to 1, the difference between creative destruction and new good development disappears. Moreover, if a larger fraction of creative destruction innovations are novel and generate growth, calibrating the model to match the fraction of growth due to creative destruction requires a lower rate of creative destruction relative to new good development. The calibrated value is consistent with more direct evidence in Garcia-Macia, Hsieh, and Klenow (2019) using data on labor flows, in which the average innovation step size from creative destruction in 1983-1993 is 1.07.

I calibrate the revenue share of large firms, which is the same in each industry, as well as the shock in Section 5.3, to match the average industry revenue share of the largest firm in 4-digit industries in Compustat. This measure likely overstates the size of the largest firm since Compustat does not include all firms. An alternative measure is the Census data on industry concentration measures, which show a smaller level of industry concentration, but a similar rise over the same time period. One downside of the Census data is that it only lists the revenue share of the top 4 firms in each industry, not the top firm. Moreover, while the Census data is in a sense more accurate because it includes more firms, it may include too many small firms that are not relevant to the mechanism

Table 1: Externally Calibrated Parameters

Parameter	Description	Value
η	Exit Rate	0.04
γ	Demand Elasticity	3.1
α	Innovation Cost Elasticity	2
ϵ	Entry Cost Elasticity	1

The exit rate is from Boar and Midrigan (2022). The demand elasticity is from Broda and Weinstein (2006), using their median estimate from 1990-2001 at the most disaggregated level. The innovation cost elasticity is from Acemoglu, Akcigit, Alp, Bloom, and Kerr (2018). The entry cost elasticity is chosen so that there is a free entry condition at the aggregate level.

Table 2: Internally Calibrated Parameters

Parameter	Description	Value
λ	Innovation Step Size	1.05
χ_C	Relative Creative Destruction Cost	0.265
χ_E	Entry Cost	2.22
χ_L	Large Firm Innovation Cost	15.14
ρ	Time Discount Rate	0.0194

Table 3: Calibration Targets

Moment Description	Data Average from 1983-1993	Model
R&D Relative to GDP	1.81%	1.81%
Creative Destruction Growth Share	26.51%	26.53%
TFP Growth Rate	1.66%	1.66%
Large Firm Market Share	40.68%	40.74%
Real Interest Rate	3.6%	3.6%

The ratio of R&D expenditures on GDP is the Business Enterprise Expenditure on R&D (BERD) relative to GDP from the OECD MSTI database. The creative destruction growth share is the fraction of growth from creative destruction from Garcia-Macia, Hsieh, and Klenow (2019). I compute this value in the model excluding innovation when large firms creatively destroy their own goods because this will appear as innovating on their own goods in the data. The TFP growth rate is from the BLS measure in Garcia-Macia, Hsieh, and Klenow (2019). The large firm market share is the sales-weighted average across 4-digit industries of the largest firm's revenue share in Compustat from Olmstead-Rumsey (2022). The real interest rate is the 1-year real interest rate from FRED.

in the model. In the model, if there are many small firms that do not innovate but simply imitate the innovations of others, then the effect may just be to lower the price index by a fixed factor, without any further impact on the decisions of the innovative firms.

5.3 Quantitative Experiment: A Rise in Large Firm Innovation

I ask whether and to what extent a rise in concentration driven by a fall in the cost of innovation for large firms can explain changes in US data since the mid-1990s. In line with the interpretation of a large firm's innovation cost as capturing the measure of entrepreneurs working within the firm, we can interpret the fall in the cost of innovation as an increase in the concentration of entrepreneurial activity within large firms. The economy begins in the balanced growth path from the calibration in Section 5.2. There is an unanticipated permanent change in χ_L in all industries so that the revenue share of the large firm in each industry in the new balanced growth path is 0.51, the sales-weighted average across 4-digit industries of the largest firm's revenue share in 2018 in Compustat from Olmstead-Rumsey (2022) (the large firm innovation cost falls to $\chi_L = 12.04$). I track the transition path of the economy as it converges to a new balanced growth path.

5.3.1 Aggregate Results:

I show the revenue share of the large firm in each industry along the transition path in Figure 3. The revenue share converges over a similar time interval as the gap between the years in the initial calibration, 1983-1993, and the target year for the shock, 2018.

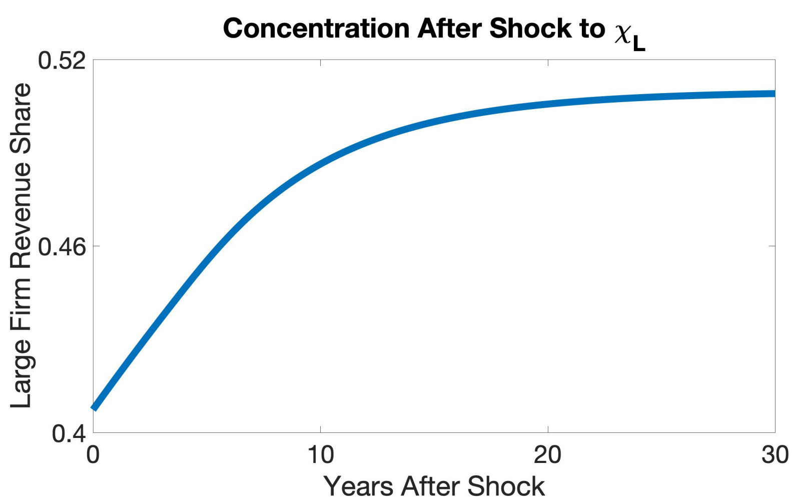

Figure 3: The revenue share of each large firm over time following a shock to χ_L.

I compare the main results concerning aggregate growth in the model to the data in Table 4, and display the model results graphically in Figure 4. The model can explain 41% of the long-run fall in growth, which in the model is due entirely to a change in the growth rate of productivity because output relative to productivity is constant along a balanced growth path. The model can explain all of the increase in the short-run growth rate in the data if we include growth in output due to changes in output relative to productivity, $C_t/Z_t = L_t^p$, as well as changes in productivity. However, the burst in growth does not last as long in the model as in the data: the peak difference in output along the transition path from the original balanced growth path occurs after 4 years and is 42% of the difference in the data after 4 years.

Table 4: Growth After Shock to χ_L

Moment Description	Data	Model
Growth Rate Burst	+0.64 ppt (38.6%) (1993-2003)	Output: +0.87 ppt (52.4%) (first year) TFP: +0.1 ppt (6.0%) (first year)
Cumulative Burst	+6.4 ppt (38.6%) (1993-2003)	Output: +1.07 ppt (16.1%) (4 years) TFP: +0.18 ppt (2.7%) (3 years)
Growth Rate Fall	-0.34 ppt (-20.5%) (2003-2013)	-0.14 ppt (-8.4%) (New BGP)

For each value, ppt is the percentage point rise, and the number in parentheses is the percent rise relative to the initial value. The data are taken from Garcia-Macia, Hsieh, and Klenow (2019). The growth rate burst in the model is the peak growth rate in the short-run following the shock. The output growth rate reflects changes in output relative to productivity, C_t/Z_t, as well as changes in TFP, Z_t. The cumulative burst is the sum of growth rates, i.e., the peak difference between the new path and the old path.

I decompose the change in the productivity growth rate over time into the composition and total innovation effects in Figure 5. Throughout, the composition effect drives down growth as large firms' revenue shares increase and innovation shifts toward creative destruction of small firms' goods. Nonetheless, growth is higher in the short-run because total innovation increases. In the long-run, growth is lower because the composition effect dominates; total innovation is only slightly higher due to an increase in output relative to the wage driven by a shift in labor away from entry and innovation.

To understand the dynamics of the total innovation effect, it is useful to look at entry, displayed in Figure 6. Immediately following the shock, the entry rate hits its lower bound of 0; the expected discounted profits of entering become negative, but the measure of small firms can only fall over time as firms exogenously exit. Thus, growth increases because large firms innovate more, and total small firm innovation is slow to fall due to an overhang of small firms.

Entry:

The large fall in entry in the short-run and the smaller fall in the long-run match the data in Decker, Haltiwanger, Jarmin, and Miranda (2016), which show that the entry rate declined sharply in the mid-to-late 1990s followed by a partial recovery before a large drop during the Great Recession.

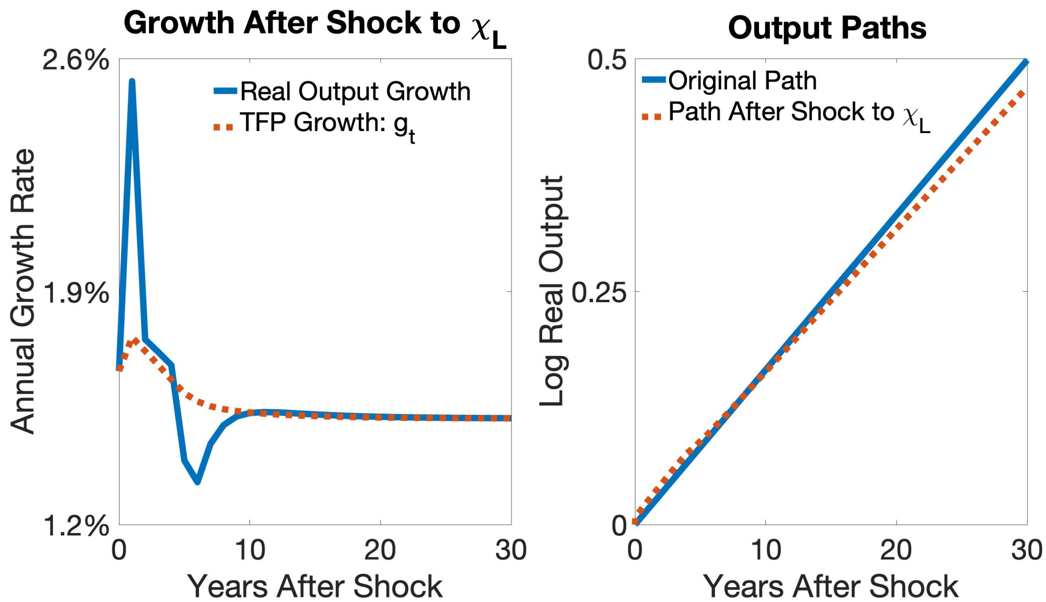

Figure 4: The left panel shows annual growth rates following the shock. The dotted orange line is the growth rate of productivity, g_t, and the solid blue line includes changes in output relative to productivity, C_t/Z_t, which is constant along a balanced growth path. The right panel shows paths of real output over time. The solid blue line is the original path the economy would have followed had it not been hit by a shock. The dotted orange line is the realized path following the shock.

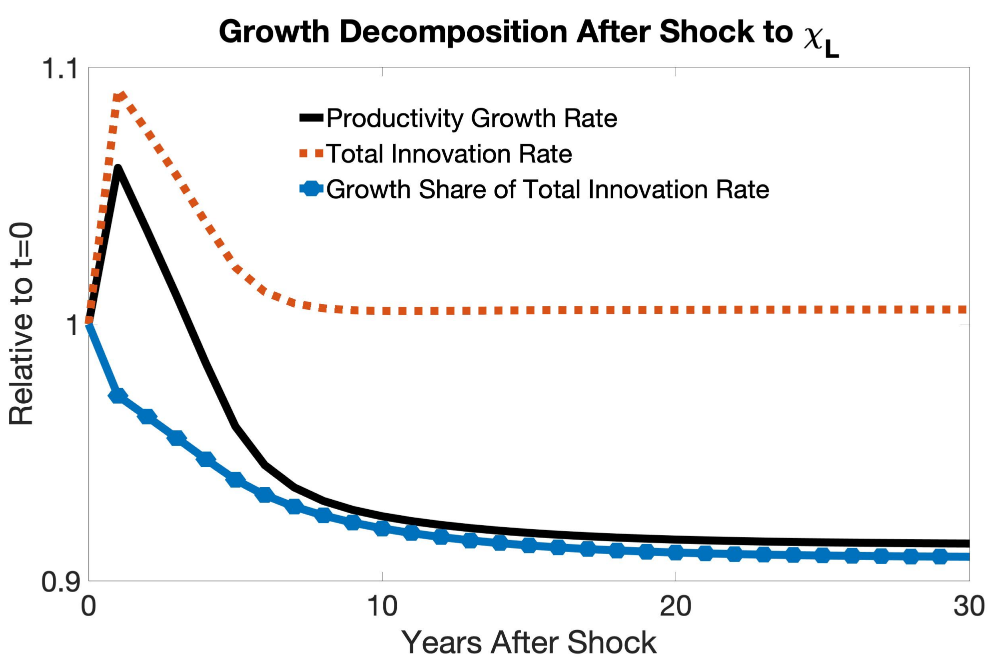

Figure 5: The solid black line depicts the annual productivity growth rate relative to before the shock. The dotted orange line and the textured blue line decompose the black line into the total innovation rate and growth over the total innovation rate, respectively, relative to before the shock.

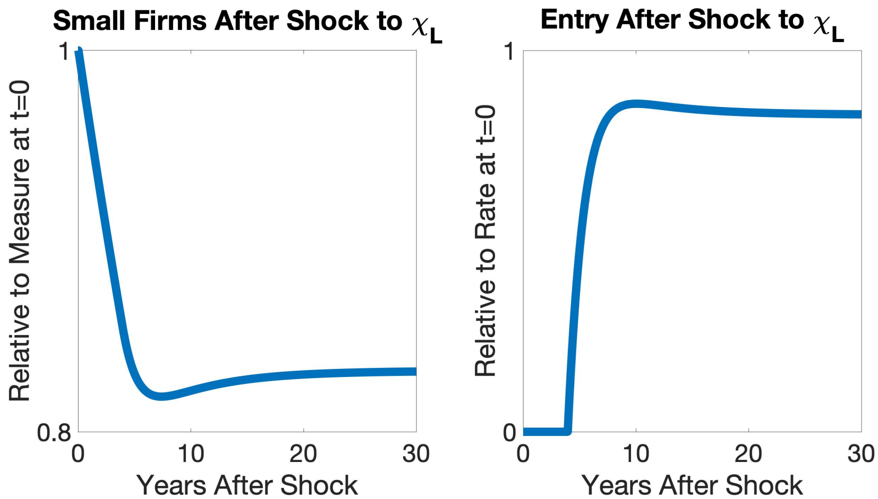

Figure 6: The left panel is the measure of small firms over time relative to the measure before the shock. The right panel is the entry rate over time relative to the entry rate before the shock.

Creative Destruction:

I show in Figure 7 that the share of growth due to creative destruction falls following the shock, as in the long-run in Garcia-Macia, Hsieh, and Klenow (2019), although by a smaller magnitude. This follows from the shift in innovation toward large firms because growth due to creative destruction is a *smaller* share of large firms' growth than of small firms' growth. This is not at odds with Figure 2 or with the analysis in Section 4.2.3 because a large firm only creatively destroys small firms' goods, whereas small firms creatively destroy all firms' goods. Although large firms focus their innovation particularly toward creative destruction of small firms' goods, their innovation is less focused on creative destruction overall.

5.3.2 Industry Level Results:

A distinguishing feature of the theory is that it implies a stronger negative relationship between growth and concentration at the aggregate level than at the industry level, following an aggregate shock to large firm innovation. I compare this prediction to the empirical work in Ganapati (2021), which shows that in US data from 1972-2012, controlling for sector and time fixed effects, a 1 percent rise in the revenue share of the largest 4 firms in a 6-digit NAICS industry over a 5-year period is associated with a 0.1 percent rise in the industry's real output. Similarly, Ganapati (2021) finds that a 1 percent rise in the revenue share of the largest 4 firms in a 6-digit NAICS industry over a 5-year period is associated with a 0.2 percent rise in the industry's real output relative to employment. To run the same regression in the model, I generate industry heterogeneity by imposing that a measure 0 of industries do not see a change in their large firm's innovation

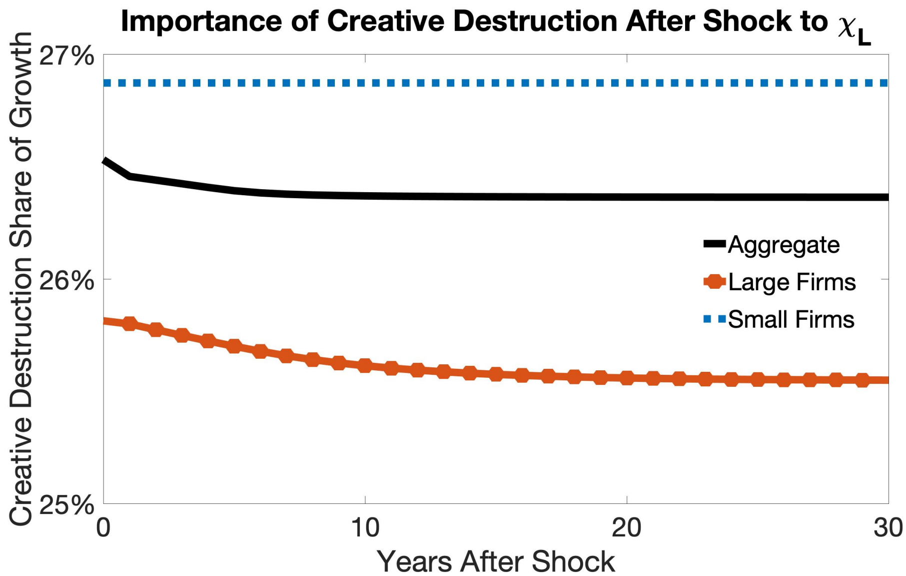

Figure 7: Each line depicts, over time following the shock, the rate at which a type of firm generates growth through creative destruction relative to the total rate at which that type of firm generates growth.

cost. I then regress the change in industry log real output on the change in industry log large firm revenue share and a time fixed effect over the three 5-year time periods during the first 15 years of the transition path following the shock, after which the economy is effectively on the new balanced growth path. I find similar results as Ganapati (2021): a 1 percent rise in the revenue share of the largest firm is associated with a 0.03 percent rise in the industry's real output. This effect is due entirely to different productivity growth rates across industries because revenue in each industry is the same.

The theory thus generates a parsimonious explanation for the short-run burst in growth as well as the positive relationship between concentration and growth across industries: the measure of small firms is slow to adjust and only adjusts at the aggregate level.

5.3.3 Welfare and Size-Dependent Taxes

Taking into account the transition path, household welfare falls by the equivalent of a permanent 5.8% drop in consumption. The decline in the long-run growth rate is ultimately the dominant effect. This suggests that on the margin, contrary to the result in Edmond, Midrigan, and Xu (2021), a tax on firms increasing in their size will improve growth and welfare. For a small tax, large firm don't change their prices since they already set their markups at the constraint implied by the second-best producer. Large firms respond to the tax by reducing innovation, leading to more small firms and growth in the long-run. Even if large firms ultimately respond by investing less in their innovative capacity, the effect on growth and welfare is positive: large firms over-invest in innovative capacity because as their innovation cost falls, their profits rise yet welfare falls.

6 Antitrust Policy: Acquisitions

I use the calibrated model to explore the effects of two different types of acquisition policies. First, for each good produced by small firms, at an exogenous Poisson arrival rate the large firm in the same industry can make a take-it-or-leave-it offer to purchase the good from the small firm. I consider the effects of a tax, τ, on these transactions so that if the relative productivity of the good is $\tilde{z}_{n,t}(j)$, then the small firm receives payment $\tilde{z}_{n,t}(j)\pi_{S,n,t}(\tilde{Z}_{L,n,t})$, and the large firm pays $(1+\tau)\tilde{z}_{n,t}(j)\pi_{S,n,t}(\tilde{Z}_{L,n,t})$. If a large firm acquires goods at rate $A_{n,t}$ with average relative productivity $\zeta_{n,t}$, then the evolution of $\tilde{Z}_{L,n,t}$ is as before in (6) with the additional term $A_{n,t}\zeta_{n,t}$.

Second, for each small firm, at an exogenous Poisson arrival rate the large firm in the same industry can make a take-it-or-leave-it offer to purchase the small firm's entrepreneur, i.e., its innovation capacity. I again consider the effects of a tax, τ, on these transactions so that the small firm receives payment $V_{S,n,t}(\tilde{Z}_{L,n,t})$, and the large firm pays $(1+\tau)V_{S,n,t}(\tilde{Z}_{L,n,t})$. If a large firm acquires small firms' entrepreneurs at rate $A_{n,t}$, then the measure of small firms declines at rate $A_{n,t}$ beyond the baseline effects of entry and exit. The large firm's measure of entrepreneurs changes at rate

$$\partial\chi_{L,n,t}^{-1/(\alpha-1)}/\partial t = A_t - \eta\left(\chi_{L,n,t}^{-1/(\alpha-1)} - \bar{\chi}_{L,n}^{-1/(\alpha-1)}\right),$$

where $\chi_{L,n,t}$ is the large firm's innovation cost at time t and $\bar{\chi}_{L,n}$ is the large firm's exogenously given innovation cost absent any acquisitions, so the second term on the right-hand side reflects the rate at which the large firm's acquisitions exit.

In each case, any taxes collected are dispersed to the representative household. If the tax is negative, then it is funded by a lump sum tax on the representative household.

The main findings are that acquisitions of small firms' *goods* increase growth and welfare if they are sufficiently valuable to the large firm. Acquisitions of small firms' *entrepreneurs* reduce growth and welfare.

6.1 Acquisitions of Small Firm Goods

The economy begins on a balanced growth path in the calibrated model either before the shock to χ_L or after. Each good produced by a small firm can be acquired by the large firm in the same industry at rate 0.05. Put another way, each large firm can acquire 5% of the goods produced by small firms in its industry per year. The initial tax rate on acquisitions is $\tau = 0.2$, which is sufficiently high so that large firms do not purchase small firms' goods, and the economy is indistinguishable from the case without any acquisitions. There is an unanticipated permanent change in the tax rate τ. I track the transition path as the economy converges to a new balanced growth path.

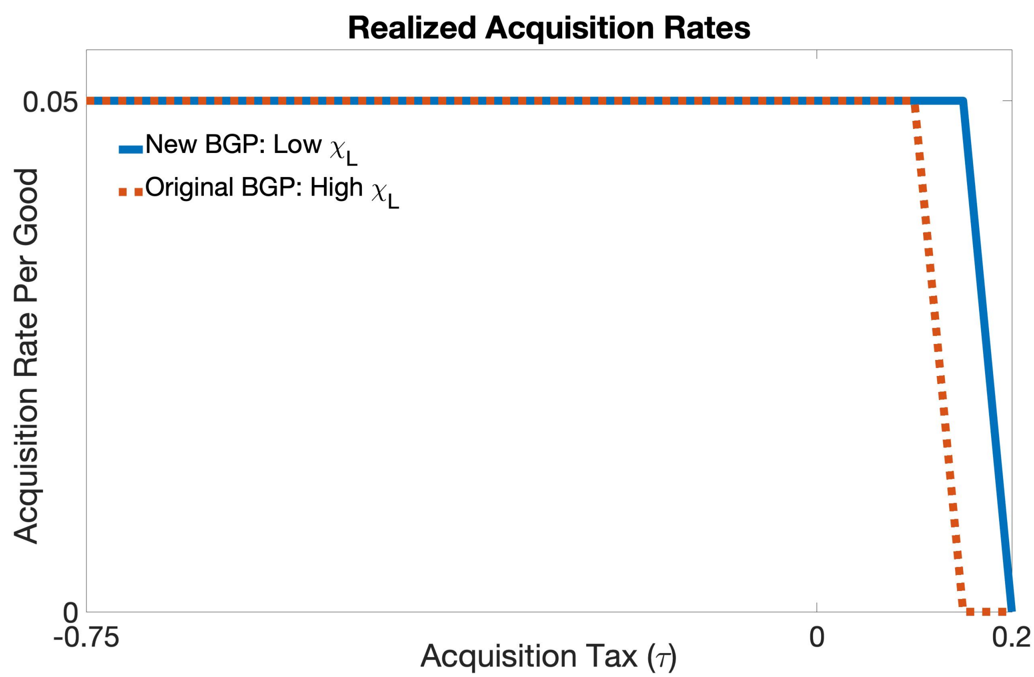

Figure 8: Each line depicts the fraction of small firm goods that large firms acquire per year in the balanced growth path as a function of the acquisition tax rate.

I show in Figure 8 that once the tax rate is sufficiently low, in the long-run, large firms switch from never acquiring to always acquiring small firms' goods when given the opportunity. Large firms acquire small firms' goods even when the tax rate is positive because they face a lower creative destruction rate.

I show the effects of changes in the tax rate on the long-run growth rate and on welfare taking into account the transition path in Figure 9. When the tax rate falls just enough so that the acquisition rate rises to its maximum level, there is a large negative effect on growth and on welfare. As the tax rate falls further and ultimately becomes a subsidy, the acquisition rate does not change, but

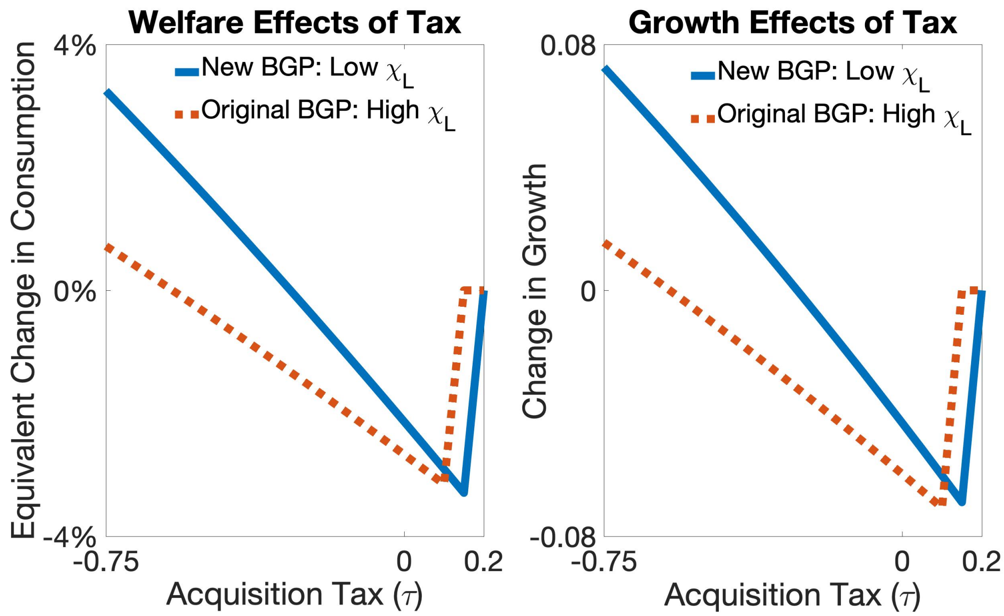

Figure 9: The left panel depicts, starting from either the original balanced growth path before the shock to χ_L or the new balanced growth path after the shock, the welfare effects of an unanticipated permanent change in the acquisition tax rate, taking into account the transition path. Welfare is computed as the equivalent permanent percentage change in final good consumption. The right panel depicts the percentage point change in the long-run growth rate as a function of the acquisition tax rate.

growth and welfare rise, eventually exceeding their values in the balanced growth path without acquisitions.

To understand the growth and welfare results, I show in Figure 10 that changes in the *rate of acquisitions* and changes in the *value of acquisitions to the large firm* have opposite effects. The rise in the acquisition rate, which occurs entirely as the tax rate falls just below 0.2, increases large firm revenue shares, which reduces their incentive to generate growth relative to their incentive to creatively destroy small firms' goods (the solid black in the left panel). On the other hand, the opportunity to acquire small firms' goods, which becomes more valuable as the tax rate falls further below 0.2, reduces the large firm's incentive to perform all types of innovation (the dotted blue line in the left panel) because the more a large firm innovates, the less relative productivity remains for them to acquire. Since large firm innovation ultimately reduces growth and welfare, as we saw in Section 5.3, increasing the *value of acquisitions* to the large firm, conditional on the *rate of acquisitions*, improves growth and welfare.

The benefits of reducing the acquisition tax or subsidizing acquisitions are larger starting from the balanced growth path with higher concentration. Small firms have less relative productivity for large firms to acquire, mitigating the effect of the rise in the acquisition rate, and more significantly, large firms innovate more, increasing the effect of a proportional fall in large firm innovation.

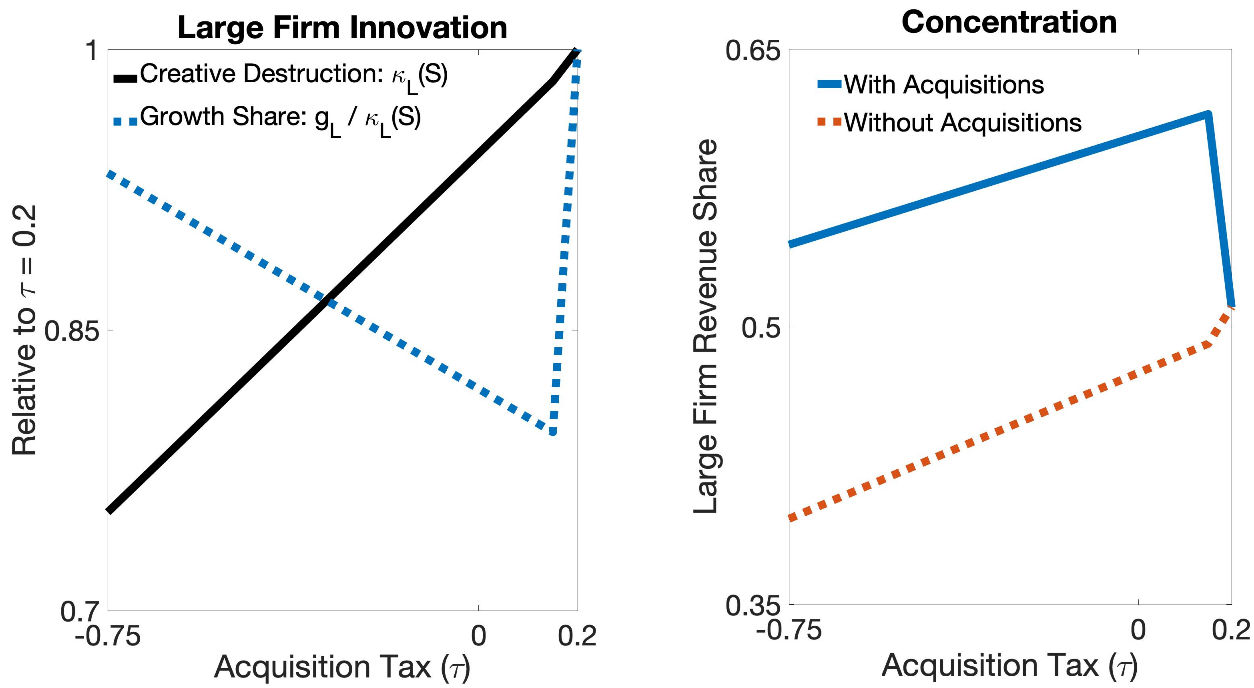

Figure 10: The left panel displays innovation rates in the new balanced growth path as a function of the acquisition tax rate relative to in the original balanced growth path: the solid black line is the rate at which the large firm creatively destroys small firms' goods and the dotted blue line is the rate at which the large firm generates growth relative to the rate at which it creatively destroys small firms' goods. In the right panel, the solid blue and dotted orange lines depict each large firm's long-run revenue share as a function of the acquisition tax, with and without the direct effects of acquisitions on the distribution, respectively. The figure uses the calibration of the model following the shock to χ_L.

6.2 Acquisitions of Small Firm Innovative Capacity

The economy begins on the balanced growth path in the calibrated model following the shock to χ_L from Section 5.3. Each small firm's entrepreneur can be acquired by the large firm in the same industry at rate 0.01. The initial tax rate on acquisitions is sufficiently high so that large firms do not purchase, and the economy is indistinguishable from the case without any acquisitions. There is an unanticipated permanent decrease in the tax rate so that large firms acquire small firms' entrepreneurs whenever given the opportunity. I track the transition path as the economy converges to a new balanced growth path.

The effect is largely the same as a fall in the large firm innovation cost, detailed in Section 5.3, except that here the fall comes at the expense of small firm innovative capacity rather than for free as in Section 5.3. Growth falls in the long-run by 0.19 percentage points and welfare, taking into account the transition path, falls by the equivalent of a permanent 6.18% drop in final good consumption. The particular choice of the acquisition tax rate does not matter as long as it is sufficiently low for large firms to acquire small firms because the value large firms receive from acquisitions does not affect their other decisions.

7 Extension: Large Firm Entry and Exit

Following the shock to the innovation cost of large firms in Section 5.3, there is a rise in large firm profits. Since potential entrants face no possibility of becoming a large firm, these profits have no impact on entry. In this section, I alter the model so that large firms exit at the small firm exogenous exit rate η, and potential entrants face the possibility of entering as a large firm with zero relative productivity in an industry that did not otherwise have one. The results from the baseline model hold.

Since large firms exit and then enter with zero relative productivity, even though exogenous parameters are the same in each industry, the distribution of industry states across the economy is no longer a single mass point. To isolate the effects of this dispersion from the other effects of endogenizing large firm entry, I develop another version of the model in which potential entrants face no possibility of becoming a large firm, but large firms exit at exogenous rate η and are immediately replaced by a new large firm with zero relative productivity. I call this model the *exogenous* large firm entry model, and call the model in which potential entrants may enter as large firms the *endogenous* large firm entry model. In both cases, when a large firm exits, they receive value 0.

Given an entry rate E_t, in the endogenous large firm entry model, large firms enter the economy at rate $E_L E_t$, where E_L is exogenously given. I hold fixed the measure of entrepreneurs per unit of entry at 1. Since a large firm has a measure $\chi_L^{-1/(\alpha-1)}$ of entrepreneurs, it follows that the entry rate of small firms into each industry is $E_S E_t$, where $E_S = 1 - \chi_L^{-1/(\alpha-1)} E_L$. To maintain that there is at most one large firm per industry, I impose that large firm entry is directed to industries without a large firm. If the measure of industries without a large firm at time t is $\Gamma_{0,t}$, then the Poisson arrival rate of a large firm into such an industry is $E_L E_t / \Gamma_{0,t}$. Thus, if a potential entrant pays the entry cost, they receive expected value

$$E_L V_{L,t}(0) + E_S \int_0^1 V_{S,t}(\tilde{Z}_{L,n,t}) dn.$$

I calibrate the endogenous and exogenous large firm entry models to the same initial balanced growth path as in Table 3. The changed parameter values are listed in Table 5. As before, each industry has the same parameters. I also calibrate E_L so that, in both the initial balanced growth path and the new balanced growth following the shock, there is a large firm in almost every industry: 93.63% and 96.96% of industries have a large firm in the pre- and post-shock balanced growth paths, respectively.

Table 5: Re-Calibrated Parameters For Models with Large Firm Entry

Parameter	Description	Endogenous Entry	Exogenous Entry
λ	Innovation Step Size	1.059	1.057
χ_C	Relative Creative Destruction Cost	0.305	0.295
χ_E	Entry Cost	2.9	3.0
χ_L	Large Firm Innovation Cost	10.4	11.5
E_L	Per Unit Large Firm Entry Rate	2.12	N/A

Table 6: Different Model Results After Shock to χ_L

Moment	Baseline	Endogenous Large Entry	Exogenous Large Entry
Concentration	+10.14 ppt (24.9%)	+10.44 ppt (25.7%)	+10.37 ppt (25.5%)
Growth	-0.14 ppt (-8.4%)	-0.16 ppt (-9.6%)	-0.18 ppt (-11.1%)
Small Firms	-16.5%	-24.0%	-25.9%
Welfare	-5.8%	-6.7%	-12.5%

Concentration is the sales-weighted average large firm industry revenue share across industries. Small firms is the measure of small firms. The changes in concentration, growth, and the measure of small firms are from the initial to the new balanced growth paths, and are computed both in percentage points (ppt) and as a percentage of the initial value. The change in welfare is from the initial balanced growth path to immediately after the shock (taking into account the transition path to the new balanced growth path), and is computed as the welfare equivalent permanent percent change in final good consumption.

As in Section 5.3, I conduct the following experiment. The economy begins on an initial balanced growth path. There is an unanticipated permanent change in χ_L in all industries so that the average revenue share of large firms in the new balanced growth path is 0.51 (χ_L falls to 8.74 in the endogenous entry case and 9.05 in the exogenous entry case). In the endogenous entry case, the measure of entrepreneurs and entry rate of large firms per unit of entry are held fixed, so the entry rate of small firms per unit of entry, E_S, falls with χ_L from 0.41 to 0.30. I track the transition path of the economy as it converges to the new balanced growth path.

I compare the main results in the large firm entry models with the results from Section 5.3 in Table 6. The results are similar. There are two main differences between the baseline model and the model with endogenous large firm entry that have opposite effects on growth and welfare. The first difference, which is the only difference between the baseline model and the exogenous large firm entry model, is that the models with large firm entry exhibit dispersion in large firm revenue shares across industries. Since the negative effects of large firms on growth are increasing and convex in large firms' revenue shares, this dispersion implies a greater cost of a rise in the average large firm revenue share. The second difference between the baseline model and the endogenous large firm entry model is that an entrant's value is split between the possibility of being a small firm and of being a large firm. Although the value of entering as a small firm is held fixed across balanced growth paths by the free entry condition in the baseline model, it falls by 2% following the shock in the endogenous large firm entry model because the value of entering as a large firm increases. The decrease in the value of being a small firm allows for an increase in the effective discount rate on small firm profits, and therefore in the total innovation rate and in growth. Nonetheless, this effect is not significant enough to overwhelm the conclusion that more large firm innovation reduces growth.

I interpret the particular way of modeling large firm entry as implying that the shock to the large firm innovation cost is an increase in the concentration of entrepreneurial activity within larger firms. One alternative is to decrease the large firm innovation cost while holding fixed the entry rate of large and small firms per unit of entry, E_L and E_S, respectively. This has the effects of the shock already studied in addition to a fall in the entry cost. In that case, the fall in the entry cost overwhelms the other effects, and growth and welfare rise. However, if large firms exit more slowly than do small firms, then they must enter less often as well, and this result no longer holds because the effects of a fall in the large firm innovation cost converge to the effects in the baseline model. Since the time discount rate ρ is strictly positive, the product of the vanishing probability and the increasing value of entering as a large firm goes to zero.

8 Conclusion

To understand the relationship between concentration and growth, and the policy implications, I study a model with one large firm and a continuum of small firms in each industry. Firms can innovate through creative destruction, developing new goods, and improving on their own goods. Large firms, to avoid cannibalization, have a strong relative preference for creatively destroying their competitors' goods. As a result, when large firms innovate more, small firms' innovations are discounted heavily relative to the overall innovation and growth rate. A widespread fall in large firm innovation costs increases concentration and reduces small firm entry, long-run growth, and taking into account the transition path, welfare. Growth rises in the short-run because the measure of small firms and therefore aggregate small firm innovation is slow to fall. Similarly, in industries in which the large firm's innovation cost falls more, there is a bigger rise in concentration and faster growth because the measure of small firms in an industry responds only to the aggregate environment. I show that these predictions match US data from the mid-1990s to the mid-2010s.

Large firm acquisitions of their competitors' goods have direct and indirect effects with opposite implications for concentration, growth, and welfare. Acquisitions directly shift revenue to large firms, strengthening their relative preference for creative destruction of their competitors' goods, and leading to a fall in growth. The indirect effect is that since acquisitions are valuable to large firms, each large firm innovates less so that more revenue share remains for it to acquire. As large firm innovation falls, it is replaced by small firm innovation, which is more geared toward growth rather than creative destruction of small firms' goods, ultimately facilitating more innovation and growth. If acquisitions are sufficiently valuable to large firms, then growth and welfare are higher in an economy with acquisitions than without. This positive effect is stronger when industries are more concentrated.

The theory and results highlight a novel way to think about the effects of market power and optimal competition policy. Large firms are harmful because of how they achieve their size through innovation. Research and development subsidies that target large firms may backfire by discouraging small firm innovation. Policies that increase concentration may be beneficial as long as they reduce large firm innovation. Facilitating acquisitions is a particularly useful policy because, unlike taxing large firms, it does not require knowledge of firms' relevant industries or their revenue shares in those industries.

Finally, although this paper focuses on growth, the theory has implications for other settings, and suggests potential avenues for future research. For example, suppose a firm can develop different types of goods, some of which are more novel to the industry, and others of which are close substitutes with the firm's competitors' goods. The same force that leads larger firms to set higher markups in static models of oligopolistic competition implies that larger firms have a stronger preference for producing the types of goods that are close substitutes with their competitors. Thus, subsidizing large high markup firms to produce more may be costly unlike in models in which firm production is one-dimensional.

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