

# Market Concentration, Growth, and Acquisitions

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## Abstract

I study an oligopolistic growth model in which firms can innovate by creatively destroying their competitors' goods, innovating on their own varieties, and developing new varieties. To avoid cannibalization, larger firms innovate disproportionately through creative destruction, which generates little growth but deters other firms from innovating. A fall in large firm innovation costs, calibrated to match the recent rise in industry concentration in the US, explains almost half the fall in growth from the 1990s to the 2010s, as well as the burst in growth during the late 1990s. Despite this link between the rise of dominant firms and the fall in growth, a substantial reduction in taxes on large firm acquisitions of their competitors' goods *increases* growth and welfare: to preserve valuable acquisition opportunities, large firms engage in less creative destruction. Dispersion in large firm innovation costs across industries yields a novel theory of the inverted-U relationship between concentration and growth.

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# 1 Introduction

Many authors have documented a rising share of revenue going to the top firms in industries at the national level in the US since the 1990s.<sup>1</sup> This trend has spurred research into its connection to the recent decline in growth, as well as the policy implications.<sup>2</sup> Over a similar time period, there was a dramatic rise in the rate at which venture capital backed startups are acquired relative to the rate at which they go public.<sup>3</sup> Two important questions emerge: Is large firm behavior behind the fall in growth? If so, should antitrust authorities limit acquisitions to reduce industry concentration and promote growth? These questions highlight the importance of understanding how large firms' market shares shape their, and their competitors', innovation incentives.

I argue that how large firms' market shares affect the *way in which they innovate* rather than just *how much they innovate*, and the resulting impact on their competitors' incentives to innovate, can help explain recent patterns in the data and has stark implications for optimal acquisition and innovation policy. The Arrow (1962) "replacement effect" states that for fear of cannibalization, large firms are reluctant to innovate. I show that this argument only applies to types of innovation that generate growth, and that large firms do not face the same disincentive when engaging in innovation that takes sales directly from their competitors. It follows that 1) a fall in large firm innovation costs, and therefore a rise in large firm innovation, *reduces* growth if their competitors' innovation rates are sufficiently responsive to the expected present discounted value of an innovation; and 2) in that case, policymakers face a novel trade-off when addressing large firm acquisitions of their smaller competitors' goods: a higher acquisition rate increases concentration and decreases growth, but a higher expected value of acquisitions to large firms reduces large firm innovation, leading to *lower* concentration and *faster* growth. A reduction in a tax on acquisitions increases growth if it has a sufficiently small effect on the acquisition rate. Moreover, since reducing large firm innovation is a benefit of acquisitions, not a cost, a fall in an acquisition tax has a stronger positive effect on growth when industries are more concentrated.

I formalize the theory in an endogenous growth model in which there is a continuum of industries, each of which consists of a single large firm and a continuum of small firms. Each firm can innovate by creatively destroying other firms' goods, developing new goods, or improving their own goods. The measure of small firms is determined by a free entry condition at the aggregate level, which implies that the value of entering is the expected value of entering into a random industry, and that

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<sup>1</sup>See Autor, Dorn, Katz, Patterson, and Van Reenen (2020), Barkai (2020).

<sup>2</sup>See Cavenaile, Celik, and Tian (2021), Aghion, Bergeaud, Boppart, Klenow, and Li (2022), Akcigit and Ates (2021), De Ridder (2021), Liu, Mian, and Sufi (2022), Olmstead-Rumsey (2022), and Garcia-Macia, Hsieh, and Klenow (2019).

<sup>3</sup>See Pellegrino (2021).

the measure of small firms is the same in each industry. I show that dispersion *across industries* in the innovation cost of the large firm generates an inverted-U shape of growth as a function of industry concentration, as observed in the data.<sup>4</sup> On the other hand, an *economy-wide* fall in large firm innovation costs has a much stronger tendency to reduce growth. I calibrate the model to US data in the early 1990s and find that a fall in large firm innovation costs that generates the rise in concentration observed in the data can explain the observed temporary burst in aggregate growth in the late 1990s, as well as 41% of the eventual fall in the average growth rate. I introduce into the calibrated model an exogenous rate at which large firms can make take-it-or-leave-it offers to their smaller competitors to acquire their goods. I evaluate the effects of a tax on these acquisitions and find that a sufficiently large subsidy leads to an increase in welfare and a higher long-run growth rate relative to the economy without acquisitions. The required subsidy to increase welfare and surpass the no-acquisition growth rate is substantially lower when industries are more concentrated following the drop in large firm innovation costs.

Compared to small firms, large firms generate growth at a low rate relative to the rate at which they creatively destroy small firms' goods. When a firm improves their own goods or develops new ones, all the revenue they gain comes from generating growth, which takes revenue proportionally from all goods in the industry. When a firm creatively destroys a competitor's good, it improves the good's quality, generating revenue through growth, but mostly takes the revenue its competitor used to receive from the good. The larger a firm's revenue share, the more they discount generating revenue through growth because the more it cannibalizes their sales.

The free entry condition at the aggregate level implies that on a balanced growth path, the sum of growth and the rate of creative destruction faced by small firms is pinned down by the cost of entry since each serves as a discount rate on small firm profits. If large firm innovation costs fall across the economy, then innovation shifts toward large firms, whose revenue shares grow even bigger, and the composition of the discount rate on small firm profits shifts away from growth and toward creative destruction.

Dispersion in large firm innovation costs across industries drives dispersion in large firm industry revenue shares. Across industries, the sum of growth and creative destruction of small firms' goods is no longer fixed. Instead, small firm innovation is a decreasing but continuous function of the total discount rate on small firm profits. To understand why growth is highest in industries with intermediate levels of concentration, consider the two extremes. If the large firm has a very high innovation cost, then its revenue share is negligible and it innovates like a small firm. A marginal

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<sup>4</sup>See Cavenaile, Celik, and Tian (2021) for a direct measure of growth as a function of industry concentration, and Aghion, Bloom, Blundell, Griffith, and Howitt (2005) for an indirect measure that uses markups as a proxy for concentration.

decrease in its innovation cost has the same effect as increasing the measure of small firms, and leads to faster growth. But after a sufficient decrease in its innovation cost, the large firm is nearly a monopolist, and has little incentive to develop new goods or improve on its old ones. The large firm maintains its dominance by creatively destroying its competitors' goods at such a fast rate that they see little benefit from innovating. Nonetheless, small firms produce so few goods that this creative destruction generates little growth.

Large firm acquisitions of their smaller competitors' goods increase concentration and shift large firms further toward destructive types of innovation, pushing down growth and welfare. However, if acquisitions are valuable to large firms, they discourage large firm innovation because the more a large firm innovates, the less market share remains for it to acquire. Thus, given an acquisition rate, reducing taxes on acquisitions shifts innovation to small firms who innovate in a more socially optimal way, increasing growth and welfare. This is especially true when industries are more concentrated because in that case, large firm innovation is particularly high and particularly geared toward creative destruction of small firms' goods.

Testable empirical predictions of the model's main mechanism are that firms can direct creative destruction efforts toward their competitors, and that smaller firms face higher discount rates on the profits from their innovations. The theory implies that this disparity increased as market concentration rose and growth fell in the US since the 1990s. Argente, Lee, and Moreira (2021) show that the revenues of high sales products depreciate more quickly than the revenues of low sales products, in line with the prediction that such products would be creatively destroyed more quickly if firms can direct their creative destruction efforts. Akcigit, Alp, and Peters (2021) show that a relatively high creative destruction rate for goods produced by firm types that innovate less, and tend to be small, can explain the high employment shares of old firms in US and Indian data. My theory provides an explanation for this disparity in creative destruction rates: firms focus their creative destruction toward their competitors' goods, so more innovative firms avoid a larger fraction of total creative destruction.

The theory also has implications for innovation policy. Research and development subsidies that target large firms may backfire and reduce growth by encouraging those firms to creatively destroy their competitors' goods, reducing smaller firms' incentives to innovate. Such subsidies are more likely to increase growth if they provide incentives to enter by targeting future large firms. It is even better to instead subsidize small firm innovation. A policy geared toward startups may accomplish these goals.

These results highlight an important subtlety in optimal competition and innovation policy. In theories that focus only on *how much* large firms produce or innovate, high markups or the Arrow

(1962) “replacement effect” imply that it is optimal to subsidize high market shares and encourage *more* large firm production or innovation.<sup>5</sup> In these models, reducing large firm innovation is a cost of acquisitions, rather than a benefit.<sup>6</sup> In this paper, taking into account *the way in which* large firms innovate, policies that encourage large firm production or innovation backfire. Policies that offer large firms a tempting alternative to innovation increase growth by shifting activity to smaller firms that innovate in a more socially optimal way.

### **Large Firm Market Shares and Innovation:**

Previous work on industries with oligopolistic competition and innovation mostly focuses on the impact of a large firm’s market share on its *magnitude* of innovation.<sup>7</sup> A notable exception is the theory put forward in Argente, Baslandze, Hanley, and Moreira (2021) and mentioned in Akcigit and Ates (2021) that large firms use patents to deter competition. Although the mechanism is different, the result is similar in that large firm innovation generates little growth relative to the rate at which it deters small firm innovation. This theory is complementary to the one proposed here because creative destruction is most relevant in industries with weak patent protection.

Previous theories of the inverted-U relationship between market concentration and growth across industries, such as in Aghion, Bloom, Blundell, Griffith, and Howitt (2005) and Cavenaile, Celik, and Tian (2021), are based on the relative sizes of the “escape competition” and the Schumpeterian effects of competition. Smaller market shares increase pressure for firms to innovate to escape their competitors, but the expectation of the competition that pushes down market shares discourages innovation. In this paper, the magnitude of large firm innovation is driven by both effects, but the magnitude of small firm innovation depends only on the Schumpeterian effect: as the expected rate of growth and creative destruction increases, small firms innovate less. The inverted-U relationship depends on the response of small firm innovation to an increase in large firm innovation, and growth is decreasing in concentration when this response is sufficiently high. An important implication is that if the entry rate and innovation are more responsive to economy-wide changes, then widespread higher concentration tends to be associated with lower growth at the aggregate level even if across industries or within an industry over time, higher concentration is associated with higher growth.

### **Large Firm Acquisitions of Small Competitors’ Goods:**

“Entry for buyout”, described in Rasmusen (1988) and more recently, Letina, Schmutzler, and Seibel (2021) and Fons-Rosen, Roldan-Blanco, and Schmitz (2022), is another theory in which a high expected value of future acquisitions increases growth even though the distributional consequences of actual acquisitions do not. If an acquired firm receives a fraction of the surplus from

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<sup>5</sup>See Edmond, Midrigan, and Xu (2021).

<sup>6</sup>See Fons-Rosen, Roldan-Blanco, and Schmitz (2022).

<sup>7</sup>See Shapiro (2012) for a discussion, and Cavenaile, Celik, and Tian (2021) for a recent example.

the acquisition, then the expectation of being acquired increases the value of entry and innovation. Yet, if large firm innovation is one-dimensional, the entry for buyout effect needs to be weighed against the negative effect on large firm innovation; large firms have less reason to innovate if it erodes potential acquisition opportunities. In this paper, acquisitions are made with take-it-or-leave-it offers, so small firms face no incentive to enter or innovate to be acquired, and the entry for buyout effect is not present. Instead, acquisitions may be useful *because* they reduce large firm innovation, not in spite of it. A distinct implication of this theory is that expected acquisition opportunities are more beneficial when concentration is higher.

“Killer acquisitions”, described in Cunningham, Ederer, and Ma (2021) and then in Letina, Schmutzler, and Seibel (2021) and Fons-Rosen, Roldan-Blanco, and Schmitz (2022), are acquisitions in which the acquiring firm does not produce the acquired good. The possibility of killer acquisitions is an additional potential cost of acquisitions because consumers cannot benefit directly from innovations that are not put into practice. Nonetheless, it only strengthens the positive effects of acquisitions on innovation discussed in this paper. The positive effect on growth of the expected value of future acquisitions is the same because if killer acquisitions are valuable to a large firm, then the large firm still reduces innovation to maintain acquisition opportunities. Moreover, the negative effect on growth of the actual acquisitions is lower because killer acquisitions are not produced and therefore have a smaller effect on large firms’ revenue shares.

### **The Rise in Concentration and The Fall in Growth:**

This paper is related to recent papers that study the effect of high productivity or superstar firms on growth, such as Aghion, Bergeaud, Boppart, Klenow, and Li (2022), De Ridder (2021), Cavenaile, Celik, and Tian (2021), and Liu, Mian, and Sufi (2022). Previous work has mostly considered models with only small firms, and so they focus on the effect of productivity dispersion across small firms, whereas I abstract from productivity dispersion and focus on the effect of large firms’ market power. In particular, Aghion, Bergeaud, Boppart, Klenow, and Li (2022) and De Ridder (2021) focus on the channel that increased competition from high productivity competitors reduces less productive firms’ markups, and therefore their incentive to grow. The channel I study in this paper is complementary in the sense that they focus on the flow profits a small firm receives from innovating, whereas I focus on the effective discount rate on small firm profits.

Liu, Mian, and Sufi (2022) study a growth model with two large firms in each industry, and find that a large firm can reduce growth by building a substantial productivity advantage over its competitor. The mechanism is that a bigger gap implies that the large firm will optimally cut its price by more in response to innovation by its competitor. On the other hand, as discussed, I focus on how a large firm’s innovation decisions affect the rate at which its competitors discount their

profits. An important difference is that my mechanism does not rely on a large firm responding directly to the actions of a single competitor. In that sense, my mechanism may be more relevant when thinking about the effect of a large firm on the innovation decisions of small firms. Finally, Cavenaile, Celik, and Tian (2021) study a growth model with large firms, but the pressure those large firms place on small firms has no effect on growth because small firms always have zero profits.

Akcigit and Ates (2021) and Olmstead-Rumsey (2022) propose theories in which exogenous changes in the economy’s innovation technology cause a decline in growth, as well as a rise in market concentration and markups. The theory I propose reverses the causality and suggests that changes in industry structure—a rise in concentration—drive a decline in growth. Moreover, it provides an alternative explanation for the fall in the effect of a patent on a firm’s market value documented in Olmstead-Rumsey (2022): the innovation is more quickly creatively destroyed by a large firm.

### **Model Building Blocks:**

The model builds on two different strands of the growth literature, one focused on models of creative destruction<sup>8</sup>, and one on expanding varieties models<sup>9</sup>. Recent work combines the two, but without large firms with positive market shares.<sup>10</sup>

The paper proceeds as follows. In section 2, I describe a simple illustrative model. In section 3, I describe the full quantitative model. In section 4, I solve the model and discuss optimal firm behavior. In section 5, I characterize analytical results, calibrate the model, and show quantitative results on the effects of a fall in large firms’ innovation costs. In section 6, I analyze the effects of acquisition policies. In section 7, I confirm the robustness of the model’s results to allowing large firm entry and exit. In section 8, I conclude.

## **2 A Simple Industry Model**

I first describe a simple industry model of creative destruction with a large firm that does not creatively destroy its own goods. The model illustrates the key mechanism of the theory. If firms can target creative destruction toward their competitors’ goods rather than their own, then large firms will generate less growth for a given rate at which they creatively destroy their competitors’

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<sup>8</sup>See Grossman and Helpman (1991b), Aghion and Howitt (1992), Klette and Kortum (2004), and Aghion, Bergeaud, Boppart, Klenow, and Li (2022).

<sup>9</sup>See Romer (1990) and Grossman and Helpman (1991a).

<sup>10</sup>See Atkeson and Burstein (2019).

goods. If small firm innovation is sufficiently responsive to the rate at which small firm goods are creatively destroyed, then an increase in innovation by large firms reduces overall growth. The simple model also demonstrates important empirical predictions of this mechanism. Firms can direct creative destruction efforts toward their competitors, and smaller firms face higher discount rates on the profits from their innovations.

Time is continuous and indexed by  $t \in [0, \infty)$ . There is a unit measure of goods, indexed by  $j \in [0, 1]$ , each of which always receives the same revenue. At each time  $t$ , there is a measure  $N_t$  of small firms, and a single large firm. Each good is produced by a single firm at each time  $t$ , with productivity  $z_t(j)$ . The producer of a good receives flow profits  $\pi$ . Firms innovate by creatively destroying each others' goods. When a firm creatively destroys a good, it becomes the sole producer of that good, and the good's productivity,  $z_t(j)$ , is multiplied by  $\lambda > 1$ .

Industry productivity is  $Z_t = \int_0^1 z_t(j) dj$ , and the industry growth rate is  $g_t = \frac{1}{Z_t} \frac{\partial Z_t}{\partial t}$ . Denote by  $S_t \in [0, 1]$  the measure of goods the large firm produces at time  $t$ , which is also the large firm's share of industry revenue.

Each small firm chooses a Poisson arrival rate at which they creatively destroy each good,  $\tilde{x}_t(j)\Delta$ , subject to the flow cost function  $\int_0^1 \tilde{x}_t(j)^2 dj$ , where  $\Delta$  is very small so that at Poisson arrival rate  $\int_0^1 \tilde{x}_t(j) dj$ , they creatively destroy a single good, and the relative probability of creatively destroying good  $j$  is proportional to  $\tilde{x}_t(j)$ . Small firms choose innovation rates to maximize the expected present discounted value of profits, where they discount future payouts by the real interest rate  $r$ .

In this simple model, I take the large firm's innovation decisions as given. The large firm creatively destroys goods produced by small firms at a flow rate rather than a Poisson arrival rate: in a finite interval of time, each small firm creatively destroys a finite *number* of goods, and the large firm creatively destroys a finite *measure* of goods. *The large firm does not creatively destroy its own goods.*

## 2.1 Long-Run Industry Concentration and Growth

I study balanced growth path Markov Perfect equilibria as a function of the large firm's exogenous innovation rate. All small firms creatively destroy all goods at the same rate,  $x_S$ , since each good yields the same flow profits. The large firm creatively destroys small firms' goods at rate  $x_L$ . The large firm's industry revenue share is constant over time at  $S$ . The industry productivity growth



rate is constant at  $g$ . I consider two cases for determining the constant measure of small firms,  $N$ . In the first case,  $N$  is given by a free entry condition that fixes the value of being a small firm producing zero goods. In the second case,  $N$  is exogenously given.

I analyze the effects of changes in the rate at which the large firm creatively destroys small firms' goods,  $x_L$ , on the large firm's industry revenue share,  $S$ , and growth,  $g$ , in the long-run. Since  $S$  is strictly increasing in  $x_L$ , we can write  $g$  as a function of  $S$ . On a balanced growth path, the large firm's industry revenue share and growth are

$$S = \frac{x_L}{Nx_S + x_L}; \quad g = (\lambda - 1)(Nx_S + (1 - S)x_L). \quad (1)$$

Small firms creatively destroy goods and generate growth at rate  $Nx_S$ , and the large firm creatively destroys goods and generates growth at rate  $(1 - S)x_L$  since it does not creatively destroy its own goods. Small firm creative destruction is given by the First Order Condition:

$$x_S = \frac{\pi/2}{r + Nx_S + x_L},$$

where  $Nx_S + x_L$  is the Poisson arrival rate at which each small firm's good is creatively destroyed.

If  $N$  is determined by the free entry condition, then since the value of being a small firm producing zero goods is pinned down by the optimal small firm creative destruction rate, it follows that for any large firm creative destruction rate,  $x_L$ , the total rate at which a small firm's good is creatively destroyed,  $Nx_S + x_L$ , is constant. From equation (1), the derivative of growth with respect to the large firm's industry revenue share is

$$g'(S) = \frac{-2S}{1 + S} \frac{g}{1 - S},$$

which is always negative. If  $N$  is exogenously given, then

$$g'(S) = \left( \frac{-2S}{1 + S} + \frac{1 + r_0}{2 + r_0} \right) \frac{g}{1 - S},$$

where  $r_0$  is the interest rate relative to the total rate of creative destruction:  $r_0 = r/(Nx_S + x_L)$ . Since  $r_0$  is strictly decreasing in  $S$ , in this case the derivative of growth with respect to the large firm's industry revenue share is negative if and only if  $S > S^*$  for some  $S^* \in (0, 1)$ .

The derivative in the free entry case, and the first term of the derivative in the exogenous  $N$  case, is the *composition effect*. Holding fixed the total rate at which small firm goods are creatively destroyed, total small firm creative destruction falls one-for-one with the rise in large firm creative destruction. Small firm creative destruction contributes to growth at rate  $\lambda - 1$  and large firm creative destruction contributes to growth at rate  $(1 - S)(\lambda - 1)$ , so the effect of the fall in the former always outweighs the effect of the rise in the latter, leading to a decreasing relationship

between growth and the large firm's industry revenue share. The second term of the derivative in the exogenous  $N$  case is the *total innovation effect*. Holding fixed the ratio of small firm creative destruction to large firm creative destruction,  $Nx_S/x_L$ , the total rate at which small firm goods are creatively destroyed, and thus growth, are increasing in large firm creative destruction. If the measure of small firms is exogenously given, then both the composition and total innovation effects are present because each small firm faces a convex innovation cost. If the large firm's revenue share is sufficiently small, then it makes little difference that the large firm does not creatively destroy its own goods, so the total innovation effect dominates the composition effect, which implies an increasing relationship between growth and the large firm's revenue share.

In the quantitative model I study in future sections, potential entrants pay entry costs not knowing into which industry they will enter. Thus, I interpret the exogenous  $N$  case as describing the relationship between concentration and growth *across industries* due to dispersion in large firm creative destruction,  $x_L$ , and the free entry case as describing the relationship between average concentration and growth *over time* due to widespread changes in  $x_L$ . Across industries, growth as a function of the large firm's revenue share exhibits an inverted-U shape. On the other hand, a widespread rise in large firm innovation over time increases large firms' revenue shares and pushes down growth.

### 3 Quantitative Model

Time is continuous and indexed by  $t \in [0, \infty)$ . There is a unit measure of industries, indexed by  $n \in [0, 1]$ , each of which consists of firms producing differentiated goods. There is a representative household who consumes the numeraire final good and inelastically supplies  $\bar{L}$  units of labor. The household's preferences are

$$\int_0^\infty e^{-\rho t} \ln(C_t) dt,$$

where  $C_t$  is consumption of the final good. Final good consumption is a Cobb-Douglas aggregate of industry consumption:

$$\ln(C_t) = \int_0^1 \ln(C_{n,t}) dn,$$

where industry consumption is an aggregate of the consumption of the differentiated goods within the industry. The household takes goods prices and the wage,  $W_t$ , as given and chooses the consumption of each good in each industry. The household can also buy and sell a risk-free real bond with net interest rate  $r_t$  (denominated in units of the final good). Going forward, I focus on a particular industry and drop industry subscripts to simplify notation.

### 3.1 Overview and Demand

Since final good consumption is a Cobb-Douglas aggregate of industry consumption, total revenue in an industry is exogenously given as  $R_t$ . The representative household's stochastic discount factor is characterized by the risk-free interest rate  $r_t$ . There is a measure  $M_t$  of goods indexed by  $j \in [0, M_t]$ . The representative consumer has CES preferences across goods with elasticity  $\gamma \geq 1$ : denote consumption of good  $j$  to be  $c_t(j)$  and consumption of the industry good to be  $C_t$ , then

$$C_t = \left( \int_0^{M_t} c_t(j)^{\frac{\gamma-1}{\gamma}} dj \right)^{\frac{\gamma}{\gamma-1}}.$$

Denote the price of good  $j$  by  $p_t(j)$ . The price of the industry good is therefore  $P_t$ , where

$$P_t = \left( \int_0^{M_t} p_t(j)^{1-\gamma} dj \right)^{\frac{1}{1-\gamma}}. \quad (2)$$

The demand curve for good  $j$  is

$$c_t(j) = p_t(j)^{-\gamma} P_t^{\gamma-1} R_t. \quad (3)$$

### 3.2 Production and Competition

There is a large firm, denoted by subscript  $L$ , and a measure  $N_t$  of small firms, indexed by  $i \in [0, N_t]$ . The large firm can produce each good  $j$  with production function

$$q_{L,t}(j) = z_{L,t}(j) l_{L,t}(j), \quad (4)$$

where  $q_{L,t}(j)$  is quantity and  $l_{L,t}(j)$  is labor, which is purchased in a perfectly competitive market at wage  $W_t$ . Each small firm  $i$  can produce each good  $j$  with production function

$$q_{i,t}(j) = z_{i,t}(j) l_{i,t}(j). \quad (5)$$

All varieties of each good produced by different firms are perfect substitutes.

In each moment, firms simultaneously choose prices to maximize static profits. Let  $\bar{z}_t(j)$  be the highest productivity for good  $j$  among all producers, i.e.,

$$\bar{z}_t(j) = \max\{\{z_{i,t}(j)\}_{i \in [0, N_t]}, z_{L,t}(j)\},$$

The evolution of productivity will be such that  $\bar{z}_t(j)$  is always achieved by only a single firm. Given that firms compete in prices and that different varieties of each good are perfect substitutes, the most productive producer, with productivity  $\bar{z}_t(j)$ , is the sole producer of good  $j$ , and their price

is constrained to be weakly less than the marginal cost of the second most productive producer of good  $j$ .

Let  $\mu_t(j)$  be the gross markup of price over marginal cost for good  $j$ :  $\mu_t(j) = \bar{z}_t(j)p_t(j)/W_t$ .

Let  $Z_t$  be an aggregate of productivity in the industry:

$$Z_t \equiv \left( \int_0^{M_t} \bar{z}_t(j)^{\gamma-1} dj \right)^{\frac{1}{\gamma-1}},$$

and define the relative productivity of good  $j$ :  $\tilde{z}_t(j) \equiv \bar{z}_t(j)/Z_t$ . Going forward, I will usually characterize a good by its relative productivity rather than its productivity.

### 3.3 Innovation

In each moment, each firm chooses two types of innovation: a rate of creative destruction for each good  $j$ , and a rate of new good development. Conditional on creatively destroying a good  $j$ , a firm's relative productivity for that good becomes  $\lambda \tilde{z}_t(j)$ , where  $\lambda > 1$ . Conditional on developing a new good, a firm's relative productivity for that good,  $\lambda \tilde{z}$ , is drawn so that the expected value of  $\tilde{z}^{\gamma-1}$  is equal to 1. Moreover, whenever a firm creatively destroys a good or develops a new good, so that the new relative productivity for that good is  $\tilde{z}_t(j)$ , all other firms are able to produce that good with relative productivity  $\tilde{z}_t(j)/\kappa$ , where  $\kappa > 1$ . Thus  $\kappa$  is the maximum possible gap between the productivities of the most productive and the second-most productive producers of a good. To ensure a balanced growth path, the costs of innovation depend on the relative productivity of the good (whether creatively destroyed or newly developed).

To be clear, a firm can creatively destroy a good that it already produces. For small firms, this possibility is irrelevant since each small firm produces finitely many goods, and creatively destroys each good at an infinitesimal rate. For the large firm, this possibility is meaningful, and implies that the innovative capacity of the economy is not mechanically reduced as the large firm's market share grows.

#### 3.3.1 Small Firms

Each small firm  $i$  chooses a creative destruction rate  $x_{C,i,t}(j)$  for each good  $j$ , where subscript  $C$  denotes *creative* destruction, and a new good development rate  $x_{D,i,t}$ , where subscript  $D$  denotes new good *development*. They creatively destroy good  $j$  at Poisson arrival rate  $x_{C,i,t}(j)\Delta$ , where  $\Delta$

is very small so that at Poisson arrival rate  $\int_0^{M_t} x_{C,i,t}(j) dj$ , they creatively destroy a single good, and the relative probability of creatively destroying good  $j$  is proportional to  $x_{C,i,t}(j)$ . A small firm develops a new good at Poisson arrival rate  $x_{D,i,t}$ . The total flow cost in units of labor is

$$\chi_S \left( x_{D,i,t}^\alpha + \chi_C \int_0^{M_t} \tilde{z}_t(j)^{\gamma-1} x_{C,i,t}(j)^\alpha dj \right),$$

where  $\chi_S > 0$  is the small firm innovation cost,  $\chi_C > 0$  is the relative cost of creative destruction compared to new good development, and  $\alpha > 1$  determines the curvature of cost in the innovation rate.

### 3.3.2 Large Firm

The large firm chooses a creative destruction rate  $x_{C,L,t}(j)$  for each good  $j$ , and a new good development rate  $x_{D,L,t}$ . They creatively destroy good  $j$  at Poisson arrival rate  $x_{C,L,t}(j)$ , and develop new goods at rate  $x_{D,L,t}$ . The total flow cost in units of labor is

$$\chi_L \left( x_{D,L,t}^\alpha + \chi_C \int_0^{M_t} \tilde{z}_t(j)^{\gamma-1} x_{C,L,t}(j)^\alpha dj \right),$$

where  $\chi_L > 0$  is the large firm's innovation cost, and the large firm faces the same relative cost of creative destruction as small firms,  $\chi_C$ .

### 3.3.3 Small Firms vs. The Large Firm

For a small firm, at a Poisson arrival rate they creatively destroy a single good or develop a single new good. Hence, in finite time, a small firm gains control over a finite number of goods. On the other hand, the large firm creatively destroys goods and develops new goods at a continuous rate. Hence, in finite time, the large firm gains control over a finite *measure* of goods. We can think of the large firm as controlling jointly the innovation technologies of a continuum of small firms. Thus, all firms face the same relative cost of creative destruction compared to new good development, and the large firm's innovation cost,  $\chi_L$ , depends on the measure of this continuum of small firms whose innovation technologies it controls.

### 3.3.4 Parameter Assumption and Discussion

To simplify analysis of the model, I make the following assumption:

**Assumption 3.1.** *The maximum productivity gap between the most productive and second-most productive producers of a good is weakly less than the creative destruction step size and the markup any firm would set if unconstrained, i.e.,  $\kappa \leq \min\{\lambda, \gamma/(\gamma - 1)\}$ .*

It follows that regardless of how a firm became the most productive producer of a good, the gap between that firm's productivity and the productivity of the second-most productive producer of the good is  $\kappa$ . I interpret this assumption as suggesting that firms can imitate each others' goods sufficiently well so that regardless of the gap between a firm's new innovation (creative destruction or new good development) and whatever came before it, the firm feels the same competitive pressure when pricing that good.

Assumption 3.1 implies that we can interpret new good development as firms innovating on their own goods. In either case, any gains enjoyed by a firm come from adding productivity to the industry, not from taking productivity from another firm, and in either case, the firm sets the same markup.

Since Assumption 3.1 implies that all firms set the same markup on all goods, I thus abstract from the effects of firms setting different markups on newly developed goods and on creatively destroyed goods, as well as the effects of large firms setting higher markups than small firms. Allowing for different markups substantially complicates the analysis. As large firms gain market share, they set higher markups, which reduces competition and encourages growth from small firms. This effect is mitigated or reversed if large firms are sufficiently more productive than small firms: they may set higher markups than small firms, but lower prices. Then, as large firms gain market share the industry price index falls, which decreases small firms' incentive to innovate.

### 3.4 Entry and Exit

Entry is undirected, so an entering firm draws an industry from the uniform distribution. At each moment in time, there is an infinite mass of potential entrants. If a potential entrant pays the cost of entry, then they draw an industry and enter as a single small firm. Otherwise, the potential entrant receives value 0. The total cost of entry is increasing in the entry rate and is  $E_t^\epsilon$  units of labor, where  $E_t$  is the entry rate and  $\epsilon \geq 1$  is the elasticity of total entry costs with respect to the entry rate. Thus, the marginal entrant faces an entry cost of  $\epsilon E_t^{\epsilon-1}$ . At the lower bound for the elasticity,  $\epsilon = 1$ , the marginal entry cost is constant and there is a free entry condition. At the upper bound for the elasticity,  $\epsilon = \infty$ , the marginal entry cost is 0 if  $E_t < 1$  and infinite if  $E_t > 1$ . In that case, the entry rate is always 1.

Each small firm exits exogenously at Poisson arrival rate  $\eta > 0$ . When a firm exits, it sells each good for which it is the most productive producer to another small firm (not the small firm that is the second-most productive producer of that good).

### 3.5 Equilibrium

At each moment in time, the goods market must clear, i.e., the amount each firm supplies of each good is equal to the representative household's demand for that good, and the labor market must clear, i.e., the labor used in production, for entry costs, and for innovation costs, must equal the labor inelastically supplied by the representative household.

Given the parameter restriction made in Assumption 3.1, I characterize each good by its type  $f \in \{S, L\}$ , which denotes whether the good's current producer is a small firm ( $S$ ) or the large firm ( $L$ ). Let  $T(j)$  be good  $j$ 's type. For each type  $f \in \{S, L\}$ , define  $\tilde{Z}_{f,t}$  to be an aggregate of the relative productivities of goods of type  $f$  at time  $t$ :

$$\tilde{Z}_{f,t} = \left( \int_{j:T(j)=f} \tilde{z}_t(j)^{\gamma-1} dj \right)^{\frac{1}{\gamma-1}}.$$

It follows that  $\tilde{Z}_{S,t}^{\gamma-1} + \tilde{Z}_{L,t}^{\gamma-1} = 1$ .

The industry state is the fraction of industry relative productivity in goods produced by the large firm,  $\tilde{Z}_{L,t}$ , which implies the value of  $\tilde{Z}_{S,t}$ . The aggregate state is the measure of small firms in each industry,  $N_t$ , and the distribution of industry states across industries.

I study Markov Perfect Equilibria in which firms' markups are given by static optimization of profits and are a function only of  $\tilde{Z}_{L,t}$ . In particular, markups are not a function of the industry aggregate of productivity,  $Z_t$ , the measure of small firms,  $N_t$ , or of time  $t$ . Firms' innovation decisions are given by dynamic optimization of expected discounted profits and are functions only of the industry state,  $\tilde{Z}_{L,t}$ , and the aggregate state when converging to a balanced growth path. Innovation decisions do not depend on  $Z_t$ . Moreover, each firm creatively destroys all goods of each type  $f$  at the same rate. Potential entrants' decisions are given by dynamic optimization of expected discounted profits net of the marginal entry cost and are functions only of the distribution of industry states across industries, and the aggregate measure of small firms.

To be clear, firms can always observe all features of the economy when optimizing, but they suppose that other firms' actions depend only on the variables mentioned above. I show that it is then

optimal for each firm also to condition their own actions only on the variables mentioned above.

I focus on balanced growth path equilibria and the convergence to a balanced growth path following unanticipated shocks. A balanced growth path is an equilibrium in which  $\tilde{Z}_{L,t}$  is constant over time in each industry, the measure of small firms  $N_t$  is constant over time,  $Z_t$  grows at a constant rate, and each firm's innovation decisions are functions only of  $\tilde{Z}_{L,t}$  in their industry.

### 3.6 Firm Problem

Before describing the firm problem, note that since small firms take industry aggregates as given, we can split their static profit maximization problem into a separate problem for each good they produce. Moreover, when innovating, a small firm's problem is the same regardless of the goods it produces.

#### 3.6.1 Static Profit Maximization: Prices

At each moment in time, firms choose prices simultaneously to maximize static profits. A small firm that is the most productive producer of a good  $j$  with relative productivity  $\tilde{z}_t(j) = \bar{z}_t(j)/Z_t$  takes as given the industry price index, the wage, and industry revenue, and chooses a markup to maximize static profits  $\pi_{S,t}(\tilde{z}_t(j); \tilde{Z}_{L,t})$  subject to the demand curve (3), the production function (5), and competition from the second-best producer:  $\mu_t(j) \leq \kappa$ . A large firm takes as given small firms' prices, the wage, and industry revenue, and chooses markups for its goods to maximize static profits  $\pi_{L,t}(\tilde{Z}_{L,t})$  subject to the demand curve (3), the production function (4), competition from the second-best producers of each of its goods, and aggregation (2), which determines the industry price index as a function of goods prices.

#### 3.6.2 Dynamic Profit Maximization: Innovation

At each moment in time, firms simultaneously choose innovation rates: a creative destruction rate for each good, and a new good development rate. In the dynamic problem, a firm takes as given its profit function from static optimization.

All small firms choose the same innovation rates. Moreover, we will see that each small firm creatively destroys all goods at the same rate. Thus, let  $x_{C,S,t}$  denote this rate of creative destruction,



and let  $x_{D,S,t}$  denote the rate at which small firms develop new goods. For large firms, let  $x_{C,L,t}(f)$  denote the rate at which a large firm creatively destroys a good that is currently produced by a type  $f \in \{S, L\}$  firm, and let  $x_{D,L,t}$  denote the rate at which a large firm develops new goods.

**Small Firms:** For a small firm to choose their optimal innovation rate, they must know the expected present discounted value of being the most productive producer of a good. They take as given the static profit function at each moment in time, the aggregate state, and the innovation rates of other firms, which imply the evolution of the industry state and the growth rate of industry productivity. The expected present discounted value of producing good  $j$  is given by the HJB equation:

$$\begin{aligned} r_t \bar{\pi}_{S,t}(\tilde{z}_t(j); \tilde{Z}_{L,t}) = & \pi_{S,t}(\tilde{z}_t(j); \tilde{Z}_{L,t}) - (x_{C,S,t} + x_{C,L,t}(S)) \bar{\pi}_{S,t}(\tilde{z}_t(j); \tilde{Z}_{L,t}) - g_t \tilde{z}_t(j) \frac{\partial \bar{\pi}_{S,t}(\tilde{z}_t(j); \tilde{Z}_{L,t})}{\partial \tilde{z}_t(j)} \\ & + \dot{\tilde{Z}}_{L,t} \frac{\partial \bar{\pi}_{S,t}(\tilde{z}_t(j); \tilde{Z}_{L,t})}{\partial \tilde{Z}_{L,t}} + \frac{\partial \bar{\pi}_{S,t}(\tilde{z}_t(j); \tilde{Z}_{L,t})}{\partial t}, \end{aligned}$$

where  $g_t$  is the growth rate of industry productivity, i.e.,  $\dot{Z}_t/Z_t$ , and a dot over a variable indicates its derivative with respect to time. The first term on the right-hand side of the first line is flow profits, the second term reflects the rate at which the good is creatively destroyed, and the third term reflects the rate at which the firm's relative productivity is depreciated by growth in industry productivity either due to creative destruction or new good development. The second line reflects changes in the expected present discounted value of profits over time due to changes in the industry state, or changes in the aggregate state when the economy is converging to a balanced growth path.

A small firm chooses innovation rates to maximize the expected present discounted value of profits using the household stochastic discount factor, i.e., the risk-free interest rate. As mentioned above, a small firm's innovation optimization problem is the same regardless of the goods they produce. The value function of a small firm with zero goods is given by the HJB equation:

$$\begin{aligned} r_t V_{S,t}(\tilde{Z}_{L,t}) = & \max_{x_{D,t}} \left\{ x_{D,t} \mathbf{E}[\bar{\pi}_{S,t}(\lambda \tilde{z}; \tilde{Z}_{L,t})] - W_t \chi_S x_{D,t}^\alpha \right\} \\ & + \max_{\{x_{C,t}(j)\}} \left\{ \int_0^{M_t} x_{C,t}(j) \bar{\pi}_{S,t}(\lambda \tilde{z}_t(j); \tilde{Z}_{L,t}) dj - W_t \chi_S \chi_C \int_0^{M_t} \tilde{z}_t(j)^{\gamma-1} x_{C,t}(j)^\alpha dj \right\} \\ & - \eta V_{S,t}(\tilde{Z}_{L,t}) + \dot{\tilde{Z}}_{L,t} \frac{\partial V_{S,t}(\tilde{Z}_{L,t})}{\partial \tilde{Z}_{L,t}} + \frac{\partial V_{S,t}(\tilde{Z}_{L,t})}{\partial t}, \end{aligned}$$

where the first line is the optimization problem for the rate at which the small firm develops a new good, and the expected value is over realizations of  $\tilde{z}$ ; the second line is the optimization problem for the rate at which the firm creatively destroys each good; and the third line reflects the firm's exit rate as well as changes in the value function over time due to changes in the industry state, or changes in the aggregate state when the economy is converging to a balanced growth path.

**Large Firm:** The large firm takes as given the aggregate state, the current industry state, as well as small firm behavior as a function of the industry state, and chooses innovation rates to maximize the expected present discounted value of profits, including innovation costs, discounting with the risk-free interest rate. The value function is given by the HJB equation:

$$r_t V_{L,t}(\tilde{Z}_{L,t}) = \pi_{L,t}(\tilde{Z}_{L,t}) + \max_{\{x_{C,L,t}(j)\}, x_{D,L,t}} \left\{ \dot{\tilde{Z}}_{L,t}(\{x_{C,L,t}(j)\}, x_{D,L,t}; \tilde{Z}_{L,t}) \frac{\partial V_{L,t}(\tilde{Z}_{L,t})}{\partial \tilde{Z}_{L,t}} - W_t \chi_L \chi_C \int_0^{M_t} \tilde{z}_t(j)^{\gamma-1} x_{C,L,t}(j)^\alpha dj - W_t \chi_L x_{D,L,t}^\alpha \right\} + \frac{\partial V_{L,t}(\tilde{Z}_{L,t})}{\partial t}.$$

The first term on the right-hand side of the first line is the large firm's flow profits; the second term and the second line are the optimization problem of the large firm choosing innovation rates: the term on the first line is the benefit through changes in  $\tilde{Z}_{L,t}$ , and the second line is the flow cost of innovation. The final line reflects changes in the value function over time due to changes in the aggregate state when the economy is converging to a balanced growth path.

### 3.7 Aggregation and Welfare

Given Assumption 3.1, which implies that all firms set a markup  $\kappa$  on all goods, it follows that the industry price index is  $P_t = \kappa W_t Z_t^{-1}$ , and consumption of the industry good is  $C_t = Z_t L_t^p$ , where

$$L_t^p \equiv \int_0^{M_t} l_t(j) dj$$

is labor used in production, with  $l_t(j)$  the labor used in production of good  $j$ .

Consider an economy in which all industries are identical. Since the final good is the numeraire, the final good price is always 1, and the wage is therefore  $W_t = Z_t/\kappa$ . The interest rate is the sum of the time discount rate and the growth rate of final good consumption:  $r_t = \rho + g_t$ . We can write the representative household's welfare as

$$\int_0^\infty e^{-\rho t} (\ln(Z_t) + \ln(L_t^p)) dt.$$

Along a balanced growth path with growth rate  $g$  and labor used in production  $L^p$ , the household's welfare at time  $t$  is

$$\frac{g}{\rho^2} + \frac{\ln(L^p)}{\rho} + \frac{\ln(Z_t)}{\rho}.$$

## 4 Firm Optimization

### 4.1 Static Optimization: Prices

For brevity, I omit the static optimization problem, and only note that all firms would set a markup weakly greater than  $\gamma/(\gamma - 1)$  if unconstrained by the second-most productive producer.<sup>11</sup> Thus, by Assumption 3.1, all firms set a markup of  $\kappa$  on all goods. The static profit function for a small firm producing a good with relative productivity  $\tilde{z}$  is thus

$$\pi_{S,t}(\tilde{z}) = \tilde{z}^{\gamma-1} \frac{\kappa - 1}{\kappa} R_t,$$

and for a large firm with relative productivity  $\tilde{Z}_{L,t}$  is

$$\pi_{L,t}(\tilde{Z}_{L,t}) = \tilde{Z}_{L,t}^{\gamma-1} \frac{\kappa - 1}{\kappa} R_t.$$

The industry revenue share of a large firm is  $\tilde{Z}_{L,t}^{\gamma-1}$ .

### 4.2 Evolution of the Industry State and Growth

To solve the dynamic firm problem, we must first understand how the evolution of the industry state and growth of industry productivity depend on the innovation decisions of firms.

The industry state  $\tilde{Z}_{L,t}$  evolves according to

$$\begin{aligned} \frac{\partial \tilde{Z}_{L,t}^{\gamma-1}}{\partial t} = & \lambda^{\gamma-1} \left( x_{D,L,t} + \left( 1 - \tilde{Z}_{L,t}^{\gamma-1} \right) x_{C,L,t}(S) + \tilde{Z}_{L,t}^{\gamma-1} x_{C,L,t}(L) \right) \\ & - \tilde{Z}_{L,t}^{\gamma-1} (N_t x_{C,S,t} + x_{C,L,t}(L)) - (\gamma - 1) \tilde{Z}_{L,t}^{\gamma-1} g_t, \end{aligned} \quad (6)$$

where  $g_t$  is the growth rate of industry productivity,  $Z_t$ :

$$\begin{aligned} (\gamma - 1)g_t \equiv \frac{\partial Z_t^{\gamma-1}/\partial t}{Z_t^{\gamma-1}} = & (\lambda^{\gamma-1} - 1) \left( N_t x_{C,S,t} + \left( 1 - \tilde{Z}_{L,t}^{\gamma-1} \right) x_{C,L,t}(S) + \tilde{Z}_{L,t}^{\gamma-1} x_{C,L,t}(L) \right) \\ & + \lambda^{\gamma-1} (N_t x_{D,S,t} + x_{D,L,t}). \end{aligned}$$

In the evolution of  $\tilde{Z}_{L,t}$  over time, the first line is the inflow due to new good development, creative destruction of small firms' goods, and creative destruction of the large firm's own goods. The first term on the second line is the outflow due to creative destruction of the large firm's goods

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<sup>11</sup>See Edmond, Midrigan, and Xu (2021) for a derivation of the optimal markup with oligopoly, nested CES demand, and Bertrand competition.

by small firms and the large firm, and the last term is the outflow due to growth in  $Z_t$ , which reduces relative productivity. In the expression for growth,  $g_t$ , the first line is growth from creative destruction: the  $-1$  in  $\lambda^{\gamma-1} - 1$  reflects the destroyed productivity of the old good; and the second line is growth from new good development in which all the productivity of new goods is novel.

### 4.3 Dynamic Optimization: Innovation

#### 4.3.1 Small Firms

We can write the expected present discounted value of small firm profits from producing a good with relative productivity  $\tilde{z}_t$  as

$$\bar{\pi}_{S,t}(\tilde{z}_t; \tilde{Z}_{L,t}) = \tilde{z}_t^{\gamma-1} \bar{\pi}_{S,t}(\tilde{Z}_{L,t}),$$

where the new definition of  $\bar{\pi}$  is the old definition with  $\tilde{z}_t = 1$ . The First Order Condition for the small firm new good development optimization problem then yields

$$x_{D,S,t}(\tilde{Z}_{L,t}) = \left( \frac{\lambda^{\gamma-1} \bar{\pi}_{S,t}(\tilde{Z}_{L,t})}{\alpha \chi_S} \right)^{\frac{1}{\alpha-1}}. \quad (7)$$

The First Order Condition for the small firm creative destruction optimization problem yields

$$x_{C,S,t}(\tilde{Z}_{L,t}) = \left( \frac{\lambda^{\gamma-1} \bar{\pi}_{S,t}(\tilde{Z}_{L,t})}{\alpha \chi_S \chi_C} \right)^{\frac{1}{\alpha-1}}, \quad (8)$$

where  $x_{C,S,t}(\tilde{Z}_{L,t})$  is the common rate at which a small firm creatively destroys all goods, and so  $M_t x_{C,S,t}(\tilde{Z}_{L,t})$  is the Poisson arrival rate at which a small firm creatively destroys a single good. A small firm creatively destroys all goods at the same rate since the cost and benefit each scale with the relative productivity of the good, and since a small firm does not internalize the different effects that creatively destroying different types of goods will have on the industry.

#### 4.3.2 Large Firm

Using the evolution of the industry state as a function of the large firm's innovation decisions, the First Order Condition for the large firm's new good development rate yields

$$x_{D,L,t}(\tilde{Z}_{L,t}) = \left( \frac{\lambda^{\gamma-1} (1 - \tilde{Z}_{L,t}^{\gamma-1})}{\chi_L \alpha} \frac{\partial V_{L,t}(\tilde{Z}_{L,t})}{\partial \tilde{Z}_{L,t}^{\gamma-1}} \right)^{\frac{1}{\alpha-1}}. \quad (9)$$

The term  $1 - \tilde{Z}_{L,t}^{\gamma-1}$  reflects the two ways in which new good development affects the industry state: it increases the productivity of goods produced by the large firm, but it also increases the total productivity in the industry,  $Z_t$ , which depreciates relative productivities. The significance of the second effect scales with the large firm's relative productivity share,  $\tilde{Z}_{L,t}^{\gamma-1}$ .

The First Order Condition for the rate at which the large firm creatively destroys small firms' goods yields

$$x_{C,L,t}(S; \tilde{Z}_{L,t}) = \left( \frac{1 + (\lambda^{\gamma-1} - 1) (1 - \tilde{Z}_{L,t}^{\gamma-1})}{\chi_L \chi_C^\alpha} \frac{\partial V_{L,t}(\tilde{Z}_{L,t})}{\partial \tilde{Z}_{L,t}^{\gamma-1}} \right)^{\frac{1}{\alpha-1}}, \quad (10)$$

and for the rate at which the large firm creatively destroys its own goods yields

$$x_{C,L,t}(L; \tilde{Z}_{L,t}) = \left( \frac{(\lambda^{\gamma-1} - 1) (1 - \tilde{Z}_{L,t}^{\gamma-1})}{\chi_L \chi_C^\alpha} \frac{\partial V_{L,t}(\tilde{Z}_{L,t})}{\partial \tilde{Z}_{L,t}^{\gamma-1}} \right)^{\frac{1}{\alpha-1}}. \quad (11)$$

When the large firm creatively destroys a small firm's good, we can decompose the effect on the industry state into two components. First, the small firm's good is essentially transferred to the large firm, generating the 1 in the numerator. Second, the relative productivity of the good is increased, which has the same effect as when the large firm creatively destroys its own good. When the large firm creatively destroys its own good it is equivalent to developing a new good, except that the quantity of novel productivity is  $\lambda^{\gamma-1} - 1$  rather than  $\lambda^{\gamma-1}$ .

**New Good Development vs. Creative Destruction:** The key mechanism in the model is that, compared to small firms, the large firm has a relative preference for creatively destroying its competitors' goods over other types of innovation. Small firm creative destruction relative to new good development is

$$\frac{x_{C,S,t}(\tilde{Z}_{L,t})}{x_{D,S,t}(\tilde{Z}_{L,t})} = \chi_C^{\frac{-1}{\alpha-1}}, \quad (12)$$

which depends only on the cost of creative destruction relative to new good development. The rate at which the large firm creatively destroys its competitors' goods relative to the rate at which it develops new goods is

$$\frac{x_{C,L,t}(S; \tilde{Z}_{L,t})}{x_{D,L,t}(\tilde{Z}_{L,t})} = \chi_C^{\frac{-1}{\alpha-1}} \left( \frac{1 + (\lambda^{\gamma-1} - 1) (1 - \tilde{Z}_{L,t}^{\gamma-1})}{\lambda^{\gamma-1} (1 - \tilde{Z}_{L,t}^{\gamma-1})} \right)^{\frac{1}{\alpha-1}}, \quad (13)$$

which is strictly increasing in its relative productivity share,  $\tilde{Z}_{L,t}$ , and strictly greater than the relative rate for small firms as long as  $\tilde{Z}_{L,t} > 0$ . The rate at which the large firm creatively destroys its competitors' goods relative to the rate at which it creatively destroys its own goods is

$$\frac{x_{C,L,t}(S; \tilde{Z}_{L,t})}{x_{C,L,t}(L; \tilde{Z}_{L,t})} = \left( \frac{1 + (\lambda^{\gamma-1} - 1) (1 - \tilde{Z}_{L,t}^{\gamma-1})}{(\lambda^{\gamma-1} - 1) (1 - \tilde{Z}_{L,t}^{\gamma-1})} \right)^{\frac{1}{\alpha-1}}, \quad (14)$$

which is also strictly increasing in  $\tilde{Z}_{L,t}$ , and strictly greater than 1. Comparing equations (12) and (13), we can see that compared to small firms, the large firm has a relative preference for creatively destroying its competitors' goods over new good development. In both cases, a small firm does not internalize the effect on other firms' relative productivities. On the other hand, the large firm prefers creative destruction of its competitors' goods because then the relative productivity of its new good comes disproportionately from small firms' goods rather than from its own goods. Similarly, whereas small firms creatively destroy other small firms' goods and the large firm's goods at the same rate, equation (14) shows that the large firm has a relative preference for creatively destroying its competitors' goods over creatively destroying its own goods.

## 5 Results

### 5.1 Growth and Concentration Along a Balanced Growth Path

I focus first on the distribution of growth across industries and across economies along a balanced growth path, in which each industry is in a steady state equilibrium with a constant  $\tilde{Z}_L$  over time. These results also provide intuition for the quantitative exercises that follow. Without solving for a Markov Perfect Equilibrium of the dynamic game, we can compute the growth rate of an industry or the economy as a function of the market share of the large firm. I omit the time  $t$  subscript.

The following theorem shows that across industries in a single balanced growth path, growth as a function of the large firm's market share exhibits an inverted-U shape.

**Theorem 5.1.** *Suppose  $\alpha \geq 2$  and the economy is on a balanced growth path. Suppose the only heterogeneity across industry parameters is in the large firm's cost of innovation,  $\chi_L$ . The long-run growth rate in an industry is a function of the long-run market share of the large firm,  $g(\tilde{Z}_L)$ . There exists a threshold market share  $Z^*$  such that  $g(\tilde{Z}_L)$  is strictly increasing if  $\tilde{Z}_L < Z^*$  and strictly decreasing if  $\tilde{Z}_L > Z^*$ .*

To gain intuition for the theorem, note that along a balanced growth path, and in the steady state equilibrium of an industry, the expected present discounted value of profits from a good produced by a small firm with relative productivity 1 is

$$\bar{\pi}_S = \frac{(1 - \kappa^{-1})R}{r + Nx_{C,S} + x_{C,L}(S) + (\gamma - 1)g}, \quad (15)$$

where heterogeneity across industries, driven by heterogeneity in  $\chi_L$ , is in the equilibrium innovation rates,  $x_{C,S}$ ,  $x_{C,L}(S)$ , and  $g$ . The effect of the large firm's market share on growth operates

through the effective discount rate on small firm profits, the denominator on the right-hand side of equation (15).

As the large firm's market share increases, there are two effects analogous to the composition and total innovation effects from Section 2, the first of which pushes growth down and the second of which pushes growth up. The first effect is due to a shift in the composition of the effective discount rate on small firm profits. We can decompose the non-interest component of the effective discount rate as

$$Nx_{C,S} + x_{C,L}(S) + (\gamma - 1)g = Nx_{C,S} + (\gamma - 1)g_S + x_{C,L}(S) + (\gamma - 1)g_L, \quad (16)$$

where  $g_S$  and  $g_L$  are growth from small and large firm innovation, respectively. In the steady state equilibrium of an industry, the terms in the decomposition are related by

$$\tilde{Z}_L^{\gamma-1}(Nx_{C,S} + (\gamma - 1)g_S) = \left(1 - \tilde{Z}_L^{\gamma-1}\right)(x_{C,L}(S) + (\gamma - 1)g_L), \quad (17)$$

where the left-hand side is the rate at which relative productivity flows from the large firm to small firms, and the right-hand side is the rate at which relative productivity flows from small firms to the large firm. Holding fixed the effective discount rate on small firm profits, equation (17) shows that as the large firm's market share increases, innovation shifts away from small firms (the first two terms on the right-hand side of equation (16)) and toward the large firm (the last two terms). Recalling the relative innovation rates of small and large firms, equations (12), (13), and (14), the large firm's ratio of growth to creative destruction of small firms' goods,  $g_L/x_{C,L}(S)$ , is lower than the ratio for small firms,  $g_S/x_{C,S}$ . Thus, the effective discount rate on small firm profits shifts away from growth and toward creative destruction.

The second effect of the large firm's market share on growth is due to an increase in the effective discount rate on small firm profits. The First Order Conditions for small firm innovation imply that small firm innovation is

$$Nx_{C,S} + (\gamma - 1)g_S = \lambda^{\gamma-1}N \left( \frac{\lambda^{\gamma-1}\bar{\pi}_S}{\alpha\chi_S} \right)^{\frac{1}{\alpha-1}} \left( 1 + \chi_C^{\frac{-1}{\alpha-1}} \right). \quad (18)$$

As the large firm's market share increases and innovation shifts away from small firms, the left-hand side of equation (18) falls. For the right-hand side to fall as well,  $\bar{\pi}_S$  must fall, so the effective discount rate on small firm profits must rise. Thus, holding fixed the composition of the effective discount rate, growth increases.

When the large firm's market share is sufficiently low, the composition effect is small and the second effect dominates; the large firm's innovation is not so tilted toward creative destruction of its competitors, and the composition of its innovation does not change much in its market share.

When the large firm's market share is sufficiently high, the opposite holds and the composition effect dominates.

From this intuition, we also have the following Theorem that compares balanced growth paths across economies.

**Theorem 5.2.** *Suppose  $\alpha \geq 2$  and a free entry condition holds, i.e.,  $\epsilon = 1$ . Suppose there is no heterogeneity across industries. Index the balanced growth path of the economy by the large firm's cost of innovation,  $\chi_L$ , and let labor supply,  $\bar{L}$ , adjust so that final good output,  $C_t$ , relative to productivity,  $Z_t$ , is constant across economies. The long-run growth rate is a strictly decreasing function of the long-run market share of the large firm,  $g(\tilde{Z}_L)$ .*

The free entry condition at the aggregate level, along with holding fixed output relative to productivity, implies that the non-interest component of the effective discount rate on small firm profits is constant across economies indexed by the large firm's cost of innovation. All that remains is the composition effect, which implies that growth falls as the large firm's market share increases. In the quantitative exercise in Section 5.3, I allow output relative to productivity to adjust, but the effect of changes in output relative to productivity on long-run growth is small relative to the composition effect of the rise in the large firm's market share.

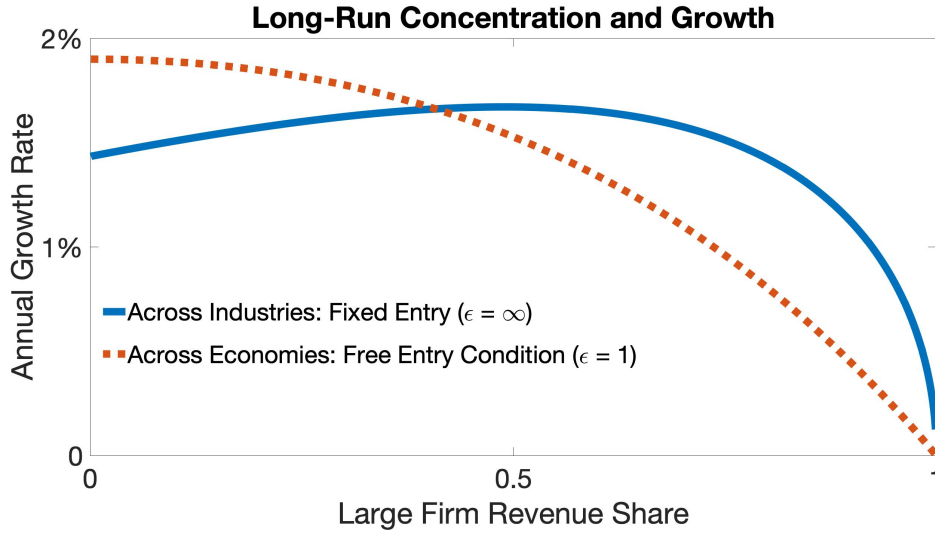


Figure 1: The lines depict growth on a balanced growth path at various levels of the large firm's market share. The solid blue line shows the growth rate across industries on a single balanced growth path, and the dotted orange line shows the growth rate across balanced growth paths in different economies, each with constant large firm market shares across industries. The figure is based on the calibration described in Section 5.2.



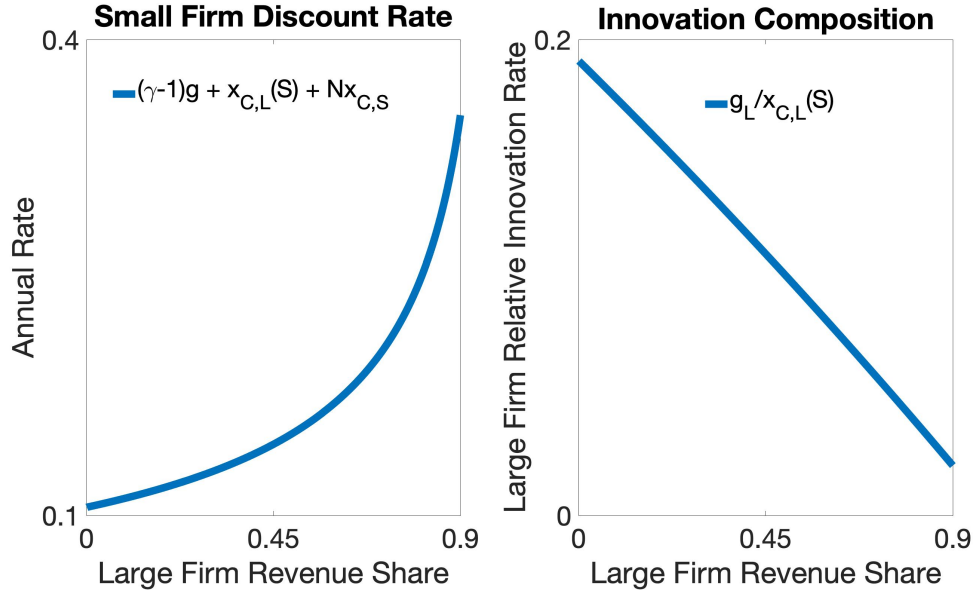


Figure 2: The left panel depicts the effective discount rate on small firm profits minus the interest rate in the steady state equilibrium of an industry as a function of the large firm's revenue share. The right panel depicts the rate at which a large firm generates growth relative to the rate at which it creatively destroys small firms' goods, as a function of its revenue share. The figure is based on the calibration described in Section 5.2.

Figure 1 shows growth as a function of the large firm's revenue share both across industries on a balanced growth path, and across balanced growth paths. Figure 2 shows the two channels described above through which the large firm's market share affects growth. In the left panel, we see that the small firm effective discount rate is higher in industries with bigger large firm revenue shares. In the right panel, we see that the rate at which a large firm generates growth relative to the rate at which it creatively destroys its competitors' goods is decreasing in its revenue share. The analogous value for small firms is the value for the large firm when its revenue share is 0. Equations (16) and (18) show that the large firm's share of the non-interest component of the effective discount rate is  $\tilde{Z}_L^{\gamma-1}$ . Along with the right panel of Figure 2, we can thus see that the composition effect implies a decreasing relationship between the large firm's revenue share and growth.

## 5.2 Calibration

I calibrate the model and solve it computationally to yield more results. I first calibrate the model to an initial balanced growth path in which all industries are identical. I set some parameters externally, and internally calibrate the rest to jointly match a set of moments in the data. The

externally calibrated parameters as well as their sources are listed in Table 1. The internally calibrated parameters are listed in Table 2. The data moments used to calibrate the internally calibrated parameters as well as their sources are listed in Table 3. I set the minimum productivity gap,  $\kappa$ , equal to the innovation step size,  $\lambda$ , which is the largest possible value given Assumption 3.1. I normalize the final good price to 1 in all periods, and set the household’s labor supply,  $\bar{L}$ , so that output in the initial balanced growth path relative to productivity,  $C_t/Z_t = R_t/Z_t$ , is 1. The units of time are years.

Table 1: Externally Calibrated Parameters

| Parameter  | Description                | Value |
|------------|----------------------------|-------|
| $\eta$     | Exit Rate                  | 0.04  |
| $\gamma$   | Demand Elasticity          | 3.1   |
| $\alpha$   | Innovation Cost Elasticity | 2     |
| $\epsilon$ | Entry Cost Elasticity      | 1     |

The exit rate is from Boar and Midrigan (2022). The demand elasticity is from Broda and Weinstein (2006), using their median estimate from 1990-2001 at the most disaggregated level. The innovation cost elasticity is from Acemoglu, Akcigit, Alp, Bloom, and Kerr (2018). The entry cost elasticity is chosen so that there is a free entry condition at the aggregate level.

The innovation cost elasticity,  $\alpha$ , which I calibrate externally to 2, is particularly important because it determines how a large firm’s innovation composition responds to its market share. I assume that creative destruction and new good development costs are independent, and that each innovation rate responds to the expenditures on that type of innovation as in the studies described in Acemoglu, Akcigit, Alp, Bloom, and Kerr (2018).

Table 2: Internally Calibrated Parameters

| Parameter | Description                        | Value  |
|-----------|------------------------------------|--------|
| $\lambda$ | Innovation Step Size               | 1.05   |
| $\chi_C$  | Relative Creative Destruction Cost | 0.265  |
| $\chi_S$  | Small Firm Innovation Cost         | 2.22   |
| $\chi_L$  | Large Firm Innovation Cost         | 15.14  |
| $\rho$    | Time Discount Rate                 | 0.0194 |

The innovation step size,  $\lambda$ , which I calibrate internally to 1.05, is important because it determines

Table 3: Calibration Targets

| Moment Description                | Data                   | Model  |
|-----------------------------------|------------------------|--------|
|                                   | Average from 1983-1993 |        |
| R&D Relative to GDP               | 1.81%                  | 1.81%  |
| Creative Destruction Growth Share | 26.51%                 | 26.53% |
| TFP Growth Rate                   | 1.66%                  | 1.66%  |
| Large Firm Market Share           | 40.68%                 | 40.74% |
| Real Interest Rate                | 3.6%                   | 3.6%   |

The ratio of R&D expenditures on GDP is the Business Enterprise Expenditure on R&D (BERD) relative to GDP from the OECD MSTI database. The creative destruction growth share is the fraction of growth from creative destruction from Garcia-Macia, Hsieh, and Klenow (2019). I compute this value in the model excluding innovation when large firms creatively destroy their own goods because this will appear as innovating on their own goods in the data. The TFP growth rate is from the BLS measure in Garcia-Macia, Hsieh, and Klenow (2019). The large firm market share is the sales-weighted average across 4-digit industries of the largest firm's revenue share in Compustat from Olmstead-Rumsey (2022). The real interest rate is the 1-year real interest rate from FRED.

the fraction of a creative destruction innovation that improves on the replaced good and is novel,  $(\lambda^{\gamma-1} - 1)/\lambda^{\gamma-1} = 0.1$ . As  $\lambda$  increases and the fraction that is novel goes to 1, the difference between creative destruction and new good development disappears. Moreover, if a larger fraction of creative destruction innovations are novel and generate growth, calibrating the model to match the fraction of growth due to creative destruction requires a lower rate of creative destruction relative to new good development. The calibrated value is consistent with more direct evidence in Garcia-Macia, Hsieh, and Klenow (2019) using data on labor flows, in which the average innovation step size from creative destruction in 1983-1993 is 1.07.

I calibrate the market share of large firms, as well as the shock in Section 5.3, to the average market share of the largest firm in 4-digit industries in Compustat. This measure likely overstates the size of the largest firm since Compustat does not include all firms. An alternative measure is the Census data on industry concentration measures, which show a smaller level of industry concentration, but a similar rise over the same time period. One downside of the Census data is that it only lists the market share of the top 4 firms in each industry, not the top firm. Moreover, while the Census data is in a sense more accurate because it includes more firms, it may include too many small firms that are not relevant to the mechanism in the model. In the model, if there are many small firms that do not innovate but simply imitate the innovations of others, then the

effect may just be to lower the price index by a fixed factor, without any further impact on the decisions of the innovative firms.

### 5.3 The Rise in Concentration and the Fall in Growth

I show that a rise in concentration driven by a fall in the cost of innovation for large firms can explain a portion of the changes in US data since the mid-1990s. In line with the interpretation of the large firm's innovation cost discussed in Section 3.3.3, we can interpret the fall in the cost of innovation as an increase in the concentration of innovative capacity within large firms. The economy begins in the balanced growth path from the calibration in Section 5.2. There is an unanticipated permanent change in  $\chi_L$  in all industries so that the revenue share of the large firm in each industry in the new balanced growth path is 0.51, the sales-weighted average across 4-digit industries of the largest firm's revenue share in 2018 in Compustat from Olmstead-Rumsey (2022) (the large firm innovation cost falls to  $\chi_L = 12.04$ ). I track the transition path of the economy as it converges to a new balanced growth path. Figure 3 shows the revenue share of the large firm in each industry along the transition path. The revenue share converges over a similar time interval as the gap between the years in the initial calibration, 1983-1993, and the target year for the shock, 2018.

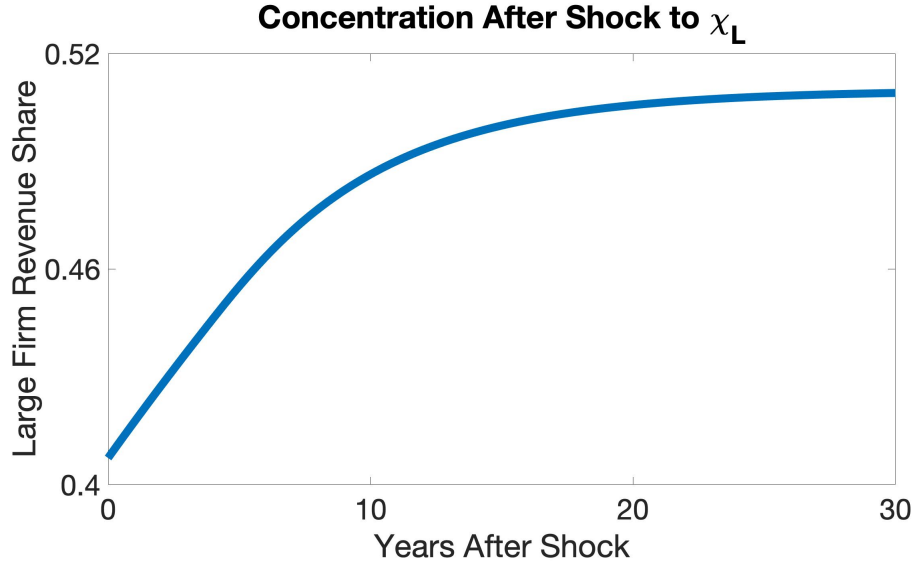


Figure 3: The line depicts the revenue share of the large firm in each industry in the economy over time following a shock to  $\chi_L$ .

Table 4 compares the main results concerning growth in the model to the data. The model can explain all of the increase in the short-run growth rate in the data if we include growth in output due to changes in output relative to productivity,  $C_t/Z_t$ , as well as changes in productivity. However,

the burst in growth does not last as long in the model as in the data: the peak difference in output along the transition path from the original balanced growth path occurs after 4 years and is 42% of the difference in the data after 4 years. The model can explain 41% of the long-run fall in growth, which in the model is due entirely to a change in the growth rate of productivity,  $Z_t$ , since output relative to productivity is constant along a balanced growth path.

Table 4: Growth After Shock to  $\chi_L$

| Moment Description | Data                           | Model   |
|--------------------|--------------------------------|---|
| Growth Rate Burst  | +0.64 ppt (38.6%) (1993-2003)  | Output: +0.87 ppt (52.4%) (first year)<br>TFP: +0.1 ppt (6.0%) (first year) |
| Cumulative Burst   | +6.4 ppt (38.6%) (1993-2003)   | Output: +1.07 ppt (16.1%) (4 years)<br>TFP: +0.18 ppt (2.7%) (3 years)      |
| Growth Rate Fall   | -0.34 ppt (-20.5%) (2003-2013) | -0.14 ppt (-8.4%) (New BGP)   |

For each value, ppt is the percentage point rise, and the number in parentheses is the percent rise relative to the initial value. The data are taken from Garcia-Macia, Hsieh, and Klenow (2019). The growth rate burst in the model is the peak growth rate in the short-run following the shock. The output growth rate reflects changes in output relative to productivity,  $C_t/Z_t$ , as well as changes in  $Z_t$ . The cumulative burst is the sum of growth rates, i.e., the peak difference between the new output or productivity path and the old path.

Figure 4 shows annual growth, in real output and in productivity, following the shock. As seen in Table 4, there is a burst in growth immediately following the shock, particularly in output growth but in productivity growth as well, ultimately followed by a decline in the long-run growth rate. Figures 5 and 6 provide a deeper look at the underlying forces. Based on the discussion in Section 5.1 on the small firm effective discount rate, the dashed blue line in Figure 5 shows that the composition effect drives down growth as the large firm's market share grows and innovation shifts toward creative destruction of small firms' goods. The dotted orange line shows that in the first few years after the shock, the effective discount rate on small firm profits rises, outweighing the composition effect, and leading to a higher growth rate. However, in the long-run, the effective discount rate is only slightly higher due a small increase in real output relative to productivity, and the composition effect dominates. Figure 6 shows that the large increase in the short-run in the effective discount rate on small firm profits is possible because the entry rate hits its lower bound of 0; the expected discounted profits of entering become negative, but the measure of small firms can only fall over time as firms exogenously exit. Thus, growth increases because large firms innovate more, and total small firm innovation is slow to fall due to an overhang of small firms.

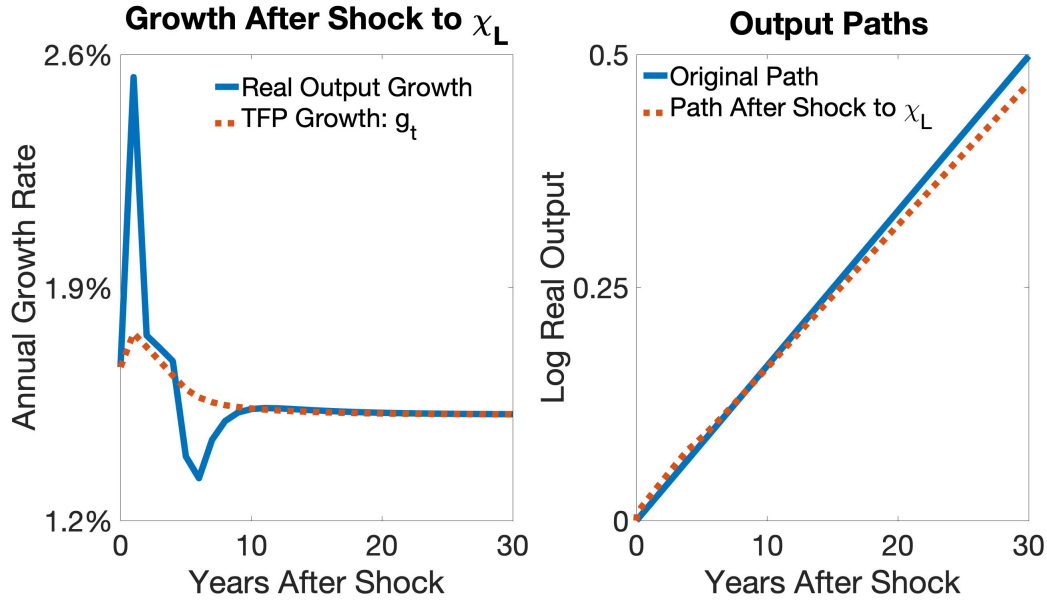


Figure 4: The left panel shows annual growth rates following the shock. The dotted orange line is the growth rate of productivity,  $g_t$ , and the solid blue line includes changes in output relative to productivity,  $C_t/Z_t$ , which is constant along a balanced growth path. The right panel shows paths of real output over time. The solid blue line is the original path the economy would have followed had it not been hit by a shock. The dotted orange line is the realized path following the shock.

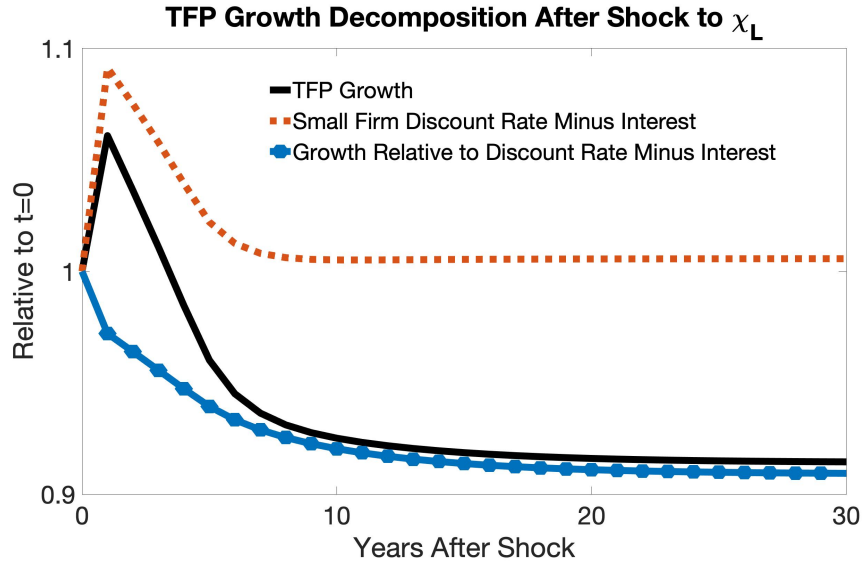


Figure 5: The solid black line depicts the annual productivity growth rate relative to in the original balanced growth path before the shock. The dotted orange line and the textured blue line decompose the black line into the non-interest component of the effective discount rate on small firm profits and growth over the non-interest component of the effective discount rate, respectively, relative to before the shock.

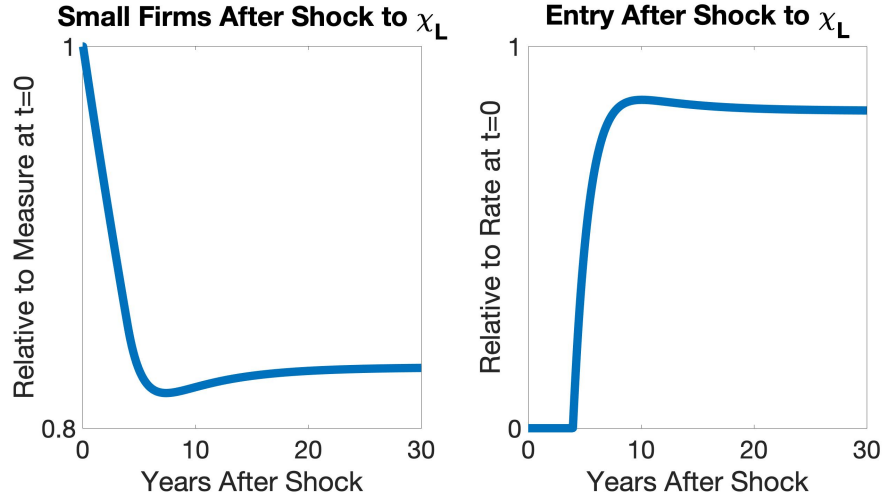


Figure 6: The left panel is the measure of small firms over time relative to the measure before the shock. The right panel is the entry rate over time relative to the entry rate before the shock.

The small increase in the long-run in real output relative to productivity is driven by the fall in long-run entry costs (as well as a fall in innovation costs), which implies an increase in labor used in production.

The large fall in entry in the short-run and the smaller fall in the long-run match the data in Decker, Haltiwanger, Jarmin, and Miranda (2016), which show that the entry rate declined sharply in the mid-to-late 1990s followed by a partial recovery before a large drop during the Great Recession.

### 5.3.1 Creative Destruction

Figure 7 shows that the share of growth due to creative destruction falls following the shock, as in the long-run in Garcia-Macia, Hsieh, and Klenow (2019), although by a smaller magnitude. More generally, we can see that a *smaller* share of large firms' growth is due to creative destruction than of small firms' growth. This is not at odds with Figure 2 or with the relative innovation rates in equations (12), (13), and (14) because a large firm only creatively destroys small firms' goods, whereas small firms creatively destroy all firms' goods. Although large firms focus their innovation particularly toward creative destruction of small firm goods, their innovation is less focused on creative destruction overall.

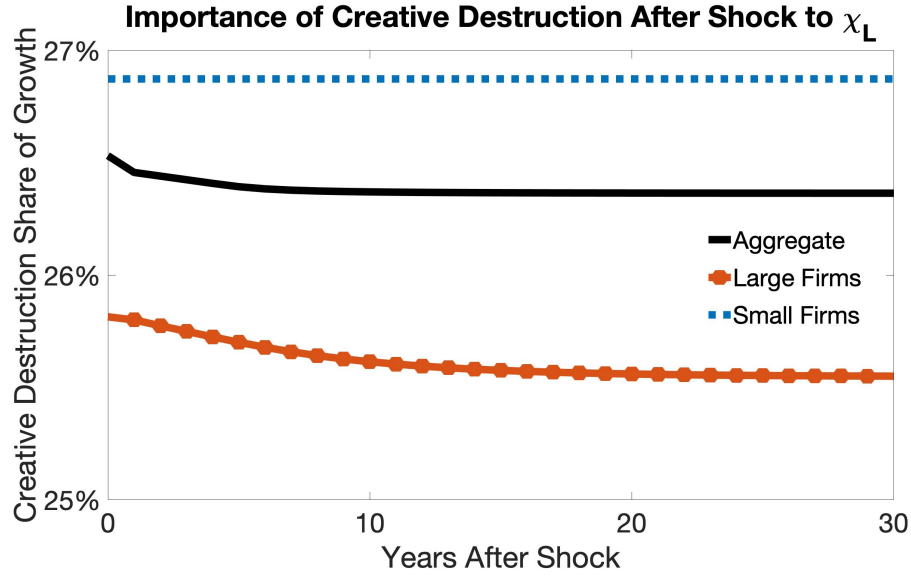


Figure 7: Each line depicts, over time following the shock, the rate at which a type of firm generates growth through creative destruction relative to the total rate at which that type of firm generates growth.

### 5.3.2 Welfare and Size-Dependent Taxes

Taking into account the transition path, welfare defined as the utility of the representative household falls by the equivalent of a permanent 5.8% drop in final good consumption. The decline in the long-run growth rate from the original balanced growth path to the new balanced growth path is ultimately the dominant effect. This suggests that on the margin, contrary to the result in Edmond, Midrigan, and Xu (2021), a tax on firms increasing in their size will improve growth and welfare. For a small tax, large firm prices are unaffected since they are already setting their markups at the constraint implied by the second-best producer. Large firms will respond to the tax by reducing innovation, leading to more small firms and growth and in the long-run. Even if large firms ultimately respond by investing less in their innovative capacity, the effect on growth and welfare is positive: large firms over-invest in innovative capacity because as their innovation cost,  $\chi_L$ , falls, their profits rise yet welfare falls.

## 6 Antitrust Policy: Acquisitions

I use the calibrated model to explore the effects of two different types of acquisition policies. First, for each good produced by small firms, at an exogenous Poisson arrival rate the large firm in the same industry can make a take-it-or-leave-it offer to purchase the good from the small firm. I consider the effects of a tax,  $\tau$ , on these transactions so that if the relative productivity



of the good is  $\tilde{z}$ , then the small firm receives payment  $\tilde{z}^{\gamma-1}\bar{\pi}_{S,t}\left(\tilde{Z}_{L,t}\right)$ , and the large firm pays  $(1+\tau)\tilde{z}^{\gamma-1}\bar{\pi}_{S,t}\left(\tilde{Z}_{L,t}\right)$ . If a large firm acquires goods at rate  $A_t$  with relative productivities  $\tilde{z}$  so that the average value of  $\tilde{z}^{\gamma-1}$  is  $\tilde{z}_{A,t}^{\gamma-1}$ , then the evolution of  $\tilde{Z}_{L,t}^{\gamma-1}$  is as before in equation (6) with the additional term  $A_t\tilde{z}_{A,t}^{\gamma-1}$ .

Second, for each small firm, at an exogenous Poisson arrival rate the large firm in the same industry can make a take-it-or-leave-it offer to purchase the small firm's innovation capacity. I again consider the effects of a tax,  $\tau$ , on these transactions so that the small firm receives payment  $V_{S,t}\left(\tilde{Z}_{L,t}\right)$ , and the large firm pays  $(1+\tau)V_{S,t}\left(\tilde{Z}_{L,t}\right)$ . If a large firm acquires small firms' innovative capacities at rate  $A_t$ , then the measure of small firms declines at rate  $A_t$  beyond the baseline effects of entry and exit. The large firm's innovation capacity changes at rate

$$\frac{\partial\chi_{L,t}^{\frac{1}{1-\alpha}}}{\partial t} = A_t\chi_S^{\frac{1}{1-\alpha}} - \eta\left(\chi_{L,t}^{\frac{1}{1-\alpha}} - \bar{\chi}_L^{\frac{1}{1-\alpha}}\right),$$

where  $\chi_{L,t}$  is the large firm's innovation cost at time  $t$ , and  $\bar{\chi}_L$  is the large firm's exogenously given innovation cost absent any acquisitions, so the second term on the right-hand side reflects the rate at which the large firm's acquisitions exit.

In each case, any taxes collected are dispersed to the representative household. If the tax is negative, then it is funded by a lump sum tax on the representative household.

Acquisitions of small firms' goods increase growth and welfare if they are sufficiently valuable to the large firm. Acquisitions of small firms' innovation capacities reduce growth and welfare.

## 6.1 Acquisitions of Small Firm Goods

The economy begins on a balanced growth path in the calibrated model either before the shock to  $\chi_L$  or after. Each good produced by a small firm can be acquired by the large firm in the same industry at rate 0.05. Put another way, each large firm can acquire 5% of the goods produced by small firms in its industry per year. The initial tax rate on acquisitions is  $\tau = 0.2$ , which is sufficiently high so that large firms do not purchase small firms' goods. Thus, the economy is indistinguishable from the case without any acquisitions. There is an unanticipated permanent change in the tax rate  $\tau$ . I track the transition path as the economy converges to a new balanced growth path.

Figure 8 shows the fraction of small firm goods that large firms acquire in the long-run as a function of the acquisition tax rate,  $\tau$ , both under the initial calibration before the shock to the large firm

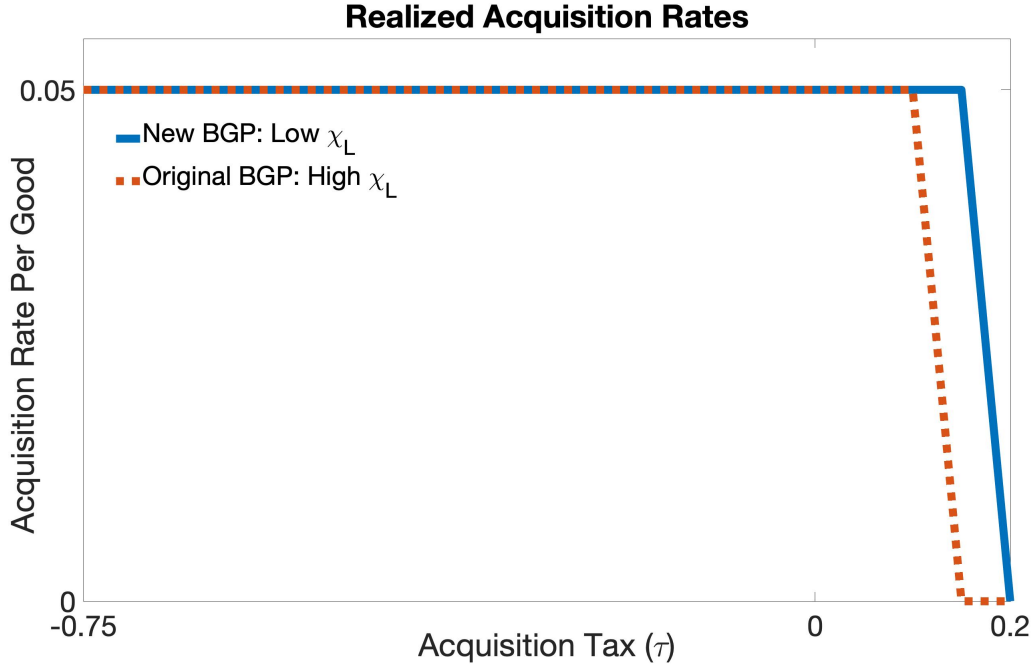


Figure 8: Each line depicts the fraction of small firm goods that large firms acquire per year in the balanced growth path as a function of the acquisition tax rate.

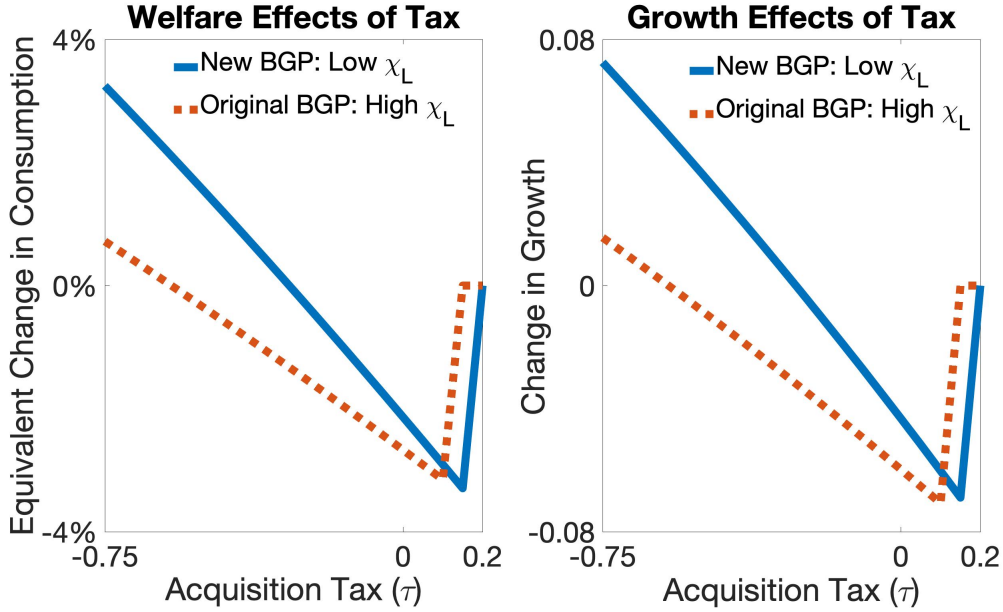


Figure 9: The left panel depicts, starting from either the original balanced growth path before the shock to  $\chi_L$  or the new balanced growth path after the shock, the welfare effects of an unanticipated permanent change in the acquisition tax rate, taking into account the transition path. Welfare is computed as the equivalent permanent percentage change in final good consumption. The right panel depicts the percentage point change in the long-run growth rate as a function of the acquisition tax rate.

innovation cost and the new calibration following the shock to the large firm innovation cost. Once the tax rate is sufficiently low, although still positive, large firms switch from never acquiring to always acquiring small firm goods when given the opportunity. Large firms acquire small firms' goods even when the tax rate is positive because they face a lower creative destruction rate.

Figure 9 shows that when the tax rate falls just enough so that the acquisition rate rises above 0, there is a large negative effect on growth and welfare. As the tax rate falls further and ultimately becomes a subsidy, the acquisition rate does not change, but growth and welfare rise, eventually exceeding their values in the balanced growth path without acquisitions.

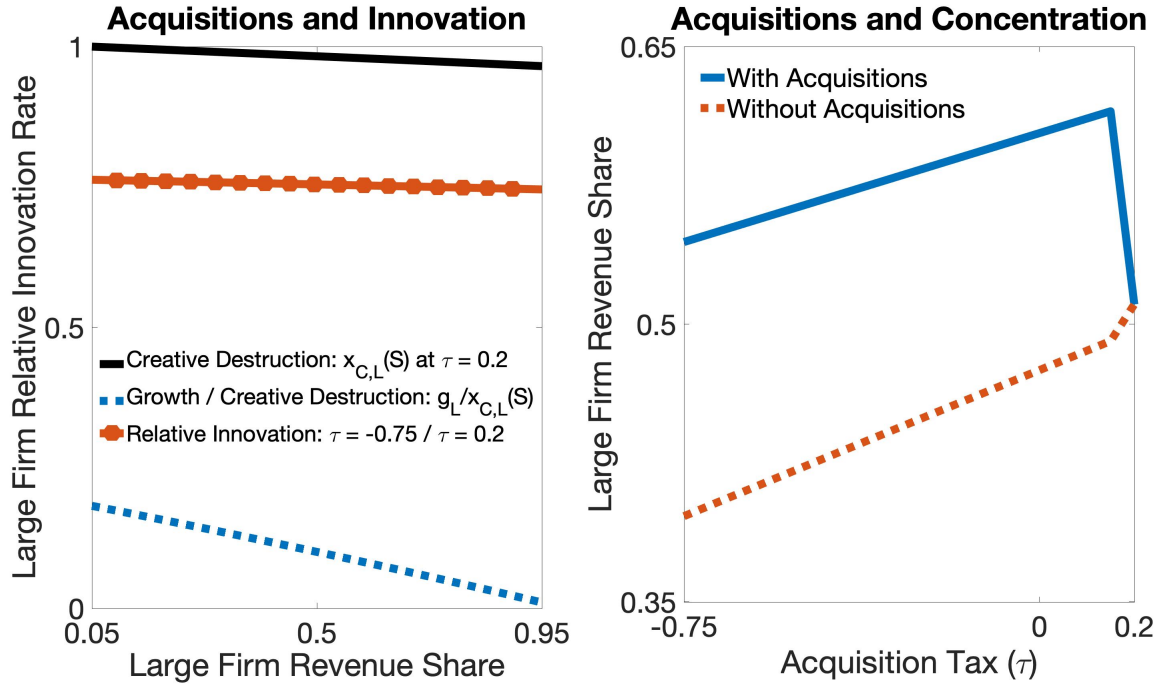


Figure 10: In the left panel, the solid black line is the rate at which the large firm creatively destroys small firms' goods as a function of its current revenue share relative to when its revenue share is 0.05. The dotted blue line is the rate at which the large firm generates growth relative to the rate at which it creatively destroys small firms' goods (the same line as in the right panel of Figure 2). The textured orange line is the rate at which the large firm performs each type of innovation when the acquisition tax is -0.75 relative to the same rate when the tax is 0.2. In the right panel, the solid blue and dotted orange lines depict each large firm's long-run revenue share as a function of the acquisition tax, with and without the effects of acquisitions on the distribution, respectively. The figure uses the calibration of the model following the shock to  $\chi_L$ .

To understand the growth and welfare results, Figure 10 shows how acquisitions and the acquisition tax rate affect large firm innovation. The main takeaway is that changes in the *rate of acquisitions* and changes in the *value of acquisitions* to the large firm have opposing effects. The rise in the acquisition rate increases large firm revenue shares, reducing their incentive to generate growth

relative to their incentive to creatively destroy small firm goods (the solid black and dotted blue lines in the left panel). On the other hand, the opportunity to acquire small firms' goods reduces the incentive to perform all types of innovation (the textured orange line in the left panel) because the more a large firm innovates, the less relative productivity remains for them to acquire. Since large firm innovation ultimately reduces growth and welfare, as we saw in Section 5.3, increasing the *value of acquisitions* to the large firm, conditional on the *rate of acquisitions*, improves growth and welfare.

The benefits of reducing the acquisition tax or subsidizing acquisitions are larger starting from the balanced growth path with more industry concentration. Small firms have less relative productivity for large firms to acquire, mitigating the effect of the rise in the acquisition rate, and, more significantly, large firms innovate more, increasing the effect of a proportional fall in large firm innovation.

## 6.2 Acquisitions of Small Firm Innovative Capacity

The economy begins on the balanced growth path in the calibrated model following the shock to  $\chi_L$  from Section 5.3. Each small firm's innovative capacity can be acquired by the large firm in the same industry at rate 0.01. The initial tax rate on acquisitions is sufficiently high so that large firms do not purchase small firms. Thus, the economy is indistinguishable from the case without any acquisitions. There is an unanticipated permanent decrease in the tax rate so that large firms acquire small firms whenever given the opportunity. I track the transition path as the economy converges to a new balanced growth path.

The effect is largely the same as a fall in the large firm innovation cost, detailed in Section 5.3, except that the increase in large firm innovative capacity comes at the expense of small firm innovative capacity rather than for free. Growth falls in the long-run by 0.19 percentage points and welfare, taking into account the transition path, falls by the equivalent of a permanent 6.18% drop in final good consumption. The particular choice of the acquisition tax rate does not matter as long as it is sufficiently low for large firms to acquire small firms because the value large firms receive from acquisitions does not affect their other decisions.

## 7 Extension: Large Firm Entry and Exit

Following the shock to the innovation cost of large firms,  $\chi_L$ , in Section 5.3, there is a rise in large firm profits. Since potential entrants face no possibility of becoming a large firm, these profits have no impact on entry. Thus, in this section, I alter the model so that large firms exit at exogenous rate  $\eta$ , the same rate as for small firms, and potential entrants face the possibility of entering as a large firm in an industry that did not otherwise have one. The results from the baseline model still hold.

Since large firms exit and then enter with zero relative productivity, even if exogenous parameters are the same in each industry, the distribution of industry states across the economy is no longer a single mass point. Instead, for each  $n \in [0, 1]$ ,  $\tilde{Z}_{L,t}(n)$  is the relative effective productivity of the large firm in industry  $n$ . When there is no large firm in industry  $n$  and when a new large firm enters industry  $n$ , then  $\tilde{Z}_{L,t}(n) = 0$ . To isolate the effects of this dispersion from the other effects of endogenizing large firm entry, I develop another version of the model in which large firms exit at exogenous rate  $\eta$ , and are immediately replaced by a new large firm with zero relative productivity. I call this model the *exogenous* large firm entry model, and call the model in which potential entrants may enter as small or large firms the *endogenous* large firm entry model. In both cases, when a large firm exits, they receive value 0.

Given an entry rate  $E_t$ , in the endogenous large firm entry model, large firms enter the economy at rate  $E_L E_t$ , where  $E_L$  is exogenously given. I hold fixed the innovative capacity per unit of entry,  $\chi_S^{\frac{1}{1-\alpha}}$ . Since a large firm has the innovative capacity of a measure  $(\chi_L/\chi_S)^{\frac{1}{1-\alpha}}$  of small firms, it follows that the entry rate of small firms into each industry is  $E_S E_t$ , where

$$E_S = 1 - \left( \frac{\chi_L}{\chi_S} \right)^{\frac{1}{1-\alpha}} E_L.$$

To maintain that there is at most one large firm per industry, I suppose that large firm entry is directed to industries without a large firm. If the measure of industries without a large firm is  $\Gamma_{0,t}$ , then the Poisson arrival rate of a large firm into such an industry is  $E_L E_t / \Gamma_{0,t}$ . Thus, if a potential entrant pays the entry cost, they receive expected value

$$E_L V_{L,t}(0) + E_S \int_0^1 V_{S,t}(\tilde{Z}_{L,t}(n)) dn.$$

I calibrate the endogenous and exogenous large firm entry models to the same initial balanced growth path as in Table 3. The changed parameter values are listed in Table 5. As before, each industry has the same parameters. I also calibrate  $E_L$  so that, in both the initial balanced growth

path and the new balanced growth following the shock, there is a large firm in almost every industry: 93.63% and 96.96% of industries have a large firm in the pre- and post-shock balanced growth paths, respectively.

Table 5: Re-Calibrated Parameters For Models with Large Firm Entry

| Parameter | Description                        | Endogenous Entry | Exogenous Entry |
|-----------|------------------------------------|------------------|-----------------|
| $\lambda$ | Innovation Step Size               | 1.059            | 1.057           |
| $\chi_C$  | Relative Creative Destruction Cost | 0.305            | 0.295           |
| $\chi_S$  | Small Firm Innovation Cost         | 2.9              | 3.0             |
| $\chi_L$  | Large Firm Innovation Cost         | 10.4             | 11.5            |
| $E_L$     | Per Unit Large Firm Entry Rate     | 2.12             | N/A             |

Table 6: Different Model Results After Shock to  $\chi_L$

| Moment        | Baseline           | Endogenous Large Entry | Exogenous Large Entry |
|---------------|--------------------|------------------------|-----------------------|
| Concentration | +10.14 ppt (24.9%) | +10.44 ppt (25.7%)     | +10.37 ppt (25.5%)    |
| Growth        | -0.14 ppt (-8.4%)  | -0.16 ppt (-9.6%)      | -0.18 ppt (-11.1%)    |
| Small Firms   | -16.5%             | -24.0%                 | -25.9%                |
| Welfare       | -5.8%              | -6.7%                  | -12.5%                |

Concentration is the sales-weighted average large firm industry revenue share across industries. Small firms is the measure of small firms. The changes in concentration, growth, and the measure of small firms are from the initial to the new balanced growth paths, and are computed both in percentage points (ppt) and as a percentage of the initial value. The change in welfare is from the initial balanced growth path to immediately after the shock (taking into account the transition path to the new balanced growth path), and is computed as the welfare equivalent permanent percent change in final good consumption.

As in Section 5.3, I conduct the following experiment. The economy begins on an initial balanced growth path. There is an unanticipated permanent change in  $\chi_L$  in all industries so that the average market share of large firms in the new balanced growth path is 0.51 ( $\chi_L$  falls to 8.74 in the endogenous entry case and 9.05 in the exogenous entry case). In the endogenous large firm entry case, the innovative capacity per unit of entry and the entry rate of large firms per unit of entry are held fixed, so the entry rate of small firms per unit of entry,  $E_S$ , falls with  $\chi_L$  from 0.41 to 0.30. I track the transition path of the economy as it converges to the new balanced growth path. Table 6 compares the main results in these models with the results from Section 5.3.

The results are similar in the models with and without large firm entry and exit. There are two main differences between the baseline model and the model with endogenous large firm entry that have opposite effects on growth and thus welfare. The first difference, which is the only difference between the baseline model and the exogenous large firm entry model, is that the models with large firm entry exhibit dispersion in large firm revenue shares across industries. Since the negative effects of large firms on growth and welfare are increasing and convex in large firms' revenue shares, this dispersion implies a greater cost of a rise in the average large firm revenue share. The second difference between the baseline model and the endogenous large firm entry model is that an entrant's innovation capacity is split between the possibility of being a small firm and of being a large firm. When  $\chi_L$  falls, the value of entering as a large firm per unit of innovative capacity,  $V_L(0)/\chi_L^{\frac{1}{1-\alpha}}$ , rises relative to the expected value of entering as a small firm per unit of innovative capacity,  $\int_0^1 V_S(\tilde{Z}_L(n)) dn/\chi_S^{\frac{1}{1-\alpha}}$ . Thus, although the value of entering as a small firm is held fixed across balanced growth paths by the free entry condition in the baseline model, it falls by 2% in the endogenous large firm entry model. An increase in the difference between the discount rates on large and small firms' innovations allows for a rise in the small firm discount rate, which implies a higher growth rate. Nonetheless, this effect is not large enough to overwhelm the conclusion that more large firm innovation reduces growth.

I interpret the particular way of modeling the large firm entry process as implying that the shock to the large firm innovation cost is an increase in the concentration of innovative capacity within larger firms. One alternative is to increase in the ease of generating new innovative capacity, i.e., decrease the large firm innovation cost while holding fixed the entry rate of large and small firms per unit of entry,  $E_L$  and  $E_S$ , respectively. This has the effects of the shock already studied in addition to a fall in the entry cost. In that case, the fall in the entry cost overwhelms the other effects, and growth and welfare rise. However, as the entry and exit rates of large firms relative to small firms become small, this result does not hold because the effects of a fall in the large firm innovation cost converge to the effects in the baseline model. Since the discount rate is strictly positive, the product of the vanishing probability and the increasing value of entering as a large firm goes to zero.

## 8 Conclusion

To understand the relationship between concentration and growth, and the policy implications, I study a model with one large firm and a continuum of small firms in each industry. Firms can innovate through creative destruction, developing new goods, and improving on their own goods.

Large firms, to avoid cannibalization, have a strong relative preference for creatively destroying their competitors' goods. As a result, when large firms innovate more, small firms' innovations are discounted heavily relative to the overall innovation and growth rate. A widespread fall in large firm innovation costs increases concentration and reduces small firm entry, long-run growth, and taking into account the transition path, welfare. Small firm innovation is less responsive to differences across industries than to widespread changes, so more concentrated industries only have lower growth if they have particularly big large firms that target their innovation strongly toward creative destruction of their competitors' goods. Thus, across industries, growth as a function of the large firm's revenue share exhibits an inverted-U shape.

Large firm acquisitions of their competitors' goods have direct and indirect effects with opposite implications for concentration, growth, and welfare. Acquisitions directly shift market share to large firms, strengthening their relative preference for creative destruction of their competitors' goods, and leading to a fall in growth. The indirect effect is that since acquisitions are valuable to large firms, each large firm innovates less so that more market share remains for it to acquire. As large firm innovation falls, it is replaced by small firm innovation, which is more geared toward growth rather than creative destruction of small firms' goods, ultimately facilitating more innovation and growth. If acquisitions are sufficiently valuable to large firms, then growth and welfare are higher in an economy with acquisitions than without. This positive effect is stronger when industries are more concentrated.

The theory and results highlight a novel way to think about the effects of market power and optimal competition policy. Large firms are harmful not because of their size, but because of how they achieve their size through innovation. Research and development subsidies that target large firms may backfire by discouraging innovation by smaller firms. Policies that increase concentration may be beneficial as long as they reduce large firm innovation. Facilitating acquisitions is a particularly useful policy because, unlike taxing large firms, it does not require knowledge of firms' relevant industries or their market shares in those industries.

Finally, although this paper focuses on growth, the theory has implications for other settings, and suggests potential avenues for future research. For example, suppose a firm can develop different types of goods, some of which are more novel to the industry, and others of which are close substitutes with the firm's competitors' goods. The same force that leads larger firms to set higher markups in static models of oligopolistic competition implies that larger firms have a stronger preference for producing the types of goods that are close substitutes with their competitors.



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