More Than the Sum of Its Parts? Markups and the Role of Establishments

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Abstract

We study how firms grow along two margins—their number of establishments and their sales at each establishment—and ask how variable markups distort each margin. Using Swedish data on the universe of firms, we find two novel facts. First, each successive establishment a firm opens is smaller than its previous establishments, conditional on establishment age. Second, there is an increasing relationship between a firm's sales per establishment and its markup, but no clear relationship between a firm's number of establishments and its markup. We develop a model of competition between potentially multi-establishment firms to rationalize our findings, and calibrate it to the Swedish economy. As in models with single-establishment firms, most losses from misallocation are because large firms produce too little and small firms too much. In particular, firms' choices of how many establishments to open are near efficient. Nonetheless, the effectiveness of firm size-dependent policy is limited by a novel trade-off between the benefit of large firms increasing production at their existing establishments and the cost of them opening new low quality establishments.

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1 Introduction

Firms expand by opening new establishments (the extensive margin) and increasing their sales per establishment (the intensive margin). Establishments can be retail stores, which give a firm access to a local market, or they can be manufacturing plants, which also allow a firm to build new products. How does firm growth differ along the extensive margin vs. the intensive margin? How do markups distort these margins? How should optimal policy take into account these distortions?

We begin our investigation with an empirical analysis using comprehensive Swedish firm and establishment-level data from 1997-2017. We document two new findings. First, each successive establishment a firm opens tends to be smaller than the firm's previous establishments, conditional on establishment age. That is, for any establishment age, a firm's second establishment is smaller than its first, its third establishment is smaller than its second, and so on, where size is measured using the wage bill or employment.[1] Second, there is a positive relationship between a firm's sales per establishment and its markup, but no clear relationship between a firm's number of establishments and its markup. Therefore, larger firms set higher markups—consistent with previous work[2]—but this is entirely because larger firms tend to sell more at each of their establishments.

Guided by these empirical findings, we develop a model of competition between potentially multi-establishment firms. In the model, endogenously variable markups across establishments generate three types of misallocation: across establishments within firms, across firms, and between production at existing establishments vs. building new establishments. We calibrate to the Swedish economy and find two main results. First, eliminating misallocation improves consumption by 7.5%, mostly by reallocating production across firms' existing establishments and mostly by reallocation across firms rather than within firms. In particular, firms' choices of how many establishments to open are near efficient. Hence, misallocation is mostly that small firms tend to have small establishments that produce too much and large firms tend to have large establishments that produce too little. Second, our establishment-level framework implies a novel trade-off for *firm size*-dependent policy that limits its effectiveness: subsidizing large firms' production is beneficial because they produce too little at their existing establishments, but doing so becomes costly as it pushes them to open lower

[1] These are the outcomes on which we have data at the establishment-level.

[2] For example, Edmond, Midrigan, and Xu (2023) document a positive firm size-markup relationship in US manufacturing data and Burstein, Carvalho, and Grassi (2024) do so in French data.

and lower quality establishments. As such, the optimal firm size-dependent policy eliminates only 50% of misallocation, whereas it eliminates 79% if we can hold fixed each firm's set of establishments. Moreover, if we design optimal firm size-dependent policy under the mistaken belief that each firm's set of establishments is fixed, then in equilibrium, misallocation only falls by 27%, about half as much as under the genuinely optimal policy.

Our model has the following features. An establishment is characterized by the quality of its differentiated variety. Firms vary in their baseline quality, which shifts the distribution of quality across their potential establishment opportunities. We can interpret an establishment as selling a unique variety or as selling the firm's variety in a unique location, in which case quality captures the local taste for the variety. To match our first empirical result, each successive establishment a firm opens is lower quality and therefore smaller than its previous establishments. To match our second empirical result, the demand system is such that each establishment faces a demand elasticity that declines with relative sales, so a higher quality establishment is larger and sets a higher markup. On the other hand, a firm's overall sales do not affect its demand elasticity. High quality firms open more establishments and their establishments tend to be higher quality, larger, and with higher markups.

We calibrate the model to match firm- and establishment-level moments in our Swedish data. In particular, we match the relationship between a firm's sales per establishment and its markup, the average wage bill of firms' later establishments relative to their earlier establishments, and the fact that multi-establishment firms are rare but earn the majority of sales. Eliminating all misallocation raises consumption by 7.5%. The largest gains by far come from reallocating production *across firms*: reallocating production across existing establishments within firms raises consumption by only 0.7%, also reallocating production across firms raises consumption by an additional 6.7%, and finally choosing each firm's number of establishments yields further consumption gains of only 0.1%. The efficient allocation shifts production toward larger establishments and firms, and closes 6.7% of establishments at multi-establishment firms. Intuitively, efficiently choosing each firm's number of establishments has little effect on consumption because given a total number of establishments, they are already efficiently allocated across firms in the competitive equilibrium. This follows from demand elasticities depending on establishment rather than firm sales, which implies that a firm's size does not distort its incentive to open establishments.

Finally, we compute the optimal firm size-dependent tax/subsidy scheme in the calibrated model, i.e., under the restriction that a firm's payment can only depend on its total sales across all its establishments. This restriction is relevant for understanding the effects of various implicit taxes/subsidies on firm size.[3] Moreover, it may be difficult for policymakers to condition on establishment-level outcomes. Optimal policy gives a relative subsidy to larger firms' production and increases consumption by 3.7%, half of the way to the first best. That firms respond by opening/closing establishments is central to the design and effectiveness of firm size-dependent policy. If we can hold fixed each firm's set of establishments, then the optimal firm size-dependent policy raises consumption by 5.9%, four-fifths of the way to the first best. But if we set this policy thinking firms' establishment choices are fixed when they are not, then consumption only rises by 2%.

Intuitively, the potency of firm size-dependent policy is limited by the following novel trade-off. Subsidizing large firm production is beneficial because they produce too little at their existing establishments. But large firms also respond to their subsidy by opening increasingly low quality establishments and producing too much at these establishments. Importantly, this trade-off is not inherent to the model but is the result of our particular calibration. In the competitive equilibrium, multi-establishment firms' marginal establishments are higher quality than any single-establishment firm's establishment. So in principle, it can be beneficial for large firms to open new establishments.

Expanding through establishments. This paper relates to a recent literature studying firm expansion through the opening of new establishments. Hsieh and Rossi-Hansberg (2023) show that the recent rise in industry concentration in the US services sector was due to the largest firms expanding by opening establishments in new locations. Moreover, these new locations tended to be in smaller cities, consistent with our theory/finding that firms start with their most promising establishment opportunities and then work their way down. Oberfield, et al. (2024) use a spatial model with heterogeneous locations to study where firms optimally open establishments. Becker, et al. (2024) use a spatial model with multi-establishment firms to study location-specific markups. We contribute to this literature by documenting our two novel facts in Swedish data. In terms of our model, we abstract from heterogeneity across locations. Instead, we focus on heterogeneity across a firm's establishments, and on the relative efficiency of a firm's intensive vs. extensive margin production decisions. Abstracting from location-specific heterogeneity allows us to keep the

[3]For example, large corporations may be better able to deal with complicated regulations and may be more visible to the government, which can convey either costs or benefits.

model tractable, and so to solve for optimal firm size-dependent policy.

Size-dependent markups and misallocation. This paper also relates to previous work studying misallocation implied by size-dependent markups. Edmond, Midrigan, and Xu (2023) do so in a setting without any distinction between firms and establishments. They find that compared to the competitive equilibrium, the first best reallocates production from small to large firms, which can be implemented with a relative subsidy for large firms' production. We build on their framework by studying how variable markups distort production along two margins: how many establishments to open and how much to produce at each. We find that the first best still reallocates from small to large firms. Moreover, there is no additional misallocation due to firms' endogenous choices of how many establishments to open. That said, firm size-dependent policy is much less effective in our model because it faces a trade-off between the benefit of large firms expanding along the intensive margin and the cost of them over-expanding along the extensive margin.

Afrouzi, Drenik, and Kim (2023) estimate that a firm's markup is increasing in its sales per customer but not in its number of customers. This is complementary to our finding that a firm's markup is increasing in its sales per establishment but not in its number of establishments because in our theory, we can interpret opening new establishments as finding new customers to sell to. Crucially, our models differ in that they suppose all a firm's customers are identical, whereas we suppose each successive establishment at a firm is lower quality. This choice allows us to match our empirical finding that each successive establishment at a firm is smaller. As a result, they find that large firms match with too few customers relative to the first best, so there is additional misallocation due to firms' endogenous extensive margin decisions. By contrast, we find almost no additional misallocation through the extensive margin because firms' establishment choices are near efficient. The near efficiency of firms' establishment choices in the competitive equilibrium is also central to the novel trade-off that firm size-dependent policy faces in our model.

The paper proceeds as follows. In Section 2, we describe our empirical analysis. In Section 3, we develop the model and in Section 4, we calibrate it. In Section 5, we use the calibrated model to evaluate misallocation and study optimal firm size-dependent policy.

2 Empirical Analysis

2.1 Data

We use data covering the universe of Swedish firms and establishments from 1997 to 2017. We merge information from two administrative datasets based on firms' tax forms. Both are from Statistics Sweden, the Swedish government agency responsible for producing official statistics. The first data set, Företagens Ekonomi, contains the annual financial accounts of all Swedish firms. This provides our firm-level outcomes at an annual frequency. The second data set, Registerbaserad arbetsmarknadsstatistik, contains the complete set of employer-employee linkages at a monthly frequency. For each worker, the data include their labor earnings and the IDs of the establishment and firm at which they are employed. We aggregate across workers within each establishment-year and merge with our firm-level data. Then for each firm, we have annual data on its number of establishments as well as the wage bill and employment at each of its establishments.

We restrict attention to firm-year observations with at least one employee and positive sales operating in the private economy, which results in a total of 4.9 million firm-year observations. Nominal variables are deflated to 2017 SEK using the GDP deflator.

2.2 Descriptive Analysis

Table 1 reports statistics of the firm size distribution pooled across years (1997 to 2017) using various measures of firm size. Most firms are small: at the 75^{th} percentile, annual sales are 7.9 million SEK ($\approx$790,000 US dollars), employment is 5 workers, and number of establishments is 1. But some firms are large, so the firm size distribution is highly right-skewed and the mean exceeds the 75^{th} percentile for each measure: mean sales are 28.1 million SEK, mean employment is 10 workers, and the mean number of establishments is 1.2.

Importance of establishments. Although multi-establishment firms are rare, they are much larger and earn the majority of sales. Table 2 reports that 2.6% of firms are multi-establishment, which rises to 8.6% if we weight the share in each industry by industry sales. The share of sales earned by multi-establishment firms is 53.1%.

The number of establishments is also an important driver of firm sales for large firms. Figure 1 plots the log of sales per establishment against the log of firm sales by sorting log firm sales into 1000 equally sized bins and taking the averages within each bin. The red dots denote the percentiles of the unweighted firm sales distribution and the sales-weighted distribution (in parentheses). For the smallest 90% of firms, log sales per establishment sits on the 45° line,

Table 1: Summary Statistics (1997-2017)

	25^{th} Pct.	50^{th} Pct.	75^{th} Pct.	Mean	SD
*Sales**	1.2	2.8	7.9	28.1	572.0
*Value added**	0.5	1.1	2.9	7.7	143.1
Employment	1	2	5	10.0	132.0
*Wage bill**	0.2	0.6	1.6	3.7	53.3
*Capital stock**	0.1	0.2	1.1	9.4	278.4
*Intermediate Inputs**	0.4	1.0	2.7	10.9	271.7
Establishments	1	1	1	1.2	4.2

Variables marked with * are in millions of 2017-SEK. The capital stock is the accumulation of past fixed asset investment minus depreciation. The columns report the 25^{th}, 50^{th}, and 75^{th} percentile for each measure, then the mean and standard deviation. The number of observations is 4,854,361.

Table 2: Multi-Establishment Firms

Share of firms (unweighted)	Share of firms (industry sales-weighted)	Share of sales
2.6%	8.6%	53.1%

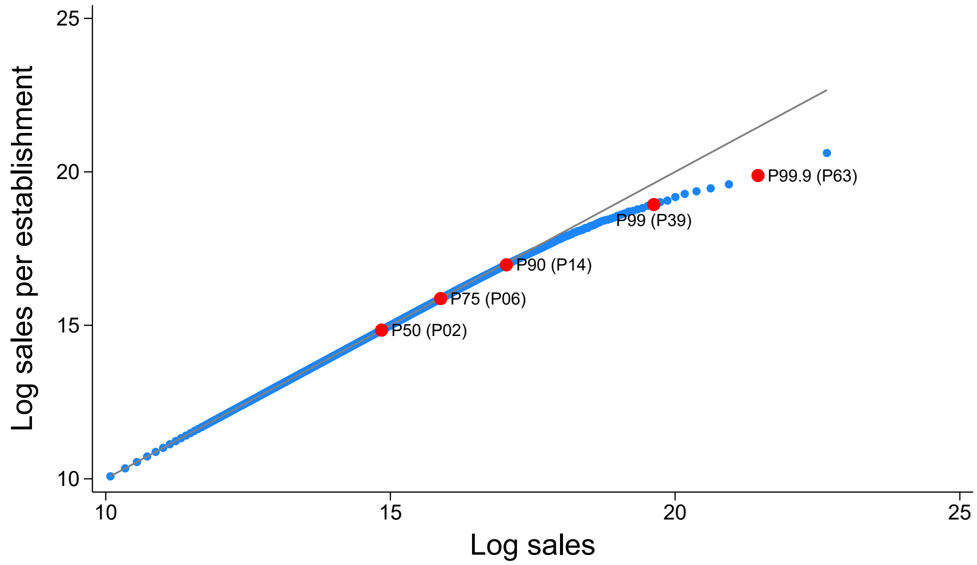
The first two columns report the share of firms with more than one establishment. We compute the share in each of 600 5-digit industries in each year, then column 1 takes the unweighted average across industry-years, and column 2 takes the sales-weighted average across industries within each year, and then the unweighted average across years. The third column reports the share of sales earned by multi-establishment firms across all years.

so it moves one-for-one with log sales, i.e., multi-establishment firms are negligible. But these firms earn only 14% of total sales. For larger firms, variation in sales is increasingly driven by variation in number of establishments rather than sales per establishment, as indicated by log sales per establishment falling below the 45° line. In particular, toward the top of the sales distribution, the slope of the log sales per establishment line falls to 0.6, which means that roughly 40% of variation in sales is due to variation in number of establishments.

2.3 Fact 1: Declining Size at Successive Establishments

As firms expand by opening new establishments, each successive establishment is smaller. Specifically, Figure 2 plots the average log wage bill (in 2017 SEK) and log employment across an establishment's life cycle conditional on survival for a firm's first five establishments.

Figure 1: Drivers of firm sales



We group all firm-year observations into 1000 equally sized bins according to log sales. Each blue dot plots the average of log sales per establishment within a bin against the average of log sales (in 2017 SEK). The solid black line is the 45° line. The red dots indicate percentiles of the unweighted and sales-weighted (in parentheses) firm sales distribution. For example, the first red dot reports that 50% of firms have log sales less than 15 and these firms earn only 2% of total sales.

To avoid selection effects, we restrict attention to firms with at least five establishments. There is a distinct size ranking for a firm's establishments: each successive establishment has lower employment and a lower wage bill than the previous establishment, conditional on establishment age. This holds across the establishment life cycle. The same pattern appears for $n = 2, 3, 4$ if we look at the first n establishments at firms with at least n establishments.

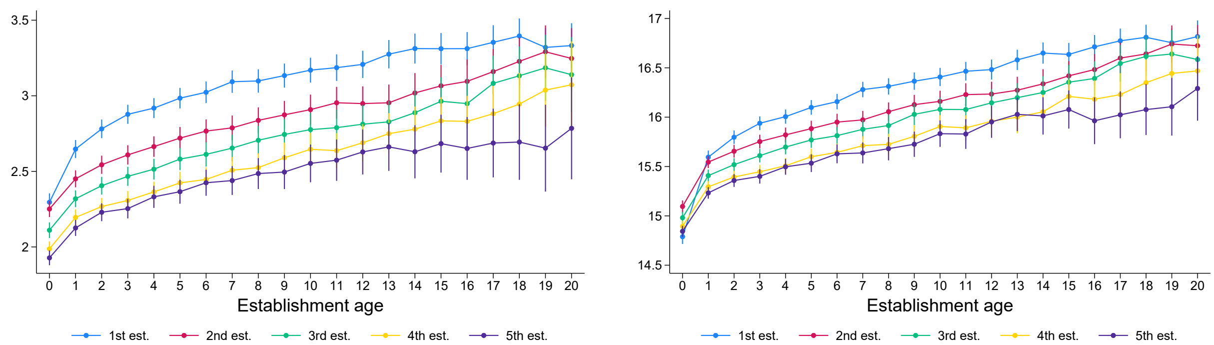
2.4 Fact 2: Drivers of Size-Dependent Firm Markups

We now study the relationship between a firm's relative sales and its relative markup. We then investigate to what extent this relationship is driven by a firm's sales per establishment or its number of establishments. To give the latter a chance, we restrict attention to firm-year observations with multiple establishments for the main analysis.

We find that firms with higher sales relative to their industry in a given year also set a higher

Figure 2: Size of successive establishments

(a) Log employment (b) Log wage bill



The average log employment and wage bill (in 2017 SEK) for the first five establishments of a firm as a function of establishment age in years, conditional on surviving to that age. Averages are computed across firms with at least five establishments. 95% confidence bands are included.

relative markup. This is consistent with previous work in other countries.[4] In particular, the first column in Table 3 reports a coefficient of 0.163 from a regression of log firm markups on log firm sales, including interacted industry and year fixed effects. We measure the firm markup as firm sales over the wage bill. The fixed effects then absorb heterogeneity in the output elasticity of labor across industry-years and normalize sales and markups relative to industry-year averages. Thus, 1% higher sales relative to the industry-year average is associated with a 0.163% higher markup relative to the industry-year average. This coefficient is highly statistically significant.

We now show that this positive firm size-markup relationship is due entirely to a positive relationship between a firm's sales per establishment and its markup. By contrast, there is no clear relationship between a firm's number of establishments and its markup. Specifically, we regress log firm markups on log number of establishments and log sales per establishment, separately and together in a horse race regression. The estimated coefficients are in columns 2-4 in Table 3.

A 1% increase in sales per establishment relative to the industry-year average is associated with a 0.246% higher markup relative to the industry-year average if we do not control

[4]For example, by Edmond, Midrigan, and Xu (2023) in US manufacturing data and by Burstein, Carvalho, and Grassi (2024) in French data.

Table 3: Firm markup regressions

	(1) Log μ	(2) Log μ	(3) Log μ	(4) Log μ
Log sales	0.163			
	(0.005)			
Log number of establishments		-0.002		-0.046
		(0.006)		(0.005)
Log sales per establishment			0.246	0.249
			(0.007)	(0.007)
Industry × year fixed effects	✓	✓	✓	✓
Only multi-establishment firms	✓	✓	✓	✓
R^2	0.590	0.508	0.632	0.634
Number of observations	124,343	124,343	124,343	124,343

Firm markup μ is measured as sales over the wage bill. Standard errors (in parentheses) are clustered at the firm level.

for log number of establishments, and a 0.249% higher markup if we do (both statistically significant). On the other hand, the relationship between a firm's relative number of establishments and its relative markup is economically and statistically insignificant if we do not control for sales per establishment, and is negative and statistically significant if we do. The coefficient on log sales per establishment is higher than on log sales (0.246 vs. 0.163) because some variation in sales is due to variation in number of establishments, which has zero or a negative association with a firm's markup. We do not want to over-interpret the distinction between a zero coefficient on log number of establishments and a slightly negative or even positive coefficient because these regressions are misspecified in the sense that our model does not predict a constant elasticity between a firm's markup and its sales per establishment or its number of establishments.

The R^2 in our regressions further demonstrate that a firm's sales per establishment beats a firm's sales in terms of predicting the firm's markup. Moreover, a firm's number of establishments does not help predict the firm's markup. Specifically, the R^2 is higher when we use sales per establishment than when we use sales, and including number of establishments in addition to sales per establishment yields almost no improvement in the R^2.

We run a variety of checks on the robustness of this finding and report the results in Table 4. Specifically, we run a horse race regression between log sales per establishment and log

Table 4: Firm markup regressions (robustness checks)

	(1)	(2)	(3)	(4)
	Log μ	Log μ	Log μ	Log μ
Log sales	-0.046			
	(0.005)			
Log number of establishments		-0.185	0.052	-0.042
		(0.004)	(0.005)	(0.005)
Log sales per establishment	0.295	0.122	0.326	0.239
	(0.009)	(0.001)	(0.002)	(0.006)
Industry × year fixed effects	✓	✓	✓	✓
Firm fixed effects			✓	
Only multi-establishment firms	✓			✓
IV				✓
R^2	0.634	0.184	0.704	0.258
Number of observations	124,343	4,798,494	4,660,949	115,622

Firm markup μ is measured as sales over the wage bill. Standard errors (in parentheses) are clustered at the firm level. Columns 2 and 3 include single-establishment as well as multi-establishment firms. IV indicates that log sales per establishment is instrumented by its one-year lagged value.

sales, we include single-establishment as well as multi-establishment observations, we add firm fixed effects, and we use one-year lagged sales per establishment to instrument for sales per establishment. The last one is designed to address the concern that our results are due to noise in sales per establishment that is not present in number of establishments: the wage bill would not react to this noise, so higher sales per establishment would be mechanically associated with higher sales relative to the wage bill, which we interpret as a higher markup. However, this concern is not borne out because across all specifications, we find a consistent positive and statistically significant relationship between a firm's relative sales per establishment and its relative markup, but no clear relationship between a firm's relative number of establishments and its relative markup. Indeed, using one-year lagged sales per establishment to instrument for sales per establishment only lowers the coefficient from 0.249 to 0.239.

3 Model

The model is static, i.e., there is a single period. A representative household consumes the numeraire final good, inelastically supplies labor, and owns all firms. There is a continuum of firms, each of which controls a continuum of establishments. Through each of its establishments, a firm uses labor to produce a differentiated variety with establishment-specific quality. Perfectly competitive final good producers aggregate these varieties into the final good. Perfectly competitive capital producers use labor to produce capital, which firms use to build establishments.

Representative household. The representative household maximizes final good consumption C subject to its budget constraint:

$$C \leq W\bar{L} + \bar{\Pi},$$

where W is the wage, $\bar{L}$ is the inelastic labor supply, $\bar{\Pi}$ are profits from firms, and the final good price is normalized to 1. We take final good consumption C as our measure of welfare.

Final good producers and demand. Perfectly competitive final good producers aggregate varieties from firms' establishments into final good output Y. They sell the final good to the representative household, so their output must equal final good consumption:

$$C = Y.$$

Specifically, final good producers purchase varieties from a double continuum of establishments: each firm $i \in [0, 1]$ sells a different variety at each of its establishments $n \in [0, N(i)]$. If final good producers purchase $y(i, n)$ from firm i's establishment n, then final good output Y is given implicitly by the Kimball (1995) aggregator:

$$\int_0^1 \int_0^{N(i)} \omega(i, n) \Upsilon \left(\frac{y(i, n)}{Y} \right) dndi = 1, \tag{1}$$

where $\omega(i, n)$ is the quality of the variety at firm i's establishment n, and where $\Upsilon(\cdot)$ is twice continuously differentiable, strictly increasing, and strictly concave. That is, given variety purchases, final good output Y is such that (1) holds. The aggregation technology is constant returns-to-scale since multiplying Y and each $y(i,n)$ by the same positive constant leaves (1) unchanged. A special case of the aggregator is CES (constant elasticity of substitution), in which case $\Upsilon(x) = x^{\frac{\sigma-1}{\sigma}}$ for $\sigma > 1$.

Final good producers choose demand $y(\cdot,\cdot)$ and final good output Y to maximize profits

$$Y - \int_0^1 \int_0^{N(i)} p(i,n) y(i,n) dndi$$

subject to the aggregation technology (1), where $p(i,n)$ is the price of the variety from firm i's establishment n. The resulting demand for each variety is given by[5]

$$p(i,n) = \omega(i,n)\Upsilon'\left(\frac{y(i,n)}{Y}\right) D \qquad D \equiv \left(\int_0^1 \int_0^{N(i)} \omega(i,n)\frac{y(i,n)}{Y}\Upsilon'\left(\frac{y(i,n)}{Y}\right) dndi\right)^{-1}, \tag{2}$$

where D is a demand index. First, an establishment's quality scales its price. Second, an establishment's price is falling in its relative output since $\Upsilon(\cdot)$ is concave. An establishment's price does not depend on output at its firm's other establishments. Finally, if $\Upsilon'(0) < \infty$, then unlike with CES, the price $p(i,n)$ remains finite at $y(i,n) = 0$.

Capital producers. Perfectly competitive capital producers convert labor into capital according to $K = L_k$, where L_k is the labor used for capital production. It follows that the capital price is the wage: $P_k = W$.

Firms and production. A unit measure of firms produce differentiated varieties through establishments. First, each firm $i \in [0,1]$ chooses a measure $N(i) \geq 1$ of establishments to open subject to a linear cost $\kappa \max\{N(i) - 1, 0\}$ in units of capital, where $\kappa > 0$ is the cost per establishment. The first unit measure of establishments $n \in [0,1]$ are free, which captures in a continuous model that a firm has an initial establishment and decides how many further establishments to open.

[5] The first order condition for $y(i,n)$ is $p(i,n) \geq (\lambda/Y)\omega(i,n)\Upsilon'(y(i,n)/Y)$, where λ is the Lagrange multiplier on the aggregation constraint and where the inequality holds with equality if $y(i,n) > 0$. For $y(i,n) = 0$, the inequality can be strict if $\Upsilon'(0) < \infty$ (unlike with CES) because then there is a finite price threshold above which demand is 0. Without loss of generality, we suppose the price is weakly below this threshold. Multiply both sides of the first order condition by $y(i,n)$, integrate across establishments, and plug in that final good producer profits are zero (implied by perfect competition) to get $Y = \lambda/D$. Plug back into the first order condition to get (2).

Firm i produces a different variety through each of its measure $N(i)$ establishments. Each variety is characterized by its quality: the establishment n variety has quality

$$\omega(i,n) = \bar{\omega}(i)\max\{n,1\}^{-\alpha}. \tag{3}$$

In words, the first unit measure of establishments $n \in [0,1]$ have quality $\omega(i,n) = \bar{\omega}(i)$, which we call firm i's baseline quality. Then quality declines at each successive establishment at rate $\alpha > 0$. So firm i's highest quality establishments are its first unit measure, $n \in [0,1]$, and its lowest quality establishment is its last, $n = N(i)$.

Firms produce all varieties with the same productivity: output of the firm i establishment n variety is the labor used for producing it, $l(i,n)$. Hence, the marginal cost of production is the wage W.

Firm i chooses its measure $N(i)$ of establishments as well as the price $p(i,n)$ and labor $l(i,n)$ for each variety $n \in [0, N(i)]$ to maximize profits:

$$\int_0^{N(i)} (p(i,n)y(i,n) - Wl(i,n))dn - P_k\kappa\max\{N(i) - 1\},$$

where for each variety n, labor $l(i,n)$ equals output $y(i,n)$, which is implied by the price $p(i,n)$ and the demand curve (2). The first term is profits from the firm's measure $N(i)$ establishments. The second term is the cost of buying capital to open establishments beyond the initial unit measure.

Establishment markups and profits. We first characterize a firm's optimal price and output at each establishment. The maximization problem at each establishment is independent because establishments are only related through economy-wide aggregates and a firm is small relative to the economy. So the markup of price over marginal cost at each establishment must satisfy the usual expression:

$$\frac{p(i,n)}{W} = \frac{\sigma(i,n)}{\sigma(i,n)-1} \qquad \sigma(i,n) \equiv \frac{-p(i,n)}{y(i,n)}\frac{\partial y(i,n)}{\partial p(i,n)} = \frac{-\Upsilon'(y(i,n)/Y)}{(y(i,n)/Y)\Upsilon''(y(i,n)/Y)}, \tag{4}$$

where $\sigma(i,n) > 1$ is the elasticity of demand for the variety at firm i's establishment n with respect to its price. Demand (2) and the optimal markup (4) jointly determine the price $p(i,n)$ and output $y(i,n)$ at each establishment.

We make the following assumption on the Kimball (1995) aggregator function $\Upsilon(\cdot)$ so that establishments with higher relative sales face a lower demand elasticity, and therefore set a higher markup. With Assumption 1, we can now state Proposition 1, which characterizes an establishment's outcomes as functions of its quality.

Assumption 1. *For all x, $\frac{-\Upsilon'(x)}{x\Upsilon''(x)}$ is strictly decreasing in x.*

Proposition 1. *An establishment's output, price, and profits are strictly increasing continuous functions $\tilde{y}(\cdot)$, $\tilde{p}(\cdot)$, and $\tilde{\pi}(\cdot)$ of its variety's quality:*

$$y(i,n) = \tilde{y}(\omega(i,n)) \qquad p(i,n) = \tilde{p}(\omega(i,n)) \qquad (p(i,n) - W)y(i,n) = \tilde{\pi}(\omega(i,n)).$$

The proposition states that any two establishments with the same quality have the same establishment level outcomes, regardless of the firm that controls them. Moreover, a higher quality establishment is larger, has a higher markup, and generates more profits. It follows that firm i's largest, highest markup, and most profitable establishments are its initial unit measure, $n \in [0,1]$, and its smallest, lowest markup, and least profitable establishment is its last, $n = N(i)$.

Intuitively, Proposition 1 holds because a higher quality shifts up an establishment's price and so the marginal benefit of production. Hence, the firm increases production. The markup must then rise because higher relative output means a lower demand elasticity. With more output and a higher markup, profits must be higher as well.

Measure of establishments. We next characterize firm i's optimal measure $N(i)$ of establishments. Since firm i is small, it takes as given establishment profits as a function of quality, $\tilde{\pi}(\cdot)$ from Proposition 1. Hence, firm i's marginal benefit of increasing $N(i)$ (opening another establishment) is simply the profits it will earn from the marginal establishment, $\tilde{\pi}(\bar{\omega}(i)N(i)^{-\alpha})$. The marginal cost is the cost of purchasing the required capital, $P_k\kappa$.

Firm i's optimal measure of establishments is given by

$$N(i) = \max\left\{\left(\frac{\bar{\omega}(i)}{\omega^*}\right)^{\frac{1}{\alpha}}, 1\right\} \qquad \text{where} \quad \tilde{\pi}(\omega^*) = P_k\kappa. \tag{5}$$

In words, ω^* is the unique quality at which establishment profits equal the cost of opening an establishment. If firm i's quality at its initial unit measure of establishments, $\bar{\omega}(i)$, is less than ω^*, then it does not open any further establishments, so $N(i) = 1$. In this case, we say firm i is a "single-establishment firm". On the other hand, if $\bar{\omega}(i) > \omega^*$, then firm i opens establishments until its marginal establishment's quality is $\bar{\omega}(i)N(i)^{-\alpha} = \omega^*$. In this case, we say firm i is a "multi-establishment firm". It follows that the marginal establishment at any multi-establishment firm has the same quality ω^*, and so the same sales, markup, and profits.

Firm size and markups. To compute firm-level outcomes, aggregate across a firm's establishments. Specifically, firm i's revenue is $R(i) = \int_0^{N(i)} p(i,n)y(i,n)dn$ and its production labor is $L(i) = \int_0^{N(i)} l(i,n)dn$. We define a firm's markup as its revenue over production costs because an establishment's markup is the same ratio but at the establishment level. Hence, a firm's markup is the cost-weighted average of its establishment markups:

$$\mu(i) \equiv \frac{R(i)}{WL(i)} = \int_0^{N(i)} \frac{p(i,n)}{W}\frac{l(i,n)}{L(i)}dn.$$

The following proposition characterizes firm outcomes as functions of baseline quality.

Proposition 2. *A firm's revenue per establishment, measure of establishments, and markup are strictly increasing continuous functions $\tilde{r}(\cdot)$, $\tilde{N}(\cdot)$, and $\tilde{\mu}(\cdot)$ of its baseline quality:*

$$R(i)/N(i) = \tilde{r}(\bar{\omega}(i)) \qquad N(i) = \tilde{N}(\bar{\omega}(i)) \qquad \mu(i) = \tilde{\mu}(\bar{\omega}(i)).$$

The proposition states that the higher a firm's baseline quality, the higher its sales per establishment, measure of establishments, and markup. Intuitively, this holds because if firm i is multi-establishment, then the qualities at its establishments beyond the initial unit measure ($n > 1$) are Pareto distributed from the multi-establishment cutoff ω^* to the firm's baseline quality $\bar{\omega}(i)$. So a higher baseline quality means an increase in the firm's initial unit measure of establishments ($n \in [0, 1]$) as well as an increase in the distribution of its other establishments' qualities. Then the proposition follows because establishment sales and markup are increasing in establishment quality.

Aggregation. The economy aggregates so that final good output is

$$Y = ZL_p,$$

where $L_p = \int_0^1 L(i)di$ is labor used for production and Z is aggregate productivity:[6]

$$Z = \left(\int_0^1 \int_0^{N(i)} (y(i,n)/Y) dndi \right)^{-1}.$$

Hence, aggregate productivity depends on each firm's measure of establishments and its relative output at each establishment.

Equilibrium. In equilibrium, the representative household maximizes consumption C subject to its budget constraint, taking as given the wage W and profits $\bar{\pi}$. Each firm i chooses their measure of establishments $N(i)$ and the price $p(i,n)$ and labor $l(i,n)$ at each establishment to maximize profits, taking as given final good output Y, the wage W, the capital price P_k, and the demand index D. Final good producers choose demand $y(i,n)$ for each establishment and final good output Y to maximize profits, taking as given the price $p(i,n)$ at each establishment. Capital producers choose capital production K and the labor used to produce capital L_k to maximize profits, taking as given the capital price P_k and the wage W. Perfect competition implies that final good and capital producers earn zero profits.

Household consumption must equal final good output: $C = Y$. Final good output Y and demand for each variety $y(\cdot,\cdot)$ must satisfy aggregation (1). Demand for each variety must equal output, which must equal the labor used to produce it: $y(i,n) = l(i,n)$. The capital used to build establishments must equal the capital produced, which must equal the labor used to produce it: $\int_0^1 (N(i) - 1)di = K = L_k$. The labor used by firms for production and by capital producers must equal the household's inelastic labor supply: $L_p + L_k = \bar{L}$.

We can find an equilibrium as follows. Given final good output, Y, and the demand index relative to the wage, D/W, use demand (2) and the optimal markup (4) to get establishment output $\tilde{y}(\cdot)$ and profits $\tilde{\pi}(\cdot)$ as functions of establishment quality. The latter implies each firm's measure of establishments $N(i)$. Then check whether the Kimball (1995) aggregator holds and the labor market clears. Hence, there are two variables (Y and D/W) that must satisfy two equations (Kimball (1995) aggregator and labor market clearing). We can then

[6] Write $Z^{-1} = L_p/Y = \int_0^1 \int_0^{N(i)} (l(i,n)/Y)dndi$. Using $y(i,n) = l(i,n)$ yields the expression for Z.

compute the remaining equilibrium variables.

4 Calibration and the Initial Equilibrium

We calibrate the model to the Swedish economy from 1997-2017. We then illustrate features of the initial equilibrium.

4.1 Calibration

Functional forms. For the Kimball (1995) aggregator function $\Upsilon(\cdot)$, we use the Klenow and Willis (2016) specification, which has derivative:

$$\Upsilon'(x) = \frac{\bar{\sigma}-1}{\bar{\sigma}} \exp\left(\frac{1-x^{\frac{\epsilon}{\bar{\sigma}}}}{\epsilon}\right),$$

where $\bar{\sigma} > 1$ and ϵ are parameters. It follows that if firm i's establishment n has relative output $x = y(i,n)/Y$, then its demand elasticity (of quantity with respect to price) and markup (see (2)) are:

$$\sigma(i,n) = \frac{-\Upsilon'(x)}{x\Upsilon''(x)} = \bar{\sigma}x^{\frac{-\epsilon}{\bar{\sigma}}} \qquad \frac{p(i,n)}{W} = \frac{\bar{\sigma}x^{\frac{-\epsilon}{\bar{\sigma}}}}{\bar{\sigma}x^{\frac{-\epsilon}{\bar{\sigma}}}-1}.$$

Hence, $\bar{\sigma}$ is the demand elasticity at relative output 1 and $\epsilon/\bar{\sigma}$ is the super elasticity—the elasticity of the demand elasticity with respect to relative output. If $\epsilon = 0$, then we have CES demand, so the demand elasticity and markup are constant. If $\epsilon > 0$, then the demand elasticity is falling in relative output, so the markup is increasing in relative output. If $\epsilon < 0$, then the demand elasticity is increasing in relative output, so the markup is decreasing. To satisfy Assumption 1 (and hit our calibration targets), we suppose $\epsilon > 0$.

The function $\Upsilon(\cdot)$ is pinned down by the derivative $\Upsilon'(\cdot)$ and setting $\Upsilon(0) = 0$:

$$\Upsilon(x) = \frac{\bar{\sigma}-1}{\epsilon} \int_0^{x^{\frac{\epsilon}{\bar{\sigma}}}} \tilde{x}^{\frac{\bar{\sigma}}{\epsilon}-1} \exp\left(\frac{1-\tilde{x}}{\epsilon}\right) d\tilde{x}.$$

In Klenow and Willis (2016), they instead set $\Upsilon(1) = 1$. If an establishment is sufficiently low quality, then it produces zero in equilibrium because $\epsilon > 0$ implies that the price remains finite as relative output goes to 0. If $\Upsilon(0) \neq 0$, then these non-producing establishments affect the Kimball (1995) aggregator and so economic outcomes. For example, if $\Upsilon(0) > 0$, then introducing non-producing establishments raises aggregate productivity. This becomes particularly relevant when we turn to optimal policy. Hence, to avoid this oddity, we set $\Upsilon(0) = 0$ so that non-producing establishments have no effect.

Next, the distribution of baseline quality across firms is Pareto, with minimum 1 and tail parameter β, i.e., the pdf is $\beta\bar{\omega}^{-\beta-1}$.

Parameters. We set the inelastic labor supply $\bar{L}$ so that final good output is $Y = 1$. This leaves five parameters to calibrate: 1) the capital cost per establishment, κ, 2) the Pareto tail parameter β for the baseline quality distribution, 3) the demand elasticity at relative output of 1, $\bar{\sigma}$, 4) the demand super elasticity, $\epsilon/\bar{\sigma}$, and 5) the elasticity α of firm i's establishment n's quality with respect to n.

Table 5: Calibration Targets

Moment	**Data**	**Model**
fraction of firms with multiple establishments	0.09	0.09
sales share of multi-establishment firms	0.53	0.53
log(markup) on log(sales/establishment), all firms	0.12	0.12
log(markup) on log(sales/establishment), multi-establishment firms	0.25	0.25
first vs. second establishment log wage bills at 10 years old	0.24	0.24

To calibrate these parameters, we exactly match five moments in our Swedish data: 1) the average share of firms that are multi-establishment in each industry (sales-weighted across industries), 2) the share of sales earned by multi-establishment firms, 3) the estimated elasticity of the markup with respect to sales per establishment (controlling for industry x year fixed effects), 4) the same elasticity but only for multi-establishment firms, and 5) the average difference in log wage bills between a firm's first establishment and second establishment at establishment age of 10 years, conditional on the firm having at least five establishments. The data moments are in Table 5 and the calibrated parameter values are in Table 6.

All five parameters are jointly determined by all five moments. Nonetheless, there is an intuitive mapping from moments to parameters. First, the demand parameters $\bar{\sigma}$ and ϵ determine the relationship between the markup and relative output at each establishment, so they mostly affect the elasticity of the markup with respect to sales per establishment for all firms and for multi-establishment firms. Specifically, if an establishment has markup μ,

Table 6: Calibrated Parameter Values

Parameter	**Value**
κ (establishment capital cost)	0.49
β (baseline quality tail parameter)	3.36
$\bar{\sigma}$ (demand elasticity at relative output of 1)	3.51
$\epsilon/\bar{\sigma}$ (demand super elasticity)	0.44
α (establishment quality elasticity)	1.27

then the elasticity of the markup with respect to its relative output is $(\mu - 1)\epsilon/\bar{\sigma}$ and the derivative of the markup with respect to $\bar{\sigma}$ (holding $\epsilon/\bar{\sigma}$ fixed) is $-\mu(\mu - 1)/\bar{\sigma}$. Hence, a higher $\epsilon/\bar{\sigma}$ pushes up the elasticity of the markup with respect to relative output equally for all establishments, and so equally for all firms. On the other hand, a higher $\bar{\sigma}$ pushes down the markup more for high markup establishments, so it pushes down the elasticity of the markup with respect to relative output particularly at large high markup establishments, and so particularly at multi-establishment firms.

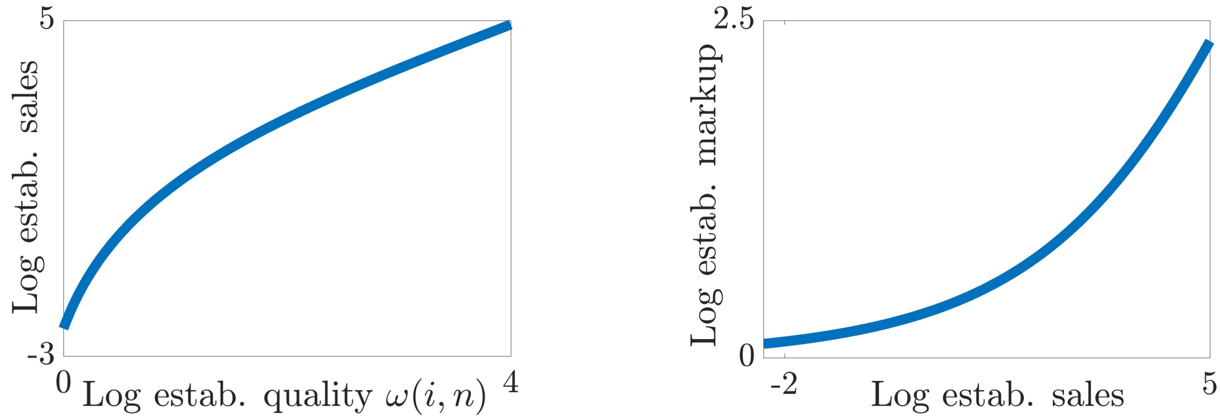
Next, the elasticity α of firm i's establishment n's quality with respect to n determines the quality of a firm's second establishment relative to its first, which is equal to $2^{-\alpha}$. Hence, it mostly determines the wage bill at a firm's second establishment relative to its first. Finally, the establishment capital cost κ and the tail parameter β of the baseline quality distribution determine the fraction of firms with multiple establishments and the sales share of multi-establishment firms. A higher κ means fewer firms are willing to pay to become multi-establishment and multi-establishment firms open fewer establishments, which reduces their sales share. A higher β pushes these moments in the same direction as a higher κ, but particularly affects the highest quality firms and so has a relatively stronger effect on the sales share of multi-establishment firms vs. the share of firms that become multi-establishment.

4.2 Initial Equilibrium

Figure 3 shows establishment-level outcomes in the calibrated economy. Higher quality establishments sell more (left panel), so they face a lower demand elasticity and set higher markups (right panel).

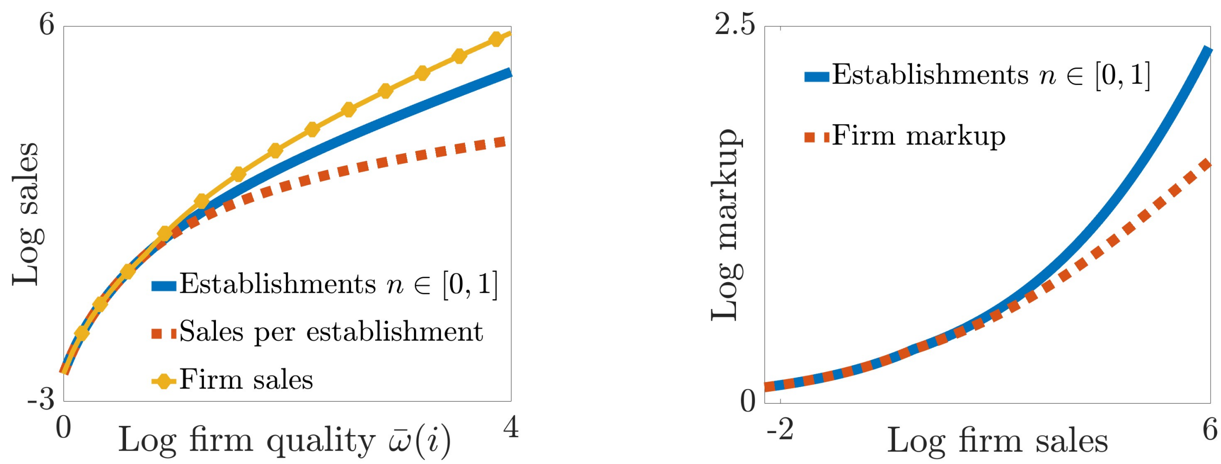
Figure 4 shows firm-level outcomes. In the left panel are three measures of a firm's sales as functions of its baseline quality: sales at its initial unit measure of establishments (solid blue line), sales per establishment across all its establishments (dotted red line), and total sales (textured yellow line). First, firms with a higher baseline quality sell more overall, and this is increasingly due to opening new establishments rather than increasing sales per establish-

Figure 3: Establishment Sales and Markups



Left panel: log establishment sales as a function of log establishment quality. Right panel: log establishment markup as a function of log establishment sales.

Figure 4: Firm Sales and Markups



Left panel, as functions of a firm's log baseline quality: the solid blue line in the middle is log sales per establishment at a firm's initial unit measure of establishments; the dotted red line on the bottom is log sales per establishment across all establishments; and the textured yellow line on the top is log firm sales. Right panel, as functions of a firm's log sales: the solid blue line on top is log establishment markup at a firm's initial unit measure of establishments; the dotted red line on the bottom is a firm's log markup (log of the cost-weighted average markup across establishments).

ment; this is the increasing gap between total sales and sales per establishment. Second, a firm's sales at its initial unit measure of establishments and its sales per establishment are both increasing in the firm's baseline quality, but sales per establishment is flatter. This is because a higher quality firm opens more establishments, which pulls down the quality of its average establishment relative to its initial (highest quality) establishment. The right panel of figure 4 illustrates the same pattern for firm markups vs. markups at a firm's initial unit measure of establishments.

5 Quantitative Results

We now turn to our main results on misallocation and firm size-dependent policy.

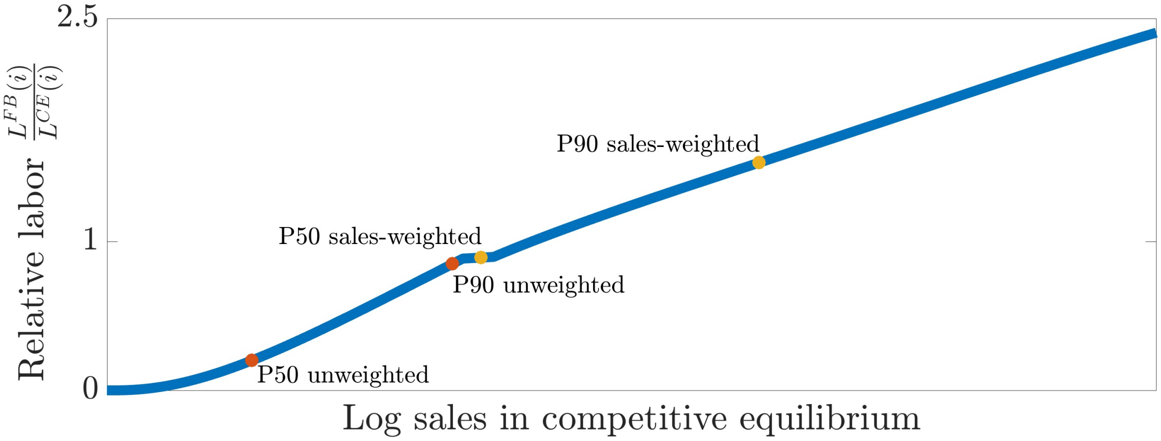
5.1 Misallocation

The first best planner chooses each firm's measure of establishments and how much to produce at each to maximize consumption (equivalently, final good output) subject to the aggregation technology (1), capital producers' technology, and the inelastic labor supply $\bar{L}$.

Consumption is 7.5% higher in the first best than in the competitive equilibrium, using our calibrated economy. This comes from a 6.9% increase in aggregate productivity Z and a shift of 0.6% of the labor supply from establishment building to production. We quantify different types of misallocation in the competitive equilibrium by decomposing the gains from moving to the first best into three steps. First, reallocate labor across each firm's establishments to maximize consumption, holding fixed each firm's measure of establishments, $N(i)$, as well as their total labor used for production, $L(i)$. This improves aggregate productivity Z and consumption C by only 0.7%. Next, reallocate labor across all establishments (within and across firms) to maximize consumption, holding fixed each firm's measure of establishments, $N(i)$. This improves aggregate productivity Z and consumption C by 7.4% relative to the competitive equilibrium. Finally, we get to the first best by also choosing each firm's measure of establishments efficiently.

According to this decomposition, misallocation of production across firms' existing establishments is the dominant source of losses in the competitive equilibrium relative to the first best. Moreover, misallocation is mostly across firms rather than within firms. In particular, Figure 5 shows that the first best reallocates labor from small to large firms: a firm's

Figure 5: Reallocation in the First Best



Firms' production labor in the first best relative to in the competitive equilibrium as a function of competitive equilibrium log sales. Px indicates the x percentile of the firm sales distribution, where the red dots are unweighted (50% of firms earn sales less than P50) and the yellow dots are sales-weighted (50% of sales are earned by firms with sales less than P50).

production labor in the first best relative to in the competitive equilibrium is an increasing function of the firm's competitive equilibrium sales.

There is little gain from efficiently choosing each firm's measure of establishments. One reason is that given a total measure of establishments, they are already efficiently allocated in the competitive equilibrium. That is, the marginal establishment at all multi-establishment firms has the same quality ω^*, and that quality is higher than the marginal establishment's quality at any single-establishment firm. Central to this result is the fact that a firm's size does not affect its incentive to open establishments.

Although efficiently choosing the set of establishments has a small effect on consumption, the first best planner makes substantial changes to the set of establishments. Specifically, they close 6.7% of establishments at multi-establishment firms, and as a result, 23% of multi-establishment firms become single-establishment. This shifts 0.6% of the labor supply from establishment building to production, but the gains are mostly offset by a fall in aggregate productivity due in part to a love-of-variety effect.

5.2 Firm Size-Dependent Policy

We now study firm size-dependent policy. Specifically, the policy is a tax $T(\cdot)$ such that firm i pays $T(R(i))$, where $R(i) = \int_0^{N(i)} p(i,n)y(i,n)$ is firm i's revenue. Taxes (or subsidies if negative) are paid lump sum to the representative household. This policy implies that firm i's marginal revenue is multiplied by $1 - T'(R(i))$.

Figure 6 shows the optimal firm size-dependent marginal tax, $\hat{T}'(R)$, as a function of firm sales. Optimal policy gives a relative marginal subsidy to large firms, thereby incentivizing a shift in production from small to large firms. We uniquely identify optimal policy $\hat{T}(\cdot)$ by setting $\hat{T}(0) = 0$ and shifting $1 - \hat{T}'(\cdot)$ until the policy is revenue-neutral.[7] Otherwise, the optimal policy is not uniquely identified because the allocation of production and establishments is unaffected by changes in the lump-sum tax, $T(0)$, or by multiplying $1 - T'(\cdot)$ by any $x > 0$.[8]

Optimal policy raises aggregate productivity Z by 5.4% and consumption C by 3.7%. Thus, even though nearly all consumption losses relative to the first best are due to misallocation across firms, the optimal firm size-dependent policy can only eliminate about half of these losses (3.7% out of 7.5%).

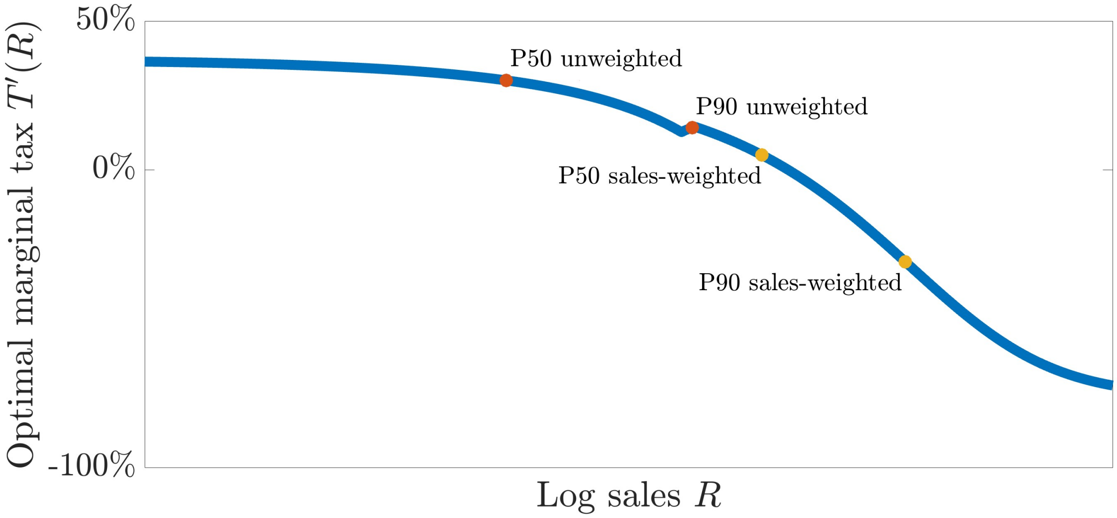
A key mechanism limiting the effectiveness of firm size-dependent policy is the following trade-off. Subsidizing large firm production is beneficial because large firms inefficiently under-produce at their existing establishments. But as we subsidize large firms more, they respond by opening lower quality establishments and producing more at these establishments. This becomes increasingly costly as the subsidy increases.

We illustrate this trade-off in Figure 7. The left panel decomposes the marginal benefit of raising the marginal tax rate $T'(R)$. The "intensive margin" effect holds fixed the measure of establishments chosen by firms with sales R, and only allows their sales at each establishment to change. The "extensive margin" effect holds fixed production at each establishment controlled by firms with sales R, but allows these firms to change their measure of establishments. In both cases, all other firms can respond fully in equilibrium. The intensive and extensive margin effects sum to zero by construction under the optimal policy. For larger firms, it is optimal to lower their tax (increase their subsidy) based on the intensive margin,

[7] Revenue-neutral means $\int_0^1 R(i)T(R(i))di = 0$.

[8] In the latter case, the equilibrium wage and capital price multiply by x as well, so firms' costs simply shift between tax vs. wage/capital payments.

Figure 6: Optimal Revenue-Neutral Firm Size-Dependent Policy



Optimal revenue-neutral firm size-dependent marginal tax rate (subsidy if negative). Px indicates the x percentile of the firm sales distribution, where red dots are unweighted (50% of firms earn sales less than P50) and yellow dots are sales-weighted (50% of sales are earned by firms with sales less than P50).

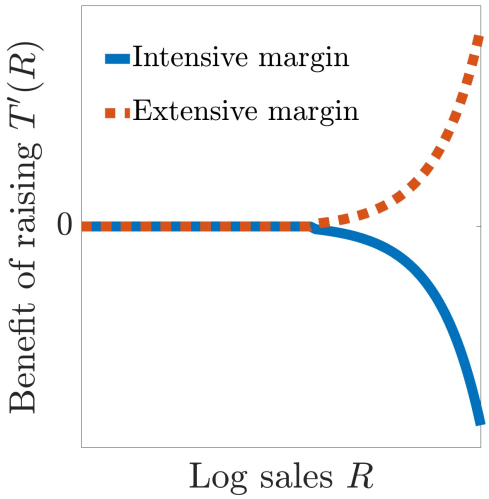
but this is balanced by the cost based on the extensive margin.

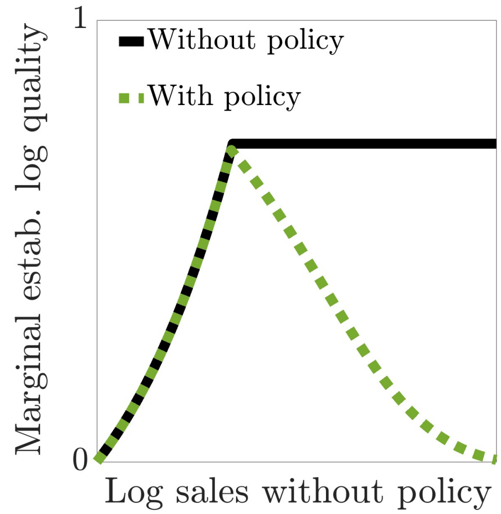
The right panel of Figure 7 demonstrates why it becomes so costly for large firms to expand by opening new establishments. Specifically, it shows the quality of a firm's marginal establishment in the competitive equilibrium (without policy) and under the optimal policy. Without policy, larger firms have higher quality marginal establishments. But policy gives a relative subsidy to large firms, so they respond by opening lower and lower quality establishments. Ultimately, under the optimal policy, the largest firms' marginal establishments are the lowest quality establishments in the economy. Moreover, large firms' relative subsidy also means that they inefficiently over-produce at these low quality establishments.

Another way to see the importance of the intensive vs. extensive margins of firms' responses to policy is to consider the optimal policy $T(\cdot)$, *taking as given each firm's measure of establishments*, i.e., by ignoring the extensive margin effect. We illustrate this optimal policy in the left panel of Figure 8. Compared to the genuinely optimal policy, it offers even more of a relative subsidy to large firms.

With a fixed set of establishments, optimal policy improves aggregate productivity Z and

Figure 7: Trade-offs of Firm Size-Dependent Policy

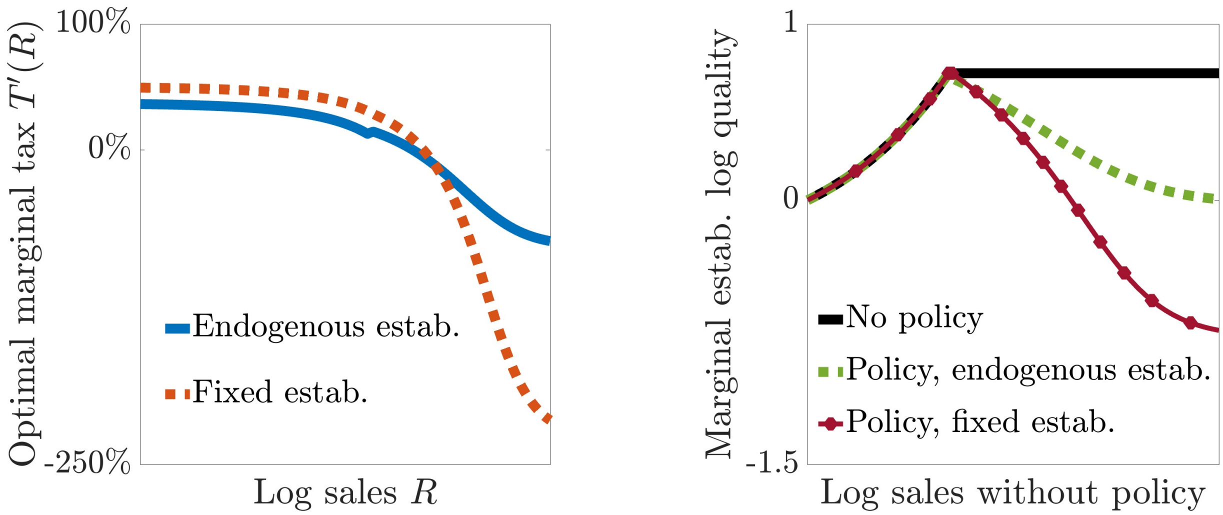




Left panel: the solid blue line is the marginal benefit of raising the marginal tax rate at sales R, holding fixed the measure of establishments chosen by firms with sales R; the dotted red line is the marginal benefit of raising the marginal tax rate at sales R, holding fixed production at each establishment chosen by firms with sales R. Both lines are computed in equilibrium under the optimal policy. Right panel: the log quality of a firm's marginal establishment, $\omega(i, N(i))$, without policy (solid black line) and under the optimal policy (dotted green line) as functions of the firm's log sales without policy.

consumption C by 5.9%, which is about four-fifths of the way to the first-best rather than half of the way under the genuinely optimal policy. However, if we set this policy and firms *do respond* by closing/opening establishments, then aggregate productivity Z increases by only 4.6% relative to no policy and consumption increases by only 2%. Hence, it is crucial to take into account the extensive margin of adjustment when designing policy: consumption rises by nearly twice as much (3.7% vs. 2%) when we incorporate all margins of adjustment into optimal policy design compared to when we ignore the extensive margin. The right panel of Figure 8 illustrates why: if we set the optimal policy ignoring the extensive margin, then large firms respond by openly dramatically lower quality establishments than they do in the competitive equilibrium and or under the genuinely optimal policy.

Figure 8: Importance of Endogenous Set of Establishments



Left panel: optimal revenue-neutral firm size-dependent marginal tax rate; the solid blue line is with each firm's measure of establishments endogenously chosen; the dotted red line is with each firm's measure of establishments fixed at its value without policy. Right panel: the log quality of a firm's marginal establishment, $\omega(i, N(i))$, without policy (solid black line), under the optimal policy (dotted green line), and in equilibrium under the fixed establishment optimal policy (textured red line) as functions of the firm's log sales without policy.

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