Market Concentration, Growth, and Acquisitions

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Abstract

I study an oligopolistic growth model in which firms can innovate by creatively destroying their competitors' goods, innovating on their own varieties, and developing new varieties. To avoid cannibalization, larger firms innovate disproportionately through creative destruction, which generates little growth but deters other firms from innovating. A fall in large firm fixed costs, calibrated to match the recent rise in industry concentration in the US, explains almost half the fall in growth from the 1990s to the 2010s, as well as the burst in growth during the late 1990s, the positive across-industry correlation between changes in concentration and growth, and the fall in growth relative to R&D expenditures. Despite this link between the rise of dominant firms and the fall in growth, a substantial reduction in taxes on large firm acquisitions of their competitors' goods *increases* growth and welfare: to preserve valuable acquisition opportunities, large firms engage in less creative destruction. Dispersion in large firm innovation costs across industries yields a novel theory of the inverted-U relationship between concentration and growth.

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1 Introduction

Many authors have documented a rising share of revenue going to the top firms in industries at the national level in the US since the 1990s.[1] This trend has spurred research into its connection to the recent decline in growth, as well as the policy implications.[2] Over a similar time period, there was a dramatic rise in the rate at which venture capital backed startups are acquired relative to the rate at which they go public.[3] Two important questions emerge: Is large firm behavior behind the fall in growth? If so, should antitrust authorities limit acquisitions to reduce industry concentration and promote growth? These questions highlight the importance of understanding how large firms' market shares shape their, and their competitors', innovation incentives.

I argue that taking into account how large firms' market shares affect the *way in which they innovate* rather than just *how much they innovate* can explain recent patterns in the data and has stark implications for optimal acquisition and innovation policy. The Arrow (1962) "replacement effect" states that for fear of cannibalization, large firms discount the value of innovation. I show that this argument only applies to the value of an innovation due to the growth it generates, and that large firms do not discount the value of an innovation that comes from taking sales directly from a competitor. Thus if firms have access to a variety of innovation technologies, large firms will tilt their efforts toward those that target their competitors' sales and generate relatively little growth. It follows that if a large firm's competitors' innovation rates are sufficiently responsive to the value of innovating, then a shock that induces large firms to innovate more *reduces* growth. In that case, policymakers face a novel trade-off when addressing large firm acquisitions of their smaller competitors' goods: a higher acquisition rate increases concentration and decreases growth, but a higher expected value of acquisitions to large firms reduces large firm innovation, leading to *lower* concentration and *faster* growth.

In Section 3, I formalize the theory in an endogenous growth model in which there is a continuum of industries, each of which consists of a single large firm and a continuum of small firms that produce a continuum of differentiated goods. Each firm can innovate by creating a new good (new good development) or by improving on an old good (creative destruction or own innovation). The measure of small firms in each industry is the same and is determined by undirected entry at the aggregate level. Revenue in each industry is fixed so that from a firm's perspective, innovation serves to shift revenue between firms, and from the household's perspective, innovation serves to

[1] See Autor, Dorn, Katz, Patterson, and Van Reenen (2020), Barkai (2020).

[2] See Cavenaile, Celik, and Tian (2021), Aghion, Bergeaud, Boppart, Klenow, and Li (2022), Akcigit and Ates (2021), De Ridder (2021), Liu, Mian, and Sufi (2022), Olmstead-Rumsey (2022), and Garcia-Macia, Hsieh, and Klenow (2019).

[3] See Pellegrino (2021).

grow industry productivity and thus real industry output.

The main mechanism, discussed in Section 4.2, concerns the difference between creative destruction and new good development / own innovation. When a firm develops a new good or improves on its own good, it takes revenue from its industry competitors by adding novel productivity to the industry, i.e., generating growth, which depreciates the value of old industry goods. When a firm creatively destroys a competitor's good, there are two effects: the innovator takes its competitor's good, which shifts revenue from the competitor without generating growth, and the innovator does so by improving on the good, which shifts revenue from all old goods by generating growth. This difference between creative destruction and new good development / own innovation has two key implications that drive the model's results. First, creative destruction imposes a large tax on firms relative to its effect on growth. It follows that the more innovation is geared toward creative destruction of small firms' goods, the more small firms are deterred from entering and innovating for a given growth rate. Second, large firms have a relative preference for creative destruction compared to small firms because they prefer to take revenue directly from a competitor rather than from all the goods in their industries, many of which they produce.

In section 5.1, I prove two qualitative results. Across industries, dispersion in large firm fixed or innovation costs drives dispersion in concentration and growth. The large firm's revenue share is a sufficient statistic for the industry growth rate, and growth as a function of concentration displays an inverted-U shape, as observed in the data.[4] On the other hand, in the long-run following a permanent aggregate shock to large firm fixed or innovation costs, if large firm revenue shares are higher, then the growth rate is lower. Intuitively, the entry rate does not respond to industry-specific shocks, so small firm innovation is more responsive to a rise in large firm innovation at the aggregate level. Thus, an aggregate rise in large firm innovation only shifts innovation toward creative destruction, which lowers growth, whereas an industry-specific rise also increases total innovation, which can increase growth. A further implication is that in the short-run following an aggregate shock, concentration and growth can both increase because the measure of small firms is slow to adjust.

In Sections 5.2 and 5.3, I calibrate the model to US data in the early 1990s—in particular, the labor flows data Garcia-Macia, Hsieh, and Klenow (2019) use to determine the share of growth due to creative destruction—and find that a fall in large firm fixed costs that generates the rise in concentration observed in the data can explain 41% of the eventual fall in the average growth rate.[5]

[4] See Cavenaile, Celik, and Tian (2021) for a direct measure of growth as a function of industry concentration, and Aghion, Bloom, Blundell, Griffith, and Howitt (2005) for an indirect measure that uses markups as a proxy for concentration.

[5] See Garcia-Macia, Hsieh, and Klenow (2019).

I compare a variety of the model's predictions to the data. The model provides a parsimonious explanation for the observed temporary burst in aggregate growth in the late 1990s,[6] as well as the positive across-industry correlation between changes in concentration and growth.[7] Each is driven by the missing response of the measure of small firms, which reacts slowly and only to aggregate shocks. Following the shock, the entry rate declines substantially, and then partially recovers in the long-run, as in the data.[8] Finally, in both the model and in the data, R&D shifts toward large firms that generate a large amount of sales relative to their R&D expenditures, yet growth relative to R&D falls.[9] Large firms are reluctant to innovate, and so their innovation is more efficient at generating revenue on the margin, but it is inefficiently focused on creative destruction and so generates little growth. This explanation for the fall in growth relative to R&D stands in contrast to an alternative explanation in De Ridder (2021) in which innovation shifts toward firms with *high* R&D expenditures relative to sales, implying a fall in its efficiency.

In Section 6, I introduce into the calibrated model the possibility that large firms can make take-it-or-leave-it offers to their smaller competitors to acquire their goods. I evaluate the effects of a tax on these acquisitions and find that a sufficiently low tax or high subsidy leads to a higher growth rate and an increase in welfare relative to the economy without acquisitions. The required subsidy to increase welfare is substantially lower in an economy calibrated to a higher level of industry concentration.

Intuitively, for a large firm, acquiring competitors' goods and innovating are substitutes. All else equal, an acquisition increases a large firm's revenue share, which shifts its innovation toward creative destruction and lowers growth. But if acquisitions are sufficiently valuable to the large firm, then the large firm reduces innovation to preserve acquisition opportunities, which increases growth. This effect is particularly powerful if industries are more concentrated because then large firm innovation is particularly destructive.

It follows that policy should not place unnecessary road blocks in the way of high surplus acquisitions, particularly in concentrated industries, but should block acquisitions that generate little surplus. Allowing acquisitions is an effective way to limit large firm innovation because unlike a tax, it does not require intimate knowledge of industry boundaries.

On the other hand, I find that allowing large firms to acquire their smaller competitors' innovation technologies reduces growth and welfare. These acquisitions only shift innovation from small to

[6] See Garcia-Macia, Hsieh, and Klenow (2019).

[7] See Ganapati (2021).

[8] See Decker, Haltiwanger, Jarmin, and Miranda (2016).

[9] See Olmstead-Rumsey (2022).

large firms, which tilts innovation toward creative destruction and leads to a lower growth rate. The theory provides a novel motivation for large firms to make these acquisitions, absent any exogenous synergies. In the calibrated model, a large firm is willing to pay more than 3 times a small firm's value, whereas in an alternative calibration without creative destruction, a large firm is willing to pay only 10% of a small firm's value, i.e., it is not willing to pay enough to acquire a small firm. Without creative destruction, an acquisition does not increase joint profits because the acquirer reduces the acquired firm's innovation, and other small firms respond by innovating more, undoing the benefit. With creative destruction, the large acquirer can prevent the acquired firm from creatively destroying its goods, which does not greatly impact other small firms, and therefore does not encourage them to substantially increase their innovation.

Testable empirical predictions of the model's main mechanism are that firms can direct their efforts at improving goods toward their competitors' goods, and that smaller firms face higher discount rates on the profits from their innovations. The theory implies that this disparity increased as market concentration rose and growth fell in the US since the 1990s. Argente, Lee, and Moreira (2021) show that the revenues of high sales products depreciate more quickly than the revenues of low sales products, in line with the prediction that such products would be creatively destroyed more quickly if firms can direct their creative destruction efforts. Akcigit, Alp, and Peters (2021) show that a relatively high creative destruction rate for goods produced by firm types that innovate less, and tend to be small, can explain the high employment shares of old firms in US and Indian data. My theory provides an explanation for this disparity: firms focus their goods improvement efforts on their competitors' goods, so more innovative firms avoid a larger fraction of total creative destruction.

These results highlight an important subtlety in optimal competition and innovation policy. In theories that focus only on *how much* large firms produce or innovate, high markups or the Arrow (1962) "replacement effect" imply that it is optimal to subsidize high market shares and encourage *more* large firm production or innovation.[10] In these models, reducing large firm innovation is a cost of acquisitions, rather than a benefit.[11] Instead, taking into account *the way in which* large firms innovate, policies that encourage large firm production or innovation backfire. Policies that offer large firms a tempting alternative to innovation increase growth by shifting activity to smaller firms that innovate in a more socially optimal way.

Large Firm Market Shares and Innovation:

Previous work on industries with oligopolistic competition and innovation mostly focuses on the

[10] See Edmond, Midrigan, and Xu (2021).

[11] See Fons-Rosen, Roldan-Blanco, and Schmitz (2022).

impact of a large firm's market share on its *magnitude* of innovation.[12] A notable exception is the theory put forward in Argente, Baslandze, Hanley, and Moreira (2021) and mentioned in Akcigit and Ates (2021) that large firms use patents to deter competition. Although the mechanism is different, the result is similar in that large firm innovation generates little growth relative to the rate at which it deters small firm innovation. This theory is complementary to the one proposed here because creative destruction is most relevant in industries with weak patent protection.

Previous theories of the inverted-U relationship between market concentration and growth across industries, such as in Aghion, Bloom, Blundell, Griffith, and Howitt (2005) and Cavenaile, Celik, and Tian (2021), are based on the relative sizes of the "escape competition" and the Schumpeterian effects of competition. Smaller market shares increase pressure for firms to innovate to escape their competitors, but the expectation of the competition that pushes down market shares discourages innovation. In this paper, I suppose that variation across industries is driven by variation in the large firm's incentives to innovate, perhaps due to granularity. The magnitude of small firm innovation depends only on the Schumpeterian effect: as the expected rate of innovation increases, small firms innovate less. In an industry with a more innovative large firm, total innovation is higher but innovation is shifted toward creative destruction, which generates less growth relative to the tax it imposes on small firms. If the large firm is relatively small, then it innovates like a small firm and the former effect dominates. If the large firm has a substantial revenue share, then the latter effect dominates. An important implication is that if small firms are more responsive to economy-wide changes, then widespread higher concentration tends to be associated with lower aggregate growth even if across industries or within an industry over time, higher concentration is associated with higher growth.

Large Firm Acquisitions of Small Competitors' Goods:
"Entry for buyout", described in Rasmusen (1988) and more recently, Letina, Schmutzler, and Seibel (2021) and Fons-Rosen, Roldan-Blanco, and Schmitz (2022), is another theory in which a high expected value of future acquisitions increases growth even though the distributional consequences of actual acquisitions do not. If an acquired firm receives a fraction of the surplus from the acquisition, then the expectation of being acquired increases the value of entry and innovation. Yet, if large firm innovation is one-dimensional, the entry for buyout effect needs to be weighed against the negative effect on large firm innovation; large firms have less reason to innovate if it erodes potential acquisition opportunities. In this paper, acquisitions are made with take-it-or-leave-it offers, so small firms face no incentive to enter or innovate to be acquired, and the entry for buyout effect is not present. Instead, acquisitions may be useful *because* they reduce large firm innovation, not in spite of it. A distinct implication of this theory is that expected acquisition

[12] See Shapiro (2012) for a discussion, and Cavenaile, Celik, and Tian (2021) for a recent example.

opportunities are more beneficial when concentration is higher.

"Killer acquisitions", described in Cunningham, Ederer, and Ma (2021) and then in Letina, Schmutzler, and Seibel (2021) and Fons-Rosen, Roldan-Blanco, and Schmitz (2022), are acquisitions in which the acquiring firm does not produce the acquired good. The possibility of killer acquisitions is an additional potential cost of acquisitions because consumers cannot benefit directly from innovations that are not put into practice. Nonetheless, it only strengthens the positive effects of acquisitions on innovation discussed in this paper. The positive effect on growth of the expected value of future acquisitions is the same because if killer acquisitions are valuable to a large firm, then the large firm still reduces innovation to maintain acquisition opportunities. Moreover, the negative effect on growth of the actual acquisitions is lower because killer acquisitions are not produced and therefore have a smaller effect on large firms' revenue shares.

The Rise in Concentration and The Fall in Growth:
This paper is related to recent papers that study the effect of high productivity or superstar firms on growth, such as Aghion, Bergeaud, Boppart, Klenow, and Li (2022), De Ridder (2021), Cavenaile, Celik, and Tian (2021), and Liu, Mian, and Sufi (2022). In terms of the theory's ability to match the data, it is the first to explain with a single shock the short-run burst in growth, the positive across-industry correlation between changes in concentration and growth, the longer-run aggregate rise in concentration and fall in growth, and the fall in growth relative to R&D despite a shift in R&D expenditures toward large firms with low R&D to sales ratios. My theory is complementary to previous work in the sense that it does not rely on a specific shock, but suggests that any shock that shifts innovation and sales from small firms to large firms will have the additional effect of shifting innovation toward creative destruction and thereby reducing growth.

Aghion, Bergeaud, Boppart, Klenow, and Li (2022) and De Ridder (2021) study the effects of productivity dispersion across small firms, whereas I abstract from productivity differences and study how a large firm's innovation is shaped by its size. They find that increased competition from high productivity competitors reduces less productive firms' markups, and therefore their incentive to grow. The channel I study in this paper is complementary in the sense that they focus on the flow profits a small firm receives from innovating, whereas I focus on the effective discount rate on small firm profits.

Liu, Mian, and Sufi (2022) study a growth model with two large firms in each industry, and find that a large firm can reduce growth by building a substantial productivity advantage over its competitor: a greater advantage implies that the large firm will optimally cut its price by more in response to innovation by its competitor. An important difference is that my mechanism does not rely on a large firm responding directly to the actions of a single competitor. Thus, it may be more relevant when thinking about an industry with both large and small firms. Finally, Cavenaile, Celik, and Tian (2021) study a growth model with large firms, but the pressure those large firms place on small firms has no effect on growth because small firms always have zero profits.

Akcigit and Ates (2021) and Olmstead-Rumsey (2022) propose theories in which exogenous changes in the economy's innovation technology cause a decline in growth, as well as a rise in market concentration. The theory I propose reverses the causality and suggests that changes in industry structure—a rise in concentration—drive a decline in growth. Moreover, it provides an alternative explanation for the fall in the effect of a patent on a firm's market value documented in Olmstead-Rumsey (2022): the innovation is more quickly creatively destroyed by a large firm.

Model Building Blocks:
The model builds on two different strands of the growth literature, one focused on models of creative destruction[13], and one on expanding varieties models[14]. Recent work combines the two, but without large firms with positive market shares.[15]

2 A Simple Industry Model

I first describe a simple industry model of creative destruction with a large firm that does not creatively destroy its own goods. The model illustrates the key mechanism of the theory. If firms can target their efforts at improving goods toward their competitors' goods (creative destruction) rather than their own, then large firms will generate less growth for a given rate at which they creatively destroy their competitors' goods. If small firm innovation is sufficiently responsive to the rate at which small firm goods are creatively destroyed, then an increase in innovation by large firms reduces overall growth. The simple model also demonstrates important empirical predictions of this mechanism. Firms can direct creative destruction efforts toward their competitors, and smaller firms face higher discount rates on the profits from their innovations.

Time is continuous and indexed by $t \in [0, \infty)$. There is a unit measure of goods, indexed by $j \in [0, 1]$, each of which always receives the same revenue. At each time t, there is a measure N_t of small firms, and a single large firm. Each good is produced by a single firm at each time t, with productivity $z_t(j)$. The producer of a good receives flow profits π. Firms innovate by

[13]See Grossman and Helpman (1991b), Aghion and Howitt (1992), Klette and Kortum (2004), and Aghion, Bergeaud, Boppart, Klenow, and Li (2022).

[14]See Romer (1990) and Grossman and Helpman (1991a).

[15]See Atkeson and Burstein (2019).

creatively destroying each others' goods. When a firm creatively destroys a good, it becomes the sole producer of that good, and the good's productivity, $z_t(j)$, is multiplied by $\lambda > 1$.

Industry productivity is $Z_t = \int_0^1 z_t(j)dj$, and the industry growth rate is $g_t = \frac{1}{Z_t}\frac{\partial Z_t}{\partial t}$. Denote by $S_t \in [0,1]$ the measure of goods the large firm produces at time t, which is also the large firm's share of industry revenue.

Each small firm chooses a distribution of creative destruction rates, $\kappa_t(j)$, so that it creatively destroys a good at Poisson arrival rate $\int_0^1 \kappa_t(j)dj$, and the relative probability it creatively destroys good j is proportional to $\kappa_t(j)$. The flow cost of innovation is $\int_0^1 \kappa_t(j)^2 dj$. Small firms choose innovation rates to maximize the expected present discounted value of profits, where they discount future payouts by the real interest rate r.

In this simple model, I take the large firm's innovation decisions as given. The large firm creatively destroys goods produced by small firms at a flow rate rather than a Poisson arrival rate: in a finite interval of time, each small firm creatively destroys a finite *number* of goods, and the large firm creatively destroys a finite *measure* of goods. *The large firm does not creatively destroy its own goods.*

2.1 Long-Run Industry Concentration and Growth

I study balanced growth path Markov Perfect equilibria as a function of the large firm's exogenous innovation rate. All small firms creatively destroy all goods at the same rate, κ_S, since each good yields the same flow profits. The large firm creatively destroys small firms' goods at the exogenously given rate κ_L. The large firm's industry revenue share and the industry growth rate are constant over time at S and g, respectively. I consider two cases for determining the constant measure of small firms, N. In the first case, N is given by a free entry condition that fixes the value of being a small firm producing zero goods. In the second case, N is exogenously given.

I analyze the effects of changes in the rate at which the large firm creatively destroys small firms' goods, κ_L, on the large firm's industry revenue share, S, and growth, g, in the long-run. Since S is strictly increasing in κ_L in equilibrium, we can write g as a function of S. On a balanced growth path, the large firm's industry revenue share and growth are

$$S = \frac{\kappa_L}{N\kappa_S + \kappa_L}; \qquad g = (\lambda - 1)(N\kappa_S + (1 - S)\kappa_L). \tag{1}$$

Small firms creatively destroy goods and generate growth at rate $N\kappa_S$, and the large firm creatively destroys goods and generates growth at rate $(1 - S)\kappa_L$ since it does not creatively destroy its own goods. Small firm creative destruction is given by the First Order Condition:

$$\kappa_S = \frac{\pi/2}{r + N\kappa_S + \kappa_L},$$

where $N\kappa_S + \kappa_L$ is the Poisson arrival rate at which each small firm's good is creatively destroyed.

If N is determined by the free entry condition, then since the value of being a small firm producing zero goods is pinned down by the optimal small firm creative destruction rate, it follows that for any exogenous large firm creative destruction rate, κ_L, the total rate at which a small firm's good is creatively destroyed, $N\kappa_S + \kappa_L$, must be the same. From equation (1), the derivative of growth with respect to the large firm's industry revenue share is then

$$g'(S) = \frac{-2S}{1+S}\frac{g}{1-S},$$

which is always negative. If N is exogenously given, then

$$g'(S) = \left(\frac{-2S}{1+S} + \frac{1+r_0}{2+r_0}\right)\frac{g}{1-S},$$

where r_0 is the interest rate relative to the total rate of creative destruction: $r_0 = r/(N\kappa_S + \kappa_L)$. Since r_0 is strictly decreasing in S, in this case the derivative of growth with respect to the large firm's industry revenue share is negative if and only if $S > S^*$ for some $S^* \in (0, 1)$.

The derivative in the free entry case, and the first term of the derivative in the exogenous N case, is the *composition effect*. Holding fixed the total rate at which small firm goods are creatively destroyed, total small firm creative destruction falls one-for-one with the rise in large firm creative destruction. Small firm creative destruction contributes to growth at rate $\lambda - 1$ and large firm creative destruction contributes to growth at rate $(1 - S)(\lambda - 1)$, so the effect of the fall in the former always outweighs the effect of the rise in the latter, leading to a decreasing relationship between growth and the large firm's industry revenue share. The second term of the derivative in the exogenous N case is the *total innovation effect*. Holding fixed the ratio of small firm creative destruction to large firm creative destruction, $N\kappa_S/\kappa_L$, the total rate at which small firm goods are creatively destroyed, and thus growth, are increasing in large firm creative destruction. If the measure of small firms is exogenously given, then both the composition and total innovation effects are present because each small firm faces a convex innovation cost. If the large firm's revenue share is sufficiently small, then it makes little difference that the large firm does not creatively destroy its own goods, so the total innovation effect dominates the composition effect, which implies an increasing relationship between growth and the large firm's revenue share.

In the quantitative model I study in future sections, potential entrants pay entry costs not knowing into which industry they will enter. Thus, I interpret the exogenous N case as describing the relationship between concentration and growth *across industries* due to dispersion in large firm creative destruction, κ_L, as well as the short-run aggregate relationship between average concentration and growth due to widespread changes in κ_L. The free entry case describes the long-run aggregate relationship between average concentration and growth *over time* due to widespread changes in κ_L. Across industries, growth as a function of the large firm's revenue share exhibits an inverted-U shape. Following an aggregate increase in large firm innovation, in the short-run, concentration and growth may both increase, but in the long-run, concentration rises while growth falls.

3 Quantitative Model

Time is continuous and indexed by $t \in [0, \infty)$. There is a unit measure of industries, each of which consists of a measure of differentiated intermediate goods. There is a representative household who consumes the numeraire final good and inelastically supplies $\bar{L}$ units of labor. There is a representative final good producer that earns zero profits and purchases differentiated goods produced within each industry to convert into the final good for sale to the household. In each industry, a single large firm and a continuum of small firms use labor to produce, develop new goods, and creatively destroy old goods.

3.1 Representative Household

The household chooses a path of final good consumption to maximize the present discounted value of its utility:

$$\int_0^\infty e^{-\rho t} \ln(C_t) dt$$

subject to the budget constraint at each time t, $C_t = W_t \bar{L}_t + \Pi_t$, where C_t is final good consumption, the final good price is normalized to 1, W_t is the wage, and Π_t is flow profits from firms. The household owns all firms in the economy and takes the wage and profits as given. The household stochastic discount factor is $e^{-\rho t}/C_t$, and I denote its negative rate of change over time by

$$r_t = \rho + \dot{C}_t / C_t, \tag{2}$$

where a dot over a variable indicates its derivative with respect to time.

3.2 Representative Final Good Producer and Demand

At each time t, the representative final good producer chooses purchases of each good in each industry and sales of the final good to maximize profits:

$$C_t - \int_0^1 \int_0^{M_{n,t}} p_{n,t}(j) c_{n,t}(j) dj dn$$

subject to the production/aggregation functions

$$ln(C_t) = \int_0^1 ln(C_{n,t}) dn \qquad C_{n,t}^{\frac{\gamma-1}{\gamma}} = \int_0^{M_{n,t}} c_{n,t}(j)^{\frac{\gamma-1}{\gamma}} dj \quad \text{for all } n \in [0,1],$$

where $M_{n,t}$ is the measure of goods available at time t in industry n, $p_{n,t}(j)$ and $c_{n,t}(j)$ are the price and real purchases of good j in industry n, respectively, and $\gamma > 1$ is the within-industry elasticity of substitution. The final good producer takes prices as given and earns zero profits. The First Order Condition for good j in industry n, along with the zero profit condition, implies the demand curve

$$c_{n,t}(j) = p_{n,t}(j)^{-\gamma} P_{n,t}^{\gamma-1} C_t, \tag{3}$$

where the industry price index is given by

$$P_{n,t}^{1-\gamma} \equiv \int_0^{M_{n,t}} p_{n,t}(j)^{1-\gamma} dj. \tag{4}$$

3.3 Intermediate Goods Producers

Each industry consists of a measure N_t of small firms, indexed by $i \in [0, N_t]$, and a single large firm, denoted by $i = L$. Since entry is undirected across industries, the measure of small firms can vary over time, but not across industries.

3.3.1 Production and Competition

At each time t, production occurs in two stages. Each firm can potentially produce a version of each good in its industry. All versions of good j in industry n are perfect substitutes, but each firm's version is produced with a version specific productivity $z_{n,i,t}(j)$, which denotes how many quality-adjusted units of the good are produced per unit of input.[16] Let $z_{n,t}(j) \equiv \max\{z_{n,i,t}(j)\}_{i\in[0,N_t]\cup\{L\}}$ be the highest productivity version of good j in industry n at time t.

[16] It is equivalent to think of version specific productivity as quality.

In the first stage of production, firms simultaneously choose for which goods in the industry they will pay a fixed cost to access their version of the good. If firm i pays the fixed cost for good j, then they can produce good j in the second stage with production function

$$q_{n,i,t}(j) = z_{n,i,t}(j) l_{n,i,t}(j). \tag{5}$$

Otherwise, they can produce good j with productivity $z_{n,t}(j)/\sigma$, where $\sigma > 1$ captures the ability of firms to imitate each other's goods. In the second stage of production, firms simultaneously choose prices for each good to maximize static profits.

I set fixed cost sufficiently low so that all goods are produced in equilibrium. In equilibrium, a firm i pays the fixed cost and has positive sales of good j only if it is the most productive producer, i.e., $z_{n,i,t}(j) = z_{n,t}(j)$. We will see that the most productive producer of each good is unique. Its markup is thus constrained to be less than σ so that its price is less than the marginal cost of other firms that can produce an imitation of its version.

To simplify the analysis, I make the following assumption, which implies that all firms set a markup of σ on all goods, as we will see in Section 4.1:

Assumption 1. $\sigma \leq \gamma/(\gamma - 1)$.

Let $Z_{n,t}$ be an aggregate of productivity in industry n:

$$Z_{n,t}^{\gamma-1} \equiv \int_0^{M_{n,t}} z_{n,t}(j)^{\gamma-1} dj,$$

and define the relative productivity of good j: $\tilde{z}_{n,t}(j) \equiv (z_{n,t}(j)/Z_{n,t})^{\gamma-1}$. Going forward, I will usually characterize a good by its relative productivity. Define Z_t to be an aggregate of productivity across the economy:

$$\ln(Z_t) \equiv \int_0^1 \ln(Z_{n,t}) dn.$$

For small firms, the per-good fixed cost is $\chi_{F,S}(z_{n,i,t}(j)/Z_{n,t})^{\gamma-1}$ units of labor, and for the large firm in industry n, the per-good fixed cost is $\chi_{F,L,n}(z_{n,L,t}(j)/Z_{n,t})^{\gamma-1}$ units of labor.

3.3.2 Innovation

At each time t, each firm chooses two types of innovation rates to maximize the expected present discounted value of profits using the household stochastic discount factor: a rate of creative destruction for each good in its industry, and a rate of new good development. Conditional on creatively destroying good j, a firm's productivity for that good becomes $\lambda z_{n,t}(j)$, where $\lambda > 1$. Conditional on developing a new good, a firm's productivity for that good, λz, is drawn so that the expected value of $z^{\gamma-1}$ is equal to $Z_{n,t}^{\gamma-1}$.

Small Firms:

A small firm's innovation technology consists of a single *entrepreneur*. It chooses a distribution of creative destruction rates for goods in its industry, $\kappa_{n,i,t}(j)$, so that it creatively destroys a good at Poisson arrival rate $\int_0^{M_{n,t}} \kappa_{n,i,t}(j)dj$, and the relative probability it creatively destroys good j is proportional to $\kappa_{n,i,t}(j)$. The flow labor cost of creative destruction is

$$\alpha^{-1}\chi_C \int_0^{M_{n,t}} \tilde{z}_{n,t}(j)\kappa_{n,i,t}(j)^{\alpha}dj,$$

where $\alpha > 1$. A small firm chooses a Poisson arrival rate at which it develops a new good, $\delta_{n,i,t}$, subject to flow labor cost $\alpha^{-1}\delta_{n,i,t}^{\alpha}$. The costs of creatively destroying each good and of developing a new good are independent, and each scales with the expected relative productivity of the innovation. Finally, χ_C is the cost of creative destruction relative to new good development.

Large Firms:

In industry n, the large firm's innovation technology consists of a *measure* $\chi_{L,n}^{-1/(\alpha-1)}$ of entrepreneurs. It thus chooses a Poisson arrival rate at which it creatively destroys *each* good in its industry, $\kappa_{n,L,t}(j)$, and a *continuous rate* at which it develops a new good, $\delta_{n,L,t}$, subject to total flow labor cost

$$\alpha^{-1}\chi_{L,n}\left(\chi_C \int_0^{M_{n,t}} \tilde{z}_{n,t}(j)\kappa_{n,L,t}(j)^{\alpha}dj + \delta_{n,L,t}^{\alpha}\right),$$

where I take as given that the large firm optimally distributes its innovation across its entrepreneurs to minimize cost.

Small vs. Large Firms:

A small firm creatively destroys a single good and develops a new good at Poisson arrival rates, and thus becomes the most-productive producer of a finite number of goods in finite time. A large firm creatively destroys a single good and develops a new good at a continuous rate, and thus becomes the most-productive producer of a finite measure of goods in finite time.

To be clear, a firm can creatively destroy a good that it already produces. For a small firm, this possibility is irrelevant since each small firm produces finitely many goods, and creatively destroys each good at an infinitesimal rate. For a large firm, this possibility is meaningful, and implies that the innovative capacity of the economy is not mechanically reduced as the large firm's share of old innovations grows.

3.3.3 Entry and Exit

Entry is undirected, so an entering firm draws an industry from the uniform distribution. At each moment in time, there is an infinite mass of potential entrants. If a potential entrant pays the cost of entry, then they draw an industry and enter as a single small firm with a 0 productivity version of each good and an entrepreneur they can use to innovate. Otherwise, the potential entrant receives value 0. The total cost of entry is increasing in the entry rate and is $\chi_E E_t^{\epsilon}$ units of labor, where E_t is the entry rate, χ_E is a cost-shifter, and $\epsilon \geq 1$ is the elasticity of total entry costs with respect to the entry rate. Thus, the marginal entrant faces an entry cost of $\chi_E \epsilon E_t^{\epsilon-1}$. At the lower bound for the elasticity, $\epsilon = 1$, the marginal entry cost is constant and there is a free entry condition. At the upper bound for the elasticity, $\epsilon = \infty$, the marginal entry cost is 0 if $E_t < 1$ and infinite if $E_t > 1$. In that case, the entry rate is always 1.

Each small firm exits exogenously at Poisson arrival rate $\eta > 0$. When a firm exits, it is still able to produce, but it loses its entrepreneur so that it can no longer innovate.

3.4 Equilibrium

At each moment in time, the goods market must clear, i.e., the amount each firm supplies of each good is equal to the representative household's demand for that good, and the labor market must clear, i.e., the labor used for production, fixed costs, innovation, and entry costs must equal the labor the household inelastically supplies.

I characterize each good by its type $f \in \{S, L\}$, which denotes whether the good's current producer is a small firm (S) or the large firm (L). Let $T_{n,t}(j)$ be good j's type in industry n at time t. For each type $f \in \{S, L\}$, define $\tilde{Z}_{f,n,t}$ to be the industry n measure of relative productivity of goods of type f at time t:

$$\tilde{Z}_{f,n,t} = \int\limits_{j:T_{n,t}(j)=f} \tilde{z}_{n,t}(j)dj,$$

where $\tilde{z}_{n,t}(j)$ is defined in Section 3.3.1. It follows that $\tilde{Z}_{S,n,t} + \tilde{Z}_{L,n,t} = 1$.

The industry state is the industry measure of relative productivity of goods of type L, $\tilde{Z}_{L,n,t}$. The aggregate state is the measure of small firms in each industry, N_t, and the distribution of industry states across industries.

I study Markov Perfect Equilibria in which firms' markups in industry n are given by static optimization of profits and are a function only of $\tilde{Z}_{L,n,t}$. In particular, markups are not a function of the industry aggregate of productivity, $Z_{n,t}$, the measure of small firms, N_t, or of time t. Firms' innovation decisions are given by dynamic optimization of expected discounted profits and are functions only of the industry state, $\tilde{Z}_{L,n,t}$, and the aggregate state when converging to a balanced growth path. Innovation decisions do not depend on the level of productivity, $Z_{n,t}$. Moreover, each firm creatively destroys all goods of each type f at the same rate. The entry rate is given by potential entrant dynamic optimization of expected discounted profits net of the marginal entry cost and is a function only of the aggregate state.

To be clear, firms can always observe all features of the economy when optimizing, but they suppose that other firms' actions depend only on the variables mentioned above. I show that it is then optimal for each firm also to condition their own actions only on the variables mentioned above.

I focus on balanced growth path equilibria and the convergence to a balanced growth path following unanticipated shocks. A balanced growth path is an equilibrium in which $\tilde{Z}_{L,n,t}$ is constant over time in each industry, the measure of small firms N_t is constant over time, and each firm's innovation decisions are functions only of $\tilde{Z}_{L,n,t}$ in their industry. It follows that aggregate productivity Z_t given by $\ln(Z_t) \equiv \int_0^1 \ln(Z_{n,t})dn$ grows at a constant rate.

3.5 Evolution of the Industry State and Growth

We will see in Section 4.2 that all small firms in industry n choose the same innovation rates. Moreover, each small firm creatively destroy all goods at the same rate. Thus, let $\kappa_{S,n,t}$ denote this rate of creative destruction, and let $\delta_{S,n,t}$ denote the rate at which small firms develop new goods. It follows that as a group, small firms in industry n creatively destroy each good at rate $N_t\kappa_{S,n,t}$, and develop a new good at rate $N_t\delta_{S,n,t}$. Let $\kappa_{L,n,t}(f)$ denote the rate at which the large firm in industry n creatively destroys a good that is currently produced by a type $f \in \{S, L\}$ firm, and let $\delta_{L,n,t}$ denote the rate at which the large firm develops new goods.

The industry state $\tilde{Z}_{L,n,t}$ evolves over time according to

$$\begin{aligned}\dot{\tilde{Z}}_{L,n,t} =& \lambda^{\gamma-1}\left(\delta_{L,n,t} + \left(1 - \tilde{Z}_{L,n,t}\right)\kappa_{L,n,t}(S) + \tilde{Z}_{L,n,t}\kappa_{L,n,t}(L)\right) \\ &- \tilde{Z}_{L,n,t}(N_t\kappa_{S,n,t} + \kappa_{L,n,t}(L)) - \tilde{Z}_{L,n,t}(\gamma - 1)g_{n,t},\end{aligned} \tag{6}$$

where $g_{n,t}$ is the growth rate of industry productivity $Z_{n,t}$:

$$(\gamma-1)g_{n,t} \equiv \frac{\partial Z_{n,t}^{\gamma-1}/\partial t}{Z_{n,t}^{\gamma-1}} =(\lambda^{\gamma-1}-1)\left(N_t\kappa_{S,n,t}+\left(1-\tilde{Z}_{L,n,t}\right)\kappa_{L,n,t}(S)+\tilde{Z}_{L,n,t}\kappa_{L,n,t}(L)\right) + \lambda^{\gamma-1}(N_t\delta_{S,n,t}+\delta_{L,n,t}). \tag{7}$$

In (6) for the evolution of the industry state over time, the first line is the inflow due to new good development, creative destruction of small firms' goods, and creative destruction of the large firm's own goods. The first term on the second line is the outflow due to creative destruction of the large firm's goods, and the last term is the outflow due to growth in $Z_{n,t}^{\gamma-1}$, which reduces relative productivity. In the expression for the industry growth rate, the first line is growth from creative destruction: the -1 in $\lambda^{\gamma-1}-1$ reflects the destroyed productivity of the old good; and the second line is growth from new good development in which all the productivity of new goods is novel.

3.6 Aggregation and Welfare

We will see in Section 4.1 that all firms set a markup σ on all goods. It follows that at time t, each industry uses the same quantity of labor in production, L_t^p, which implies that aggregate final good consumption is $C_t = Z_t L_t^p$, industry n consumption is $C_{n,t} = Z_{n,t}L_t^p$, and recalling that the final good price is normalized to 1, the wage is $W_t = Z_t/\sigma$. Household welfare is

$$\int_0^\infty e^{-\rho t}\left(\ln(Z_t)+\ln\left(L_t^p\right)\right)dt = \frac{\ln(Z_0)}{\rho}+\frac{\int_0^\infty \rho e^{-\rho t}g_t dt}{\rho^2}+\frac{\int_0^\infty \rho e^{-\rho t}\ln\left(L_t^p\right)dt}{\rho}, \tag{8}$$

where g_t is the growth rate of aggregate productivity Z_t given by $g_t = \int_0^1 g_{n,t}dn$. Welfare depends on current productivity and weighted averages of future growth and labor used in production. Since growth in one period raises consumption in all future periods, it is discounted by ρ^2 rather than ρ.

Along a balanced growth path, the labor used in production is constant at L^p, and final good consumption and the wage grow at the aggregate productivity growth rate g. Welfare is

$$\ln(Z_0)/\rho + g/(\rho^2) + \ln(L^p)/\rho.$$

3.7 Model Discussion

Before solving the model, I discuss some of the main modeling choices. Fixed costs and innovation costs scale with the relative productivity of the good. A version of this is necessary for the existence of a balanced growth path, and it implies that firms need not keep track of the distribution of productivity across goods, but just the fraction produced by the large firm. Otherwise, firms would pay fixed costs for some goods, but not others, or firms would creatively destroy some goods more aggressively than others.

The large firm fixed cost may vary across industries and from the small firm fixed cost to capture that large firms may face additional costs from maintaining many goods or may face lower per-good costs by paying a fixed cost at the firm level, and that this force may depend on the industry. The large firm innovation cost may vary across industries to capture dispersion across industries due to granularity: some industries randomly have a particularly innovative large firm.

We can interpret new good development as firms innovating on the goods for which they already have the most productive version. In either case, any gains enjoyed by a firm come from adding productivity to the industry, not from taking productivity from another firm. With own good innovation rather than new good development, we may expect that the innovation cost should scale inversely with the number or measure of goods for which the firm already has the most productive version. In that case, the results are largely unchanged, except that in the main experiment in Section 5.3, the size of the required shock is smaller, and there is a fall in the labor used for innovation costs (there is an endogenous shift in innovation technology to large firms). Moreover, Theorems 1 and 2 continue to hold.

The important consequence of Assumption 1 is that large and small firms set the same markup and that a large firm's markup does not depend on its industry revenue share. Otherwise, as large firms gain revenue share, they set higher markups, which reduces competition and encourages growth from small firms. This effect is mitigated or reversed if large firms have a higher firm specific process efficiency.[17] In that case, large firms may set higher markups than small firms, but lower prices. As large firms gain revenue share, the industry price index falls, which decreases small firms' incentive to innovate. Allowing for process efficiency variation complicates the analysis because if it is sufficiently big, then a large firm may continue to be the equilibrium producer of a good even after its most productive version is creatively destroyed. I abstract from these issues and focus on the endogenously different ways in which small and large firms innovate.

A consequence of imposing that all entrants are small is that the value of being large does not factor into the value of entering. This choice makes sense if large firms exit at much lower rates because they are then over represented in the cross section relative to their salience for a potential entrant. For example, if 1% of firms in a steady state are large and they exit half as quickly, then

[17] See Aghion, Bergeaud, Boppart, Klenow, and Li (2022).

only 0.5% of entrants are large. Moreover, as large firm exit rates go to 0, their discounted profits do not become arbitrarily big as long as the interest rate is above 0. A similar point stands if new firms take time to become large. Finally, ignoring large firm profits in the entry decision implies that in the main experiment in Section 5.3, it is not particularly important to consider whether large firms have to pay higher firm level fixed costs to lower their per-good fixed costs.

4 Firm Optimization

Before describing the firm problem, note that since small firms take industry aggregates as given, we can split their static profit maximization problem into a separate problem for each good. Moreover, when innovating, a small firm's problem is the same regardless of the goods it produces.

4.1 Static Profit Maximization: Prices

At each time t, firms choose prices simultaneously to maximize static profits. A small firm that is the most productive producer of a good with relative productivity $\tilde{z}_{n,t}(j)$ takes as given the industry price index, the wage, and industry revenue, and chooses a price to maximize static profits subject to the demand curve (3), the production function (5), and competition from the second-best producer, which we can write as $p_{n,t}(j) \leq W_t\sigma/\tilde{z}_{n,t}(j)$. A large firm takes as given small firms' prices, the wage, and industry revenue, and chooses prices for its goods to maximize static profits subject to the demand curve (3), the production function (5), competition from the second-best producers of each of its goods, and aggregation (4), which determines the industry price index as a function of goods prices.

All firms would set a markup weakly greater than $\gamma/(\gamma-1)$ if unconstrained by the second-best producer.[18] Thus, by Assumption 1, all firms set a markup of σ on all goods. The static profits for a small firm producing a good with relative productivity $\tilde{z}_{n,t}(j)$ is thus $\tilde{z}_{n,t}(j)((1-\sigma^{-1})C_t-\chi_{F,S}W_t)$. The static profits of a large firm with relative productivity $\tilde{Z}_{L,n,t}$ is $\tilde{Z}_{L,n,t}((1-\sigma^{-1})C_t-\chi_{F,L,n}W_t)$. The industry revenue share of a large firm is $\tilde{Z}_{L,n,t}$.

[18] See Edmond, Midrigan, and Xu (2021) for a derivation of the optimal markup with oligopoly, nested CES demand, and Bertrand competition.

4.2 Dynamic Profit Maximization: Innovation

At each moment in time, firms simultaneously choose innovation rates: a creative destruction rate for each good, and a new good development rate. In the dynamic problem, a firm takes as given its profit function from static optimization.

4.2.1 Small Firms

For a small firm to choose their optimal innovation rate, they must know the expected present discounted value of being the most productive producer of a good. The expected present discounted value of producing good j in industry n at time t is $\tilde{z}_{n,t}(j)\pi_{S,n,t}(\tilde{Z}_{L,n,t})$, given by the HJB equation:

$$\begin{aligned} r_t\pi_{S,n,t}(\tilde{Z}_{L,n,t}) =& (1-\sigma^{-1})C_t - \chi_{F,S}W_t - (N_t\kappa_{S,n,t} + \kappa_{L,n,t}(S) + (\gamma-1)g_{n,t})\pi_{S,n,t}(\tilde{Z}_{L,n,t}) \\ &+ \dot{\tilde{Z}}_{L,n,t}\pi'_{S,n,t}(\tilde{Z}_{L,n,t}) + \dot{\pi}_{S,n,t}(\tilde{Z}_{L,n,t}), \end{aligned} \tag{9}$$

where r_t is the discount rate implied by the household stochastic discount factor from equation (2). The first two terms on the right-hand side of the first line are flow profits including fixed costs, and the third term reflects the rate at which the good is creatively destroyed and the rate at which the good's relative productivity is depreciated by growth in industry productivity (recall that relative productivity is $\tilde{z}_{n,t}(j) \equiv (z_{n,t}(j)/Z_{n,t})^{\gamma-1}$ with $z_{n,t}(j)$ fixed over time). The second line captures the effects of changes over time in the industry state or in the aggregate state when the economy is converging to a balanced growth path.

A small firm chooses innovation rates to maximize the expected present discounted value of profits using the household stochastic discount factor, taking as given other firms' innovation rates, the industry and aggregate state, and the evolution of the states over time. As mentioned above, a small firm's innovation optimization problem is the same regardless of the goods they produce. The value function of a small firm with zero goods is given by the HJB equation:

$$\begin{aligned} r_tV_{S,n,t}(\tilde{Z}_{L,n,t}) =& \max_{\{\kappa(j)\}}\left\{\int_0^{M_{n,t}}\kappa(j)\lambda^{\gamma-1}\tilde{z}_{n,t}(j)\pi_{S,n,t}(\tilde{Z}_{L,n,t})dj - W_t\alpha^{-1}\chi_C\int_0^{M_{n,t}}\tilde{z}_{n,t}(j)\kappa(j)^{\alpha}dj\right\} \\ &+ \max_{\delta}\left\{\delta\lambda^{\gamma-1}\pi_{S,n,t}(\tilde{Z}_{L,n,t}) - W_t\alpha^{-1}\delta^{\alpha}\right\} - \eta V_{S,n,t}(\tilde{Z}_{L,n,t}) + \dot{\tilde{Z}}_{L,n,t}V'_{S,n,t}(\tilde{Z}_{L,n,t}) + \dot{V}_{S,n,t}(\tilde{Z}_{L,n,t}). \end{aligned} \tag{10}$$

The right-hand side of the first line is the optimization problem for the rate at which the small firm creatively destroys each good, the first term on the second line is the optimization problem for the rate at which the firm develops a new good, which uses the fact that the new good's expected relative productivity is $\lambda^{\gamma-1}$, and the remaining terms reflect the firm's exit rate and changes over time in the industry state and in the aggregate state when the economy is converging to a balanced growth path.

The First Order Conditions give the optimal new good development rate and the single rate at which a small firm creatively destroys each good in its industry:

$$\delta_{S,n,t} = W_t^{\frac{-1}{\alpha-1}} \left(\lambda^{\gamma-1}\pi_{S,n,t}(\tilde{Z}_{L,n,t})\right)^{\frac{1}{\alpha-1}}$$
$$\kappa_{S,n,t} = (W_t\chi_C)^{\frac{-1}{\alpha-1}} \left(\lambda^{\gamma-1}\pi_{S,n,t}(\tilde{Z}_{L,n,t})\right)^{\frac{1}{\alpha-1}}. \tag{11}$$

A small firm values equally—condition on relative productivity—a good gained through new good development, through creatively destroying a small competitor's good, and through creatively destroying a large competitor's good because it does not internalize the different effects these innovations have on the industry state or growth. A small firm creatively destroys all goods at the same rate because the cost and benefit of an innovation both scale with its relative productivity.

4.2.2 Large Firms

A large firm chooses innovation rates to maximize the expected present discounted value of profits using the household stochastic discount factor, taking as given the aggregate state and its evolution over time, the current industry state, and small firm innovation rates as a function of the industry and aggregate state. The value function is given by the HJB equation:

$$\begin{aligned} r_t V_{L,n,t}(\tilde{Z}_{L,n,t}) &= \tilde{Z}_{L,n,t}((1-\sigma^{-1})C_t - \chi_{F,L,n}W_t) + \dot{V}_{L,n,t}(\tilde{Z}_{L,n,t}) \\ &+ \max_{\{\kappa(j)\},\delta} \left\{ \dot{\tilde{Z}}_{L,n,t}(\{\kappa(j)\},\delta;\tilde{Z}_{L,n,t})V'_{L,n,t}(\tilde{Z}_{L,n,t}) - W_t\alpha^{-1}\chi_{L,n}\left(\chi_C\int_0^{M_{n,t}}\tilde{z}_{n,t}(j)\kappa(j)^\alpha dj + \delta^\alpha\right)\right\}. \end{aligned}$$

The first term on the right-hand side of the first line is flow profits including fixed costs, and the second term reflects the changes over time in the aggregate state when the economy is converging to a balanced growth path. The second line is the optimization problem of the large firm choosing innovation rates; the benefit is through changes in the industry state, which depends on the large firm's innovation rates and the current industry state through the innovation rates of small firms.

The First Order Conditions give the optimal innovation rates:

$$\delta_{L,n,t} = (W_t\chi_{L,n})^{\frac{-1}{\alpha-1}} \left(\lambda^{\gamma-1}(1-\tilde{Z}_{L,n,t})V'_{L,n,t}(\tilde{Z}_{L,n,t})\right)^{\frac{1}{\alpha-1}}$$
$$\kappa_{L,n,t}(L) = (W_t\chi_{L,n}\chi_C)^{\frac{-1}{\alpha-1}} \left((\lambda^{\gamma-1}-1)(1-\tilde{Z}_{L,n,t})V'_{L,n,t}(\tilde{Z}_{L,n,t})\right)^{\frac{1}{\alpha-1}}$$
$$\kappa_{L,n,t}(S) = (W_t\chi_{L,n}\chi_C)^{\frac{-1}{\alpha-1}} \left([1+(\lambda^{\gamma-1}-1)(1-\tilde{Z}_{L,n,t})]V'_{L,n,t}(\tilde{Z}_{L,n,t})\right)^{\frac{1}{\alpha-1}}. \tag{12}$$

The benefit to the large firm of developing a new good or creatively destroying one of its own goods is that the innovation generates growth, which the large firm discounts by $1 - \tilde{Z}_{L,n,t}$ because it adds to industry productivity and depreciates other goods' relative productivities. The benefit of creatively destroying a small competitor's good is in part that the innovation generates growth, but also that the innovation transfers the good's pre-innovation productivity from small firms to the large firm, which the large firm does not discount because it does not add to industry productivity.

4.2.3 Creative Destruction vs. Growth and Small vs. Large Firms

We will see throughout the rest of the paper that a key determinant of the equilibrium growth rate is the ratio of the rate at which firms generate growth to the rate at which they creatively destroy each of their competitors' goods, i.e., the *composition* of innovation. When a firm's good is creatively destroyed, it loses its relative productivity from that good, and the productivity of the good increases, which adds to industry productivity, and depreciates the relative productivity of all goods in the industry. From a firm's perspective, growth and the non-novel part of creative destruction both depreciate the relative productivity of its goods, and they only care about the sum of the two. The household disagrees because only growth reduces prices, whereas the non-novel part of creative destruction simply shifts relative productivity from one firm to another.

The rate at which a small firm depreciates a competitor's relative productivity through growth relative to through creative destruction is

$$\frac{(\gamma-1)g_{S,n,t}}{\kappa_{S,n,t}} = \lambda^{\gamma-1}\chi_C^{\frac{1}{\alpha-1}} + \lambda^{\gamma-1} - 1, \tag{13}$$

where $(\gamma-1)g_{S,n,t} = \lambda^{\gamma-1}\delta_{S,n,t} + (\lambda^{\gamma-1}-1)\kappa_{S,n,t}$ is the rate at which a small firm generates growth in $Z_{n,t}^{\gamma-1}$. The same ratio for a large firm is

$$\begin{aligned}\frac{(\gamma-1)g_{L,n,t}}{\kappa_{L,n,t}(S)} =& \lambda^{\gamma-1}\chi_C^{\frac{1}{\alpha-1}}\left(\frac{\lambda^{\gamma-1}(1-\tilde{Z}_{L,n,t})}{\lambda^{\gamma-1}(1-\tilde{Z}_{L,n,t})+\tilde{Z}_{L,n,t}}\right)^{\frac{1}{\alpha-1}} \\ &+ (\lambda^{\gamma-1}-1)\left(1-\tilde{Z}_{L,n,t}+\tilde{Z}_{L,n,t}\left(\frac{(\lambda^{\gamma-1}-1)(1-\tilde{Z}_{L,n,t})}{(\lambda^{\gamma-1}-1)(1-\tilde{Z}_{L,n,t})+1}\right)^{\frac{1}{\alpha-1}}\right),\end{aligned} \tag{14}$$

which is strictly greater than $(\gamma-1)g_{S,n,t}/\kappa_{S,n,t}$ if $\tilde{Z}_{L,n,t} > 0$. The two terms in each of (13) and (14) are the ratio only including growth due to new good development and the ratio only including growth due to creative destruction, in that order. In each case, the term for large firms is lower than for small firms because the bigger a large firm's industry revenue share, the more they discount the value of growth, and the lower their incentive to develop new goods or creatively destroy their own goods relative to their incentive to creatively destroy their competitors' goods.

5 Results: The Effects of Large Firm Innovation

In this section, I show results that characterize the effect of changes in large firm parameters (fixed costs or innovation costs) on industry concentration, growth, and welfare. I begin with qualitative results concerning long-run effects. I then calibrate the model, and analyze the effects of a fall in large firm fixed costs, which I compare to the data.

5.1 Qualitative Results: Concentration and Growth in the Long-Run

I focus first on the distribution of growth across industries and across economies along a balanced growth path. Without solving for a Markov Perfect Equilibrium of the dynamic game, I can characterize the relationship between large firm industry revenue shares and growth. These results also provide intuition for the quantitative exercises that follow. The key implication of the qualitative results, Theorems 1 and 2, is that if a shock increases large firm innovation without directly affecting small firms, then growth may rise in the short-run (when the measure of small firms is fixed) or in the industry in which the shock hits hardest, but growth will fall in the long-run if the shock is to the aggregate economy.

The following theorem, displayed graphically by the solid blue line in Figure 1, shows that across industries in a single balanced growth path, the industry growth rate can be written as a function of the large firm's industry revenue share, and this function exhibits an inverted-U shape. Throughout the section, I omit time t subscripts for variables that are constant over time. The proof, as well as the proof of Theorem 2, is in Appendix A.

Theorem 1. *Suppose the economy is on a balanced growth path. The long-run large firm industry revenue share is a sufficient statistic for the industry growth rate, i.e., if two industries have the same $\tilde{Z}_{L,n}$, then they have the same g_n. The growth rate is a continuously differentiable function of the large firm's revenue share $g(\tilde{Z}_{L,n})$ such that $g(0) > 0$ and $g'(0) > 0$. If $\alpha = 2$, then $\lim_{z \to 1}(g(z)) = 0$ and there exists a threshold revenue share $z^* \in (0, 1)$ such that for all $z < z^*$, $g'(Z) > 0$, and for all $z > z^*$, $g'(Z) < 0$.*

The second half of the theorem focuses on the case in which $\alpha = 2$, which I calibrate the model to based on the studies discussed in Acemoglu, Akcigit, Alp, Bloom, and Kerr (2018). The theorem, as well as Proposition 2 in the Appendix thus provide an alternative justification for the choice of $\alpha = 2$. If $\alpha = 2$, then across industries, growth as a function of concentration exhibits an inverted-U, and if α is much bigger than 2 (even greater than 2.5), then growth diverges to infinity as the large firm industry revenue share approaches 1.[19]

To gain intuition for the theorem note that along a balanced growth path, in industry n, the expected present discounted value of profits for a small firm from a good with relative productivity 1, over the wage, is

$$\frac{\pi_{S,n,t}}{W_t} = \frac{(\sigma - 1)L^p - \chi_{F,S}}{\rho + N(\kappa_{S,n} + (\gamma - 1)g_{S,n}) + \kappa_{L,n}(S) + (\gamma - 1)g_{L,n}}, \tag{15}$$

and the large firm's revenue share is

$$\tilde{Z}_{L,n} = \frac{\kappa_{L,n}(S) + (\gamma - 1)g_{L,n}}{N(\kappa_{S,n} + (\gamma - 1)g_{S,n}) + \kappa_{L,n}(S) + (\gamma - 1)g_{L,n}}. \tag{16}$$

The relationship across industries between the large firm's revenue share and the industry growth rate operates through the effective discount rate on small firm profits, the denominator on the right-hand side of (15). There are two effects analogous to the composition and total innovation effects from Section 2, the first of which implies a decreasing relationship between concentration and growth, and the second of which implies an increasing relationship.

We can see the composition effect, displayed graphically in the left panel of Figure 2, by holding fixed total innovation—the denominator of the right-hand side of (15) or (16). A higher large firm revenue share has two effects, each of which lowers the industry growth rate based on the analysis in Section 4.2.3. First, the large firm's innovation shifts away from growth and toward creative destruction of its competitors' goods. Second, the large firm's share of total innovation increases, shifting total innovation away from growth and toward creative destruction of small firms' goods.

We can see the total innovation effect, displayed graphically in the right panel of Figure 2, by holding fixed the composition of innovation—the fraction of the denominator of the right-hand side of (16) that is growth. A higher large firm revenue share implies higher total innovation and therefore a higher industry growth rate because small firm innovation is decreasing in $\pi_{S,n,t}/W_t$; if small firm innovation fell by enough to imply lower total innovation, then $\pi_{S,n,t}/W_t$ would increase and, by (11), small firms would optimally innovate more.

If the large firm's revenue share is sufficiently low, then the composition effect is small and the total innovation effect dominates; the large firm's innovation is not so shifted toward creative destruction of its competitors' goods, and the composition of its innovation does not change much in its revenue share. If the large firm's revenue share is sufficiently high, then the composition effect dominates because the large firm innovates almost entirely through creative destruction of its competitors' goods, so an increase in large firm innovation does not lead to higher growth.

[19] See Cavenaile, Celik, and Tian (2021) and Aghion, Bloom, Blundell, Griffith, and Howitt (2005) for empirical support for the inverted-U.

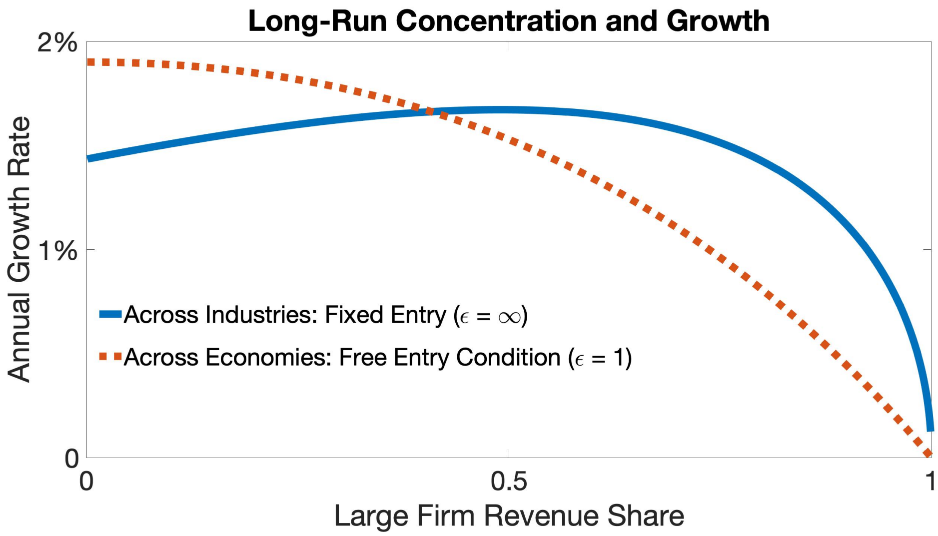

Figure 1: The lines depict growth on a balanced growth path at various levels of the large firm's industry revenue share. The solid blue line shows the growth rate across industries on a single balanced growth path, and the dotted orange line shows the growth rate across balanced growth paths in different economies, each with constant large firm revenue shares across industries. The figure is based on the calibration described in Section 5.2.

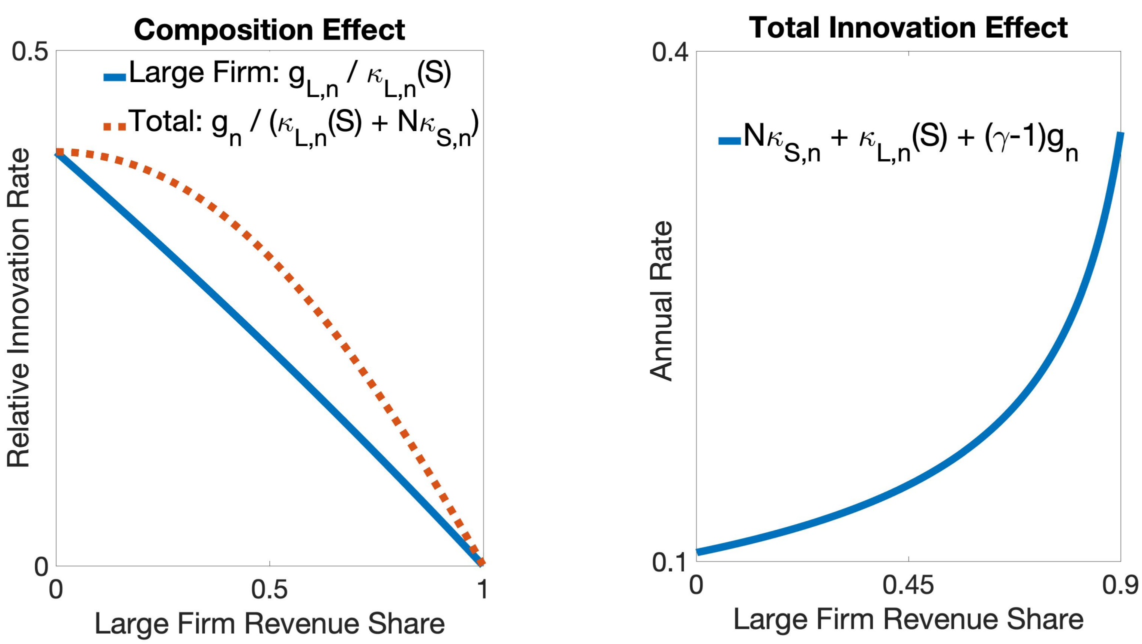

Figure 2: The left panel depicts growth relative to creative destruction of small firms' goods in an industry on a balanced growth path as a function of the large firm's revenue share, both for the large firm (the solid blue line) and for all firms (the dotted orange line). The right panel depicts the total innovation rate in an industry on a balanced growth path as a function of the large firm's revenue share. The figure is based on the calibration described in Section 5.2.

From this intuition, we also have the following theorem, displayed graphically by the dotted orange line in Figure 1, that compares balanced growth paths across economies.

Theorem 2. *Suppose a free entry condition holds, i.e., $\epsilon = 1$, and restrict attention to economies in which all industries are identical in equilibrium, i.e., $\chi_{F,L,n}$ and $\chi_{L,n}$ are constant across industries. Consider the balanced growth paths of two economies that differ in the large firm fixed cost and innovation cost, $\chi_{F,L}$ and χ_L, and let labor supply $\bar{L}$ adjust so that the labor used in production, L^p, in the two balanced growth paths is the same. The large firm industry revenue share is a sufficient statistic for the aggregate growth rate, and if one balanced growth path has a strictly higher large firm industry revenue share, then it has a strictly lower growth rate.*

Intuitively, the total innovation effect in this case is zero, and all that remains is the composition effect and the resulting negative relationship between concentration and growth. The free entry condition at the aggregate level, as well as holding fixed the labor used in production, implies that the effective discount rate on small firm profits is the same in each balanced growth path. In the quantitative exercise in Section 5.3, I allow the labor used in production to adjust, but the effect on long-run growth is insignificant relative to the composition effect of the rise in concentration.

5.2 Calibration

Table 1: Externally Calibrated Parameters

Parameter	Description	Value
η	Exit Rate	0.04
γ	Demand Elasticity	3.1
λ	Innovation Step Size	1.07
α	Innovation Cost Elasticity	2
ϵ	Entry Cost Elasticity	1

The exit rate is from Boar and Midrigan (2022). The demand elasticity is from Broda and Weinstein (2006), using their median estimate from 1990-2001 at the most disaggregated level. The innovation step size is the average step size in the 1983-1993 period in Garcia-Macia, Hsieh, and Klenow (2019). The innovation cost elasticity is from Acemoglu, Akcigit, Alp, Bloom, and Kerr (2018). The entry cost elasticity is chosen so that there is a free entry condition at the aggregate level.

I calibrate the model and solve it computationally to yield more results. I first calibrate the model to an initial balanced growth path in which all industries are identical, and in which small and large firms have the same fixed cost χ_F. I set some parameters externally, and internally calibrate the rest to jointly match a set of moments in the data. The externally calibrated parameters as well as their sources are listed in Table 1. The internally calibrated parameters are listed in Table 2. The data moments used to calibrate the internally calibrated parameters as well as their sources are listed in Table 3. I set the household's labor supply, $\bar{L}$, so that in the initial balanced growth path, output relative to productivity, $C_t/Z_t = L^p$, is 1. The units of time are years.

The innovation cost elasticity, α, which I calibrate externally to 2, is important because it determines how a large firm's innovation composition responds to its industry revenue share. I assume that creative destruction and new good development costs are independent, and that each innovation rate responds to the expenditures on that type of innovation as in the studies described in Acemoglu, Akcigit, Alp, Bloom, and Kerr (2018).

The innovation step size λ and the demand elasticity γ, which I calibrate externally to 1.07 and 3.1, respectively, are important because they determine the fraction of a creative destruction innovation that improves on the replaced good and is thus novel, $(\lambda^{\gamma-1} - 1)/\lambda^{\gamma-1} = 0.19$. As λ increases and the fraction that is novel goes to 1, the distinction between creative destruction and new good development disappears. Moreover, if a larger fraction of creative destruction innovations are novel and generate growth, calibrating the model to match the fraction of growth due to creative destruction requires a lower rate of creative destruction relative to new good development.

Table 2: Internally Calibrated Parameters

Parameter	Description	Value
σ	Imitation Discount	1.3
χ_F	Fixed Cost	0.24
χ_C	Relative Creative Destruction Cost	0.265
χ_E	Entry Cost	2.22
χ_L	Large Firm Innovation Cost	15.14
ρ	Time Discount Rate	0.0194

The mapping between the internally calibrated parameters and the target moments in the data is largely as follows. The TFP growth rate and the real interest rate exactly determine the discount rate. The large firm industry revenue share and the TFP growth rate pin down the large firm innovation cost and the entry cost: the entry cost affects small firm innovation and the large firm innovation cost affects large firm innovation. The share of growth due to creative destruction determines the relative cost of creative destruction. The aggregate markup exactly determines the

Table 3: Calibration Targets

Moment Description	Data Average from 1983-1993	Model
R&D Relative to GDP	1.81%	1.81%
Aggregate Markup	1.3	1.3
Creative Destruction Growth Share	26.5%	26.5%
TFP Growth Rate	1.66%	1.66%
Large Firm Revenue Share	40.7%	40.7%
Real Interest Rate	3.6%	3.6%

The ratio of R&D expenditures to GDP is the Business Enterprise Expenditure on R&D (BERD) relative to GDP from the OECD MSTI database. The aggregate markup is the cost-weighted average markup estimated using Compustat data in De Loecker, Eeckhout, and Unger (2020). The creative destruction growth share is the fraction of growth from creative destruction from Garcia-Macia, Hsieh, and Klenow (2019). I compute this value in the model excluding large firm creative destruction of their own goods, which appears as innovating on their own goods in the data. The TFP growth rate is from the BLS measure in Garcia-Macia, Hsieh, and Klenow (2019) (excluding public, educational, agricultural, and mining sectors). The large firm revenue share is the sales-weighted average across 4-digit industries of the largest firm's revenue share in Compustat from Olmstead-Rumsey (2022). The real interest rate is the 1-year real interest rate from FRED.

imitation discount. R&D expenditures relative to GDP determines a firm profits from innovation relative to sales, and thus pins down the fixed cost given the imitation discount. The distinction between the fixed cost and the imitation discount, identified by the aggregate markup given R&D expenditures relative to GDP, is not significant. The fixed cost is only present so that I can lower the fixed cost for large firms in the main experiment in Section 5.3.

I calibrate the revenue share of large firms, which is the same in each industry, as well as the shock in Section 5.3, to match the average industry revenue share of the largest firm in 4-digit industries in Compustat. This measure likely overstates the size of the largest firm since Compustat does not include all firms. An alternative measure is the Census data on industry concentration measures, which show a smaller level of industry concentration, but a similar rise over the same time period. One downside of the Census data is that it only lists the revenue share of the top 4 firms in each industry, not the top firm. Moreover, while the Census data is in a sense more accurate because it includes more firms, it may include too many small firms that are not relevant to the mechanism in the model. The discount a large firm applies to generating growth depends on its share of innovations. Olmstead-Rumsey (2022) shows that the average share of R&D expenditures by the largest firm in 4-digit industries in Compustat closely tracks the average sales share.

5.3 Quantitative Experiment: A Rise in Large Firm Innovation

I ask whether and to what extent a rise in concentration driven by a fall in large firm per-good fixed costs can explain changes in US data since the mid-1990s. We can interpret the fall in large firm fixed costs as capturing a shift from per-good costs to firm wide fixed costs due to the rise in information technology.[20] That said, the specific nature of the shock is not important as long as it drives an increase in large firm innovation and does not otherwise affect small firms. For example, an alternative shock that will have the same effect on all variables except the split between R&D and fixed cost expenditures is a fall in large firm innovation costs, which we can interpret as an increase in the concentration of entrepreneurial activity within large firms.

The economy begins in the balanced growth path from the calibration in Section 5.2. There is an unanticipated permanent fall in $\chi_{F,L}$ in all industries so that the revenue share of the large firm in each industry in the new balanced growth path is 0.51, the sales-weighted average across 4-digit industries of the largest firm's revenue share in 2018 in Compustat from Olmstead-Rumsey (2022) (the large firm fixed cost falls to 0.19). I track the transition path of the economy as it converges to a new balanced growth path.

5.3.1 Industry Concentration and Aggregate Growth

I show the revenue share of the large firm in each industry along the transition path in Figure 3. The revenue share converges over a similar time interval as the gap between the years in the initial calibration, 1983-1993, and the target year for the shock, 2018.

I compare the main results concerning aggregate growth in the model to the data in Table 4, and display the model results graphically in Figure 4. The model can explain 41% of the long-run fall in growth, which in the model is due entirely to a change in the growth rate of productivity because output relative to productivity is constant along a balanced growth path. The model can explain all of the increase in the short-run growth rate in the data if we include growth in output due to changes in output relative to productivity, $C_t/Z_t = L_t^p$, as well as changes in productivity. However, the burst in growth does not last as long in the model as in the data: the peak difference

[20] See Aghion, Bergeaud, Boppart, Klenow, and Li (2022) for a discussion.

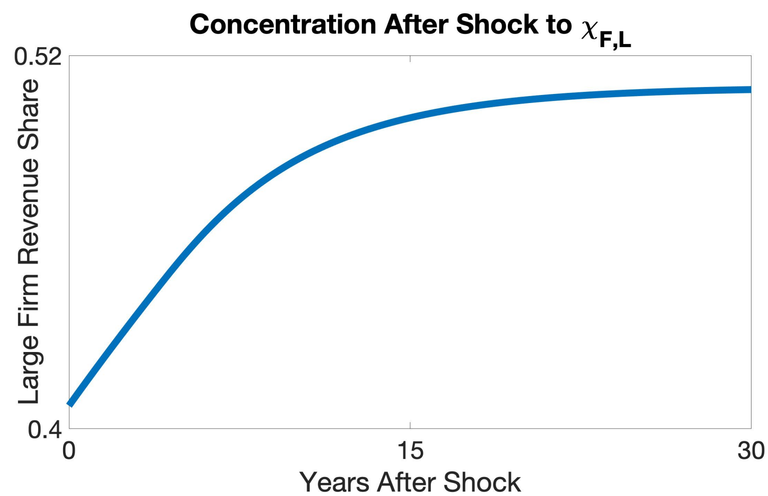

Figure 3: The revenue share of each large firm over time following a shock to $\chi_{F,L}$.

in output along the transition path from the original balanced growth path occurs after 4 years and is 42% of the difference in the data after 4 years.

Table 4: Growth After Shock to $\chi_{F,L}$

Moment Description	Data	Model
Growth Rate Burst	+0.64 ppt (38.6%) (1993-2003)	Output: +0.87 ppt (52.4%) (first year)
		TFP: +0.1 ppt (6.0%) (first year)
Cumulative Burst	+6.4 ppt (38.6%) (1993-2003)	Output: +1.07 ppt (16.1%) (4 years)
		TFP: +0.18 ppt (2.7%) (3 years)
Growth Rate Fall	-0.34 ppt (-20.5%) (2003-2013)	-0.14 ppt (-8.4%) (New BGP)

For each value, ppt is the percentage point rise, and the number in parentheses is the percent rise relative to the initial value. The data are taken from Garcia-Macia, Hsieh, and Klenow (2019). The growth rate burst in the model is the peak growth rate in the short-run following the shock. The output growth rate reflects changes in output relative to productivity, C_t/Z_t, as well as changes in TFP, Z_t. The cumulative burst is the sum of growth rates, i.e., the peak difference between the new path and the old path.

I decompose the change in the productivity growth rate over time into the composition and total innovation effects in Figure 5. Throughout, the composition effect drives down growth as large firms' revenue shares increase and innovation shifts toward creative destruction of small firms' goods. Growth is higher in the short-run because total innovation increases. In the long-run, growth is lower because the composition effect dominates; total innovation is only slightly higher due to an increase in output relative to the wage driven by a shift in labor toward production.

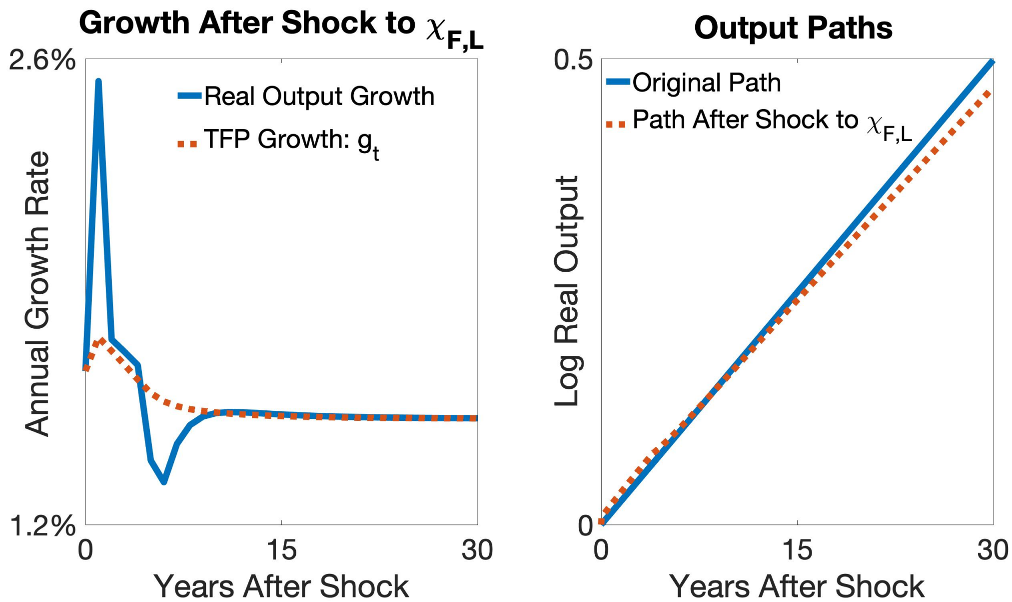

Figure 4: The left panel shows annual growth rates following the shock. The dotted orange line is the growth rate of productivity, g_t, and the solid blue line includes changes in output relative to productivity, C_t/Z_t, which is constant along a balanced growth path. The right panel shows paths of real output over time. The solid blue line is the original path the economy would have followed had it not been hit by a shock. The dotted orange line is the realized path following the shock.

To understand the dynamics of the total innovation effect, it is useful to look at entry, displayed in Figure 6. Immediately following the shock, the entry rate hits its lower bound of 0; the expected discounted profits of entering become negative, but the measure of small firms can only fall over time as firms exogenously exit. Thus, growth increases because large firms innovate more, and total small firm innovation is slow to fall due to an overhang of small firms.

5.3.2 Comparing Model Predictions to the Data

I compare several features of the economy's response to the shock to the data.

Entry:
The large fall in entry in the short-run and the smaller fall in the long-run match the data in Decker, Haltiwanger, Jarmin, and Miranda (2016), which show that the entry rate declined sharply in the mid-to-late 1990s followed by a partial recovery before a large drop during the Great Recession.

Creative Destruction:
I show in Figure 7 that the share of growth due to creative destruction falls following the shock, as

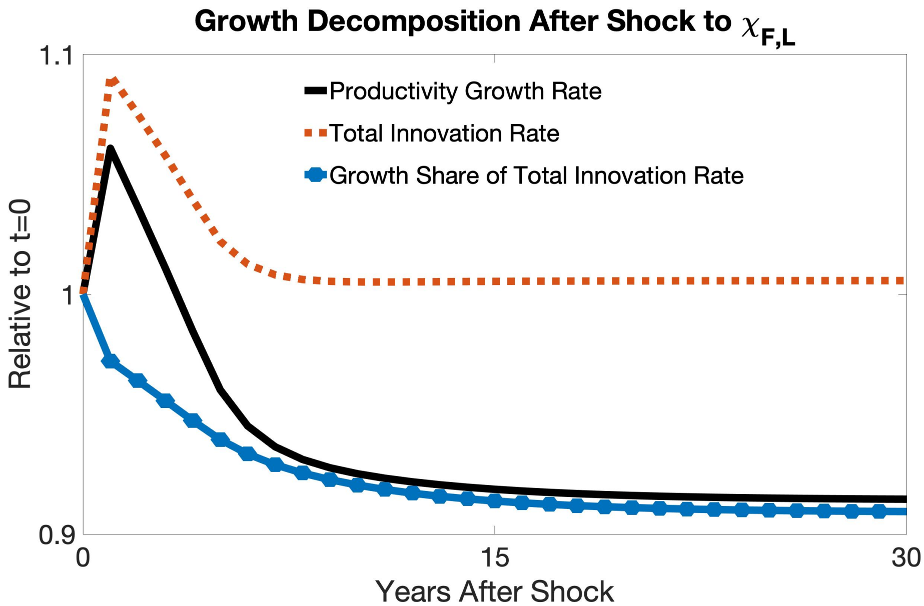

Figure 5: The solid black line depicts the annual productivity growth rate relative to before the shock. The dotted orange line and the textured blue line decompose the black line into the total innovation rate and growth over the total innovation rate, respectively, relative to before the shock.

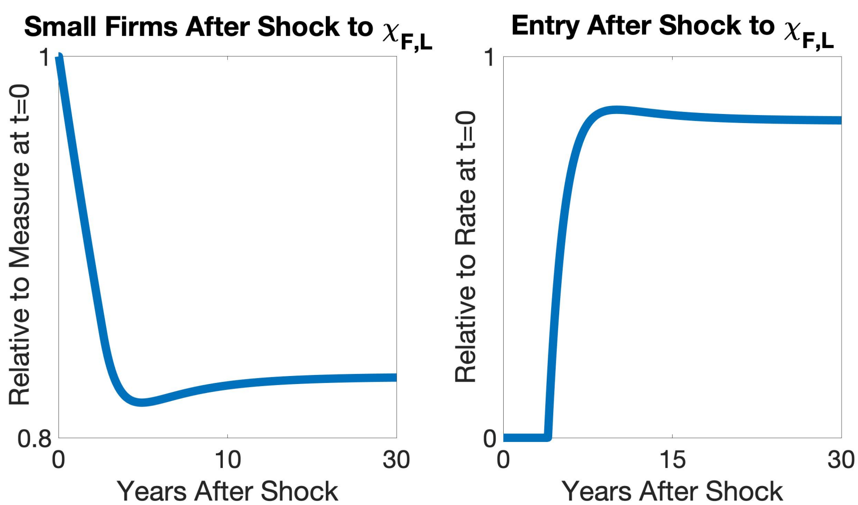

Figure 6: The left panel is the measure of small firms over time relative to the measure before the shock. The right panel is the entry rate over time relative to the entry rate before the shock.

in the long-run in Garcia-Macia, Hsieh, and Klenow (2019), although by a smaller magnitude. This follows from the shift in innovation toward large firms because growth due to creative destruction is a *smaller* share of large firms' growth than of small firms' growth. This is not at odds with Figure 2 or with the analysis in Section 4.2.3 because a large firm only creatively destroys small firms' goods, whereas small firms creatively destroy all firms' goods. Although large firms focus their innovation particularly toward creative destruction of small firms' goods, their innovation is less focused on creative destruction overall.

Finally, Garcia-Macia, Hsieh, and Klenow (2019) find that creative destruction fell as a share of growth in part because in the later time period, a larger share of job creation is at firms where employment grew by less than a factor of 3. Part of this shift is unrelated to the creative destruction share of growth, as defined in the model, if job creation shifted toward large firms, like revenue shares, and large firms experience small employment fluctuations of magnitude less than a factor of 3.

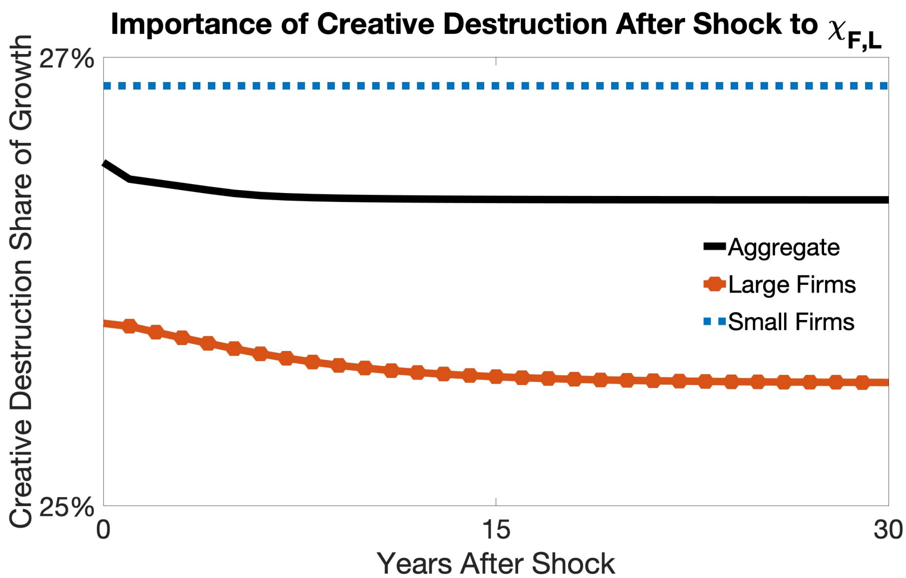

Figure 7: Each line depicts, over time following the shock, the rate at which a type of firm generates growth through creative destruction relative to the total rate at which that type of firm generates growth.

Growth Relative to R&D Expenditures:

Olmstead-Rumsey (2022) shows that relative to other firms with R&D expenditures in Compustat, industry leaders have a low ratio of R&D over revenue. Moreover, R&D expenditures shifted toward industry leaders over a similar time period and of a similar magnitude as the shift in revenue. Finally, R&D expenditures relative to GDP increased as the aggregate growth rate fell. It is thus puzzling that R&D expenditures increased and shifted toward firms that are apparently more efficient at innovating, yet growth fell.

The model offers an explanation for these facts. First, in the data, the average R&D expenditures over revenue among industry leaders relative to the average among non-leaders with positive R&D expenditures is 0.4, and in the model it is 0.67. In the model, all firms generate sales through R&D. Large firms discount the value of growth, and innovate relatively little. As a result, their innovation is particularly effective, and they are able to generate a relatively large amount of revenue given their innovation expenditures. Second, in the data R&D expenditures as a share of GDP rose while growth fell, and the growth rate relative to R&D expenditures over GDP fell from 0.91 in 1983-1993 to 0.69 in 2003-2013. In the model, innovation expenditures as a share of

GDP are flat while growth falls, and the growth rate relative to innovation expenditures over GDP falls from 0.91 on the initial balanced growth path to 0.83 on the balanced growth path following the shock. In the model, large firm innovation is particularly efficient at generating revenue, but not at generating growth because large firms endogenously focus their innovation on creatively destroying their competitors' goods.

This explanation of the fall in growth relative to R&D expenditures over GDP stands in contrast to the one proposed by De Ridder (2021) in which innovation shifts toward firms with *high* R&D expenditures relative to sales, implying a fall in its efficiency.

Industry Concentration and Industry Growth Rates:
A distinguishing feature of the theory is that it implies a stronger negative relationship between growth and concentration at the aggregate level than at the industry level, following an aggregate shock to large firm innovation. I compare this prediction to the empirical work in Ganapati (2021), which shows that in US data from 1972-2012, controlling for sector and time fixed effects, a 1 percent rise in the revenue share of the largest 4 firms in a 6-digit NAICS industry over a 5-year period is associated with a 0.1 percent rise in the industry's real output. Similarly, Ganapati (2021) finds that a 1 percent rise in the revenue share of the largest 4 firms in a 6-digit NAICS industry over a 5-year period is associated with a 0.2 percent rise in the industry's real output relative to employment. To run the same regression in the model, I generate industry heterogeneity by imposing that a measure 0 of industries do not see a change in their large firm's innovation cost. I then regress the change in industry log real output on the change in industry log large firm revenue share and a time fixed effect over the three 5-year time periods during the first 15 years of the transition path following the shock, after which the economy is effectively on the new balanced growth path. I find similar results as Ganapati (2021): a 1 percent rise in the revenue share of the largest firm is associated with a 0.03 percent rise in the industry's real output. This effect is due entirely to different productivity growth rates across industries because revenue in each industry is the same.

The theory thus generates a parsimonious explanation for the short-run burst in growth as well as the positive relationship between concentration and growth across industries: the measure of small firms is slow to adjust and only adjusts at the aggregate level.

5.3.3 Welfare and Size-Dependent Taxes

Taking into account the transition path, household welfare falls by the equivalent of a permanent 5.8% drop in consumption. The decline in the long-run growth rate is ultimately the dominant effect. This suggests that on the margin, contrary to the result in Edmond, Midrigan, and Xu (2021), a tax on firms increasing in their size will improve growth and welfare. For a small tax, large firm don't change their prices since they already set their markups at the constraint implied by the second-best producer. Large firms respond to the tax by reducing innovation, leading to more small firms and growth in the long-run. Even if large firms ultimately respond by investing less in their innovative capacity, the effect on growth and welfare is positive: large firms over-invest in innovative capacity because as their innovation cost falls, their profits rise yet welfare falls.[21]

6 Antitrust Policy: Acquisitions

I use the calibrated model to explore the effects of two different types of acquisition policies. First, for each good produced by small firms, at an exogenous Poisson arrival rate the large firm in the same industry can make a take-it-or-leave-it offer to purchase the most productive version of the good from the small firm. I consider the effects of a tax, τ, on these transactions so that if the relative productivity of the good is $\tilde{z}_{n,t}(j)$, then the small firm receives payment $\tilde{z}_{n,t}(j)\pi_{S,n,t}(\tilde{Z}_{L,n,t})$, and the large firm pays $(1+\tau)\tilde{z}_{n,t}(j)\pi_{S,n,t}(\tilde{Z}_{L,n,t})$. If a large firm acquires goods at rate $A_{n,t}$ with average relative productivity $\zeta_{n,t}$, then the evolution of $\tilde{Z}_{L,n,t}$ is as before in (6) with the additional term $A_{n,t}\zeta_{n,t}$.

Second, for each small firm, at an exogenous Poisson arrival rate the large firm in the same industry can make a take-it-or-leave-it offer to purchase the small firm's entrepreneur, i.e., its innovation capacity. I again consider the effects of a tax, τ, on these transactions so that the small firm receives payment $V_{S,n,t}(\tilde{Z}_{L,n,t})$, and the large firm pays $(1+\tau)V_{S,n,t}(\tilde{Z}_{L,n,t})$. If a large firm acquires small firms' entrepreneurs at rate $A_{n,t}$, then the measure of small firms declines at rate $A_{n,t}$ beyond the baseline effects of entry and exit. The large firm's measure of entrepreneurs changes at rate

$$\partial\chi_{L,n,t}^{-1/(\alpha-1)}/\partial t = A_{n,t} - \eta\left(\chi_{L,n,t}^{-1/(\alpha-1)} - \bar{\chi}_{L,n}^{-1/(\alpha-1)}\right),$$

where $\chi_{L,n,t}$ is the large firm's innovation cost at time t and $\bar{\chi}_{L,n}$ is the large firm's exogenously given innovation cost absent any acquisitions, so the second term on the right-hand side reflects the rate at which the large firm's acquisitions exit.

In each case, any taxes collected are dispersed to the representative household. If the tax is negative, then it is funded by a lump sum tax on the representative household.

The main findings are that acquisitions of small firms' *goods* increase growth and welfare if they

[21] A fall in large firm innovation costs generates nearly the same results as a fall in large firm fixed costs.

are sufficiently valuable to the large firm. Acquisitions of small firms' *entrepreneurs* reduce growth and welfare. Moreover, absent any tax or subsidy, a large firm is willing to acquire small firm entrepreneurs. In an alternative model without creative destruction calibrated to match the same moments (other than the creative destruction share of growth), a large firm's value of an entrepreneur acquisition is only 10% of the entrepreneur's value, i.e., the acquisition doesn't occur absent a substantial subsidy.

6.1 Acquisitions of Small Firm Goods

The economy begins on the initial balanced growth path calibrated in Section 5.2. Each good produced by a small firm can be acquired by the large firm in the same industry at rate 0.05. Put another way, each large firm can acquire 5% of the goods produced by small firms in its industry per year. The initial tax rate on acquisitions is $\tau = 0.2$, which is sufficiently high so that large firms do not purchase small firms' goods, and the economy is indistinguishable from the case without any acquisitions. There is an unanticipated permanent fall in the tax rate τ. I track the transition path as the economy converges to a new balanced growth path.

Before discussing the results in detail, I note that a large acquisition subsidy is required for acquisitions to improve growth and welfare. I interpret this result as stating how valuable acquisitions must be to the large firm. In a richer model, there may be a variety of reasons why an innovation is more profitable with a large firm than with a small firm, in which case a smaller subsidy (or even just not too high a tax) will be required.

The solid black line in Figure 8 shows that once the tax rate is sufficiently low, in the long-run, large firms switch from never acquiring to always acquiring small firms' goods when given the opportunity. Large firms acquire small firms' goods even when the tax rate is positive because they face a lower creative destruction rate.

The solid black lines in Figure 9 show the effects of changes in the tax rate on the long-run growth rate and on welfare taking into account the transition path. When the tax rate falls just enough so that the acquisition rate rises to its maximum level, there is a large negative effect on growth and welfare. As the tax rate falls further and ultimately becomes a subsidy, the acquisition rate does not change, but growth and welfare rise, eventually exceeding their values in the balanced growth path without acquisitions.

To understand the growth and welfare results, I show in Figure 10 that changes in the *rate of*

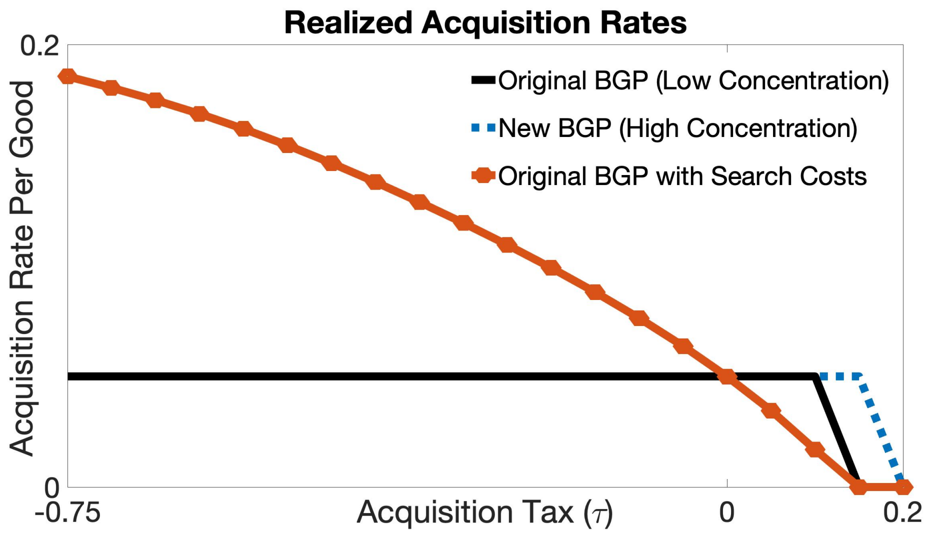

Figure 8: Each line depicts the fraction of small firm goods that large firms acquire per year in the balanced growth path as a function of the acquisition tax rate.

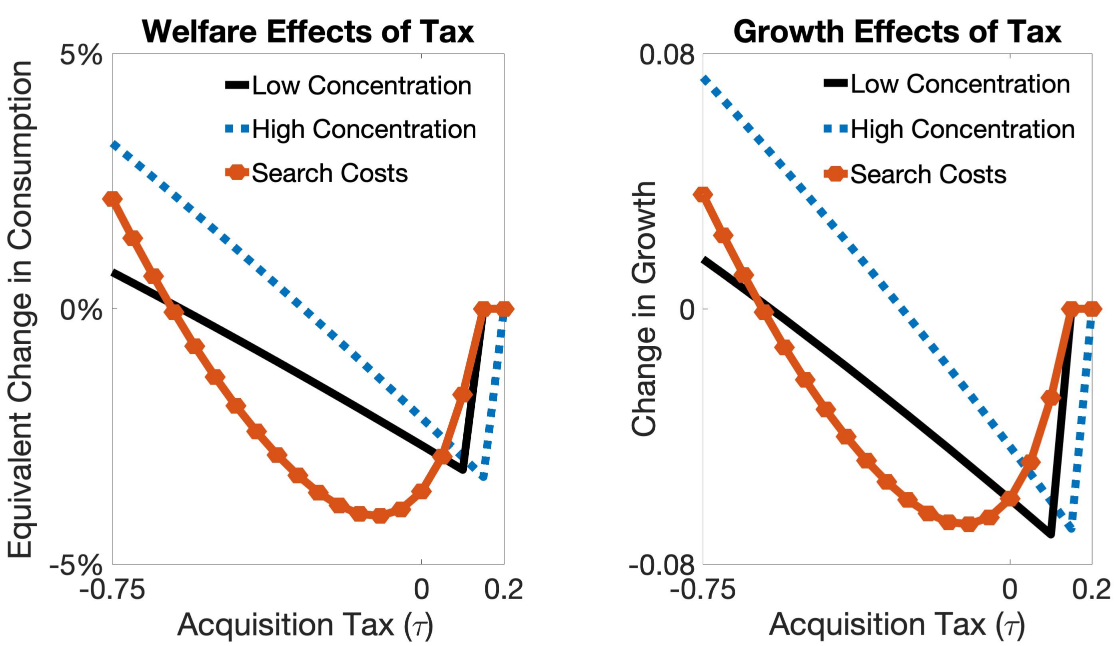

Figure 9: Each line in the left panel depicts the welfare effects – computed as the equivalent permanent percentage change in final good consumption – of an unanticipated permanent fall in the acquisition tax rate, taking into account the transition path. Each line in the right panel depicts the percentage point change in the long-run growth rate.

acquisitions and changes in the *value of acquisitions to the large firm* have opposite effects. The rise in the acquisition rate, which occurs entirely as the tax rate falls just below 0.2, increases large firm revenue shares, which reduces their incentive to generate growth relative to their incentive to creatively destroy small firms' goods (the dotted blue line in the left panel). On the other hand,

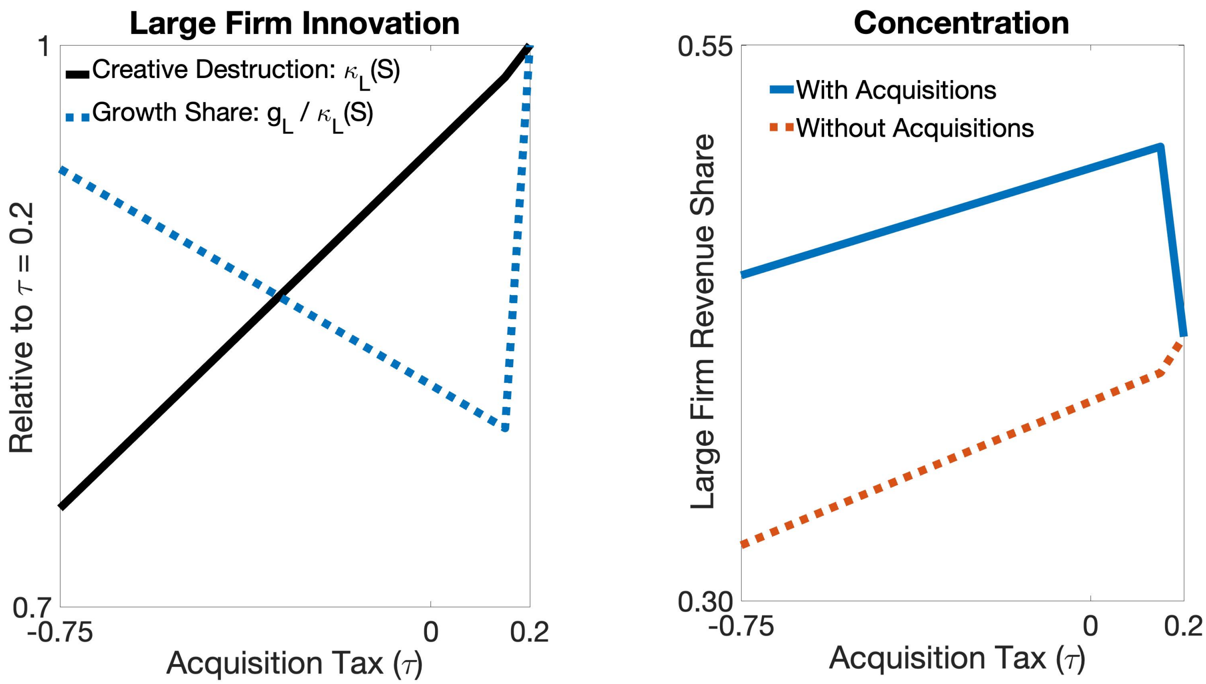

Figure 10: The left panel displays innovation rates on the new balanced growth path following a permanent fall in the acquisition tax rate relative to on the balanced growth path before the shock: the solid black line is the rate at which the large firm creatively destroys small firms' goods and the dotted blue line is the rate at which the large firm generates growth relative to the rate at which it creatively destroys small firms' goods. In the right panel, the solid blue and dotted orange lines depict each large firm's long-run revenue share following a permanent fall in the acquisition tax rate, with and without the direct effects of acquisitions on the distribution, respectively. The figure uses the initial calibration of the model from Section 5.2.

the opportunity to acquire small firms' goods, which becomes more valuable as the tax rate falls further below 0.2, reduces the large firm's incentive to perform all types of innovation (the solid black line in the left panel) because the more a large firm innovates, the less relative productivity remains for them to acquire. Since large firm innovation ultimately reduces growth and welfare, as we saw in Section 5.3, increasing the *value of acquisitions* to the large firm, conditional on the *rate of acquisitions*, improves growth and welfare.

I explore the effects of acquisitions further by performing the same experiment with two alternative models/calibrations. First, the dotted blue lines in Figures 8 and 9 show the results starting from the balanced growth path following a decrease in the large firm innovation cost χ_L that matches the rise in concentration as in the experiment described in Section 5.3. I shock the large firm innovation cost rather than the fixed cost to increase industry concentration without generating an exogenous reason for large firms to acquire small firms' goods, which makes the experiment more comparable to the initial acquisition experiment.

The benefits of reducing the acquisition tax are larger when concentration is higher. Small firms have less relative productivity for large firms to acquire, mitigating the effect of the rise in the acquisition rate, and more significantly, large firms innovate more, increasing the effect of a proportional fall in large firm innovation.

Second, the textured orange lines in Figures 8 and 9 show the results starting from the initial calibration in Section 5.2, but in a model in which a large firm must pay search costs to find acquisition opportunities. Formally, if the large firm gets the opportunity to acquire good j at rate $\tilde{A}_{n,t}(j)$, then it must pay total search costs $\int_0^{M_{n,t}} \tilde{z}_{n,t}(j)\frac{\omega}{2}\tilde{A}_{n,t}(j)^2 dj$ in units of labor. In words, for each good in its industry, to encounter the good, the large firm must pay a convex search cost that scales with the good's relative productivity. If a large firm encounters a good, then it makes a take-it-or-leave-it offer to the producer of the most productive version of the good, subject to the acquisition tax rate τ as in the initial experiment. Thus, the more valuable acquisition opportunities are to the large firm, the more opportunities the large firm encounters. I calibrate the cost-shifter ω so that if the acquisition tax rate is $\tau = 0$, then the long-run realized acquisition rate is 0.05 as in the other experiments.

Now that the acquisition opportunity rate is endogenous, lowering the tax rate leads to more opportunities. The marginal opportunity always generates zero value for the large firm taking into account search costs, so there is always a force implying that a lower tax rate increases large firm revenue shares and reduces growth and welfare. However, if the tax rate is sufficiently low, then valuable acquisition opportunities reduce the large firm's incentive to innovate, which increases growth and welfare. In that case, the ability of the large firm to increase the rate at which it receives valuable acquisition opportunities amplifies the beneficial effects of acquisitions, and leads to higher growth and welfare.

6.2 Acquisitions of Small Firm Entrepreneurs

I conduct two experiments. First, I show that acquisitions of small firms' entrepreneurs are bad for growth and welfare. The economy begins on the initial balanced growth path calibrated in Section 5.2. Each small firm's entrepreneur can be acquired by the large firm in the same industry at rate 0.01. The initial tax rate on acquisitions is sufficiently high so that large firms do not purchase, and the economy is indistinguishable from the case without any acquisitions. There is an unanticipated permanent decrease in the tax rate so that large firms acquire small firms' entrepreneurs whenever given the opportunity. I track the transition path as the economy converges to a new balanced growth path.

The effect is largely the same as a fall in the large firm fixed cost, detailed in Section 5.3, or a similar fall in the large firm innovation cost, except here the shock that induces an increase in large firm innovation comes directly at the expense of the measure of small firms rather than for free. Growth falls in the long-run by 0.19 percentage points and welfare, taking into account the transition path, falls by the equivalent of a permanent 6.18% drop in final good consumption. The particular choice of the acquisition tax rate does not matter as long as it is sufficiently low for large firms to acquire small firms because the value large firms receive from acquisitions does not affect their other decisions.

Second, I show that the preference of large firms for creatively destroying their competitors' goods – the key mechanism in the model – provides a novel motivation for large firms to acquire their competitors in the absence of any exogenous synergies. The large firm in a single industry has a one time opportunity to acquire 1% of the small entrepreneurs in its industry. Regardless of its decision, these entrepreneurs exit exogenously over time, and the industry converges back to its steady state. The aggregate economy is unaffected. I consider two calibrations for this experiment. In one case, the economy begins on the initial balanced growth path calibrated in Section 5.2. The large firm is willing to pay 3.67 times the small entrepreneurs' value of remaining unacquired, i.e., the acquisition occurs absent a large tax. In the other case, I recalibrate the economy to match the same moments in the data, except I set $\chi_C = \infty$ so that there is no creative destruction. The large firm is then only willing to pay 10% of the small entrepreneurs' value of remaining unacquired, i.e., the acquisition does not occur absent a large subsidy.

Intuitively, in both cases, if other small firms did not react to the acquisition, then joint surplus would be positive because firms can better maximize joint profits if they coordinate. However, in equilibrium, other small firms innovate more because they expect the joint innovation rate of the large firm and the acquired small firms to be lower following the acquisition. In the economy without creative destruction, this reaction overwhelms the benefit to the large firm from reducing the innovation of the acquired small firms. In the economy with creative destruction, this reaction is smaller because the large firm does not reduce the rate at which the acquired small firms creatively destroy other small firms' goods. Moreover, by reducing the rate at which the acquired small firms creatively destroy the large firm's goods, the large firm benefits a lot relative to the reaction it triggers because creative destruction imposes a large tax on the targeted firm relative to the growth it generates.

7 Conclusion

To understand the relationship between concentration and growth, and the policy implications, I study a model with one large firm and a continuum of small firms in each industry. Firms can innovate through creative destruction, developing new goods, and improving on their own goods. Large firms, to avoid cannibalization, have a strong relative preference for creatively destroying their competitors' goods. As a result, when large firms innovate more, small firms' innovations are discounted heavily relative to the overall innovation and growth rate. A widespread fall in large firm fixed costs stimulates large firm innovation, which increases concentration and reduces small firm entry, long-run growth, and taking into account the transition path, welfare. Growth rises in the short-run and in industries with a bigger fall in large firm fixed costs because the measure of small firms, which affects small firm innovation, is slow to fall and only responses to the aggregate environment. The aggregate growth rate falls despite flat innovation expenditures and a shift toward large firms whose innovation is relatively efficient at generating revenue; large firms focus their innovation on creative destruction, which creates relatively little growth. I show that these predictions match US data from the mid-1990s to the mid-2010s.

Large firm acquisitions of their competitors' goods have direct and indirect effects with opposite implications for concentration, growth, and welfare. Acquisitions directly shift revenue to large firms, strengthening their relative preference for creative destruction, and leading to a fall in growth. The indirect effect is that since acquisitions are valuable to large firms, each large firm innovates less so that more revenue share remains for it to acquire. As large firm innovation falls, it is replaced by small firm innovation, which is less geared toward creative destruction, ultimately facilitating more innovation and growth. If acquisitions are sufficiently valuable to large firms, then growth and welfare are higher in an economy with acquisitions than without. This positive effect is stronger when industries are more concentrated.

The theory and results highlight a novel way to think about the effects of market power and optimal competition policy. Large firms are harmful because of how they achieve their size through innovation. Research and development subsidies that target large firms may backfire by discouraging small firm innovation. Policies that increase concentration may be beneficial as long as they reduce large firm innovation. Facilitating acquisitions is a particularly useful policy because, unlike taxing large firms, it does not require knowledge of firms' relevant industries or their revenue shares in those industries.

Finally, although this paper focuses on growth, the theory has implications for other settings, and suggests potential avenues for future research. For example, suppose a firm can develop different types of goods, some of which are more novel to the industry, and others of which are close substitutes with the firm's competitors' goods. The same force that leads larger firms to set higher markups in static models of oligopolistic competition implies that larger firms have a stronger preference for producing the types of goods that are close substitutes with their competitors. Thus, subsidizing large high markup firms to produce more may be costly unlike in models in which firm production is one-dimensional.

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A Proofs

To make the proofs of Theorems 1 and 2 more readable, I split them into a few steps. I begin with the following proposition.

Proposition 1. *On a balanced growth path equilibrium, the long-run industry revenue share of the large firm in an industry is a sufficient statistic for that industry's growth rate in the sense that two industries with the same $\tilde{Z}_{L,n}$ have the same g_n. Moreover, we can write the industry growth rate as a continuously differentiable function $g(\tilde{Z}_{L,n})$ defined on $[0,1)$ such that $g(0) > 0$ and $g'(0) > 0$.*

Proof. Throughout the proof, I drop time t subscripts when possible because the theorem is concerned with a balanced growth path. To prove the theorem, I decompose the industry growth rate into total innovation and the growth share of total innovation, i.e., the composition of innovation: $(\gamma - 1)g_n = I_n(\gamma - 1)g_n/I_n$, where total innovation is

$$I_n \equiv N\kappa_{S,n} + \kappa_{L,n}(S) + (\gamma - 1)g_n. \tag{17}$$

I first show that total innovation and the growth share of innovation are differentiable functions of the large firm industry revenue share, $\tilde{Z}_{L,n}$. In particular, they are not functions of the large firm cost of innovation, $\chi_{L,n}$, or the large firm fixed cost, $\chi_{F,L,n}$. It follows that the growth rate is as well, and its derivative with respect to the large firm industry revenue share is

$$\frac{\partial(\gamma - 1)g_n}{\partial \tilde{Z}_{L,n}} = \frac{(\gamma - 1)g_n}{I_n}\frac{\partial I_n}{\partial \tilde{Z}_{L,n}} + I_n\frac{\partial(\gamma - 1)g_n/I_n}{\partial \tilde{Z}_{L,n}}. \tag{18}$$

I use small firm optimal innovation to show that total innovation is a continuously differentiable function of $\tilde{Z}_{L,n}$ for all $\tilde{Z}_{L,n} \in [0,1)$. Small firm innovation is a function of the present discounted profits a small firm earns from a good with relative productivity 1 divided by the wage. Present discounted profits at time t are given by the HJB equation (9). On a balanced growth path, the interest rate is $r = \rho + g$, aggregate output is $C_{t'} = Z_t e^{g(t'-t)} L^p$ for all $t' \geq t$, and the first term on the second line is zero because the industry state is constant over time. We can thus guess and verify that present discount profits are

$$\pi_{S,n,t} = (1 - \sigma^{-1})C_t/(\rho + I_n) \tag{19}$$

because then $\dot{\pi}_{S,n,t} = g\pi_{S,n,t}$. Since the wage is $W_t = Z_t/\sigma$, it follows that present discounted profits over the wage is

$$\pi_{S,n,t}/W_t = (\sigma - 1)L^p/(\rho + I_n), \tag{20}$$

which is not a function of t. Equation (6) for the evolution of the industry state over time, evaluated on a balanced growth path, shows that we can write total innovation in terms of only small firm innovation:

$$I_n = \lambda^{\gamma-1} N(\kappa_{S,n} + \delta_{S,n})/(1 - \tilde{Z}_{L,n}).$$

Thus, using equation (11) for optimal small firm innovation, it follows that

$$I_n = \frac{\lambda^{\gamma-1} N}{1 - \tilde{Z}_{L,n}} \left(\chi_C^{\frac{-1}{\alpha-1}} + 1 \right) \left(\frac{\lambda^{\gamma-1}(\sigma - 1)L^p}{\rho + I_n} \right)^{\frac{1}{\alpha-1}}. \tag{21}$$

Thus, I_n is a strictly increasing continuously differentiable function of $\tilde{Z}_{L,n}$ on $[0, 1)$.

To see that the growth share of innovation is a differentiable function of the large firm industry revenue share, use equation (6) for the evolution of the industry state over time, evaluated on a balanced growth path, to write total innovation as $I_n = I_{L,n}/\tilde{Z}_{L,n} = I_{S,n}/(1 - \tilde{Z}_{L,n})$, where $I_{L,n}$ and $I_{S,n}$ are total innovation from the large firm and small firms, respectively:

$$I_{L,n} \equiv \kappa_{L,n}(S) + (\gamma - 1)g_{L,n} \qquad\qquad I_{S,n} \equiv N(\kappa_{S,n} + (\gamma - 1)g_{S,n}).$$

It follows that the growth share of innovation is

$$\frac{(\gamma - 1)g_n}{I_n} = \tilde{Z}_{L,n} \frac{(\gamma - 1)g_{L,n}}{I_{L,n}} + (1 - \tilde{Z}_{L,n}) \frac{N(\gamma - 1)g_{S,n}}{I_{S,n}}. \tag{22}$$

Equation (13) shows that $(\gamma - 1)g_{S,n}/I_{S,n}$ is a strictly positive constant, and equation (14) shows that $(\gamma - 1)g_{L,n}/I_{L,n}$ is a strictly decreasing continuously differentiable function of $\tilde{Z}_{L,n}$ on $[0, 1)$ that is equal to $(\gamma - 1)g_{S,n}/I_{S,n}$ at $\tilde{Z}_{L,n} = 0$. Thus, the growth share of innovation is a strictly decreasing continuously differentiable function of $\tilde{Z}_{L,n}$ on $[0, 1)$.

Now, I show that as $\tilde{Z}_{L,n}$ goes to 0, the industry growth rate converges to a strictly positive number. Equation (21) shows that as $\tilde{Z}_{L,n}$ goes to 0, total innovation I_n converges to a strictly positive number. Equation (22) then shows that the industry growth rate converges to the product of that strictly positive number and $N(\gamma - 1)g_{S,n}/I_{S,n}$, which is strictly positive.

Next, I show that as $\tilde{Z}_{L,n}$ goes to 0, the derivative of the industry growth rate with respect to $\tilde{Z}_{L,n}$ converges to a strictly positive number. Since total innovation I_n is strictly increasing, and the growth share of innovation at $\tilde{Z}_{L,n} = 0$ is $N(\gamma - 1)g_{S,n}/I_{S,n}$, which is strictly positive, it follows that the first term in the derivative in (18) is strictly positive at $\tilde{Z}_{L,n} = 0$. Equation (14) shows that the derivative of $(\gamma - 1)g_{L,n}/I_{L,n}$ with respect to $\tilde{Z}_{L,n}$ is a finite number at $\tilde{Z}_{L,n} = 0$. Thus, (22) shows that the derivative of $(\gamma - 1)g_n/I_n$ is equal to 0 at $\tilde{Z}_{L,n} = 0$. It follows from (18) that the derivative of the industry growth rate is strictly positive at $\tilde{Z}_{L,n} = 0$. ■

To proceed in the proof of Theorem 1, I first show that if α (the curvature of the innovation cost function) is sufficiently small (including 2), then the industry growth rate goes to 0 as the large firm industry revenue share goes to 1. I then show that if $\alpha = 2$, then a function with the same sign as the derivative of the industry growth rate with respect to the large firm industry revenue share is strictly decreasing. I finally prove that Theorem 1 follows.

Proposition 2. *If* $\alpha \in (1, (3+\sqrt{5})/2)$*, then* $\lim_{z \to 1}(g(z)) = 0$*. If* $\alpha > (3+\sqrt{5})/2$*, then* $\lim_{z \to 1}(g(z)) = \infty$*.*

Proof. First, I show that as $\tilde{Z}_{L,n}$ goes to 1, total innovation I_n converges to the product of a strictly positive finite number and $(1 - \tilde{Z}_{L,n})^{\frac{1-\alpha}{\alpha}}$. Equation (21) shows that as $\tilde{Z}_{L,n}$ goes to 1, I_n diverges to positive infinity. Multiplying each side of (21) by $(\rho + I_n)^{\frac{1}{\alpha-1}}(1 - \tilde{Z}_{L,n})$ then shows that as $\tilde{Z}_{L,n}$ goes to 1, $I_n^{\frac{\alpha}{\alpha-1}}(1 - \tilde{Z}_{L,n})$ converges to a strictly positive finite number. The result follows.

Next, equations (22) and (14) show that as $\tilde{Z}_{L,n}$ goes to 1, the growth share of innovation $(\gamma - 1)g_n/I_n$ converges to the product of a strictly positive finite number and $(1 - \tilde{Z}_{L,n})^{\frac{1}{\alpha-1}}$ because that is the lowest power of $1 - \tilde{Z}_{L,n}$ contained in any term.

It follows that the industry growth rate g_n converges to the product of a strictly positive finite number and $(1 - \tilde{Z}_{L,n})^{\frac{1}{\alpha-1}+\frac{1-\alpha}{\alpha}}$. The exponent on $1 - \tilde{Z}_{L,n}$ is

$$\frac{1}{\alpha - 1} + \frac{1-\alpha}{\alpha} = \frac{(\alpha - \alpha_1^*)(\alpha_2^* - \alpha)}{(\alpha - 1)\alpha},$$

where

$$\alpha_1^* = (3 - \sqrt{5})/2 \qquad \alpha_2^* = (3 + \sqrt{5})/2.$$

Since $(3 - \sqrt{5})/2 < 1$, it follows that for all $\alpha \in (1, \alpha_2^*)$, the industry growth rate converges to 0 as $\tilde{Z}_{L,n}$ goes to 1, and for all $\alpha > \alpha_2^*$, the industry growth rate diverges to infinity as $\tilde{Z}_{L,n}$ goes to 1. ■

Proposition 3. *If* $\alpha = 2$*, then there exists a strictly decreasing differentiable function* $G(\cdot)$ *defined on* $[0, 1)$ *and a strictly positive differentiable function* $h(\cdot)$ *defined on* $[0, 1)$ *such that for all* $z \in [0, 1)$*,* $g'(z) = h(z)G(z)$*.*

Proof. Given $\alpha = 2$, equation (14) simplifies to

$$\frac{(\gamma - 1)g_{L,n}}{\kappa_{L,n}} = \left(\lambda^{\gamma-1}\chi_C + \lambda^{\gamma-1} - 1\right) \frac{\lambda^{\gamma-1}(1 - \tilde{Z}_{L,n}}{\lambda^{\gamma-1}(1 - \tilde{Z}_{L,n}) + \tilde{Z}_{L,n}}. \tag{23}$$

From (21), total innovation is given by the quadratic equation

$$I_n^2 + \rho I_n - A/(1 - \tilde{Z}_{L,n}) = 0,$$

where $A \equiv \lambda^{\gamma-1}(1/\chi_C + 1)\lambda^{\gamma-1}(\sigma - 1)L^p$. The quadratic equation has a unique positive solution, which must therefore be I_n:

$$I_n = -\rho/2 + \sqrt{(\rho/2)^2 + A/(1 - \tilde{Z}_{L,n})}.$$

Next, let B denote small firm growth relative to creative destruction from (13):

$$B \equiv (\gamma - 1)g_{S,n}/\kappa_{S,n} = \lambda^{\gamma-1}\chi_C + \lambda^{\gamma-1} - 1.$$

Then, from (22) and (23), the growth share of total innovation is

$$\frac{(\gamma-1)g_n}{I_n} = \tilde{Z}_{L,n}\left(B^{-1}\frac{\tilde{Z}_{L,n}}{\lambda^{\gamma-1}(1-\tilde{Z}_{L,n})} + B^{-1} + 1\right)^{-1} + (1-\tilde{Z}_{L,n})(B^{-1}+1)^{-1}.$$

Differentiating total innovation and the growth share of total innovation with respect to the large firm industry revenue share yields

$$\frac{\partial I_n}{\partial \tilde{Z}_{L,n}} = \frac{1}{1-\tilde{Z}_{L,n}}\frac{A/(1-\tilde{Z}_{L,n})}{2\sqrt{(\rho/2)^2 + A/(1-\tilde{Z}_{L,n})}}$$

and

$$\begin{aligned}\frac{\partial(\gamma-1)g_n/I_n}{\partial \tilde{Z}_{L,n}} = &\left(B^{-1}\frac{\tilde{Z}_{L,n}}{\lambda^{\gamma-1}(1-\tilde{Z}_{L,n})} + B^{-1} + 1\right)^{-1} - (B^{-1}+1)^{-1} \\ &- \frac{1}{1-\tilde{Z}_{L,n}}\left(B^{-1}\frac{\tilde{Z}_{L,n}}{\lambda^{\gamma-1}(1-\tilde{Z}_{L,n})} + B^{-1} + 1\right)^{-2} B^{-1}\frac{\tilde{Z}_{L,n}}{\lambda^{\gamma-1}(1-\tilde{Z}_{L,n})}.\end{aligned}$$

Thus, writing the derivative as

$$\frac{\partial(\gamma-1)g_n}{\partial \tilde{Z}_{L,n}} = \frac{(\gamma-1)g_n}{I_n}\frac{\partial I_n}{\partial \tilde{Z}_{L,n}} + I_n\frac{\partial(\gamma-1)g_n/I_n}{\partial \tilde{Z}_{L,n}}$$

and multiplying by $(1-\tilde{Z}_{L,n})\left(B^{-1}\frac{\tilde{Z}_{L,n}}{\lambda^{\gamma-1}(1-\tilde{Z}_{L,n})} + B^{-1} + 1\right)\left(\frac{1}{B+1}\frac{\tilde{Z}_{L,n}}{\lambda^{\gamma-1}} + 1 - \tilde{Z}_{L,n}\right)$, which is strictly positive for all $\tilde{Z}_{L,n} \in [0,1)$, yields a function $G(\tilde{Z}_{L,n})$ that has the same sign as the derivative of growth with respect to $\tilde{Z}_{L,n}$:

$$\begin{aligned}G(\tilde{Z}_{L,n}) = &\left(\frac{1}{B+1}\frac{\tilde{Z}_{L,n}}{\lambda^{\gamma-1}} + 1 - \tilde{Z}_{L,n}\right)\left(1 + \frac{1}{B+1}\frac{\tilde{Z}_{L,n}}{\lambda^{\gamma-1}}\right)\frac{A/(1-\tilde{Z}_{L,n})}{2\sqrt{(\rho/2)^2 + A/(1-\tilde{Z}_{L,n})}} \\ &- \left(\frac{1}{B+1}\frac{\tilde{Z}_{L,n}}{\lambda^{\gamma-1}} + 1 - \tilde{Z}_{L,n} + 1\right)\frac{1}{B+1}\frac{\tilde{Z}_{L,n}}{\lambda^{\gamma-1}}\left(-\rho/2 + \sqrt{(\rho/2)^2 + A/(1-\tilde{Z}_{L,n})}\right).\end{aligned}$$

The derivative of $G(\tilde{Z}_{L,n})$ is

$$
\begin{aligned}
G'(\tilde{Z}_{L,n}) = &\left(-1 + 2\left(\frac{1}{B+1}\frac{\tilde{Z}_{L,n}}{\lambda^{\gamma-1}} + 1 - \tilde{Z}_{L,n}\right)\frac{1}{B+1}\frac{1}{\lambda^{\gamma-1}}\right)\frac{A/(1-\tilde{Z}_{L,n})}{2\sqrt{(\rho/2)^2 + A/(1-\tilde{Z}_{L,n})}} \\
&+ \left(\frac{1}{B+1}\frac{\tilde{Z}_{L,n}}{\lambda^{\gamma-1}} + 1 - \tilde{Z}_{L,n}\right)\left(1 + \frac{1}{B+1}\frac{\tilde{Z}_{L,n}}{\lambda^{\gamma-1}}\right)\frac{A/(1-\tilde{Z}_{L,n})}{2\sqrt{(\rho/2)^2 + A/(1-\tilde{Z}_{L,n})}}\frac{1 - \frac{1}{2}\frac{A/(1-\tilde{Z}_{L,n})}{(\rho/2)^2 + A/(1-\tilde{Z}_{L,n})}}{1-\tilde{Z}_{L,n}} \\
&- 2\left(\frac{1}{B+1}\frac{\tilde{Z}_{L,n}}{\lambda^{\gamma-1}} + 1 - \tilde{Z}_{L,n}\right)\frac{1}{B+1}\frac{1}{\lambda^{\gamma-1}}\left(-\rho/2 + \sqrt{(\rho/2)^2 + A/(1-\tilde{Z}_{L,n})}\right) \\
&- \left(\frac{1}{B+1}\frac{\tilde{Z}_{L,n}}{\lambda^{\gamma-1}} + 2 - \tilde{Z}_{L,n}\right)\frac{1}{B+1}\frac{\tilde{Z}_{L,n}}{\lambda^{\gamma-1}}\frac{A/(1-\tilde{Z}_{L,n})}{2\sqrt{(\rho/2)^2 + A/(1-\tilde{Z}_{L,n})}}\frac{1}{1-\tilde{Z}_{L,n}}.
\end{aligned}
$$

I show that $G'(\tilde{Z}_{L,n}) < 0$, and so complete the proof, in two steps. First, the positive term on the first line is outweighed by the third line because

$$
-\rho/2 + \sqrt{(\rho/2)^2 + A/(1-\tilde{Z}_{L,n})} > \frac{A/(1-\tilde{Z}_{L,n})}{2\sqrt{(\rho/2)^2 + A/(1-\tilde{Z}_{L,n})}}.
$$

To see that the inequality holds, add $\rho/2$ to each side, multiply each side by $2\sqrt{(\rho/2)^2 + A/(1-\tilde{Z}_{L,n})}$, subtract $A/(1-\tilde{Z}_{L,n})$ from each side, square each side, and subtract $\rho^4/4 + \rho^2 A/(1-\tilde{Z}_{L,n})$ from each side, to get that the inequality is equivalent to $(A/(1-\tilde{Z}_{L,n}))^2 > 0$. Second, we can write the beginning of the second line of $G'(\tilde{Z}_{L,n})$ as

$$
\left(\frac{1}{B+1}\frac{\tilde{Z}_{L,n}}{\lambda^{\gamma-1}} + 1 - \tilde{Z}_{L,n}\right)\left(1 + \frac{1}{B+1}\frac{\tilde{Z}_{L,n}}{\lambda^{\gamma-1}}\right) = 1 - \tilde{Z}_{L,n} + \left(\frac{1}{B+1}\frac{\tilde{Z}_{L,n}}{\lambda^{\gamma-1}} + 2 - \tilde{Z}_{L,n}\right)\frac{1}{B+1}\frac{\tilde{Z}_{L,n}}{\lambda^{\gamma-1}}.
$$

The $1-\tilde{Z}_{L,n}$ term is outweighed by the negative term in the first line of $G'(\tilde{Z}_{L,n})$ because the final term on the second line is strictly less than $1/(1-\tilde{Z}_{L,n})$. The remaining term is outweighed by the fourth line of $G'(\tilde{Z}_{L,n})$ for the same reason. ■

Finally, I complete the proof of Theorem 1.

Proof. Since $g'(0) > 0$ and $g(0) > \lim_{z\to 1} g(z)$, it is sufficient to show that $g(\cdot)$ is single peaked, i.e., there exists a z^* such that $g'(z) > 0$ if and only if $z < z^*$. Since $g(\cdot)$ is continuously differentiable it is sufficient to show that if $g'(z) = 0$, then $g''(z) < 0$. If $g'(z) = 0$, then $g'\prime(z) = h'(z)g'(z)/h(z) + h(z)G'(z)$. Since $h(z) > 0$ and $G(\cdot)$ is strictly decreasing, it follows that the first term is zero and the second term is strictly negative. Thus if $g'(z) = 0$, then $g'\prime(z) < 0$, completing the proof of Theorem 1. ■

I now use the proof of Proposition 1 to prove Theorem 2.

Proof. Throughout the proof, I drop time and industry subscripts when possible because the theorem is concerned with a balanced growth path in which all industries are identical.

In the proof of Proposition 1, I decompose the growth rate on a balanced growth path into total innovation I, defined in (17), and the growth share of total innovation $(\gamma-1)g/I$. I show that the latter is a strictly decreasing continuously differentiable function of the large firm industry revenue share $\tilde{Z}_L$ on $[0,1)$, and does not depend separately on the large firm cost of innovation or fixed cost. It is therefore sufficient to show that on a balanced growth path with identical industries, the common value of total innovation I is not a function of the common large firm industry revenue share $\tilde{Z}_L$ (or the large firm cost of innovation or fixed cost). To do so, I use the small firm free entry condition.

The cost of entry at time t is $\chi_E W_t$, and the value of entering is the value of being a small firm with zero goods at time t, $V_{S,t}$. Hence, the value of being a small firm with zero goods relative to the wage on a balanced growth path is given by $V_{S,t}/W_t = \chi_E$, which is a constant.

The value of being a small firm with zero goods at time t is given by the HJB equation (10). Optimal small firm creative destruction and new good development are given by (11). Plugging in (20) for the present discounted value of profits relative to the wage on a balanced growth path, we have that small firm innovation rates are not a function of time t:

$$\delta_S = \left(\lambda^{\gamma-1}\frac{(\sigma-1)L^p}{\rho+I}\right)^{\frac{1}{\alpha-1}}$$
$$\kappa_S = \chi_C^{\frac{-1}{\alpha-1}}\left(\lambda^{\gamma-1}\frac{(\sigma-1)L^p}{\rho+I}\right)^{\frac{1}{\alpha-1}}.$$

Equation (19) for small firm profits on a balanced growth path shows that for all $t' \geq t$,

$$\pi_{S,t'} = \frac{(1-\sigma^{-1})C_t}{\rho+I}e^{g(t'-t)}$$

because aggregate output on a balanced growth path is $C_t' = Z_t e^{g(t'-t)}L^p$. We can thus guess and verify that the value of being a small firm with zero goods at time t is given by

$$(\rho+\eta)V_{S,t} = (1-1/\alpha)\left(\chi_C^{\frac{-1}{\alpha-1}}+1\right)\left(\lambda^{\gamma-1}\frac{(\sigma-1)L^p}{\rho+I}\right)^{\frac{1}{\alpha-1}}\lambda^{\gamma-1}\frac{(1-\sigma^{-1})C_t}{\rho+I}$$

because then $\dot{V}_{S,t} = gV_{S,t}$ (recall that $r=\rho+g$). Since the wage is $W_t = Z_t/\sigma$, and using that $V_{S,t}/W_t = \chi_E$, it follows that

$$(\rho+\eta)\chi_E = (1-1/\alpha)\left(\chi_C^{\frac{-1}{\alpha-1}}+1\right)\left(\lambda^{\gamma-1}\frac{(\sigma-1)L^p}{\rho+I}\right)^{\frac{\alpha}{\alpha-1}},$$

which determines total innovation I as a function of exogenous parameters not including the large firm cost of innovation or fixed cost. $\blacksquare$