Market Concentration, Growth, and Acquisitions

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Abstract

I study an oligopolistic growth model in which firms can innovate by creatively destroying their competitors' goods, improving their own goods, and developing new ones. A large firm is equivalent to a mass of small firms that can coordinate their activities to maximize joint profits. Larger firms adapt their innovation mix to avoid cannibalization, and as a result they impose a high rate of creative destruction risk on their competitors without generating much growth. A tax on large firm acquisitions of their smaller competitors' goods may backfire by encouraging large firms to creatively destroy those goods instead. In a special case of the model with only creative destruction, I prove conditions so that a rise in large firm profitability leads to a fall in growth, and so that a fall in taxes on acquisitions leads to an *increase* in growth. In the full quantitative model, a fall in large firm fixed costs, calibrated to match the recent rise in concentration in the US, explains 41% of the fall in growth from the 1990s to the 2010s, as well as the burst in growth during the late 1990s, the positive across-industry correlation between changes in concentration and growth, and the fall in growth relative to R&D expenditures. Dispersion in large firm innovation costs or profitability across industries yields a novel theory of the inverted-U relationship between concentration and growth.

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1 Introduction

Many authors have documented a rising share of revenue going to the top firms in industries at the national level in the US since the 1990s.[1] This trend has spurred research into its connection to the recent decline in growth, as well as the policy implications.[2] Over a similar time period, there was a dramatic rise in the rate at which venture capital backed startups are acquired relative to the rate at which they go public.[3] Two questions emerge: Is large firm behavior behind the fall in growth? If so, should antitrust authorities limit acquisitions to reduce industry concentration and promote growth?

To answer these questions, I study an oligopolistic growth model in which firms can improve on old goods and develop new ones. I take as the key feature of a large firm that they control a significant portion of their industry's innovation and as a result earn a significant portion of its sales. Specifically, the only meaningful ex-ante firm heterogeneity is in innovation costs; a large firm is equivalent to a group of smaller firms that can coordinate their innovation activities to maximize joint profits. I find that if firms can creatively destroy each others' goods and innovation rates are sufficiently responsive to the profits firms earn from innovating, then a shock that increases large firm innovation incentives leads to a fall in growth. In that case, acquisitions that increase concentration are costly but policies to limit them may backfire by encouraging large firms to innovate more, which leads to lower growth. On the other hand, a tax on large firms' sales reduces their innovation and increases growth. In the quantitative model, a rise in large firm profitability calibrated to generate the observed rise in concentration in the US from the mid 1990s to the late 2010s explains 41% of the fall in the long-run growth rate, as well as the short-run growth burst in the late 1990s, the positive correlation across industries between changes in concentration and growth, the fall in the entry rate, and the rise in R&D expenditures relative to GDP despite the fall in growth.

The results follow from two implications of the Arrow (1962) "replacement effect" that incumbents are reluctant to innovate in ways that cannibalize their sales from past innovations. First, within an industry, larger firms face lower rates of creative destruction. When a firm creatively destroys a good, they make an improved version that replaces the

[1]See Autor, Dorn, Katz, Patterson, and Van Reenen (2020), Barkai (2020).

[2]See Cavenaile, Celik, and Tian (2021), Aghion, Bergeaud, Boppart, Klenow, and Li (2022), Akcigit and Ates (2021), De Ridder (2021), Liu, Mian, and Sufi (2022), Olmstead-Rumsey (2022), and Garcia-Macia, Hsieh, and Klenow (2019).

[3]See Pellegrino (2021).

old one. Incumbents prefer to target their competitors' goods rather than replace their own. A large innovative firm controls a substantial share of its industry's innovation and products, so their preference for directing creative destruction elsewhere leaves their goods effectively missing a portion of the industry creative destruction rate. Second, larger firms have a stronger preference for creatively destroying their competitors' goods over improving on their own goods or developing new ones. The latter types of innovation generate sales only through growth, which takes sales from the other goods in the industry, many of which the large firm produces; creatively destroying a good takes sales from the producer of that good beyond the growth it creates.

Each of the two effects implies that if a firm is larger within its industry, then relative to the growth it generates, it imposes a higher rate of creative destruction risk on its competitors. Thus, if a large firm's incentives to innovate increase and its competitors' innovation rates are responsive, growth falls. Although the large firm does not generate much more growth, it deters its competitors from doing so by imposing on them a high rate of creative destruction risk.

The first mechanism is in line with the finding in Akcigit and Kerr (2018) that in the US, larger firms' patents face lower rates of external citations. Moreover, it can explain the high sales-to-R&D ratios of industry leaders Olmstead-Rumsey (2022) documents; to illustrate, in the limit if a firm faces no depreciation rate on its goods' sales, then its innovation problem is still well-defined as long as the interest rate is positive, but in the long-run, its stock of sales from past innovations is infinite.[4] In support of the second mechanism, Garcia-Macia, Hsieh, and Klenow (2019) estimate that creative destruction is responsible for a minority of growth in the US, but a majority of the rate at which firms discount their innovations because most of the between-firm flows creative destruction generates is the reallocation of goods, not growth.

I begin in Section 3 with a qualitative model of an industry with a single large firm and a continuum of small firms that innovate only by creatively destroying their competitors' goods. The difference between the large firm and the mass of small firms is that the large

[4]Argente, Lee, and Moreira (2021) provide empirical evidence for the ability of firms to direct their creative destruction away from their own goods: the revenues of high sales products depreciate more quickly than the revenues of low sales products, in line with the prediction that such products would be creatively destroyed more quickly. Akcigit, Alp, and Peters (2021) argue that a relatively high creative destruction rate for goods produced by firm types that innovate less, and tend to be small, can explain the high employment shares of old firms in US and Indian data.

firm does not creatively destroy its own goods, whereas small firms creatively destroy each others' goods because they cannot collude. Thus only the first mechanism described above is present. I prove an analytical condition in terms of observable equilibrium outcomes such that a rise in large firm innovation incentives leads to a fall in the long-run growth rate. In particular, growth falls if small firm innovation is sufficiently elastic with respect to the profits a small firm earns from innovating. In the limit, a monopolist deters all small firm innovation with a rapid creative destruction rate, but generates no growth because there are no small firm goods to creatively destroy.

The large firm can search for acquisition opportunities in which it makes a take-it-or-leave-it offer to a small firm for one of its good subject to a tax. I prove an analytical condition in terms of observable equilibrium outcomes such that a fall in the tax leads to a *rise* in the long-run growth rate, i.e., encouraging acquisitions increases growth. In particular, growth rises only if a rise in large firm innovation incentives leads to a fall in growth, if the elasticity of innovation with respect to the profits from innovating is sufficiently high relative to the elasticity of the acquisition rate with respect to the surplus from an acquisition, and if industry concentration is sufficiently high. This result follows from two competing effects of acquisitions. A lower tax increases the acquisition rate, which decreases growth by shifting goods to the large firm where they experience a slower rate of creative destruction. However, a lower tax also increases the surplus of each acquisition, which reduces the large firm's incentive to innovate because creatively destroying its competitors' goods and acquiring them are substitutes. An econometrician looking at the impact of exogenous acquisitions on growth would only estimate the first effect.

I develop the quantitative model in Section 4, in which firms also innovate by improving on their own goods and creating new ones, and thus both mechanisms described above are active. Moreover, I embed the industry setup into a macroeconomic model that consists of a continuum of industries, each of which contains a single large firm and a measure of small firms. In Section 5, I calibrate the model and conduct the key experiment: a rise in large firm profitability due to a fall in large firm per-good fixed costs. Although creative destruction is responsible for a minority of growth, its presence drives the quantitatively significant results—in an equivalent model without creative destruction, a rise in large firm profitability leads to a small *increase* in growth—because it is responsible for a *majority* of sales/labor flows between firms. The disparity between creative destruction's growth and flows shares implies that whereas own good improvement and new good development generate sales only through growth, creative destruction of a competitor's good generates sales mostly, though not entirely, through business stealing; the innovator takes over all production of the targeted good by making a small improvement on the previous version. Therefore, large firms tilt their innovation toward creative destruction to avoid cannibalization, which imposes risk on small firms and so deters small firm innovation, without generating much growth.

The model generates the documented short-run burst in growth and positive correlation across industries between rising concentration and growth because small firm innovation is less responsive in the short-run and to industry-specific shocks. Long-run growth falls despite a rise in R&D because firm-level innovation costs are convex, so as large firms innovate more, their R&D becomes less efficient. It follows that the results hold whether labor is fully flexible across entry, innovation, and production, or completely fixed. In the first case, a free entry condition makes small firm innovation perfectly elastic; a rise in large firm innovation implies an increase in the creative destruction risk small firms face, so for the discount rate small firms apply to innovations to remain constant, the growth rate must fall. In the second case, a rise in large firm innovation implies a shift in the fixed supply of innovation labor from small firms to large firms, and so a fall in its efficiency. The results are also robust to different calibrations.

I focus on a shock to large firm profitability because it best matches the full set of empirical observations related to the fall in growth. More generally, the model's two main mechanisms imply that other shocks that increase concentration will have an additional negative effect on growth.

I analyze the effects of a tax on acquisitions in the quantitative model in Section 6. A sufficiently low tax leads to faster growth than in an economy without acquisitions: to preserve valuable acquisition opportunities, large firms reduce innovation. The break even tax rate at which growth is the same with and without acquisitions is higher—acquisitions are more easily beneficial—if the acquisition rate is less elastic, and after the rise in large firm profitability and concentration. It follows that policy should not place unnecessary road blocks in the way of high surplus acquisitions, particularly in concentrated industries, but should block marginal low surplus acquisitions. Allowing high value acquisitions is an effective way to limit large firm innovation because it does not require intimate knowledge of industry boundaries or of which firms are large.

These results highlight an important subtlety in optimal competition and innovation policy. In theories that focus only on *how much* large firms produce or innovate, high markups or the Arrow (1962) "replacement effect" imply that it is optimal to encourage *more* large firm production or innovation.[5] According to these theories, reducing large firm innovation is a cost of acquisitions, rather than a benefit.[6] Instead, taking into account the multidimensional nature of innovation, I find that policies that encourage large firm production or innovation end up encouraging activity that imposes a strong deterrent on competition without adding much social value.

Large Firms and Innovation:

Previous work on oligopolistic competition and innovation mostly focuses on the impact of a large firm's market share on its *magnitude* of innovation, which is not important for the theory I propose because I specifically consider a rise in large firm innovation incentives.[7] A notable exception is the theory put forward in Argente, Baslandze, Hanley, and Moreira (2021) and mentioned in Akcigit and Ates (2021) that large firms use patents to deter competition. Although the mechanism is different, the implications for the effects of large firm innovation are similar. This theory is complementary to the one I propose because creative destruction is most relevant in industries with weak patent protection.

In the context of the recent rise in concentration and fall in growth, the most similar papers are Aghion, Bergeaud, Boppart, Klenow, and Li (2022), De Ridder (2021), and Liu, Mian, and Sufi (2022). In the first two, all firms have infinitesimal market shares, and a rise in innovation by high process productivity firms shifts goods to firms that are difficult to compete with, which reduces the return to creatively destroying those goods, and thus results in less innovation overall. As such, they find similar results from the rise of high productivity firms that I find from the rise of large, but equally productive, firms. In Liu, Mian, and Sufi (2022), two large firms compete in each industry, and if one becomes sufficiently dominant, then the growth rate falls because the dominant firm optimally cuts their price dramatically in response to innovation by their competitor. By contrast, my mechanism does not rely on a large firm responding directly to the actions of a single competitor. Thus, it may be more relevant when thinking about an industry with both large and small firms, especially in light of the evidence in Amiti, Itskhoki, and Konings (2019) on how responsive a firm's price is to a change in its competitors' prices.

Although none of the papers mentioned thus far study acquisitions, in each case ac-

[5] See Edmond, Midrigan, and Xu (2021).

[6] See Fons-Rosen, Roldan-Blanco, and Schmitz (2022).

[7] See Shapiro (2012) for a discussion, and Cavenaile, Celik, and Tian (2021) for a recent example.

quisitions by large or high productivity firms would be strictly bad for growth because it does not matter how those firms gain their goods. Once they gain market share, they deter innovation by reducing the *flow profits* their competitors can earn. By contrast, I find that the way large firms innovate leads to lower growth because they add to the *discount rate* their competitors use for an innovation. Thus, even if acquisitions lead to higher concentration, they can increase growth if they are associated with a sufficient drop in large firm innovation.

Finally, the model generates an alternative explanation of the inverted-U relationship between concentration and growth across industries, estimated empirically in Aghion, Bloom, Blundell, Griffith, and Howitt (2005) and Cavenaile, Celik, and Tian (2021). If variation in large firm profitability or innovation capabilities drives variation across industries, then the inverted-U emerges because an increase in large firm innovation decreases growth only if concentration is sufficiently high so that large firm innovation is sufficiently distorted. Previous theories of the inverted-U are based instead on two effects of competition: some is necessary to encourage dominant firms to innovate and escape, but too much discourages any innovation. A crucial difference is that in this paper, there can be an inverted-U across industries even if a rise in large firm innovation incentives always decreases growth at the aggregate level because the response of small firms is vital and small firms are more responsive to aggregate shocks.

Large Firm Acquisitions of Small Competitors' Goods:

"Entry for buyout", described in Rasmusen (1988) and more recently, Letina, Schmutzler, and Seibel (2021) and Fons-Rosen, Roldan-Blanco, and Schmitz (2022), is another theory in which a high expected value of future acquisitions increases growth even though the distributional consequences of actual acquisitions do not: firms enter and innovate to be acquired because they receive a fraction of the surplus. Yet, if large firm innovation is one-dimensional, then the entry for buyout effect needs to be weighed against the *negative* effect on large firm innovation. In this paper, acquisitions are made with take-it-or-leave-it offers, so the entry for buyout effect is absent. Instead, acquisitions may lead to higher growth precisely *because* they reduce large firm innovation. A distinct implication is that acquisitions have more potential to increase growth if concentration is higher.

Model Building Blocks:

The model builds on two different strands of the growth literature, one focused on models of creative destruction[8], and one on expanding varieties models[9]. Recent work combines the two, but without large firms with positive market shares.[10]

2 Industry Model

I begin with a model of an industry, taking macroeconomic aggregates as given. I study a special case of the industry model, in which I can prove analytical results, and then embed it in a macroeconomic model, which I solve quantitatively and compare to data.

There is a unit measure of industries, indexed by $n \in [0, 1]$. In this section, focus on a particular industry n. Time is continuous and indexed by $t \in [0, \infty)$. At each time t, there is a measure N_t of small firms, indexed by $i \in [0, N_t]$, and a single large firm denoted by $i = L$. They produce a measure $M_{n,t}$ of intermediate goods, indexed by $j \in [0, M_{n,t}]$, which they sell to a representative final good producer with R_t to spend. Firms hire labor at wage W_t to produce and innovate, and purchase an input for fixed costs at price normalized to 1. They maximize expected discounted profits with discount rate r_t.

Although I focus on one industry in this section, I include a subscript n for industry-specific variables so that it is clear which variables are determined at the macroeconomic level, i.e., are exogenous to the industry: N_t, R_t, W_t, and r_t.

2.1 Static Block

2.1.1 Demand

At each time t, the final good producer takes as given a price offered by each firm for each intermediate good, $\{p_{n,t}(i, j)\}$. All versions of a good j are perfect substitutes, so it purchases only the cheapest one, with price $p_{n,t}(j) = \min\{p_{n,t}(i, j)\}_{i\in[0,N_t]\cup\{L\}}$. If multiple versions have price $p_{n,t}(j)$, then any split of purchases across those versions is optimal. The final good producer chooses demand of each good $\{y_{n,t}(j)\}$ to maximize an industry aggregate $Y_{n,t}$ defined by

$$Y_{n,t}^{\frac{\gamma-1}{\gamma}} = \int_0^{M_{n,t}} y_{n,t}(j)^{\frac{\gamma-1}{\gamma}} dj, \tag{1}$$

[8]See Grossman and Helpman (1991b), Aghion and Howitt (1992), Klette and Kortum (2004), and Aghion, Bergeaud, Boppart, Klenow, and Li (2022).

[9]See Romer (1990) and Grossman and Helpman (1991a).

[10]See Atkeson and Burstein (2019).

subject to spending R_t:

$$\int_0^{M_{n,t}} p_{n,t}(j) y_{n,t}(j) dj = R_t,$$

where $\gamma > 1$ is the constant elasticity of substitution across goods.

The First Order Condition for good j and budget constraint imply the demand curve[11]

$$y_{n,t}(j) = p_{n,t}(j)^{-\gamma} P_{n,t}^{\gamma-1} R_t,$$

where $P_{n,t}$ is the industry price index so that $R_t = P_{n,t} Y_{n,t}$:

$$P_{n,t}^{1-\gamma} = \int_0^{M_{n,t}} p_{n,t}(j)^{1-\gamma} dj. \tag{2}$$

2.1.2 Production and Competition

At each time t, production occurs in two stages. Each firm can potentially produce a version of each good in its industry with a version specific productivity $z_{n,t}(i,j)$. Let $z_{n,t}(j) \equiv \max\{z_{n,t}(i,j)\}_{i \in [0,N_t] \cup \{L\}}$ be the highest productivity available for good j at time t. We will see later that for each j, t, there is a unique firm with $z_{n,t}(i,j) = z_{n,t}(j)$.

Let $Z_{n,t}$ be an industry aggregate of productivity:

$$Z_{n,t}^{\gamma-1} = \int_0^{M_{n,t}} z_{n,t}(j)^{\gamma-1} dj,$$

and define the relative productivity of a version of a good:

$$\tilde{z}_{n,t}(i,j) \equiv (z_{n,t}(i,j)/Z_{n,t})^{\gamma-1} \qquad \tilde{z}_{n,t}(j) \equiv (z_{n,t}(j)/Z_{n,t})^{\gamma-1}.$$

In the first stage of production, firms simultaneously choose for which goods they will pay a fixed cost. If firm i pays the fixed cost for good j, then they can produce it in the second stage using labor with production function

$$q_{n,t}(i,j) = z_{n,t}(i,j) l_{n,t}(i,j).$$

Otherwise, they can produce good j with productivity $z_{n,t}(j)/\sigma$, where $\sigma > 1$ captures the ability of firms to imitate each other's versions. The fixed cost for a version scales with

[11] The First Order Condition yields $Y_{n,t}^{\frac{1}{\gamma}} y_{n,t}(j)^{\frac{-1}{\gamma}} = \zeta p_{n,t}(j)$, where ζ is the Lagrange multiplier on the budget constraint. Aggregating across all j and satisfying the budget constraint yields $\zeta = Y_{n,t}^{\frac{1}{\gamma}} P_{n,t}^{\frac{1-\gamma}{\gamma}} R_t^{\frac{-1}{\gamma}}$.

its relative productivity, and is $\tilde{z}_{n,t}(i,j) f_S$ and $\tilde{z}_{n,t}(i,j) f_{L,n}$ for small firms and the large firm, respectively. In the second stage of production, fixed cost payments are common knowledge and firms simultaneously choose prices for each good to maximize static profits subject to selling less than the quantity demanded given prices.

Finally, the large firm's revenue is subject to a tax τ_R; it is multiplied by $1-\tau_R$.

2.1.3 Static Optimization and Equilibrium

At each time t, firms choose fixed cost payments and prices to maximize static profits in a subgame perfect Nash equilibrium of the two stage game. I focus on the equilibrium in which for each good j, the sole producer is the unique firm with the most productive version; it pays the fixed cost, sets a markup of σ, and thus earns flow profits

$$\tilde{z}_{n,t}(j)\left(\left(1-\sigma^{-1}\right)R_t - f_S\right) \qquad\qquad \tilde{z}_{n,t}(j)\left(\left(1-\sigma^{-1}-\tau_R\right)R_t - f_{L,n}\right) \tag{3}$$

if small or large, respectively. Profits scale with relative productivity $\tilde{z}_{n,t}(j)$: variable profits scale given CES demand and a constant markup, and fixed costs scale by definition.

The equilibrium strategies are for each good: 1) only the firm with the most productive version pays the fixed cost; 2) for any first stage actions, the most productive producer in the second stage sets its price equal to the second-most productive producer's marginal cost, and other producers set their prices equal to their own marginal costs; 3) if multiple firms set the same price, demand is split evenly among those with the lowest marginal cost. Assumptions 1 and 2 hold for all t, and ensure that these strategies are best responses:

Assumption 1. $\sigma \leq \gamma/(\gamma-1)$.

Assumption 2. $\left(1-\sigma^{-1}\right)R_t \geq f_S$ *and* $\left(1-\left(1-\sigma^{1-\gamma}\right)\mathcal{L}_{n,t}\right)\left(1-\sigma^{-1}-\tau_R\right)R_t \geq f_{L,n}$.

I solve the static optimization problem formally in Appendix A.1. Since only the most productive producer of a good pays the fixed cost, absent imitation they want to set a markup of at least $\gamma/(\gamma-1)$. Assumption 1 thus implies that imitation binds and pins down the markup. Assumption 2 ensures that the firm with the most productive version of a good earns positive profits from paying the fixed cost, and for the large firm takes into account that paying the fixed cost for one good lowers the sales of its other goods.

2.1.4 Aggregation

Given that all goods are sold with a markup σ, industry output is $Y_{n,t} = Z_{n,t}R_t/(\sigma W_t)$. In that sense, $Z_{n,t}$ is the "correct" measure of industry productivity.

2.2 Dynamic Block

2.2.1 Innovation

Each firm contains entrepreneurs it uses to innovate. At each time t, an entrepreneur's firm chooses a new good development rate δ and for each good j in the industry, a creative destruction rate $\kappa(j)$. At Poisson arrival rate δ, the firm develops a new good; the productivity z of its version is drawn so that the expected value of $(z/Z_{n,t})^{\gamma-1}$ is 1, and all other firms' versions have productivity 0. At Poisson arrival rate $\int_0^{M_{n,t}} \kappa(j)dj$, the firm creatively destroys an old good, and the relative probability it creatively destroys good j is proportional to $\kappa(j)$. Upon creatively destroying good j, its version's productivity jumps to z, which is drawn so that $z > z_{n,t}(j)$, and the expected value of $z^{\gamma-1}$ is $(\lambda z_{n,t}(j))^{\gamma-1}$, where $\lambda > 1$ is the innovation step size. Whether a firm develops a new good or creatively destroys an old one, it becomes the (temporary) sole producer of that good.

The flow labor cost of an entrepreneur's innovation is

$$\alpha \int_0^{M_{n,t}} \tilde{z}_{n,t}(j) X_S(\kappa(j))dj + X_S(\delta) \qquad \alpha \int_0^{M_{n,t}} \tilde{z}_{n,t}(j)\beta_n X_L(\kappa(j))dj + \beta_n X_L(\delta)$$

if it is controlled by a small firm or the large firm, respectively, where $\alpha > 0$ is the relative cost of creative destruction and $\beta_n > 0$ is an industry-specific large firm innovation cost-shifter. Each innovative activity has an independent cost that scales with the expected relative productivity of a successful innovation. For $I \in \{S, L\}$, the innovation cost function $X_I(\cdot)$ is twice continuously differentiable, strictly increasing, and weakly convex, with $X_I(0) = 0$ and a continuous marginal innovation cost elasticity for all $\kappa \geq 0$:

$$\epsilon_I(\kappa) \equiv \kappa X_I''(\kappa)/X_I'(\kappa).$$

The marginal innovation cost is increasing in the total rate, and it is costly to focus on one type of innovation or on creatively destroying a subset of goods.

2.2.2 Small vs. Large Firms

A small firm contains a single entrepreneur, so at a Poisson arrival rate it develops a new good or creatively destroys an old one. Thus, in equilibrium, it produces a finite number of goods. The large firm contains a unit measure of entrepreneurs, so at a *continuous rate* it develops new goods and creatively destroys old ones; it creatively destroys *each* *good* at a Poisson arrival rate. Thus, in equilibrium, it produces a *measure* of goods. This distinction is the only substantive difference between small and large firms.

2.2.3 Acquisitions

At each time t, the large firm chooses an acquisition opportunity rate $A_t(j)$ for each good $j \in [0,1]$. At Poisson arrival rate $A_t(j)$, it encounters the firm with the most productive version of good j, and makes a take-it-or-leave-it offer subject to an acquisition tax rate $\tau_A > -1$. If the other firm accepts, then its version is transferred to the large firm, the large firm pays the offered price, and pays τ_A times the price to the tax authority.

The flow labor cost of searching for potential acquisitions is $\int_0^{M_{n,t}} \tilde{z}_{n,t}(j) X_A(A_t(j))dj$, where $X_A(\cdot)$ is twice continuously differentiable, strictly increasing, and weakly convex, with $X_A(0) = 0$ and a continuous marginal search cost elasticity for all $A \geq 0$:

$$\epsilon_A(A) \equiv AX_A''(A)/X_A'(A).$$

2.2.4 Dynamic Equilibrium

At each time t, the large firm chooses innovation rates, acquisition search rates, and acquisition price offers, and small firms choose innovation rates and acquisition price acceptance strategies to maximize expected present discounted profits using interest rate r_t. I focus on Markov perfect equilibria in which the industry state is the large firm's share of industry relative productivity (or equivalently revenue):

$$\mathcal{L}_{n,t} \equiv \int_{j \in J_{n,t}} \tilde{z}_{n,t}(j)dj,$$

where $J_{n,t}$ is the set of goods of which the large firm has the most productive version, and the aggregate state is the measure of small firms N_t, expenditures R_t, the wage W_t, and the interest rate r_t. Moreover, each small firm develops new goods at the rate $\delta_{S,n,t}(\mathcal{L}_{n,t})$, creatively destroys each of its competitors' goods at the single rate $\kappa_{S,n,t}(\mathcal{L}_{n,t})$, and accepts any acquisition offer weakly above its value of a good; the large firm develops new goods at rate $\delta_{L,n,t}(\mathcal{L}_{n,t})$, creatively destroys each of its competitors' goods and its own goods at the single rates $\kappa_{L,n,t}(S;\mathcal{L}_{n,t})$ and $\kappa_{L,n,t}(L;\mathcal{L}_{n,t})$, respectively, searches for an acquisition opportunity for each of its competitors' goods at the single rate $A_{n,t}(\mathcal{L}_{n,t})$, and in an acquisition opportunity, always offers the small firm's value of the good. In each case, the subscript t captures the dependence on the aggregate state. A small firm chooses a different creative destruction rate for the goods it currently produces, but this rate is irrelevant because it produces a measure 0 of goods.

I solve the dynamic optimization problem in Sections 2.2.6 and 2.2.7.

2.2.5 Evolution of the Industry State and Growth

In equilibrium, the large firm revenue share evolves over time according to

$$\begin{aligned}\dot{\mathcal{L}}_{n,t} =&(\kappa_{L,n,t}(S;\mathcal{L}_{n,t}) + A_{n,t}(\mathcal{L}_{n,t}) + (\gamma-1)g_{L,n,t}(\mathcal{L}_{n,t}))(1-\mathcal{L}_{n,t}) \\ &- N_t(\kappa_{S,n,t}(\mathcal{L}_{n,t}) + (\gamma-1)g_{S,n,t}(\mathcal{L}_{n,t}))\mathcal{L}_{n,t},\end{aligned} \tag{4}$$

where a dot over a variable indicates its derivative with respect to time t, and where $g_{n,t}(\mathcal{L}_{n,t}) \equiv \dot{Z}_{n,t}/Z_{n,t}$ is the growth rate of industry productivity, which is the sum of growth due to large firm innovation, $g_{L,n,t}(\mathcal{L}_{n,t})$, and to small firm innovation, $N_t g_{S,n,t}(\mathcal{L}_{n,t})$:

$$\begin{aligned}(\gamma-1)g_{L,n,t}(\mathcal{L}_{n,t}) &= \left(\lambda^{\gamma-1}-1\right)\left(\kappa_{L,n,t}(S;\mathcal{L}_{n,t})(1-\mathcal{L}_{n,t}) + \kappa_{L,n,t}(L;\mathcal{L}_{n,t})\mathcal{L}_{n,t}\right) + \delta_{L,n,t}(\mathcal{L}_{n,t}) \\ (\gamma-1)g_{S,n,t}(\mathcal{L}_{n,t}) &= \left(\lambda^{\gamma-1}-1\right)\kappa_{S,n,t}(\mathcal{L}_{n,t}) + \delta_{S,n,t}(\mathcal{L}_{n,t}).\end{aligned} \tag{5}$$

When a firm develops a new good or creatively destroys one of its own, it gains sales only through adding to industry productivity, i.e., generating growth, which takes sales from all other goods by depreciating their relative productivities. Each good loses the same fraction, so the large firm bears a share $\mathcal{L}_{n,t}$ of the cost. In the other extreme, in an acquisition, the large firm takes sales from the previous producer without generating any growth, and thus without any effect on other goods. Finally, creatively destroying a competitor's good yields a combination of the two: the innovator improves on the good, which generates growth and multiplies its sales by $\lambda^{\gamma-1}-1$, but also takes the sales the good had before it was improved on.

Therefore, there are two reasons for the $1-\mathcal{L}_{n,t}$ on the first line of equation (4): the first two terms in parentheses are only targeted at small firms that control a share $1-\mathcal{L}_{n,t}$ of sales, whereas the third term—growth—is discounted because a fraction $\mathcal{L}_{n,t}$ of the sales it generates is cannibalization. A similar logic applies to the $\mathcal{L}_{n,t}$ on the second line.

2.2.6 Small Firm Dynamic Optimization

A small firm's innovation optimization problem is the same regardless of the goods it produces because it takes industry aggregates as given. At each time t, a small firm chooses innovation rates $\delta, \{\kappa(j)\}$ to maximize

$$\left(\int_0^{M_{n,t}} \kappa(j)\lambda^{\gamma-1}\tilde{z}_{n,t}(j)dj + \delta\right)\Pi_{n,t}(\mathcal{L}_{n,t}) - W_t\left(\alpha\int_0^{M_{n,t}} \tilde{z}_{n,t}(j)X_S(\kappa(j))dj + X_S(\delta)\right),$$

where $\tilde{z}_{n,t}(j)\Pi_{n,t}(\mathcal{L}_{n,t})$ is the expected present discounted profits from being the most productive producer of good j at time t, and the value per unit of relative productivity, $\Pi_{n,t}(\cdot)$, is given by the HJB equation:

$$\begin{aligned} r_t\Pi_{n,t}(\mathcal{L}_{n,t}) =& \underbrace{\left(1-\sigma^{-1}\right)R_t - f_S}_{\text{flow profits}} - (\underbrace{N_t\kappa_{S,n,t}(\mathcal{L}_{n,t}) + \kappa_{L,n,t}(S;\mathcal{L}_{n,t})}_{\text{creative destruction risk}} + \underbrace{(\gamma-1)g_{n,t}}_{\text{growth}})\Pi_{n,t}(\mathcal{L}_{n,t}) \\ &+ \dot{\mathcal{L}}_{n,t}\Pi'_{n,t}(\mathcal{L}_{n,t}) + \dot{\Pi}_{n,t}(\mathcal{L}_{n,t}). \end{aligned} \tag{6}$$

The right-hand side of the first line is flow profits and the rate at which a small firm's good's value depreciates: at a Poisson arrival rate, the good is transferred in its entirety to another firm (creative destruction risk), and at a continuous rate, industry growth erodes the good's relative productivity. The second line is the change in the value over time due to changes in the industry and aggregate state.

The First Order Conditions yield the optimal new good development rate and the optimal single rate at which a small firm creatively destroys each good in its industry:

$$W_tX'_S(\delta_{S,n,t}(\mathcal{L}_{n,t})) \geq \Pi_{n,t}(\mathcal{L}_{n,t}) \qquad \alpha W_tX'_S(\kappa_{S,n,t}(\mathcal{L}_{n,t})) \geq \lambda^{\gamma-1}\Pi_{n,t}(\mathcal{L}_{n,t}). \tag{7}$$

Each holds with equality if the innovation rate is strictly positive. The creative destruction rate is the same for all goods because costs and benefits scale with relative productivity.

2.2.7 Large Firm Dynamic Optimization

I split the large firm optimization problem into two steps. First, taking as given the evolution of its revenue share, $\dot{\mathcal{L}}_{n,t}$, the large firm chooses δ, $\{\kappa(j)\}$, and $\{A(j)\}$ to minimize cost. The large firm chooses one creative destruction rate and one acquisition rate for all its competitors' goods because costs are strictly increasing and convex, and because benefits and costs both scale with relative productivity. Let ζ be the Lagrange multiplier on the $\dot{\mathcal{L}}_{n,t}$ constraint. The First Order Conditions yield the optimal rates:

$$\begin{aligned} \beta_nW_tX'_L(\delta_{L,n,t}(\mathcal{L}_{n,t})) &\geq (1-\mathcal{L}_{n,t})\zeta \\ \beta_n\alpha W_tX'_L(\kappa_{L,n,t}(L;\mathcal{L}_{n,t})) &\geq \left(\lambda^{\gamma-1}-1\right)(1-\mathcal{L}_{n,t})\zeta \\ \beta_n\alpha W_tX'_L(\kappa_{L,n,t}(S;\mathcal{L}_{n,t})) &\geq \left(\left(\lambda^{\gamma-1}-1\right)(1-\mathcal{L}_{n,t})+1\right)\zeta \\ W_tX'_A(A_{n,t}(\mathcal{L}_{n,t})) &\geq \zeta - (1+\tau_A)\Pi_{n,t}(\mathcal{L}_{n,t}), \end{aligned} \tag{8}$$

where each inequality holds with equality if the rate is strictly positive, and where I impose that the large firm always acquires a good (at minimum price) conditional on getting the opportunity because otherwise it would not search.

The marginal benefits on the right-hand sides reflect that the large firm discounts the value of an innovation based on the share that comes from growth because that share cannibalizes the large firm's sales of its other goods (as discussed in Section 2.2.5). Thus, it fully discounts developing a new good and creatively destroying one of its own by $1 - \mathcal{L}_{n,t}$, does not discount an acquisition at all, and only discounts the portion of the value of creatively destroying a competitor's good that comes from improving on the good.

Expressions (8) imply a unique solution given $\dot{\mathcal{L}}_{n,t}$ because each rate is increasing in ζ. Let $\bar{X}_{n,t}\left(\dot{\mathcal{L}}_{n,t}; \mathcal{L}_{n,t}\right)$ be the implied cost of innovation and acquisition search/payments.[12]

Second, the large firm chooses $\dot{\mathcal{L}}_{n,t}$. The HJB equation for the large firm's expected discounted profits, $V_{n,t}(\mathcal{L}_{n,t})$, is

$$\begin{aligned} r_t V_{n,t}(\mathcal{L}_{n,t}) =& \mathcal{L}_{n,t}\left(\left(1-\sigma^{-1}-\tau_R\right) R_t - f_{L,n}\right) + \max_{\dot{\mathcal{L}}}\left\{\dot{\mathcal{L}} V'_{n,t}(\mathcal{L}_{n,t}) - \bar{X}_{n,t}\left(\dot{\mathcal{L}}; \mathcal{L}_{n,t}\right)\right\} \\ &+ \dot{V}_{n,t}(\mathcal{L}_{n,t}), \end{aligned} \tag{9}$$

where $\dot{V}_{n,t}(\cdot)$ is the effect of changes in the aggregate state over time. If any innovation or acquisition rate is strictly positive, then $\zeta = V'_{n,t}(\mathcal{L}_{n,t})$.

2.2.8 Evolution of the Measure of Small Firms

For now, I do not specify the entry/exit process that drives the evolution of the measure of small firms N_t. Although exit affects a small firm's value, it does not affect its innovation because I make one of the following equivalent modeling choices: 1) N_t is the measure of small firms *with entrepreneurs*, and when one exits, the firm survives until its goods are creatively destroyed; 2) when a firm exits, it sells its goods to another small firm.

3 Special Case: Only Creative Destruction

To illustrate a key mechanism and derive analytical results, I first study a special case of the industry model with only creative destruction. Firms cannot develop new goods, and I set the elasticity of substitution across goods, γ, to 1. When a firm creatively destroys good j in industry n, their new version has productivity $\lambda z_{n,t}(j)$. Total expenditures, the

[12] $\bar{X}_{n,t} = \beta_n[\alpha((1-\mathcal{L}_{n,t})X_L(\kappa(S)) + \mathcal{L}_{n,t}X_L(\kappa(L))) + X_L(\delta)] + (1-\mathcal{L}_{n,t})(X_A(A) + A(1+\tau_A)\Pi_{n,t}(\mathcal{L}_{n,t}))$.

wage, and the interest rate are fixed over time at R, W, and r, and I set the relative cost of creative destruction α, the measure of small firms N_t, and the measure of goods $M_{n,t}$ to 1. I drop the industry subscript n to eliminate unnecessary notation.

3.1 Firm Optimization and Equilibrium

The key implication of $\gamma = 1$ is that each good receives revenue R, regardless of its relative productivity. It follows that growth has no direct effect on sales: 1) if a firm creatively destroys its own good, it does not gain sales; 2) if a firm creatively destroys a competitor's good, it only gains sales by taking the good, and the improvement is irrelevant; 3) the value of a firm's good depreciates over time only due to creative destruction risk.

There is now a Markov perfect equilibrium without an industry state. Small firms creatively destroy their competitors' goods at rate κ_S, and the large firm creatively destroys its competitors' goods at rate κ_L and acquires them at rate A. Firms do not creatively destroy their own goods. The large firm revenue share $\mathcal{L}_t$ is no longer relevant for innovation decisions because there is no cannibalization. However, it affects the growth rate.

Adapting equations (4) and (5) yields

$$\dot{\mathcal{L}}_t = (\kappa_L + A)(1 - \mathcal{L}_t) - \kappa_S \mathcal{L}_t \qquad\qquad g_t = \ln(\lambda)((1 - \mathcal{L}_t)\kappa_L + \kappa_S).$$

The growth rate reflects that large firm innovation targets only a fraction of the goods in the industry, whereas each small firm targets all goods but a set of measure zero. I focus on balanced growth path equilibria in which the large firm revenue share, and so the growth rate, are constant over time:

$$\mathcal{L} = (\kappa_L + A)/(\kappa_L + A + \kappa_S) \qquad\qquad g = \ln(\lambda)\kappa_S(2\kappa_L + A + \kappa_S)/(\kappa_L + A + \kappa_S). \tag{10}$$

3.2 Large Firm Innovation and Growth

I first set acquisitions to 0 and study the long-run effects of shocks and policy changes that only directly affect large firm innovation incentives: the large firm innovation cost shifter β, fixed cost f_L, and revenue tax rate τ_R. To do so, I vary β, f_L, and τ_R, and characterize the implied relationship between equilibrium large firm innovation κ_L and the balanced growth path growth rate g. Small firm optimization yields

$$WX_S'(\kappa_S) \geq \Pi = \frac{(1 - \sigma^{-1})R - f_S}{r + \kappa_L + \kappa_S}, \tag{11}$$

which holds with equality if $\kappa_S > 0$, and implies that equilibrium small firm innovation κ_S is a decreasing function of equilibrium κ_L. Equations (10) thus imply that balanced growth path g is a function of equilibrium κ_L. Moreover, neither function depends directly on β, f_L, or τ_R. To map the results into the effects of β, f_L, and τ_R, Proposition 1 in Section 3.3 provides a condition under which equilibrium κ_L is a continuous increasing function of $((1 - \sigma^{-1} - \tau_R)R - f_L)/\beta$.

Throughout, I suppose $\kappa_L < \kappa_L^*$, where $\kappa_L^* \in [0, \infty]$ is the large firm innovation rate above which in equilibrium, $\kappa_S = 0$, and so $g = 0$. As such, equilibrium κ_S and balanced growth path g are continuously differentiable functions of equilibrium κ_L. Decompose g and its derivative with respect to equilibrium κ_L:

$$g = \underbrace{\bar{\kappa}}_{\text{Discount}} \underbrace{\frac{g}{\bar{\kappa}}}_{\text{Composition}} \qquad\qquad \frac{\partial g}{\partial \kappa_L} = \underbrace{\frac{\partial \bar{\kappa}}{\partial \kappa_L} \frac{g}{\bar{\kappa}}}_{\text{Discount Effect}} + \underbrace{\bar{\kappa} \frac{\partial (g/\bar{\kappa})}{\partial \kappa_L}}_{\text{Composition Effect}} ,$$

where $\bar{\kappa} \equiv \kappa_L + \kappa_S$ is the creative destruction risk component of the equilibrium discount rate on a small firm's good (the denominator on the right-hand side of inequality (11)), and $g/\bar{\kappa} = \ln(\lambda)(1 - \mathcal{L}^2)$ is a function of κ_L/κ_S, the composition of creative destruction risk between large and small firm innovation.

The "Discount Effect" is positive. More large firm innovation implies less small firm innovation, but still a higher equilibrium small firm discount rate because innovation costs are convex. Holding fixed its composition, more creative destruction implies more growth.

The "Composition Effect" is negative. Holding fixed small firm creative destruction risk, more large firm innovation shifts its source in equilibrium toward the large firm. Immediately, large firm goods face a lower relative rate of creative destruction, and in the long-run, they are a larger share of the industry (thus, the squared $\mathcal{L}$), each of which implies lower balanced growth path growth relative to small firm creative destruction risk.

Combining the two effects, $\partial g/\partial \kappa_L$ is the product of a strictly positive function and a sufficient statistic for whether following a change in large firm innovation incentives, more large firm innovation is associated with a higher balanced growth path growth rate:[13]

$$\epsilon_S(\kappa_S) - \frac{2\mathcal{L}}{1 - \mathcal{L}} \frac{\bar{\kappa}}{r + \bar{\kappa}}, \tag{12}$$

which depends on three measurable equilibrium outcomes; in particular, $\epsilon_S(\kappa_S)$ is the *inverse elasticity of small firm innovation with respect to the value of innovating*. Thus,

[13]The strictly positive function is $\ln(\lambda)(1 - \mathcal{L})^2 \frac{r + \bar{\kappa}}{\epsilon_S(\kappa_S)(r + \bar{\kappa}) + \kappa_S}$.

large firm innovation reduces growth if small firms are sufficiently responsive to the discount rate, if the large firm earns a sufficient share of industry revenue, and if creative destruction risk is a sufficiently significant component of the small firm discount rate.

The following theorem characterizes the effects of non-marginal changes in large firm innovation incentives. If those incentives drive variation across industries or over time, then it characterizes the long-run relationship between industry concentration and growth because concentration is increasing in large firm innovation.

Theorem 1. *Vary β, f_L, and τ_R. If in equilibrium, $\kappa_L < \kappa_L^*$ (so $\kappa_S > 0$ and $g > 0$), then balanced growth path g is a continuously differentiable function of κ_L such that:*

1. *At $\kappa_L = 0$, $\partial g/\partial \kappa_L \geq 0$, and strictly so if and only if $\epsilon_S(\kappa_S) > 0$.*
2. *There exists a $\kappa_L' < \kappa_L^*$ such that for all $\kappa_L \in (\kappa_L', \kappa_L^*)$, $\partial g/\partial \kappa_L < 0$.*
3. *If $\epsilon_S(\cdot)$ is constant, then g is single-peaked, and so increasing then decreasing.*

If the large firm has an insignificant revenue share, then more large firm innovation implies faster growth because the large firm innovates like a small firm and targets all goods, so the composition effect is zero. In the other limit, the growth rate is decreasing in large firm innovation and goes to zero because the large firm deters small firm innovation with a fast creative destruction rate, but in the long-run, there are no small firm goods to creatively destroy. The final property of g states that if the elasticity of small firm innovation with respect to the discount rate is constant, then this pattern fully characterizes the relationship between large firm innovation and growth: g exhibits an inverse-U shape.

3.3 Equilibrium Uniqueness

The following proposition provides a condition for the equilibrium to be unique given parameters, and characterizes properties of the equilibrium as a function of parameters.

Proposition 1. *Suppose $X_L(\cdot)$ and $X_A(\cdot)$ are strictly convex and for all κ_L, $\epsilon_L(\kappa_L)(1 + \epsilon_S(\kappa_S)) \geq 1$, where κ_S is given by κ_L and small firm optimization (inequality (11)). There is a unique equilibrium. If there are no acquisitions ($A = 0$), then equilibrium κ_L is a continuous increasing function of $\left(\left(1 - \sigma^{-1} - \tau_R\right) R - f_L\right)/\beta$. Moreover,*

1. *Equilibrium κ_L is increasing in τ_A, and strictly so if κ_L and A are strictly positive.*
2. *Equilibrium A is decreasing in τ_A, and strictly so if A is strictly positive.*

3. *If equilibrium κ_L, κ_S, and A are strictly positive, then κ_L, κ_S, A, and balanced growth path g are continuously differentiable with respect to τ_A.*

3.4 Acquisition Policy

I study the long-run effects of the acquisition tax rate τ_A, focusing on the role of large firm innovation. To have a unique equilibrium, I impose the conditions in Proposition 1. For τ_A to affect the balanced growth path g, suppose κ_L, κ_S, and A are strictly positive.

Decompose τ_A's effect on the balanced growth path growth rate:

$$\frac{\partial g}{\partial \tau_A} = \underbrace{\frac{\partial A}{\partial \tau_A}\frac{\partial g}{\partial A}}_{\text{Acquisition Effect}} + \underbrace{\frac{\partial \kappa_L}{\partial \tau_A}\frac{\partial g}{\partial \kappa_L}}_{\text{Innovation Effect}},$$

where $\partial g/\partial A$ is taken holding innovation rates fixed, and $\partial g/\partial \kappa_L$ is taken holding acquisitions fixed, but not small firm innovation, (the derivative from Section 3.2 with $A > 0$).

The "Acquisition Effect", which is positive, is the effect an econometrician would estimate following an exogenous acquisition. An increase in τ_A decreases the surplus from each acquisition, which decreases the equilibrium acquisition rate, reduces the large firm's revenue share, and so increases the balanced growth path growth rate. Put simply, actual acquisitions increase concentration, and so reduce growth.

The "Innovation Effect" has the same sign as $\partial g/\partial \kappa_L$, studied for $A = 0$ in Section 3.2. For the large firm, the outside option to creatively destroying a competitor's good is acquiring it. An increase in τ_A increases the equilibrium incentive to innovate because it reduces the surplus of an acquisition. It follows that encouraging acquisitions can be good for growth only if large firm innovation reduces growth. Put simply, the expectation of future high surplus acquisitions reduces large firm innovation, which may increase growth.

If $\epsilon_S(\kappa_S) = 0$, then $\partial g/\partial \tau_A$ is the product of a strictly positive function and a sufficient statistic for whether an increase in τ_A leads to faster growth on the balanced growth path:[14]

$$\frac{1}{\epsilon_A(A)} - \frac{1}{1-\mathcal{L}}\frac{2\kappa_L + A}{\epsilon_L(\kappa_L)\,(r+\bar{\kappa}) - \kappa_L}\left(1 + \frac{(1+\tau_A)\Pi}{\mathcal{S}}\right)^{-1}, \tag{13}$$

which depends on measurable equilibrium outcomes; $\mathcal{S}$ is the surplus of an acquisition, which accrues to the large firm, $(1+\tau_A)\Pi$ is the tax-inclusive acquisition price, and $\epsilon_L(\kappa_L)$ and $\epsilon_A(A)$ are the *inverse elasticities of large firm innovation and the acquisition rate with*

[14] See Appendix A.4 for the derivation.

respect to the value of innovating and of acquiring, respectively. Thus, discouraging acquisitions reduces growth if large firm innovation is sufficiently responsive, if the acquisition rate is sufficiently unresponsive, and if the acquisition surplus is sufficiently high.

In Appendix A.4, I develop the more complicated analog to expression (13) $\epsilon_S(\kappa_S) > 0$. The additional result is that for an increase in τ_A to reduce growth, small firm innovation must be sufficiently responsive, as expected given the results in Section 3.2.

The following theorem supposes that variation in large firm innovation incentives drives variation across industries or over time, and characterizes the relationship between industry concentration and the effect of a change in the acquisition tax rate.

Theorem 2. *Suppose $\epsilon_L(\cdot)$ and $\epsilon_A(\cdot)$ are constants, and $\epsilon_S(\cdot) = 0$. Vary f_L and τ_R. There is an $\mathcal{L}^*$ such that $\partial g/\partial \tau_A < 0$ if and only if on the balanced growth path, $\mathcal{L} > \mathcal{L}^*$ and $\Pi > W X_S'(0)$.*

Following an increase in large firm innovation incentives and the resulting rise in industry concentration, discouraging acquisitions is more likely to reduce growth. To see why, the second of equations (10) implies that if $\epsilon_S(\cdot) = 0$, then

$$\partial g/\partial A = -(1-\mathcal{L})\kappa_L/(\kappa_L + A + \kappa_S) \qquad \partial g/\partial \kappa_L = -(2\kappa_L + A)/(\kappa_L + A + \kappa_S), \quad (14)$$

where the latter includes the implied equilibrium change in κ_S. As $\mathcal{L}$ goes to 1, $\partial g/\partial A$ becomes arbitrarily small relative to $\partial g/\partial \kappa_L$ because mechanically, the effect of A and κ_L on $\mathcal{L}$ goes to 0, but the effect of κ_S does not, which drives $\partial g/\partial \kappa_L$.

4 Quantitative Macroeconomic Model

I now embed the industry model from Section 2 into a macroeconomic model. There is a unit measure of industries, indexed by $n \in [0, 1]$, each of which is modeled as in Section 2. A representative final good producer aggregates the goods in all industries into the final good to sell to a representative household for consumption and to firms to cover fixed costs. The household consumes, inelastically supplies $\bar{L}$ labor, owns all the firms, and has access to a risk-free bond with zero supply. Exogenous exit and undirected endogenous entry determine the measure of small firms, which is the same in each industry.

4.1 Representative Household and the Interest Rate

The household chooses a path of consumption $\{C_t\}$ and bond holdings $\{B_t\}$ to maximize present discounted utility subject to the budget constraint at all t:

$$\max \int_0^\infty e^{-\rho t} \ln(C_t) dt \qquad \text{s.t.} \quad P_t C_t + \dot{B}_t = W_t \bar{L} + D_t + r_t B_t \quad \text{for all } t,$$

where $\rho > 0$ is the time discount rate, and the household takes as given the final good price P_t, the wage W_t, flow profits from firms D_t, and the net interest rate r_t.

Bond market clearing implies that for all t, the net interest rate must equal the negative rate of change of the stochastic discount factor over time: $r_t = \rho + \dot{C}_t/C_t + \dot{P}_t/P_t$.

4.2 Representative Final Good Producer

At each time t, the final good producer chooses final good output Y_t and demand for each industry composite good $Y_{n,t}$ (equation (1) in Section 2.1.1) to maximize profits subject to Cobb-Douglas aggregation, taking as given the price of the final good, P_t, and of each industry composite good, $P_{n,t}$ (determined by cost minimization in equation (2)):

$$\max \left\{ P_t Y_t - \int_0^1 P_{n,t} Y_{n,t} dn \right\} \qquad \text{s.t.} \quad \ln(Y_t) = \int_0^1 \ln(Y_{n,t}) dn.$$

The First Order Condition for $Y_{n,t}$ implies constant expenditures across industries: $R_t = P_t Y_t$. Zero profits implies the final good price: $\ln(P_t) = \int_0^1 \ln(P_{n,t}) dn$.

4.3 Entry and Exit

At each time t, each of an infinite mass of potential entrants can receive value 0 or pay $\xi > 0$ units of labor to draw an industry from the uniform distribution and enter as a small firm with an entrepreneur and a 0 productivity version of each good. Potential entrants maximize expected discounted profits using the interest rate to discount payoffs. For the problem to be well-defined, the value of entry net the cost must be weakly negative.

Each small firm entrepreneur exits exogenously at Poisson arrival rate $\eta > 0$, after which its firm can produce, but not innovate. Thus, the measure of small firm *entrepreneurs*, which is relevant, does not equal the measure of producing small firms, which is irrelevant. The former, N_t, is the same in each industry because entry is *undirected*, and evolves over time due to entry e_t and exit according to $\dot{N}_t = e_t - \eta N_t$.

4.4 Aggregation and Normalization

At each time t, production labor L_t^p is the same in each industry, industry n output is $Y_{n,t} = Z_{n,t}L_t^p$, aggregate output is $Y_t = Z_t L_t^p$, and the final good price is $P_t = \sigma W_t / Z_t$, where Z_t is aggregate productivity:[15] $\ln(Z_t) = \int_0^1 \ln(Z_{n,t})dn$.

I normalize $P_t Z_t = 1$ for all t so that $W_t = \sigma^{-1}$, and expenditures are $R_t = L_t^p$. Aggregate productivity growth is $g_t \equiv \dot{Z}_t / Z_t = \int_0^1 g_{n,t} dn$.

4.5 Fixed Costs and Innovation Cost Functions

A unit of fixed cost requires Z_t units of the final good. With the normalization, fixed costs are as in Section 2.1.2: $\tilde{z}_{n,t}(i,j)f_S$ and $\tilde{z}_{n,t}(i,j)f_{L,n}$ for small and large firms, respectively.

I set the innovation cost function so the marginal cost has a constant elasticity $\epsilon > 0$:

$$X_S(x) = X_L(x) = (\epsilon + 1)^{-1} x^{\epsilon+1}.$$

We can interpret the cost shifter β_n as the large firm has a measure $\beta_n^{-1/\epsilon}$ of entrepreneurs.

4.6 Equilibrium

Firms in industries play Markov perfect equilibria (Sections 2.1.3 and 2.2.4), and potential entrants, the household, and the final good producer optimize, taking as given the path of the aggregate state, $\{R_t, N_t, W_t, r_t\}$. At each t, final good producers earn zero profits, the value of entry is weakly less than the cost, the tax authority uses lump sum taxes/transfers to balance its budget, and markets clear for bonds, labor (production, innovation, and entry equal supply), and the final good (consumption and fixed costs equal supply).

I study balanced growth paths and convergence to them following unanticipated shocks. A balanced growth path is an equilibrium in which each industry's state and the aggregate state are constant over time. As such, aggregate expenditures, production labor, and the growth rates of each $Z_{n,t}$ and Z_t are constant, and the interest rate is $r = \rho$.

[15] Since each firm sets a markup of σ, it follows that 1) $P_{n,t} = \sigma W_t / Z_{n,t}$ and $P_t = \sigma W_t / Z_t$; 2) each firm's revenue relative to production labor is σW_t, so $P_{n,t}Y_{n,t}/L_{n,t}^p = \sigma W_t = P_t Y_t / L_t^p$. Combining the two yields $Y_{n,t} = Z_{n,t}L_{n,t}^p$ and $Y_t = Z_t L_t^p$. From $P_{n,t}Y_{n,t} = P_t Y_t$, it follows that $L_{n,t}^p = L_t^p$.

4.7 Welfare

The measure of welfare is that of the household, which depends on current productivity and weighted averages of future growth, production labor, and fixed costs:

$$\int_0^\infty e^{-\rho t}\left(ln(Z_t)+\ln\left(L_t^p-\int_0^1((1-\mathcal{L}_{n,t})f_S+\mathcal{L}_{n,t}f_{L,n})dn\right)\right)dt$$

$$=\frac{ln(Z_0)}{\rho}+\frac{\int_0^\infty \rho e^{-\rho t}g_t dt}{\rho^2}+\frac{\int_0^\infty \rho e^{-\rho t}\ln\left(L_t^p-\int_0^1((1-\mathcal{L}_{n,t})f_S+\mathcal{L}_{n,t}f_{L,n})dn\right)dt}{\rho}, \quad (15)$$

Growth at t is discounted by ρ^2 because it raises consumption in all $t'>t$. On a balanced growth path, welfare is $\ln(Z_0)/\rho+g/\rho^2+\ln\left(L^p-\int_0^1((1-\mathcal{L}_n)f_S+\mathcal{L}_n f_{L,n})dn\right)/\rho$.

4.8 Entrant Optimization

A small firm's value of innovating does not depend on the goods it produces. Thus, the value of entering industry n at time t is $E_{n,t}(\mathcal{L}_{n,t})$, which is given by the HJB equation:

$$\begin{aligned} r_t E_{n,t}(\mathcal{L}_{n,t}) &= \left(\kappa_{S,n,t}(\mathcal{L}_{n,t})\lambda^{\gamma-1}+\delta_{S,n,t}(\mathcal{L}_{n,t})\right)\Pi_{n,t}(\mathcal{L}_{n,t})+\dot{\mathcal{L}}_{n,t}E'_{n,t}(\mathcal{L}_{n,t})+\dot{E}_{n,t}(\mathcal{L}_{n,t}) \quad (16)\\ &\quad -W_t(\epsilon+1)^{-1}\left(\alpha\kappa_{S,n,t}(\mathcal{L}_{n,t})^{\epsilon+1}+\delta_{S,n,t}(\mathcal{L}_{n,t})^{\epsilon+1}\right)-\eta E_{n,t}(\mathcal{L}_{n,t}). \end{aligned}$$

The right-hand side is the benefit and cost from innovation (Section 2.2.6), the risk of exit, and the effects of changes over time in the industry and aggregate state. The value of entry net the cost is $\int_0^1 E_{n,t}(\mathcal{L}_{n,t})dn-W_t\xi$.

4.9 Model Discussion

Before proceeding to the results, I discuss some of the main modeling choices.

The presence of imitation and Assumption 1 imply a constant markup across firms and goods. One effect is the number of industry states is 1 rather than 3, and new good development is equivalent to own good improvement. More deeply, this choice abstracts from the relationship between static efficiency and growth. If large firms set higher markups, then an increase in their revenue share increases prices relative to productivity, which implies *higher* profits and small firm innovation incentives. However, large firms also may produce with low marginal costs relative to productivity,[16] which has the opposite effect.

[16]Due to a higher process productivity as in Aghion, Bergeaud, Boppart, Klenow, and Li (2022) or more intangibles as in De Ridder (2021).

A consequence of making all entrants permanently small is that large firm profits do not affect entry. One justification is that if large firms exit more slowly or if firms take time to become large, then due to discounting, large firms are over represented in the cross section relative to their salience for a potential entrant. Second, this is the correct approach for the main experiment in Section 5.3 if the interpretation of the shock is that large firms pay higher firm-level fixed costs to achieve lower per-good fixed costs.

5 Results: The Effects of Large Firm Innovation

I characterize the effect of changes to large firm innovation incentives (the large firm fixed cost, revenue tax rate, and innovation cost) on industry concentration, growth, and welfare. I present qualitative results concerning long-run effects, then calibrate the model and compare quantitative results to US data. Throughout, the acquisition rate is 0.

5.1 Concentration and Growth in the Long-Run

I use small firm optimization and large firm *relative* innovation rates to characterize the long-run relationship between large firm revenue shares and growth. The following theorem, analogous to Theorem 1 and displayed graphically in Figure 1, shows that if innovation costs are quadratic, then across industries, the growth rate is a function of the large firm industry revenue share that exhibits an **inverted-U shape**. On the other hand, an aggregate increase in large firm innovation incentives always decreases the long-run growth rate, leaving aside quantitatively small general equilibrium labor market effects.

Theorem 3. *There are continuously differentiable functions $g_I(\cdot)$ and $g_A(\cdot)$ such that 1) on a balanced growth path, the industry n growth rate is $g_I(\mathcal{L}_n)$; 2) if all industries are the same, the large firm fixed cost f_L, revenue tax rate τ_R, and innovation cost shifter β vary, and labor supply $\bar{L}$ adjusts so that balanced growth path production labor L^p is constant, then the long-run aggregate growth rate is $g_A(\mathcal{L})$. The following hold:*

1. *$g_I(0) > 0$, $g_I'(0) > 0$, $g_A(0) > 0$, and $g_A'(0) = 0$.*
2. *If $\epsilon = 1$, then $\lim_{\mathcal{L}\to 1}(g_I(\mathcal{L})) = 0$ and there exists a threshold $\mathcal{L}^* \in (0, 1)$ such that $g_I'(\mathcal{L}) > 0$ for $\mathcal{L} < \mathcal{L}^*$ and $g_I'(\mathcal{L}) < 0$ for $\mathcal{L} > \mathcal{L}^*$.*
3. *For all $\mathcal{L} > 0$, $g_A'(\mathcal{L}) < 0$, and $\lim_{\mathcal{L}\to 1}(g_A(\mathcal{L})) = 0$.*

As in Section 3.2, decompose the effect of large firm innovation incentives on the long-run growth rate into the discount and composition effects. On a balanced growth path, using equation (6), a small firm's value of an innovation is

$$\Pi_n(\mathcal{L}_n) = \frac{\left(1-\sigma^{-1}\right) L^p - f_S}{\rho + N(\kappa_{S,n}(\mathcal{L}_n) + (\gamma-1)g_{S,n}(\mathcal{L}_n)) + \kappa_{L,n}(S;\mathcal{L}_n) + (\gamma-1)g_{L,n}(\mathcal{L}_n)}. \qquad (17)$$

The denominator is the small firm discount rate, which includes depreciation from growth and creative destruction risk. The "discount effect" is the effect on the discount rate, holding fixed the share of the discount rate minus ρ due to growth depreciation; the "composition effect" is the effect on that share, holding fixed the discount rate.

Across industries, the discount effect is strictly positive because N is fixed and innovation costs are convex: more large firm innovation implies a higher discount rate and so more growth. Across balanced growth paths, the discount effect is zero due to free entry.

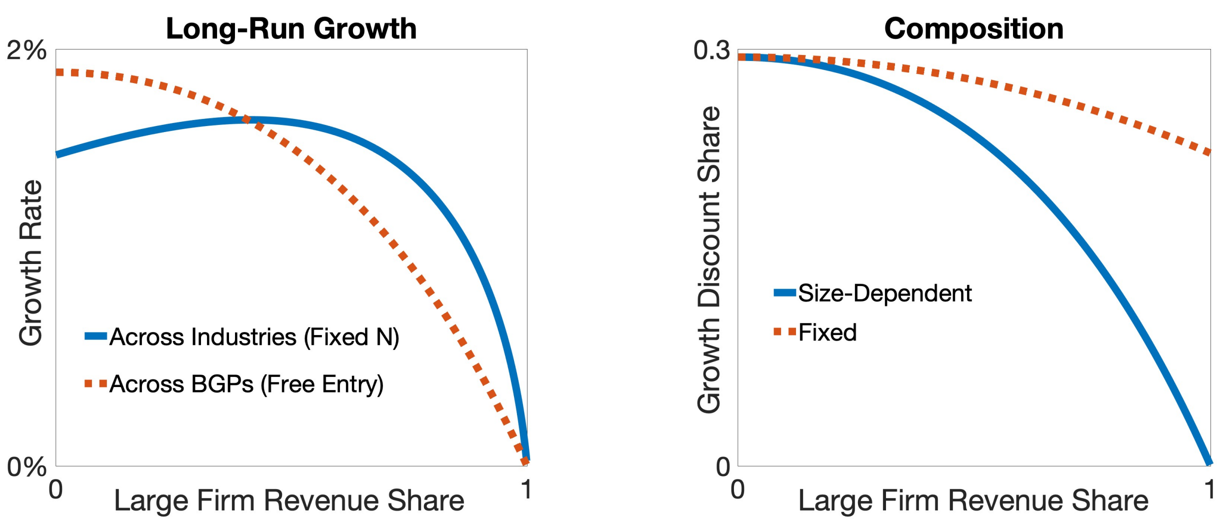

Figure 1: Both figures use the calibration in Section 5.2. Left panel: the solid blue and dotted orange lines display $g_I(\cdot)$ and $g_A(\cdot)$, respectively, from Theorem 3. Right panel: the solid blue and dotted orange lines display growth relative to the small firm discount rate (minus ρ) in equilibrium, and with large firm relative innovation rates set as if $\mathcal{L}_n = 0$, respectively.

The composition effect is strictly negative; the more large firms innovate, the more the small firm discount rate is achieved through creative destruction risk rather than depreciation from growth. First, the sole force in Section 3.2: the reluctance of firms to creatively destroy their own goods is only relevant for large firms that are responsible for non-negligible shares of innovation and sales in their industries. Large firm goods stagnate, which reduces growth but not small firm creative destruction risk. Second, large firms have a size-dependent relative preference for creatively destroying competitors' goods:

$$\frac{\kappa_{L,n}(L;\mathcal{L}_n)}{\kappa_{L,n}(S;\mathcal{L}_n)} = \mathcal{D}(\mathcal{L}_n)\left(\frac{\lambda^{\gamma-1}-1}{\lambda^{\gamma-1}}\right)^{1/\epsilon} \qquad \frac{\delta_{L,n}(\mathcal{L}_n)}{\kappa_{L,n}(S;\mathcal{L}_n)} = \mathcal{D}(\mathcal{L}_n)\left(\frac{\alpha}{\lambda^{\gamma-1}}\right)^{1/\epsilon},$$

where each is the product of $\mathcal{D}(\mathcal{L}_n) \equiv \left(\frac{1-\mathcal{L}_n}{(1-\lambda^{1-\gamma})(1-\mathcal{L}_n)+\lambda^{1-\gamma}}\right)^{1/\epsilon}$ and the small firm relative innovation rate. $\mathcal{D}(\cdot)$ is strictly decreasing from $\mathcal{D}(0) = 1$ to $\mathcal{D}(1) = 0$, and is the discount a large firm applies to generating growth rather than creatively destroying a competitor's good; a fraction $\lambda^{1-\gamma}$ of the sales gained through creative destruction is taken from the competitor, and thus does not cannibalize the sales of the large firm's other goods. I show the relative importance of the two mechanisms in the right panel of Figure 1.

5.1.1 Labor Market Effects and Approximating Long-Run Effects of Shocks

In the quantitative experiments, shocks that shift innovation to large firms result in an increase in aggregate expenditures to increase production labor and clear the labor market. Omitted from Theorem 3, this pushes up the return to innovating. Put another way, it is costly for small firms to replace large firm innovation. Nonetheless, the effect on the long-run growth rate is not significant; the composition effect dominates.

It follows that given the effect of a shock to large firm innovation incentives (f_L, τ_R, or β) on the long-run large firm revenue share, $g_A(\cdot)$ from Theorem 3 provides a good approximation of the effect on long-run growth. Computing $g_A(\cdot)$ only requires values for the cost elasticity ϵ, the relative creative destruction cost α, the innovation step size λ, the elasticity of substitution γ, and the initial large firm revenue share $\mathcal{L}$ and growth rate g. Thus, the size of the fall in g in the main experiment in Section 5.3 depends almost entirely on ϵ, α, λ, and γ because I calibrate the shock to match a given increase in $\mathcal{L}$.

5.2 Calibration

I calibrate a balanced growth path with identical industries. I set some parameters externally (Table 1) and internally calibrate the rest to jointly match empirical moments (Table 2). I set labor supply $\bar{L}$ so aggregate expenditure $R = L^p$ is 1. Small and large firms have the same fixed cost. The revenue tax rate is 0. The units of time are years.

5.2.1 Externally Calibrated Parameters

The exit rate η is the annual employment-weighted average firm exit rate from Boar and Midrigan (2022).[17] The demand elasticity γ is the median estimate from Broda and Weinstein (2006) at the most disaggregated level in the earliest time period, which is apt because I use the most disaggregated industry definition available for the large firm revenue share measure. The marginal innovation cost elasticity ϵ captures two elasticities: of a firm's total innovation rate with respect to its value or cost, and of a firm's *relative* innovation with respect to the value or cost of one type of innovation *relative* to another. Acemoglu, Akcigit, Alp, Bloom, and Kerr (2018) discuss a variety of studies that estimate the former elasticity to be 1. The proof of Theorem 3 shows that 1 is also a good target for the latter:[18] if variation in large firm innovation incentives drives variation in concentration across industries, then to match the inverted-U relationship between growth and concentration documented in Cavenaile, Celik, and Tian (2021), the elasticity of relative innovation rates cannot be much greater than 1 (even 1.5); otherwise, an industry's growth rate diverges to infinity as its large firm's revenue share goes to 1 because the large firm uses a high rate of growth as well as creative destruction to maintain its dominance.

Table 1: Externally Calibrated Parameters

Parameter	Description	Value
η	Exit Rate	0.04
γ	Demand Elasticity	3.1
ϵ	Innovation Cost Elasticity	1

5.2.2 Internally Calibrated Parameters

I match the BLS measure of aggregate productivity growth from Garcia-Macia, Hsieh, and Klenow (2019), the 1-year real interest rate from FRED, the cost-weighted average markup estimated in Compustat data by De Loecker, Eeckhout, and Unger (2020), and

[17] η should be the rate at which an entrepreneur stops innovating in their industry; closing one firm to start another should not count as an exit. Nonetheless, its effects on the long-run results are small because it only impacts the shift in labor from entry to production, which is insignificant.

[18] See Proposition 2 in Appendix A.6.

Business Enterprise Expenditures on R&D/GDP from the OECD MSTI database, all averages from 1983-1993. The first two determine the time discount rate ρ: the real interest rate is $\rho + g$.[19] The markup is the imitation discount σ. R&D/GDP (innovation costs over nominal output) determines profits over sales, and so the fixed cost given σ. The distinction between the fixed cost and σ is only relevant so I can lower f_L in experiments.

I set λ so the average growth contribution of a creative destruction innovation, $\frac{\lambda^{\gamma-1}-1}{\gamma-1}$, matches the Akcigit and Kerr (2018) estimate of the average growth contribution of an external patent (that mostly cites other firms' patents).[20] Thus, 13% of a good's sales immediately after it is creatively destroyed are novel, so creative destruction risk is 74% of the small firm discount rate even though only 34% of growth is due to creative destruction.

I match $\mathcal{L}$ to the sales-weighted average industry revenue share of the largest firm in 4-digit industries in Compustat from Olmstead-Rumsey (2022). As a robustness check in Section 5.4.3, I instead use Census industry concentration data, which include all firms rather than just public ones. Compustat is less complete but may better capture the relevant moment for cannibalization, which is a large firm's share of innovations rather than sales. $\mathcal{L}$ and g pin down the large firm innovation cost shifter β and entry cost ξ.

Table 2: Internally Calibrated Parameters and Data Moments

Parameter	Value	Moment	Value
ρ – Time Discount Rate	0.0194	Real Interest Rate	3.6%
σ – Imitation Discount	1.3	Markup	1.3
f_S, f_L – Fixed Cost	0.183	R&D/GDP	1.81%
λ – Innovation Step Size	1.067	External Innovation Step	0.069
ξ – Entry Cost	4.233	TFP Growth Rate	1.66%
β – Large Firm Innovation Cost	28.36	Large Firm Revenue Share	40.7%
α – Creative Destruction Cost	0.3114	Large Job Destruction Rate	25.57%

To calibrate α, I use large job destruction flows—the share of aggregate employment lost over a 5 year period at firms whose employment shrank by at least two-thirds—

[19] A real bond in the model must compensate for ρ and declining marginal utility due to growth in C_t.

[20] Their analogous estimate for internal innovations is lower at 0.051, and so implies a lower λ and a stronger composition effect, whereas Garcia-Macia, Hsieh, and Klenow (2019) estimate a slightly higher 0.081 using labor flows data through the lens of a growth model different from the one in this paper.

computed in Census data by Garcia-Macia, Hsieh, and Klenow (2019) to estimate the rate at which a small firm's good is creatively destroyed. The model does not identify these flows because it does not identify how many goods each small firm produces.[21] To be conservative, I find a maximum value of α by supposing that all creative destruction of small firm goods leads to large flows, which are thus $(1-\mathcal{L})(1-e^{-5(\kappa_S+\kappa_L(S))})$.[22] For consistency and again to be conservative, I use the Census concentration measure (see Section 5.4.3) for $\mathcal{L}$, which yields a higher α than does the Compustat measure. I use an alternative calibration strategy in Section 5.4.3 as a robustness check.

The calibrated values for ϵ, α, λ, and γ, and so $g_A(\cdot)$, do not depend on values for other parameters because ϵ, α, λ, γ, $\mathcal{L}$, and g imply relative innovation rates, which are sufficient, without solving for a dynamic equilibrium. I use $g_A(\cdot)$ to confirm that the choice of α is conservative in the sense that lower values yield larger falls in long-run growth.

5.3 Quantitative Experiment: A Rise in Large Firm Innovation

I ask whether a rise in concentration driven by a fall in large firm fixed costs can explain changes in US data since the mid-1990s. I interpret the shock as a shift from per-good to firm wide fixed costs—a rise in span of control—due to the rise in information technology.[23] I show in Section 5.4 that a fall in large firm innovation costs yields similar results.

The economy begins on the balanced growth path from Section 5.2. There is an unanticipated permanent fall in f_L to 0.17 in all industries, calibrated so the new balanced growth path large firm revenue share is 0.51, the 2018 sales-weighted average revenue share of the largest firm across 4-digit industries in Compustat from Olmstead-Rumsey (2022).

5.3.1 Industry Concentration and Aggregate Growth

I compare the effects on growth to data from Garcia-Macia, Hsieh, and Klenow (2019) in Table 3. The shock explains 41% of the long-run fall in the growth rate, and all of the

[21]The number of goods per firm depends on the productivity distribution of innovations and whether firms develop new goods or improve on their old goods, neither of which is specified. Each small firm may have many small goods and face a zero probability of shrinking by more than two-thirds in 5 years.

[22]I suppose job losses from depreciation due to growth, which on their own amount to a share $1-e^{-5(\gamma-1)g} < 2/3$, do not lead to large flows. For growth depreciation to generate large flows, a small firm must have multiple goods, which then implies that some creative destruction does not lead to large flows. I exclude innovation labor, which is small, so I can calibrate α without computing a dynamic equilibrium.

[23]See Aghion, Bergeaud, Boppart, Klenow, and Li (2022) for a discussion.

short-run increase, mostly due to a temporary shift in labor from entry to production. The cumulative growth burst is smaller than in the data because it peaks in the model after 3 years, whereas in the data is an average over 10 years. The large firm revenue share and the growth rate (Figure 2) converge over a similar time interval as the gap between the initial calibration years, 1983-1993, and the target year for the shock, 2018.

Table 3: Growth After a Fall in f_L

Moment	Data	Model
Growth Burst	+0.64 ppt (39%) (1993-2003)	GDP: +0.77 ppt (46%) (first year) TFP: +0.12 ppt (7%) (first year)
Cumulative Burst	+6.4 ppt (39%) (1993-2003)	GDP: +0.91 ppt (18%) (3 years) TFP: +0.21 ppt (4%) (3 years)
Growth Fall	-0.34 ppt (-20%) (2003-2013)	-0.14 ppt (-8%) (New BGP)

ppt is percentage point rise, and in parentheses is the percent rise. Growth burst is the peak change in the growth rate. Cumulative burst is the peak difference in accumulated growth. GDP uses output over $\bar{L}$ to measure productivity. Growth fall is the long-run change.

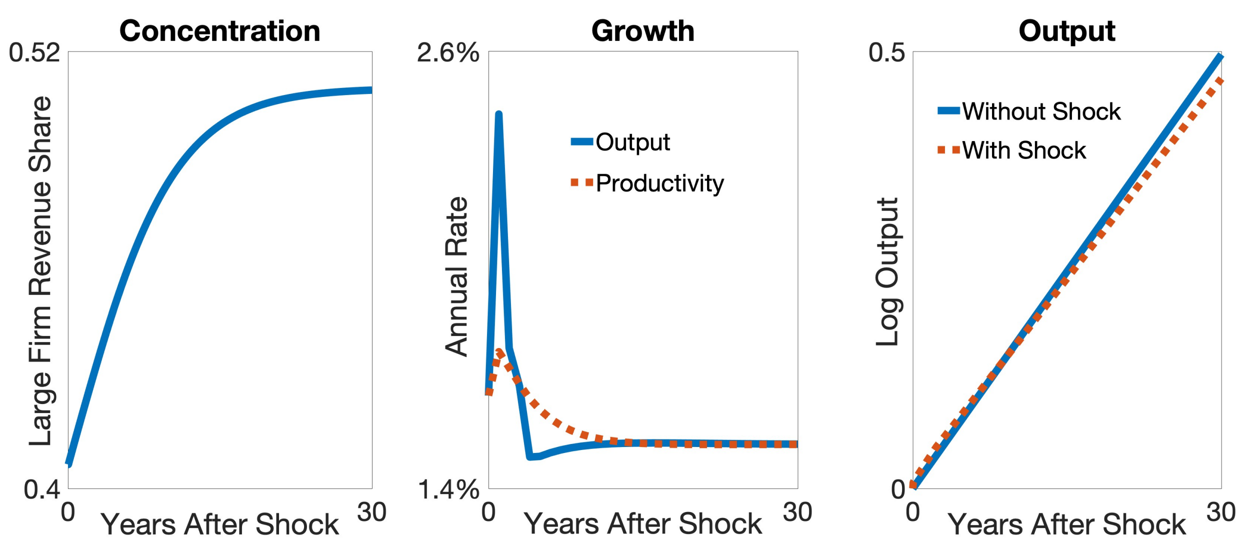

Figure 2: Transition paths after the shock to f_L. Left panel: $\mathcal{L}_t$. Middle panel: the dotted orange and solid blue lines are annual g_t and the growth rate of $Y_t/\bar{L}$. Right panel: the dotted orange and solid blue lines are $\ln(Y_t)$ in equilibrium and in a counterfactual without the shock.

I decompose the time path of the productivity growth rate in Figure 3. The negative composition effect is present immediately because innovation shifts to large firms, and grows as large firm revenue shares increase as well. The small firm discount rate increases dramatically in the short-run—flow profits rise because goods expenditures increase to clear the labor market, and the net value of entry falls below zero because there are excess small firms—but by less in the long-run because the permanent increase in expenditures is small. Production labor would need to rise by 10% to cancel the composition effect.

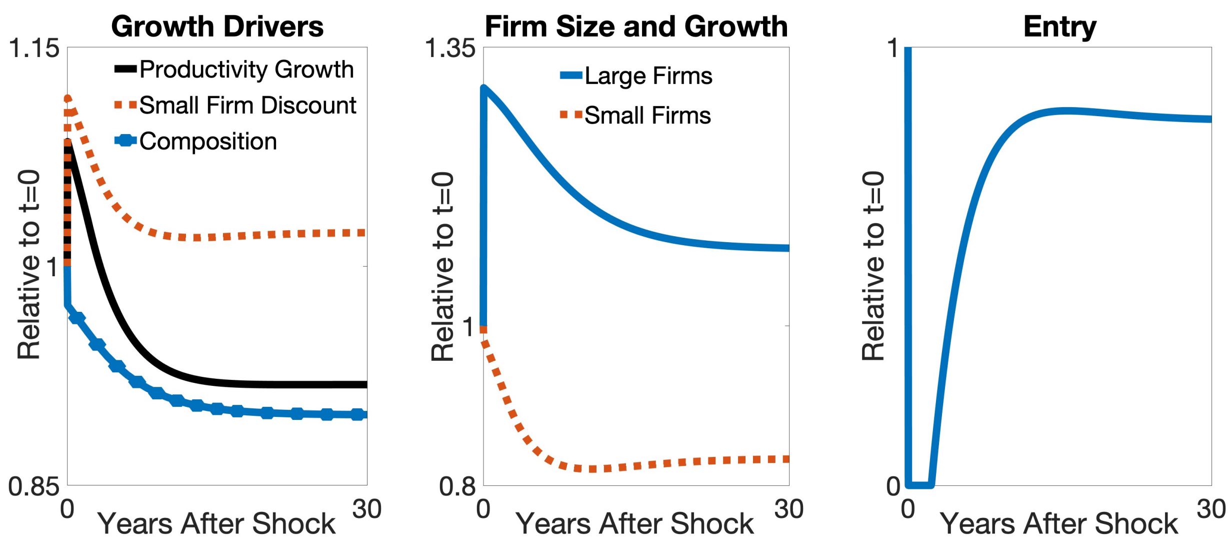

Figure 3: Variables relative to their values in the initial balanced growth path. Left panel: the solid black, dotted orange, and textured blue lines are g_t/g and its decomposition into the small firm discount rate (minus ρ) and growth over the discount rate. Middle panel: the solid blue and dotted orange lines are $g_{L,t}/g_L$ and $g_{S,t}/g_S$. Right panel: e_t/e.

5.3.2 Welfare and Size-Dependent Taxes

Including the transition path, the present discounted values of consumption (welfare) and output fall by the equivalent of a permanent 4.8% and 5.7% drop, respectively; output shifts from fixed costs to consumption. On its own, the decline in the long-run growth rate implies the equivalent of a 6.9% permanent fall in consumption.

On the margin, decreasing the large firm revenue tax τ_R decreases welfare because it has the same effect as a fall in f_L, but without increasing consumption relative to output.

5.3.3 Labor Flows

The model matches the dynamics of large job destruction flows documented in Garcia-Macia, Hsieh, and Klenow (2019), and their resulting estimates of the share of growth due to creative destruction. Empirical flows rise by 4% from 1983-1993 to 1993-2003 and fall by 13% from 1983-1993 to 2003-2013, whereas the model measure described in Section 5.2, $(1-\mathcal{L})\left(1-e^{-5(\kappa_S+\kappa_L(S))}\right)$ (using the model value of $\mathcal{L}$), rises by 9% immediately after the shock and falls by 15% across balanced growth paths. The estimated creative destruction growth share is 0.27 in 1983-1993, rises by 5% from 1983-1993 to 1993-2003, and falls by 17% from 1983-1993 to 2003-2013, whereas in the model, the share of growth due to creative destruction *of small firm goods* (which generates large flows) is 0.25 in the initial balanced growth path, rises by 3% immediately after the shock, and falls by 7% across balanced growth paths. Large flows increase in the short-run because innovation shifts to creative destruction, but fall in the long-run because employment shifts to large firms that do not experience large flows. Including small firm creative destruction of large firm goods, the growth share still falls in the long-run—though by less than 1%—because large firms experience a slower rate of creative destruction.

The model mechanism does not identify other statistics used in Garcia-Macia, Hsieh, and Klenow (2019) that depend on the productivity distribution of innovations such as large job *creation* flows. Moreover, it says nothing about new good development vs. own good improvement because they have identical effects on sales and growth.

5.3.4 Growth Relative to R&D

The model matches the simultaneous rise in R&D/GDP and fall in growth in US data, which yield a fall in the growth rate relative to R&D/GDP from 0.91 in 1983-1993 to 0.69 in 2003-2013. In the model, across balanced growth paths, R&D/GDP rises from 1.81% to 1.82%, and the growth rate relative to R&D/GDP falls from 0.91 to 0.84. After the shock, large firms innovate more, and convex costs imply a fall in innovation efficiency.

Creative destruction is crucial to prevent a counteracting effect. Large firms are relatively efficient at developing new goods because to avoid cannibalization, they do relatively little; their marginal cost is 0.67 and 0.74 compared to that of small firms in the initial and new balanced growth paths, respectively. The opposite holds for creatively destroying competitors' goods because large firms face lower creative destruction rates and thus innovate more; their marginal cost is 1.08 and then 1.41 compared to that of small firms.

5.3.5 Industry Concentration and Industry Growth Rates

The model matches the finding in Ganapati (2021) that across industries in the US, rising concentration is associated with *faster* growth. With sector and time fixed effects, a 1 percent rise in the revenue share of the largest 4 firms in a 6-digit NAICS industry over a 5-year period is associated with a 0.1 percent rise in real output and a 0.2 percent rise in real output relative to employment. I run the same regression during the transition path by creating a measure 0 control group of industries with a constant large firm fixed cost. A 1 percent rise in an industry's large firm revenue share over a 5-year period is associated with a 0.04 percent rise in real output and in real output relative to employment.

The theory thus generates a parsimonious explanation for the short-run burst in growth as well as the positive relationship between concentration and growth across industries: small firm innovation is less responsive in the short-run and to industry-specific shocks.

5.3.6 Entry

The fall in entry (Figure 3) matches empirical results in Decker, Haltiwanger, Jarmin, and Miranda (2016):[24] the employment share of firms less than 5 years old fell by 20% during the early 1990s, and then 5% in the early 2000s; in the model, the same share (including production and innovation labor) falls by 16% across balanced growth paths.

5.4 Different Shocks and Calibrations

5.4.1 A Fall in Large Firm Innovation Costs

I redo the main experiment in Section 5.3 but change the large firm innovation cost shifter β rather than the fixed cost f_L. The results are in Table 4. Innovation costs relative to output decrease (unlike in the data) because innovation *rates* are similar to in Section 5.3, but innovation is cheaper. Long-run growth falls by less than in Section 5.3 because goods expenditures rise by more to clear the labor market given lower innovation costs.

5.4.2 Fixed Innovation Labor

Without entry/exit, a rise in large firm innovation incentives increases growth because small firms are not sufficiently responsive. An alternative way to make small firms re-

[24]Garcia-Macia, Hsieh, and Klenow (2019) find similar results, but their third time period overlaps with the Great Recession.

sponsive is to fix both the measure of small firms and innovation labor. I fix the labor used for each of entry, innovation, and production to their values in the initial balanced growth path, and redo the main experiment from Section 5.3 with relative wages for entry and innovation labor so that each labor market clears. The results are in Table 4.

The long-run growth rate falls, although by less than in Section 5.3 because convex costs imply that small firm innovation becomes more efficient as it decreases. Thus, a rise in large firm innovation incentives leads to lower growth if the distribution of labor across uses is fully flexible or fully fixed. The model's main mechanism is again crucial because, as discussed in Section 5.3.4, it drives the fall in average innovation efficiency.

Table 4: Alternative Shocks and Calibrations

Experiment	Growth	R&D/GDP	ξ	β	α	f_S, f_L	f_L^*
Original Experiment	1.52%	1.82%	4.233	28.36	0.3114	0.183	0.17
Innovation Cost (β)	1.54%	1.67%	4.233	28.36	0.3114	0.183	22 (β^*)
Fixed Labor	1.59%	1.98%	4.233	28.36	0.3114	0.183	0.158
Census Calibration	1.61%	1.85%	4.19	40.2	0.3114	0.186	0.178
Inverted-U Calibration	1.6%	1.86%	9.7	44.6	4.16	0.17	0.143

Each row is for a different experiment; it shows the internally calibrated parameters that sometimes vary, and long-run growth and R&D/GDP following the shock. The new value of the shocked parameter, which is β in the second case and f_L in the others, is in the last column.

5.4.3 Alternative Calibrations

I redo the main experiment from Section 5.3 under two alternative calibrations; the new parameter values and results are in Table 4. First, I match the large firm revenue share in the initial and new balanced growth paths to the sales-weighted average revenue share of the largest 4 firms in 6-digit NAICS industries in the US in 1997 (30.6%) and 2012 (35.9%), respectively, computed by Barkai (2020) in Census data using consistently defined industries over time; I use the same 2-digit sectors as Garcia-Macia, Hsieh, and Klenow (2019). I begin in 1997 because there is a change in industry definitions, but the concentration increase would likely be larger if I started in 1983-1993 because Autor, Dorn, Katz, Patterson, and Van Reenen (2020) find that concentration has been increasing in Census data since the 1980s. The fall in growth is smaller than in Section 5.3 because the lower large firm revenue share implies a weaker composition effect.

Second, instead of large job destruction flows, I use the inverted-U relationship between growth and the large firm revenue share across industries from Theorem 3, $g_I(\cdot)$, to calibrate the creative destruction cost α. Cavenaile, Celik, and Tian (2021) estimate inverted-U relationships between innovation measures and HHI (sum of squared revenue shares) in 4-digit industries in Compustat data, with sector and year fixed effects. I match the location of the peak of $g_I(\cdot)$ to the patent-maximizing large firm revenue share implied by their regression of industry patents on HHI and HHI squared.[25] The fall in growth is smaller than in Section 5.3 because α is higher, which implies less creative destruction.

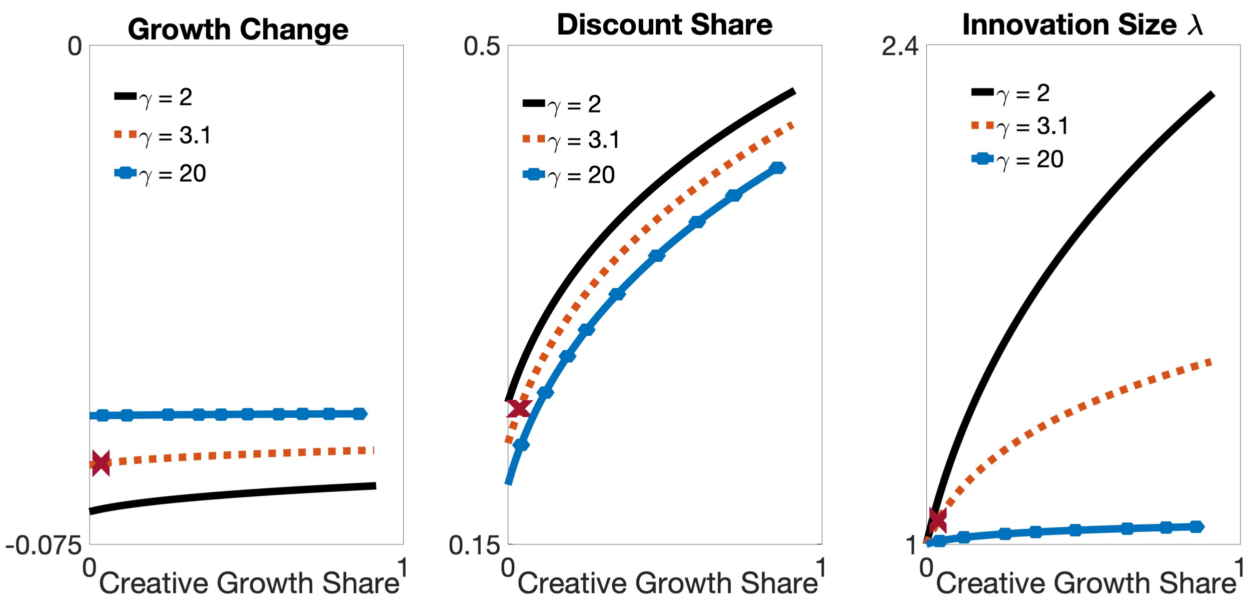

Figure 4: Each line varies λ, indexed by the creative destruction share of growth, for a fixed γ, with α set to match the target location of the peak of $g_I(\cdot)$. The x on each dotted line indicates the calibration in Table 4. Left panel: $g_A(0.51) - g_A(0.407)$ in percentage points. Middle panel: the creative destruction share of the small firm discount (minus ρ). Right panel: λ.

Under this calibration strategy, the size of the fall in growth is robust. Recall from the ends of Sections 5.1 and 5.2 that given values for parameters ϵ, α, λ, and γ, for equilibrium outcomes $\mathcal{L}$ and g on the initial balanced growth path, and for $\mathcal{L}$ on the new balanced growth path, we can use $g_A(\cdot)$ from Theorem 3 to approximate g on the

[25] I match the maximizing HHI rather than the regression coefficients because $g_I(\cdot)$ is not a second-order polynomial, so to do the latter would require specifying an HHI distribution across industries.

new balanced growth path. Moreover, the same set of values from the initial balanced growth path is sufficient to compute $g_I(\cdot)$ from Theorem 3. Thus, I take ϵ and the initial values for $\mathcal{L}$ and g from the calibration in Section 5.2, and for any values of γ and λ, I calibrate α to match the inverted-U from Cavenaile, Celik, and Tian (2021). I then compute $g_A(0.51) - g_A(0.407)$ to approximate the change in long-run growth following the shock. I vary γ between 2 and 20, and for each γ, vary λ over its feasible range from 1 to the maximum above which the desired α does not exist. As Figure 4 illustrates, $g_A(0.51)-g_A(0.407)$ ranges from -0.055 to -0.07 percentage points, and variation is mostly due to the choice of γ. The share of growth due to creative destruction is not important, but the share of the small firm discount rate due to creative destruction risk is relevant.

6 Antitrust Policy: Acquisitions

Can acquisitions (Section 2.2.3) increase growth in the calibrated model, and what determines whether that is the case? To analyze the effects of acquisitions, I parameterize the search function $X_A(A) = \omega_1 A^{\omega_2^{-1}+1}$ with a cost shifter $\omega_1 > 0$, and the *elasticity of the acquisition rate with respect to the acquisition surplus*, $\omega_2 > 0$.

6.1 The Effects of the Tax Rate on Growth

I first compute the balanced growth path growth rate for various tax rates in Figure 5, using the calibration following the decrease in f_L in the main experiment in Section 5.3, under three search cost functions with different values for ω_2, the elasticity of the acquisition rate with respect to the surplus of an acquisition: 1) $\omega_1 = 3.8$ and $\omega_2 = 1$; 2) $\omega_1 = 694$ and $\omega_2 = 1/3$; 3) search is free if $A \leq 0.05$ and infinite if $A > 0.05$. In each case, I set the cost shifter ω_1 so the balanced growth path acquisition rate is 0.05 when the tax rate is at its minimum -1.

If the tax rate is just below 0.6 so that acquisitions occur, but generate little surplus, then the acquisition effect (Section 3) dominates and the long-run growth rate is lower than in the economy without acquisitions. Acquisitions push up large firm revenue shares, which has little effect on the rate at which large firms creatively destroy their competitors' goods, but reduces the rate at which they generate growth (Figure 6). As the tax rate falls further, acquisitions generate more surplus, so the innovation effect is stronger and eventually the long-run growth rate begins to increase. Large firms reduce all types of

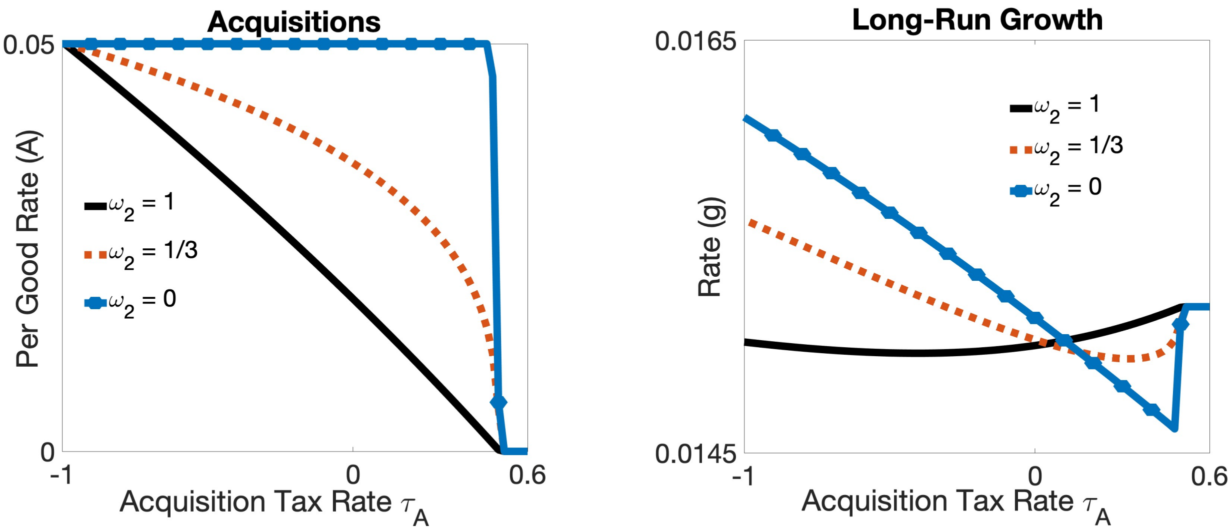

Figure 5: Variables on balanced growth paths as a function of τ_A, using the calibration following the drop in f_L in Section 5.3, and cost functions with different elasticities ω_2, and cost shifters ω_1 so that $A = 0.05$ at $\tau_A = -1$. Left panel: acquisition rate A. Right panel: growth rate g.

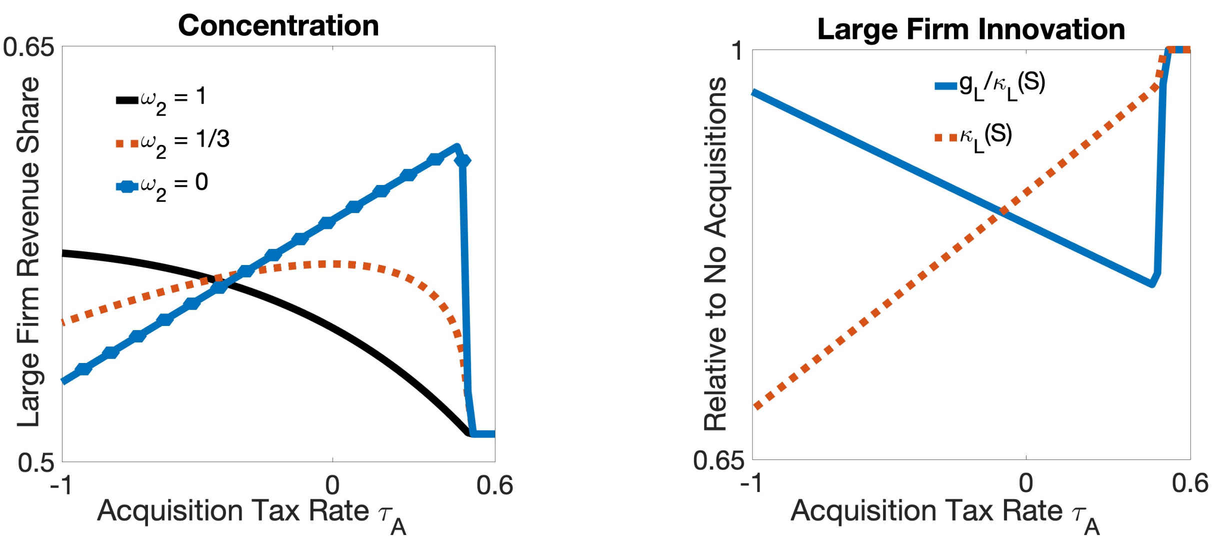

Figure 6: Left panel: see the caption in Figure 5; the large firm revenue share $\mathcal{L}$. Right panel: using the cost function that is 0 for $A \leq 0.05$ and infinite otherwise, the balanced growth path $g_L/\kappa_L(S)$ and $\kappa_L(S)$ as a function of τ_A relative to their values at $\tau_A = 0.6$.

innovation to preserve valuable acquisition opportunities (Figure 6). As expected from expression (13) in Section 3, a high acquisition surplus more readily increases growth if the acquisition rate is less elastic with respect to the surplus.

The tax rate at which reduce the tax on the margin increases long-run growth is much higher than the tax rate at which growth is faster in the economy with acquisitions than in the economy without. Thus, if taxing all acquisitions out of existence is not a desirable policy, then even at relatively high tax rates, the effect on large firm innovation bolsters the argument for lowering the tax further and encouraging more acquisitions.

6.2 The Acquisition Rate Elasticity and Concentration

I analyze when it is more beneficial to lower the acquisition tax rate. I vary the elasticity of the acquisition rate with respect to the surplus of an acquisition, ω_2, and using the calibration in Section 5.2 both before the main experiment in Section 5.3 and after, I compute the break even tax rate τ_A and acquisition surplus at which the long-run growth rate is equal in the economy with acquisitions and without (Figure 7). I set the cost shifter ω_1 so that the acquisition rate is always low to eliminate non-linear effects from changes in large firm revenue shares. As expected from Section 3.4, the break even tax rate is higher—acquisitions more easily increase growth—if the acquisition rate elasticity is lower, and after the rise in concentration due to the fall in the large firm fixed cost.

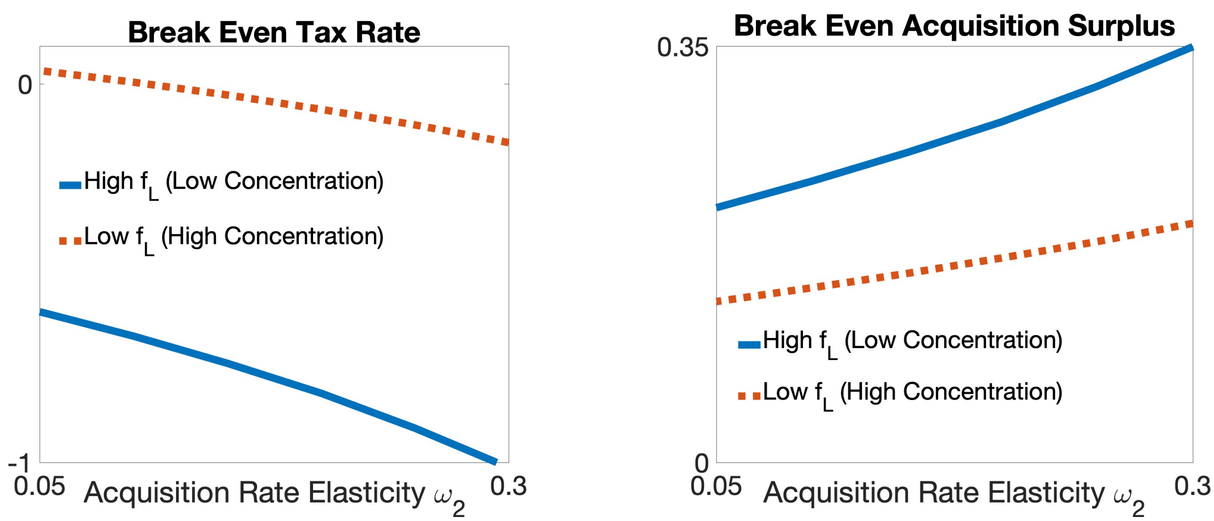

Figure 7: In each panel, the solid blue line uses the calibration from Section 5.2, and the dotted orange line uses the same but after the main experiment in Section 5.3. Left panel: the value of τ_A at which the economy with acquisitions has the same long-run growth rate as the one without, as a function of ω_2. Right panel: the surplus of an acquisition at the break even τ_A.

Another way to see this result is that when the break even tax rate is high, the required acquisition surplus for acquisitions to increase growth is lower. If the acquisition rate is more elastic with respect to the surplus, then total search costs are larger relative to the marginal search cost, so a high surplus conditional on finding an acquisition opportunity is not a powerful incentive to reduce innovation because acquisition opportunities are costly to find. Following the rise in concentration, as we saw in Section 3.4, the innovation effect is mechanically stronger relative to the acquisition effect, so a given surplus has a more beneficial effect on growth. Finally, the lower break even surplus in this case demonstrates that the higher break even tax rate is not just compensating for the inherently higher acquisition surplus due to large firms' lower fixed costs.

7 Conclusion

I study the implications of the concentration of innovative activity. I use a model with one large firm and a continuum of small firms in each industry. Firms can innovate through improving on old goods and developing new ones. The defining feature of a large firm is that it has the innovation technology of a measure of small firms, and thus behaves as a mass of small firms that can coordinate their innovative activities to maximize their joint profits. I show that this feature is sufficient for a rise in large firm profitability to explain the rise in concentration, the fall in growth, and related changes to the US economy since the mid-1990s.

Large firm acquisitions of their competitors' goods have distributional and incentive effects with opposite implications for concentration, growth, and welfare. Acquisitions directly shift revenue to large firms, which strengthens their relative preference for creative destruction and leaves more goods stagnating. As a result, growth falls. On the other hand, The expectation of future valuable acquisitions pushes each large firm to innovate less so that more revenue share remains for it to acquire. As large firm innovation falls, it is replaced by small firm innovation, which is less geared toward creative destruction and includes creative destruction of large firm goods. As a result, growth increases. The second effect is more likely to dominate if innovation is more elastic, the acquisition rate is less elastic, the acquisition rate is high, and following the recent changes in the economy that pushed up industry concentration.

The theory and results highlight a novel way to think about the effects of market power and optimal competition policy. Large firms are harmful because of how they achieve their size through innovation. Research and development subsidies that target large firms may backfire by discouraging small firm innovation, which is more efficient at generating growth. Policies that increase concentration may be beneficial as long as they reduce large firm innovation. Facilitating acquisitions is a particularly useful policy because, unlike taxing large firms, it does not require knowledge of firms' relevant industries or their revenue shares in those industries.

Finally, although this paper focuses on growth, the theory has implications for other settings, and suggests potential avenues for future research. For example, suppose a firm can develop different types of goods, some of which are more novel to the industry, and others of which are close substitutes with the firm's competitors' goods. The same force that leads larger firms to set higher markups in static models of oligopolistic competition implies that larger firms have a stronger preference for producing the types of goods that are close substitutes with their competitors. Thus, subsidizing large high markup firms to produce more may be costly unlike in models in which firm production is one-dimensional.

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A Proofs and Derivations

A.1 Proof of Static Equilibrium (Section 2.1.3)

In the second stage, a firm without the most productive active version of good j has no hope of earning strictly positive profits, so it is optimal to set price equal to marginal cost. A firm with the most productive active version would set a markup of at least $\gamma/(\gamma-1)$ if unconstrained by other producers of good j, so by Assumption 1, pricing below other producers' marginal costs is a binding constraint.[26]

In the first stage, a firm without the most productive version of good j will earn zero profits in the second stage if it pays the fixed cost, so it is optimal not to. If the small firm with the most productive version pays the fixed cost, it earns profits $\tilde{z}_{n,t}(j)\left(1-\sigma^{-1}\right)R_t$, so it is optimal to do so if Assumption 2 holds. A large firm's fixed cost decision is more complicated because paying the fixed cost for some of its goods reduces the relative productivity of its other goods. If the large firm pays fixed costs for a fraction x of its

[26]See Edmond, Midrigan, and Xu (2021) for a derivation of the optimal markup with oligopoly, nested CES demand, and Bertrand competition.

relative productivity, then its share of industry relative productivity is

$$\tilde{\mathcal{L}}_{n,t}(x) \equiv \frac{x\mathcal{L}_{n,t}}{1-(1-x)\left(1-\sigma^{1-\gamma}\right)\mathcal{L}_{n,t}}$$

because its versions are replaced by imitations with productivity discounted by σ. It earns total profits across both stages $\tilde{\mathcal{L}}_{n,t}(x)\left(1-\sigma^{-1}-\tau_R\right)R_t - x\mathcal{L}_{n,t}f_{L,n}$, which is strictly concave in x. The large firm finds it optimal to pay the fixed cost for all goods for which it has the most productive version if the first derivative of total profits at $x=1$ is positive, which is the case if Assumption 2 holds.

A.2 Proof of Theorem 1

I use (12), which has the same sign as $\partial g/\partial \kappa_L$, to prove the theorem. First, the theorem is vacuously true is $\kappa_L^* = 0$, i.e., if there is an equilibrium without any innovation.

Suppose $\kappa_L^* > 0$ in which case in equilibrium, $\bar{\kappa} > 0$. For the first property, if $\kappa_L = 0$, then since $\kappa_L^* > 0$, it follows that $\kappa_S > 0$, and so $\mathcal{L} = 0$. Thus, the second term in (12) is 0, and the result follows. For the second and third properties, observe that as equilibrium κ_L increases due to changes in β, f_L, and τ_R, the balanced growth path $\frac{2\mathcal{L}}{1-\mathcal{L}}\frac{\bar{\kappa}}{r+\bar{\kappa}}$ increases; κ_S must decrease (so $\mathcal{L}$ must increase) and $\bar{\kappa}$ must increase because if $\bar{\kappa}$ decreases, then κ_S must increase, which contradicts the fall in $\bar{\kappa}$. Thus, the second property holds because as κ_L goes to κ_L^*, κ_S goes to 0, which implies that the first term in (12) converges to $\epsilon_S(0)$, which is finite, and that the second term in (12) goes to infinity ($\mathcal{L}$ converges to 1).

A.3 Proof of Proposition 1

Use HJB equation (9) to solve for the equilibrium large firm value of a good:

$$\bar{V} \equiv V'(\mathcal{L}) = \frac{(1-\sigma^{-1}-\tau_R)R - f_L + W\beta X_L(\kappa_L) + WX_A(A) + A(1+\tau_A)\Pi}{r+\kappa_L+A+\kappa_S}. \tag{18}$$

Holding fixed parameters, use the large firm FOCs (8) with $\zeta = \bar{V}$ (and the strict convexity of $X_L(\cdot)$ and $X_A(\cdot)$), and small firm optimization (11) to write equilibrium large firm innovation κ_L, small firm innovation κ_S, the acquisition surplus $\mathcal{S} \equiv \bar{V} - (1+\tau_A)\Pi$, the small firm value of a good Π, the acquisition rate A, and so the right-hand side of (18) as continuous functions of $\bar{V}$ that are differentiable everywhere except at finitely many points. Let $T(\bar{V})$ be the function on the right-hand side. To prove that there is a unique equilibrium, it is thus sufficient to show that there is a unique $\bar{V}$ such that $T(\bar{V}) = \bar{V}$.

First, $\bar{V} > 0$ in equilibrium because $(1 - \sigma^{-1} - \tau_R)R - f_L > 0$ by Assumption 2, and the remaining terms on the right-hand side of (18) are weakly positive. It follows that $T(\bar{V})$ is bounded below by a strictly positive number, so that as $\bar{V}$ goes to 0, $T(\bar{V}) > \bar{V}$. To prove existence and uniqueness, it is thus sufficient to show that the derivative of $T(\bar{V})$ is bounded above by a number strictly below 1.

Canceling terms by using that if $\partial\kappa_L/\partial\bar{V} \neq 0$, then the FOC for κ_L holds with equality, and the same for A, we have that

$$T'(\bar{V}) = (A(1 + \tau_A)\partial\Pi/\partial\bar{V} - \bar{V}\partial\kappa_S/\partial\bar{V})/(r + \kappa_L + A + \kappa_S). \tag{19}$$

Differentiating Π and both sides of the FOCs for κ_L and κ_S with respect to $\bar{V}$ yields

$$\frac{\partial\Pi}{\partial\bar{V}} = \frac{-\Pi}{r + \bar{\kappa}}\frac{\partial\bar{\kappa}}{\partial\bar{V}} \qquad \frac{\partial\kappa_L}{\partial\bar{V}} = \frac{1}{\epsilon_L(\kappa_L)}\frac{\kappa_L}{\bar{V}} \qquad \frac{\partial\kappa_S}{\partial\bar{V}} = -\left(\epsilon_S(\kappa_S)\frac{r + \bar{\kappa}}{\kappa_S} + 1\right)^{-1}\frac{\partial\kappa_L}{\partial\bar{V}}, \tag{20}$$

where $\bar{\kappa} \equiv \kappa_L + \kappa_S$. Using $\partial\bar{\kappa}/\partial\bar{V} \geq 0$ (since we saw in Appendix A.2 that $\bar{\kappa}$ is increasing in κ_L), it follows that $T'(\bar{V})$ is less than

$$\frac{1}{\epsilon_S(\kappa_S)(r + \bar{\kappa})/\kappa_S + 1}\frac{1}{\epsilon_L(\kappa_L)}\frac{\kappa_L}{r + \kappa_L + A + \kappa_S},$$

which is strictly less than 1 because $r > 0$ and $(\epsilon_S(\kappa_S) + 1)\epsilon_L(\kappa_L) \geq 1$ by assumption.

For the next claim in the proposition, suppose $A = 0$ and define $\tilde{V} \equiv \bar{V}/\beta$ and $\Upsilon \equiv ((1 - \sigma^{-1} - \tau_R)R - f_L)/\beta$. Dividing both sides of the FOC for κ_L by β shows that the equilibrium κ_L is a continuous increasing function of the equilibrium $\tilde{V}$ that does not otherwise depend on Υ. Thus, to show that the equilibrium κ_L is a continuous increasing function of Υ, it is sufficient to show that the same holds for the equilibrium $\tilde{V}$.

From small firm optimization, the equilibrium κ_S is also a continuous function of the equilibrium $\tilde{V}$ that does not otherwise depend on Υ. Let $\tilde{T}(\tilde{V}, \Upsilon)$ be the right-hand side of (18) divided by β, where I make the dependence on Υ explicit because Υ will vary. From the first part of the proof and since $\tilde{T}$ is linear in Υ, $\tilde{T}$ is continuous in each of its arguments and differentiable everywhere except at finitely many points. Moreover, for each Υ, there is a unique equilibrium, which is characterized by a $\tilde{V}$ such that $\tilde{T}(\tilde{V}, \Upsilon) = \tilde{V}$. Totally differentiating with respect to Υ yields in equilibrium,

$$\partial\tilde{V}/\partial\Upsilon = (1 - \partial\tilde{T}(\tilde{V}, \Upsilon)/\partial\tilde{V})^{-1}\partial\tilde{T}(\tilde{V}, \Upsilon)/\partial\Upsilon.$$

The argument that $T'(\bar{V})$ is bounded above by a number strictly below 1 shows that the same is true for $\partial\tilde{T}(\tilde{V}, \Upsilon)/\partial\tilde{V}$. It follows that $\partial\tilde{V}/\partial\Upsilon > 0$ because $\partial\tilde{T}(\tilde{V}, \Upsilon)/\partial\Upsilon > 0$.

For the remaining claims, using the large firm FOCs and small firm optimization, it is sufficient to show that equilibrium $\bar{V}$ and $\mathcal{S}$ are strictly increasing and decreasing in τ_A, respectively (acquisitions depend on $\mathcal{S}$). The same argument as for $\partial\tilde{V}/\partial\Upsilon$ shows that

$$\frac{\partial\bar{V}}{\partial\tau_A} = \frac{(1-T'(\bar{V}))^{-1}A\Pi}{r+\kappa_L+A+\kappa_S} = \frac{A\Pi}{r+\kappa_L+\kappa_S+A(1-(1+\tau_A)\partial\Pi/\partial\bar{V})+\bar{V}\partial\kappa_S/\partial\bar{V}},$$

where the second equality uses (19). Thus,

$$\frac{\partial\mathcal{S}}{\partial\tau_A} = \left(1-(1+\tau_A)\frac{\partial\Pi}{\partial\bar{V}}\right)\frac{\partial\bar{V}}{\partial\tau_A}-\Pi = \frac{-(r+\kappa_L+\kappa_S+\bar{V}\partial\kappa_S/\partial\bar{V})\Pi}{r+\kappa_L+\kappa_S+A(1-(1+\tau_A)\partial\Pi/\partial\bar{V})+\bar{V}\partial\kappa_S/\partial\bar{V}}.$$

The result follows because (20) and $(\epsilon_S(\kappa_S)+1)\epsilon_L(\kappa_L)\geq 1$ imply $\bar{V}\partial\kappa_S/\partial\bar{V}\geq -\kappa_L$.

A.4 Derivation of $\partial g/\partial\tau_A$

By assumption, the large firm FOCs (8) hold with equality (with $\zeta = \bar{V} \equiv V'(\mathcal{L})$). Differentiating each side of the FOC for κ_L with respect to $\bar{V}$ and for A with respect to $\mathcal{S}\equiv\bar{V}-(1+\tau_A)\Pi$ yield that $1/\epsilon_L(\kappa_L)$ and $1/\epsilon_A(A)$ are the elasticities of equilibrium κ_L and A with respect to equilibrium $\bar{V}$ and $\mathcal{S}$, respectively. Then, $\partial\bar{V}/\partial\tau_A$ and $\partial\mathcal{S}/\partial\tau_A$ from the end of Appendix A.3, and $\partial g/\partial A$ and $\partial g/\partial\kappa_L$ (the former holds innovation fixed and the latter includes the derivative of equilibrium κ_S with respect to equilibrium κ_L) from (14) in Section 3.4 imply that the derivative of balanced growth path g with respect to τ_A is the product of

$$\mathcal{L}(1-\mathcal{L})\frac{\kappa_L}{\kappa_L+A}\frac{1}{\mathcal{S}}\left(r+\kappa_L+\kappa_S-\frac{1}{\epsilon_S(\kappa_S)(r+\bar{\kappa})/\kappa_S+1}\frac{1}{\epsilon_L(\kappa_L)}\kappa_L\right)\frac{\partial\bar{V}}{\partial\tau_A},$$

which is strictly positive because $\epsilon_L(\kappa_L)(\epsilon_S(\kappa_S)+1)\geq 1$, and

$$\frac{(1-\mathcal{L})((1-\mathcal{L})\kappa_L+A)(\epsilon_S(\kappa_S)(r+\bar{\kappa})/\kappa_S+1)-((1+\mathcal{L}^2)\,\kappa_L+A)}{\mathcal{L}(1-\mathcal{L})[(r+\kappa_L+\kappa_S)\epsilon_L(\kappa_L)(\epsilon_S(\kappa_S)(r+\bar{\kappa})/\kappa_S+1)-\kappa_L]}\frac{\mathcal{S}}{\bar{V}}+\frac{1}{\epsilon_A(A)}, \qquad (21)$$

where algebra shows that $\mathcal{S}/\bar{V} = (1+(1+\tau_A)\Pi/\mathcal{S})^{-1}$.

For $\partial g/\partial\tau_A$ to be strictly negative, the numerator of the first fraction in (21) must be strictly negative; the denominator is strictly positive because $\epsilon_L(\kappa_L)(\epsilon_S(\kappa_S)+1)\geq 1$. If the numerator is strictly negative, then $\partial g/\partial\tau_A<0$ only if the product of the first two fractions is sufficiently negative to outweigh $1/\epsilon_A(A)$. The numerator is strictly negative if and only if $\epsilon_S(\kappa_S)$ is sufficiently small. If that is the case, then the product of the first two fractions is strictly increasing in $\epsilon_S(\kappa_S)$ (an increase in $\epsilon_S(\kappa_S)$ makes the numerator less negative and increases the denominator), is strictly increasing in $\epsilon_L(\kappa_L)$ (an increase in $\epsilon_L(\kappa_L)$ increases the denominator), and is strictly decreasing in $\mathcal{S}/((1+\tau_A)\Pi)$. Finally, expression (13) follows from setting $\epsilon_S(\kappa_S)=0$.

A.5 Proof of Theorem 2

It is sufficient to show that $\mathcal{L}$, κ_L, A, and the second term in (13) (not including the minus) are increasing in $\aleph \equiv (1-\sigma^{-1}-\tau_R)R - f_L$, and do not otherwise depend on f_L and τ_R. In that case, if $\mathcal{L}$ is sufficiently high, then κ_L and A are strictly positive, which implies that (13) is a sufficient statistic for whether $\partial g/\partial \tau_A < 0$ ($\kappa_S = 0$ does not bind by assumption), and that (13) is strictly negative.

The same argument as for $\partial \tilde{V}/\partial \Upsilon$ in Appendix A.3 shows that $\bar{V}$ only depends on f_L and τ_R through $\aleph$, and that in equilibrium,

$$\partial \bar{V}/\partial \aleph = (1 - T'(\bar{V})^{-1}/(r + \kappa_L + A + \kappa_S) > 0.$$

Since $\epsilon_S(\cdot) = 0$, it follows from (20) in Appendix A.3 that $\bar{\kappa}$ and Π are constant in $\bar{V}$, which implies that $\mathcal{S}$ only depends on f_L and τ_R through $\aleph$, and that $\partial \mathcal{S}/\partial \aleph = \partial \bar{V}/\partial \aleph$. It follows from the large firm FOCs (8) and from (20) in Appendix A.3 that κ_L and A are increasing in $\aleph$, and that κ_S is decreasing in $\aleph$. Thus, $\mathcal{L}$ is increasing in $\aleph$. Finally, it follows that the second term in (13) (not including the minus) is increasing in $\aleph$ because $\epsilon_S(\cdot) = 0$ implies that $\epsilon_L(\cdot) \geq 1$, and so $\epsilon_L(\kappa_L)(r + \bar{\kappa}) > \kappa_L$.

A.6 Proof of Theorem 3

Throughout the proof, I drop time t subscripts when possible. I begin by proving the results concerning growth across industries on a single balanced growth path. I first show that we can decompose the industry growth rate into two continuously differentiable functions of the large firm industry revenue share, a strictly increasing $D(\cdot)$ and strictly decreasing $\tilde{g}(\cdot)$, so that for all n, $(\gamma - 1)g_n = D(\mathcal{L}_n)\tilde{g}(\mathcal{L}_n)$, and $D(\mathcal{L}_n)$ is the non-interest component of the discount rate on a small firm's good:

$$D(\mathcal{L}_n) = N(\kappa_{S,n}(\mathcal{L}_n) + (\gamma - 1)g_{S,n}(\mathcal{L}_n)) + \kappa_{L,n}(S; \mathcal{L}_n) + (\gamma - 1)g_{L,n}(\mathcal{L}_n. \tag{22}$$

Consider $D(\cdot)$, and suppose equation (22) holds. Using HJB equation (6) for the present discounted profits a small firm earns from a good, we can guess and verify that on a balanced growth path, those profits relative to the wage are a constant $\tilde{\Pi}_n$ ($\Pi_{n,t}(\mathcal{L}_n)$ grows at the same rate as the wage) given by equation (17), reproduced below, because on a balanced growth path, the interest rate is $r = \rho + g$, aggregate output is $C_{t'} = Z_t e^{g(t'-t)} L^p$ for all $t' \geq t$, the industry state is constant, and the real wage is $W_t/P_t = Z_t/\sigma$:

$$\tilde{\Pi}_n \equiv \Pi_{n,t}(\mathcal{L}_n)/W_t = ((\sigma - 1)L^p - f_S)/(\rho + D(\mathcal{L}_n)).$$

It then follows from equation (4) for the evolution of the industry state over time, and the second equation in (5) for growth due to small firms that on a balanced growth path, $D(\mathcal{L}_n) = N(\lambda^{\gamma-1}\kappa_{S,n}(\mathcal{L}_n)+\delta_{S,n}(\mathcal{L}_n))/(1-\mathcal{L}_n)$, which along with equations (7) for optimal small firm innovation, implies that

$$(1-\mathcal{L}_n)D(\mathcal{L}_n)(\rho+D(\mathcal{L}_n))^{1/\epsilon} = N((\lambda^{\gamma-1})^{1+1/\epsilon}\alpha^{-1/\epsilon}+1)((\sigma-1)L^p - f_S)^{1/\epsilon}, \tag{23}$$

where the right-hand side is a strictly positive number. Eliminating the n subscripts on the left-hand side, equation (23) implicitly defines the function $D(\cdot)$ with the desired properties.

Second, consider $\tilde{g}(\cdot)$. Use equation (4) for the evolution of the industry state over time, evaluated on a balanced growth path, to write

$$\frac{(\gamma-1)g_n}{D(\mathcal{L}_n)} = (1-\mathcal{L}_n)\frac{(\gamma-1)g_{S,n}(\mathcal{L}_n)}{\kappa_{S,n}(\mathcal{L}_n)+(\gamma-1)g_{S,n}(\mathcal{L}_n)} + \mathcal{L}_n\frac{(\gamma-1)g_{L,n}(\mathcal{L}_n)}{\kappa_{L,n}(S;\mathcal{L}_n)+(\gamma-1)g_{L,n}(\mathcal{L}_n)}. \tag{24}$$

It follows from the second equation in (5) for growth due to small firms, and equations (7) for optimal small firm innovation that the first term on the right-hand side of equation (24) is $(1-\mathcal{L}_n)B/(B+1)$, where

$$B \equiv \lambda^{\gamma-1} - 1 + (\alpha\lambda^{1-\gamma})^{1/\epsilon}$$

is the rate at which a small firm generates growth relative to the rate at which it creatively destroys a good. It follows from the first equation in (5) and equations (8) that the second term on the right-hand side is the product of $\mathcal{L}_n$ and

$$\begin{aligned}\frac{(\gamma-1)g_{L,n}(\mathcal{L}_n)}{\kappa_{L,n}(S;\mathcal{L}_n)+(\gamma-1)g_{L,n}(\mathcal{L}_n)} &= 1-\left(1+\frac{(\gamma-1)g_{L,n}(\mathcal{L}_n)}{\kappa_{L,n}(S;\mathcal{L}_n)}\right)^{-1} \qquad (25)\\ = 1-\Bigg(1+(1-\mathcal{L}_n)(\lambda^{\gamma-1}-1) &+ \left(\frac{(1-\mathcal{L}_n)}{(1-\mathcal{L}_n)\lambda^{\gamma-1}+\mathcal{L}_n}\right)^{1/\epsilon}\left(\mathcal{L}_n(\lambda^{\gamma-1}-1)^{1+1/\epsilon}+\alpha^{1/\epsilon}\right)\Bigg)^{-1},\end{aligned}$$

where the last line, dropping the n subscripts, is a strictly decreasing continuously differentiable function of $\mathcal{L}$ that goes from $B/(B+1)$ at 0 to 0 at 1. Thus, we can define the function $\tilde{g}(\cdot)$ with the desired properties.

It follows that the growth rate across industries is characterized by a continuously differentiable function of the large firm industry revenue share, $g_I(\cdot)$, given by $(\gamma-1)g_I(\mathcal{L}) = D(\mathcal{L})\tilde{g}(\mathcal{L})$. Moreover, $g_I(0) > 0$ because equation (23) shows that $D(0) > 0$, and the first term on the right-hand side of equation (24) is strictly greater than 0. Next, $g_I'(0) > 0$

because equation (23) shows that $D'(0) > 0$, and $\tilde{g}'(0) = 0$ (the first term on the right-hand side of equation (24) not including $1 - \mathcal{L}_n$ is equal to the second term not including $\mathcal{L}_n$).

Instead of the second item in Theorem 3 (concerning $\epsilon = 1$), I prove a stronger proposition:

Proposition 2. *If $\epsilon \in (0, (3+\sqrt{5})/2 - 1)$, then $\lim_{\mathcal{L}\to 1}(g_I(\mathcal{L})) = 0$. If $\epsilon > (3+\sqrt{5})/2 - 1$, then $\lim_{\mathcal{L}\to 1}(g_I(\mathcal{L})) = \infty$.*

Proof. First, equation (23) shows that as $\mathcal{L}$ goes to 1, $D(\mathcal{L})$ diverges to positive infinity. Therefore, as $\mathcal{L}$ goes to 1, $\rho + D(\mathcal{L})$ converges to $D(\mathcal{L})$, which implies that $(1-\mathcal{L})D(\mathcal{L})^{1+1/\epsilon}$ converges to a strictly positive number, and so $D(\mathcal{L})$ converges to the product of a strictly positive number and $(1-\mathcal{L})^{-\epsilon/(\epsilon+1)}$.

Next, equations (24) and (25) show that as $\mathcal{L}$ goes to 1, $\tilde{g}(\mathcal{L})$ converges to 0, which implies that $\tilde{g}(\mathcal{L})$ converges to the product of a strictly positive number and $(1-\mathcal{L})^{\min\{1,1/\epsilon\}}$ because that is the lowest power of $1-\mathcal{L}$ contained in any term.

It follows that as $\mathcal{L}$ goes to 1, $g_I(\mathcal{L})$ converges to the product of a strictly positive number and $(1-\mathcal{L})^{\min\{1,1/\epsilon\}-\epsilon/(\epsilon+1)}$. If $\epsilon > 1$, then the exponent on $1-\mathcal{L}$ is $(\epsilon - \epsilon_1^*)(\epsilon_2^* - \epsilon)/(\epsilon(\epsilon+1))$, where

$$\epsilon_1^* = (3-\sqrt{5})/2 - 1 \qquad\qquad \epsilon_2^* = (3+\sqrt{5})/2 - 1.$$

Since $\epsilon_1^* < 0$ and $\epsilon_2^* > 1$, it follows that for all $\epsilon \in (0, \epsilon_2^*)$, the exponent on $1-\mathcal{L}$ is strictly positive, so $g_I(\mathcal{L})$ converges to 0 as $\mathcal{L}$ goes to 1. For all $\epsilon > \epsilon_2^*$, the exponent on $1-\mathcal{L}$ is strictly negative, so $g_I(\mathcal{L})$ diverges to infinity as $\mathcal{L}$ goes to 1. ■

I now complete the proof of the parts of Theorem 3 concerning $g_I(\cdot)$. Set $\epsilon = 1$. Since $g_I(0) > 0$, $g_I'(0) > 0$, $\lim_{\mathcal{L}\to 1}(g_I(\mathcal{L})) = 0$, and $g_I(\cdot)$ is continuously differentiable, it is sufficient to show that $g_I(\cdot)$ has at most one critical point. Equation (23) shows that

$$D'(\mathcal{L}) = \frac{D(\mathcal{L})(\rho + D(\mathcal{L}))}{(\rho + 2D(\mathcal{L}))(1-\mathcal{L})},$$

and equation (24) along with the associated derivations shows that

$$\tilde{g}(\mathcal{L}) = \frac{B + \lambda^{1-\gamma}\mathcal{L}B/(B+1)}{B + 1 + \lambda^{1-\gamma}\mathcal{L}/(1-\mathcal{L})} \qquad \tilde{g}'(\mathcal{L}) = \frac{\tilde{g}(\mathcal{L})}{1-\mathcal{L}}\left(\frac{\lambda^{1-\gamma}(1-\mathcal{L})}{B+1+\lambda^{1-\gamma}\mathcal{L}} - \frac{\lambda^{1-\gamma}/(1-\mathcal{L})}{B+1+\lambda^{1-\gamma}\mathcal{L}/(1-\mathcal{L})}\right).$$

It follows from multiplying $(\gamma - 1)g_I'(\mathcal{L})$ by $(1-\mathcal{L})/((\gamma-1)g_I(\mathcal{L}))$ and adding and subtracting one to $D'(\mathcal{L})\tilde{g}(\mathcal{L})$ that at a critical point of $g_I(\cdot)$,

$$-\frac{D(\mathcal{L})}{\rho + 2D(\mathcal{L})} + \frac{\lambda^{1-\gamma}(1-\mathcal{L})}{B+1+\lambda^{1-\gamma}\mathcal{L}} + \frac{B+1-\lambda^{1-\gamma}}{B+1+\lambda^{1-\gamma}\mathcal{L}/(1-\mathcal{L})} = 0.$$

The left-hand side is strictly decreasing in $\mathcal{L}$ because $D(\cdot)$ is strictly increasing and the second and third terms are strictly positive ($\lambda^{1-\gamma} < 1$). Thus, there is at most one solution.

Finally, I prove the parts of Theorem 3 concerned with the aggregate growth rate across balanced growth paths. The arguments are the same as for the growth rate across industries on a single balanced growth path, except that now the function $D(\cdot)$ is constant. Thus, the results follow because $\tilde{g}(\cdot)$ is continuously differentiable, $\tilde{g}(\mathcal{L})$ converges to 0 as $\mathcal{L}$ goes to 1, $\tilde{g}'(0) = 0$, and $\tilde{g}'(\mathcal{L}) < 0$ for all $\mathcal{L} > 0$.

I use the free entry condition to show that $D(\cdot)$ is constant in this case. Using HJB equation (16) for the value of entering industry n and equations (7) for optimal small firm innovation, we can guess and verify that on a balanced growth path, that value relative to the wage is a constant ($E_{n,t}(\mathcal{L}_n)$ grows at the same rate as the wage),

$$\tilde{E}_n \equiv E_{n,t}(\mathcal{L}_n)/W_t = (\epsilon/(\epsilon+1)) \left((\lambda^{\gamma-1})^{1+1/\epsilon}\alpha^{-1/\epsilon} + 1\right) \tilde{\Pi}_n^{1+1/\epsilon}/(\rho+\eta),$$

because on a balanced growth path, the interest rate is $r = \rho + g$, $\Pi_{n,t}(\mathcal{L}_n)/W_t$ is the constant $\tilde{\Pi}_n$, and the industry state is constant. On a balanced growth path with a strictly positive entry rate in which all industries are identical, the free entry condition fixes for all n, the value of entering industry n relative to the wage: $\tilde{E}_n = \xi$. It follows that for any variation in the large firm fixed cost f_L, revenue tax rate τ_R, and innovation cost shifter β, the balanced growth path present discounted profits a small firm earns from a good relative to the wage, $\tilde{\Pi}_n$, is constant. Thus, given constant labor used for production L^p, the discount rate on a small firm's good must be constant as well.