Optimal Vaccine Policies: Spillovers and Incentives

Nikhil Vellodi, Joshua Weiss*

[Current Draft]: January 20, 2021 (click here for the latest version)

Abstract

We offer a novel theoretical framework to study optimal vaccination policies. The key features of the model are that agents: 1) differ both in their potential exposure (x) to others and vulnerability (y) to severe illness, 2) exert negative externalities through interaction, and 3) can take voluntary preventative measures, for instance self-isolation. Our main result is a complete characterization of the second-best policy. Three striking features emerge. First, it is *non-monotone* – people with intermediate y are vaccinated more than those with either low or high y. Second, it exhibits an *exposure premium* among those who do not self-isolate – people with higher x require lower overall risk, xy, to be vaccinated. Third, for those who voluntarily self-isolate, it is invariant to y, depending only upon x. Numerical results demonstrate that policies vaccinating only the most vulnerable perform significantly worse than other simple heuristics, especially when supplies are limited.

*Vellodi: Paris School of Economics, Weiss: Federal Reserve Bank of Richmond and the Institute for International Economic Studies (IIES), Stockholm University. We thank Miquel Oliu-Barton, Francis Bloch, Paula Onuchic, Bary Pradelwski, Ludovic Renou, Evan Sadler and Olivier Tercieux for helpful comments. Vellodi acknowledges support from the EUR grant ANR-17-EURE-0001.

1 Introduction

Vaccine policies are a crucial ingredient in the eradication of pandemics. Given the scarcity of supply, and the urgent need for roll-out, the prudent prioritization of vaccine allocation is of paramount importance. Prevailing views revolve around two key considerations, vulnerability risk and transmission risk. Sensible arguments can be made for allocating the vaccine to those most at risk from suffering severe adverse symptoms conditional upon infection, as well as those who would likely spread the disease through a high rate of contact with others. (see Section 5 for a detailed discussion of these varying approaches) Thus far, however, little attention has been devoted to the role that incentives and behavior play in designing optimal vaccine policy. A recent report by the World Health Organization documents the growing importance and policy relevance of various behavioral incentives couched under the term "lockdown fatigue", referring to the rejection of mandatory isolation requirements.[1] As such, prudent policy design should account for agency in the design of second-best policies, rather than mandating rules and assuming they are fully implemented.

A burgeoning literature aims to incorporate behavior into standard *susceptible-infected-recovered* (SIR) dynamic epidemiological models, primarily to study the evolution of contagion rates.[2] Such models offer a strong methodology for generating robust quantitative predictions, but often become analytically unwieldy when combined with other considerations such as multi-dimensional heterogeneous agents. This complexity is necessary to ascertain the relative importance of vulnerability versus transmission risk in vaccine prioritization policies.

To this end, we offer a simple, tractable model that abstracts from dynamic considerations found in SIR models, but admits multi-dimensional heterogeneity. The model comprises a unit measure of individuals, indexed by both their potential exposure to other agents, x, and their vulnerability, y. Formally, x denotes an individual's probability of entering an *interaction pool*, wherein infection occurs through pairwise interactions, and y denotes their likelihood of suffering

[1] See https://tinyurl.com/y43jpxne.

[2] See Funk et al. (2010) for a comprehensive review.

a severe reaction to the disease *conditional* upon contracting it. As a consequence, holding y fixed, an individual with a higher x not only faces a higher risk of an adverse outcome, but also exerts a greater negative externality on others by increasing the overall size of the interaction pool. Finally, upon receiving an interaction opportunity, an individual may incur a fixed cost c to self-isolate instead.[3]

Our main result – Propositions 1 and 2 combined – provides a full, analytic characterization of the optimal allocation of vaccines taking as given incentives to voluntarily self-isolate, i.e. the *second-best* policy. It displays three striking features. First, it is *non-monotone* – people with intermediate y are vaccinated more than those with either low or high y. Such individuals over-interact relative to the social optimum, so they are particularly costly to society when unvaccinated. Second, among those not self-isolating, the optimal policy exhibits an *exposure premium* – people with higher x require a lower overall risk threshold xy to be vaccinated. This feature reflects the social value of vaccinating individuals that tend to infect others. Third, for those who self-isolate, the policy depends only upon x and not upon y. The latter two properties reveal two different ways that x is more relevant for vaccine allocation than y. For those who self-isolate, y loses its significance relative to x because individuals face no risk of infection yet their cost of avoiding interaction is increasing in the likelihood of interaction, x. For those who do not self-isolate, x and y both contribute equally to their overall risk, but only x contributes to the negative externality they impose on others. Grouping agents by behavior, the second and third features describe *within-group* allocation, while the first feature describes *between-group* allocation. On this final point, since the interacting group contains those behaving inefficiently, it is this group that receives an increased quantity of vaccines, thus generating the non-monotonicity. Beyond this main contribution, we provide further analytical insights. For instance, by characterizing the first-best allocation (vaccine allocation combined with a mandatory lockdown rule) – Proposition 3 – we highlight the key distortions that private incentives entail, most importantly socially excessive interaction by agents with intermediate y.

[3] We extend the analysis to other forms of behavior, such as vaccine hesitancy.

In addition to these theoretical findings, in Section 5 we perform various numerical exercises, not only to further explore key properties of the optimal policy, but also to compare it to commonly considered heuristics, such as: 1) y-policies that vaccinate only the most vulnerable, 2) x-policies that vaccinate only the most interactive, 3) xy-policies that vaccinate according to effective risk, and 4) random policies that do not correlate vaccines with any observable characteristics. We consider various performance criteria, including (utilitarian) welfare, death rates, isolation costs, and a novel measure that computes the additional quantity of vaccine required to render a policy welfare-equivalent to the second-best. This final measure informs the decision between taking time to evaluate and implement optimal policies and implementing off-the-shelf heuristics.[4]

Several policy-relevant insights emerge. Most importantly, y-policies perform significantly worse than other heuristics, and this performance gap is largest when vaccine supply is heavily constrained or the disease is particularly contagious or deadly. In those cases, y-policies end up targeting only self-isolating individuals and generate no spillovers. This result is of significant practical relevance, as many countries adopt such policies at the start of vaccine roll-out, precisely when supply is at its most limited. Beyond this, we find that vaccine allocation should be geographically dispersed rather than concentrated and should favor regions that either do not or struggle to implement mandatory lockdown policies. Finally, as vulnerability and exposure become more negatively correlated – an empirically relevant case – the welfare losses from heuristic policies relative to the second-best are exacerbated. These results can only obtain in settings that explicitly account for both heterogeneous vulnerability and exposure, as we do here. As such, we believe our results, both analytical and numerical, will help provide qualitative insights into hitherto under-explored features of vaccine allocation.

1.1 Related Literature

Our paper lies at the intersection of two separate approaches to modeling policy interventions during epidemics: the *networks*-based approach and the *dynamic contagion*-based approach. In

[4]Such a trade-off is relevant in the case of COVID-19 since many vaccines remain unused as of the writing of this paper. Moreover, most of the currently available vaccines expire relatively rapidly.

the networks-based approach, there is a literature in computer science that studies the problem of targeted interventions on random / adaptive networks.[5] These papers typically abstract from distortions to optimal policies due to equilibrium behavior. Within economics, papers such as Galeotti et al. (2020) and Ballester et al. (2006) explicitly consider incentives and behavior in network models. In particular, Galeotti et al. (2020) introduces a far richer network of interactions, and a far more general payoff structure, with leading applications to supply chains and social media, where local effects are strong. In contrast, our model involves random matching, and thus cannot capture such local effects. That said, random matching is an established approach to modeling epidemics – it forms the core of SIR-type models – where interactions with passing acquaintances are an important form of transmission. Finally, these papers do not have multi-dimensional types, while the tractability of our model allows us to obtain stronger characterizations. On this note, our model of interaction combines random matching with heterogeneous and partially endogenous contact rates, but crucially also spillover effects – an individual's contact rate is determined by the overall market thickness, itself determined by other individuals' contact rates. To the best of our knowledge, it constitutes in itself a theoretical contribution to the search and matching literature. In particular, our paper is related to Farboodi et al. (2020), who introduce heterogeneous contact rates into a decentralized asset market with random matching and also study how agency costs shape the endogenous distribution of contact rates.

The dynamic contagion-based approach typically takes the well-established SIR model of dynamic contagion and enriches it with agent incentives and behavior.[6] These papers are primarily concerned with making quantitative, dynamic predictions as well as evaluating inter-temporal policy trade-offs. Such concerns are crucial ingredients when making quantitative forecasts. We argue that agent heterogeneity is equally important, and thus propose a simple framework to explore this complementary theme while abstracting from complex dynamics.

Our work relates to the "risk compensation" phenomenon, which describes how mitigating the

[5]See Pastor-Satorras and Vespignani (2002), Shaw and Schwartz (2008), Gross et al. (2006), Epstein et al. (2008).

[6]See Brotherhood et al. (2020), Greenwood et al. (2019), and Geoffard and Philipson (1996).

downsides to risky actions might not reduce the prevalence of adverse outcomes if it is outweighed by greater risk-taking.[7] Such papers typically study how responses to policy alter the effectiveness of interventions, typically finding that they might mitigate efficacy. In contrast, we focus on how incentives can re-direct the targeting of interventions between the different characteristics of agents, a feature we believe is novel to this literature.

Finally, of most pressing applied concern is recent work considering the design of vaccine policies, and in particular in response to the COVID-19 epidemic. A large pre-COVID-19 literature exists looking at vaccination policy and equilibrium behavior, as summarized in Chen and Toxvaerd (2014). Our paper shares certain similarities with these, in particular demonstrating how second-best outcomes exhibit inefficiently low self-isolation or vaccine uptake. To the best of our knowledge, ours is the first paper to explicitly consider both margins of heterogeneity, and their interaction with optimal vaccination policies. Regarding COVID-19 in particular, Babus et al. (2020) consider a framework with multi-dimensional agents, but absent both contact externalities and behavioral considerations. Pathak et al. (2020) take a very different approach, bringing ethical considerations into a multi-priority approach. Finally, Rowthorn and Toxvaerd (2020) insert vaccination policy into a dynamic SIR framework.

2 Baseline Model

We first introduce the baseline model without behavioral considerations. A unit measure of agents are indexed by two-dimensional type $(x, y) \in [0,1]^2$ drawn from a continuously differentiable distribution F with full support and with density function f. Each agent (x, y) is placed in an *interaction pool* with probability x. Once in the pool, they meet another agent with probability μ, where μ is the mass of agents in the pool. If an interaction occurs, each agent contracts the virus with probability $\alpha > 0$, and conditional upon infection, an agent (x, y) dies with probability y. Thus, we refer to x as an agent's *exposure type* and y as an agent's *vulnerability type*. Agents not placed in the pool cannot contract the virus. Each agent receives a terminal payoff of 0 if

[7] See Talamás and Vohra (2020), Kremer (1996), and Greenwood et al. (2019) for an extensive discussion.

they survive, and $-b < 0$ if they die. Denote by $\lambda \triangleq \alpha\mu$ the *transmission rate*, so that $x\lambda$ is the likelihood that type (x, y) contracts the virus. Simple algebra confirms that an agent's ex-ante expected payoff is

$$u(x, y) = -b\lambda xy. \tag{1}$$

Note that $u(x, y)$ is decreasing in λ, x, y and crucially is determined by the product xy – which we refer to as the agent's *risk-type* – rather than each component separately.

Vaccination Policy – A social planner aims to maximize welfare by allocating a fixed supply $\beta \in (0, 1)$ of vaccine.[8] If an agent is vaccinated, any interaction involving that agent cannot cause infection, i.e. $\alpha = 0$ for this agent's interactions. Formally, a *vaccine policy* is a mapping $v : [0, 1]^2 \to [0, 1]$ where $v(x, y)$ is the probability that an individual with type (x, y) is vaccinated and such that v is feasible:

$$\int_0^1 \int_0^1 v(x, y)f(x, y)dxdy \leqslant \beta. \tag{2}$$

Under policy v, the transmission rate becomes

$$\lambda = \alpha \int_0^1 \int_0^1 \big(1 - v(x, y)\big)xf(x, y)dxdy \tag{3}$$

and ex-ante individual expected utility is

$$u(x, y) = -b\lambda xy\big(1 - v(x, y)\big).$$

Since the distribution over types, F, has a pdf, we can restrict attention to vaccination policies such that for all (x, y), $v(x, y) \in \{0, 1\}$.

Voluntary Self-Isolation – We now extend the model by considering behavioral incentives and their implications for optimal policy. We first focus on incentives to voluntarily self-isolate and later turn to incentives to accept vaccination given possible adverse side-effects. Upon receiving an opportunity to enter the interaction pool, an agent can now pay a cost $c > 0$ to avoid doing so.

[8] In this baseline, this is identical to minimizing total deaths.

A *strategy profile* is a mapping $\sigma : [0,1]^2 \to [0,1]$, where $\sigma(x,y)$ is the probability an agent with type (x,y) does not pay the cost. The transmission rate is now a function of agents' strategies:

$$\lambda = \alpha \int_0^1 \int_0^1 \sigma(x,y)\big(1 - v(x,y)\big) x f(x,y) dx dy, \tag{4}$$

where we leave the dependence of strategies and the transmission rate on the vaccination policy implicit. Note that an individual's interaction strategy can but need not depend explicitly on whether they are vaccinated; all types are vaccinated either with probability 0 or probability 1, so conditioning on an individual's type is sufficient.

Equilibrium – Upon receiving an opportunity to enter the interaction pool, an unvaccinated individual with type (x,y) faces the optimization problem

$$\max\{-c, -\lambda y b\},$$

where the first term is the cost of isolating and the second term is the expected cost of death that results from interacting. An unvaccinated individual's optimal strategy is thus

$$\sigma^*(x,y) \begin{cases} = 1 & y < y^* \\ \in [0,1] & y = y^* \\ = 0 & y > y^*, \end{cases}$$

where

$$y^* \triangleq c/(\lambda b). \tag{5}$$

In words, each unvaccinated individual self-isolates and avoids the interaction pool if they are sufficiently vulnerable. All individuals use the same threshold y^* and their strategies do not depend on the frequency with which they receive interaction opportunities, x. Given σ^*, an unvaccinated individual gets utility

$$u(x, y) = \begin{cases} -b\lambda xy & y < y^* \\ -cx & y \geq y^*. \end{cases}$$

A vaccinated individual has no reason to avoid interaction, always interacts when given the opportunity, and gets utility 0. Equilibrium is defined in the usual fashion for anonymous large games: given the aggregate distribution over actions, each agent is best-responding, while the distribution itself is consistent with the individual strategies of agents.

Definition 1. Fix $v \in V$. A *v-equilibrium* is a collection $\{\sigma, \lambda\}$ such that equations 4 and 5 are satisfied.

Since externalities in our model are purely negative, the induced game is one of strategic substitutes and consequently admits a unique equilibrium.

Lemma 1. *For all $v \in V$, there exists a unique v-equilibrium.*

Given a policy v, define welfare $\mathcal{W}$ to be the integral over all agents' utility:

$$\mathcal{W} = \int_0^1 \int_0^1 u(x, y) f(x, y) dx dy, \tag{6}$$

where we leave the dependence of welfare on vaccination policy implicit.

2.1 Model Discussion

The separation of an agent's unconditional risk type into their likelihood of interaction, x, and their likelihood of death conditional on infection, y, is crucial to better understand their distinct roles. The marginal distribution $F(\cdot, y)$ models a network of interactions in a tractable manner, in the spirit of random graphs but taken in the mean-field limit.[9] The assumption that F is smooth is

[9]Specifically, $F(\cdot, y)$ resembles the *degree distribution* of a random graph, but is technically different insofar as matching is modelled as random and bilateral in our setting.

made for simplicity, and is not necessary for the qualitative insights we provide. The model could easily be generalized to allow that the contact rate within the interaction pool be $\Theta(\mu)$, where Θ is a continuous, strictly increasing function. The idea that viral contraction occurs simply through pairwise interaction is a reduced form for an un-modelled presence of contagious agents already present within the interaction pool. It is also designed to resemble standard SIR models, in an attempt to bridge the two approaches. Equation 1 captures cleanly the fundamental notion that an agent's overall risk of death is the product of their rate of exposure, their conditional likelihood of death, and aggregate behavior.

Regarding self-isolation, the idea that an agent pays the cost to avoid interaction only after receiving an interaction opportunity gives rise to y-ordered preferences over self-isolation, as x linearly scales both the benefit of avoiding interaction and the cost. This gives rise to the natural prediction that more vulnerable people are more likely to self-isolate, regardless of their interaction rates. It also implies that the planner prefers to vaccinate high x individuals because any strategy they play while unvaccinated is more costly. We can alternatively model self-isolation by only allowing individuals to pay the interaction avoidance, c, prior to receiving any interaction opportunities. In this case, the cost of self-isolation does not scale with x and the planner is more inclined to vaccinate individuals with high xy rather than just high x.

3 Optimal Policy

3.1 Policy Examples

Before proceeding to solve for second- and first-best policies, it is instructive to define some natural policies, some of which form the backbone of real-world policies. In Section 5, we compare their performance to optimal policies. Denote by V the set of all policies.

Definition 2. A policy $v \in V$ is *monotone on* $S \subset [0,1]^2$ if $v(x,y) = 1$ and $(x,y) \in S$ implies that $v(x',y') = 1$ for all $(x',y') \in S$ such that $x' \geqslant x$ and $y' \geqslant y$. A policy $v \in V$ is *monotone* if $v(x,y) = 1$ implies that $v(x',y') = 1$ for all (x',y') such that $x' \geqslant x$ and $y' \geqslant y$.

We now describe some leading examples of monotone policies.

Definition 3. A monotone policy v is a *y-policy* if $v(x, y) = 1$ for some x implies that $v(x', y') = 1$ for all $y' \geqslant y$, $x' \in [0, 1]$. A monotone policy v is an *x-policy* if $v(x, y) = 1$ for some y implies that $v(x', y') = 1$ for all $x' \geqslant x$, $y' \in [0, 1]$.

These policies give priority to a particular type dimension. Arguably the most common policy sought in practice is a y-policy, which would allocate the vaccine only to the most vulnerable, while x-policies aim more toward achieving *herd immunity*.

Definition 4. A monotone policy v is *risk ranking* or an *xy-policy* if $v(x, y) = 1$ implies $v(x', y') = 1$ for all (x', y') such that $x'y' \geqslant xy$.

Risk ranking policies are characterized by an *iso-risk* threshold such that agents are vaccinated if and only if their risk type, xy, is greater than the threshold.

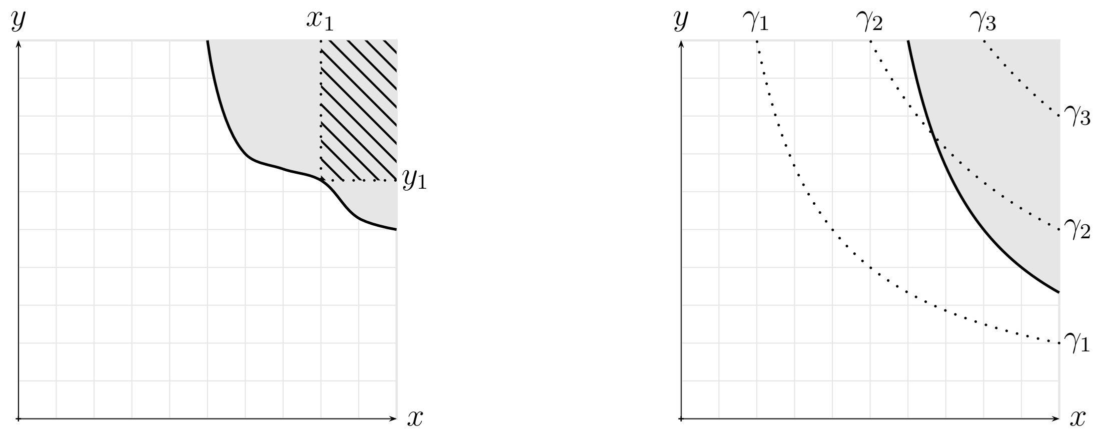
Definition 5. A policy $v \in V$ is an *x-threshold policy* if there exists an *x-threshold function* $h : [0, 1] \to [0, 1]$ such that $v(x, y) = 1$ if and only if $x \geqslant h(y)$. A policy $v \in V$ is a *y-threshold policy* if there exists a *y-threshold function* $h : [0, 1] \to [0, 1]$ such that $v(x, y) = 1$ if and only if $y \geqslant h(x)$.

Note that every monotone policy is both an x- and y-threshold policy with a decreasing threshold function. For example, an xy-policy is characterized by the x-threshold function $h(y) = A/y$; risk, xy, is constant along the boundary. In the rest of the paper, we often refer to x- or y-threshold functions as just threshold functions when the difference is clear given the context.

Definition 6. A policy $v \in V$ that is monotone on S exhibits an *exposure premium on S* if, on S, it is characterized by a y-threshold function $h(\cdot)$ such that $xh(x)$ is strictly decreasing in x. A monotone policy $v \in V$ exhibits an *exposure premium* if it has a y-threshold function $h(\cdot)$ such that $xh(x)$ is strictly decreasing in x.[10]

[10] An equivalent definition exists in terms of x-threshold functions.

Figure 1: Policy Examples



Left panel: monotone policy. Black line: threshold function. Dashed area: $\{(x, y)|x \geqslant x_1, y \geqslant y_1\}$. Grey shaded area: $v(x, y) = 1$. Right panel: exposure premium policy. Black line: x- or y-threshold function. Dotted lines: xy-policies with iso-risk thresholds $\gamma_1 = 0.2, \gamma_2 = 0.5, \gamma_3 = 0.8$. Grey shaded area: $v(x, y) = 1$.

In words, if a monotone policy exhibits an exposure premium, then individuals with higher x are vaccinated with lower levels of risk overall, xy. Thus, the policy places a premium on an individual's exposure, x, relative to their vulnerability, y.

To illustrate the definition of exposure premium, consider the vaccination boundary under a monotone policy traced out by a differentiable y-threshold function $h(\cdot)$. Along the boundary, the marginal rate of substitution of x for y (MRS) is $-h'(x)$. On the other hand, the MRS along an iso-risk curve is y/x. It is readily checked that the exposure premium property is equivalent in this case to the MRS being greater along the policy boundary than along an iso-risk curve.[11] Thus, a policy exhibits an exposure premium if it is more willing to substitute x for y than simply ranking by risk would imply. We might expect this to hold when a policy places a premium on vaccinating individuals that tend to infect others.

[11] To wit, take $(x_0, h(x_0))$ on the boundary. Then the MRS of the iso-risk curve that intersects h at x_0 is $h(x_0)/x_0$. The exposure premium property thus boils down to checking that $h(x_0) + x_0 h'(x_0) < 0$.

3.2 Second-Best Policy

We now turn to the main question of the paper: what is the optimal vaccine policy taking incentives into account. That is, what is the *second-best policy*? Given the equilibrium strategies uncovered in equation 5, the second-best problem reduces to choosing $y^* \in [0,1]$, $\lambda \in [0,1]$, and $v : [0,1]^2 \to [0,1]$ to maximize the Lagrangian

$$\begin{aligned}\mathcal{L} =& \mathcal{W} + \gamma_1 \left(\beta - \int_0^1 \int_0^1 v(x,y) f(x,y) dx dy \right) + \gamma_2 \left(y^* - \min \left\{ \frac{c}{\lambda b}, 1 \right\} \right) \\ &+ \gamma_3 \left(\lambda - \alpha \int_0^{y^*} \int_0^1 (1 - v(x,y)) x f(x,y) dx dy \right), \end{aligned} \tag{7}$$

where $\mathcal{W}$ is given by equation (6) and the constraints are the vaccine supply constraint, the incentive constraint derived from equation (5), and the transmission rate constraint derived from equation (4).

We begin by showing that the planner's optimal vaccination policy is an x-threshold policy with a threshold function that is decreasing in y below y^* and constant above y^*.

Proposition 1. *The planner's problem is equivalent to one in which they choose a decreasing continuous function* $g : [0,1] \to [0,1]$ *and real numbers* $g^* \in [0,1]$, $y^* \in [0,1]$, *and* $\lambda \in [0,1]$ *to maximize the Lagrangian*

$$\begin{aligned}\mathcal{L} =& \mathcal{W} + \gamma_1 \left(\beta - \int_0^{y^*} \int_{g(y)}^1 f(x,y) dx dy - \int_{y^*}^1 \int_{g^*}^1 f(x,y) dx dy \right) + \gamma_2 \left(y^* - min \left\{ \frac{c}{\lambda b}, 1 \right\} \right) \\ &+ \gamma_3 \left(\lambda - \alpha \int_0^{y^*} \int_0^{g(y)} x f(x,y) dx dy \right), \end{aligned}$$

where

$$\mathcal{W} = -\lambda b \int_0^{y^*} \int_0^{g(y)} y x f(x,y) dx dy - c \int_{y^*}^1 \int_0^{g^*} x f(x,y) dx dy.$$

To understand this result, note that the interaction threshold, y^*, is the individually optimal threshold for voluntary self-isolation. As such, the second-best policy discriminates between agents depending on their incentives to self-isolate. Above y^*, individuals self-isolate and the value of vaccination is in avoiding the cost associated with self-isolation. This cost scales with x, but is invariant to y. Thus, the optimal policy is an x-policy. Below y^*, individuals interact and face the risk of infection, so the cost of leaving them unvaccinated scales with y. Moreover, the private cost of infection and the public spillover cost through λ both scale with x. Thus, the optimal policy is a monotone policy with a decreasing x-threshold function $g(\cdot)$; a higher y implies a higher benefit of vaccination, so a lower x is allowed.

Note that the constraints $\lambda \in [0,1]$ and $y^* \in [0,1]$ do not bind as long as the constraints multiplying the Lagrangian multipliers γ_2 and γ_3 hold. To understand the planner's problem in more detail, it is useful to consider the first derivatives of the Lagrangian.

First, for any $y \leq y^*$,

$$\frac{\partial \mathcal{L}}{\partial g(y)} = \left[-\lambda b y g(y) + \gamma_1 - \gamma_3 \alpha g(y)\right] f\big(g(y), y\big). \tag{8}$$

The marginal benefit of increasing $g(y)$ is the value of relaxing the vaccine supply constraint, γ_1. The marginal cost is the sum of two components. An individual with type $(g(y), y)$ – who interacts even when unvaccinated since $y \leq y^*$ – dies with probability $\lambda y g(y)$, which generates utility loss b. Second, the measure of unvaccinated interacting individuals increases by $g(y)$, which tightens the equilibrium transmission rate constraint by $\alpha g(y)$ at cost γ_3.

Next, for the threshold g^* that prevails for $y > y^*$, we have

$$\frac{\partial \mathcal{L}}{\partial g^*} = (\gamma_1 - c g^*) \int_{y^*}^{1} f(g^*, y) dy. \tag{9}$$

As before, the marginal benefit of increasing the threshold g^* is the value of relaxing the vaccine supply constraint, γ_1. In this case, the marginal cost is the increase in interaction avoidance costs, cg^*, since individuals with type $y > y^*$ prefer to self-isolate when unvaccinated.

For the transmission rate λ, we have

$$\frac{\partial \mathcal{L}}{\partial \lambda} = \begin{cases} -b\int_0^{y^*}\int_0^{g(y)} yxf(x,y)dxdy + \gamma_2 \frac{c}{\lambda^2 b} + \gamma_3 & \lambda > c/b \\ -b\int_0^{y^*}\int_0^{g(y)} yxf(x,y)dxdy + \gamma_3 & \lambda < c/b \end{cases} \tag{10}$$

The marginal benefit of increasing λ is the value of relaxing the equilibrium transmission rate constraint, γ_3, as well as the value of relaxing the incentive compatibility constraint, γ_2, multiplied by the effect of a change in λ on the individually optimal threshold y^*. The marginal cost is an increase in the transmission and therefore death rate for interacting individuals. When $\lambda < c/b$, the individually optimal threshold y^* is equal to 1, so small changes in λ have no effect on y^*.

Finally, for the interaction threshold y^*, if $y^* < 1$, we have

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial y^*} =& -\lambda b y^* \int_0^{g(y^*)} xf(x,y^*)dx + c\int_0^{g^*} xf(x,y^*)dx \\ & -\gamma_1\left(\int_{g(y^*)}^1 f(x,y^*)dx - \int_{g^*}^1 f(x,y^*)dx\right) + \gamma_2 - \gamma_3\alpha\int_0^{g(y^*)} xf(x,y^*)dx \\ =& c\int_{g(y^*)}^{g^*} xf(x,y^*)dx - \gamma_1\int_{g(y^*)}^{g^*} f(x,y^*)dx + \gamma_2 - \gamma_3\alpha\int_0^{g(y^*)} xf(x,y^*)dx,\end{aligned} \tag{11}$$

where the second equality uses the definition of y^* in equation (5) to plug in for $\lambda b y^*$. Consider the four terms in the final line. The first two terms result from the potential discontinuity in the vaccination policy at y^*, i.e. $g(y^*) \neq g^*$. Individuals with $y = y^*$ and x between $g(y^*)$ and g^* go from being unvaccinated and not interacting – with value $-cx$ to the planner – to vaccinated and interacting – with value $-\gamma_1$ to the planner – or vice versa, depending on which threshold is higher. The final two terms are the marginal benefit of relaxing the incentive compatibility constraint and tightening the equilibrium transmission rate constraint because more individuals are below the interaction threshold and therefore join the interaction pool.

We now complete the characterization of the second-best policy in Proposition 2, which combined with Proposition 1 constitutes our main result.

Proposition 2. *Define*

$$y_0 \triangleq \frac{\gamma_1 - \gamma_3\alpha}{\lambda b}. \tag{12}$$

Then, $y_0 < y*$ and for all $y \leqslant y_0$, $g(y) = 1$, i.e. no individuals with type $y \leqslant y_0$ are vaccinated. For all $y \in (y_0, y^*]$,

$$g(y) = \frac{\gamma_1}{\lambda by + \gamma_3\alpha}, \tag{13}$$

which is strictly positive, strictly decreasing, and approaches 1 as y approaches the lower limit y_0. The optimal vaccination policy is monotone and exhibits an exposure premium on $[0, y^*]$ and on $(y^*, 1]$.

If $y^* < 1$, then the vaccination threshold for $y > y^*$ is

$$g^* = min\left\{\frac{\gamma_1}{c}, 1\right\}.$$

Moreover, g^* is strictly greater than $g(y^*)$, the vaccination threshold at y^*.

The optimal policy displays three striking features. First, if $y^* < 1$, it is *non-monotone*. Specifically, we show that $g(y^*) < g^*$, which implies that the policy tends to vaccinate people with an intermediate y (just below y^*) rather than a higher or lower y. Intuitively, while high y agents efficiently self-isolate and low y agents efficiently do not, agents with intermediate y interact when they should self-isolate. They are thus particularly costly to society when unvaccinated and it is valuable to vaccinate them.

Second, the optimal policy exhibits an *exposure premium* on $\{(x, y) \mid y \leqslant y^*\}$. If the planner only took into account an individual's private value of vaccination, then the optimal policy for $y \leq y^*$ would be an xy-policy. Taking into account the spillover effect, the planner is willing to vaccinate high x individuals with relatively low risk levels, xy.

Third, as discussed previously, the policy is *invariant* to y on $\{(x, y) \mid y > y^*\}$ but does depend on x. Moreover, it exhibits an exposure premium for $y > y^*$, but for a different reason than for $y \leqslant y^*$. Below y^*, the policy exhibits an exposure premium because x matters more than y due to spillovers. Above y^*, the policy exhibits an exposure premium because self-isolating behavior eliminates the importance of changes in vulnerability, y.

Taken together, the second and third features illustrate how to optimally allocate vaccines *within* groups as defined by behavior. On the other hand, the first feature illustrates how to allocate vaccines *between* these two groups based on the incentives that affect individuals' interaction decisions. Since individuals over interact, the optimal policy allocates more vaccines to the interacting group, generating the non-monotonicity at y^*.

Figure 2 demonstrates these features graphically.

3.3 First-Best Policy

To further understand the distortions imposed by incentives, we solve the *first-best problem*, in which the planner ignores the incentive constraint for interaction decisions. This is best thought of as a policy that jointly prescribes vaccine allocation as well as *mandatory lockdown*, but rather than apply uniformly across all individuals as is often the case in practice, the rules can be variably enforced based on observable measures of risk and sociability.[12] The planner's problem is then to choose a vaccine policy $v : [0,1]^2 \to [0,1]$, an interaction policy $\sigma : [0,1]^2 \times \{0,1\} \to [0,1]$, and a transmission rate $\lambda \in [0,1]$ to maximize the Lagrangian

$$\begin{aligned}\mathcal{L} = \mathcal{W} &+ \gamma_1 \left(\beta - \int_0^1 \int_0^1 v(x,y) f(x,y) dx dy \right) \\ &+ \gamma_3 \left(\lambda - \alpha \int_0^1 \int_0^1 (1 - v(x,y)) \sigma(x,y,0) x f(x,y) dx dy \right), \qquad (14)\end{aligned}$$

where an unvaccinated individual with type (x,y) interacts with probability $\sigma(x,y,0)$, a vaccinated individual interacts with probability $\sigma(x,y,1)$, and welfare is defined as before. We have the following result that shows how the planner's problem differs from before.

Proposition 3. *The planner's problem is equivalent to one in which they choose a decreasing continuous function $g : [0,1] \to [0,1]$ and real numbers $y^* \in [0,1]$ and $\lambda \in [0,1]$ to maximize the*

[12] As we show later, the lockdown rule can be implemented with a simple, constant tax on interactions.

Lagrangian

$$\mathcal{L} = \mathcal{W} + \gamma_1 \left(\beta - \int_0^{y^*} \int_{g(y)}^1 f(x,y)dxdy - \int_{y^*}^1 \int_{g(y^*)}^1 f(x,y)dxdy \right) + \gamma_3 \left(\lambda - \alpha \int_0^{y^*} \int_0^{g(y)} xf(x,y)dxdy \right),$$

where

$$\mathcal{W} = -\lambda b \int_0^{y^*} \int_0^{g(y)} yxf(x,y)dxdy - c \int_{y^*}^1 \int_0^{g(y^*)} xf(x,y)dxdy.$$

The optimal interaction threshold is given by

$$y^* = min \left\{ \frac{c - \gamma_3 \alpha}{\lambda b}, 1 \right\}$$

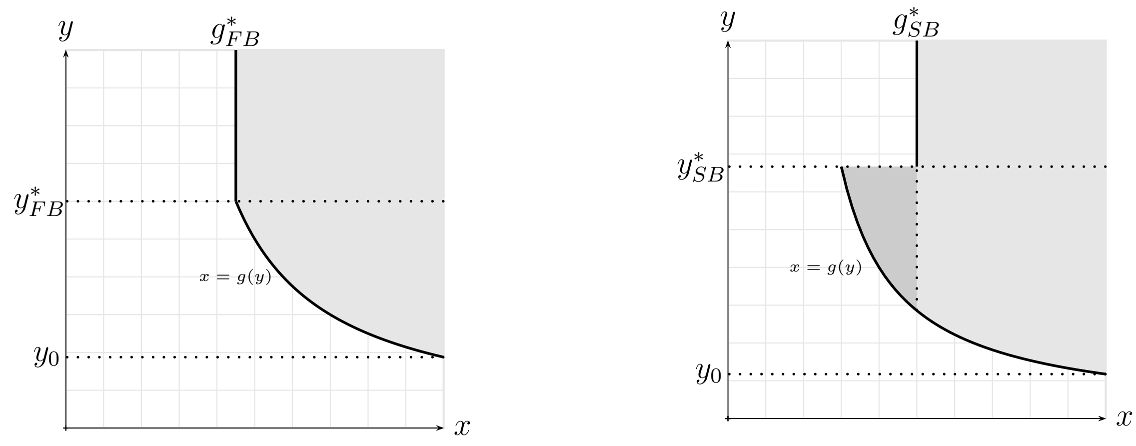
and the vaccination threshold function is

$$g(y) = min \left\{ \frac{\gamma_1}{\lambda b y + \gamma_3 \alpha}, 1 \right\},$$

which implies an optimal vaccination policy that is monotone and exhibits an exposure premium everywhere. Finally, if $y^* < 1$, then $g(y^*) = \gamma_1 / c < 1$.

The key qualitative difference between the first- and second-best vaccination policies is that the threshold function is continuous in the former case but discontinuous in the latter case at the interaction threshold, y^*. By extension, the first-best policy is monotone while the second-best policy is not. As mentioned earlier, the discontinuity at y^* in the second-best policy is driven by the fact that the threshold y^* is inefficiently high. In response, the planner reallocates vaccines toward individuals who inefficiently over-interact (the dark shaded area in Figure 2). This is no longer the case in the first-best in which y^* is chosen efficiently. On the other hand, the second and third features of the second-best policy we described earlier – constant threshold function above y^* and decreasing threshold function that implies an exposure premium below y^* – still hold; they only depend on the existence of self-isolating and interacting individuals and are not related to

Figure 2: Optimal Policies



Left panel: first-best policy. Black line $g(y) = x$. Shaded area: $v(x, y) = 1$. Right panel: second-best policy. Black line $g(y) = x$. Shaded area: $v(x, y) = 1$. Dark shaded area: additional vaccination due to incentive distortions.

the efficiency of the decision to interact.

The following result states that the first-best interaction policy can be simply implemented with a positive flat interaction tax.

Corollary 1. *Suppose in the solution to the planner's first best problem, the interaction threshold is y^* and the equilibrium transmission rate is λ. The planner can decentralize the optimal interaction policy with a flat interaction tax equal to $c - \lambda b y^*$ per interaction if $y^* < 1$ and 0 otherwise. If $y^* < 1$, then the optimal tax is strictly positive. If $y^* = 1$, then the planner's first-best problem is the same as the second-best and the optimal interaction policy is trivially decentralized.*

Weak Incentives for Self-Isolation – If the cost of interaction avoidance relative to the cost of interaction, $c/(\alpha b)$, or the fraction of the population that is vaccinated, β, is sufficiently high, then $y^* = 1$ under any policy.[13] In this case, it is as if we are in the baseline model in which all individuals always interact when given the opportunity. Second-best and first-best policies are the same since the planner sets y^* at the individually optimal level, 1, even when unconstrained. The optimal policy is monotone and exhibits an exposure premium, much as in the right panel of

[13]For example, if $c/(\alpha b) > 1$, then even an individual facing certain death upon infection would choose to interact; if β is sufficiently large then $\lambda < c/b$ for any policy that satisfies the vaccine supply constraint.

Figure 1.

4 Other Applications

4.1 Vaccine Hesitancy

In reality, people might be unwilling to be vaccinated. There are important qualitative differences between self-isolation and vaccine hesitancy. Crucially, in the former, *all individuals* have agency over their choices and thus outcomes, while in the latter only those who are offered a vaccine can exercise their will. This difference leads to qualitative differences in second-best policies, and thus has direct policy implications.

Two leading explanations for vaccine hesitancy exist. First, people might believe the efficacy of the vaccine to be limited. Second, people might believe that the vaccine entails unintended, negative side-effects. We focus on the latter.[14] To this end, we extend the model to include a fixed cost $p > 0$ from taking the vaccine.[15] For simplicity, in this subsection, we suppose all individuals interact when given the opportunity, i.e. $c = \infty$. A *strategy profile* is a mapping $\sigma : [0, 1]^2 \to \{0, 1\}$.

Simple algebra demonstrates that type (x, y) accepts the vaccine if and only if

$$\underbrace{b\lambda xy}_{\text{benefit}} \geqslant \underbrace{p}_{\text{cost}} \tag{15}$$

Equilibrium strategies are thus described by a scalar A^* such that $\sigma(x, y) = 1$ if and only if $xy \geqslant A^*$ and that solves

$$\frac{p}{bA^*}\begin{cases} \geqslant \lambda & A^* = 1 \\ = \lambda & A^* \in (0,1) \\ \leqslant \lambda & A^* = 0 \end{cases}$$

[14] The former can easily be accommodated in the baseline analysis.

[15] The cost p can be micro-founded as follows: suppose there is a hidden, aggregate state $\theta \in \{0, \bar{\theta}\}$ such that if an agent accepts vaccination, their terminal payoff is $-\theta$, and agents share a common belief $p = \mathbb{E}(\theta)$.

the indifference condition for the marginal individual. If the indifference condition holds with equality, then from the definition of λ, we have that

$$\frac{p}{bA^*} = \alpha \left[\int_{xy<A^*} xf(x,y)dxdy + \int_{xy\geqslant A^*} x\big(1-v(x,y)\big)f(x,y)dxdy \right]. \tag{16}$$

Lemma 2. *For all $v \in V$, a unique v-equilibrium σ^* exists, featuring vaccination acceptance rules defined by an iso-risk threshold A^* such that a type (x,y) individual refuses vaccination if and only if $xy < A^*$. Furthermore, $A^* > 0$, i.e. a positive measure of agents would refuse vaccination under σ^*.*

One immediate prediction is that individuals with high x but very low y and vice versa are unlikely to accept vaccination. This is in line with recent surveys that find that the very old and the very young are more likely than others to refuse vaccination. That said, some surveys report that hesitancy is simply decreasing in y. The analysis could easily be adjusted to accommodate this empirical regularity by assuming that agents are offered the vaccine only after entering (or not) the interaction pool.[16]

The planner's problem is then to choose $A^* \in [0,1]$, $\lambda \in [0,1]$, and a vaccination policy $v : [0,1]^2 \to [0,1]$ to maximize the Lagrangian

$$\begin{aligned}\mathcal{L} = \mathcal{W} &+ \gamma_1 \left(\beta - \int_0^1 \int_0^1 v(x,y)f(x,y)dxdy \right) + \gamma_2 \left(A^* - \min\left\{ \frac{p}{\lambda b}, 1 \right\} \right) \\ &+ \gamma_3 \left(\lambda - \alpha \left[\int_{(x,y):xy\geqslant A^*} x(1-v(x,y))f(x,y)dxdy + \int_{(x,y):xy<A^*} xf(x,y)dxdy \right] \right), \end{aligned} \tag{17}$$

where welfare is

$$\mathcal{W} = -\int_{(x,y):xy\geqslant A^*} \left((1-v(x,y))\lambda byx + v(x,y)p\right) f(x,y)dxdy - \int_{(x,y):xy<A^*} \lambda byxf(x,y)dxdy.$$

Proposition 4. *The second-best policy is an x-threshold policy characterized by an x-threshold*

[16] Specifically, the optimal policy would consist of a binding $y = A^*$ curve, rather than an $xy = A^*$ curve, with the remaining construction of $g(y)$ virtually unchanged.

function $g : [0,1] \to [0,1]$*, i.e.*

$$v(x,y)\sigma(x,y) = \begin{cases} 0 & x < g(y) \\ 1 & x \geqslant g(y) \end{cases}$$

Define

$$\hat{y} \triangleq \max\left\{0, \frac{\gamma_1 - \gamma_3\alpha + p}{b\lambda}\right\} \tag{18}$$

and

$$\bar{y} \triangleq \frac{\gamma_3 \alpha A^*}{\gamma_1}. \tag{19}$$

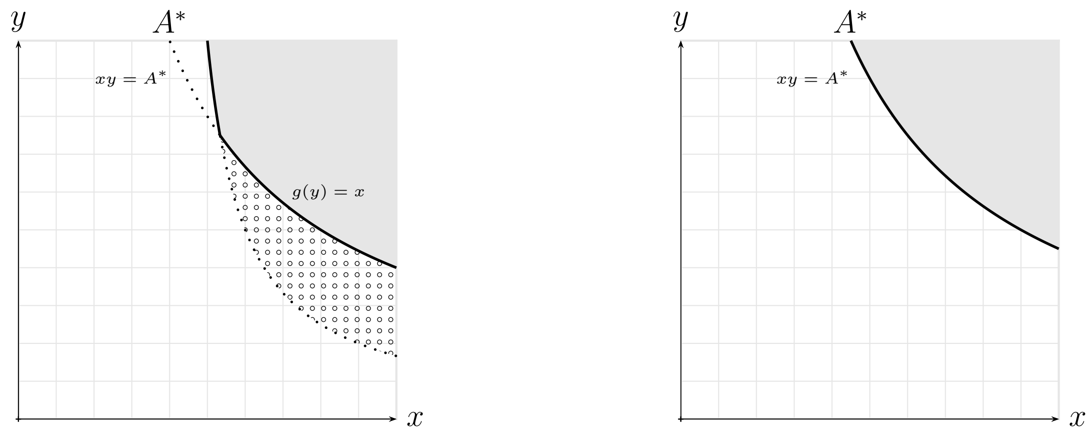
The threshold function g *is given by*

$$g(y) = \begin{cases} 0 & y < \hat{y} \\ A^*/y & y \in [\hat{y}, \bar{y}) \\ \frac{\gamma_1 + p}{\lambda b y + \gamma_3 \alpha} & y \geqslant max\{\hat{y}, \bar{y}\} \end{cases}$$

The optimal policy is monotone, is an xy-policy with iso-risk threshold A^* *for all* $y \in [\hat{y}, \bar{y})$*, and exhibits an exposure premium for all* $y \geqslant max\{\hat{y}, \bar{y}\}$*.*

The proposition shows that for $y \geqslant \bar{y}$, vaccine hesitancy is irrelevant because all those offered a vaccine strictly prefer to accept. Yet, for $y \in [\hat{y}, \bar{y})$, vaccine hesitancy is a binding constraint and the vaccine is simply offered to all who will take it. This is because the optimal policy exhibits an exposure premium, while vaccine hesitancy depends only on the unconditional risk level, xy. The marginal individual vaccinated with a high y has a relatively low x, which implies a low spillover and therefore a relatively high risk level, xy. The opposite is true for the marginal individual vaccinated with a low y. Figure 3 demonstrates the policy graphically, showing explicitly how inefficient vaccine rejection occurs by those with high x and low y.

Figure 3: Second-best policies under vaccine hesistancy and one-way efficacy



Left panel: second-best under vaccine hesitancy. Black line: optimal policy threshold. Dotted area: inefficient vaccine rejection. Grey shaded area: $v(x, y) = 1$. Right panel: second-best under one-way efficacy, xy-policy. Grey shaded area: $v(x, y) = 1$.

4.2 Silent-spreading

Thus far, we have assumed that a vaccinated agent cannot contract the disease in the first place, and hence can neither experience severe symptoms nor transmit the illness to others, i.e. that the vaccine is *two-way*. As of the writing of this paper, it is not clear whether this assumption will hold in the case of COVID-19 vaccines. Of the available vaccines, only the Oxford-AstraZeneca vaccine ran bi-weekly PCR tests that indicate the assumption is valid.[17] Furthermore, some experts remain uncertain that the vaccines developed are two-way.[18] We may analyse a *one-way* vaccine by supposing that vaccination sends an individual's y to 0, but leaves the possibility of transmission unaffected. That is, a vaccinated individual is protected, but interaction with them entails unmitigated risk for unvaccinated agents. Again assuming full interaction ($c = \infty$) for simplicity, the optimal vaccine policy in such a case would be an xy-policy (that does *not* exhibit an exposure premium), as the value of vaccination is purely determined by an agent's risk, xy. Taking this and the main result together, it is clearly imperative that the two-way feature of a

[17]See https://tinyurl.com/y8n2ayrh for a detailed discussion.

[18]Professor Robert Read states: "*The reason [not to vaccinate young people] is that a) they don't get such a severe disease and b) we haven't been able to demonstrate yet that the vaccines have any impact at all on transmission.*" See https://tinyurl.com/y5d7ucr6.

vaccine is carefully verified.

5 Practical Guidelines

Our analysis offers clear, practical guidelines to government health agencies tasked with developing vaccine prioritization schemes. How are countries approaching this problem in reality? There is wide-spread consensus that front-line health workers – who arguably rank highly on both fronts – should be among the very first to be vaccinated.[19] Beyond this consensus, prioritization approaches tend to differ, sometimes dramatically, between countries. For instance, most Western countries such as the US, UK, Canada and Europe are vaccinating based on y, whereas countries such as Indonesia and Russia are vaccinating the youth. In the case of the former, Amin Soebandrio, director at the Eijkman Institute for Molecular Biology in Jakarta, states:[20]

> "*Our aim is herd immunity...with the most active and exposed group of population – those 18 to 59 – vaccinated, they form a fortress to protect the other groups. It's less effective when we use our limited number of vaccines on the elderly when they're less exposed.*"

These comments raise another interesting point of contention, namely that the optimal vaccination policy should be carefully calibrated to the limitations on supply, and that with very limited supply, priority should be re-balanced toward high x agents, rather than high y. In contrast, European Union guidelines suggest the opposite: when supply is limited, priority should be given to high y people.[21]

With these observations in hand, we perform a series of numerical calculations, not only to uncover deeper properties of the second-best policy not amenable to analytic derivation, but also to compare it to commonly considered heuristics as alluded to above. These include: 1) y-policies, which vaccinate only the most vulnerable; 2) x-policies, which vaccinate only the most interactive;

[19]Healthcare workers also ranked highly for other reasons. Most obviously, they reduce overall mortality rates. Beyond this, they are often given priority as a way of acknowledging their sacrifice and social service prior to the vaccine existing. See Pathak et al. (2020) for a discussion.

[20]See `https://tinyurl.com/y8vvpfkm` and `https://tinyurl.com/yybj498g`.

[21]See `https://tinyurl.com/y8n2ayrh`.

3) xy-policies, which vaccinate those who face the most risk overall; and 4) random policies under which vaccinations are uncorrelated with observable characteristics. Throughout these exercises, we assume x and y are drawn independently from standard uniform distributions, unless otherwise stated.[22]

Before proceeding, we show that other than the choice of distribution F, there are essentially only two parameters in the model; the one we discuss shortly and β. As such, since we vary both in our numerical exercises, they show the full range of model outcomes across the entire parameter space taking as given our choice for F.

Particularly for the final set of figures, we define a key invariant of the model – the *survival value index (SVI)* as $\alpha b/c$ – which is essentially the value of life, normalized by the cost of remaining infection-free, i.e. self-isolating. Crucially, welfare comparisons depend only upon this invariant, rather than on each of its deep parameters separately.[23]

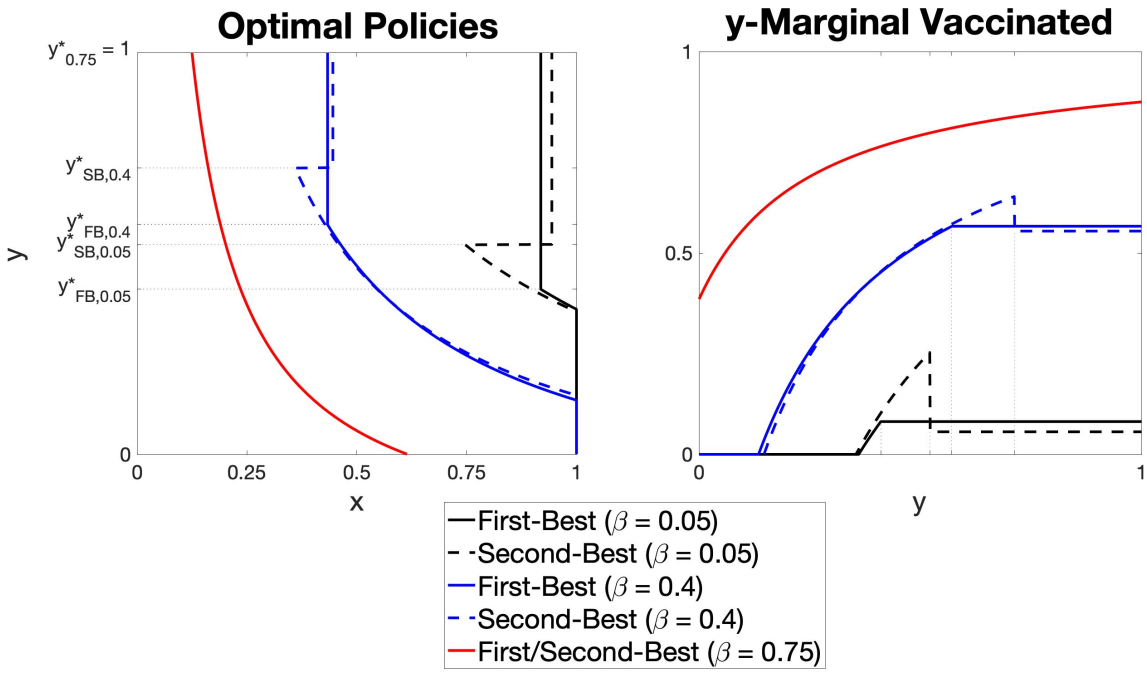
Lemma 3. *The optimal vaccination policy and equilibrium welfare normalized by c do not depend on α, b, or c independently, but only on the survival value index (SVI), $\alpha b/c$.*

We begin with a series of graphs comparing the first- and second-best policies. Figure 4 shows the basic features of the first- and second-best policies. Note in the panel entitled "Optimal Policies" that the first-best policies demonstrate a greater exposure premium than the second-best; spillovers are lessened in the second-best problem because behavior, y^*, responds to changes in the environment. The "y-Marginal Vaccinated" panel plots the y-marginal conditional on vaccination, showing clearly the non-monotonicity of the second-best policy – intermediate y agents constitute the greatest proportion of vaccinated agents, whereas the first-best is monotone. Figure 5 compares the performance of the first- and second-best policies. The panel entitled "Incentive Distortion" measures the performance of the second-best – without mandatory lockdown policy – relative to the first-best. In the "Policy Distortion" panel, we measure the impact that ignoring behavior

[22] As such, we avoid drawing inferences regarding magnitudes, focusing on qualitative insights.

[23] Note that changes in α, b, or c, holding $\alpha b/c$ fixed, can have effects on observable variables. For example, a higher b but lower α implies a higher utility loss from death, but a lower risk of death contingent on interaction. The planner and individuals do not care about the difference, but the lower α case has a lower observed death rate.

Figure 4: Optimal Policies: Basics

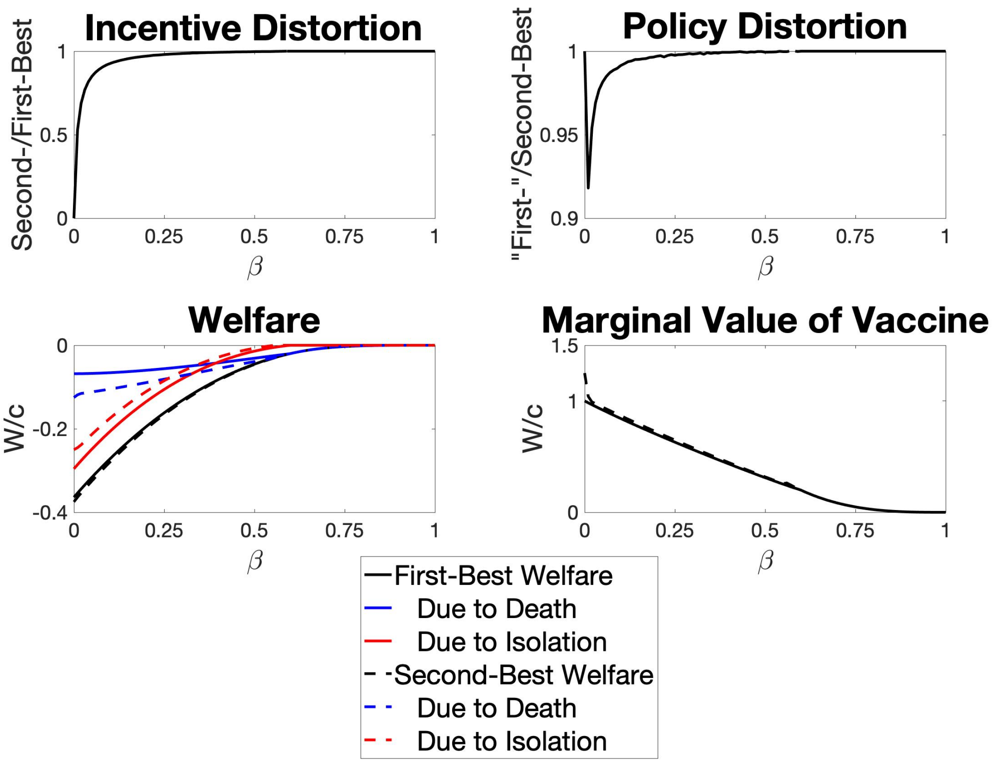


Left panel: x-threshold functions under the first- and second-best policies at various levels of the percentage of the population that can be vaccinated, β. Individuals to the right of the appropriate curve are vaccinated. Right panel: the marginal distributions of y conditional on being vaccinated under the first- and second-best policies at various levels of β. Both panels reflect that $\beta = 0.75$ is sufficiently large so that the first- and second-best policies are the same.

has on welfare by studying a novel policy wherein the planner solves the second-best problem, but assuming that agents internalize externalities and choose y^* efficiently. Thus, the planner sets policy erroneously believing that agents self-isolate according to the optimal mandatory lockdown (first-best) rule. These first two panels show that the distinction between the first- and second-best (and policymaker awareness of which holds) is particularly important when vaccine supply is low. The panel entitled "Welfare" decomposes welfare losses into losses from deaths and losses from self-isolation.[24] First-best losses are driven primarily by isolation rather than death, reflecting that private incentives to self-isolate are inefficiently weak. Finally, the "Marginal Value of Vaccine" panel demonstrates both that there are diminishing returns to vaccination, and that the marginal value is greater under the second-best. This implies that vaccine supply should be geographically

[24] Throughout this section, welfare is normalized by c so that it only depends on $\alpha b/c$, not on each parameter separately.

Figure 5: Optimal Policies: Analysis

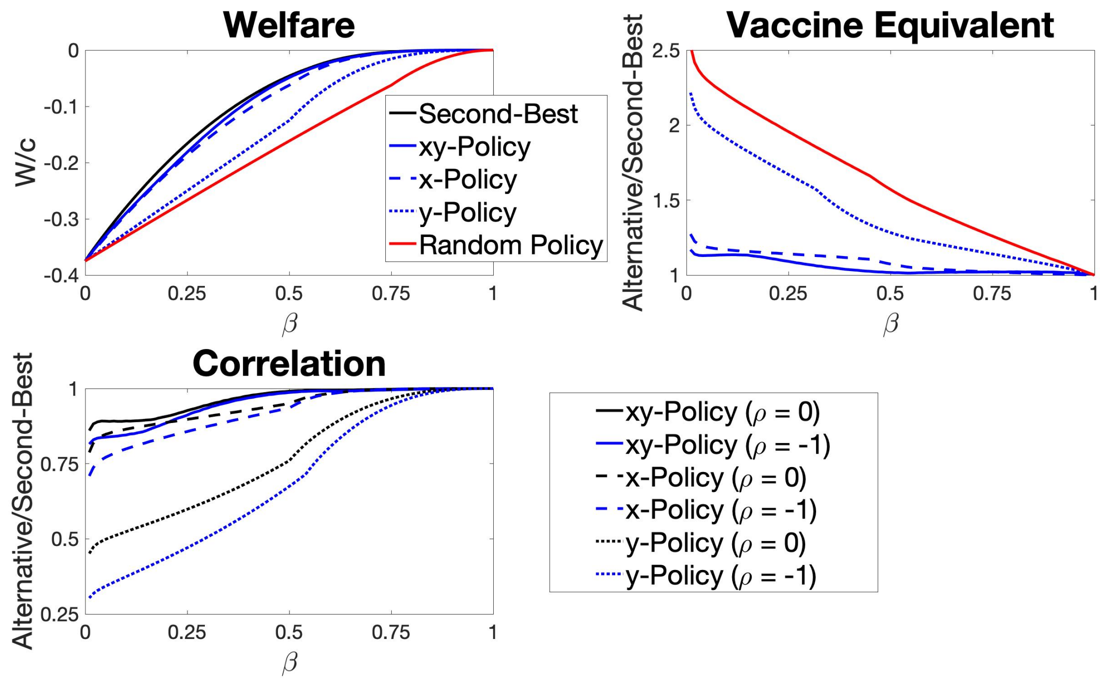


Top-left panel: the improvement in welfare (compared to no vaccines) under the second-best relative to under the first-best at various levels of the percentage of the population that can be vaccinated, β. Top-right panel: the improvement in welfare (compared to no vaccines) under the misperceived first-best relative to under the second-best at various levels of β. The misperceived first-best is derived by optimizing assuming individuals will self-isolate efficiently as in the first-best even though they self-isolate selfishly as in the second-best. Bottom-left panel: welfare (normalized by c) and its components under the first- and second-best policies at various levels of β. Bottom-right panel: the marginal normalized welfare value of a vaccine under the first- and second-best at various levels of β.

dispersed, not concentrated, and should favor regions that cannot implement mandatory lockdown policies.

We turn next to heuristic policies, wherein we compare the second-best against x-, y-, xy-, and random policies. Figure 6 compares welfare across these policies by looking at: 1) absolute welfare in "Welfare"; 2) a novel performance measure that shows the relative quantity of vaccines required so that each heuristic policy performs as well as the second-best in "Vaccine Equivalent"; and 3) in "Correlation", the welfare improvement (compared to no vaccines) under heuristic policies relative to under the second-best for the baseline type distribution and an alternative distribution with

Figure 6: Heuristic Policies: Welfare

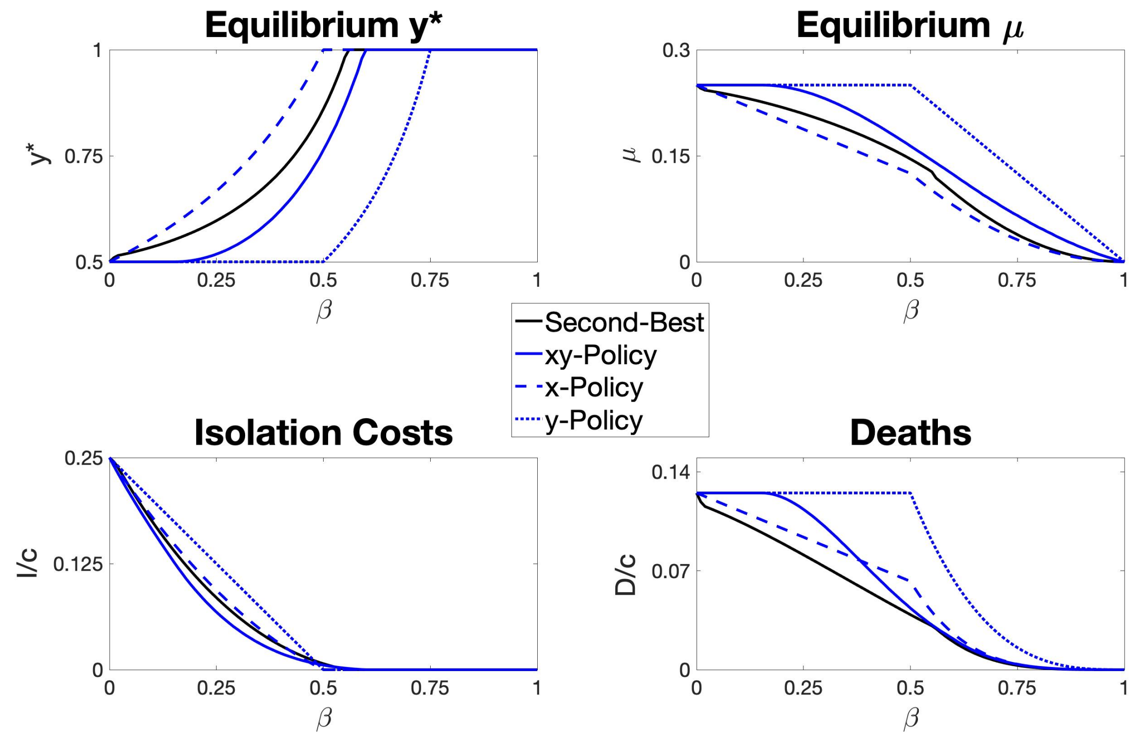


Top-left panel: welfare (normalized by c) under the second-best and heuristic policies. Top-right panel: the multiple of vaccines required so that each heuristic policy delivers the same welfare as the second-best. For example, a value of 1.5 means a 50% increase in vaccines is required. Bottom-left panel: the improvement in welfare (compared to no vaccines) under each heuristic policy relative to the improvement under the second-best for the baseline distribution with independent uniforms ($\rho = 0$) and an alternative distribution with uniform marginals but a negative correlation between x and y ($\rho = -1$).

uniform marginals but a negative correlation between x and y.[25] The most striking observation is that y-policies perform significantly worse than other heuristics, and that this performance loss is greatest when supply is limited. This is of significant practical relevance, as the majority of countries not only adopt such policies, but do so at the start of vaccine roll-out when supply is most constrained. To better understand this result, consider Figure 7, which shows equilibrium interaction thresholds, y^*, interaction pool sizes normalized by α, μ, and the components of welfare losses under the second-best and heuristic policies. A policy reduces isolation costs through direct and indirect channels: allowing those vaccinated to interact freely and encouraging interaction by reducing the size of the *unvaccinated* interaction pool. y-policies work through the direct channel,

[25]Specifically, we impose the following functional form for the pdf of the distribution: $f(x, y) = 1+\rho(1-2x)(1-2y)$, where $\rho \in [-1, 1]$. A little algebra shows that the marginal distributions of x and y are uniform, that the conditional distributions have affine pdfs, and that the correlation between x and y is $\rho/3$.

Figure 7: Heuristic Policies: Details

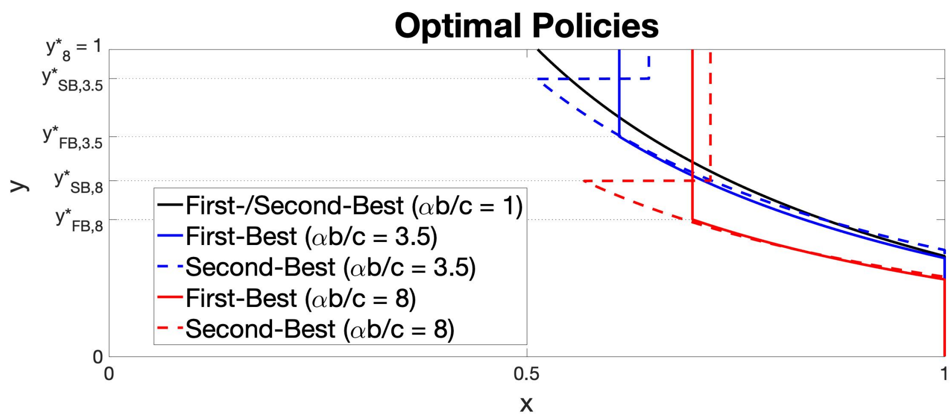


Top panels: the equilibrium interaction threshold, y^*, and unvaccinated interaction pool size, μ, under the second-best and heuristic policies at various levels of the percentage of the population that can be vaccinated, β. Bottom panels: the components of welfare losses (normalized by c), isolation costs or death, under the second-best and heuristic policies at various levels of β. Here, an improvement in welfare is a fall in isolation costs or deaths.

but shut the indirect channel down; targeting the self-isolating leaves the interaction threshold and unvaccinated interaction pool unchanged. Crucially, when vaccine supplies, β, are low and many are self-isolating, the indirect channel is of first-order importance. Policies that target high x individuals thus perform far better, even at reducing isolation costs among the most vulnerable. The "Correlation" panel shows that this issue is exacerbated when x and y are negatively correlated because the y-policy ends up targeting low x individuals who are not particularly valuable to vaccinate. Finally, the novel measure in "Vaccine Equivalent" captures the trade-off between implementing optimal policy – which might require costly or delayed implementation – or using well-established heuristics that are easier to roll-out.

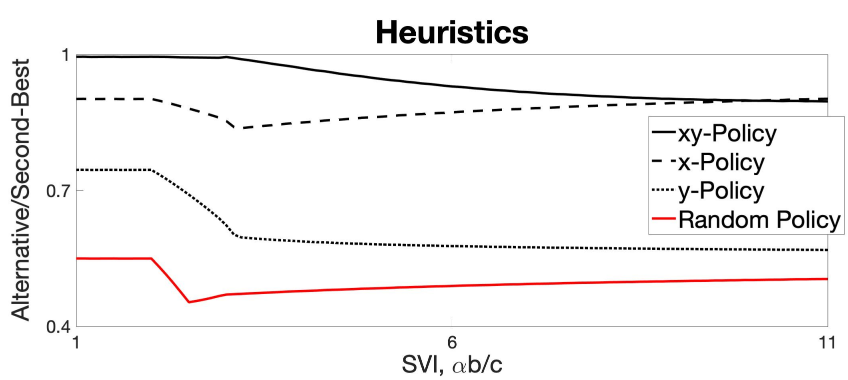
In each of the last three figures, we vary the SVI and show how key outcomes and measure change. In Figure 8, we show the x-threshold functions under first- and second-best policies. In

Figure 8: Infectiousness: Basics



x-threshold functions under the first- and second-best policies at various levels of $\alpha b/c$ (SVI). Individuals to the right of the appropriate curve are vaccinated. An SVI of 8 is sufficiently large so that the first- and second-best policies are the same.

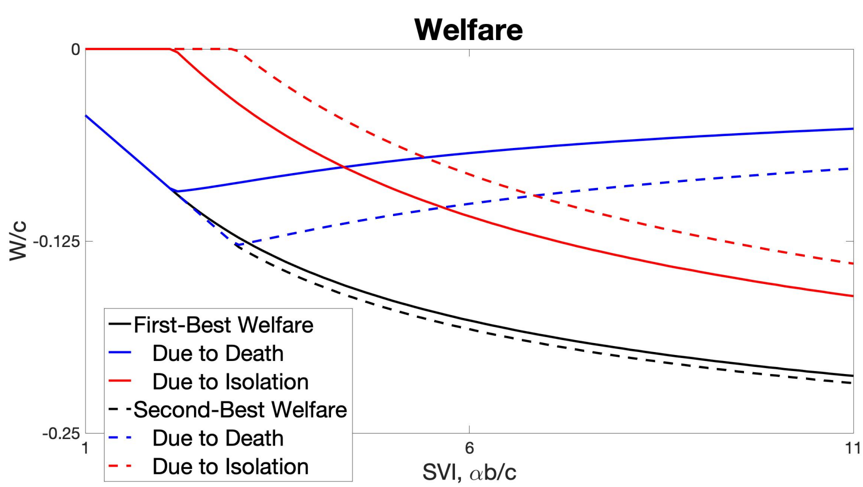
Figure 9: Infectiousness: Heuristics



The improvement in welfare (compared to no vaccines) under each heuristic policy relative to the improvement under the second-best at various levels of $\alpha b/c$ (SVI).

Figure 9, we show the improvement in welfare (compared to no vaccines) under heuristic policies relative to under the second-best. In Figure 10, we show normalized welfare and its components under the first- and second-best policies. It is perhaps easiest to view an increase in the SVI as being driven by α, holding b and c fixed. It then corresponds to an increase in the infectiousness of the virus, holding individual-specific fixed risk, vaccine supply, the value of life, and the cost of self-isolation. As such, these exercises speak directly to current concerns regarding new variants of COVID-19 that exhibit these features. An important takeaway is that as infectiousness, α, increases, policy must take into account the response of behavior, y^*. More individuals find interaction too costly and instead self-isolate. It is valuable to vaccinate such individuals, but recall that the differences in y among them do not matter since they no longer face any risk of infection. As such, the optimal policy spreads vaccines across a large range of y, targeting only those with the highest x. We can see this result in Figure 8 as well as in Figure 9, which shows that y-policies perform particularly poorly as infectiousness rises. Finally, in Figure 10, we can see that individual behavior and optimal policy respond so aggressively to the increase in infectiousness that the welfare loss from death actually falls while interaction avoidance costs rise.

Figure 10: Infectiousness: Welfare



Welfare (normalized by c) and its components under first- and second-best policies at various levels of $\alpha b/c$ (SVI).

References

Babus, A., S. Das, and S. Lee, "The Optimal Allocation of Covid-19 Vaccines," *Covid Economics*, 2020, *44*.

Ballester, C., A. Calvó-Armengol, and Y. Zenou, "Who's Who in Networks. Wanted: The Key Player," *Econometrica*, 2006, *74* (5), 1403–1417.

Brotherhood, L., P. Kircher, C. Santos, and M. Tertilt, "An economic model of the Covid-19 pandemic with young and old agents: Behavior, testing and policies," *working paper*, 2020.

Chen, F. and F. Toxvaerd, "The economics of vaccination," *Journal of Theoretical Biology*, 2014, *363*, 105–117.

Epstein, J.M., J. Parker, D. Cummings, and R.A. Hammond, "Coupled contagion dynamics of fear and disease: mathematical and computational explorations," *PLoS ONE*, 2008, *3* (12).

Farboodi, M., G. Jarosch, and R. Shimer, "The Emergence of Market Structure," *working paper*, 2020.

Funk, S., M. Salathé, and V.A.A. Jansen, "Modelling the influence of human behaviour on the spread of infectious diseases: a review," *Journal of the Royal Society*, 2010, *7* (50).

Galeotti, A., B. Golub, and S. Goyal, "Targetting Interventions in Networks," *Econometrica*, 2020, *88* (6), 2445–2471.

Geoffard, P.-Y. and T. Philipson, "Rational Epidemics and Their Public Control," *International Economic Review*, 1996, *37* (3).

Greenwood, J., P. Kircher, C.Santos, and M. Tertilt, "An Equilibrium Model of the African HIV/AIDS Epidemic," *Econometrica*, 2019, *87* (4).

Gross, T., C.J.D D'Lima, and B. Blasius, "Epidemic dynamics on an adaptive network," *Phys Rev Lett*, 2006, *96* (20).

Kremer, M., “Integrating behavioral choice into epidemiological models of AIDS,” *Quarterly Journal of Economics*, 1996, *111*, 549–573.

Pastor-Satorras, R. and A. Vespignani, “Immunization of complex networks,” *Phys Rev E*, 2002, *65* (3).

Pathak, P.A., T. Sönmez, M.U. Ünver, and M.B. Yenmez, “Fair Allocation of Vaccines, Ventilators and Antiviral Treatments: Leaving No Ethical Value Behind in Health Care Rationing,” *working paper*, 2020.

Rowthorn, R. and F. Toxvaerd, “On the Management of Population Immunity,” *working paper*, 2020.

Shaw, L.B. and I.B. Schwartz, “Fluctuating epidemics on adaptive networks,” *Phys Rev E*, 2008, *77* (6).

Talamás, E. and R. Vohra, “Free and perfectly safe but only partially effective vaccines can harm everyone,” *Games and Economic Behaviour*, 2020, *122.*

A Proofs

A.1 Lemma 1

Recall that the threshold is $y^* = c/(\lambda b)$, which is decreasing in λ. It follows that the right-hand-side of equation 4 is decreasing in λ since the integrand is always positive and as λ increases, y^* decreases, and the integral is over a smaller set. The right-hand-side is also continuous by the Fundamental Theorem of Calculus. Furthermore, it is strictly positive when $\lambda = 0$ and is always strictly less than 1. Since the left-hand-side is λ, the result follows from the Intermediate Value Theorem.

A.2 Proposition 1

First, we have the following lemma that shows that the planner always strictly prefers more vaccines.

Lemma A.1. *The Lagrange multiplier on the vaccine supply constraint, γ_1, is strictly positive.*

Proof. If there is a strictly positive measure of self-isolating individuals, then they can be vaccinated for a strictly positive private benefit without any spillovers. As such, the marginal benefit of a vaccine must be strictly positive. If there is not a strictly positive measure of self-isolating individuals, then since $\beta < 1$, there is a strictly positive measure of unvaccinated interacting individuals. They can be vaccinated for a strictly positive private benefit with spillovers that yield a strictly positive public benefit. To see this note that as the equilibrium interaction rate falls, other unvaccinated interacting individuals' infection rates fall, which raises welfare, and at most a measure 0 of isolating individuals can begin interacting since there are at most a measure 0 of isolating individuals. □

We now prove the following proposition that is also useful for proving Proposition 2.

Proposition A.1. *The optimal policy is characterized by a function $g : [0,1] \to [0,1]$ and a number $\bar{g}$ such that*

$$v(x,y) = \begin{cases} 0 & x \leq g(y); y \leq y^* \\ 1 & x > g(y); y \leq y^* \\ 0 & x \leq g^*; y > y^* \\ 1 & x > g^*; y > y^* \end{cases}$$

i.e. a type (x,y) individual with $y \leq y^*$ is vaccinated if and only if $x > g(y)$ and an individual with $y > y^*$ is vaccinated if and only if $x > g^*$. Moreover, the threshold function below y^* is given by

$$g(y) = min\left\{\frac{\gamma_1}{\lambda b y + \gamma_3 \alpha}, 1\right\} \tag{A.1}$$

and the threshold above y^* is given by

$$g^* = min\left\{\frac{\gamma_1}{c}, 1\right\}. \tag{A.2}$$

Proof. First, consider type (x, y) with $y > y^*$. The first derivative of the planner's Lagrangian in equation (7) with respect to $v(x, y)$ is

$$(cx - \gamma_1)f(x, y),$$

where the first term is the benefit that the individual no longer has to pay the interaction avoidance cost and the second term is the cost of providing the vaccine. Observe that the second derivative is always 0. If $\gamma_1/c > 1$, then since $x \leq 1$, the planner's derivative is always strictly negative and they optimally set $v(x, y) = 0$. If $\gamma_1/c = 1$, then the planner's derivative is strictly negative if $x < 1$, but always equal to 0 if $x = 1$. Since $x = 1$ with probability 0, we set $v(x, y) = 0$ regardless. If $\gamma_1/c < 1$, then the planner's derivative is always strictly negative if $x < \gamma_1/c$, equal to 0 if $x = \gamma_1/c$, and strictly positive if $x > \gamma_1/c$. As such, from the definition of $\bar{g}$ in equation (A.2), the planner optimally sets $v(x, y) = 1$ if and only if $x > \bar{g}$.

Next, consider type (x, y) with $y \leq y^*$. In this case, the first derivative of the planner's Lagrangian with respect to $v(x, y)$ is

$$(\lambda byx - \gamma_1 + \gamma_3\alpha x)f(x, y), \tag{A.3}$$

where the first term is the benefit that the individual can no longer become infected, the second term is the cost of providing the vaccine, and the third term captures the externality of lowering the equilibrium transmission rate by vaccinating an individual who was interacting when unvaccinated. Again, observe that the second derivative is always 0. Suppose $\lambda by + \gamma_3\alpha \geq \gamma_1$. It follows that the derivative in equation (A.3) is strictly increasing in x. Moreover, the derivative is strictly negative if $x < \gamma_1/(\lambda by + \gamma_3\alpha)$, strictly positive if the opposite strict inequality holds, and equal to 0 if the inequality holds with equality. It follows from the definition of $g(\cdot)$ in equation (A.1) that the planner optimally sets $v(x, y) = 1$ if and only if $x > g(y)$.

Finally, consider type (x, y) with $y \leq y^*$ and $\lambda by + \gamma_3\alpha < \gamma_1$. If $\lambda by + \gamma_3\alpha \geq 0$, then the derivative in equation (A.3) is weakly increasing in x and is strictly negative if $x = 1$, and so is strictly negative for any x. If $\lambda by + \gamma_3\alpha < 0$, then the derivative is strictly decreasing in x. Since the derivative is strictly negative if $x = 0$, it follows that the derivative is strictly negative for any x. As such, in any case, the planner optimally sets $v(x, y) = 0$. This is consistent with $g(y)$ because $\gamma_1/(\lambda by + \gamma_3\alpha) > 1$, so $g(y) = 1$. □

We immediately have the following result.

Corollary 2. *The threshold function g is continuous and decreasing.*

Proposition 1 follows from plugging in the vaccination policy

$$v(x, y) = \begin{cases} 0 & x \leq g(y); y \leq y^* \\ 1 & x > g(y); y \leq y^* \\ 0 & x \leq g^*; y > y^* \\ 1 & x > g^*; y > y^* \end{cases}$$

A.3 Proposition 2

Following Proposition A.1, all that remains to prove in the first part of Proposition 2 is that $y_0 < y^*$, $g(y)$ is strictly positive for all $y \leq y^*$, and $g(\cdot)$ exhibits an exposure premium $[0, y^*]$ and on $(y^*, 1]$.

We begin by proving a lemma necessary for further characterizing the second best because it shows how to deal with the case that $\lambda = c/b$. Call Problem 1 the planner's problem in which we assume $y^* < 1$ and given by $y^* = c/(\lambda b)$. As such, Problem 1 is only defined for $\lambda > c/b$. Call Problem 2 the planner's problem in which we assume $y^* = 1$. Even though this is only the case for $\lambda \leq c/b$, define Problem 2 and welfare for all policies $g(\cdot)$ and λ, assuming that y^* is fixed at 1. We say that a potential solution to Problem 2 is feasible if $\lambda \leq c/b$.

Lemma A.2. *Let P_1 be the set of solutions to Problem 1 and P_2 be the set of solutions to Problem 2 with $\lambda \leq c/b$. The optimal policy is the choice from the union of these sets that maximizes welfare.*

Proof. Observe that Problem 2 always has a solution. First, we show that if a solution to Problem 2 has $\lambda > c/b$, then there is a strictly better solution to Problem 1 and it is the optimal policy. Suppose Problem 2 has such an infeasible solution. The welfare in this solution is weakly higher than the welfare from any feasible solution with $y^* = 1$, i.e. with $\lambda \leq c/b$. Now, suppose the planner implements the same policy in the true planner's problem that sets $y^* = \min\{c/(\lambda b), 1\}$ – by the same policy, we mean vaccinating the same set of types (x, y), so the vaccination policy is unaffected by changes in λ and y^*. We can see that the policy leads to $\lambda > c/b$ and $y^* < 1$. If not, then $y^* = 1$ and as in Problem 2, we must have $\lambda > c/b$, which implies $y^* < 1$, a contradiction. Moreover, the policy yields higher welfare than in Problem 2. To see this note that there are two effects on welfare from reducing y^* below 1 while holding the policy fixed: the direct effect of the change in y^* and the indirect effect through the implied change in λ. For the indirect effect, as y^* falls below 1, λ falls as well, which strictly increases welfare. For the direct effect, the partial derivative of welfare with respect to y^* is $-\lambda b y^* g(y^*) + cg(y^*)$, which is weakly negative if $y^* > c/(\lambda b)$. It follows that as y^* decreases from 1 until the incentive compatibility constraint, $y^* = c/(\lambda b)$, is satisfied, welfare increases. As such, we have a policy that generates $y^* < 1$ that yields higher welfare than any feasible policy that generates $y^* = 1$. It follows that the optimal policy leads to $y^* < 1$ and so is a well-defined solution to Problem 1.

Next, suppose no solutions to Problem 2 have $\lambda > c/b$. It follows that any solutions to Problem 2 (there must be at least one) are feasible and yield higher welfare than any policy that generates $y^* = 1$. If Problem 1 has a solution, then it yields higher welfare than any policy that generates $y^* < 1$. It follows that the optimal policy is the welfare-maximizing choice from the set of solutions to Problem 1 and from the set of solutions to Problem 2 with $\lambda \leq c/b$. If, on the other hand, Problem 1 does not have a solution, then for any policy that yields $y^* < 1$, there is another policy that yields $y^* < 1$ and strictly higher welfare. As such, the optimal policy has $y^* = 1$. It follows that the optimal policy is a feasible solution to Problem 2 and all feasible solutions to Problem 2 yield the same welfare as the optimal policy. □

A takeaway from Lemma A.2 is that if the optimal policy has $y^* = 1$, then it is a solution to the planner's problem assuming y^* is fixed at 1, in which case we can ignore the planner's choice of y^* and the derivative of the Lagrangian with respect to λ is the same as when $\lambda < c/b$:

$$\frac{\partial \mathcal{L}}{\partial \lambda} = -b \int_0^{y^*} \int_0^{g(y)} yxf(x,y)dxdy + \gamma_3.$$

We next show that the planner never vaccinates individuals with $x = 0$.

Lemma A.3. *For all $y \leq y^*$, the threshold function, $g(y)$, is strictly positive.*

Proof. It follows from equation (8) that for all $y \leq y^*$, if $g(y) = 0$, then the derivative of the Lagrangian with respect to $g(y)$ is strictly positive. □

We can now show that the remaining Lagrangian multipliers are strictly positive, i.e. that the planner strictly prefers to reduce y^* and interaction relative to the individually optimal level and to reduce the transmission rate λ relative to the equilibrium level.

Lemma A.4. *The Lagrange multiplier on the equilibrium transmission rate constraint, γ_3, is strictly positive. If $y^* < 1$, then the Lagrange multiplier on the incentive compatibility constraint for y^*, γ_2, is strictly positive.*

Proof. First, suppose $y^* = 1$. Lemma A.2 shows that the First Order Condition for λ is

$$\gamma_3 = b \int_0^{y^*} \int_0^{g(y)} yxf(x,y)dxdy.$$

Since $y^* > 0$ and Lemma A.3 shows that $g(y) > 0$ for all $y \leq y^*$, it follows that $\gamma_3 > 0$.

Next, suppose $y^* < 1$. Plugging in the definition of y^* from equation (5) into the derivative of the Lagrangian with respect to $g(y^*)$ yields

$$\frac{\partial \mathcal{L}}{\partial g(y^*)} = (\gamma_1 - (c + \gamma_3\alpha)g(y^*))f(g(y^*), y^*). \tag{A.4}$$

Suppose $\gamma_1 = (c + \gamma_3\alpha)g(y^*)$. Plugging in for c in the final line of the derivative of the Lagrangian with respect to y^* yields

$$\frac{\partial \mathcal{L}}{\partial y^*} = \left(\frac{\gamma_1}{g(y^*)} - \gamma_3\alpha\right) \int_{g(y^*)}^{g^*} xf(x,y^*)dx - \gamma_1 \int_{g(y^*)}^{g^*} f(x,y^*)dx + \gamma_2 - \gamma_3\alpha \int_0^{g(y^*)} xf(x,y^*)dx.$$

If $g^* \geq g(y^*)$, then for all $x \in [g(y^*), g^*]$, $x/g(y^*) \geq 1$. It follows that

$$\frac{\gamma_1}{g(y^*)} \int_{g(y^*)}^{g^*} xf(x,y^*)dx \geqslant \gamma_1 \int_{g(y^*)}^{g^*} f(x,y^*)dx.$$

If $g^* < g(y^*)$, then for all $x \in [g^*, g(y^*)]$, $x/g(y^*) < 1$. In that case, it follows that

$$
\begin{aligned}
\frac{\gamma_1}{g(y^*)} \int_{g(y^*)}^{g^*} x f(x, y^*) dx &= -\frac{\gamma_1}{g(y^*)} \int_{g^*}^{g(y^*)} x f(x, y^*) dx \\
&> -\gamma_1 \int_{g^*}^{g(y^*)} f(x, y^*) dx \\
&= \gamma_1 \int_{g(y^*)}^{g^*} f(x, y^*) dx.
\end{aligned}
$$

In either case, we have that

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial y^*} &\geqslant -\gamma_3 \alpha \int_{g(y^*)}^{g^*} x f(x, y^*) dx + \gamma_2 - \gamma_3 \alpha \int_0^{g(y^*)} x f(x, y^*) dx \\
&= \gamma_2 - \gamma_3 \alpha \int_0^{g^*} x f(x, y^*) dx.
\end{aligned}
$$

Since the derivative of $\mathcal{L}$ with respect to y^* must be equal to 0, it follows that if γ_3 is weakly negative, then γ_2 must be weakly negative as well. Setting the derivative of $\mathcal{L}$ with respect to λ in equation (10) equal to 0 shows that one of γ_2 and γ_3 must be strictly positive. It follows that $\gamma_3 > 0$. Finally, regardless of whether $g^* \geqslant g(y^*)$ or the opposite holds, the sum of the first two terms in the last line of equation (11) for the derivative of $\mathcal{L}$ with respect to y^* are negative, which implies that

$$
\frac{\partial \mathcal{L}}{\partial y^*} \leqslant \gamma_2 - \gamma_3 \alpha \int_0^{g(y^*)} x f(x, y^*) dx.
$$

Since $\gamma_3 > 0$ and $g(y^*) > 0$, it follows that $\gamma_2 > 0$ as well.

Next, suppose $\gamma_1 > (c + \gamma_3 \alpha) g(y^*)$. It follows from equation (A.4) that the derivative of $\mathcal{L}$ with respect to $g(y^*)$ is strictly positive, so $g(y^*) = 1$. It follows that $-c > \gamma_3 \alpha - \gamma_1$ and, recalling that $g^* \leqslant 1$, we have that

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial y^*} &= -c \int_{g^*}^{1} x f(x, y^*) dx + \gamma_1 \int_{g^*}^{1} f(x, y^*) dx + \gamma_2 - \gamma_3 \alpha \int_0^1 x f(x, y^*) dx \\
&> (\gamma_3 \alpha - \gamma_1) \int_{g^*}^{1} x f(x, y^*) dx + \gamma_1 \int_{g^*}^{1} f(x, y^*) dx + \gamma_2 - \gamma_3 \alpha \int_0^1 x f(x, y^*) dx \\
&= \gamma_2 - \gamma_3 \alpha \int_0^{g^*} x f(x, y^*) dx.
\end{aligned}
$$

The rest of the argument follows as in the previous paragraph.

Finally, we cannot have $\gamma_1 < (c + \gamma_3 \alpha) g(y^*)$ because in that case, the derivative of $\mathcal{L}$ with respect to $g(y^*)$ is strictly negative. It then must be that $g(y^*) = 0$, which is not the case. □

We can now complete the proof of the first half of Proposition 2 with the following two corollaries.

Corollary 3. *The threshold below which $g(y) = 1$, y_0, is strictly below y^*.*

Proof. Since $g(y_0) = 1$ and $g(\cdot)$ is decreasing, it is sufficient to show that $g(y^*) < 1$. Plugging in the definition y^* from equation (5), we have from equation (A.1) in Proposition A.1 that

$$g(y^*) = \min\left\{\frac{\gamma_1}{c + \gamma_3\alpha}, 1\right\}.$$

Since γ_1 and γ_3 are strictly positive, it follows that $g(y^*) < \gamma_1/c$. As such if $g(y^*) = 1$, then $\gamma_1/c > 1$. If $y^* < 1$, then it follows from equation (A.2) in Proposition A.1 that $g^* = 1$. Since $g(\cdot)$ is decreasing, we then have that no individuals are vaccinated, which violates the vaccine supply constraint and so cannot be the case. On the other hand, if $y^* = 1$, then since $g(\cdot)$ is decreasing, we again have that no individuals are vaccinated, which again cannot be the case. □

Corollary 4. *For all $y \leqslant y^*$, the optimal vaccination policy implied by $g(\cdot)$ is monotone and exhibits an exposure premium. For all $y > y^*$, the optimal vaccination policy is monotone and exhibits an exposure premium.*

Proof. To see that the optimal policy is monotone on $y \leqslant y^*$, note that the threshold function is decreasing. Take an (x, y) such that $y \leqslant y^*$ and $v(x, y) = 1$. If $x' \geqslant x$ and $y' \in [y, y^*]$, then $g(y') \leqslant g(y) \leqslant x \leqslant x'$, so $v(x', y') = 1$ as well.

To see that the optimal policy exhibits an exposure premium on $y \leqslant y^*$, use the optimal $g(\cdot)$ from equation (A.1) in Proposition A.1 to take the derivative of $yg(y)$ with respect to y. If $y < y_0$, then the derivative is 1. If $y \in [y_0, y^*]$, then

$$\frac{\partial yg(y)}{\partial y} = \left(1 - \frac{\lambda by}{\lambda by + \gamma_3\alpha}\right) g(y),$$

which is strictly positive since $\gamma_3 > 0$.

To see that the optimal policy is monotone and exhibits an exposure premium for $y > y^*$, note that it is an x-policy and that since the threshold is constant, the derivative of y times the threshold is 1, which is positive. □

Now, following Proposition A.1, all that remains to prove in the second half of Proposition 2 is that if $y^* < 1$, then $g^* > g(y^*)$.

Corollary 5. *If $y^* < 1$, then $g^* > g(y^*)$.*

Proof. Suppose $y^* < 1$. If $g^* = 1$, then since Lemma A.3 shows that $g(y^*) < 1$, we are done. Suppose $g^* < 1$, so $g^* = \gamma_1/c$. Plugging in the definition of y^* from equation (5) into equation (A.1) from Proposition A.1 for $g(y^*)$ yields

$$g(y^*) = \frac{\gamma_1}{c + \gamma_3\alpha},$$

where we also use $g(y^*) < 1$. Since γ_1 and γ_3 are strictly positive, it follows that $g(y^*) < \gamma_1/c = g^*$. □

A.4 Proposition 3

First, since the Lagrangian is linear in v and σ and since individual types are drawn from a continuous distribution F with a pdf f, we can restrict our attention to pure vaccination and interaction policies, i.e. for all (x, y), $v(x, y) \in \{0, 1\}$ and $\sigma(x, y) \in \{0, 1\}$.

Now, for a type (x, y) the planner's vaccination and interaction policy decision is to maximize over the four possible choices for $\sigma(x, y)$ and $v(x, y)$ in $\{0, 1\}^2$. Define $d_{i,j}$ to be the difference between the Lagrangian when $\sigma(x, y) = i$ and $v(x, y) = j$ and the Lagrangian when $\sigma(x, y) = v(x, y) = 0$. We have that $d_{0,0} = 0$, $d_{0,1} = -\gamma_1 dF(x, y)$,

$$d_{1,0} = (xc - \lambda xyb)f(x, y) - \gamma_3 \alpha x f(x, y)$$

and if $\sigma(x, y) = v(x, y) = 1$, the difference is

$$d_{1,1} = xcf(x, y) - \gamma_1 f(x, y).$$

Since $d_{0,1} < 0$, it cannot be optimal for the planner to set $\sigma(x, y) = 0$ and $v(x, y) = 1$. If $y < (c - \gamma_3\alpha)/(\lambda b)$, then $d_{1,0} > 0$ and $d_{1,1} > d_{1,0}$ if and only if $x > \gamma_1/(\lambda yb + \gamma_3\alpha)$. If the opposite inequality for y holds, then $d_{1,0} < 0$ and $d_{1,1} > 0$ if and only if $x > \gamma_1/c$. It follows that if $y < (c - \gamma_3\alpha)/(\lambda b)$, then the individual interacts, $\sigma(x, y) = 1$, regardless of x and the individual is vaccinated, $v(x, y) = 1$, if and only if $x > \gamma_1/(\lambda yb + \gamma_3\alpha)$. If $y > (c - \gamma_3\alpha)/(\lambda b)$, then the individual interacts and is vaccinated, $\sigma(x, y) = v(x, y) = 1$, if $x > \gamma_1/c$. Otherwise, the individual does not interact and is not vaccinated. Therefore, an unvaccinated individual interacts if and only if

$$y > y^* = \min\left\{\frac{c - \gamma_3\alpha}{\lambda b}, 1\right\}$$

and regardless of x. If $y \leqslant y^*$, then an individual is vaccinated if and only if

$$x > g(y) = \min\left\{\frac{\gamma_1}{\lambda by + \gamma_3\alpha}, 1\right\}.$$

If $y > y^*$, then an individual is vaccinated if and only if $x > g(y^*) = \gamma_1/c$.

Finally, the same arguments as in the proofs of Lemmas A.3 and A.4 show that γ_1 and γ_3 are strictly positive. Since the x-threshold function is continuous, the same argument as in the proof of Corollary 4 shows that the optimal vaccination policy is monotone and exhibits an exposure premium everywhere.

A.5 Corollary 1

Given a flat interaction tax, τ, an individual wants to interact if and only if $\lambda by + \tau < c$. If $y^* < 1$, then $\lambda by^* = c - \gamma_3\alpha$. Since $\gamma_3 > 0$, the result follows. If $y^* = 1$, then $\lambda b \leq c - \gamma_3\alpha$. Since $\gamma_3 > 0$, it follows that all individuals want to interact, as in the first best.

A.6 Lemma 2

The left-hand-side of equation 16, $p/(bA^*)$, is decreasing and continuous in A^*, goes to p/b as A^* goes to 1, and goes to positive infinity as A^* goes to 0. The right-hand-side is increasing and continuous in A^*, and is always weakly positive. It follows that if the right-hand-side is strictly greater than p/b at $A^* = 1$, then there exists an unique equilibrium with $A^* \in (0, 1)$. Otherwise, the left-hand-side is always strictly greater than the right-hand-side for $A^* < 1$ and the unique equilibrium has $A^* = 1$. We can never have $A^* = 0$ because types in an open neighborhood of $(0, 0)$ gain almost nothing from accepting vaccination, but lose $p > 0$.

A.7 Proposition 4

If $A^* = 1$, then the proposition trivially holds with $\bar{y} = 1$. As such, for the remainder of the proof, suppose $A^* \in (0, 1)$, which implies that $A^* = p/(\lambda b) < 1$.

First, the Lagrange multiplier on the vaccine supply constraint, γ_1, is weakly positive; it may be 0 if on the margin, the planner vaccinates an individual who does not accept. Now, suppose a type (x, y) individual will accept a vaccine if offered. The first derivative of the Lagrangian with respect to $v(x, y)$ is

$$\left[\lambda byx - p - \gamma_1 + \gamma_3 \alpha x\right] f(x, y),$$

where the first two terms are the direct benefit and cost of the vaccine to the individual, the third term is the cost of providing the vaccine, and the fourth term captures the externality of lowering the equilibrium transmission rate. The second derivative with respect to $v(x, y)$ is always 0. If $y \geqslant \hat{y}$, where $\hat{y}$ is defined in equation (18) in the proposition, then $\lambda by + \gamma_3 \alpha \geqslant \gamma_1 + p > 0$. It follows that the derivative is strictly increasing in x, is strictly negative if $x < (\gamma_1 + p)/(\lambda by + \gamma_3 \alpha)$, strictly positive if the opposite strict inequality holds, and equal to 0 if the inequality holds with equality. On the other hand, if $y < \hat{y}$ and $\lambda by + \gamma_3 \alpha \geqslant 0$, then the derivative is weakly increasing in x and strictly negative at $x = 1$, so it is strictly negative for all x. Finally, if $y < \hat{y}$ and $\lambda by + \gamma_3 \alpha < 0$, then the derivative is strictly decreasing in x and strictly negative at $x = 0$, so it is strictly negative for all x. It follows from the definition of $g(\cdot)$ in the proposition that if an individual with $y < \hat{y}$ is willing to accept a vaccine, then the planner optimally sets $v(x, y) = 0$ and if an individual with $y \geqslant \hat{y}$ is willing to accept a vaccine, then the planner optimally sets $v(x, y) = 1$ if and only if $x \geqslant g(y)$. If an individual is not willing to accept a vaccine, then the planner optimally sets $v(x, y) = 1$ if and only if $x \leqslant tA^*/y$, where $t \in [0, 1]$. If $\gamma_1 > 0$, then $t = 0$. These individuals are vaccinated simply to satisfy the vaccine supply constraint. We can equally suppose that the planner need not satisfy the constraint with equality.

What remains to prove the proposition is to show that, for $\bar{y}$ defined in equation (19) in the proposition, $g(y)y \geqslant A^*$ if and only if $y \geqslant \bar{y}$ and for all $y \geqslant \max\{\hat{y}, \bar{y}\}$, the policy exhibits an exposure premium.

To prove these results, we first rewrite the planner's problem as choosing $A^* \in [0, 1]$, $\lambda \in [0, 1]$, a continuous and decreasing x-threshold function $g:[0,1]\to[0,1]$, and $t\in[0,1]$ to maximize

$$\mathcal{L}=\mathcal{W}+\gamma_1\left(\beta-\int_0^1\int_{\max\{g(y),A^*/y\}}^1 f(x,y)dxdy-\int_0^1\int_0^{tA^*/y} f(x,y)dxdy\right)+\gamma_2\left(A^*-\min\left\{\frac{p}{\lambda b},1\right\}\right)$$
$$+\gamma_3\left(\lambda-\alpha\int_0^1\int_0^{\max\{g(y),A^*/y\}} xf(x,y)dxdy\right),$$

where welfare is

$$\mathcal{W}=-\lambda b\int_0^1\int_0^{\max\{g(y),A^*/y\}} yxf(x,y)dxdy-\int_0^1\int_{\max\{g(y),A^*/y\}}^1 pf(x,y)dxdy.$$

Before we can proceed, we first have to show that the Lagrange multiplier on the vaccine hesitancy constraint, γ_2, is weakly positive and on the equilibrium transmission rate constraint, γ_3, is strictly positive. To that end, observe that the derivative of welfare with respect to A^* is 0 since A^* is optimally chosen to equate λbA^* with p. We then have that

$$\frac{\partial\mathcal{L}}{\partial A^*}=\gamma_1\int_{y:A^*/y>g(y)}\frac{1}{y}f(A^*/y,y)dy+\gamma_2-\gamma_3\alpha\int_{y:A^*/y>g(y)}\frac{A^*}{y^2}f(A^*/y,y)dy.$$

If the set of y such that $A^*/y>g(y)$ has measure 0, then the First Order Condition for A^* implies that $\gamma_2=0$. If that set has strictly positive measure, then for any y in that set, we can increase $g(y)$ to A^*/y without affecting any other variables, including welfare, at which point the derivative of the Lagrangian with respect to $g(y)$ is

$$\frac{\partial\mathcal{L}}{\partial g(y)}=\gamma_1 f(A^*/y,y)-\gamma_3\alpha\frac{A^*}{y}f(A^*/y,y)dy,$$

where we use that the derivative of welfare with respect to $g(y)$, like the derivative of welfare with respect to A^*, is 0. For the policy to be optimal, it must be that this derivative is weakly negative. Plugging into the First Order Condition for A^*, which sets the derivative of the Lagrangian with respect to A^* equal to 0, implies that $\gamma_2\geqslant 0$. In any case, we have that $\gamma_2\geqslant 0$.

If $\gamma_2>0$, then since $\gamma_1\geqslant 0$, the First Order Condition for A^* implies that $\gamma_3>0$. For $\gamma_2=0$, consider the derivative of the Lagrangian with respect to λ:

$$\frac{\partial\mathcal{L}}{\partial\lambda}=-b\int_0^1\int_0^{\max\{g(y),A^*/y\}} yxf(x,y)dxdy+\gamma_2\frac{p}{\lambda^2 b}+\gamma_3.$$

The First Order Condition sets this derivative equal to 0, which implies along with $\gamma_2=0$ that $\gamma_3>0$. As such, in any case, we have that $\gamma_3>0$.

Now, to see the remaining results – that there exists the desired $\bar{y}$ such that $g(y)y\geqslant A^*$ if and only if $y\geqslant\bar{y}$ and that the optimal policy exhibits an exposure premium for all $y\geqslant\max\{\hat{y},\bar{y}\}$ – consider the derivative of $g(y)y$ with respect to y:

$$\frac{\partial g(y)y}{y} = \frac{(\gamma_1 + p)\gamma_3\alpha}{(\lambda b y + \gamma_3\alpha)^2}.$$

The derivative is strictly positive since $\gamma_1 \geqslant 0$ and $\gamma_3 > 0$. Both results follow. Moreover, if $\bar{y} > 0$, then it is given by $g(\bar{y})\bar{y} = A^*$. Thus, for all $y \in [\hat{y}, \bar{y})$, the policy is an xy-policy with iso-risk threshold A^*.

A.8 Lemma 3

Define $\tilde{\gamma}_1 \triangleq \gamma_1/c$, $\tilde{\gamma}_3 \triangleq \gamma_3/b$, and $\mu \triangleq \lambda/\alpha$. The system of First Order Conditions for the Problem 1 version of the second-best problem (described in Subsection A.3 and in which we assume $y^* < 1$) can be written as the following system of six equations:

$$\beta = \int_0^{y^*} \int_{g(y)}^1 f(x,y)dxdy + \int_{y^*}^1 \int_{g^*}^1 f(x,y)dxdy, \tag{A.5}$$

$$\mu = \int_0^{y^*} \int_0^{g(y)} xf(x,y)dxdy, \tag{A.6}$$

$$y^* = \frac{c}{\alpha b}\frac{1}{\mu}, \tag{A.7}$$

$$g(y) = \min\left\{\frac{c}{\alpha b}\frac{\tilde{\gamma}_1}{\mu y + \tilde{\gamma}_3}, 1\right\}, \tag{A.8}$$

$$g^* = \min\{\tilde{\gamma}_1, 1\}, \tag{A.9}$$

and

$$\tilde{\gamma}_3 = \frac{\int_{g(y^*)}^{g^*} xf(x,y^*)dx - \tilde{\gamma}_1 \int_{g(y^*)}^{g^*} f(x,y^*)dx + \left(\frac{\alpha b}{c}\right)^2 \mu^2 \int_0^{y^*}\int_0^{g(y)} yxf(x,y)dxdy}{\left(\frac{\alpha b}{c}\right)^2 \mu^2 + \frac{\alpha b}{c}\int_0^{g(y^*)} xf(x,y^*)dx}, \tag{A.10}$$

which we solve for μ, y^*, $g(\cdot)$, g^*, $\tilde{\gamma}_1$, and $\tilde{\gamma}_3$. The final equation comes from setting

$$\frac{\partial \mathcal{L}}{\partial y^*} - \frac{\lambda^2 b}{c}\frac{\partial \mathcal{L}}{\partial \lambda} = 0;$$

we can then use the First Order Condition for either λ or y^* to back out γ_2. Finally, note that

$$\mathcal{W} = -c\left(\frac{\alpha b}{c}\mu \int_0^{y^*}\int_0^{g(y)} yxf(x,y)dxdy + \int_{y^*}^1\int_0^{g^*} xf(x,y)dxdy\right),$$

so welfare scales with the interaction avoidance cost, c, but only $\alpha b/c$ affects welfare comparisons across policy choices.

For Problem 2, a similar argument applies except we no longer need equation (A.7) for y^* or equation

(A.9) for g^*, and equation (A.10) for $\tilde{\gamma}_3$ becomes

$$\tilde{\gamma}_3 = \int_0^1 \int_0^{g(y)} yxf(x,y)dxdy.$$

For the first-best, a similar argument applies except we no longer need equation (A.9) for g^*, we replace equation (A.5) with

$$\beta = \int_0^{y^*} \int_{g(y)}^1 f(x,y)dxdy + \int_{y^*}^1 \int_{g(y^*)}^1 f(x,y)dxdy, \tag{A.11}$$

we replace equation (A.7) with

$$y^* = \min\left\{\frac{\frac{c}{\alpha b} - \tilde{\gamma}_3}{\mu}, 1\right\}, \tag{A.12}$$

and we replace equation (A.10) with

$$\tilde{\gamma}_3 = \int_0^{y^*} \int_0^{g(y)} yxf(x,y)dxdy. \tag{A.13}$$

B Computational Methods

To compute the first- and second-best optimal policies, we compute solutions to the systems of equations described in Subsection A.8. In each case, we show that the system of equations minus one has an unique solution given a guess for $\tilde{\gamma}_3/\mu$. We then iterate over $\tilde{\gamma}_3/\mu$ until the final equation is satisfied. To solve the second-best problem, we conduct this procedure for Problem 1 (assuming $y^* < 1$) and Problem 2 (assuming $y^* = 1$) and choose the solution that achieves the highest welfare. To solve the first-best problem, we simply choose the solution that achieves the highest welfare. In all our numerical experiments, the two sides of the final equation are continuous and monotone in $\tilde{\gamma}_3/\mu$ and in opposite directions, so there is clearly an unique solution to the system of First Order Conditions.

B.1 Second-best Policy

Lemma B.1. *Taking as given $\tilde{\gamma}_3/\mu$, the system of the first five equations in Lemma 3, equations (A.5)-(A.9), has an unique solution for the remaining five unknowns, μ, y^*, $\tilde{\gamma}_1$, $g(\cdot)$, and g^*.*

Proof. First, we show that taking as given $\tilde{\gamma}_1/\mu$ and $\tilde{\gamma}_3/\mu$, the system of four equations, (A.6) - (A.9), i.e. the five equations minus the vaccine supply constraint, has an unique solution for the remaining four unknowns, μ, y^*, $g(y)$, and g^*. Take as given $\tilde{\gamma}_1/\mu$ and $\tilde{\gamma}_3/\mu$. For each value of μ, we know y^* from equation (A.7), $g(y)$ from (A.8), and g^* from (A.9). Hence, it is sufficient to show that there cannot be two solutions to the system of four equations with different values for μ. Suppose there are two solutions (whose variables are indexed by subscripts 1 and 2) with $\mu_1 > \mu_2$. It follows that $y_1^* < y_2^*$, and $g_1 = g_2$. It follows that the right-hand-side of equation (A.6) for the equilibrium value of μ is strictly smaller in the first solution than in the second. This contradicts $\mu_1 > \mu_2$. As such, taking as given $\tilde{\gamma}_1/\mu$ and $\tilde{\gamma}_3/\mu$, the system of four equations, (A.6) - (A.9), has an unique solution for the four unknowns, μ, y^*, $g(y)$, and g^*.

Moreover, suppose we choose two different values $(\tilde{\gamma}_1/\mu)_1 > (\tilde{\gamma}_1/\mu)_2$. Since $\tilde{\gamma}_3/\mu$ is fixed, we have that $g_1 > g_2$. It must be that $\mu_1 > \mu_2$. If $\mu_1 \leq \mu_2$, then $y_1^* \geq y_2^*$, which, along with $g_1 > g_2$, implies an increase in the right-hand-side of equation (A.6) for μ, contradicting $\mu_1 \leq \mu_2$.

Now, take as given $\tilde{\gamma}_3/\mu$ and consider the system of five equations, (A.5)-(A.9), with five unknowns, $\tilde{\gamma}_1/\mu$, μ, y^*, $g(y)$, and g^*. Since the list four equations uniquely define μ, y^*, $g(y)$, and g^* given $\tilde{\gamma}_1/\mu$, it is sufficient to show that there cannot be two solutions with different values for $\tilde{\gamma}_1/\mu$. Suppose there are with $(\tilde{\gamma}_1/\mu)_1 > (\tilde{\gamma}_1/\mu)_2$. We know that $g_1 > g_2$ and $\mu_1 > \mu_2$, which implies that $y_1^* < y_2^*$ and, along with $(\tilde{\gamma}_1/\mu)_1 > (\tilde{\gamma}_1/\mu)_2$, implies that $g_1^* > g_2^*$. The change from μ_1 to μ_2 has no direct effect on the right-hand-side of the vaccine supply constraint, equation (A.5), but the other three changes each imply that the right-hand-side is strictly lower in the first case than in the second (to see the effect of the change in y^*, recall that $g(y^*) < g^*$). Since the left-hand-side, β, is the same in both cases, this cannot be. It follows that given $\tilde{\gamma}_3/\mu$, the system of five equations, (A.5)-(A.9), has an unique solution for the five unknowns, $\tilde{\gamma}_1/\mu$, μ, y^*, $g(y)$, and g^*.

Furthermore, suppose there are two solutions to the full system of six equations, (A.5)-(A.10), with $(\tilde{\gamma}_3/\mu)_1 > (\tilde{\gamma}_3/\mu)_2$. It must be that $(\tilde{\gamma}_1/\mu)_1 > (\tilde{\gamma}_1/\mu)_2$. Suppose not. Then $g_1 < g_2$ and it follows that $\mu_1 < \mu_2$. Otherwise, if $\mu_1 \geq \mu_2$, then $y_1^* \leq y_2^*$, which, along with $g_1 < g_2$, implies that the right-hand-side of the equilibrium transmission rate constraint, (A.6), is strictly smaller in the first case than in the second, which contradicts $\mu_1 \geq \mu_2$. As such, $\mu_1 < \mu_2$, which implies that $y_1^* > y_2^*$ and, along with $(\tilde{\gamma}_1/\mu)_1 \leq (\tilde{\gamma}_1/\mu)_2$, implies that $g_1^* < g_2^*$. It follows that the right-hand-side of the vaccine supply constraint, (A.5), is strictly larger in the first case than in the second, which cannot be since β is the same across the two cases. As such, we have that $(\tilde{\gamma}_1/\mu)_1 > (\tilde{\gamma}_1/\mu)_2$.

Finally, we show that $\tilde{\gamma}_3/\mu \in [0, 1]$. Since we showed in Lemma A.4 that $\gamma_3 > 0$, it follows that $\tilde{\gamma}_3/\mu > 0$. Moreover, consider the sixth equation in our system, (A.10) for the combined First Order Conditions for λ and y^*. Since $\tilde{\gamma}_1 \geq g^*$, we can see that

$$\int_{g(y^*)}^{g^*} x f(x, y^*) dx < \tilde{\gamma}_1 \int_{g(y^*)}^{g^*} f(x, y^*) dx.$$

Moreover, $\int_0^{g(y^*)} x f(x, y^*) dx > 0$. It then follows from plugging in the equilibrium value of μ that

$$\frac{\tilde{\gamma}_3}{\mu} < \frac{\int_0^{y^*} \int_0^{g(y)} y x f(x, y) dx dy}{\int_0^{y^*} \int_0^{g(y)} x f(x, y) dx dy},$$

where the right-hand-side is the average value of y among those in the interaction pool. Since all individuals have $y \leq 1$, it follows that this average is less than 1. □

B.2 First-best Policy

Lemma B.2. *Take as given $\tilde{\gamma}_3/\mu$. There is an unique solution to the system of four equations (excluding equation* (A.13) *for $\tilde{\gamma}_3$),* (A.6)*,* (A.8)*,* (A.11)*, and* (A.12)*, for the remaining four unknowns, $\tilde{\gamma}_1$, μ, y^*, and $g(\cdot)$.*

Proof. We first show that taking as given $\tilde{\gamma}_3/\mu$ and $\tilde{\gamma}_1/\mu$, there is an unique solution to the system of three equations (excluding the vaccine supply constraint), (A.6), (A.8), and (A.12), for the remaining three unknowns, μ, y^*, and $g(\cdot)$. Note that g is fully determined by $\tilde{\gamma}_1/\mu$ and $\tilde{\gamma}_3/\mu$. It is then sufficient to note that equation (A.6) implies that μ is strictly increasing in y^* and equation (A.12) implies that y^* is decreasing in μ.

To complete the proof of the lemma, it is sufficient to show that there cannot be two solutions to the system of four equations, (A.6), (A.8), (A.11), and (A.12), with different values for $\tilde{\gamma}_1/\mu$. Suppose there are two such solutions, indexed by subscripts 1 and 2, with $(\tilde{\gamma}_1/\mu)_1 > (\tilde{\gamma}_1/\mu)_2$. It follows that $g_1 > g_2$. If $\mu_1 \leq \mu_2$, then equation (A.12) shows that $y_1^* \geq y_2^*$. But then equation (A.6) implies that $\mu_1 > \mu_2$, a contradiction. It follows that $\mu_1 > \mu_2$ and $y_1^* < y_2^*$. Now, the right-hand-side of equation (A.11) is strictly decreasing in g and strictly increasing in y^*. To see the second claim, observe that the derivative of the right-hand-side with respect to y^* is $-\int_{y^*}^{1} g'(y^*) f(g(y^*), y) dy$, which is strictly positive since the derivative of g with respect to y is always strictly negative. It follows that the right-hand-side is strictly smaller in the first solution than in the second, which cannot be the case since the left-hand-side, β, is the same in both cases. $\square$