Market Concentration, Growth, and Acquisitions

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Abstract

I study an oligopolistic growth model in which firms can innovate by creatively destroying their competitors' goods, improving their own goods, and developing new ones. A large firm is equivalent to a mass of small firms that can coordinate their activities to maximize joint profits. Larger firms adapt their innovation mix to avoid cannibalization, and as a result they impose a high rate of creative destruction risk on their competitors without generating much growth. A tax on large firm acquisitions of their smaller competitors' goods may backfire by encouraging large firms to creatively destroy those goods instead. In a special case of the model with only creative destruction, I prove conditions so that a rise in large firm profitability leads to a fall in growth, and so that a fall in taxes on acquisitions leads to an *increase* in growth. In the full quantitative model, a fall in large firm fixed costs, calibrated to match the recent rise in concentration in the US, explains 41% of the fall in growth from the 1990s to the 2010s, as well as the burst in growth during the late 1990s, the positive across-industry correlation between changes in concentration and growth, and the fall in growth relative to R&D expenditures. Dispersion in large firm innovation costs or profitability across industries yields a novel theory of the inverted-U relationship between concentration and growth.

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1 Introduction

Many authors have documented a rising share of revenue going to the top firms in industries at the national level in the US since the 1990s.[1] This trend has spurred research into its connection to the recent decline in growth, as well as the policy implications.[2] Over a similar time period, there was a dramatic rise in the rate at which venture capital backed startups are acquired relative to the rate at which they go public.[3] Two questions emerge: Is large firm behavior behind the fall in growth? If so, should antitrust authorities limit acquisitions to reduce industry concentration and promote growth?

To answer these questions, I study an oligopolistic growth model in which firms can improve on old goods and develop new ones. I take as the key feature of a large firm that they control a significant portion of their industry's innovation and as a result earn a significant portion of its sales. Specifically, the only meaningful ex-ante firm heterogeneity is in innovation costs; a large firm is equivalent to a group of smaller firms that can coordinate their innovation activities to maximize joint profits. I find that if firms can creatively destroy each others' goods and innovation rates are sufficiently responsive to the profits firms earn from innovating, then a shock that increases large firm innovation incentives leads to a fall in growth. In that case, acquisitions that increase concentration are costly but policies to limit them may backfire by encouraging large firms to innovate more, which leads to lower growth. On the other hand, a tax on large firms' sales reduces their innovation and increases growth. In the quantitative model, a rise in large firm profitability calibrated to generate the observed rise in concentration in the US from the mid 1990s to the late 2010s explains 41% of the fall in the long-run growth rate, as well as the short-run growth burst in the late 1990s, the positive correlation across industries between changes in concentration and growth, the fall in the entry rate, and the rise in R&D expenditures relative to GDP despite the fall in growth.

The results follow from two implications of the Arrow (1962) "replacement effect" that incumbents are reluctant to innovate in ways that cannibalize their sales from past innovations. First, within an industry, larger firms face lower rates of creative destruction. When a firm creatively destroys a good, they make an improved version that replaces the old one. Incumbents prefer to target their competitors' goods rather than replace their own. A large innovative firm controls a substantial share of its industry's innovation and products, so their preference for directing creative destruction

[1]See Autor, Dorn, Katz, Patterson, and Van Reenen (2020), Barkai (2020).

[2]See Cavenaile, Celik, and Tian (2021), Aghion, Bergeaud, Boppart, Klenow, and Li (2022), Akcigit and Ates (2021), De Ridder (2021), Liu, Mian, and Sufi (2022), Olmstead-Rumsey (2022), and Garcia-Macia, Hsieh, and Klenow (2019).

[3]See Pellegrino (2021).

elsewhere leaves their goods effectively missing a portion of the industry creative destruction rate. Second, larger firms have a stronger preference for creatively destroying their competitors' goods over improving on their own goods or developing new ones. The latter types of innovation generate sales only through growth, which takes sales from the other goods in the industry, many of which the large firm produces; creatively destroying a good takes sales from the producer of that good beyond the growth it creates.

Each of the two effects implies that if a firm is larger within its industry, then relative to the growth it generates, it imposes a higher rate of creative destruction risk on its competitors. Thus, if a large firm's incentives to innovate increase and its competitors' innovation rates are responsive, growth falls. Although the large firm does not generate much more growth, it deters its competitors from doing so by imposing on them a high rate of creative destruction risk.

The first mechanism is in line with the finding in Akcigit and Kerr (2018) that in the US, larger firms' patents face lower rates of external citations. Moreover, it can explain the high sales-to-R&D ratios of industry leaders Olmstead-Rumsey (2022) documents; to illustrate, in the limit if a firm faces no depreciation rate on its goods' sales, then its innovation problem is still well-defined as long as the interest rate is positive, but in the long-run, its stock of sales from past innovations is infinite.[4] In support of the second mechanism, Garcia-Macia, Hsieh, and Klenow (2019) estimate that creative destruction is responsible for a minority of growth in the US, but a majority of the rate at which firms discount their innovations because most of the between-firm flows creative destruction generates is the reallocation of goods, not growth.

I begin in Section 2 with a qualitative model of an industry with a single large firm and a continuum of small firms that innovate only by creatively destroying their competitors' goods. The difference between the large firm and the mass of small firms is that the large firm does not creatively destroy its own goods, whereas small firms creatively destroy each others' goods because they cannot collude. Thus only the first mechanism described above is present. I prove an analytical condition in terms of observable equilibrium outcomes such that a rise in large firm innovation incentives leads to a fall in the long-run growth rate. In particular, growth falls if small firm innovation is sufficiently elastic with respect to the profits a small firm earns from innovating. In the limit, a monopolist deters all small firm innovation with a rapid creative destruction rate, but

[4] Argente, Lee, and Moreira (2021) provide empirical evidence for the ability of firms to direct their creative destruction away from their own goods: the revenues of high sales products depreciate more quickly than the revenues of low sales products, in line with the prediction that such products would be creatively destroyed more quickly. Akcigit, Alp, and Peters (2021) argue that a relatively high creative destruction rate for goods produced by firm types that innovate less, and tend to be small, can explain the high employment shares of old firms in US and Indian data.

generates no growth because there are no small firm goods to creatively destroy.

The large firm can search for acquisition opportunities in which it makes a take-it-or-leave-it offer to a small firm for one of its good subject to a tax. I prove an analytical condition in terms of observable equilibrium outcomes such that a fall in the tax leads to a *rise* in the long-run growth rate, i.e., encouraging acquisitions increases growth. In particular, growth rises only if a rise in large firm innovation incentives leads to a fall in growth, if the elasticity of innovation with respect to the profits from innovating is sufficiently high relative to the elasticity of the acquisition rate with respect to the surplus from an acquisition, and if industry concentration is sufficiently high. This result follows from two competing effects of acquisitions. A lower tax increases the acquisition rate, which decreases growth by shifting goods to the large firm where they experience a slower rate of creative destruction. However, a lower tax also increases the surplus of each acquisition, which reduces the large firm's incentive to innovate because creatively destroying its competitors' goods and acquiring them are substitutes. An econometrician looking at the impact of exogenous acquisitions on growth would only estimate the first effect.

I develop the quantitative model in Section 3, in which firms also innovate by improving on their own goods and creating new ones, and thus both mechanisms described above are active. Moreover, I embed the industry setup into a macroeconomic model that consists of a continuum of industries, each of which contains a single large firm and a measure of small firms. In Section 4, I calibrate the model and conduct the key experiment: a rise in large firm profitability due to a fall in large firm per-good fixed costs. Although creative destruction is responsible for a minority of growth, its presence drives the quantitatively significant results—in an equivalent model without creative destruction, a rise in large firm profitability leads to a small *increase* in growth—because it is responsible for a *majority* of sales/labor flows between firms. The disparity between creative destruction's growth and flows shares implies that whereas own good improvement and new good development generate sales only through growth, creative destruction of a competitor's good generates sales mostly, though not entirely, through business stealing; the innovator takes over all production of the targeted good by making a small improvement on the previous version. Therefore, large firms tilt their innovation toward creative destruction to avoid cannibalization, which imposes risk on small firms and so deters small firm innovation, without generating much growth.

The model generates the documented short-run burst in growth and positive correlation across industries between rising concentration and growth because small firm innovation is less responsive in the short-run and to industry-specific shocks. Long-run growth falls despite a rise in R&D because firm-level innovation costs are convex, so as large firms innovate more, their R&D becomes less efficient. It follows that the results hold whether labor is fully flexible across entry, innovation, and production, or completely fixed. In the first case, a free entry condition makes small firm innovation perfectly elastic; a rise in large firm innovation implies an increase in the creative destruction risk small firms face, so for the discount rate small firms apply to innovations to remain constant, the growth rate must fall. In the second case, a rise in large firm innovation implies a shift in the fixed supply of innovation labor from small firms to large firms, and so a fall in its efficiency. The results are also robust to different calibrations.

I focus on a shock to large firm profitability because it best matches the full set of empirical observations related to the fall in growth. More generally, the model's two main mechanisms imply that other shocks that increase concentration will have an additional negative effect on growth.

I analyze the effects of a tax on acquisitions in the quantitative model in Section 5. A sufficiently low tax leads to faster growth than in an economy without acquisitions: to preserve valuable acquisition opportunities, large firms reduce innovation. The break even tax rate at which growth is the same with and without acquisitions is higher—acquisitions are more easily beneficial—if the acquisition rate is less elastic, and after the rise in large firm profitability and concentration. It follows that policy should not place unnecessary road blocks in the way of high surplus acquisitions, particularly in concentrated industries, but should block marginal low surplus acquisitions. Allowing high value acquisitions is an effective way to limit large firm innovation because it does not require intimate knowledge of industry boundaries or of which firms are large.

These results highlight an important subtlety in optimal competition and innovation policy. In theories that focus only on *how much* large firms produce or innovate, high markups or the Arrow (1962) "replacement effect" imply that it is optimal to encourage *more* large firm production or innovation.[5] According to these theories, reducing large firm innovation is a cost of acquisitions, rather than a benefit.[6] Instead, taking into account the multidimensional nature of innovation, I find that policies that encourage large firm production or innovation end up encouraging activity that imposes a strong deterrent on competition without adding much social value.

Large Firms and Innovation:
Previous work on oligopolistic competition and innovation mostly focuses on the impact of a large firm's market share on its *magnitude* of innovation, which is not important for the theory I propose because I specifically consider a rise in large firm innovation incentives.[7] A notable exception is the

[5]See Edmond, Midrigan, and Xu (2021).
[6]See Fons-Rosen, Roldan-Blanco, and Schmitz (2022).
[7]See Shapiro (2012) for a discussion, and Cavenaile, Celik, and Tian (2021) for a recent example.

theory put forward in Argente, Baslandze, Hanley, and Moreira (2021) and mentioned in Akcigit and Ates (2021) that large firms use patents to deter competition. Although the mechanism is different, the implications for the effects of large firm innovation are similar. This theory is complementary to the one I propose because creative destruction is most relevant in industries with weak patent protection.

In the context of the recent rise in concentration and fall in growth, the most similar papers are Aghion, Bergeaud, Boppart, Klenow, and Li (2022), De Ridder (2021), and Liu, Mian, and Sufi (2022). In the first two, all firms have infinitesimal market shares, and a rise in innovation by high process productivity firms shifts goods to firms that are difficult to compete with, which reduces the return to creatively destroying those goods, and thus results in less innovation overall. As such, they find similar results from the rise of high productivity firms that I find from the rise of large, but equally productive, firms. In Liu, Mian, and Sufi (2022), two large firms compete in each industry, and if one becomes sufficiently dominant, then the growth rate falls because the dominant firm optimally cuts their price dramatically in response to innovation by their competitor. By contrast, my mechanism does not rely on a large firm responding directly to the actions of a single competitor. Thus, it may be more relevant when thinking about an industry with both large and small firms, especially in light of the evidence in Amiti, Itskhoki, and Konings (2019) on how responsive a firm's price is to a change in its competitors' prices.

Although none of the papers mentioned thus far study acquisitions, in each case acquisitions by large or high productivity firms would be strictly bad for growth because it does not matter how those firms gain their goods. Once they gain market share, they deter innovation by reducing the *flow profits* their competitors can earn. By contrast, I find that the way large firms innovate leads to lower growth because they add to the *discount rate* their competitors use for an innovation. Thus, even if acquisitions lead to higher concentration, they can increase growth if they are associated with a sufficient drop in large firm innovation.

Finally, the model generates an alternative explanation of the inverted-U relationship between concentration and growth across industries, estimated empirically in Aghion, Bloom, Blundell, Griffith, and Howitt (2005) and Cavenaile, Celik, and Tian (2021). If variation in large firm profitability or innovation capabilities drives variation across industries, then the inverted-U emerges because an increase in large firm innovation decreases growth only if concentration is sufficiently high so that large firm innovation is sufficiently distorted. Previous theories of the inverted-U are based instead on two effects of competition: some is necessary to encourage dominant firms to innovate and escape, but too much discourages any innovation. A crucial difference is that in this paper, there can be an inverted-U across industries even if a rise in large firm innovation incentives always decreases growth at the aggregate level because the response of small firms is vital and small firms are more responsive to aggregate shocks.

Large Firm Acquisitions of Small Competitors' Goods:
"Entry for buyout", described in Rasmusen (1988) and more recently, Letina, Schmutzler, and Seibel (2021) and Fons-Rosen, Roldan-Blanco, and Schmitz (2022), is another theory in which a high expected value of future acquisitions increases growth even though the distributional consequences of actual acquisitions do not: firms enter and innovate to be acquired because they receive a fraction of the surplus. Yet, if large firm innovation is one-dimensional, then the entry for buyout effect needs to be weighed against the *negative* effect on large firm innovation. In this paper, acquisitions are made with take-it-or-leave-it offers, so the entry for buyout effect is absent. Instead, acquisitions may lead to higher growth precisely *because* they reduce large firm innovation. A distinct implication is that acquisitions have more potential to increase growth if concentration is higher.

Model Building Blocks:
The model builds on two different strands of the growth literature, one focused on models of creative destruction[8], and one on expanding varieties models[9]. Recent work combines the two, but without large firms with positive market shares.[10]

2 Qualitative Industry Model

I first study an oligopolistic growth model with a single industry in which firms can only innovate through creative destruction. The model illustrates the key mechanisms of the theory and is sufficiently tractable to yield analytical results.

[8]See Grossman and Helpman (1991b), Aghion and Howitt (1992), Klette and Kortum (2004), and Aghion, Bergeaud, Boppart, Klenow, and Li (2022).

[9]See Romer (1990) and Grossman and Helpman (1991a).

[10]See Atkeson and Burstein (2019).

2.1 Model

2.1.1 Overview

Time is continuous and indexed by $t \in [0, \infty)$. At each time t, there is a unit measure of small firms indexed by $i \in [0, 1]$, and a single large firm denoted by $i = L$. They produce a unit measure of intermediate goods, indexed by $j \in [0, 1]$, which they sell to a representative household with R to spend. Firms hire labor at wage W to produce and innovate, and maximize expected discounted profits with discount rate r.

This simplified model is in partial equilibrium: I take as given expenditures on each good R, the wage W, and the discount rate r.

2.1.2 Static Block

Demand:
At each time t, the household takes as given a price offered by each firm for each intermediate good, $\{p_t(i, j)\}$. All versions of a good j are perfect substitutes, so the household purchases only the cheapest one, with price $p_t(j) = \min\{p_t(i, j)\}_{i \in [0,1] \cup \{L\}}$, and splits purchases evenly if multiple versions have price $p_t(j)$. The household chooses consumption of each version of each good $\{c_t(j)\}$ to maximize a Cobb-Douglas aggregate C_t defined by

$$\ln(C_t) = \int_0^1 \ln(c_t(j))dj,$$

subject to spending R:

$$\int_0^1 p_t(j)c_t(j)dj = R.$$

It follows that the household spends R on each good: $c_t(j) = R/p_t(j)$. Consumption of each firm's output of each good, $\{c_t(i, j)\}$ is implied.

Production and Competition:
At each time t, production occurs in two stages. Each firm can potentially produce a version of each good in its industry with a version specific productivity $z_t(i, j)$. Let $z_t(j) \equiv \max\{z_t(i, j)\}_{i \in [0,1] \cup \{L\}}$ be the highest productivity version of good j at time t. Given the innovation process that I describe in the next subsection, for each j, t, there is a unique firm with $z_t(i, j) = z_t(j)$.

In the first stage of production, firms simultaneously choose for which goods they will pay a fixed cost to access their version of the good. If firm i pays the fixed cost for good j, then they can produce it in the second stage using labor with production function

$$q_t(i,j) = z_t(i,j)l_t(i,j).$$

Otherwise, they can produce good j with productivity $z_t(j)/\sigma$, where $\sigma > 1$ captures the ability of firms to imitate each other's versions of a good. The fixed cost is f_S and f_L units of labor for small firms and the large firm, respectively. In the second stage of production, fixed cost payments are common knowledge and firms simultaneously choose prices for each good to maximize static profits subject to the household's demand given prices: $q_t(i,j) \leq c_t(i,j)$.

Finally, the large firm's revenue is subject to a tax τ_R so that it is multiplied by $1 - \tau_R$.

I study equilibria in which only the most productive producer of good j pays the fixed cost, and for any first stage actions by their competitors, they set their price equal to the marginal cost of the second-most productive producer. Thus, in equilibrium, they set a markup of price over marginal cost equal to σ. These strategies form an equilibrium if and only if the most productive producer has a sufficiently low fixed cost so that they earn positive profits. Note that there are no equilibria in which any firm other than the most productive producer pays the fixed cost as long as σ is less than the innovation step size, which I define in the next subsection. This is always the case in the calibrated model.

To ensure the desired equilibrium exists, I make the following assumption.

Assumption 1. *The most productive producer of a good earns strictly positive profits:*

$$\left(1 - \sigma^{-1}\right) R > max\{\tau_R R + f_L W, f_S W\}.$$

Aggregation:

Given that all goods are sold with a markup σ, household industry consumption is $C_t = Z_t R/(\sigma W)$, where industry productivity is given by

$$\ln(Z_t) = \int_0^1 \ln(z_t(j))dj.$$

2.1.3 Dynamic Block

Innovation:

Each firm contains entrepreneurs it uses to innovate. At each time t, for each of its entrepreneurs, firm i chooses a creative destruction rate $\kappa_t(i,j)$ for each good $j \in [0,1]$. Each of its entrepreneurs creatively destroys a good at Poisson arrival rate $\int_0^1 \kappa_t(i,j)dj$, and the relative probability it creatively destroys good j is proportional to $\kappa_t(i,j)$. If a firm creatively destroys a good, then the productivity of its version of the good jumps to $\lambda z_t(j)$, where $\lambda > 1$ is the innovation step size, i.e., the firm becomes the sole producer of the good.

A small firm contains a single entrepreneur, creatively destroys goods at a Poisson arrival rate, and in equilibrium produces a finite number of goods. A small firm's flow labor cost of innovation is $\int_0^1 X_S(\kappa_t(i,j))dj$, where $X_S(\cdot)$ is twice continuously differentiable, strictly increasing, and weakly convex, with $X_S(0) = 0$. Moreover, the elasticity of the marginal innovation cost,

$$\epsilon_S(\kappa) \equiv \frac{\kappa X_S''(\kappa)}{X_S'(\kappa)},$$

is well-defined and continuous for all $\kappa \geq 0$.

The large firm contains a measure of entrepreneurs, creatively destroys *each good* at a Poisson arrival rate, and so creatively destroys goods at a continuous rate, and in equilibrium produces a finite *measure* of goods. If the large firm creatively destroys each good j at Poisson arrival rate $\kappa_t(L,j)$, then its flow labor cost of innovation is $\beta \int_0^1 X_L(\kappa_t(L,j))dj$, where $\beta > 0$ is a cost shifter, and $X_L(\cdot)$ is twice continuously differentiable, strictly increasing, and weakly convex, with $X_L(0) = 0$. Moreover, the elasticity of the marginal innovation cost,

$$\epsilon_L(\kappa) \equiv \frac{\kappa X_L''(\kappa)}{X_L'(\kappa)},$$

is well-defined and continuous for all $\kappa \geq 0$.

That a small firm contains a single entrepreneur and the large firm contains a measure is the only substantive difference between large and small firms.

Acquisitions:
At each time t, the large firm chooses an acquisition opportunity rate $A_t(j)$ for each good $j \in [0,1]$. At Poisson arrival rate $A_t(j)$, the large firm encounters the firm with the most productive version of good j. The firms play a two stage game in which the large firm makes a take-it-or-leave-it offer subject to an acquisition tax rate τ_A, and the other firm chooses whether to accept. If the other firm accepts, then its version is transferred to the large firm, the large firm pays the offered price, and pays $\tau_A > -1$ times the price to the tax authority. If the surplus of an acquisition is strictly positive, then there is a unique equilibrium in which the large firm offers the other firm's value, and the other firm accepts. If the surplus is strictly negative, then in all equilibria, there is no acquisition. Finally, if the surplus is zero, then I suppose there is no acquisition; this choice does not affect the results that follow.

The flow labor cost of searching for potential acquisitions is $\int_0^1 X_A(A_t(j))dj$, where $X_A(\cdot)$ is twice continuously differentiable, strictly increasing, and weakly convex, with $X_A(0) = 0$. Moreover, the elasticity of the marginal search cost,

$$\epsilon_A(A) \equiv \frac{AX''_A(A)}{X'_A(A)},$$

is well-defined and continuous for all $A \geq 0$.

Equilibrium:
At each time t, firms choose fixed cost payments and prices to maximize static profits in a subgame perfect Nash equilibrium of the production game. The large firm chooses innovation rates, acquisition search rates, and acquisition price offers, and small firms choose innovation rates and acquisition price acceptance strategies to maximize expected present discounted profits. I focus on Markov perfect equilibria without any state in which a small firm creatively destroy all goods it doesn't currently produce at the same rate κ_S (all small firms choose the same rate), the large firm creatively destroys all goods it doesn't currently at the same rate κ_L, and the large firm searches for the opportunity to acquire each good it doesn't currently at the same rate A. Moreover, if $A > 0$, then the large firm makes an offer the small firm accepts with probability 1. **Firms do not creatively destroy goods they currently produce.**

Denote by $\mathcal{L}_t \in [0, 1]$ the measure of goods for which the large firm has the most productive version at time t, which is also the large firm's share of industry revenue. The large firm revenue share evolves over time according to

$$\dot{\mathcal{L}}_t = (\kappa_L + A)(1 - \mathcal{L}_t) - \kappa_S \mathcal{L}_t, \tag{1}$$

where a dot over a variable indicates the derivative with respect to time t.

The growth rate of industry output, $\dot{C}_t/C_t$, is the growth rate of industry productivity:

$$g_t \equiv \dot{Z}_t/Z_t = (\lambda - 1)((1 - \mathcal{L}_t)\kappa_L + \kappa_S). \tag{2}$$

The growth rate reflects that the large firm potentially creatively destroys only a fraction of the goods in the economy, whereas each small firm potentially creatively destroys all but a set with measure zero of the goods.

I focus on balanced growth path equilibria in which the large firm revenue share, and thus the growth rate, are constant over time. In that case,

$$\mathcal{L} = (\kappa_L + A)/(\kappa_L + A + \kappa_S). \tag{3}$$

2.1.4 Firm Optimization

Small Firm Innovation:

The value of a small firm currently producing n goods is the sum of two components: n times the expected present discounted profits from producing a single good, Π, and the value of an entrepreneur's innovation technology at a small firm producing zero goods. Only Π is relevant to a small firm's innovation decision, and it is given by the Hamilton-Jacobi-Bellman (HJB) equation:

$$r\Pi = \left(1 - \sigma^{-1}\right) R - f_S W - (\kappa_L + \kappa_S)\Pi, \tag{4}$$

where the terms on the right-hand side are flow profits and the rate at which the good is creatively destroyed. The possibility of an acquisition does not affect a small firm's value because it receives exactly its value of the good.

At each time t, a small firm chooses innovation rates $\{\kappa(j)\}$ to maximize

$$\int_0^1 (\kappa(j)\Pi - W X_S(\kappa(j)))dj.$$

The First Order Condition for each $\kappa(j)$ shows that a small firm creatively destroys each good at the same rate κ_S given by

$$X_S'(\kappa_S) \geq \Pi/W, \tag{5}$$

where the inequality is an equality if $\kappa_S > 0$, and the expected present discounted profits from producing a single good are

$$\Pi = \frac{\left(1 - \sigma^{-1}\right) R - f_S W}{r + \kappa_L + \kappa_S}. \tag{6}$$

To allow for the possibility of an equilibrium with $\kappa_S > 0$, I make the following assumption.

Assumption 2. *If innovation rates are zero, small firms find it profitable to innovate:*

$$X_S'(0) < \frac{\left(1 - \sigma^{-1}\right) R - f_S W}{r}.$$

Large Firm Innovation and Acquisitions:

I split the large firm optimization problem into two steps. First, taking as given a rate at which its revenue share increases, $\partial \mathcal{L}_t / \partial t$, the large firm chooses $\kappa(j)$, $A(j)$, and $\tilde{A}(j)$ for all goods j it does not currently produce to minimize cost, where $\tilde{A}(j)$ is the probability, conditional on finding a potential acquisition, the large firm offers Π and acquires the good; otherwise, the large firm makes an offer that is rejected. The large firm optimally creatively destroys and searches for all its competitors' goods at single rates κ_L and A, and sets $\tilde{A}(j) = 1$ if $A(j) > 0$ because $X_L(\cdot)$ and

$X_A(\cdot)$ are strictly increasing and convex. Second, the large firm optimally chooses κ_L and A. The HJB equation for the large firm's value of producing a fraction $\mathcal{L}_t$ of the industry goods is

$$
\begin{aligned}
rV(\mathcal{L}_t) = & \left(\left(1-\sigma^{-1}-\tau_R\right)R - f_L W\right)\mathcal{L}_t \\
& + \max_{\kappa_L, A}\left\{\dot{\mathcal{L}}_t(\kappa_L, A)V'(\mathcal{L}_t) - W\beta X_L(\kappa_L) - WX_A(A) - A(1+\tau_A)\Pi\right\},
\end{aligned} \tag{7}
$$

where $\mathcal{L}_t(\cdot,\cdot)$ is the rate at which $\mathcal{L}_t$ increases as a function of the large firm's actions, taking as given small firm innovation. The terms on the right-hand side of the HJB equation are flow profits and the innovation/acquisition optimization problem.

Taking as given creative destruction and acquisition rates that do not depend on $\mathcal{L}_t$, κ_L and A, I guess and verify that the marginal value of a good does not either. Setting $V'(\mathcal{L}_t) = \bar{V}$ and differentiating each side of equation (7) yields

$$
\bar{V} = \frac{\left(1-\sigma^{-1}-\tau_R\right)R - f_L W + W\beta X_L(\kappa_L) + WX_A(A) + A(1+\tau_A)\Pi}{r+\kappa_L+A+\kappa_S}. \tag{8}
$$

The First Order Conditions for κ_L and A show that they do not depend on $\mathcal{L}_t$ and are given by

$$
\beta X_L'(\kappa_L) \geq \bar{V}/W \tag{9}
$$

which holds with equality if $\kappa_L > 0$, and

$$
X_A'(A) \geq (\bar{V} - (1+\tau_A)\Pi)/W, \tag{10}
$$

which holds with equality if $A > 0$.

2.2 Results

I characterize the effects of various shocks or policy changes on the long-run growth rate. I begin with changes in parameters that only directly affect large firm innovation incentives—the large firm's innovation cost shifter β, fixed cost f_L, and revenue tax rate τ_R. I then consider changes in the acquisition tax rate τ_A.

Some results depend on taking derivatives of equilibrium outcomes with respect to exogenous parameters. Thus, I begin with the following proposition concerning the uniqueness of equilibria.

Proposition 1. *Small firm optimization—inequality* (5) *and equation* (6)*—implicitly define a continuous decreasing function $\kappa_S(\kappa_L)$ that does not depend on large firm variables β, $X_L(\cdot)$, f_L,* *or τ_R. There is a $\kappa_L^* > 0$, possibly infinite, such that $\kappa_S(\cdot)$ is continuously differentiable on $[0, \kappa_L^*)$, and $\kappa_S(\kappa_L) > 0$ if and only if $\kappa_L < \kappa_L^*$.*

Suppose τ_A is sufficiently large so that the acquisition rate A is always zero, $X_L(\cdot)$ is strictly convex, and for all κ_L, $\epsilon_L(\kappa_L)(\epsilon_S(\kappa_S(\kappa_L)) + 1) \geq 1$. There is a unique equilibrium κ_L, which is a continuous increasing function of $\left(\left(1 - \sigma^{-1} - \tau_R\right) R - f_L W\right) / \beta$ that is strictly increasing if $\kappa_L > 0$, and is continuously differentiable everywhere except at $\kappa_L = \kappa_L^*$.

2.2.1 Large Firm Innovation and Growth

I study the long-run effects of shocks and policy changes that only directly affect large firm innovation incentives: the large firm's innovation cost shifter, β, fixed cost, f_L, and revenue tax rate τ_R. Throughout, I suppose the acquisition tax rate τ_A and search cost function $X_A(\cdot)$ are such that $A = 0$ is optimal, i.e., there are no acquisitions. Using the first part of Proposition 1, one way to understand the effects of changes in large firm variables is to study the effects of exogenous changes in κ_L on small firm optimal innovation. For example, if an increase in κ_L leads to a higher long-run growth rate g taking into account small firm optimization, and if a combination of changes to β, f_L, and τ_R leads to an equilibrium with a higher κ_L, then that equilibrium has a higher g. The second part of Proposition 1 shows how to map the results into results about the effects of particular changes in β, f_L, and τ_R.

I now characterize the effect of large firm innovation on the long-run growth rate. I focus on $\kappa_L < \kappa_L^*$, defined in Proposition 1, so that $\kappa_S(\kappa_L)$ is continuously differentiable, which implies that the derivatives I take throughout are well-defined. In Theorem 1, I cover the case $\kappa_L \geq \kappa_L^*$ (for finite κ_L^*) as well.

To build intuition, I write the long-run growth rate as the product of two terms:

$$g = \underbrace{\bar{\kappa}}_{\text{Discount}} \underbrace{\frac{g}{\bar{\kappa}}}_{\text{Composition}}, \tag{11}$$

where the first term, $\bar{\kappa} \equiv \kappa_L + \kappa_S$, is the rate at which a small firm's good is creatively destroyed, which is the non-interest component of the discount rate on a small firm's good. If small firms can tolerate a higher discount rate, then there is more innovation and growth. The second term is the growth rate relative to the discount rate, or the discount rate's "composition", which depends only on relative innovation rates:[11] $g/\bar{\kappa} = (\lambda - 1)(1 - \mathcal{L}^2)$. Large firm innovation produces little growth relative to its effect on the discount rate because it only targets small firms' goods.

[11]Divide equation (2) by $\bar{\kappa}$, and use equation (3) to replace $\kappa_L/\bar{\kappa}$ with $\mathcal{L}$ and $\kappa_S/\bar{\kappa}$ with $1 - \mathcal{L}$.

I use equation (11) to decompose the effect of a change in large firm innovation:

$$\frac{\partial g}{\partial \kappa_L} = \underbrace{\frac{\partial \bar{\kappa}}{\partial \kappa_L}\frac{g}{\bar{\kappa}}}_{\text{Discount Effect}} + \underbrace{\bar{\kappa}\frac{\partial (g/\bar{\kappa})}{\partial \kappa_L}}_{\text{Composition Effect}} .$$

The "Discount Effect" is positive.[12] An increase in large firm innovation increases the small firm discount rate, and so the growth rate, because small firms face convex innovation costs. The "Composition Effect" is negative.[13] An increase in large firm innovation shifts the source of creative destruction to the large firm, which in the long-run increases the large firm's share of industry goods. Each reduces the growth rate because large firm innovation only targets small firm goods, whereas small firm innovation targets all goods.

Combining the two effects, $\partial g/\partial \kappa_L$ is the product of a strictly positive function and[14]

$$\epsilon_S(\kappa_S) - \frac{2\mathcal{L}}{1-\mathcal{L}}\frac{\bar{\kappa}}{r+\bar{\kappa}}, \tag{12}$$

which is therefore a sufficient statistic based on measurable equilibrium outcomes for whether an increase in large firm innovation leads to an increase in the long-run growth rate. In particular, $\epsilon_S(\kappa_S)$ is the *inverse elasticity of small firm innovation with respect to the value of innovating.* If small firm innovation is sufficiently elastic, then an increase in large firm innovation leads to lower growth because more elastic small firm innovation implies a weaker discount effect and a stronger composition effect. Moreover, expression (12) shows that for an increase in κ_L to decrease growth, concentration must be sufficiently high because it drives the composition effect, and creative destruction must be a sufficient share of the small firm discount because otherwise, a small percentage increase in the small firm discount implies a large percentage increase in innovation.

The following theorem characterizes the relationship between large firm innovation and the effect of a marginal increase in large firm innovation. If variation in large firm innovation drives variation across industries, economies, or over time, then the theorem also characterizes the relationship between concentration and growth.

Theorem 1. *Vary κ_L and use $\kappa_S(\kappa_L)$ defined in Proposition 1 to determine small firm innovation.*

g is a continuous function of κ_L that is continuously differentiable everywhere except at κ_L^* with the following properties:

[12]This follows from equation (26) in the proof of Proposition 1 in the appendix, which shows that $\kappa_S'(\kappa_L) \geq -1$, and strictly so if $\epsilon_S(\kappa_S) > 0$.

[13]This holds because $\kappa_S(\kappa_L)$ is decreasing.

[14]Use $\kappa_S'(\kappa_L)$ given in equation (26) in the proof of Proposition 1 in the appendix to take the derivatives of $\mathcal{L}$ and $\bar{\kappa}$ with respect to κ_L. The strictly positive function multiplying expression (12) is $(\lambda-1)(1-\mathcal{L})^2\frac{r+\bar{\kappa}}{\epsilon_S(\kappa_S)(r+\bar{\kappa})+\kappa_S}$.

1. *$g > 0$ if and only if $\kappa_L < \kappa_L^*$.*

2. *$\partial g/\partial \kappa_L \geq 0$ at $\kappa_L = 0$, and strictly so if and only if $\epsilon_S(\kappa_S(0)) > 0$.*

3. *There exists a $\kappa_L' < \kappa_L^*$ such that for all $\kappa_L \in (\kappa_L', \kappa_L^*)$, $\partial g/\partial \kappa_L < 0$.*

4. *If $\epsilon_S(\cdot)$ is constant, then there is a single $\kappa_L < \kappa_L^*$ at which $\partial g/\partial \kappa_L = 0$.*

The first three properties of g, taken together, state that as large firm innovation starts at 0 and increases, initially the long-run growth rate increases, but then eventually decreases and goes to 0 as the large firm takes over the industry. The final property of g states that if the elasticity of small firm innovation is constant, then this pattern fully characterizes the relationship between large firm innovation and growth in the sense that g exhibits an inverse-U shape.

Intuitively, if the large firm produces an insignificant share of industry goods, then it innovates like a small firm and targets all goods, so the composition effect is zero. In the other limit, the growth rate is zero because the large firm deters small firm innovation by creatively destroying small firm goods at a high rate, but there are no small firm goods to creatively destroy.

Finally, suppose variation in large firm innovation drives variation in concentration and growth. To maximize the long-run growth rate, a policymaker should discourage innovation by sufficiently large firms. If a small firm free entry condition fixes the value of a small firm innovation ($\epsilon_S^{-1} = \infty$), then higher concentration is associated with lower growth. If small firm innovation is less elastic ($\epsilon_S^{-1} < \infty$), then growth as a function of concentration has an inverted-U shape.

2.2.2 Acquisition Policy

Proposition 2. *Suppose $X_L(\cdot)$ and $X_A(\cdot)$ are strictly convex and for all κ_L, $\epsilon_L(\kappa_L)(1+\epsilon_S(\kappa_S(\kappa_L)) \geq 1$. There is a unique equilibrium, and κ_L, A, and g are continuous functions of τ_A with the following properties:*

1. *κ_L is increasing in τ_A and strictly so if $\kappa_L, A > 0$.*

2. *A is decreasing in τ_A and strictly so if $A > 0$.*

3. *If $\kappa_L, \kappa_S, A > 0$, then κ_L, A, and g are continuously differentiable with respect to τ_A.*

If $A = 0$ (and not at the boundary), then changes in the acquisition tax rate have no effect. If $\kappa_L = 0$, then the large firm has no effect on growth, which is $(\lambda - 1)\kappa_S(0)$. If $\kappa_S = 0$, then growth is 0. Thus, I focus on the case in which $\kappa_S, \kappa_L, A > 0$, so that changes in the acquisition tax rate can have an effect on the long-run growth rate.

I study the long-run effects of changes to the acquisition tax rate τ_A. To guarantee a unique equilibrium and that large firm decisions are continuously differentiable with respect to relevant exogenous parameters, I use Proposition 1 and impose throughout that $X_L(\cdot)$ and $X_A(\cdot)$ are strictly convex and for all κ_L, $\epsilon_L(\kappa_L)(1+\epsilon_S(\kappa_S(\kappa_L))) \geq 1$. I decompose the effect of a change in the tax rate on the long-run growth rate (see Appendix A.4 for derivations):

$$\frac{\partial g}{\partial \tau_A} = \underbrace{\frac{\partial\left(\bar{V}-(1+\tau_A)\Pi\right)}{\partial \tau_A}\frac{\partial A}{\partial\left(\bar{V}-(1+\tau_A)\Pi\right)}\frac{\partial g}{\partial A}}_{\text{Acquisition Effect}} + \underbrace{\frac{\partial \bar{V}}{\partial \tau_A}\frac{\partial \kappa_L}{\partial \bar{V}}\frac{\partial g}{\partial \kappa_L}}_{\text{Innovation Effect}},$$

where $\partial g/\partial A$ is taken holding innovation rates fixed, and $\partial g/\partial \kappa_L$ is taken holding the acquisition rate fixed, but taking into account optimal small firm innovation $\kappa_S(\kappa_L)$. In each term, the first derivative is the equilibrium response of the value of acquiring or innovating, the second derivative is the response of the acquisition or innovation rate to the value of acquiring or innovating determined by the First Order Condition for each (equations (9) and (10)), and the third derivative is the effect on growth of changes in the acquisition or innovation rate.

The "Acquisition Effect" is positive. An increase in the tax rate decreases the surplus from each acquisition, which decreases the acquisition rate, reduces the large firm's share of industry goods, and increases the long-run growth rate. The sign of the "Innovation Effect" is the sign of $\partial g/\partial \kappa_L$. The decrease in the surplus from acquisitions leads to an increase in the marginal value of a good, and so in large firm innovation because the large firm's desire to creatively destroy its competitors' goods depends on the outside option, which is acquiring those goods instead. Thus, *encouraging acquisitions is good for growth exactly when large firm innovation is bad for growth.*

If $\kappa_L, \kappa_S(\kappa_L), A > 0$ and $\epsilon_S(\kappa_S) = 0$, then $\partial g/\partial \tau_A$ is the product of a strictly positive function and

$$\frac{1}{\epsilon_A(A)} - \frac{1}{1-\mathcal{L}}\frac{2\kappa_L + A}{\epsilon_L\,(r+\bar{\kappa}) - \kappa_L}\left(1 + \frac{(1+\tau_A)\Pi}{\bar{V}-(1+\tau_A)\Pi}\right)^{-1}, \tag{13}$$

which is therefore a sufficient statistic based on measurable equilibrium outcomes for whether an increase in the acquisition tax rate leads to an increase in the long-run growth rate. In particular, $\epsilon_L(\kappa_L)$ and $\epsilon_A(A)$ are the *inverse elasticities of large firm innovation with respect to the value of innovating and of the acquisition rate with respect to the value from an acquisition*, respectively. If the elasticity of large firm innovation is sufficiently high or the elasticity of the acquisition rate is sufficiently low, then an increase in the tax rate reduces growth; a higher innovation elasticity implies a stronger innovation effect, and a lower acquisition elasticity implies a weaker acquisition effect. For example, as the acquisition elasticity goes to 0, an increase in the acquisition tax rate always reduces growth because it has no effect on the acquisition rate, but lowers the value from acquisitions, which reduces the large firm's outside option to innovating, and results in more large firm innovation. In the appendix, I develop the more complicated analog to the expression in (13) in the general $\epsilon_S(\kappa_S)$ case. The intuition and results concerning $\epsilon_A(A)$ and $\epsilon_L(\kappa_L)$ are the same, and a lower $\epsilon_S(\kappa_S)$ lowers the threshold for $\epsilon_A(A)$, i.e., makes it easier for it to be the case that encouraging acquisitions increases growth.

The following theorem, similar to Theorem 1 supposes that variation in large firm innovation drives variation in industry concentration, and characterizes the relationship between concentration and the effect of an increase in the acquisition tax rate.

Theorem 2. *Suppose $\epsilon_S(\cdot)$, $\epsilon_L(\cdot)$, and $\epsilon_A(\cdot)$ are constants, with $\epsilon_S = 0$. Vary the large firm fixed cost f_L and revenue tax rate τ_R, holding other parameters fixed. There exists a long-run large firm revenue share cutoff $\mathcal{L}^*$ such that the acquisition rate A is strictly positive if and only if $\mathcal{L} > \mathcal{L}^*$. If $\mathcal{L}^* < 1$, then there exists a cutoff $\mathcal{L}^{**} \in (max\{\mathcal{L}^*, 0\}, 1)$ such that $\partial g/\partial \tau_A > 0$ if $\mathcal{L} < \bar{\mathcal{L}}$ and $\partial g/\partial \tau_A < 0$ if $\mathcal{L} > \bar{\mathcal{L}}$.*

Intuitively, given $\epsilon_S = 0$, if $\mathcal{L}$ is high, then the small firm discount is the same, but the large firm discount rate is lower given the shift in innovation to creative destruction that only targets small firms. Thus, the effect of an acquisition on the large firm revenue share, which depends on the small firm discount rate and drives the acquisition effect, is the same, but the effect of a valuable future acquisition opportunity on the value of innovating, which depends on the large firm discount rate and drives the innovation effect, is higher.

3 Quantitative Model

I now develop the richer quantitative model. There is a unit measure of industries, indexed by $n \in [0, 1]$, each of which consists of a measure of differentiated intermediate goods, indexed by $j \in [0, M_{n,t}]$. There is a representative household who consumes the intermediate goods in each industry, and inelastically supplies $\bar{L}$ units of labor. In each industry, a single large firm and a continuum of small firms use labor to produce, develop new goods, and improve on old goods, and the intermediate goods from each industry to cover fixed costs. The household owns all firms and has access to a risk-free bond in zero net supply.

The wage, interest rate, and expenditures on goods are determined in general equilibrium.

3.1 Representative Household Problem

The household chooses a path of consumption bundles to maximize the present discounted value of utility, taking prices as given. The household problem can be split into two steps: first the household chooses final good consumption to maximize the present discounted value of utility, and then at each time t, chooses a consumption bundle to minimize cost subject to achieving the specified level of final good consumption. Thus, in the first stage, the household maximizes

$$\int_0^\infty e^{-\rho t} \ln(C_t) dt,$$

subject to the budget constraint

$$P_t C_t + \dot{B}_t = W_t \bar{L} + D_t + r_t B_t,$$

where $\rho > 0$ is the time discount rate, C_t is final good consumption, W_t is the wage, D_t is flow profits from firms, B_t is bond holdings, r_t is the net interest rate, and P_t is the final good price that the household anticipates as the outcome of the second stage of its optimization problem. In the second stage, at each time t, the household chooses consumption of each good in each industry to minimize cost

$$\int_0^1 \int_0^{M_{n,t}} p_{n,t}(j) c_{n,t}(j) dj dn$$

subject to the aggregation functions

$$ln(C_t) = \int_0^1 ln(C_{n,t}) dn \qquad C_{n,t}^{\frac{\gamma-1}{\gamma}} = \int_0^{M_{n,t}} c_{n,t}(j)^{\frac{\gamma-1}{\gamma}} dj \quad \text{for all } n \in [0, 1],$$

where $c_{n,t}(j)$ is consumption of good j in industry n, $p_{n,t}(j)$ is that good's price, and $\gamma > 1$ is the elasticity of substitution across goods within an industry. Thus, the final good is a Cobb-Douglass aggregate of industry goods, each of which is a Constant Elasticity of Substitution aggregate of the differentiated intermediate goods within the industry.

3.2 Household Optimization and Demand

From the first stage of the household problem, the stochastic discount factor is $e^{-\rho_t}/C_t$. For the bond market to clear with zero net supply, i.e., $B_t = 0$ for all t, the net interest rate must equal the negative rate of change of the stochastic discount factor over time:

$$r_t = \rho + \dot{C}_t / C_t. \tag{14}$$

The second stage of the household problem yields its demand for each intermediate good at each time t. The First Order Condition for good j in industry n, along with aggregation, implies the demand curve[15]

$$c_{n,t}(j) = p_{n,t}(j)^{-\gamma} P_{n,t}^{\gamma-1} P_t C_t, \tag{15}$$

where the industry and final good price indices are given by

$$ln(P_t) = \int_0^1 ln(P_{n,t})dn \qquad P_{n,t}^{1-\gamma} \equiv \int_0^{M_{n,t}} p_{n,t}(j)^{1-\gamma}dj \quad \text{for all } n \in [0,1]. \tag{16}$$

3.3 Intermediate Goods Producers

3.3.1 Production and Competition

Each industry consists of a measure of small firms and a single large firm. Static production and competition are as in Section 2.

Let $Z_{n,t}$ be an aggregate of productivity in industry n and Z_t be aggregate productivity:

$$Z_{n,t}^{\gamma-1} \equiv \int_0^{M_{n,t}} z_{n,t}(j)^{\gamma-1}dj \qquad \ln(Z_t) \equiv \int_0^1 \ln(Z_{n,t})dn,$$

and define the relative productivity of firm i's version and the most productive version of a good:

$$\tilde{z}_{n,t}(i,j) \equiv (z_{n,t}(i,j)/Z_{n,t})^{\gamma-1} \qquad \tilde{z}_{n,t}(j) \equiv (z_{n,t}(j)/Z_{n,t})^{\gamma-1}.$$

The differences from Section 2 are that the fixed cost is denoted in units of the final good—using the same aggregation functions as for the household—the fixed cost a firm must pay for a good scales with its version's relative productivity and with aggregate productivity, and the large firm's fixed cost varies across industries: the fixed cost for a small firm is $P_t Z_t f_S \tilde{z}_{n,t}(i,j)$, and for a large firm is $P_t Z_t f_{L,n} \tilde{z}_{n,t}(i,j)$.

I make the following assumption, which implies that all firms set a markup of σ because a firm's price for a good is always constrained by the ability of its competitors to imitate its version.

Assumption 3. $\sigma \leq \gamma/(\gamma-1)$.

[15]The First Order Condition yields $p_{n,t}(j) = \zeta C_{n,t}^{\frac{1-\gamma}{\gamma}} c_{n,t}(j)^{\frac{-1}{\gamma}}$, where ζ is the Lagrange multiplier on the constraint that final good consumption equals C_t. Industry aggregation and setting aggregate expenditures to $P_t C_t$ implies that $\zeta = P_t C_t$. Industry aggregation then implies that $C_{n,t} = P_t C_t / P_{n,t}$.

Moreover, Assumption 1 no longer applies, but I suppose f_S and f_L are sufficiently low so that there is an equilibrium in which only the most productive producer of a good pays the fixed cost.[16]

3.3.2 Innovation

As in Section 2, each firm contains entrepreneurs it uses to innovate. Now, an entrepreneur can also develop a new good at a Poisson arrival rate. Conditional on developing a new good, a firm's productivity z for that good is drawn so that the expected relative productivity $(z/Z_{n,t})^{\gamma-1}$ is 1. All other firms' versions of a new good have productivity zero.

Each small firm contains a single entrepreneur. It creatively destroys a good at Poisson arrival rate $\int_0^{M_{n,t}} \kappa(j)dj$, where the relative probability it creatively destroys good j is proportional to $\kappa(j)$, and develops a new good at Poisson arrival rate δ. The flow labor cost is

$$(\epsilon+1)^{-1}\left(\alpha\int_0^{M_{n,t}} \tilde{z}_{n,t}(j)\kappa(j)^{\epsilon+1}dj+\delta^{\epsilon+1}\right),$$

where $\epsilon > 0$ is the marginal innovation cost elasticity (now constant), and $\alpha > 0$ is the relative cost of creative destruction. Each innovative activity has an independent cost that scales with the expected relative productivity of a successful innovation.

The large firm in industry n contains a measure $\beta_n^{-1/\epsilon}$ of entrepreneurs. It creatively destroys *each* good in its industry at a Poisson arrival rate, and develops a new good at a *continuous rate*. Cost minimization implies that it assigns the same innovation rates to each of its entrepreneurs, so that the flow labor cost is the same as for a small firm, but multiplied by the cost shifter β_n.

3.3.3 Acquisitions

Acquisitions are as in Section 2, except that the cost of searching for good j scales with its relative productivity. A large firm can only acquire goods within its industry.

[16]In practice, it is sufficient to impose this is the case on the initial calibrated balanced growth path because I consider shocks that raise expenditures on goods, and so the return to paying the fixed cost.

3.3.4 Entry and Exit

The measure of small firm entrepreneurs is no longer exogenous. At each time t, there is an infinite mass of potential entrants. Each potential entrant can receive value 0 or pay $\xi > 0$ units of labor to draw an industry from the uniform distribution and enter as a small firm with an entrepreneur and a 0 productivity version of each good.

Each small firm entrepreneur exits exogenously at Poisson arrival rate $\eta > 0$. After losing its entrepreneur, a small firm can produce, but not innovate. Thus, the measure of small firm *entrepreneurs* in an industry, which is relevant, is not equal to the measure of small firms producing in equilibrium, which is not relevant.

The measure of small firm entrepreneurs at time t, N_t, is the same in each industry because entry is *undirected*. It evolves over time due to entry e_t and exit according to

$$\dot{N}_t = e_t - \eta N_t.$$

3.4 Equilibrium

At each time t, firms choose fixed cost payments and prices to maximize static profits in a subgame perfect Nash equilibrium of the two stage production game. The large firm chooses innovation rates, acquisition search rates, and acquisition price offers, small firms choose innovation rates and acquisition price acceptance strategies, and potential entrants choose entry strategies to maximize expected present discounted profits using the interest rate to discount future payoffs. In the two stage production game, I focus on the equilibrium in which for each good, only the firm with the most productive version pays the fixed cost, and they set a markup of price over marginal cost equal to σ. In the dynamic game, I focus on Markov perfect equilibria in which the industry state is the large firm's share of industry relative productivity (or equivalently revenue):

$$\mathcal{L}_{n,t} \equiv \int_{j \in J_{n,t}} \tilde{z}_{n,t}(j) dj,$$

where $J_{n,t}$ is the set of goods in industry n of which the large firm has the most productive version, and the aggregate state is the measure of small firm entrepreneurs N_t, the distribution of industry states, and aggregate productivity Z_t. Firm actions can be a function of the industry and aggregate state, *other than* Z_t. Moreover, in industry n, all small firms creatively destroy all goods at the same rate $\kappa_{S,n,t}(\mathcal{L}_{n,t})$, develop new goods at the same rate $\delta_{S,n,t}(\mathcal{L}_{n,t})$, and accept acquisition offers weakly above their values; and the large firm creatively destroys all its competitors' goods at the same rate $\kappa_{L,n,t}(S;\mathcal{L}_{n,t})$, creatively destroys all its own goods at the same rate $\kappa_{L,n,t}(L;\mathcal{L}_{n,t})$, develops new goods at rate $\delta_{L,n,t}(\mathcal{L}_{n,t})$, and searches for each acquisition opportunity at the same rate $A_{n,t}(\mathcal{L}_{n,t})$—and always offers the small firm's value of the good, which is accepted. In each case, the t subscript captures the dependence on the aggregate state other than Z_t.

At each t, each intermediate good's market must clear, i.e., supply from firms equals the demand from the representative household for consumption and from other firms for fixed costs; and the labor market must clear, i.e., the labor used for production, innovation, acquisition search, and entry costs must equal the labor the household inelastically supplies.

I focus on balanced growth path equilibria and the convergence to a balanced growth path following an unanticipated shock. A balanced growth path is an equilibrium in which each industry's state and the aggregate state other than Z_t are constant over time, and Z_t grows at a constant rate.

3.5 Evolution of the Industry State and Industry Growth

In equilibrium, the industry state evolves over time according to

$$\begin{aligned}\dot{\mathcal{L}}_{n,t} =&(\kappa_{L,n,t}(S;\mathcal{L}_{n,t}) + (\gamma-1)g_{L,n,t}(\mathcal{L}_{n,t}) + A_{n,t}(\mathcal{L}_{n,t}))(1-\mathcal{L}_{n,t}) \\ &- N_t(\kappa_{S,n,t}(\mathcal{L}_{n,t}) + (\gamma-1)g_{S,n,t}(\mathcal{L}_{n,t}))\mathcal{L}_{n,t},\end{aligned} \tag{17}$$

where $g_{n,t}(\mathcal{L}_{n,t}) \equiv \dot{Z}_{n,t}/Z_{n,t}$ is the growth rate of industry productivity, which is the sum of growth due to large firm innovation, $g_{L,n,t}(\mathcal{L}_{n,t})$, and due to small firm innovation, $N_t g_{S,n,t}(\mathcal{L}_{n,t})$:

$$\begin{aligned}(\gamma-1)g_{L,n,t}(\mathcal{L}_{n,t}) &= \left(\lambda^{\gamma-1}-1\right)\left(\kappa_{L,n,t}(S;\mathcal{L}_{n,t})(1-\mathcal{L}_{n,t}) + \kappa_{L,n,t}(L;\mathcal{L}_{n,t})\mathcal{L}_{n,t}\right) + \delta_{L,n,t}(\mathcal{L}_{n,t}) \\ (\gamma-1)g_{S,n,t}(\mathcal{L}_{n,t}) &= \left(\lambda^{\gamma-1}-1\right)\kappa_{S,n,t}(\mathcal{L}_{n,t}) + \delta_{S,n,t}(\mathcal{L}_{n,t}).\end{aligned} \tag{18}$$

The difference between equation (17) and equation (1) in Section 2 is that now growth affects the distribution of sales. For example, if the large firm generates novel productivity either through developing a new good or creatively destroying an old one, it takes relative productivity from all the old goods in the industry, a fraction $1-\mathcal{L}_{n,t}$ of which comes from small firms. Equation (18) reflects that when a firm creatively destroys a good, only the improvement contributes to growth.

3.6 Aggregation and Welfare

At each time t, all industries use the same quantity of labor in production, L_t^p, and industry n output is $Y_{n,t} = Z_{n,t}L_t^p$.[17] On a balanced growth path, L_t^p is constant over time, and the growth rate of industry n output is g_n. Aggregating across industries implies that aggregate output is $Y_t = Z_t L_t^p$. The growth rate of aggregate productivity is $g_t = \int_0^1 g_{n,t}dn$, which is the growth rate of aggregate output on a balanced growth path. The real wage is $W_t/P_t = Z_t/\sigma$.[18]

Household welfare is

$$\int_0^\infty e^{-\rho t}\left(ln(Z_t) + \ln\left(L_t^p - \int\limits_0^1((1-\mathcal{L}_{n,t})f_S + \mathcal{L}_{n,t}f_{L,n})dn\right)\right)dt$$

$$= \frac{ln(Z_0)}{\rho} + \frac{\int_0^\infty \rho e^{-\rho t}g_t dt}{\rho^2} + \frac{\int_0^\infty \rho e^{-\rho t}\ln\left(L_t^p - \int\limits_0^1((1-\mathcal{L}_{n,t})f_S + \mathcal{L}_{n,t}f_{L,n})dn\right)dt}{\rho}, \tag{19}$$

which depends on current productivity and weighted averages of future growth, production labor, and fixed costs. Since growth in one period raises consumption in all future periods, it is discounted by ρ^2 rather than ρ.

On a balanced growth path, welfare is $\ln(Z_0)/\rho + g/(\rho^2) + \ln\left(L^p - \int\limits_0^1((1-\mathcal{L}_n)f_S + \mathcal{L}_n f_{L,n})dn\right)/\rho$.

3.7 Firm Optimization

Before describing the firm problem, note that since small firms take industry aggregates as given, we can split their static profit maximization problem into a separate problem for each good. Moreover, when innovating, a small firm's problem is the same regardless of the goods it produces.

3.7.1 Static Profit Maximization

At each time t, firms play the following subgame perfect Nash equilibrium of the two stage production game. In the second stage, given fixed cost payments, if a firm has the most productive active

[17] Constant markups and industry aggregation (expression (16)) imply that $P_{n,t} = \sigma W_t/Z_{n,t}$, so it follows from footnote 15 that $Y_{n,t} = (\sigma W_t)^{-1}Z_{n,t}P_tY_t$. The production function and the demand curve (equation (15)) for good j, along with the expression for $P_{n,t}$ imply that $l_{n,t}(j) = y_{n,t}(j)/z_{n,t}(j) = (\sigma W_t)^{-1}\tilde{z}_{n,t}(j)^{\gamma-1}P_tY_t$. Aggregating up yields the result.

[18] Aggregating the expression for $P_{n,t}$ in footnote 17 across industries using expression (16) yields the result.

version of good j, then it sets its price for good j equal to the minimum marginal cost across all other firms' active versions; otherwise, it sets its price equal to its own marginal cost. In the first stage, only the firm with the most productive version of good j pays the fixed cost for good j.

In the second stage, a firm without the most productive active version of good j has no hope of earning strictly positive profits, so it is optimal to set price equal to marginal cost. A firm with the most productive active version would set a markup of at least $\gamma/(\gamma-1)$ if unconstrained by other producers of good j, so by Assumption 3, pricing below other producers' marginal costs is a binding constraint.[19] In the first stage, a firm without the most productive version of good j will earn zero profits in the second stage if it pays the fixed cost, so it is optimal not to. A small firm with the most productive version earns positive profits across both stages from paying the fixed cost—and so finds it optimal to do so—if $\left(1-\sigma^{-1}\right)Y_t/Z_t \geq f_S$. A large firm's fixed cost decision is more complicated because paying the fixed cost for some of its goods reduces the relative productivity of its other goods. If the large firm pays fixed costs for a fraction x of its relative productivity, then its share of industry relative productivity is

$$\tilde{\mathcal{L}}_{n,t}(x) \equiv \frac{x\mathcal{L}_{n,t}}{1-(1-x)\left(1-\sigma^{1-\gamma}\right)\mathcal{L}_{n,t}}$$

because its versions are replaced by their imitations, and it earns total profits across both stages $\tilde{\mathcal{L}}_{n,t}(x)\left(1-\sigma^{-1}-\tau_R\right)P_tY_t - x\mathcal{L}_{n,t}P_tZ_tf_{L,n}$, which is strictly concave in x. The large firm finds it optimal to pay the fixed cost for all goods for which it has the most productive version if the first derivative of total profits at $x=1$ is positive: $\left(1-\left(1-\sigma^{1-\gamma}\right)\mathcal{L}_{n,t}\right)\left(1-\sigma^{-1}-\tau_R\right)Y_t/Z_t \geq f_{L,n}$.

3.7.2 Dynamic Profit Maximization

At each time t, firms simultaneously choose innovation rates, taking as given profit functions from static optimization.

Small Firms:
As in Section 2, the value of a small firm is the sum of the expected discounted profits from each of the goods for which it currently has the most productive version, and if the small firm has an entrepreneur, the value of a small firm with an entrepreneur that does not have the most productive version of any goods. The former determines the value of innovating, and the latter determines the value of entry. The expected discounted profits from being the most productive producer of good j in industry n at time t is $\tilde{z}_{n,t}(j)\Pi_{n,t}(\mathcal{L}_{n,t})$, where the time t subscript in $\Pi_{n,t}(\cdot)$

[19] See Edmond, Midrigan, and Xu (2021) for a derivation of the optimal markup with oligopoly, nested CES demand, and Bertrand competition.

captures the dependence on the aggregate state, and where $\Pi_{n,t}(\cdot)$ is given by the HJB equation:

$$r_t\Pi_{n,t}(\mathcal{L}_{n,t}) = \left(1-\sigma^{-1}\right)P_tY_t - P_tZ_tf_S - (N_t\kappa_{S,n,t}(\mathcal{L}_{n,t}) + \kappa_{L,n,t}(S;\mathcal{L}_{n,t}) + (\gamma-1)g_{n,t})\Pi_{n,t}(\mathcal{L}_{n,t}) + \dot{\mathcal{L}}_{n,t}\Pi'_{n,t}(\mathcal{L}_{n,t}) + \dot{\Pi}_{n,t}(\mathcal{L}_{n,t}). \tag{20}$$

Relative to equation (4) in Section 2, equation (20) includes the effects of changes in the industry and aggregate state, and since $\gamma > 1$, the rate at which industry productivity growth depreciates the good's relative productivity.

At each time t, a small firm chooses innovation rates $\{\kappa(j)\}, \delta$ to maximize

$$\left(\int_0^{M_{n,t}} \kappa(j)\lambda^{\gamma-1}\tilde{z}_{n,t}(j)dj + \delta\right)\Pi_{n,t}(\mathcal{L}_{n,t}) - W_t(\epsilon+1)^{-1}\left(\alpha\int_0^{M_{n,t}} \tilde{z}_{n,t}(j)\kappa(j)^{\epsilon+1}dj + \delta^{\epsilon+1}\right).$$

The First Order Conditions give the optimal single rate at which a small firm creatively destroys each good in its industry, and the optimal new good development rate:

$$\kappa_{S,n,t}(\mathcal{L}_{n,t}) = \left(\lambda^{\gamma-1}\Pi_{n,t}(\mathcal{L}_{n,t})/(W_t\alpha)\right)^{1/\epsilon} \qquad \delta_{S,n,t}(\mathcal{L}_{n,t}) = (\Pi_{n,t}(\mathcal{L}_{n,t})/W_t)^{1/\epsilon}. \tag{21}$$

A small firm values equally—conditional on relative productivity—a good gained through new good development, creatively destroying a small competitor's good, and creatively destroying a large competitor's good because it does not internalize the different effects these innovations have on the industry state or growth. Moreover, it creatively destroys all goods at the same rate because both the cost and benefit of an innovation scale with its relative productivity.

The value function of a small firm with an entrepreneur that does not have the most productive version of any goods is given by the HJB equation:

$$r_tE_{n,t}(\mathcal{L}_{n,t}) = \left(\kappa_{S,n,t}(\mathcal{L}_{n,t})\lambda^{\gamma-1} + \delta_{S,n,t}(\mathcal{L}_{n,t})\right)\Pi_{n,t}(\mathcal{L}_{n,t}) - \eta E_{n,t}(\mathcal{L}_{n,t}) - W_t(\epsilon+1)^{-1}\left(\alpha\kappa_{S,n,t}(\mathcal{L}_{n,t})^{\epsilon+1} + \delta_{S,n,t}(\mathcal{L}_{n,t})^{\epsilon+1}\right) + \dot{\mathcal{L}}_{n,t}E'_{n,t}(\mathcal{L}_{n,t}) + \dot{E}_{n,t}(\mathcal{L}_{n,t}). \tag{22}$$

The right-hand side is the benefit and cost from innovation, the risk of exit, and the effects of changes over time in the industry and aggregate state.

At each time t, the value of entry net of the cost, $\int_0^1 E_{n,t}(\mathcal{L}_{n,t})dn - \xi W_t$, is weakly negative. If it is strictly negative, then the entry rate e_t is 0.

Large Firms:

I split the large firm optimization problem into two steps, as in Section 2. First, taking as given $\dot{\mathcal{L}}_{n,t}$, the large firm chooses δ and $\kappa(j)$ and $A(j)$ for all goods j in the industry to minimize cost. As in Section 2, the large firm always acquires a good conditional on getting the opportunity because search is costly. The large firm creatively destroys and searches for all its competitors' goods at single rates, and creatively destroys all its own goods at a single rate because the respective costs are strictly increasing and convex, and because both the benefit and cost of creatively destroying a good scale with its relative productivity. If ζ is the Lagrange multiplier on the $\dot{\mathcal{L}}_{n,t}$ constraint, then the First Order Conditions yield the optimal innovation rates:

$$\begin{aligned}\delta_{L,n,t}(\mathcal{L}_{n,t}) &= ((1-\mathcal{L}_{n,t})\zeta/(W_t\beta_n))^{1/\epsilon}\\ \kappa_{L,n,t}(L;\mathcal{L}_{n,t}) &= \left(\left(\lambda^{\gamma-1}-1\right)(1-\mathcal{L}_{n,t})\zeta/(W_t\beta_n\alpha)\right)^{1/\epsilon}\\ \kappa_{L,n,t}(S;\mathcal{L}_{n,t}) &= \left(\left(\left(\lambda^{\gamma-1}-1\right)(1-\mathcal{L}_{n,t})+1\right)\zeta/(W_t\beta_n\alpha)\right)^{1/\epsilon},\end{aligned} \tag{23}$$

and the optimal acquisition rate:

$$X_A'(A_{n,t}(\mathcal{L}_{n,t})) \geq (\zeta - (1+\tau_A)\Pi_{n,t}(\mathcal{L}_{n,t}))/W_t,$$

which holds with equality if $A_{n,t}(\mathcal{L}_{n,t}) > 0$.

Developing a new good or creatively destroying one of its own increases industry productivity, which the large firm discounts by $1-\mathcal{L}_{n,t}$ because it depreciates all goods' relative productivities. Creatively destroying a small firm's good increases industry productivity, but also transfers the good's pre-innovation productivity to the large firm. The large firm does not discount the latter.

Second, the large firm chooses a Lagrange multiplier ζ. The HJB equation for the large firm's expected discounted profits, $V_{n,t}(\mathcal{L}_{n,t})$, is

$$\begin{aligned}r_tV_{n,t}(\mathcal{L}_{n,t}) =& \mathcal{L}_{n,t}\left(\left(1-\sigma^{-1}-\tau_R\right)P_tY_t - P_tZ_tf_{L,n}\right) + \dot{V}_{n,t}(\mathcal{L}_{n,t})\\ &+ \max_{\zeta}\left\{\dot{\mathcal{L}}_{n,t}(\zeta;\mathcal{L}_{n,t})V_{n,t}'(\mathcal{L}_{n,t}) - X_{n,t}(\zeta;\mathcal{L}_{n,t}) - (1-\mathcal{L}_{n,t})A(1+\tau_A)\Pi_{n,t}(\mathcal{L}_{n,t})\right\},\end{aligned}$$

where $\dot{\mathcal{L}}_{n,t}(\cdot;\cdot)$ and $X_{n,t}(\cdot;\cdot)$ are the rate at which the large firm gains revenue share and the flow innovation/search cost, respectively, implied by ζ and optimal small firm innovation, and $\dot{V}_{n,t}(\cdot)$ is the effect of changes in the aggregate state over time. The Lagrange multiplier is $\zeta = V_{n,t}'(\mathcal{L}_{n,t})$.

3.8 Model Discussion

Before proceeding to the results, I discuss some of the main modeling choices.

We can interpret new good development as firms innovating on the goods for which they already have the most productive version. In either case, any sales gained come from adding productivity to the industry, not from taking productivity from another firm.

A consequence of imposing that all entrants are small is that the value of being large does not factor into the value of entering. This choice makes sense if large firms exit at much lower rates because they are then over represented in the cross section relative to their salience for a potential entrant. For example, if 1% of firms in a steady state are large and they exit half as quickly, then only 0.5% of entrants are large. Moreover, discounting implies that the halved exit rate does not fully compensate for the halved probability of entering as a large firm. A similar point stands if new firms take time to become large. Finally, ignoring large firm profits in the entry decision is the correct approach for the main experiment in Section 4.3 if the interpretation of the fall in f_L is that large firms have to pay higher firm level fixed costs to lower their per-good fixed costs.

4 Results: The Effects of Large Firm Innovation

I characterize the effect of changes in large firm innovation incentives on industry concentration, growth, and welfare. As in Section 2, variation in the large firm fixed cost, revenue tax rate, or innovation cost, which only affect small firms through their effects on large firm innovation, drive variation in outcomes. I begin with qualitative results concerning long-run effects. I then calibrate the model and compare quantitative results to the data. Throughout, the acquisition rate is 0.

4.1 Qualitative Results: Concentration and Growth in the Long-Run

Similar to in Section 2, I use small firm optimization and the large firm's *relative* innovation rates from equations (23) to characterize the long-run relationship between large firm revenue shares and growth. I show that the results from Section 2 hold under certain conditions, and provide intuition for the quantitative exercises that follow. Moreover, I use the results for one of the calibration methods. I omit time t subscripts for variables that are constant over time.

The following theorem, displayed graphically in Figure 1, shows that if innovation costs are quadratic, then across industries, the growth rate is a function of the large firm's industry revenue share that exhibits an **inverted-U shape**. On the other hand, an aggregate increase in large firm innovation incentives always leads to a fall in the long-run growth rate, leaving aside effects through the labor market, which I discuss below and in the calibrated model are not significant.

Theorem 3. *There are two continuously differentiable functions $g_I(\cdot)$ and $g_A(\cdot)$ on $[0, 1)$. First, on a balanced growth path, the industry n growth rate is $g_I(\mathcal{L}_n)$. Second, if all industries are the same, the large firm fixed cost f_L, revenue tax rate τ_R, and innovation cost shifter β vary, and labor* *supply $\bar{L}$ adjusts so that balanced growth path production labor L^p is constant, then the long-run aggregate growth rate is $g_A(\mathcal{L})$, where $\mathcal{L}$ is the large firm revenue share. The following hold:*

1. $g_I(0) > 0$, $g_I'(0) > 0$, $g_A(0) > 0$, *and* $g_A'(0) = 0$.
2. *If* $\epsilon = 1$, *then there exists a threshold* $\mathcal{L}^* \in (0,1)$ *such that* $g_I'(\mathcal{L}) > 0$ *for* $\mathcal{L} < \mathcal{L}^*$ *and* $g_I'(\mathcal{L}) < 0$ *for* $\mathcal{L} > \mathcal{L}^*$, *and* $\lim_{\mathcal{L}\to 1}(g_I(\mathcal{L})) = 0$.
3. *For all* $\mathcal{L} > 0$, $g_A'(\mathcal{L}) < 0$, *and* $\lim_{\mathcal{L}\to 1}(g_A(\mathcal{L})) = 0$.

Small Firm Discount Rate:

The theorem is intuitive in light of Theorem 1 because across industries, ϵ takes the place of a constant ϵ_S, and across balanced growth paths, the free entry condition makes small firm innovation perfectly elastic, as with $\epsilon_S = 0$. To understand the result better in this richer setting, again decompose the effect of large firm innovation on the long-run growth rate, taking into account small firm optimization, into the discount rate and composition effects. On a balanced growth path, using equation (20), a small firm's value of innovating relative to the wage is

$$\tilde{\Pi}_n \equiv \frac{\Pi_{n,t}(\mathcal{L}_n)}{W_t} = \frac{(\sigma-1)L^p - f_S}{\rho + N(\kappa_{S,n}(\mathcal{L}_n) + (\gamma-1)g_{S,n}(\mathcal{L}_n)) + \kappa_{L,n}(S;\mathcal{L}_n) + (\gamma-1)g_{L,n}(\mathcal{L}_n)}, \tag{24}$$

where the denominator (without ρ) is the discount rate. The discount rate effect is the same as before: across industries, higher large firm innovation implies a higher discount rate and, holding its composition fixed, a higher growth rate because N is fixed and small firms face convex innovation costs; across balanced growth paths, the free entry condition implies a fixed discount rate.

Composition Effect:

The composition effect is again negative, but now also results from large firms' endogenously size-dependent relative innovation rates:

$$\frac{\kappa_{L,n}(L;\mathcal{L}_n)}{\kappa_{L,n}(S;\mathcal{L}_n)} = \mathcal{D}(\mathcal{L}_n)\left(\frac{\lambda^{\gamma-1}-1}{\lambda^{\gamma-1}}\right)^{1/\epsilon} \qquad \frac{\delta_{L,n}(\mathcal{L}_n)}{\kappa_{L,n}(S;\mathcal{L}_n)} = \mathcal{D}(\mathcal{L}_n)\left(\frac{\alpha}{\lambda^{\gamma-1}}\right)^{1/\epsilon},$$

where $\mathcal{D}(\mathcal{L}_n) \equiv \left(\frac{1-\mathcal{L}_n}{(1-\lambda^{1-\gamma})(1-\mathcal{L}_n)+\lambda^{1-\gamma}}\right)^{1/\epsilon}$. Each relative innovation rate is the product of two terms, the latter of which is the small firm relative innovation rate; a firm of any size discounts creatively destroying its own good. The function $\mathcal{D}(\cdot)$ is strictly decreasing from 1 to 0, and is the discount a large firm applies to gaining sales purely through generating growth rather than creatively destroying a competitor's good; a fraction $\lambda^{1-\gamma}$ of the productivity gained through creative destruction is taken from the competitor, and thus not discounted. These size-dependent relative innovation rates are relevant because the small firm discount rate now contains both creative destruction risk and depreciation due to growth. The bigger a large firm's revenue share, the more it shifts its innovation toward creative destruction of its small competitors' goods, and the more the small firm discount rate is achieved through creative destruction risk rather than depreciation due to growth. I show the relative importance of this mechanism for the composition effect in the right panel of Figure 1.

Labor Market Effects:
In the quantitative experiments, shocks that shift innovation to large firms reduce the labor used for entry and innovation. For the labor market to clear, expenditures on goods must rise to increase production labor. Omitted from Theorem 3, this effect pushes up the return to innovating. Moreover, it increases output for any given level of productivity. Thus, it is costly for small firms to replace large firm innovation. Nonetheless, the discount rate and composition effects discussed above dominate; the labor effects on the long-run growth rate are not significant.

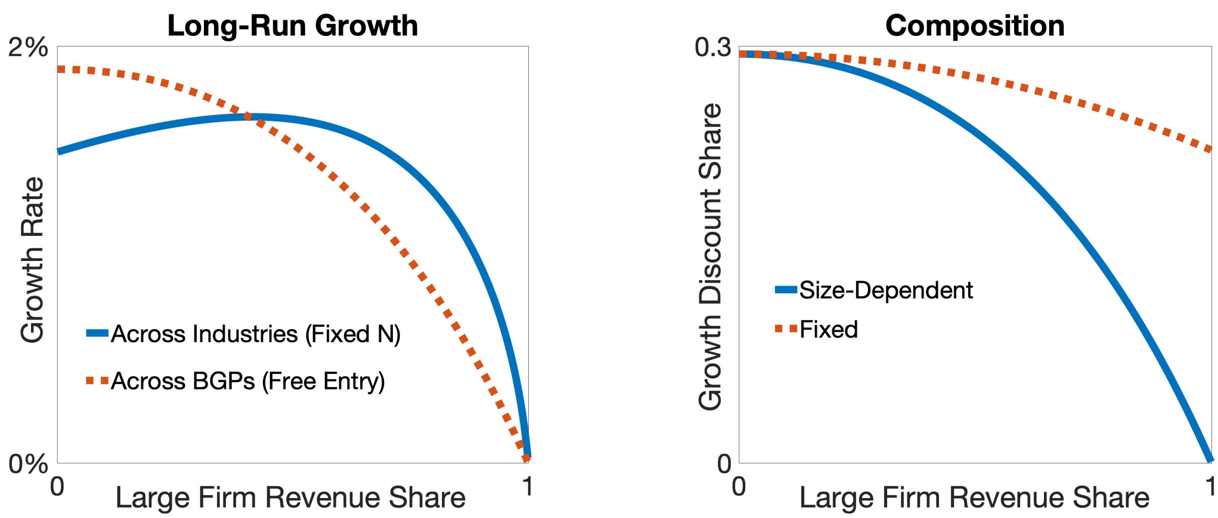

Figure 1: Both figures use the calibration in Section 4.2. Left panel: the solid blue line and the dotted orange line display the functions $g_I(\cdot)$ and $g_A(\cdot)$, respectively, from Theorem 3. Right panel: the solid blue line and the dotted orange line display growth relative to the small firm discount rate (minus ρ) in equilibrium and setting large firm relative innovation rates to their values at $\mathcal{L}_n = 0$, respectively.

4.2 Calibration

I calibrate the model to a balanced growth path in which all industries are identical, and small and large firms have the same fixed cost. I set some parameters externally, listed in Table 1, and internally calibrate the rest to jointly match moments in the data, listed in Table 2. I set labor supply $\bar{L}$ so that output relative to productivity, $Y_t/Z_t = L^p$, is 1. I set the revenue tax rate to 0. The units of time are years.

Externally Calibrated Parameters:

I take the exit rate η to be the annual employment-weighted average firm exit rate from Boar and Midrigan (2022). Ideally, η would match the rate at which an innovative entrepreneur stops innovating in the industry; for example, closing one firm to start another should not count as an exit. Nonetheless, its effects on the long-run results in the main experiment are small because it only impacts the shift in labor from entry to production, which is insignificant. I take the demand elasticity γ to be the median estimate from Broda and Weinstein (2006) at the most disaggregated level in the earliest time period, which is in line with the calibration strategy for the large firm's revenue share that uses the most disaggregated industry definition available. Finally, the marginal innovation cost elasticity ϵ captures two elasticities: of a firm's total innovation rate with respect to its value or cost, and of a firm's *relative* innovation with respect to the value or cost of one type of innovation *relative* to another. Acemoglu, Akcigit, Alp, Bloom, and Kerr (2018) discuss a wide range of studies that estimate the former elasticity to be 1. The proof of Theorem 3 provides an argument that 1 is a good target for the latter as well: Proposition 3 in Appendix A.6 states that if variation in large firm innovation incentives drives variation in concentration across industries, then to match the inverted-U relationship between growth and concentration across industries documented in Cavenaile, Celik, and Tian (2021), the elasticity of relative innovation rates cannot be much greater than 1—even if the elasticity is greater than 1.5, then an industry's growth rate diverges to infinity as its large firm's revenue share goes to 1 because the large firm uses a high rate of growth as well as creative destruction to maintain its dominance.

Table 1: Externally Calibrated Parameters

Parameter	Description	Value
η	Exit Rate	0.04
γ	Demand Elasticity	3.1
ϵ	Innovation Cost Elasticity	1

Internally Calibrated Parameters:

I take the TFP growth rate to be the BLS measure from Garcia-Macia, Hsieh, and Klenow (2019), the real interest rate to be the 1-year rate from FRED, the markup to be the cost-weighted average estimated with Compustat data by De Loecker, Eeckhout, and Unger (2020), and R&D/GDP to be Business Enterprise Expenditures on R&D relative to GDP from the OECD MSTI database, all averages from 1983-1993. The first two determine the time discount rate ρ through $r = \rho + g$. The markup is the imitation discount σ. R&D/GDP determines firm profits relative to sales, and thus the fixed cost $f_S = f_L$ given σ. The distinction between the fixed cost and σ is not significant; I set $f_L > 0$ so that I can lower it in the quantitative experiments.

The innovation step size λ, the equilibrium large firm revenue share $\mathcal{L}$, and the relative cost of creative destruction α are crucial for the size of the composition effect, which is the key mechanism for the main results. I calibrate λ so that the average growth contribution of a creative destruction innovation, $(\lambda^{\gamma-1} - 1)/(\gamma - 1)$, is equal to the Akcigit and Kerr (2018) estimate using patent data of the average growth contribution of an external patent (that mostly cites other firms' patents).[20] The result is that 13% of the sales a firm gains from a creative destruction innovation comes from generating growth, so that the risk of creative destruction is 74% of the small firm discount rate (not including ρ) even though the share of growth due to creative destruction is only 34%.

I match $\mathcal{L}$ to the sales-weighted average industry revenue share of the largest firm in 4-digit industries in Compustat from Olmstead-Rumsey (2022). An alternative measure uses Census industry concentration data, which is more complete because it contains all firms. However, the relevant moment is a large firm's share of innovations rather than sales because the former is the fraction of old innovations replaced by a new one that are under the large firm's control. Relative to the Compustat data, the Census data may contain many irrelevant small firms that only imitate other firms' innovations. As a robustness check in Section 4.4, I instead use the Census industry concentration data, which imply a lower value. The large firm industry revenue share and the TFP growth rate pin down the large firm innovation cost shifter β and the small firm entry cost ξ.

Table 2: Internally Calibrated Parameters and Data Moments

Parameter	Value	Moment	Value (Data)
ρ – Time Discount Rate	0.0194	Real Interest Rate	3.6%
σ – Imitation Discount	1.3	Markup	1.3
f_S, f_L – Fixed Cost	0.183	R&D/GDP	1.81%
λ – Innovation Step Size	1.067	External Innovation Step	0.069
ξ – Entry Cost	4.233	TFP Growth Rate	1.66%
β – Large Firm Innovation Cost Shifter	28.36	Large Firm Revenue Share	40.7%
α – Relative Creative Destruction Cost	0.3114	Large Job Destruction Rate	25.57%

To calibrate α I use large job destruction flows—the share of aggregate employment lost over a 5 year period at firms whose employment shrank by at least two-thirds—computed from Census employment data in Garcia-Macia, Hsieh, and Klenow (2019) to estimate the rate at which a

[20] Their analogous estimate for internal innovations is lower at 0.051, and so implies a lower λ and a stronger composition effect, whereas Garcia-Macia, Hsieh, and Klenow (2019) estimate a slightly higher 0.081. Akcigit and Kerr (2018) use patent citations data, whereas Garcia-Macia, Hsieh, and Klenow (2019) estimate their value using labor flows data through the lens of growth model different from the one in this paper.

small firm's good is creatively destroyed. The calibrated model does not uniquely determine large job destruction flows because the productivity distribution of new goods is unspecified, and as discussed in Section 3.8, we can interpret new good development as an improvement on a firm's old good; the less productive each good is and the more new good development entails the creation of new goods, the more goods each small firm produces, and the lower the job destruction rate. I take a conservative approach and estimate a maximum value of α by supposing that all creative destruction of small firm goods leads to large negative job flows, i.e., those flows are $(1-\mathcal{L})(1-e^{-5(\kappa_S+\kappa_L(S))})$.[21][22] For consistency, I use the Census concentration measure discussed in Section 4.4 for $\mathcal{L}$, which yields a higher α than does the Compustat measure. Note that given α, λ, γ, $\mathcal{L}$, and g, we can compute $\kappa_S + \kappa_L(S)$ on a balanced growth path using optimal relative innovation rates, without computing a dynamic equilibrium.

4.3 Quantitative Experiment: A Rise in Large Firm Innovation

I ask whether and to what extent a rise in concentration driven by a fall in large firm fixed costs can explain changes in US data since the mid-1990s. I interpret the fall in large firm fixed costs as capturing a shift from per-good costs to firm wide fixed costs due to the rise in information technology.[23] I explore the effects of falls in the large firm innovation cost or revenue tax, which are nearly identical, in Section 4.4.

The economy begins on the balanced growth path calibrated in Section 4.2. There is an unanticipated permanent fall in f_L to 0.17 in all industries, which is calibrated so that the large firm revenue share in the new balanced growth path is 0.51, the sales-weighted average across 4-digit industries of the largest firm's revenue share in 2018 in Compustat from Olmstead-Rumsey (2022).

4.3.1 Industry Concentration and Aggregate Growth

I compare the effects of the shock on growth to data from Garcia-Macia, Hsieh, and Klenow (2019) in Table 3. The shock explains 41% of the fall in the long-run growth rate in the data, due entirely to a change in g because Y_t/Z_t is constant in the long-run. The shock explains all of the increase in the short-run growth rate of output relative to inputs, mostly due to a temporary large shift

[21] I suppose job losses from depreciation due to growth, which on their own amount to a share $1-e^{-5(\gamma-1)g} < 2/3$, do not lead to large destruction flows. For growth depreciation to generate large flows, a small firm must have multiple goods, which then implies that some creative destruction does not lead to large destruction flows.

[22] I exclude innovation labor, which is small, so that I can calibrate α without computing a dynamic equilibrium.

[23] See Aghion, Bergeaud, Boppart, Klenow, and Li (2022) for a discussion.

in labor from entry costs to production. The cumulative growth burst is smaller than in the data because the peak increase in output relative to its counterfactual path without the shock occurs after 3 years, whereas the burst in the data is an average over a 10 year period.

Table 3: Growth After a Fall in f_L

Moment Description	Data	Model
Growth Rate Burst	+0.64 ppt (38.6%) (1993-2003)	Output: +0.77 ppt (46.3%) (first year) TFP: +0.12 ppt (7.2%) (first year)
Cumulative Burst	+6.4 ppt (38.6%) (1993-2003)	Output: +0.91 ppt (18.3%) (3 years) TFP: +0.21 ppt (4.3%) (3 years)
Growth Rate Fall	-0.34 ppt (-20.5%) (2003-2013)	-0.14 ppt (-8.4%) (New BGP)

ppt is the percentage point rise, and in parentheses is the percent rise. Growth rate burst is the peak growth rate following the shock. Cumulative burst is the peak difference between the new and old paths.

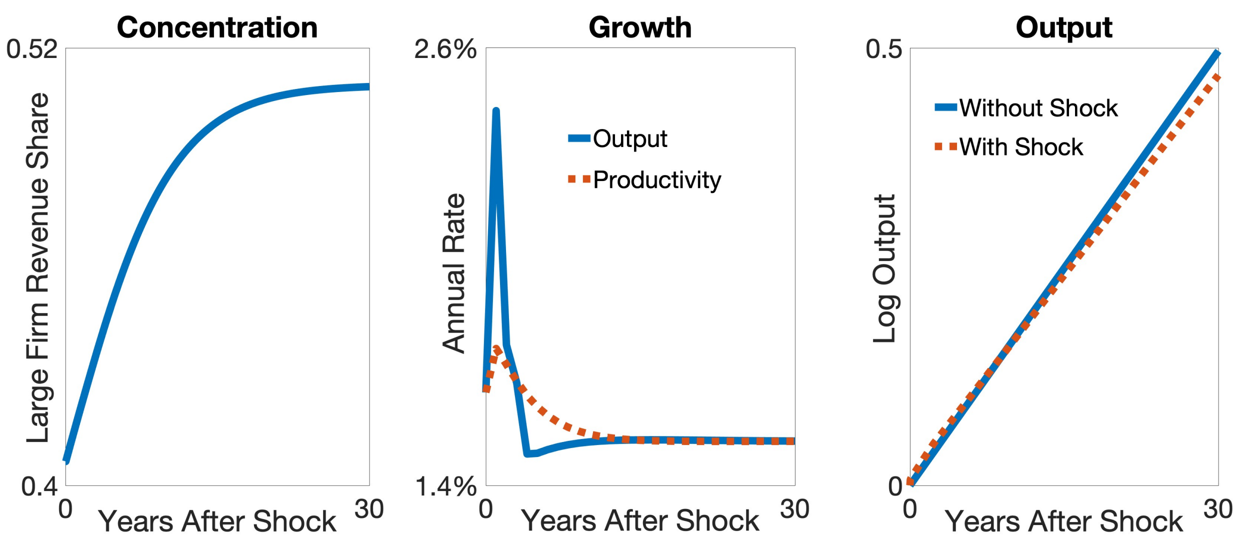

Figure 2: Transition paths following the shock to f_L. Left panel: the large firm revenue share $\mathcal{L}_t$. Middle panel: the dotted orange line is g_t; the solid blue line is the growth rate of Y_t. Right panel: the dotted orange line is $\ln(Y_t)$ following the shock; the solid blue line is the counterfactual $\ln(Y_t)$ without the shock.

The large firm revenue share and the growth rate, depicted in Figure 2, converge over a similar time interval as the gap between the years in the initial calibration, 1983-1993, and the target year for the shock, 2018.

I decompose the change in the productivity growth rate over time from the initial balanced growth path into the composition and discount rate effects in Figure 3. Growth is higher in the short-run because the small firm discount rate is higher: the measure of small firms is slow to fall, so the small firm innovation elasticity is finite, and the temporary large increase in output relative to the wage pushes up the return to innovating. Growth is lower in the long-run because the composition effect dominates: the free entry condition makes small innovation infinitely elastic, and the small firm discount rate is only slightly higher due to a small increase in output relative to the wage.

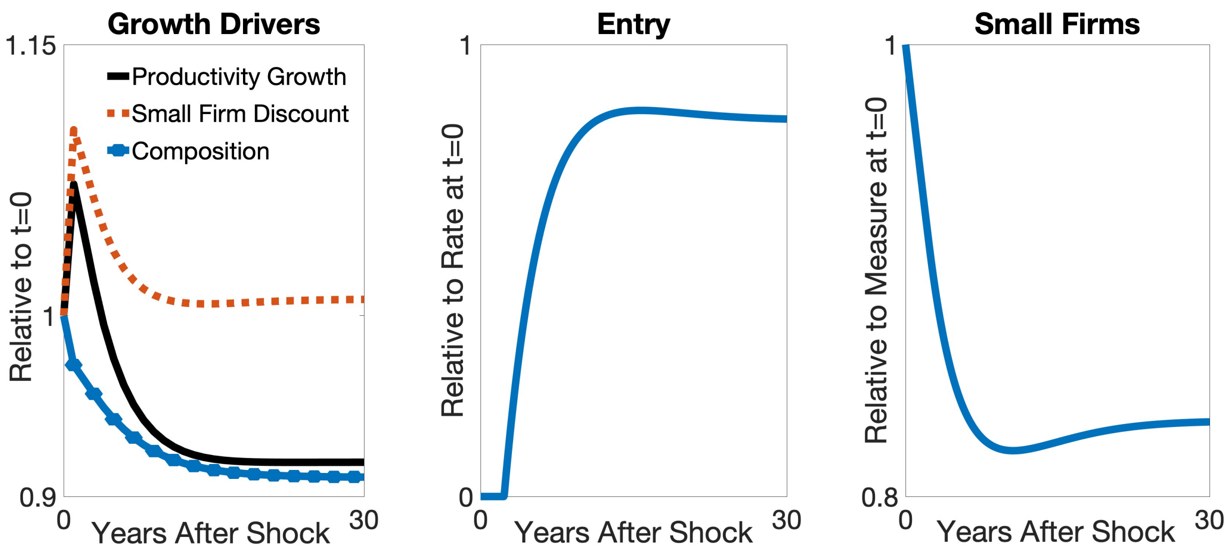

Figure 3: Left panel: the solid black line is g_t/g, where g is on the original balanced growth path; the dotted orange and textured blue lines decompose g_t/g into the small firm discount rate (minus ρ) and growth over the discount rate, respectively, relative to their values on the original balanced growth path. Middle panel: the relative entry rate e_t/e. Right panel: the relative measure of small firms N_t/N.

4.3.2 Welfare and Size-Dependent Taxes

Taking into account the transition path, the present discounted value of output falls by the equivalent of a permanent 5.7% drop. The present discounted value of consumption—household welfare—falls by the equivalent of a permanent 4.8% drop; consumption rises relative to output because fixed costs are lower. The decline in the long-run growth rate is the dominant effect: on its own, it implies the equivalent of a 6.9% permanent fall in consumption across balanced growth paths.

The effects of a fall in the large firm revenue tax rate τ_R and fixed cost f_L are identical, except that the former does not lead to a rise in consumption relative to output. It follows that on the margin, a rise in τ_R increases welfare.

4.3.3 Comparing Model Predictions to the Data

Entry:
The large fall in entry in the short-run and the smaller fall in the long-run, depicted in Figure 3, match the data in Decker, Haltiwanger, Jarmin, and Miranda (2016), which show that the entry rate declined sharply in the mid-to-late 1990s followed by a partial recovery before a large drop during the Great Recession.

Large Job Destruction Flows and Creative Destruction:
The model matches the dynamics of large job destruction flows documented in Garcia-Macia, Hsieh, and Klenow (2019): $(1-\mathcal{L})\left(1-e^{-5(\kappa_S+\kappa_L(S))}\right)$ (using the model value of $\mathcal{L}$) falls by 15% across balanced growth paths, whereas empirical large job destruction flows fall by 13% from 1983-1993 to 2003-2013; the model measure rises by 9% immediately following the shock, and the empirical measure rises by 4% from 1983-1993 to 1993-2003. Although the small firm discount rate shifts toward creative destruction, they represent a lower share of total employment, i.e., more labor is employed by large firms for whom creative destruction does not cause large flows.

If we use these these large flows to infer growth due to creative destruction, then the model generates the rise and fall documented in Garcia-Macia, Hsieh, and Klenow (2019).

Growth Relative to R&D:
The model can explain the simultaneous rise in R&D expenditures relative to GDP and fall in growth. In the data, R&D/GDP rose while growth fell, and the growth rate relative to R&D/GDP fell from 0.91 in 1983-1993 to 0.69 in 2003-2013. In the model, across balanced growth paths, innovation expenditures as a share of GDP rise slightly from 1.81% to 1.82%, and the growth rate relative to R&D/GDP falls from 0.91 to 0.84. Following the shock, large firms innovate more because they earn higher profit margins. As their innovation increases, its efficiency falls because innovation costs are convex.

Industry Concentration and Industry Growth Rates:
The model matches the finding in Ganapati (2021) that across industries in the US, rising concentration is associated with *faster* growth. Specifically, Ganapati (2021) estimates that, controlling for sector and time fixed effects, a 1 percent rise in the revenue share of the largest 4 firms in a 6-digit NAICS industry over a 5-year period is associated with a 0.1 percent rise in real output and a 0.2 percent rise in real output relative to employment.

I run the same regression in the model by creating a measure 0 control group of industries in which the large firm fixed cost does not change. I regress the change in industry log real output on the change in industry log large firm revenue share and a time fixed effect over the three 5-year time periods during the first 15 years of the transition path, after which industry concentration is effectively constant. A 1 percent rise in the revenue share of the largest firm in an industry is associated with a 0.04 percent rise in real output and in real output relative to employment.

The theory thus generates a parsimonious explanation for the short-run burst in growth as well as the positive relationship between concentration and growth across industries: the small firm innovation elasticity is lower in the short-run and in response to industry-specific shocks.

4.4 Different Shocks and Calibrations

4.4.1 A Fall in Large Firm Innovation Costs

Starting from the initial balanced growth path, I calibrate a fall in the large firm innovation cost shifter β to generate the same rise in the large firm revenue share across balanced growth paths as in the main experiment. Table 4 lists the results for the long-run growth rate and R&D relative to output. The latter falls counter to the data because if innovation rates are similar to in the main experiment, then innovation expenditures are lower. The only difference across experiments for the long-run growth rate operates through the labor market; the fall in innovation labor implies an increase in production labor, and therefore a rise in expenditures on goods, which pushes up the return to innovating and the equilibrium small firm discount rate.

4.4.2 Fixed Innovation Labor

Without the free entry condition, the discount rate effect dominates; a rise in large firm innovation incentives increases growth because small firms are not sufficiently responsive. An alternative way to make small firm innovation more responsive to large firm innovation is to fix both the measure of small firms and innovation labor. Starting from the initial balanced growth path, I fix the labor used for each of entry, innovation, and production. There is a relative wage for entry labor and innovation labor so that each labor market clears. I lower the large firm fixed cost f_L to generate the same rise in the large firm revenue share across balanced growth paths as in the main experiment in Section 4.3, and show the results in Table 4.

The long-run growth rate falls, although by less than in the main experiment because even though innovation labor is fixed, convex costs imply that small firms become more efficient innovators as they lose labor to large firms. Thus, a rise in large firm innovation incentives leads to lower growth if the distribution of labor across uses (entry, innovation, and production) is fully flexible or fully fixed.

Table 4: Alternative Shocks and Calibrations

Experiment	Growth	R&D/GDP	ξ	β	α	f_S, f_L	f_L^*
Original Experiment	1.52%	1.82%	4.233	28.36	0.3114	0.183	0.17
Innovation Cost (β)	1.54%	1.67%	4.233	28.36	0.3114	0.183	22.1 (β^*)
Fixed Labor	1.59%	1.98%	4.233	28.36	0.3114	0.183	0.158
Census Calibration	1.61%	1.85%	4.19	40.2	0.3114	0.186	0.178
Inverted-U Calibration	1.6%	1.86%	9.7	44.6	4.16	0.17	0.143

Each row is for a different experiment; it shows the internally calibrated parameters that sometimes vary, and long-run growth and R&D/GDP following the shock. The new value of the shocked parameter, which is β in the second case and f_L in the others, is in the last column.

4.4.3 Alternative Calibrations

I run the main experiment under two alternative calibrations. In Table 4, I display the different parameters, and show the results.

First, I match the large firm revenue share in the initial and new balanced growth paths to the sales-weighted average revenue share of the largest 4 firms in 6-digit NAICS industries in the US in 1997 (30.6%) and 2012 (35.9%), respectively, computed by Barkai (2020) using consistently defined industries over time. I restrict the data to the same 2-digit sectors used by Garcia-Macia, Hsieh, and Klenow (2019) for the growth and labor flows data moments. Although the initial time period in the calibration in 1983-1993, the concentration data has a break before 1997 so that consistently defined industries are not available. Nonetheless, Autor, Dorn, Katz, Patterson, and Van Reenen (2020) suggests that this restriction understates the rise in concentration and so the resulting fall in growth because concentration has been increasing in Census data at least since the 1980s. The effect on growth is quantitatively smaller because the lower large firm revenue share implies a weaker composition effect.

Second, I use the inverted-U relationship between growth and the large firm revenue share implied by Theorem 3, $g_I(\cdot)$, to calibrate α instead of large job destruction flows. Specifically, Cavenaile, Celik, and Tian (2021) find an inverted-U relationship between a variety of innovation measures and HHI in Compustat data at the 4-digit industry level, controlling for sector and year fixed effects. I match the location of the peak of $g_I(\cdot)$ to the patent-maximizing large firm revenue share implied by their regression of industry patents on HHI (the large firm's revenue share squared) and HHI squared. I match the maximizing HHI rather than the regression coefficients because $g_I(\cdot)$ is not a second-order polynomial, so to do the latter would require specifying some distribution of HHI across industries. The effect on growth is quantitatively smaller because the resulting α is higher, which implies a much lower significance of creative destruction.

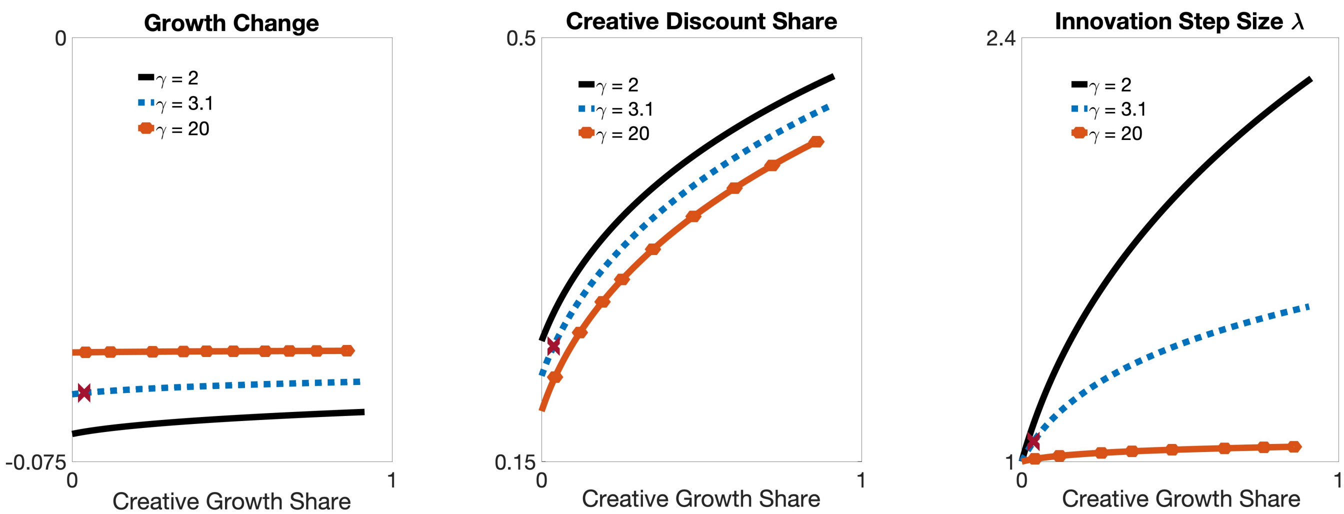

Figure 4: Each line varies λ for a fixed γ, setting α to match the target location of the peak of $g_I(\cdot)$. The x-axis is the resulting share of growth due to creative destruction. The x on each dotted line is from the calibration with $\gamma = 3.1$ and $\lambda = 1.067$. Left panel: $g_A(0.51) - g_A(0.407)$ in percentage points. Middle panel: the share of the small firm discount (not including ρ) that is creative destruction. Right panel: λ.

An advantage of this calibration strategy is that it makes the main result nearly constant across a wide range of parameter values. I take $\epsilon = 1$ and the growth rate and large firm revenue share in the initial balanced growth path from the calibration in Section 4.2. Values for γ, λ, and α then determine the functions from Theorem 3: $g_I(\cdot)$, which is the inverted-U, and $g_A(\cdot)$, which is a close approximation for the true long-run aggregate growth rate following a change in f_L because labor market effects are insignificant. For values of γ and λ, I calibrate α to match the target location of the peak of $g_I(\cdot)$ from Cavenaile, Celik, and Tian (2021), and compute $g_A(0.51) - g_A(0.407)$. I vary γ between 2 and 20, and λ over its entire feasible range for each γ from 1 to the maximum above which the desired α no longer exists. The result, displayed in Figure 4, is that the fall in the growth rate is almost entirely determined by the location of the inverted-U's peak, given initial values of concentration and growth; $g_A(0.51) - g_A(0.407)$ ranges from 0.055 to 0.07 percentage points. The share of growth due to creative destruction is particularly unimportant.

Finally, I use the same method to verify that the calibrated value of α in Section 4.2 is conservative in the sense that lower values of α lead to a bigger fall in growth following a drop in f_L.

5 Antitrust Policy: Acquisitions

Can acquisitions increase growth in the calibrated model, and what determines when that is the case? As in the model in Section 2, a large firm can get an acquisition opportunity for each good it doesn't produce subject to a search cost. In an acquisition opportunity, the large firm makes a take-it-or-leave-it offer subject to a tax on any payments. Any taxes collected or subsidies given are dispersed to or taken from the representative household. The search function is characterized by two parameters: a cost shifter and a constant elasticity with respect to the search rate. It follows that the elasticity of the acquisition rate with respect to the surplus of an acquisition, i.e., the large firm's value of an acquisition opportunity, is constant.

The Effects of the Tax Rate on Growth:
To understand how acquisitions and the acquisition tax rate affect growth, I first show the long-run growth rate for various tax rates in Figure 5, using the calibration following the shock to f_L in the main experiment in Section 4.3. I use two search cost functions: the elasticity of $X_A(A)$ is 3, and I set a cost shifter so that A is about 0.05 at the largest possible subsidy, $\tau_A = -1$; I take the limit as the elasticity of $X_A(A)$ goes to 0 so that the cost is 0 if $A < \bar{A}$ and infinite if $A > \bar{A}$. In the first case, the elasticity of A with respect to an acquisition's surplus is 0.5, and in the second case it is 0 except where the surplus is 0.

In both cases, if the tax rate is low enough so that acquisitions occur, but high enough so that they generate little surplus, the acquisition effect defined in Section 2 dominates and the long-run growth rate is lower than without acquisitions. Acquisitions push up large firm revenue shares, so the small firm discount shifts from growth to creative destruction risk because large firms cut back on developing new goods and creatively destroying their own, but mostly maintain the rate at which they creatively destroy competitors' goods. In the case with an acquisition rate elasticity of 0.5, search costs also pull labor away from production, which lowers the return to innovating and slows growth. If the tax rate is lower, then eventually the innovation effect dominates and the long-run growth rate is higher than without acquisitions. The expected surplus from future acquisitions reduces all types of large firm innovation, including creative destruction of competitors' goods.

Although a subsidy is required for the economy with acquisitions to grow faster than the economy without, it is beneficial to reduce the tax on the margin as long as the tax rate is below a positive threshold. Thus, if taxing all acquisitions out of existence is not a desirable policy, then the effect

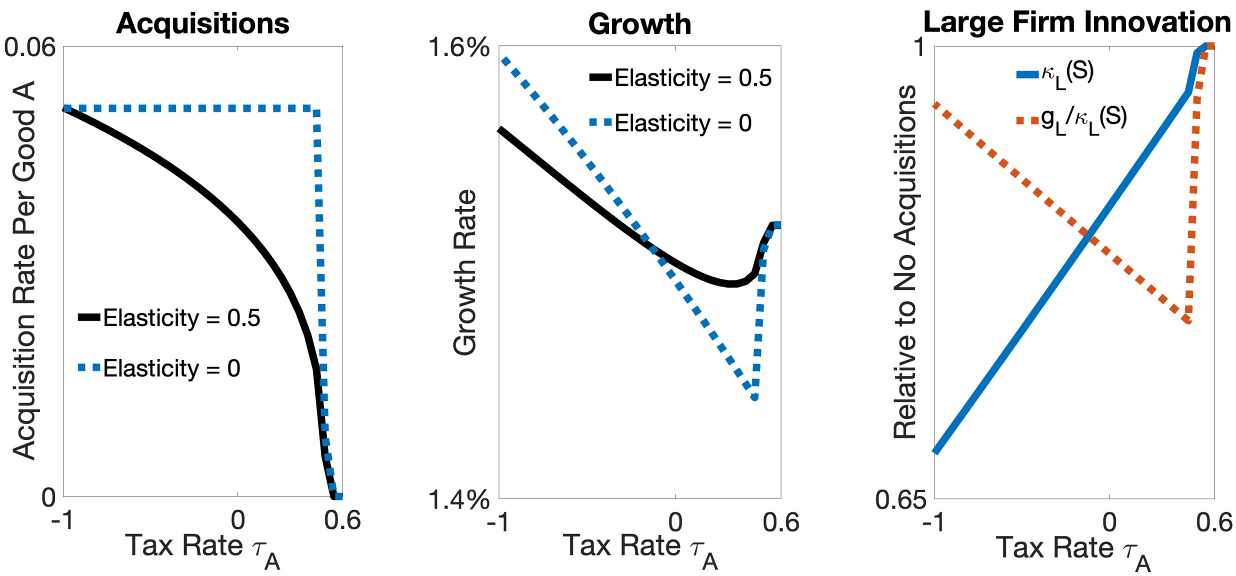

Figure 5: Each panel displays results on a balanced growth path as a function of τ_A, using the calibration following the drop in f_L in Section 4.3. The left and middle panels display results for cost functions such that the elasticity of the acquisition rate is 0.5 (solid black line) and 0 (dotted blue line). The right panel is only for the latter function. Left panel: the acquisition rate. Middle panel: the growth rate. Right panel: the large firm creative destruction rate (solid blue line) and growth over the creative destruction rate (dotted orange line) relative to their values on the balanced growth path without acquisitions.

on large firm innovation may bolster the argument for lowering the tax rate further and allowing more acquisitions.

The Acquisition Rate Elasticity and Concentration:
When is it more beneficial to lower the tax rate on acquisitions? For a variety of acquisition search cost functions and using the calibration in Section 4.2 both before the main experiment and after, I compute the break even tax rate τ_A at which the long-run growth rate is higher in the economy with acquisitions than without. In line with the results in Section 2.2.2, I find that the break even rate is lower—acquisitions more easily increase growth—if the acquisition rate elasticity is lower (the cost function elasticity is higher) and after the fall in f_L that increases concentration. I display the results in Figure 6.

To understand the results, it is useful to look at the surplus of an acquisition at the break even tax rate, which is also the marginal search cost. If the acquisition rate elasticity or large firm fixed cost is lower, then the surplus of an acquisition is lower at the break even rate. It follows that the break even rate must be higher to reduce the surplus, particularly after the fall in f_L pushes up the surplus. If the acquisition rate elasticity is lower, a smaller surplus is required for acquisitions to lead to higher growth because the search cost elasticity is higher, which implies

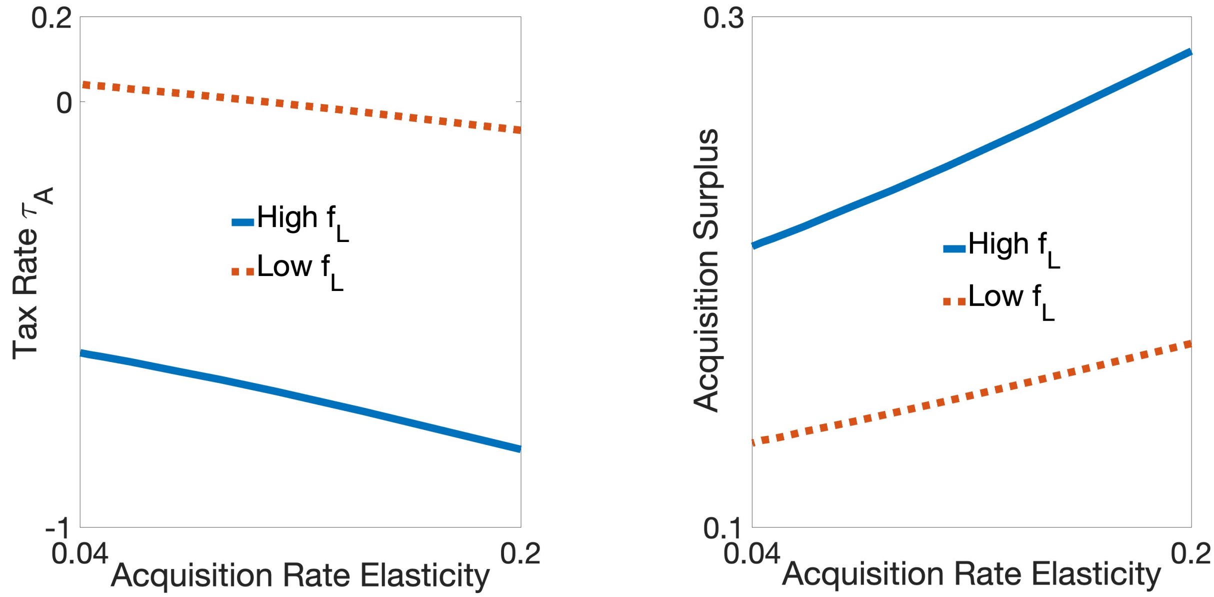

Figure 6: In each panel, the solid blue line uses the calibration from Section 4.2, and the dotted orange line uses the same but after the fall in f_L from the main experiment in Section 4.3. Left panel: for a variety of inverse marginal search cost elasticities, the break even tax rate at which the growth rate is the same as in the economy without acquisitions. Right panel: for a variety of inverse marginal search cost elasticities, the surplus of an acquisition at the break even tax rate.

a smaller total cost for a given marginal cost, and so a greater incentive to reduce innovation to preserve acquisition opportunities and a smaller shift in labor away from production. On the other hand, after the fall in f_L, a lower acquisition surplus is sufficient for acquisitions to lead to higher growth because the small firm discount is the same but the large firm discount rate is lower given the shift in innovation toward creative destruction targeting only small firms. Thus, the effect of an acquisition on the large firm revenue share, which depends on the small firm discount rate, is mostly the same, but the effect of a valuable future acquisition opportunity, which depends on the large firm discount rate, is higher.

6 Conclusion

I study the implications of the concentration of innovative activity. I use a model with one large firm and a continuum of small firms in each industry. Firms can innovate through improving on old goods and developing new ones. The defining feature of a large firm is that it has the innovation technology of a measure of small firms, and thus behaves as a mass of small firms that can coordinate their innovative activities to maximize their joint profits. I show that this feature is sufficient for a rise in large firm profitability to explain the rise in concentration, the fall in growth, and related changes to the US economy since the mid-1990s.

Large firm acquisitions of their competitors' goods have distributional and incentive effects with opposite implications for concentration, growth, and welfare. Acquisitions directly shift revenue to large firms, which strengthens their relative preference for creative destruction and leaves more goods stagnating. As a result, growth falls. On the other hand, The expectation of future valuable acquisitions pushes each large firm to innovate less so that more revenue share remains for it to acquire. As large firm innovation falls, it is replaced by small firm innovation, which is less geared toward creative destruction and includes creative destruction of large firm goods. As a result, growth increases. The second effect is more likely to dominate if innovation is more elastic, the acquisition rate is less elastic, the acquisition rate is high, and following the recent changes in the economy that pushed up industry concentration.

The theory and results highlight a novel way to think about the effects of market power and optimal competition policy. Large firms are harmful because of how they achieve their size through innovation. Research and development subsidies that target large firms may backfire by discouraging small firm innovation, which is more efficient at generating growth. Policies that increase concentration may be beneficial as long as they reduce large firm innovation. Facilitating acquisitions is a particularly useful policy because, unlike taxing large firms, it does not require knowledge of firms' relevant industries or their revenue shares in those industries.

Finally, although this paper focuses on growth, the theory has implications for other settings, and suggests potential avenues for future research. For example, suppose a firm can develop different types of goods, some of which are more novel to the industry, and others of which are close substitutes with the firm's competitors' goods. The same force that leads larger firms to set higher markups in static models of oligopolistic competition implies that larger firms have a stronger preference for producing the types of goods that are close substitutes with their competitors. Thus, subsidizing large high markup firms to produce more may be costly unlike in models in which firm production is one-dimensional.

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A Proofs and Derivations

A.1 Proof of Proposition 1

I begin with the first part of the proposition. Combining the First Order Condition for the small firm innovation rate κ_S (inequality (5)) and the expression for the expected present discounted value a small firm earns from a good Π (equation (6)) yields

$$W X_S'(\kappa_S)(r + \kappa_L + \kappa_S) \geq \left(1 - \sigma^{-1}\right) R - f_S W, \tag{25}$$

where the inequality holds with equality if $\kappa_S > 0$. The left-hand side is strictly increasing in κ_S and goes to infinity as κ_S goes to infinity because $X_S(\cdot)$ is strictly increasing and convex. Thus, there is a unique solution for the small firm innovation rate that depends on the large firm innovation rate κ_L: $\kappa_S(\kappa_L)$. Moreover, $\kappa_S(\kappa_L)$ is decreasing because the left-hand side is increasing in κ_L and κ_S.

For the remainder of the proof of the first part of the proposition, consider two cases. Suppose $X_S'(0) = 0$. Inequality (25) cannot hold at $\kappa_S = 0$ because the right-hand side is strictly positive by Assumption 1. It follows that for all κ_L, $\kappa_S(\kappa_L) > 0$, which implies that $X_S'(\kappa_S(\kappa_L)) > 0$ and that inequality (25) holds with equality. Totally differentiating each side with respect to κ_L and using the definition of $\epsilon_S(\kappa_S)$ yields the derivative

$$\kappa_S'(\kappa_L) = \frac{-\kappa_S(\kappa_L)}{\epsilon_S(\kappa_S(\kappa_L))(r + \kappa_L + \kappa_S(\kappa_L)) + \kappa_S(\kappa_L)}, \tag{26}$$

which is continuous in κ_L because $\epsilon_S(\cdot)$ is continuous by assumption and because the existence of $\kappa_S'(\kappa_L)$ implies that $\kappa_S(\kappa_L)$ is continuous. Thus, setting $\kappa_L^* = \infty$, for all $\kappa_L < \kappa_L^*$, $\kappa_S(\kappa_L) > 0$ and has a continuous derivative.

Next, suppose $X_S'(0) > 0$. Define κ_L^* by making inequality (25) hold with equality at $\kappa_S = 0$. It follows that $\kappa_L^* > 0$ because at $\kappa_L = 0$, the left-hand side is strictly less than the right-hand side by Assumption 2. Moreover, $\kappa_S(\kappa_L) > 0$ if and only if $\kappa_L < \kappa_L^*$: if $\kappa_L < \kappa_L^*$, then $\kappa_S(\kappa_L) > 0$ because the left-hand side is strictly less than the right-hand side at $\kappa_S = 0$, and if $\kappa_L \geq \kappa_L^*$, then $\kappa_S(\kappa_L) = 0$ because the inequality holds at $\kappa_S = 0$. Finally, $\kappa_S(\kappa_L)$ is continuously differentiable on $[0, \kappa_L^*)$ because there $\kappa_S(\kappa_L) > 0$ and inequality (25) holds with equality, so totally differentiating each side with respect to κ_L yields the continuous derivative in equation (26).

I now prove the second part of the proposition. Suppose there are no acquisitions ($A = 0$), that $X_L(\cdot)$ is strictly convex, and that for all κ_L, $\epsilon_L(\kappa_L)(\epsilon_S(\kappa_S(\kappa_L)) + 1) \geq 1$. I show that we can write κ_L as a continuously differentiable function of the marginal value of a good to the large firm relative to the innovation cost shifter, $\bar{V}/\beta$. I use this function and equation (8) for $\bar{V}$ to show that the equilibrium value of $\bar{V}/\beta$ is a function of $\left(\left(1 - \sigma^{-1} - \tau_R\right) R - f_L W\right)/\beta$ with the right properties so that the equilibrium values of κ_L and g are functions of the same with the desired properties.

As a preliminary step, equation (8) for $\bar{V}$ and Assumption 1 imply that $\bar{V} > 0$ in any equilibrium, which I take as given for the remainder of the proof.

The First Order Condition for κ_L (inequality (9)) implicitly defines a function $\kappa_L\left(\bar{V}/\beta\right)$ because, due to the strict convexity of $X_L(\cdot)$, the left-hand side is strictly increasing in κ_L and goes to infinity as κ_L goes to infinity. To see that $\kappa_L(\cdot)$ is continuously differentiable, consider two cases. If $X_L'(0) = 0$, then the First Order Condition cannot hold at $\kappa_L = 0$ because $\bar{V} > 0$. It follows that $\kappa_L\left(\bar{V}/\beta\right) > 0$, which implies that the First Order Condition holds with equality and, given $\epsilon_L(\cdot) > 0$, implies that

$$W\beta X_L''\left(\left(\kappa_L\left(\bar{V}/\beta\right)\right)\right) = \epsilon_L\left(\kappa_L\left(\bar{V}/\beta\right)\right)\frac{\bar{V}}{\kappa_L\left(\bar{V}/\beta\right)} > 0.$$

Therefore, differentiating both sides of the First Order Condition with respect to $\bar{V}/\beta$ yields the derivative

$$\kappa_L'\left(\bar{V}/\beta\right) = \frac{1}{\epsilon_L\left(\kappa_L\left(\bar{V}/\beta\right)\right)}\frac{\kappa_L\left(\bar{V}/\beta\right)}{\bar{V}/\beta}, \tag{27}$$

which is continuous in $\bar{V}/\beta$ because $\epsilon_L(\cdot)$ is continuous by assumption and because the existence of $\kappa_L'\left(\bar{V}/\beta\right)$ implies that $\kappa_L\left(\bar{V}/\beta\right)$ is continuous. On the other hand, suppose $X_L'(0) > 0$. If $\bar{V} < W\beta X_L'(0)$, then the First Order Condition holds with a strict inequality, which implies that $\kappa_L\left(\bar{V}/\beta\right) = \kappa_L'\left(\bar{V}/\beta\right) = 0$. If $\bar{V} > W\beta X_L'(0)$, then the First Order Condition holds with equality, $\kappa_L\left(\bar{V}/\beta\right) > 0$, and $\kappa_L'\left(\bar{V}/\beta\right)$ is given by equation (27). It follows that $\kappa_L'\left(W X_L'(0)\right) = 0$ because as $\bar{V}$ goes to $W\beta X_L'(0)$ from either direction, $\kappa_L'\left(\bar{V}/\beta\right)$ converges to 0. Thus, $\kappa_L(\cdot)$ is continuously differentiable.

We can write equation (8) for $\bar{V}$ (with $A = 0$) using $\kappa_L(\cdot)$ and $\kappa_S(\cdot)$:

$$\left(r + \kappa_L\left(\bar{V}/\beta\right) + \kappa_S\left(\kappa_L\left(\bar{V}/\beta\right)\right)\right)\bar{V}/\beta - WX_L\left(\kappa_L\left(\bar{V}/\beta\right)\right) = \left((1-\sigma^{-1}-\tau_R)R - f_L W\right)/\beta. \quad (28)$$

The right-hand side is constant in $\bar{V}/\beta$ and strictly greater than 0 by Assumption 1. The left-hand side is continuous in $\bar{V}/\beta$ and continuously differentiable everywhere except at the single point where $\kappa_L\left(\bar{V}/\beta\right) = \kappa_L^*$. Moreover, as $\bar{V}/\beta$ goes to 0, the First Order Condition for κ_L implies that $\kappa_L\left(\bar{V}/\beta\right)$ goes to 0 because $X_L(\cdot)$ is strictly increasing, which implies that $X_L\left(\kappa_L\left(\bar{V}/\beta\right)\right)$ goes to 0 because $X_L(\cdot)$ is continuous and $X_L(0) = 0$. Therefore, as $\bar{V}/\beta$ goes to 0, the left-hand side of equation (28) goes to 0. Finally, I show that the derivative of the left-hand side, where it exists, is bounded below by a strictly positive number, which implies that a unique solution $\bar{V}/\beta$ always exists because the left-hand side goes to infinity as $\bar{V}/\beta$ goes to infinity, implies that this solution is strictly increasing in the right-hand side, and implies that everywhere but at a single point, the left-hand side as a function of $\bar{V}/\beta$ has a continuously differentiable inverse. Taking the derivative and plugging in that if $\kappa_L'\left(\bar{V}/\beta\right) \neq 0$, then $WX_L'\left(\kappa_L\left(\bar{V}/\beta\right)\right) = \bar{V}/\beta$ yields

$$r + \kappa_L\left(\bar{V}/\beta\right) + \kappa_S\left(\kappa_L\left(\bar{V}/\beta\right)\right) + \kappa_S'\left(\kappa_L\left(\bar{V}/\beta\right)\right)\kappa_L'\left(\bar{V}/\beta\right)\bar{V}/\beta. \quad (29)$$

If the last term in the sum is not equal to 0, then $\kappa_L\left(\bar{V}/\beta\right) > 0$ and $\kappa_S\left(\kappa_L\left(\bar{V}/\beta\right)\right) > 0$, which implies that the derivatives are given by equations (26) and (27), which implies that expression (29) is weakly greater than

$$r + \kappa_L\left(\bar{V}/\beta\right) + \kappa_S\left(\kappa_L\left(\bar{V}/\beta\right)\right) - \frac{1}{\epsilon_S\left(\kappa_S\left(\kappa_L\left(\bar{V}/\beta\right)\right)\right) + 1}\frac{1}{\epsilon_L\left(\kappa_L\left(\bar{V}/\beta\right)\right)}\kappa_L\left(\bar{V}/\beta\right),$$

which is weakly greater than $r > 0$ by the premise of the proposition. Thus, the equilibrium value of $\bar{V}/\beta$ is a strictly increasing continuous function of $\left(\left(1-\sigma^{-1}-\tau_R\right)R - f_L W\right)/\beta$ that is continuously differentiable everywhere except at the single point where $\kappa_L\left(\bar{V}/\beta\right) = \kappa_L^*$.

Finally, $\kappa_L(\cdot)$ implies that the equilibrium value of κ_L is an increasing continuous function of $\left(\left(1-\sigma^{-1}-\tau_R\right)R - f_L W\right)/\beta$ that is strictly increasing if $\kappa_L > 0$, and that is continuously differentiable everywhere except at $\kappa_L = \kappa_L^*$.

A.2 Proof of Theorem 1

Equation (2) for the growth rate, evaluated on a balanced growth path, equation (3) for the long-run large firm revenue share, and the function $\kappa_S(\kappa_L)$ imply that if $A = 0$, then the long-run growth rate is a function of κ_L:

$$g(\kappa_L) = (\lambda - 1)\kappa_S(\kappa_L)\left(1 + \frac{\kappa_L}{\kappa_L + \kappa_S(\kappa_L)}\right),$$

which is well-defined for all $\kappa_L \geq 0$ because $\kappa_S(0) > 0$ by Assumption 2. Moreover, $g(\cdot)$ is continuous everywhere and continuously differentiable everywhere except at $\kappa_L = \kappa_L^*$ because that is the case for $\kappa_S(\cdot)$. The derivative is

$$g'(\kappa_L) = (\lambda - 1)\kappa_S'(\kappa_L)\left(1 + \left(\frac{\kappa_L}{\kappa_L + \kappa_S(\kappa_L)}\right)^2\right) + (\lambda - 1)\left(\frac{\kappa_S(\kappa_L)}{\kappa_L + \kappa_S(\kappa_L)}\right)^2,$$

where $\kappa_S'(\kappa_L)$ is given in equation (26).

The first property of $g(\cdot)$ in the theorem holds because $g(\kappa_L) > 0$ if and only if $\kappa_S(\kappa_L) > 0$, which holds if and only if $\kappa_L < \kappa_L^*$.

The second property of $g(\cdot)$ holds because $g'(0) = (\lambda - 1)\left(1 + \kappa_S'(0)\right)$, and $\kappa_S'(0) \geq -1$ and strictly so if and only if $\epsilon_S(\kappa_S(0)) > 0$.

For the third property of $g(\cdot)$, consider two cases. If $\epsilon_S(0) = 0$, then as κ_L goes to κ_L^* from below, $\kappa_S(\kappa_L)$ converges to 0 and $\kappa_S'(\kappa_L)$ converges to -1 by the continuity of $\epsilon_S(\cdot)$ (even though $\kappa_S'(\kappa_L^*)$ does not exist), which implies that $g'(\kappa_L)$ converges to $-2(\lambda - 1)$. If $\epsilon_S(0) > 0$, then as κ_L goes to κ_L^* from below, $\kappa_S(\kappa_L)$ converges to 0, which implies that $g'(\kappa_L)$ converges to

$$-2(\lambda - 1)\frac{\kappa_S(\kappa_L)}{\epsilon_S(0)\left(r + \kappa_L^*\right)} + (\lambda - 1)\left(\frac{\kappa_S(\kappa_L)}{\kappa_L^*}\right)^2,$$

which is strictly negative for κ_L sufficiently close to κ_L^*. In either case, the property follows from the continuity of $g'(\kappa_L)$ on $[0, \kappa_L^*)$.

For the fourth property, for $\kappa_L < \kappa_L^*$, divide $g'(\kappa_L)$ by $-(\lambda - 1)\kappa_S'(\kappa_L)$, which is strictly positive, to see that $g'(\kappa_L) = 0$ if and only if

$$1 + \left(\frac{\kappa_L}{\kappa_L + \kappa_S(\kappa_L)}\right)^2 = \frac{\kappa_S(\kappa_L)}{\kappa_L + \kappa_S(\kappa_L)}\left(\epsilon_S\left(\frac{r}{\kappa_L + \kappa_S(\kappa_L)} + 1\right) + \frac{\kappa_S(\kappa_L)}{\kappa_L + \kappa_S(\kappa_L)}\right).$$

The left-hand side is strictly increasing in κ_L because $\kappa_S'(\kappa_L) < 0$, and the right-hand side is strictly decreasing in κ_L because $\kappa_S'(\kappa_L) \in [-1, 0)$, which implies that $\kappa_L + \kappa_S(\kappa_L)$ is increasing in κ_L. Therefore, there is at most one $\kappa_L < \kappa_L^*$ such that $g'(\kappa_L) = 0$. Finally, the second and third properties of $g(\cdot)$ in the theorem imply that there is at least one $\kappa_L < \kappa_L^*$ such that $g'(\kappa_L) = 0$.

A.3 Proof of Proposition 2

Suppose $X_L(\cdot)$ and $X_A(\cdot)$ are strictly convex and for all κ_L, $\epsilon_L(\kappa_L)(\epsilon_S(\kappa_S(\kappa_L)) + 1) \geq 1$. I follow a similar argument as in the proof of the second part of Proposition 1. I show that we can write the expected discounted profits a small firm earns from a good, Π, as a continuous function of the marginal value of a good to the large firm $\bar{V}$ that is continuously differentiable everywhere except at a single point. I then use Π to write the surplus from an acquisition S as a continuous function of $\bar{V}$ and τ_A that is continuously differentiable everywhere except at a single point. Next, I show that we can write the acquisition rate A as a continuous function of S that is continuously differentiable everywhere except at a single point. I use these functions, along with $\kappa_S(\cdot)$ from Proposition 1, $\kappa_L(\cdot)$ from the proof of Proposition 1, and equation (8) for $\bar{V}$ to show that the equilibrium value of $\bar{V}$ is a function of τ_A with the right properties so that the equilibrium values of A, κ_L, and g are functions of τ_A with the desired properties.

As in the proof of the second part of Proposition 1, equation (8) for $\bar{V}$ and Assumption 1 imply that $\bar{V} > 0$ in any equilibrium, which I impose throughout the proof. The functions $\kappa_S(\kappa_L)$ from Proposition 1 and $\kappa_L\left(\bar{V}/\beta\right)$ from the proof of the second part of Proposition 1, and equation (6) for the expected discounted profits a small firm earns from a good define a continuous function $\Pi\left(\bar{V}\right)$ on $\bar{V} > 0$, which is continuously differentiable everywhere except at the single point where $\kappa_L\left(\bar{V}/\beta\right) = \kappa_L^*$. Moreover, where it exists, $\Pi'\left(\bar{V}\right) \leq 0$ because Π depends inversely on $\kappa_L + \kappa_S$, and the proof of Proposition 1 shows that $\kappa_L'\left(\bar{V}/\beta\right) \geq 0$ and $\kappa_S'(\kappa_L) \geq -1$.

The function $\Pi\left(\bar{V}\right)$ defines a function for $\bar{V} > 0$ and $\tau_A > -1$ for the surplus from an acquisition, $S\left(\bar{V}, \tau_A\right) \equiv \bar{V} - (1+\tau_A)\Pi\left(\bar{V}\right)$, which is continuously differentiable with respect to τ_A, continuous everywhere in $\bar{V}$, and continuously differentiable with respect to $\bar{V}$ everywhere except at the single point where $\kappa_L\left(\bar{V}/\beta\right) = \kappa_L^*$.

The First Order Condition for the acquisition rate A (inequality (10)) implicitly defines a function of the surplus from an acquisition, $A(S)$, because due to the strict convexity of $X_A(\cdot)$, the left-hand side is strictly increasing in A and goes to infinity as A goes to infinity. Moreover, $A(\cdot)$ is continuous because $X_A'(\cdot)$ is continuous and strictly increasing, and so has a continuous inverse. Finally, I show that the derivative $A'(S)$ exists and is continuous everywhere except where $S = WX_A'(0)$. If $S < WX_A'(0)$, then the First Order Condition holds with a strict inequality, which implies that $A(S) = A'(S) = 0$. If $S > WX_A'(0) \geq 0$, then $A(S) > 0$ and the First Order Condition holds with equality, which given $\epsilon_A(\cdot) > 0$, implies that

$$WX_A''(A(S)) = \epsilon_A(A(S))\frac{S}{A(S)} > 0.$$

Thus, differentiating both sides of the First Order Condition yields the continuous derivative:

$$A'(S) = \frac{1}{\epsilon_A(A(S))}\frac{A(S)}{S}. \tag{30}$$

Now, define $T\left(\bar{V}\right)$ to be the terms in equation (8) not including A:

$$T\left(\bar{V}\right) \equiv \left(r+\kappa_L\left(\bar{V}/\beta\right)+\kappa_S\left(\kappa_L\left(\bar{V}/\beta\right)\right)\right)\bar{V}-W\beta X_L\left(\kappa_L\left(\bar{V}/\beta\right)\right)-(1-\sigma^{-1}-\tau_R)R+f_L W, \quad (31)$$

which, from the proof of the second part of Proposition 1, is strictly negative as $\bar{V}$ goes to 0, is continuous, and is continuously differentiable with a derivative greater than r everywhere except where $\kappa_L\left(\bar{V}/\beta\right)=\kappa_L^*$. We can write equation (8) for $\bar{V}$ using $T(\cdot)$, $\Pi(\cdot)$, $A(\cdot)$, and $S(\cdot,\cdot)$:

$$T\left(\bar{V}\right)+A\left(S\left(\bar{V},\tau_A\right)\right)\bar{V}-WX_A\left(A\left(S\left(\bar{V},\tau_A\right)\right)\right)-A\left(S\left(\bar{V},\tau_A\right)\right)(1+\tau_A)\Pi\left(\bar{V}\right)=0. \quad (32)$$

The left-hand side is differentiable with respect to $\bar{V}$ and τ_A everywhere except where $\kappa_L\left(\bar{V}/\beta\right)=\kappa_L^*$ and $S\left(\bar{V},\tau_A\right)=WX_A'(0)$. Taking the derivatives and plugging in that if $A'(S)\neq 0$, then $S=WX_A'(A(S))$ yields

$$\begin{aligned}\frac{\partial LHS}{\partial\bar{V}}&=T'\left(\bar{V}\right)+A\left(S\left(\bar{V},\tau_A\right)\right)\left(1-(1+\tau_A)\Pi'\left(\bar{V}\right)\right)\\ \frac{\partial LHS}{\partial\tau_A}&=-A\left(S\left(\bar{V},\tau_A\right)\right)\Pi\left(\bar{V}\right).\end{aligned} \quad (33)$$

The first derivative is greater then $r>0$ because $T'\left(\bar{V}\right)\geq r$ and $\Pi'\left(\bar{V}\right)\leq 0$.

Equation (32) defines $\bar{V}$ as a continuous function of τ_A because the left-hand side is continuous in $\bar{V}$ and τ_A, and for a fixed τ_A, the left-hand side is strictly increasing in $\bar{V}$, goes to $T\left(\bar{V}\right)$, which goes to a strictly negative number as $\bar{V}$ goes to 0, and goes to infinity as $\bar{V}$ goes to infinity. Thus, there is a unique equilibrium value $\bar{V}$, which is continuous in τ_A, and using $\kappa_L(\cdot)$, $\kappa_S(\cdot)$, $\Pi(\cdot)$, $S(\cdot,\cdot)$, and $A(\cdot)$, there are unique equilibrium values κ_L, κ_S, Π, S, and A, which are also continuous in τ_A. The equilibrium value of $\bar{V}$ is increasing in τ_A because it is continuous in τ_A and differentiable everywhere except at finitely many points with derivative

$$\frac{\partial\bar{V}}{\partial\tau_A}=\frac{A\Pi}{T'\left(\bar{V}\right)+A\left(1-(1+\tau_A)\Pi'\left(\bar{V}\right)\right)},$$

which is positive because $T'(\cdot)>0$ and $\Pi'(\cdot)\leq 0$. Moreover, $\bar{V}$ is strictly increasing in τ_A if $A>0$ because in that case, the derivative is strictly positive. The equilibrium value of S is strictly decreasing in τ_A because it is continuous in τ_A and differentiable everywhere except at finitely many points with derivative

$$\frac{\partial S}{\partial\tau_A}=\frac{\partial\bar{V}}{\partial\tau_A}\left(1-(1+\tau_A)\Pi'\left(\bar{V}\right)\right)-\Pi=\frac{-T'\left(\bar{V}\right)\Pi}{T'\left(\bar{V}\right)+A\left(1-(1+\tau_A)\Pi'\left(\bar{V}\right)\right)}, \quad (34)$$

which is strictly negative.

It follows from $\kappa_L(\cdot)$ that the equilibrium value of κ_L is increasing in τ_A, and strictly so if $\kappa_L, A>0$ because in that case, $\bar{V}$ is strictly increasing in τ_A and $\kappa_L\left(\bar{V}\right)$ is strictly increasing. It follows from

$A(\cdot)$ that the equilibrium value of A is decreasing in τ_A, and strictly so if $A > 0$ because in that case, $A(S)$ is strictly increasing.

Finally, suppose $\kappa_S, \kappa_L, A > 0$. The equilibrium values of $\bar{V}$ and S are continuously differentiable with respect to τ_A because in that case, $T(\cdot)$ and $\Pi(\cdot)$ are continuously differentiable. Moreover, the equilibrium values of κ_L and A are continuously differentiable with respect to τ_A because in that case, $\kappa_L(\cdot)$ and $A(\cdot)$ are continuously differentiable.

Finally, the results concerning the long-run growth rate g follow from equation (2) for the growth rate, evaluated on a balanced growth path, and equation (3) for the long-run large firm revenue share $\mathcal{L}$, which imply that, as long as $\kappa_L + A + \kappa_S > 0$ (always the case in equilibrium), g is a continuously differentiable function of κ_S, κ_L, and A that does not depend otherwise on τ_A.

A.4 Derivation of $\partial g/\partial\tau_A$

If κ_S, κ_L, and A are strictly positive, then using $\kappa_L'\left(\bar{V}/\beta\right)$ given by equation (27), the derivative of the equilibrium value of κ_L with respect to τ_A is

$$\frac{\partial\kappa_L}{\partial\tau_A} = \frac{1}{\epsilon_L(\kappa_L)}\frac{\kappa_L}{\bar{V}}\frac{\partial\bar{V}}{\partial\tau_A}. \tag{35}$$

Using $A'(S)$ given by equation (30), the derivative of the equilibrium value of $S = \bar{V} - (1+\tau_A)\Pi$ given by equation (34), $T'\left(\bar{V}\right)$ given by equation (29), $\kappa_S'(\kappa_L)$ given by equation (26), and $\kappa_L'\left(\bar{V}/\beta\right)$ given by equation (27), the derivative of the equilibrium value of A with respect to τ_A is

$$\frac{\partial A}{\partial\tau_A} = -\frac{1}{\epsilon_A(A)}\frac{1}{\bar{V} - (1+\tau_A)\Pi}\left(r + \kappa_L + \kappa_S - \frac{\kappa_S}{\epsilon_S(\kappa_S)(r+\kappa_L+\kappa_S)+\kappa_S}\frac{1}{\epsilon_L(\kappa_L)}\kappa_L\right)\frac{\partial\bar{V}}{\partial\tau_A}. \tag{36}$$

Using equation (2), evaluated on a balanced growth path, for the growth rate g, and equation (3) for the long-run large firm revenue share $\mathcal{L}$, the partial derivative of the growth rate with respect to the acquisition rate is

$$\frac{\partial g}{\partial A} = -\mathcal{L}(1-\mathcal{L})\frac{\kappa_L}{\kappa_L + A}, \tag{37}$$

which is strictly negative. The partial derivative of the growth rate with respect to the large firm innovation rate, taking into account small firm optimization through $\kappa_S(\kappa_L)$, is

$$\frac{\partial g}{\partial\kappa_L} = (1-\mathcal{L})\left(1 - \mathcal{L}\frac{\kappa_L}{\kappa_L + A}\right) - \left(\mathcal{L}^2\frac{\kappa_L}{\kappa_L + A} + 1\right)\frac{\kappa_S}{\epsilon_S(\kappa_S)(r+\kappa_L+\kappa_S)+\kappa_S}. \tag{38}$$

Combining equations (35), (36), (37), and (38), the derivative of the equilibrium value of g with respect to τ_A is the product of

$$\mathcal{L}(1-\mathcal{L})\frac{\kappa_L}{\kappa_L + A}\frac{1}{\bar{V} - (1+\tau_A)\Pi}\left(r + \kappa_L + \kappa_S - \frac{\kappa_S}{\epsilon_S(\kappa_S)(r+\kappa_L+\kappa_S)+\kappa_S}\frac{1}{\epsilon_L(\kappa_L)}\kappa_L\right)\frac{\partial\bar{V}}{\partial\tau_A},$$

which is strictly positive because $\epsilon_L(\kappa_L)(\epsilon_S(\kappa_S)+1) \geq 1$, and

$$\frac{(1-\mathcal{L})((1-\mathcal{L})\kappa_L + A)(\epsilon_S(\kappa_S)(r+\kappa_L+\kappa_S)+\kappa_S) - ((1+\mathcal{L}^2)\,\kappa_L + A)\,\kappa_S}{\mathcal{L}(1-\mathcal{L})[(r+\kappa_L+\kappa_S)\epsilon_L(\kappa_L)(\epsilon_S(\kappa_S)(r+\kappa_L+\kappa_S)+\kappa_S) - \kappa_L\kappa_S]}\frac{\bar{V}-(1+\tau_A)\Pi}{\bar{V}} + \frac{1}{\epsilon_A(A)}. \tag{39}$$

The numerator in the first fraction in expression (39) is strictly negative if and only if, holding other variables constant, $\epsilon_S(\kappa_S)$ is sufficiently small, and the denominator is strictly positive because $\epsilon_L(\kappa_L)(\epsilon_S(\kappa_S)+1) \geq 1$. If the numerator is negative, then the first fraction is strictly increasing in $\epsilon_S(\kappa_S)$ because an increase in $\epsilon_S(\kappa_S)$ makes the numerator less negative and increases the denominator, and is strictly increasing in $\epsilon_L(\kappa_L)$ because an increase in $\epsilon_L(\kappa_L)$ increases the denominator. Thus, if $\epsilon_S(\kappa_S)$ is sufficiently large, then the first fraction is positive, and the derivative of g with respect to τ_A is positive. If $\epsilon_S(\kappa_S)$ is sufficiently small, then the first fraction is strictly negative, and the derivative of g with respect to τ_A is strictly negative if and only if $\epsilon_A(A)$ is greater than a threshold. The threshold is strictly increasing in $\epsilon_S(\kappa_S)$ and $\epsilon_L(\kappa_L)$. Finally, expression (13) follows from setting $\epsilon_S(\kappa_S) = 0$.

A.5 Proof of Theorem 2

Suppose $\epsilon_S(\cdot)$, $\epsilon_L(\cdot)$, and $\epsilon_A(\cdot)$ are constants with $\epsilon_S = 0$, $\epsilon_L \geq 1$, and $\epsilon_A > 0$. It follows that $X_A'(0) = 0$ because otherwise $\epsilon_A(0) = 0$.

I first show that if $\mathcal{L} \in (0,1)$ and there is an increase in f_L and/or τ_R, then the equilibrium $\mathcal{L}$ is strictly lower, the equilibrium marginal value of a good to the large firm, $\bar{V}$, is strictly lower, the equilibrium creative destruction rate of a small firm's good and expected discounted profits a small firm earns from a good, $\kappa_L + \kappa_S$ and Π, are constant, the equilibrium large firm innovation rate and acquisition rate, κ_L and A, are weakly lower, and the equilibrium sum $\kappa_L + A$ is strictly lower. I next show the existence of $\mathcal{L}^*$. Finally, I show the existence of $\mathcal{L}^{**}$.

Suppose $\mathcal{L} \in (0,1)$. It follows that one of κ_L and A is strictly positive, and $\kappa_S > 0$. The proof of Proposition 2 shows that equation (32) must hold in equilibrium. It follows from the definition of $T(\cdot)$ in equation (31) that for a fixed $\bar{V}$, the left-hand side of equation (32) is strictly decreasing in $(1-\sigma^{-1}-\tau_R)\,R - f_L W$, and does not otherwise depend on τ_R or f_L. Moreover, expression (33) shows that the left-hand side of equation (32) is strictly increasing in $\bar{V}$. It follows that if $(1-\sigma^{-1}-\tau_R)\,R - f_L W$ is strictly higher, then the equilibrium value of $\bar{V}$ is strictly higher. Thus, using $\kappa_L\left(\bar{V}/\beta\right)$ defined implicitly by the First Order Condition for κ_L in the proof of the second part of Proposition 1, the equilibrium value of κ_L is higher, and strictly so if $\kappa_L > 0$. Next, $\epsilon_S = 0$ and $\kappa_S > 0$ imply that $\kappa_S'(\kappa_L) = -1$. It follows that the equilibrium value of the creative destruction rate of a small firm's good, $\kappa_L + \kappa_S$, and so the equilibrium value of the expected present discounted profits a small firm earns from a good, Π, are constant. Therefore, the equilibrium value of the surplus from an acquisition, $\bar{V} - (1 + \tau_A)\Pi$, is strictly higher, which implies, using $A(S)$—the acquisition rate as a function of the surplus from an acquisition—defined implicitly by the First Order Condition for A, that the equilibrium value of A is higher, and strictly so if $A > 0$. Finally, it follows that the equilibrium value of the long-run large firm revenue share $\mathcal{L}$ is strictly higher because if $\kappa_L > 0$, then the numerator of $1 - \mathcal{L}$, κ_S, is strictly lower, and the denominator, $\kappa_L + A + \kappa_S$, is higher, or if $A > 0$, then the numerator is lower and the denominator is strictly higher.

The equilibrium variables mentioned are continuous functions of $(1 - \sigma^{-1} - \tau_R)\, R - f_L W$ because the proof of Proposition 2 shows that equation (32) is continuous in $(1 - \sigma^{-1} - \tau_R)\, R - f_L W$ and in $\bar{V}$, and that all the functions mentioned are continuous.

It follows that there exists an $\mathcal{L}^*$ such that $A > 0$ if and only if $\mathcal{L} > \mathcal{L}^*$ because if $\mathcal{L}$ is higher, then so is $(1 - \sigma^{-1} - \tau_R)\, R - f_L W$, and thus so is A.

From the derivation in Appendix A.4, the sign of $\partial g / \partial \tau_A$ is the sign of expression (13). From the above arguments, it follows that if $\mathcal{L}$ is strictly larger, then so is $(1 - \sigma^{-1} - \tau_R)\, R - f_L W$. Thus, expression (13) is strictly lower because $1 - \mathcal{L}$ is strictly lower, $2\kappa_L + A$ is strictly higher, $\epsilon_L(r + \kappa_L + \kappa_S)$ is constant, $-\kappa_L$ is lower, Π is constant, and $\bar{V} - (1 + \tau_A)\Pi$ is strictly higher. Hence, if $\mathcal{L}^* < 1$, then there exists an $\mathcal{L}^{**}$ such that $\partial g / \partial \tau_A < 0$ if and only if $\mathcal{L} > \mathcal{L}^{**}$. Moreover, $\mathcal{L}^{**} > \mathcal{L}^*$ because if $\mathcal{L}^* \geq 0$ and $\mathcal{L} = \mathcal{L}^*$, then $\bar{V} - (1 + \tau_A)\Pi = 0$ (since $X'_A(0) = 0$), which implies that expression (13) is strictly positive. Next, $\mathcal{L}^{**} > 0$ because if $\mathcal{L} = 0$, then $2\kappa_L + A = 0$, which implies that expression (13) is strictly positive. Finally, $\mathcal{L}^{**} < 1$ because if $\mathcal{L} = 1$, then $1 - \mathcal{L} = 0$, which implies that expression (13) is strictly negative.

A.6 Proof of Theorem 3

Throughout the proof, I drop time t subscripts when possible. I begin by proving the results concerning growth across industries on a single balanced growth path. I first show that we can decompose the industry growth rate into two continuously differentiable functions of the large firm industry revenue share, a strictly increasing $D(\cdot)$ and strictly decreasing $\tilde{g}(\cdot)$, so that for all n, $(\gamma - 1)g_n = D(\mathcal{L}_n)\tilde{g}(\mathcal{L}_n)$, and $D(\mathcal{L}_n)$ is the non-interest component of the discount rate on a small firm's good:

$$D(\mathcal{L}_n) = N(\kappa_{S,n}(\mathcal{L}_n) + (\gamma - 1)g_{S,n}(\mathcal{L}_n)) + \kappa_{L,n}(S; \mathcal{L}_n) + (\gamma - 1)g_{L,n}(\mathcal{L}_n. \tag{40}$$

Consider $D(\cdot)$, and suppose equation (40) holds. Using HJB equation (20) for the present discounted profits a small firm earns from a good, we can guess and verify that on a balanced growth path, those profits relative to the wage are a constant $\tilde{\Pi}_n$ ($\Pi_{n,t}(\mathcal{L}_n)$ grows at the same rate as the wage) given by equation (24), reproduced below, because on a balanced growth path, the interest rate is $r = \rho + g$, aggregate output is $C_{t'} = Z_t e^{g(t'-t)} L^p$ for all $t' \geq t$, the industry state is constant, and the real wage is $W_t/P_t = Z_t/\sigma$:

$$\tilde{\Pi}_n \equiv \Pi_{n,t}(\mathcal{L}_n)/W_t = ((\sigma-1)L^p - f_S)/(\rho + D(\mathcal{L}_n)).$$

It then follows from equation (17) for the evolution of the industry state over time, and the second equation in (18) for growth due to small firms that on a balanced growth path, $D(\mathcal{L}_n) = N(\lambda^{\gamma-1}\kappa_{S,n}(\mathcal{L}_n) + \delta_{S,n}(\mathcal{L}_n))/(1-\mathcal{L}_n)$, which along with equations (21) for optimal small firm innovation, implies that

$$(1-\mathcal{L}_n)D(\mathcal{L}_n)(\rho + D(\mathcal{L}_n))^{1/\epsilon} = N((\lambda^{\gamma-1})^{1+1/\epsilon}\alpha^{-1/\epsilon} + 1)((\sigma-1)L^p - f_S)^{1/\epsilon}, \tag{41}$$

where the right-hand side is a strictly positive number. Eliminating the n subscripts on the left-hand side, equation (41) implicitly defines the function $D(\cdot)$ with the desired properties.

Second, consider $\tilde{g}(\cdot)$. Use equation (17) for the evolution of the industry state over time, evaluated on a balanced growth path, to write

$$\frac{(\gamma-1)g_n}{D(\mathcal{L}_n)} = (1-\mathcal{L}_n)\frac{(\gamma-1)g_{S,n}(\mathcal{L}_n)}{\kappa_{S,n}(\mathcal{L}_n) + (\gamma-1)g_{S,n}(\mathcal{L}_n)} + \mathcal{L}_n\frac{(\gamma-1)g_{L,n}(\mathcal{L}_n)}{\kappa_{L,n}(S;\mathcal{L}_n) + (\gamma-1)g_{L,n}(\mathcal{L}_n)}. \tag{42}$$

It follows from the second equation in (18) for growth due to small firms, and equations (21) for optimal small firm innovation that the first term on the right-hand side of equation (42) is $(1-\mathcal{L}_n)B/(B+1)$, where

$$B \equiv \lambda^{\gamma-1} - 1 + (\alpha\lambda^{1-\gamma})^{1/\epsilon}$$

is the rate at which a small firm generates growth relative to the rate at which it creatively destroys a good. It follows from the first equation in (18) and equations (23) that the second term on the right-hand side is the product of $\mathcal{L}_n$ and

$$\begin{aligned}\frac{(\gamma-1)g_{L,n}(\mathcal{L}_n)}{\kappa_{L,n}(S;\mathcal{L}_n) + (\gamma-1)g_{L,n}(\mathcal{L}_n)} &= 1 - \left(1 + \frac{(\gamma-1)g_{L,n}(\mathcal{L}_n)}{\kappa_{L,n}(S;\mathcal{L}_n)}\right)^{-1} \\ &= 1 - \left(1 + (1-\mathcal{L}_n)(\lambda^{\gamma-1}-1) + \left(\frac{(1-\mathcal{L}_n)}{(1-\mathcal{L}_n)\lambda^{\gamma-1} + \mathcal{L}_n}\right)^{1/\epsilon}\left(\mathcal{L}_n(\lambda^{\gamma-1}-1)^{1+1/\epsilon} + \alpha^{1/\epsilon}\right)\right)^{-1},\end{aligned} \tag{43}$$

where the last line, dropping the n subscripts, is a strictly decreasing continuously differentiable function of $\mathcal{L}$ that goes from $B/(B+1)$ at 0 to 0 at 1. Thus, we can define the function $\tilde{g}(\cdot)$ with the desired properties.

It follows that the growth rate across industries is characterized by a continuously differentiable function of the large firm industry revenue share, $g_I(\cdot)$, given by $(\gamma-1)g_I(\mathcal{L}) = D(\mathcal{L})\tilde{g}(\mathcal{L})$. Moreover, $g_I(0) > 0$ because equation (41) shows that $D(0) > 0$, and the first term on the right-hand side of equation (42) is strictly greater than 0. Next, $g_I'(0) > 0$ because equation (41) shows that $D'(0) > 0$, and $\tilde{g}'(0) = 0$ (the first term on the right-hand side of equation (42) not including $1-\mathcal{L}_n$ is equal to the second term not including $\mathcal{L}_n$).

Instead of the second item in Theorem 3 (concerning $\epsilon = 1$), I prove a stronger proposition:

Proposition 3. *If $\epsilon \in (0, (3+\sqrt{5})/2 - 1)$, then $\lim_{\mathcal{L}\to 1}(g_I(\mathcal{L})) = 0$. If $\epsilon > (3+\sqrt{5})/2 - 1$, then $\lim_{\mathcal{L}\to 1}(g_I(\mathcal{L})) = \infty$.*

Proof. First, equation (41) shows that as $\mathcal{L}$ goes to 1, $D(\mathcal{L})$ diverges to positive infinity. Therefore, as $\mathcal{L}$ goes to 1, $\rho + D(\mathcal{L})$ converges to $D(\mathcal{L})$, which implies that $(1-\mathcal{L})D(\mathcal{L})^{1+1/\epsilon}$ converges to a strictly positive number, and so $D(\mathcal{L})$ converges to the product of a strictly positive number and $(1-\mathcal{L})^{-\epsilon/(\epsilon+1)}$.

Next, equations (42) and (43) show that as $\mathcal{L}$ goes to 1, $\tilde{g}(\mathcal{L})$ converges to 0, which implies that $\tilde{g}(\mathcal{L})$ converges to the product of a strictly positive number and $(1-\mathcal{L})^{\min\{1,1/\epsilon\}}$ because that is the lowest power of $1-\mathcal{L}$ contained in any term.

It follows that as $\mathcal{L}$ goes to 1, $g_I(\mathcal{L})$ converges to the product of a strictly positive number and $(1-\mathcal{L})^{\min\{1,1/\epsilon\}-\epsilon/(\epsilon+1)}$. If $\epsilon > 1$, then the exponent on $1-\mathcal{L}$ is $(\epsilon-\epsilon_1^*)(\epsilon_2^*-\epsilon)/(\epsilon(\epsilon+1))$, where

$$\epsilon_1^* = (3-\sqrt{5})/2 - 1 \qquad \epsilon_2^* = (3+\sqrt{5})/2 - 1.$$

Since $\epsilon_1^* < 0$ and $\epsilon_2^* > 1$, it follows that for all $\epsilon \in (0, \epsilon_2^*)$, the exponent on $1-\mathcal{L}$ is strictly positive, so $g_I(\mathcal{L})$ converges to 0 as $\mathcal{L}$ goes to 1. For all $\epsilon > \epsilon_2^*$, the exponent on $1-\mathcal{L}$ is strictly negative, so $g_I(\mathcal{L})$ diverges to infinity as $\mathcal{L}$ goes to 1. ∎

I now complete the proof of the parts of Theorem 3 concerning $g_I(\cdot)$. Set $\epsilon = 1$. Since $g_I(0) > 0$, $g_I'(0) > 0$, $\lim_{\mathcal{L}\to 1}(g_I(\mathcal{L})) = 0$, and $g_I(\cdot)$ is continuously differentiable, it is sufficient to show that $g_I(\cdot)$ has at most one critical point. Equation (41) shows that

$$D'(\mathcal{L}) = \frac{D(\mathcal{L})(\rho + D(\mathcal{L}))}{(\rho + 2D(\mathcal{L}))(1-\mathcal{L})},$$

and equation (42) along with the associated derivations shows that

$$\tilde{g}(\mathcal{L}) = \frac{B + \lambda^{1-\gamma}\mathcal{L}B/(B+1)}{B+1+\lambda^{1-\gamma}\mathcal{L}/(1-\mathcal{L})} \qquad \tilde{g}'(\mathcal{L}) = \frac{\tilde{g}(\mathcal{L})}{1-\mathcal{L}}\left(\frac{\lambda^{1-\gamma}(1-\mathcal{L})}{B+1+\lambda^{1-\gamma}\mathcal{L}} - \frac{\lambda^{1-\gamma}/(1-\mathcal{L})}{B+1+\lambda^{1-\gamma}\mathcal{L}/(1-\mathcal{L})}\right).$$

It follows from multiplying $(\gamma-1)g_I'(\mathcal{L})$ by $(1-\mathcal{L})/((\gamma-1)g_I(\mathcal{L}))$ and adding and subtracting one to $D'(\mathcal{L})\tilde{g}(\mathcal{L})$ that at a critical point of $g_I(\cdot)$,

$$-\frac{D(\mathcal{L})}{\rho+2D(\mathcal{L})}+\frac{\lambda^{1-\gamma}(1-\mathcal{L})}{B+1+\lambda^{1-\gamma}\mathcal{L}}+\frac{B+1-\lambda^{1-\gamma}}{B+1+\lambda^{1-\gamma}\mathcal{L}/(1-\mathcal{L})}=0.$$

The left-hand side is strictly decreasing in $\mathcal{L}$ because $D(\cdot)$ is strictly increasing and the second and third terms are strictly positive ($\lambda^{1-\gamma}<1$). Thus, there is at most one solution.

Finally, I prove the parts of Theorem 3 concerned with the aggregate growth rate across balanced growth paths. The arguments are the same as for the growth rate across industries on a single balanced growth path, except that now the function $D(\cdot)$ is constant. Thus, the results follow because $\tilde{g}(\cdot)$ is continuously differentiable, $\tilde{g}(\mathcal{L})$ converges to 0 as $\mathcal{L}$ goes to 1, $\tilde{g}'(0)=0$, and $\tilde{g}'(\mathcal{L})<0$ for all $\mathcal{L}>0$.

I use the free entry condition to show that $D(\cdot)$ is constant in this case. Using HJB equation (22) for the value of entering industry n and equations (21) for optimal small firm innovation, we can guess and verify that on a balanced growth path, that value relative to the wage is a constant ($E_{n,t}(\mathcal{L}_n)$ grows at the same rate as the wage),

$$\tilde{E}_n\equiv E_{n,t}(\mathcal{L}_n)/W_t=(\epsilon/(\epsilon+1))\left((\lambda^{\gamma-1})^{1+1/\epsilon}\alpha^{-1/\epsilon}+1\right)\tilde{\Pi}_n^{1+1/\epsilon}/(\rho+\eta),$$

because on a balanced growth path, the interest rate is $r=\rho+g$, $\Pi_{n,t}(\mathcal{L}_n)/W_t$ is the constant $\tilde{\Pi}_n$, and the industry state is constant. On a balanced growth path with a strictly positive entry rate in which all industries are identical, the free entry condition fixes for all n, the value of entering industry n relative to the wage: $\tilde{E}_n=\xi$. It follows that for any variation in the large firm fixed cost f_L, revenue tax rate τ_R, and innovation cost shifter β, the balanced growth path present discounted profits a small firm earns from a good relative to the wage, $\tilde{\Pi}_n$, is constant. Thus, given constant labor used for production L^p, the discount rate on a small firm's good must be constant as well.