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MECE 5397

Project A – Poisson Equation

Write a computer code to solve the two-dimensional Poisson equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -F(x,y) \quad (29)$$

The domain of interest is the rectangle

$$a_x < x < b_x\,, \qquad a_y < y < b_y \quad (30)$$

and the boundary conditions

$$u(x = a_x, y) = \phi_{ab}(y)\,, \qquad u(x = b_x, y) = \psi_{ab}(y)\,, \quad (31)$$

$$\left.\frac{\partial u}{\partial y}\right|_{y=a_y} = 0\,, \qquad \left.\frac{\partial u}{\partial y}\right|_{y=b_y} = 0\,, \quad (32)$$

$$a_x = a_y = -\pi\,, \qquad b_x = b_y = \pi \quad (33)$$

$$\phi_{ab}(y) = (y - a_y)^2 \sin\frac{\pi(y - a_y)}{2(b_y - a_y)}\,, \qquad \psi_{ab}(y) = \cos[\pi(y - a_y)]\cosh(b_y - y) \quad (34)$$

$$F(x,y) = \sin\left[\pi\frac{x - a_x}{b_x - a_x}\right]\cos\left[\frac{\pi}{2}\left(2\frac{y - a_y}{b_y - a_y} + 1\right)\right] \quad (35)$$

Use ghost node(s) for Neumann condition(s).
After carrying out all the simulations needed for the report, run one last simulation with $F = 0$ and include the results in the report.

- Using Stream Plots
- Surface plots