GAs To Solve The Travelling Salesman Problem

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This report investigates the *Travelling Salesman Problem* through the use of a genetic algorithm (GA) employing the *inver-over* crossover operator, a combination of scramble and swap mutations, and a combination of $\mu + \lambda$ and replacement survival selection operators.

1. PROBLEM SPECIFICATIONS

Wolfram MathWorld defines the Travelling Salesman Problem (TSP) as "a problem in graph theory requiring the most efficient (*i.e.*, least total distance) Hamiltonian cycle a salesman can take through each of n cities," for which "no general method of solution is known." [6] The website also remarks that the problem is assigned to the class of NP-hard (non-deterministic polynomial time) problems.

Given three datasets, for territories Western Sahara, Uruguay, and Canada of size 29, 734, and 4, 663 cities, respectively, the objective has been to design a genetic algorithm from scratch in the Python programming language to solve the TSP for each locale using some advanced technique(s).

The first advanced technique employed in the algorithm is the *inver-over* hybrid crossover/mutation operator.

2. THE INVER-OVER OPERATOR

The inver-over operator can be regarded as either a crossover or mutation operator because it takes information from other individuals in the GA's mating pool, yet bases a single offspring off of a single primary parent, much the same way a mutation operator is unary, performing mutation on a single individual [4]. Throughout this report, the inver-over operator will be referenced as a crossover operator as this was

the inver-over algorithm's purpose in the GA developed.

2.1. Usefulness

As *inver-over* embodies aspects of both crossover and mutation operators, the operator is designed to provide middle-ground to algorithms relying primarily on crossover for variation (as this is computationally expensive) and those relying on mutation for variety, since this is often ineffective in escaping local minima¹ [4].

2.2. Representation

A set P of individuals of length n, each represented as a sequence $i_k = \langle C_{c_k(0)}, C_{c_k(1)}, C_{c_k(2)}, \dots, C_{c_k(n-1)} \rangle$, where $c_k \colon \{0,1,\dots,n-1\} \to \{1,2,\dots,n\}$ is a bijection between indices of i_k , $0 \leqslant k < \operatorname{card}(P)$, and city indices as provided in the data file, is used to denote the population. Let $M \subsetneq P$ be the mating pool containing individuals $i_{m(0)}, i_{m(1)}, \dots, i_{m(\operatorname{card}(M)-1)}$ with $m \colon \{0,\dots,\operatorname{card}(M)-1\} \to \{0,\dots,\operatorname{card}(P)-1\}$ an injective function mapping individuals in the mating pool M to their equal 'self' in the population P.

Fitness of an individual i_k , denoted by fitness (i_k) in the Algorithm 1, is then determined by the formula in Equation (1).

$$fitness(i_m) = \sum_{j=0}^{n-1} \left\| \overrightarrow{C_j C_{j+1}} \right\|, \qquad (1)$$

¹Local minima occur when the natural selection in the GA narrows the gene pool towards an ostensibly optimal solution and eliminates individuals that otherwise would evolve to become the true optimal solution.

where

$$\left\| \overrightarrow{C_j C_{j+1}} \right\| = \sqrt{(x_{C_{j+1}} - x_{C_j})^2 + (y_{C_{j+1}} - y_{C_j})^2}$$

is the Eucliean distance between cities C_j and C_{j+1} .² Hence a lower fitness score is better.

2.3. Algorithm

The algorithm (Algorithm 1) used mirrors that depicted in the article by Tao and Michalewicz [4].

Algorithm 1. The inver-over operator

Require:

- $\triangleright M$ be the mating pool
- $\triangleright 0 \leqslant i < \operatorname{card}(M)$ be the index of the initial parent
- $\triangleright n$ be the number of cities in the tour

Ensure:

▶ New offspring individual created

```
1: \mathbf{var}\ child := M[i]

    Copy the parent
    Output
    Description:
    O
     2: var unused := \{x \in \mathbb{Z} \mid 0 \le x < n-1\}
     3: var p := \frac{1}{2}

    Select randomly

     4: \mathbf{var} \ c := \mathrm{rand}(unused)
     5: unused := unused \setminus \{c\}
     6: while card(unused) > 0 do
                                  if \operatorname{rand}\{x \in \mathbb{R} \mid 0 \leqslant x < 1\} < p then
     7:
                                                   c' := rand(unused)
     8:
     9:
                                  else
                                                  newPar := rand(M)
 10:
                                                  newParC := where(newPar = child[c])
11:
                                                                                                                                                          \triangleright Index j in newPar
12:
                                                   c' := newParentC + 1
                                  unused := unused \setminus \{c'\}
13:
                                  if child[c \pm 1] = child[c'] then
14:
15:
                                                  break from the while loop
                                  child[c:c'] := child[c':c]
16:
                                                                                                                                                                                                                      ▷ Invert
                                   c := c'
 17:
18: if fitness(child) \geqslant \text{fitness}(M[i]) then
19:
                                   return child
20: else
21:
                                  return M[i]
```

Here M[i] is the primary parent from which the child is based. One can observe that all changes are

then made to this individual, like a mutation, yet they involve other individuals, denoted newPar, in the mating pool.

We borrow the following example of a single iteration of the algorithm from Tao and Michalewicz.

- 1. Let $child = \langle 2, 3, 9, 4, 1, 5, 8, 6, 7 \rangle$ and the current city index c is 1 so child[c] = 3.
- 2. (a) Suppose the random number generated by $\operatorname{rand}\{x \in \mathbb{R} \mid 0 \leq x < 1\}$ does not exceed p. Another city index c' from the child is selected, say c' = 6 so $\operatorname{child}[c'] = 8$. The section of child after indices c and c' (i.e., $\langle 9, 4, 1, 5, 8 \rangle$) is inverted, leaving $\operatorname{child} = \langle 2, 3, 8, 5, 1, 4, 9, 6, 7 \rangle$.
 - (b) Otherwise, another individual is (randomly) selected from the mating pool to become the new parent. Let this be $\langle 1,6,4,3,5,7,9,2,8 \rangle$. Here the city next to child[c]=3 is city 5. So, the segment of child to invert is that which starts after city 3 and terminates after city 5 (i.e., $\langle 9,4,1,5 \rangle$). This leaves $child=\langle 2,3,5,1,4,9,8,6,7 \rangle$.

As in Tao and Michalewicz, we remark that in either case the resulting string is intermediate in the sense that the above inversion operator is applied several times before an offspring is evaluated. After a number of iterations, suppose $child = \langle 9, 3, 6, 8, 5, 1, 4, 2, 7 \rangle$ and c = 2 so child[c] = 6.

- 3. (a) If $\operatorname{rand}\{x \in \mathbb{R} \mid 0 \leqslant x < 1\}$ is greater than p, the city following $\operatorname{child}[c] = 6$ is selected from a randomly chosen individual in the mating pool M. Let this city be city 8. As 8 follows 6 in child , the algorithm terminates.
 - (b) Otherwise, a randomly selected city is chosen. This may also be 8, in which case the algorithm also terminates. If it is not 8, the algorithm continues as described in (2).

3. OPTIMIZATION

3.1. The NumPy Module

Before any benchmarking was done, an effort was made to use the NumPy [2] module's interfaces as

² Take indices j modulo n so when $j=n-1, (n-1)+1\equiv 0\pmod n$ and we compute the distance from the last city back to the first.

much as possible over raw Python data types. For example, when possible, numpy.ndarray was chosen over Python's list data structure. The SciPy³ documentation states:

...the fact that [Python lists] can contain objects of differing types mean that Python must store type information for every element, and must execute type dispatching code when operating on each element. This also means that very few list operations can be carried out by efficient C loops—each iteration would require type checks and other Python API bookkeeping. [1]

As the problem being solved requires extensive accessing and mutating of arrays containing a single data type (*i.e.*, real numbers), NumPy provided a very easily-implemented speed boost.

3.2. Cheaply Computing Distance

Suppose that the distance between cities C_j and C_{j+1} is not less than 1 and the same applies for cities C_k and C_{k+1} . That is,

$$1 \leqslant \left\| \overrightarrow{C_j C_{j+1}} \right\|$$

$$= \sqrt{(x_{C_{j+1}} - x_{C_j})^2 + (y_{C_{j+1}} - y_{C_j})^2}$$

and

$$1 \leqslant \left\| \overrightarrow{C_k C_{k+1}} \right\|$$

$$= \sqrt{(x_{C_{k+1}} - x_{C_k})^2 + (y_{C_{k+1}} - y_{C_k})^2}$$

Then $\left\|\overrightarrow{C_j}\overrightarrow{C_{j+1}}\right\| < \left\|\overrightarrow{C_k}\overrightarrow{C_{k+1}}\right\|$ if, and only if, $\left\|\overrightarrow{C_j}\overrightarrow{C_{j+1}}\right\|^2 < \left\|\overrightarrow{C_j}\overrightarrow{C_{j+1}}\right\|^2$. Hence, by adding an assert statement to our code to ensure that

$$(x_{C_{j+1}} - x_{C_j})^2 + (y_{C_{j+1}} - y_{C_j})^2 \geqslant 1,$$

we can omit the expensive square root operation $\frac{1}{2}(n^2-n)$ times⁴ and still maintain accurate distance comparisons and fitness scores.

After the final iteration of the GA, the true distance must still be calculated using Equation (1).

3.3. Maximizing Hardware Utilization

At the time of writing, nearly all modern computers utilize multiple processor cores (or in some cases multiple processors, each with multiple cores) to accomplish tasks⁵. Natively, the Python language uses only *one* core, leaving the rest underutilized.

Immediately, multi-threaded processing using Python's multiprocessing dummy module was investigated to take full advantage of the computer's hardware. Multithreading was chosen over multiprocessing as this does not impose the requirement that functions be serializable (referred to by the term *pickleable* in Python). This allows local helper functions and lambda expressions to be used. This resulted in *worse* performance because, as QuantStart explains,

...the Python interpreter is not thread safe. This means that there is a globally enforced lock [Global Interpreter Lock] when trying to safely access Python objects from within threads. At any one time only a single thread can acquire a lock for a Python object or C API. The interpreter will reacquire this lock for every 100 bytecodes of Python instructions and around (potentially) blocking I/O operations. Because of this lock CPU-bound code will see no gain in performance when using the Threading library...[3]

Instead, multiprocessing was used to create several Python instances when necessary, each with their own lock (and dedicated thread) that does not interfere with the other processes. Unfortunately, this meant a large amount of code needed to be restructured to ensure that all functions and the data passed to them could be pickled. Implementation was done using the Pool class and the Process class from Python's multiprocessing module. A caveat of this is that separate multiprocessing processes cannot share memory without a shared data structure [5]. As NumPy arrays were the preferred data structure, several functions needed to be refactored to return the result of a computation (which then is inserted in

³NumPy is part of the SciPy Stack.

⁴If distances are computed only once (see sub-section 3.4). Otherwise the number of expensive calculations may be higher.

⁵Specifications of several of Memorial's LabNet computers used for computation are included in Appendix B.3.

some data structure in the main program) rather than modifying the data structure within the function.

These changes shortened execution of the algorithm for the Uruguay dataset from nearly 5.5 hours to less than 1.5 hours on a computer with a quad-core processor. Monitoring of Windows' taskmgr.exe and Unix's htop programs revealed that CPU utilization, while spread evenly among processor cores, still was not consistently near 100%, as one may expect.

Attempting program execution on several different computers and servers led to the conclusion that the limiting factor at this point was not the CPU but rather the speed of the RAM in the computer⁶.

We also remark that nodes with less cores at higher speeds (i.e., en2048ece04.engr.mun.ca, which has a quad-core 3.2GHz processor) fared much better than nodes with many cores at slightly lower speeds (i.e., reliant.cs.mun.ca, which has two dual-threaded quad-core processors each clocking 2.67GHz).

3.4. Pre-computing All Distances

Initially, distance between cities C_j and C_{j+1} was computed each time it was needed to score the fitness of an individual. Even with the 'cheap' computation technique described in sub-section 3.2, this resulted in significant needless computation as some values inevitably would be computed more than once. To avoid duplicate calculations, there were two possible approaches to take:

- (i) Develop a data structure to compute distances as needed and store the values for future reference
- (ii) Pre-compute all distances

The first approach avoids the cost of pre-computing distances that are never used by using a just-in-time approach and then storing the computed value. A major drawback of this is that is nullifies any advantage of multiprocessing since this approach computes small sets of distances very sporadically instead of all of the distances at one time. For this reason, the second option was chosen.

4. ALGORITHM & IMPLEMENTATION

4.1. Program Entry and Preprocessing

The program entry point is in main.py; other local modules used in the algorithm are contained in the following directory structure.

```
main.py

➢ mutation/
                                                                                                                                                        init__.py

    crossover/

                 init__.py
                                                                                                                                                        scramble.py
                 inver_over.py
                                                                                                                                                        swap.py

  fitness/

                                                                                                                                     ▶ population/
                 init__.py
                                                                                                                                                        init__.py
                 distance.py
                                                                                                                                                        andidates.py
                 fitness.py
                                                                                                                                                        initialization.py

    ga_helper/

                 init__.py

    selection/

                 forfspring.py

    b timing/
    b timing/
    b timing/
    c timin
                                                                                                                                                        parent.py
                 init__.py
                                                                                                                                                        survival.py
```

Initialization is done in main.py through a call to initialization.init_file() in the population module, which takes a path to a file and returns a numpy.ndarray object containing the x- and y-coordinate pairs of the cities in the data file.

```
1 def init_file(populationFilePath):
2    if not os.path.isfile(populationFilePath):
3      raise FileNotFoundError
4    return np.loadtxt(
5      populationFilePath,
6      delimiter=' ',
7      usecols=(1, 2))
```

The array returned from this function is then passed to the <code>Distances()</code> class constructor in the <code>fitness.distance</code> module and the <code>gen_all()</code> method is called on this object to generate all $\frac{1}{2}(n^2-n)$ distances.

```
8 def gen_all(self):
9
      city pairs = (
          (start_idx, end_idx)
10
          for start_idx in \leftarrow
            range(self._cities.shape[0])
12.
          for end_idx in range(
13
              start_idx + 1,
              self._cities.shape[0]))
14
15
      with mp.Pool() as pool:
          [self._add_edge(*data_point)
16
17
              for data_point in pool.starmap(
                 self._add_edge_map_helper,
city_pairs)]
18
```

The _weighted_adjacency instance variable of the object is then passed to the main function;

⁶Due to the size of the data being stored in memory, CPU L1, L2, and L3 caches (see Appendix B.3) were all too small to store the entire data structure; hence it needed to be stored in RAM, and thus limited by the operating frequency of the RAM.

this way main() can run multiple times without reloading and recomputing the same data.

4.2. GA Initialization

Immediately in main (), variables such as generation limit G, candidate pool size CP, mating pool size MP, and so on are defined.

Candidates are then chosen by choosing CP enumerations from the set of permutations of $\{0,1,\ldots,n-1\}$, where n is the number of cities. Following this part of the algorithm's initialization, the adjacent_distance() function is invoked on for the population. This function calls single_cand_adjacent_distance,

```
20 def single_cand_adjacent_distance(distances, \leftrightarrow
     city_idx, true_distance=False):
21
      dist = []
22
      if true_distance:
23
          for i in range(city_idx.shape[0]):
24
              start_idx = city_idx[i]
25
              end idx = city idx[(i+1)]
                city_idx.shape[0]]
26
              d = distances[start_idx][end_idx] ←
                ** 0.5
27
              dist.append(d)
28
      else:
29
          for i in range(city_idx.shape[0]):
30
              start_idx = city_idx[i]
              end_idx = city_idx[(i+1) % \leftrightarrow
31
                city_idx.shape[0]]
32
              d = distances[start_idx][end_idx]
33
              dist.append(d)
      return np.array(dist)
```

which uses the adjacency matrix constructed in preprocessing to form an array of pairwise distances between each adjacent set of cities in the individual. Note that end_idx is defined as city_idx[(i+1) % city_idx.shape[0]], i.e., the index of city_idx being accessed is found modulo city_idx.shape[0] = n so this computes distance from the last city on the tour back to the first. From the returned sequences of distances, we compute overall fitness of each individual in the pool (using Equation (1) with the optimization discussed in sub-section 3.2).

4.3. GA Operators

Within a loop that iterates G times, the first operation performed is parent selection using the multi-pointer selection (MPS) operator. MPS randomly selects an individual from one of MP evenly spaced segments from the candidate pool and adds it to the parent pool. The operator then chooses MP-1 other individuals

at regular intervals of CP/MP. MPS was primarily chosen to minimize the risk of elitist selection emerging in our algorithm (*i.e.*, selecting only the fittest candidates), which leads to a high chance of the algorithm peaking at a local minima extremely early.

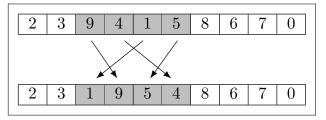
Inver-over is then introduced as the crossover operator, which in addition to the mutation operators discussed below, alters the offspring of the parent candidates selected by MPS. This operation prevents the GA from plateauing at a local minima while promoting the exploration aspect of the algorithm by inverting a random segment at the crossover phase, based on the individuals in the mating pool.

Initial implementations of the algorithm used scramble as a mutation operator and $\mu + \lambda$ survival selection. Preliminary tests showed that the randomness introduced by the swap mutation could be harmful to a fit individual more often than it was beneficial.

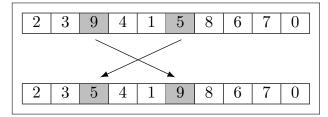
Instead, a system was developed that considers the previous two generations' top fitnesses when deciding which operator(s) to use for the next generation. This is done by comparing variables current_best_fitness and previous_best_fitness; if optimal generational fitness is not improving then introduce more randomness, otherwise maintain moderate randomness

Varied amounts of randomness were selected by adding swap mutation — which simply swaps two cities on the tour, depicted in Figure 1b — instead of just scramble. In such cases where generational fitness was not improving, scramble mutation (Figure 1a) was favoured 4: 1, the mutation rate was increased, crossover rate was decreased 1:2, severity of mutation⁷ was increased fourfold, and survivor selection was done favouring the replacement operator, rather than $\mu + \lambda$. Replacement was favoured 3: 1 here because the new (more random) individuals may initially score lower (i.e., higher Hamiltonian cycle distance) than existing individuals, however their varied genetic make-up could allow them to escape local minima more effectively. Such a scenario with $\mu + \lambda$ would result in many or all of the newer, less fit individuals immediately being removed from the

⁷See sub-sub-section 4.3.1.



(a) Scramble Mutation



(b) Swap Mutation

Fig. 1. Two Types of Mutation Implemented

population. When generational fitness is improving, the two mutation types are favoured evenly, mutation rate is lowered back to 10%, crossover rate is reset to 90%, and survival selection favours $\mu + \lambda~75\%$ of the time.

4.3.1. Severity in the Scramble Mutation

As the scramble mutation is now being used when randomness is desirable, the decision to capitalize on this operator's strength was made. The implementation of this involves a mutation factor $f_m \in (0,1)$.

```
35 def scramble_individual(individual, ←
     mutation_factor=0.25):
36
      mutant = individual
37
      n = mutant.shape[0]
38
      mutation_thresold = mutation_factor * n
39
40
      start_idx = end_idx = 0
      while end_idx-start_idx < mutation_thresold:
41
          start_idx = np.random.randint(0, n - 3)
42.
43
          end idx = \leftrightarrow
            np.random.randint(start_idx+2, n-1)
44
      np.random.shuffle(mutant[start_idx:end_idx])
      return mutant
```

A mutation threshold is then defined by $t_m := f_m \cdot n$. Two indices, start_idx and end_idx, both between 0 and n-1, are selected at random from the individual. If end_idx - start_idx < t_m then the algorithm chooses new indices until this requirement is satisfied.

Consequently, when high entropy is required a mutation factor can be chosen to enforce this (e.g.,

if $f_m = 0.4$ then no less than 40% of the individual will be scrambled).

5. RESULTS

With the optimizations discussed in Section 3.4, preprocessing speeds increased dramatically from nearnegligible time to load the file to the times listed in Table 1. Note that there are only three entries per data set as pre-processing was done outside the main () function to save computation (*i.e.*, pre-processing was done once for every 10 trials). Execution of

Trial	Western Sahara	Uruguay	Canada
1	112.342	769.979	31302.409
2	110.887	774.185	30910.104
3	110.485	725.295	30977.966

Table 1. Distance Pre-computation Times (ms)

the GA was done with 5,000 generations for each dataset with each generation taking approximately 400, 1,200, and 4,600 milliseconds for Western Sahara, Uruguay, and Canada datasets, respectively. For the smallest dataset, Western Sahara, 5,000 proved to be a rather large number of iterations as the last generation with improved fitness hovers around 1,500 (Table 2).

Trial	Last	Best	 Trial	Last	Best	
HHAI	Improve.	Fitness	Iriai	Improve.	Fitness	
1	1308	27620	16	1056	29337	
2	4012	30664	17	1407	27768	
3	2889	27768	18	1686	28728	
4	3271	28513	19	1078	28765	
5	4103	27768	20	2371	28061	
6	3158	28202	21	1534	28249	
7	1716	28202	22	610	28735	
8	1059	29154	23	4890	27620	
9	1948	29098	24	2484	28249	
10	2199	30048	25	803	29535	
11	4639	29177	26	990	28231	
12	1710	28039	27	1224	30709	
13	839	27748	28	810	30502	
14	729	29686	29	683	28222	
15	597	29373	30	4582	28755	

Table 2. Last Generation With Improvement (Western Sahara)

Consequently, Figure 2 illustrates a plateau fairly early in the execution. This means that in some cases, similar results could be obtained much faster, with much less computation. On the other hand, some trials have improvement as far as nearly 4,900 generations into execution. Such trials sometimes are the ones with the most optimal fitness (e.g., trials 5 and 23 had improvement in generation 4, 103 and 4, 890, respectively and finished with best fitnesses of 27, 768 and 27, 620). According to the University of Waterloo's Faculty of Mathematics, optimal fitness for this tour is 27,603 [7]. Therefore, several of the trials logged in Table 2 show the algorithm's effectiveness in reaching near-optimal solutions. Given time to run more generations, optimal solutions should be achieved.

Uruguay, on the other hand, did not plateau in the same way that Western Sahara did (Figure 3).

Trial	Last	Best	Trial	Last	Best
	Improve.	Fitness	IIIai	Improve.	Fitness
1	4995	682092	16	4993	687626
2	4987	686624	17	4996	683809
3	4991	675225	18	4989	691854
4	4993	675225	19	4995	705421
5	4974	685579	20	4995	672055
6	4990	673494	21	4996	686104
7	4996	689418	22	4967	685925
8	4996	678421	23	4990	674031
9	4995	659951	24	4981	687605
10	4999	687908	25	4999	707432
11	4987	680266	26	4991	681644
12	4988	699569	27	4995	671644
13	4998	702853	28	4987	702787
14	4993	696526	29	4993	690934
15	4995	701105	30	4999	694009

Table 3. Last Generation With Improvement (Uruguay)

This can be seen both by the way that Figure 3 maintains a relatively steep slope all the way to the last generation, as well as the high frequency of improvements above 4, 980 generations in Table 3. Consequently, the algorithm, given more time to compute more generations, should yield similar results as were demonstrated for Western Sahara.

Execution for the Canada dataset ran successfully

for a number of trials, however, due to the technique of running ten trials of the algorithm per execution, technical errors mid-way through execution spoiled the data for all trials.

6. CONCLUSION

Results gathered from execution of the algorithm with the Western Sahara dataset demonstrate the developed algorithm's efficacy in solving the TSP. One improvement that the results from the Western Sahara dataset indicate is a need for more effective local minima escaping techniques. Table 2 shows that in some cases a high number of generations was required to achieve a near-optimal fitness, however Figure 2 demonstrates a severe plateau of best generational fitness around 500 generations.

A more effective technique to escape local minima would reduce the number of generations with no improvement while still striving for near-optimal solutions.

While not as optimal, the results from the Uruguay dataset do make sense and provide more useful information of where improvements to the algorithm could be made. Extrapolation from Figure 3 indicates that running more generations would likely result in a plateau of the best generational fitness much nearer the true optimal solution. However, as execution for all datasets was done with a constant population and mating pool size while the number of possible individuals grew at a rate of n! for n cities, future tests should be done with population and mating pool sizes that are not kept constant, but rather increase with the tour length. This would allow more variety in the population and hence could promote more diverse offspring, capable of escaping local minima faster and without reliance on severe mutation for randomness.

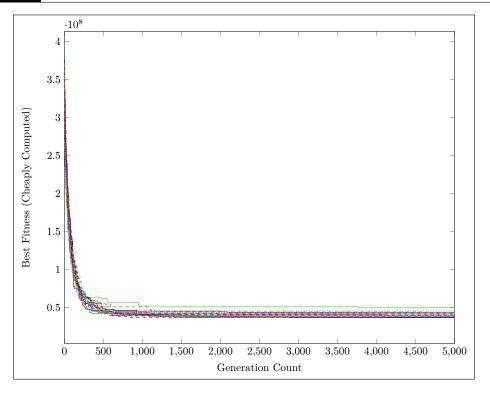


Fig. 2. Fitness Changes over 5,000 Generations (Western Sahara)

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A. SOURCE CODE

Source code and log files used in this report are available to the public from the project's GitHub repository, located at https://github.com/jwfh/cs3201-final-project.

B. COMPUTER SPECIFICATIONS

As discussed in Section 3.3, hardware limitations of the computers used for algorithm execution were something that needed to be considered and worked with in this project. Therefore, to facilitate an accurate and comprehensive report, hardware specifications for all terminals used are included in this section.

B.1. Hosts Used

A number of hosts belonging to Memorial's LabNet network were used to run the GA developed for this report. These hosts are listed below.

```
devastator.cs.mun.ca
en2048ece03.engr.mun.ca
en2048ece04.engr.mun.ca
en2048ece05.engr.mun.ca
en2048ece07.engr.mun.ca
en2048ece12.engr.mun.ca
laserbeak.cs.mun.ca
overkill.cs.mun.ca
pounce.cs.mun.ca
starscream.cs.mun.ca
```

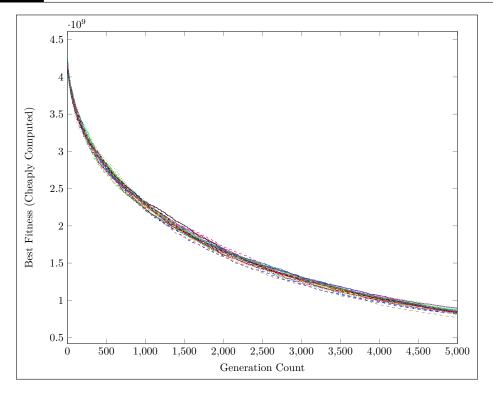


Fig. 3. Fitness Changes over 5, 000 Generations (Uruguay)

Several other nodes were tested and not used due to lesser hardware capabilities.

```
excalibur.cs.mun.ca
excelsior.cs.mun.ca
intrepid.cs.mun.ca
reliant.cs.mun.ca
valiant.cs.mun.ca
```

B.2. Processor Speed

Specifications of the processor(s) in all nodes used for testing are included in Table 4 on page 10.

B.3. Memory Specifications

Due to conjecture made in Section 3.3 that one of the limiting factors of the program execution time was random access memory (RAM) speed, an effort was made to retrieve these specifications for the report. Unfortunately, due to system restrictions placed on LabNet computers, standard users do not have sudo privileges. As a result, access to the dmidecode and lshw commands needed to read information about system hardware is restricted.

Hostname	Sockets	Cores per Socket	Threads per Core	CPU max. MHz	Cache		
Hostilaille					L1	L2	L3
devastator	1	4	1	3800	32K	256K	6144K
en2048ece03	1	4	1	3600	32K	256K	6144K
en2048ece04	1	4	1	3600	32K	256K	6144K
en2048ece05	1	4	2	4000	32K	256K	8192K
en2048ece07	1	4	2	4000	32K	256K	8192K
en2048ece12	1	4	2	4000	32K	256K	8192K
excalibur	2	4	2	2668	32K	256K	8192K
excelsior	2	4	2	2668	32K	256K	8192K
laserbeak	1	4	1	3800	32K	256K	6144K
intrepid	2	4	2	2668	32K	256K	8192K
overkill	1	4	1	3800	32K	256K	6144K
pounce	1	4	1	3800	32K	256K	6144K
reliant	2	4	2	2668	32K	256K	8192K
starscream	1	4	1	3800	32K	256K	6144K
valiant	2	4	2	2668	32K	256K	8192K

Table 4. Computer CPU Specifications