Using a Neural Network Trained only on Integer Order Systems to Identify Fractional Order Dynamics

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***Abstract*— This paper presents a standard type neural network that identifies the order of the dynamics of a unit step response. The system is trained on only first and second order systems, yet identifies fractional order responses with a fairly high degree of accuracy. The details of the structure of the neural network, the training method and the training sets, as well as statistics describing the accuracy of the fractional predictions are presented. This demonstrates the potential for practicing engineers to use similar machine learning tools trained on "standard" systems with the ability to distinguish when features such as fractional order dynamics are significant and warrant deeper consideration for the design or control of such a system.**

I. INTRODUCTION

Fractional calculus and fractional order dynamics are increasingly important in modern engineered systems. Unlike integer order derivatives, fractional order derivatives, and hence the dynamics that depend on them, are *nonlocal*. As such, many modern, large scale engineered systems may exhibit fractional order dynamics and responses. In instances where significant fractional order dynamics are present, control algorithms which directly address the fractional nature of the system may be superior. Therefore, tools to readily identify if significant fractional order dymancs are present are needed.

There is a vast literature on fractional calculus. Some textbooks include [1]–[3]. Fractional-order control is also a topical area such as in [4], [5]. An excellent review article illustrating the very broad range of applications of fractional calculus and control in science and engineering is [6].

Our main interets are identifying cases where fractional order models may provide useful "reduced order" models for large scale systems [7], [8] and for exact models for many large scale systems [9]–[13]. While this paper does not build upon it, our closest publication to this would be [14] where we created a symmetric neural network with a sequential set of identical layers. When it was trained on first derivatives of functions, the middle layer could represent the half derivative.

There are many different definitions of the fractional derivative.[1] A common feature of these is replacing factorial functions appearing in many integer-order representations of the derivative with gamma functions. The Riemann-Liouville, Caputo and the Grünwald-Letnikov definitions are perhaps the most common examples of fractional derivative definitions, and the reader is referred to the references [15]–[18] for descriptions and definitions of each.

II. Neural Network and Training

The neural network presented in this paper is illustrated in Figure 1. The input to the network is the unit step response of a linear system in the time range of $0 \leq t \leq 10$ discretized into time steps of $\Delta t = 0.1$ [s], which gives 101 input nodes. These are fed to a first hidden layer with 64 nodes, a second hidden layer with 16 nodes, a third hidden layer of 16 nodes and a single output node. Each hidden layer has the ReLU() activation function and bias weights.

This network is not trained as a classifier because we want it to be able to generalize first and second order systems to fractional orders between them. The network was implemented in python using the torch library and pytorch_lightning tools. The loss function is the mean squared error loss function,`mse_loss()`, and the optimization method adopted was Adam optimizer, `torch.optim.Adam()` with a learning rate of 0.001. A branch of our github repositiry that shoudl repeatably replicate the results presented in this paper is at ZZZZZZ.

An individual element of the training set was the step response for a first or second order system. The manner in which they were generated was:

- Select a value from a uniform random distribution, and if the value is less than 0.5, then the step response will be for a second order system, and if not, then it will be for a first order system.
- Select two numbers, c_1 and c_2 from a uniform distribution with values between 0.01 and 4.
  - If the response is for a first order system, then the transfer function is

$$G(s) = \frac{c_2}{c_1 s + c_2}.$$

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[1]The Wikipedia page for fractional calculus lists over 20 different definitions of fractional derivatives.

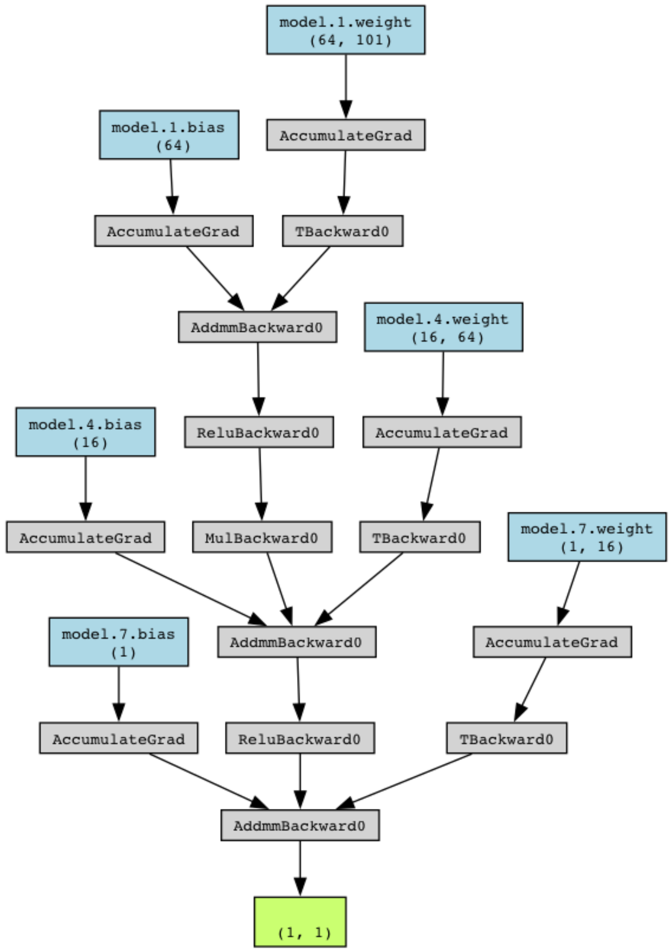

Fig. 1. The neural network.

– If the response is for a second order system, then the selected transfer function is

$$G(s) = \frac{c_2}{c_1 s^2 + c_2}.$$

- The unit step response for the transfer function is generated using the `control.step_response()` function from the python control system library. It is sampled every 0.1 second so that the length of the response vector is 101.
- Using this method we generate a set of 100,000 first or second order step responses with approximately the same number of first and second order responses.
- The training set is split into three subsets: 60,000 training elements, 20,000 validation elements and 20,000 testing elements.
- For training, the training set is shuffled at the beginning of each epoch and the optimization method is applied to change the weights in the network.
- At the end of the epoch, the network is run on the validation set to compute an error for data points the network was not trained on. Evidence of overtraining would be if the validation set error decreases and then increases.

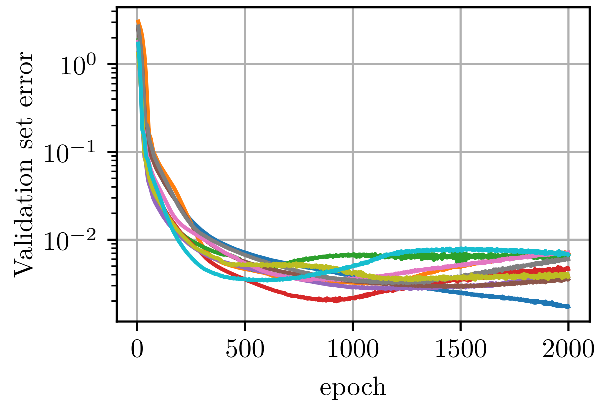

Fig. 2. Neural network output error on validation (not training) set verus training epoch.

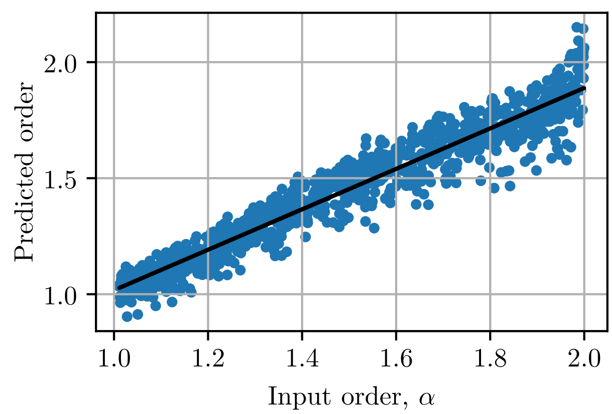

Fig. 3. Comparison of actual fractional orders and predicted orders.

- At the end of all the training, the error for the testing set is computed.

Figure 2 illustrates the error on the validation set for 10 training runs for the network versus epoch. It appears that if the validation error increases, it tends to start to do so around 1000 epochs; otherwise it tends to stop changing around 1000 epochs (there seems to be one exception). As such, we will fix the number of training epochs at 1000.

III. Using the Integer Trained Network on Fractional Order Step Responses

In order to determine how well the trained network will generalize from integer order training data to fractional order step responses,

more text

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$$\alpha + \beta = \chi \tag{1}$$

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TABLE I
An Example of a Table

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Fig. 4. Inductance of oscillation winding on amorphous magnetic core versus DC bias magnetic field

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VI. CONCLUSIONS

A conclusion section is not required. Although a conclusion may review the main points of the paper, do not replicate the abstract as the conclusion. A conclusion might elaborate on the importance of the work or suggest applications and extensions.

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