Flow Nets

Today's agenda

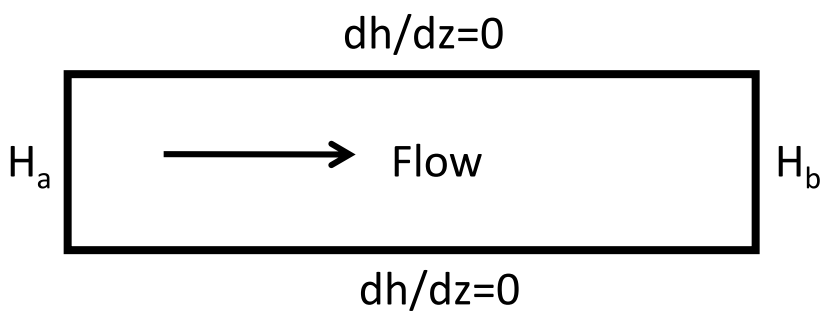
- Flow Nets
- Flow net examples

Consider a 2D flow system:

Assumptions:

- Aquifer is homogeneous
- Aquifer is fully saturated
- Aquifer is isotropic
- No change in potential field with time – steady-state
- Soil and water are incompressible
- Flow is laminar and Darcy's law is applicable
- All model parameters and boundary conditions are known

This 2D system is governed by Darcy's law and Conservation of Mass (we'll later see that this makes the groundwater flow equation – the Laplace equation).



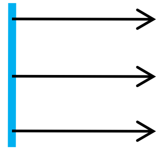
In order to solve this – to calculate heads and flow – we need boundary conditions.

Solving a flow net allows you to calculate Q graphically – flow nets are graphical solutions to the partial differential equations for groundwater flow systems!

There are 3 types of boundary conditions:

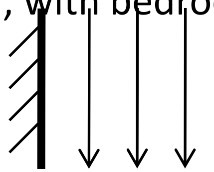
1. Constant hydraulic head – head is a known, specified value at a location or along a portion of a boundary.
    - Represents/implies an infinite source or sink of water
    - Flow lines intersect at right angles (boundary is an equipotential line)
    - Examples: lake, river

Constant head

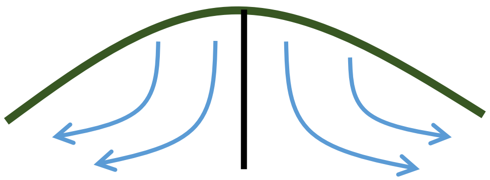


2. Zero gradient or no-flow across a portion of a boundary
    - Represents an impermeable barrier or a line of symmetry across which no flow occurs
    - Flow lines are parallel (boundary is a flow line)
    - Examples: contact (e.g., with bedrock), fault, flow divide

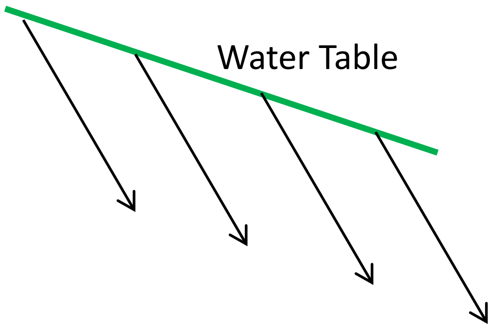
No-flow: Impermeable



No-flow: Divide



3. Water table boundary (unconfined aquifers only)
    - Neither a flow line or an equipotential line
    - Head is known but may be variable
    - If there is no recharge, flow lines are parallel to it
    - If there is recharge, flow lines are oblique



Flow Nets:

A flow net is a graphical representation of a potential field and flow system consisting of equipotential lines and flow lines, and which can be constructed according to geometric rules. It is a graphical solution to the groundwater flow equation that can be used to calculate flow rates and directions.

Definitions and Rules:

- Equipotential: line of constant potential (head)
- Flow lines: directions of flow through a steady flow potential field
- Constant-head boundaries are equipotential lines
- Equipotential lines are ~parallel and never intersect
- No-flow boundaries are flowlines
- Flowlines are ~parallel
- Flow lines cross equipotential lines at right angles in isotropic media
    —Flow lines never intersect
    —Equipotential lines are perpendicular to no-flow boundaries
    —Equipotential lines are parallel to constant-head boundaries
- Flow lines and equipotential lines form ***curvilinear squares*** (you can draw circles inside tangent to all sides, and lines passing through the center are approximately the same length)
- Partial streamtubes can occur, as can partial squares at the boundaries
- Streamtubes must start and stop at constant-head or water table boundaries.

Henzl and Rozhon, 2007

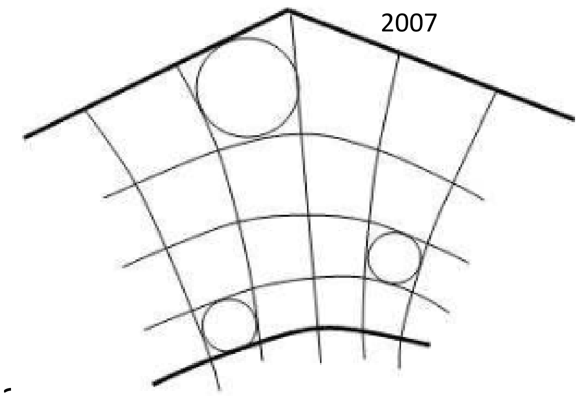


Fig.1: Curvilinear squares

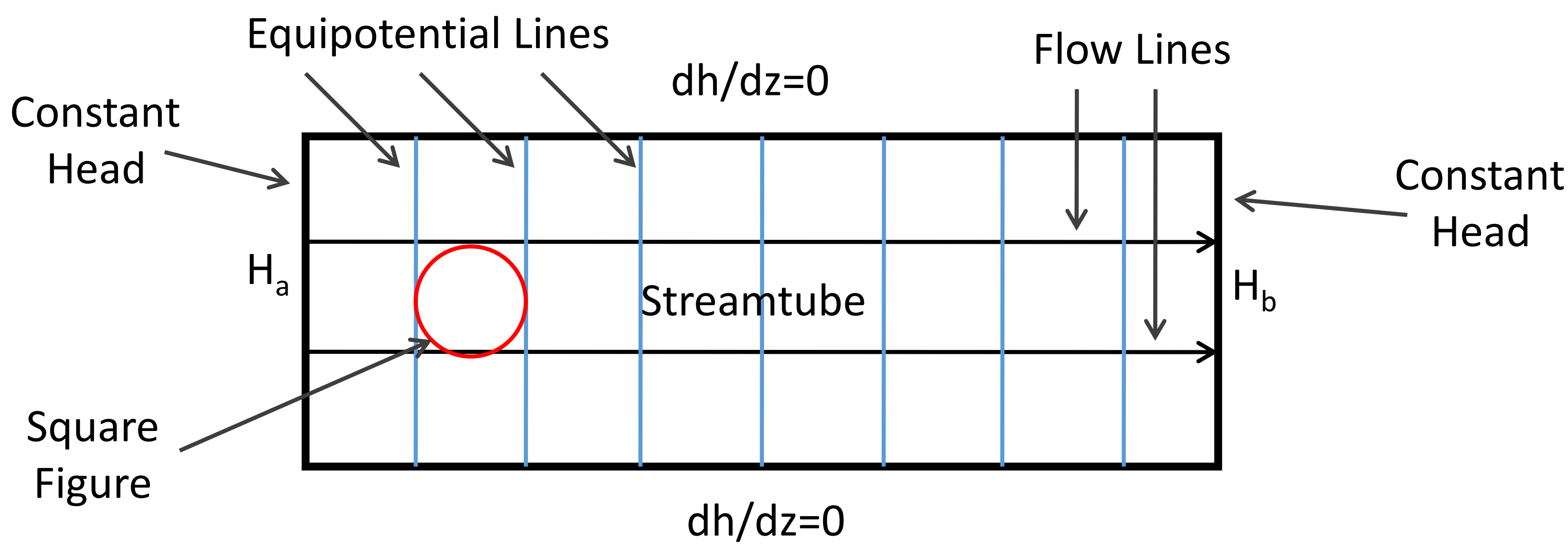
Streamtube: a graphical pipe containing a certain discharge of water. If the flow net is constructed with square figures, then each streamtube transmits an identical discharge of water.

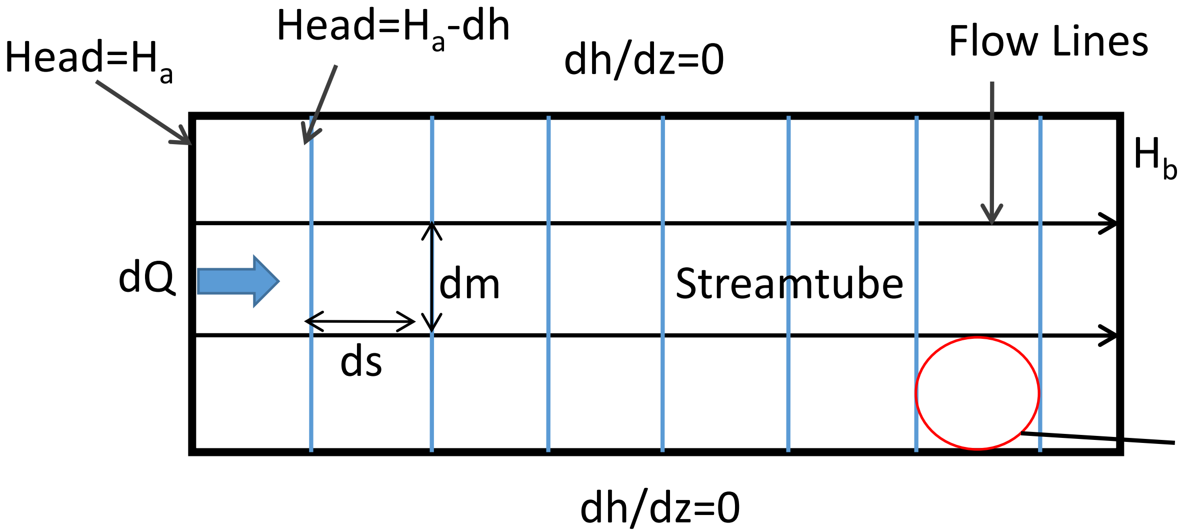
Procedure for constructing a flow net:

1. **Determine the boundary conditions** and the likely direction of flow. Find the known equipotentials.
2. **Determine where the water is coming from and where it is going** by finding the constant head or water table boundaries and the start and end of the flow lines.

On a diagram with axes on the same scale:

3. **Draw a trial set of flow lines** where the outer flow lines are parallel to the no-flow boundaries. Make them evenly spaced.
4. **Draw a trial set of equipotential lines** starting at one end of the system and going to the other. Equipotentials will be perpendicular to the flow lines (isotropic systems). If there is a water table, there will be equipotentials corresponding to the elevation of the water table. **Try to make square figures** when drawing equipotential lines.
5. **Adjust** equipotential and flow lines to make square figures.



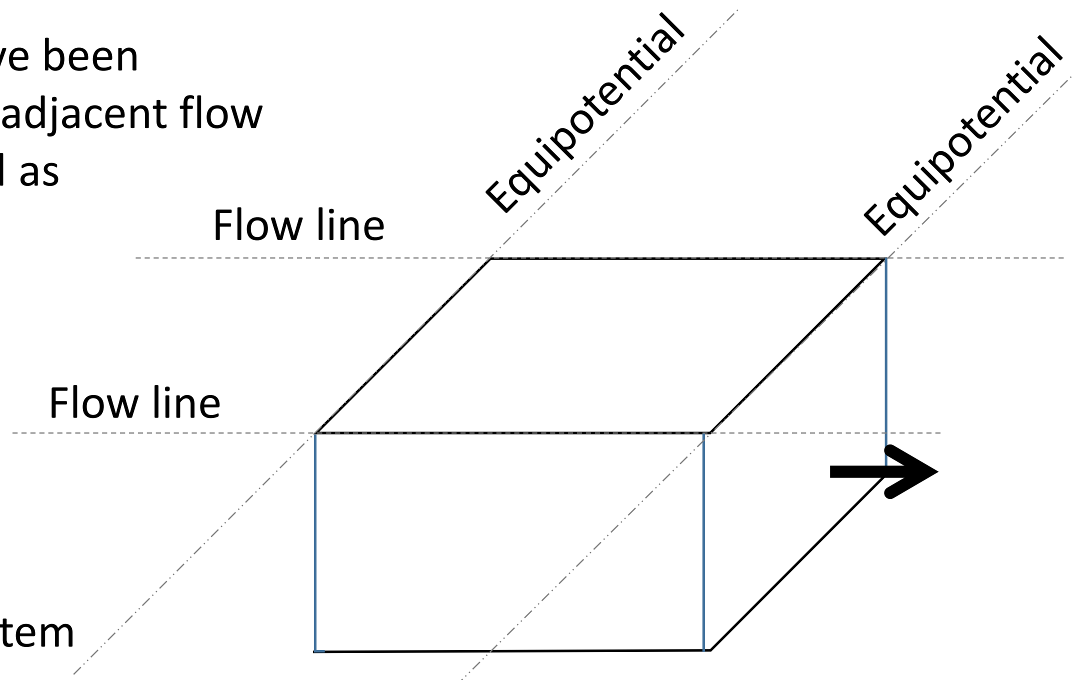


Assume w=1, unit width into page

Why are these circles? Preserves dQ along a streamtube

For any of the flow channels: that have been constructed (i.e. the spaces between adjacent flow lines), discharge (Q) can be calculated as

$$\mathbf{Q} = \frac{n_t}{n_d} K\Delta H$$



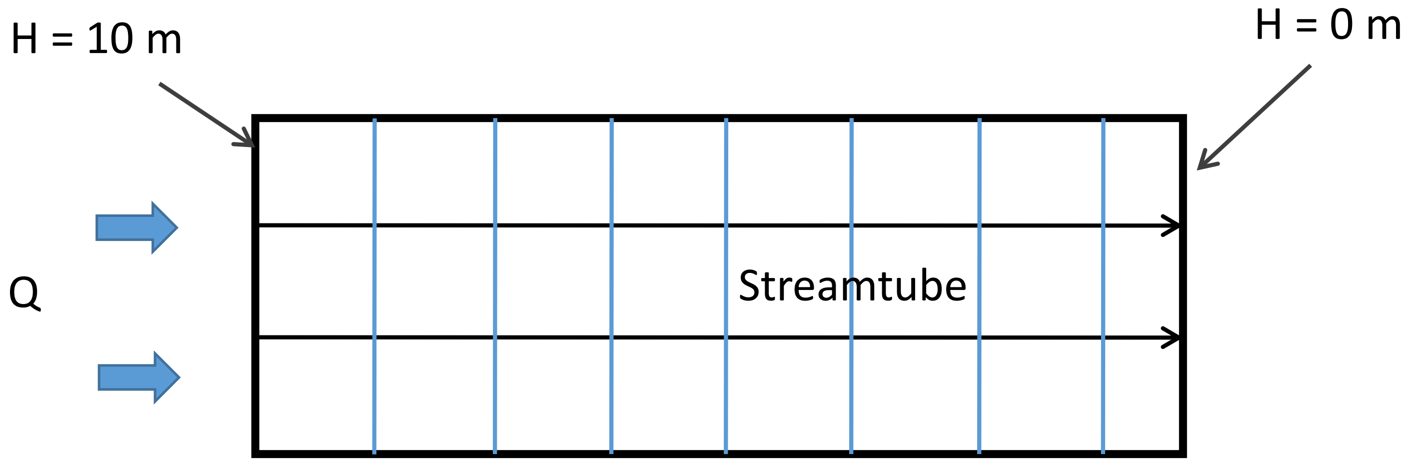
n_t is the number of stream tubes
n_d is the number of head drops
K is hydraulic conductivity
dh is the total head drop across the system

If the total head drop across the system is 10m, there are n_d divisions of head (head drops), and K=2 m/d, then:

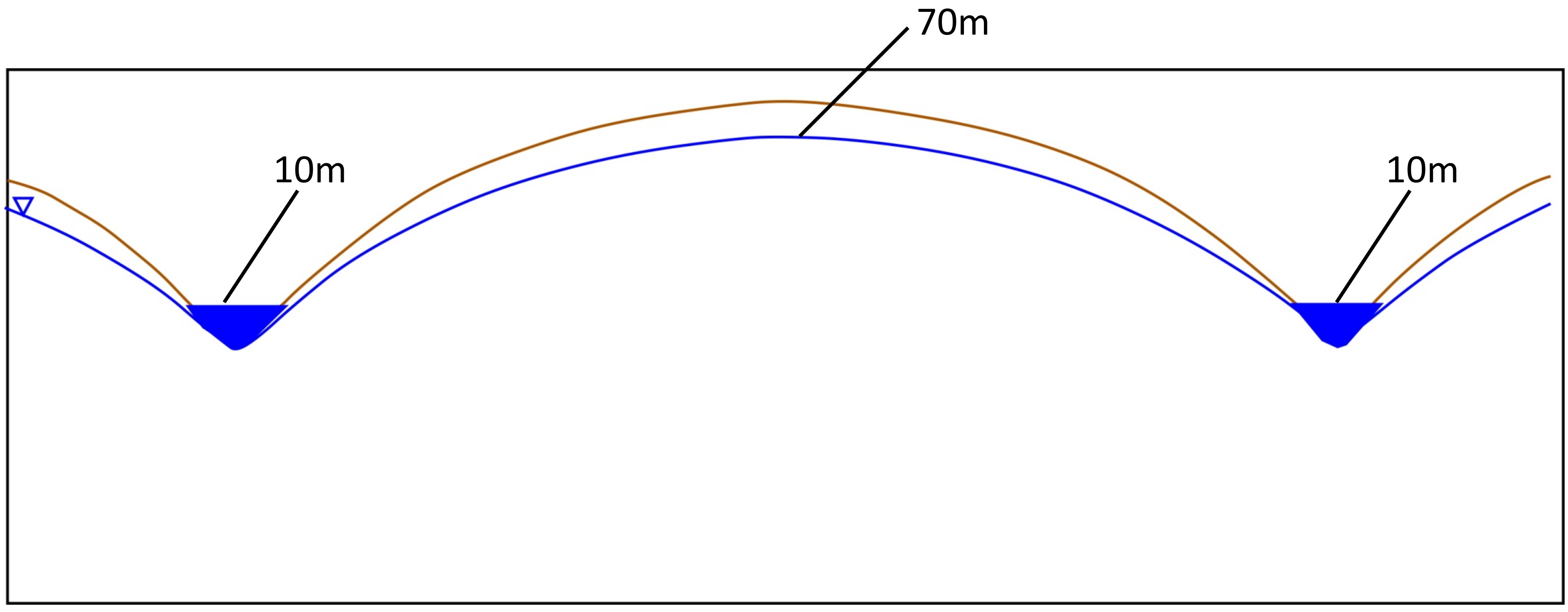
$$\mathbf{Q} = \frac{n_t}{n_d} K \Delta H$$

n_t is the number of stream tubes
n_d is the number of head drops
K is hydraulic conductivity
dh is the total head drop across the system

$$\mathbf{Q} = \frac{(3)(2)(10)}{7.5} = 8\ \frac{m^3}{d}$$



Some Examples…



Two Rivers Example

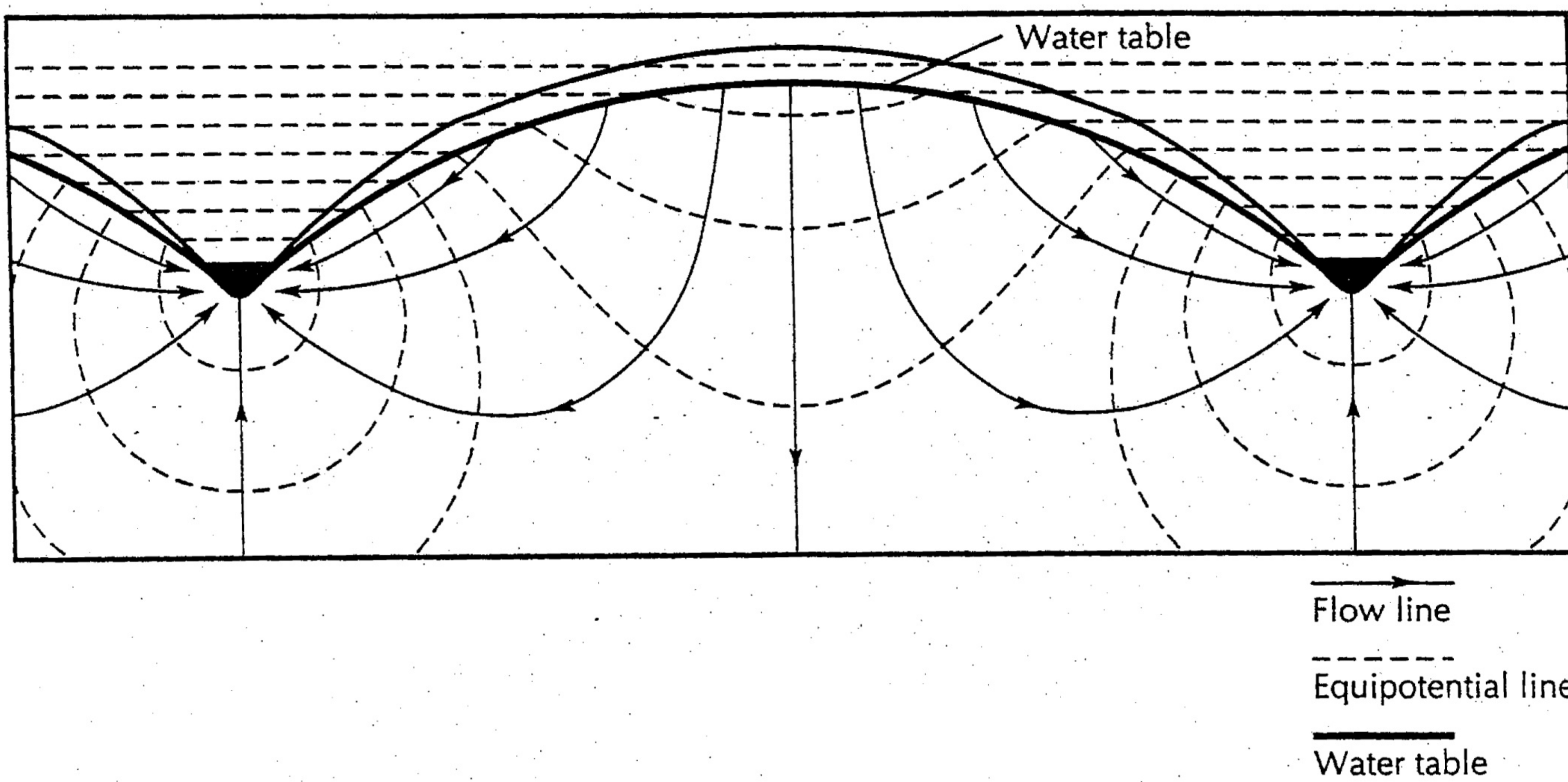


FIGURE 8.1 Cross-sectional flow net in an isotropic, homogeneous aquifer. The aquifer is much deeper than the diagram. Source: M. K. Hubbert, *Journal of Geology* 48, no. 8 (1940): 795–944. Used with permission of the University of Chicago Press.

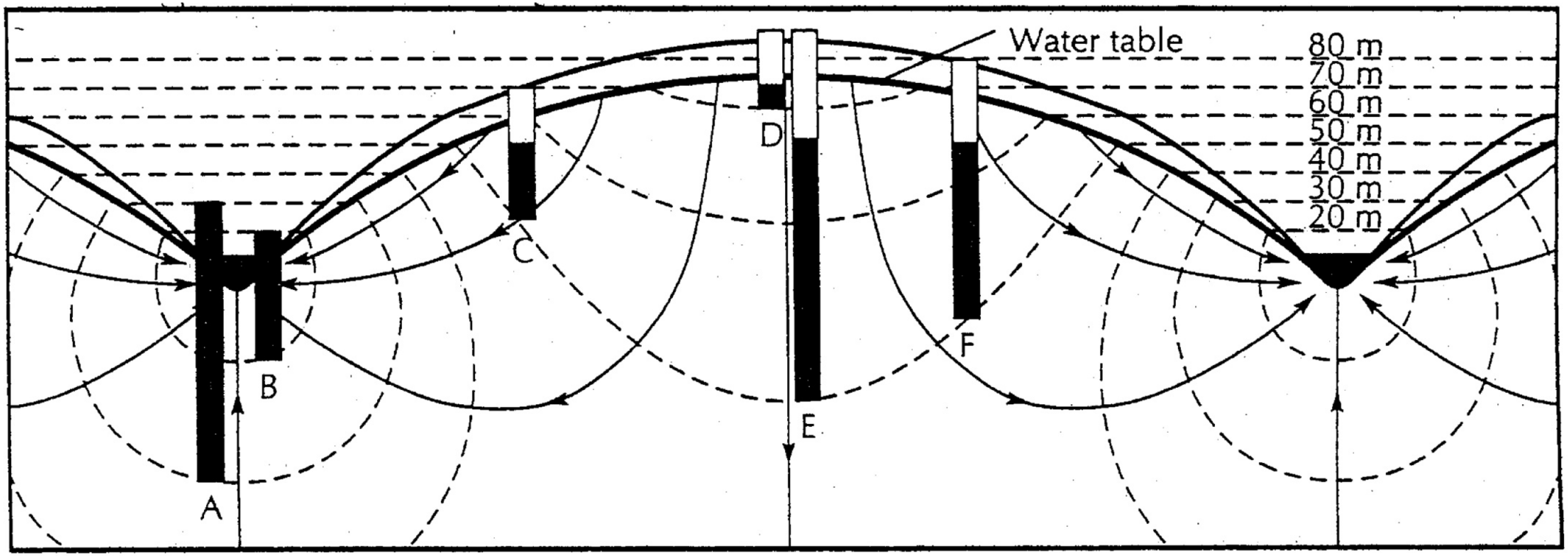


FIGURE 8.2 Piezometers superimposed on Figure 8.1. The water level in the piezometer will rise to the elevation of the hydraulic head, which is represented by the equipotential line at the open end of the piezometer. Source: Modified from M. K. Hubbert, *Journal of Geology* 48, no. 8 (1940): 795–944. Used with permission of the University of Chicago Press.

Recharge: *flow directed down*
Discharge: *flow directed up*

Note: Artesian conditions can be either geologically controlled (Province of Artois – confined aquifer) or topographically controlled (as above).

Draw a *quantitatively accurate* flow net for the case below. Assume the material is homogeneous and isotropic. Show flow direction arrows on the flow lines.

If K = 3 x 10^{-5} m/s, what is the volumetric flow rate through the medium per unit thickness?