

**Conjecture.** For any integer  $n$  that has 5 as the last digit,  $n^2$  will have 25 as the last two digits.

**Proof.** Let  $n$  be an integer with 5 as the last digit.  
Then  $n = 10j + 5$  for some integer  $j$ .  
Therefore:

$$\begin{aligned}n^2 &= (10j + 5)(10j + 5) \\&= 100j^2 + 100j + 25 \\&= 100j(j + 1) + 25\end{aligned}$$

Let  $\ell = j(j + 1)$ . Then we have:

$$n^2 = 100\ell + 25$$

which is to say:

$$n^2 \equiv 25 \pmod{100}$$

Since congruence mod 100 preserves the last two decimal digits,  $n^2$  ends in 25.

□