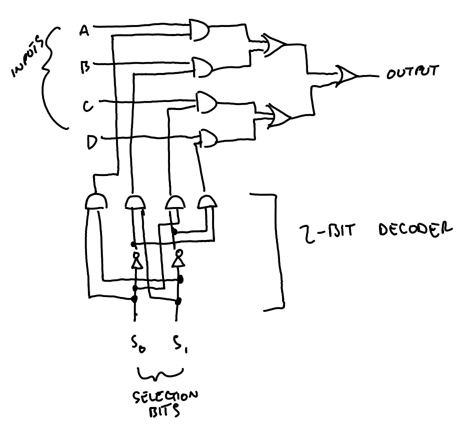
Lab 1 - Practice for Logic Design

Friday, January 29, 2021 10:02 AM

1. Draw a complete schematic diagram of the 4x1 multiplexor, using only three basic gates (2-input AND, 2-input OR, and negator).



2. Assume that signal X is 3 bits wide (x_2, x_1, x_0).
   Write logic equations for F1~F4 based on the following functionalities.
   - F1: X contains only one 0.
   - F2: X contains an even number of 0's.
   - F3: X, when interpreted as an unsigned binary number, is less than 4.
   - F4: X, when interpreted as a signed (two's complement) number, is negative.

   Now, rewrite your logic equation for F2 using only XOR functions.

F1:

X_0	X_1	X_2	Y
0	1	1	1
1	0	1	1
0	1	1	1

$$(\bar{x}_0\cdot x_1\cdot x_2)+(x_0\cdot\bar{x}_1\cdot x_2)+(x_0\cdot x_1\cdot\bar{x}_2)$$

F2:

X_0	X_1	X_2	Y
0	0	1	1
0	1	0	1
1	0	0	1
1	1	1	1

$$(x_0\cdot x_1\cdot x_2)+(\bar{x}_0\cdot\bar{x}_1\cdot x_2)+(\bar{x}_0\cdot x_1\cdot\bar{x}_2)+(x_0\cdot\bar{x}_1\cdot\bar{x}_2)$$

F3:

X_0	X_1	X_2	Y
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1

$$(\bar{x}_0\cdot\bar{x}_1\cdot\bar{x}_2)+(\bar{x}_0\cdot\bar{x}_1\cdot x_2)+(\bar{x}_0\cdot x_1\cdot\bar{x}_2)+(\bar{x}_0\cdot x_1\cdot x_2)$$

$$\bar{x}_0\cdot\left((\bar{x}_1\cdot\bar{x}_2)+(\bar{x}_1\cdot x_2)+(x_1\cdot\bar{x}_2)+(x_1\cdot x_2)\right)$$

$$\bar{x}_0\cdot\left(\bar{x}_1\cdot(\overset{1}{\cancel{\bar{x}_2+x_2}})+x_1\cdot(\overset{1}{\cancel{\bar{x}_2+x_2}})\right)$$

$$\bar{x}_0\cdot(\overset{1}{\cancel{\bar{x}_1+x_1}})$$

$$\bar{x}_0$$

F4:

X_0	X_1	X_2	Y
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

$$(x_0\cdot\bar{x}_1\cdot\bar{x}_2)+(x_0\cdot\bar{x}_1\cdot x_2)+(x_0\cdot x_1\cdot\bar{x}_2)+(x_0\cdot x_1\cdot x_2)$$

$$x_0\cdot\left((\bar{x}_1\cdot\bar{x}_2)+(\bar{x}_1\cdot x_2)+(x_1\cdot\bar{x}_2)+(x_1\cdot x_2)\right)$$

$$x_0\cdot\left(\bar{x}_1\cdot(\overset{1}{\cancel{\bar{x}_2+x_2}})+x_1\cdot(\overset{1}{\cancel{\bar{x}_2+x_2}})\right)$$

$$x_0\cdot(\overset{1}{\cancel{\bar{x}_1+x_1}})$$

$$x_0$$

F2: W/ XOR

X_0	X_1	X_2	Y
0	0	1	1
0	1	0	1
1	0	0	1
1	1	1	1

$$x_0\oplus x_1\oplus x_2$$

3. Consider the following two equivalent logic equations for E.

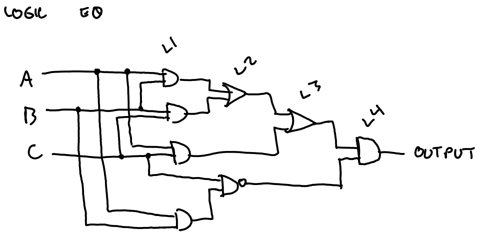
   $E = ((A\cdot B) + (A\cdot C) + (B\cdot C))\cdot\overline{(A\cdot B\cdot C)}$

   $E = (A\cdot B\cdot\overline{C}) + (A\cdot\overline{B}\cdot C) + (\overline{A}\cdot B\cdot C)$

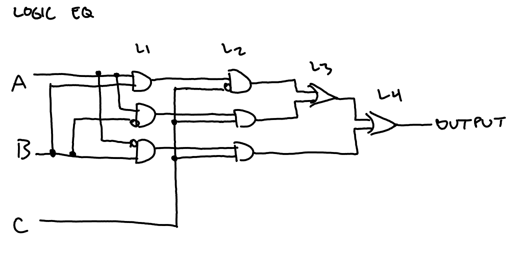
   Draw a schematic diagram for each equation with minimum number of 2-input gates.

   Answer which equation is more efficient in terms of the number of 2-input gates.
   Note that the negator is not counted for this.

FIRST LOGIC EQ



SECOND LOGIC EQ



BOTH EQUATIONS HAVE THE SAME EFFICIENCY AS
THEY HAVE THE SAME NUMBER OF LOGIC GATES
AND HAVE 4-LEVEL LOGIC.

4. Prove that the two equations for E (shown in #3) are equivalent by using DeMorgan's laws and other basic laws (if needed).

$$
\begin{aligned}
(A\cdot B\cdot \bar{C}) + (A\cdot \bar{B}\cdot C) + (\bar{A}\cdot B\cdot C) &= ((A\cdot B) + (A\cdot C) + (B\cdot C))\cdot \overline{(A\cdot B\cdot C)} \\
&= ((A\cdot B) + (A\cdot C) + (B\cdot C))\cdot (\bar{A}+\bar{B}+\bar{C}) \\
&= (((A\cdot B)\cdot(\bar{A}+\bar{B}+\bar{C})) + ((A\cdot C)\cdot(\bar{A}+\bar{B}+\bar{C})) + ((B\cdot C)\cdot(\bar{A}+\bar{B}+\bar{C}))) \\
&= (((\cancel{A\cdot \bar{A}\cdot B}) + (\cancel{A\cdot B\cdot \bar{B}}) + (A\cdot B\cdot \bar{C})) + ((\cancel{A\cdot C\cdot \bar{A}}) + (A\cdot C\cdot \bar{B}) + (\cancel{A\cdot C\cdot \bar{C}})) + ((B\cdot C\cdot \bar{A}) + (\cancel{B\cdot C\cdot \bar{B}}) + (\cancel{B\cdot C\cdot \bar{C}}))) \\
&= ((0 + 0 + \cancel{1}) + (0 + \cancel{1} + 0) + (\cancel{1} + 0 + 0)) \\
&= (A\cdot B\cdot \bar{C}) + (A\cdot \bar{B}\cdot C) + (\bar{A}\cdot B\cdot C)
\end{aligned}
$$

$\therefore$ EQUIVALENT

5. One logic function that is used for a variety of purposes is *exclusive_OR (XOR)*. The output of the 2-input XOR function is true only if exactly one of the inputs is true. Prove that the following two logic equations for the 2-input XOR function is equivalent by using the DeMorgan's laws and other basic laws (if needed).

$XOR = (A\cdot\bar{B}) + (\bar{A}\cdot B)$

$XOR = (A+B)\cdot\overline{(A\cdot B)}$

$$
\begin{aligned}
(A\cdot\bar{B}) + (\bar{A}\cdot B) &= (A+B)\cdot\overline{(A\cdot B)} \\
&= (A+B)\cdot(\bar{A}+\bar{B}) \\
&= (\bar{A}\cdot(A+B)) + (\bar{B}\cdot(A+B)) \\
&= (\underbrace{\cancel{A\cdot\bar{A}}}_{0} + \bar{A}B) + (A\bar{B} + \underbrace{\cancel{B\bar{B}}}_{0}) \\
&= (\bar{A}\cdot B) + (A\cdot\bar{B})
\end{aligned}
$$

$\therefore$ EQUIVALENT

6. From #5, extend your practice to the 3-input XOR function by showing its truth table, logic equation and schematic diagram (using only 2-input basic gates).

A	B	C	$A\oplus B$	$(A\oplus B)\oplus C$
0	0	0	0	0
0	0	1	0	1
0	1	0	1	1
0	1	1	1	0
1	0	0	1	1
1	0	1	1	0
1	1	0	0	0
1	1	1	0	1

$(\bar{A}\cdot\bar{B}\cdot C) + (\bar{A}\cdot B\cdot\bar{C}) + (A\cdot\bar{B}\cdot\bar{C}) + (A\cdot B\cdot C)$