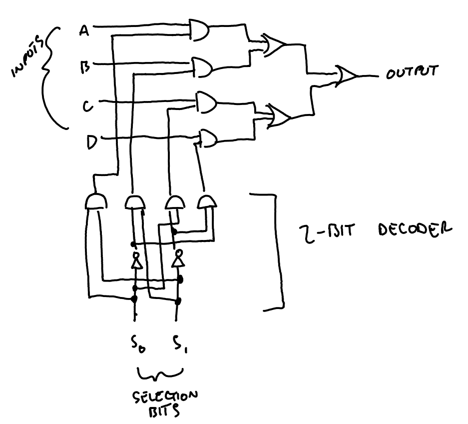
Lab 1 - Practice for Logic Design

Friday, January 29, 2021 10:02 AM

1. Draw a complete schematic diagram of the 4x1 multiplexor, using only three basic gates (2-input AND, 2-input OR, and negator).



2. Assume that signal X is 3 bits wide (x_2, x_1, x_0).
   Write logic equations for F1~F4 based on the following functionalities.

   F1: X contains only one 0.
   F2: X contains an even number of 0's.
   F3: X, when interpreted as an unsigned binary number, is less than 4.
   F4: X, when interpreted as a signed (two's complement) number, is negative.

   Now, rewrite your logic equation for F2 using only XOR functions.

F1:

X_0	X_1	X_2	Y
0	1	1	1
1	0	1	1
0	1	1	1

$$(\bar{x}_0\cdot x_1\cdot x_2) + (x_0\cdot\bar{x}_1\cdot x_2) + (x_0\cdot x_1\cdot\bar{x}_2)$$

F2:

X_0	X_1	X_2	Y
0	0	1	1
0	1	0	1
1	0	0	1
1	1	1	1

$$(x_0\cdot x_1\cdot x_2) + (\bar{x}_0\cdot\bar{x}_1\cdot x_2) + (\bar{x}_0\cdot x_1\cdot\bar{x}_2) + (x_0\cdot\bar{x}_1\cdot\bar{x}_2)$$

F3:

X_0	X_1	X_2	Y
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1

$$(\bar{X}_0\cdot\bar{X}_1\cdot\bar{X}_2) + (\bar{X}_0\cdot\bar{X}_1\cdot X_2) + (\bar{X}_0\cdot X_1\cdot\bar{X}_2) + (\bar{X}_0\cdot X_1\cdot X_2)$$

$$\bar{X}_0\cdot\left((\bar{X}_1\cdot\bar{X}_2) + (\bar{X}_1\cdot X_2) + (X_1\cdot\bar{X}_2) + (X_1\cdot X_2)\right)$$

$$\bar{X}_0\cdot\left(\bar{X}_1\cdot\overset{1}{(\cancel{\bar{X}_2+X_2})} + X_1\cdot\overset{1}{(\cancel{\bar{X}_2+X_2})}\right)$$

$$\bar{X}_0\cdot\overset{1}{(\cancel{\bar{X}_1+X_1})}$$

$$\bar{X}_0$$

F4:

X_0	X_1	X_2	Y
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

$$(X_0\cdot\bar{X}_1\cdot\bar{X}_2) + (X_0\cdot\bar{X}_1\cdot X_2) + (X_0\cdot X_1\cdot\bar{X}_2) + (X_0\cdot X_1\cdot X_2)$$

$$X_0\cdot\left((\bar{X}_1\cdot\bar{X}_2) + (\bar{X}_1\cdot X_2) + (X_1\cdot\bar{X}_2) + (X_1\cdot X_2)\right)$$

$$X_0\cdot\left(\bar{X}_1\cdot\overset{1}{(\cancel{\bar{X}_2+X_2})} + X_1\cdot\overset{1}{(\cancel{\bar{X}_2+X_2})}\right)$$

$$X_0\cdot\overset{1}{(\cancel{\bar{X}_1+X_1})}$$

$$X_0$$

F2 : W/ XOR

X_0	X_1	X_2	Y
0	0	1	1
0	1	0	1
1	0	0	1
1	1	1	1

$$X_0 \oplus X_1 \oplus X_2$$

3. Consider the following two equivalent logic equations for E.

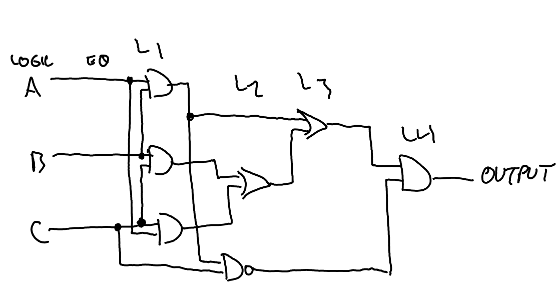
   $E = ((A\cdot B) + (A\cdot C) + (B\cdot C)) \cdot \overline{(A\cdot B\cdot C)}$

   $E = (A\cdot B\cdot\overline{C}) + (A\cdot\overline{B}\cdot C) + (\overline{A}\cdot B\cdot C)$

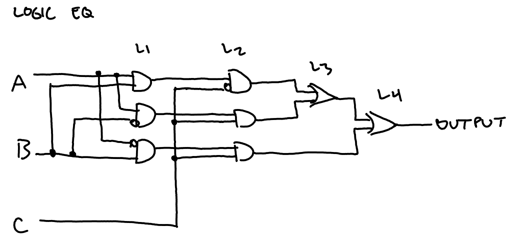
   Draw a schematic diagram for each equation with minimum number of 2-input gates.

   Answer which equation is more efficient in terms of the number of 2-input gates.
   Note that the negator is not counted for this.

FIRST LOGIC EQ



SECOND LOGIC EQ



THE FIRST LOGIC EQUATION IS MORE EFFICIENT.
WHILE BOTH EQUATIONS USE 4-LEVEL LOGIC THE FIRST LOGIC EQUATION CAN BE BUILT W 7 GATES WHILE THE SECOND REQUIRES 8.

4. Prove that the two equations for E (shown in #3) are equivalent by using DeMorgan's laws and other basic laws (if needed).

$$
\begin{aligned}
(A\cdot B\cdot \bar{C}) + (A\cdot \bar{B}\cdot C) + (\bar{A}\cdot B\cdot C) &= ((A\cdot B) + (A\cdot C) + (B\cdot C))\cdot \overline{(A\cdot B\cdot C)} \\
&= ((A\cdot B) + (A\cdot C) + (B\cdot C))\cdot (\bar{A}+\bar{B}+\bar{C}) \\
&= (((A\cdot B)\cdot(\bar{A}+\bar{B}+\bar{C})) + ((A\cdot C)\cdot(\bar{A}+\bar{B}+\bar{C})) + ((B\cdot C)\cdot(\bar{A}+\bar{B}+\bar{C}))) \\
&= (((A\cdot\cancel{\bar{A}}\cdot B) + (A\cdot\cancel{B\cdot\bar{B}}) + (A\cdot B\cdot\bar{C})) + ((A\cdot\cancel{C\cdot\bar{A}}) + (A\cdot C\cdot\bar{B}) + (A\cdot\cancel{C\cdot\bar{C}})) + ((B\cdot C\cdot\bar{A}) + (B\cdot\cancel{C\cdot\bar{B}}) + (\cancel{B\cdot C\cdot\bar{C}}))) \\
&= ((0 + 0 + \cancel{}) + (0 + \cancel{} + 0) + (\cancel{} + 0 + 0)) \\
&= (A\cdot B\cdot\bar{C}) + (A\cdot\bar{B}\cdot C) + (\bar{A}\cdot B\cdot C)
\end{aligned}
$$

$\therefore$ EQUIVALENT

5. One logic function that is used for a variety of purposes is *exclusive_OR (XOR).* The output of the 2-input XOR function is true only if exactly one of the inputs is true. Prove that the following two logic equations for the 2-input XOR function is equivalent by using the DeMorgan's laws and other basic laws (if needed).

XOR = $(A\cdot\bar{B}) + (\bar{A}\cdot B)$

XOR = $(A+B)\cdot\overline{(A\cdot B)}$

$$
\begin{aligned}
(A\cdot\bar{B}) + (\bar{A}\cdot B) &= (A+B)\cdot\overline{(A\cdot B)} \\
&= (A+B)\cdot(\bar{A}+\bar{B}) \\
&= (\bar{A}\cdot(A+B)) + (\bar{B}\cdot(A+B)) \\
&= (\underbrace{\cancel{A\cdot\bar{A}}}_{0} + \bar{A}B) + (A\bar{B} + \underbrace{\cancel{B\cdot\bar{B}}}_{0}) \\
&= (\bar{A}\cdot B) + (A\cdot\bar{B})
\end{aligned}
$$

$\therefore$ EQUIVALENT

6. From #5, extend your practice to the 3-input XOR function by showing its truth table, logic equation and schematic diagram (using only 2-input basic gates).

A	B	C	$A\oplus B$	$(A\oplus B)\oplus C$
0	0	0	0	0
0	0	1	0	1
0	1	0	1	1
0	1	1	1	0
1	0	0	1	1
1	0	1	1	0
1	1	0	0	0
1	1	1	0	1

$(\bar{A}\cdot\bar{B}\cdot C) + (\bar{A}\cdot B\cdot\bar{C}) + (A\cdot\bar{B}\cdot\bar{C}) + (A\cdot B\cdot C)$