Project Euler Solutions Document

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1 Multiples of 3 and 5

Problem: If we list all the natural numbers below 10 that are multiples of 3 or 5, we get 3, 5, 6 and 9. The sum of these multiples is 23. Find the sum of all the multiples of 3 or 5 below 1000.

Answer: 233498
Method: Three counters from 1 to n. The first, i, counting normally and the second two modulo 3 and 5.when the modulo counts reset, i is added to the sum. An if statement stops i being added twice if the modulo counters both reset, for example at $i = 15$.

2 Even Fibonacci numbers

Problem: Each new term in the Fibonacci sequence is generated by adding the previous two terms. By starting with 1 and 2, the first 10 terms will be:

$$1, 2, 3, 5, 8, 13, 21, 34, 55, 89, \ldots$$

By considering the terms in the Fibonacci sequence whose values do not exceed four million, find the sum of the even-valued terms.

Answer: 5702886
Method: Rather than check parity of each entry, notice that every third term is even. A modulo three counter moves as the sequence is generated and added every third (and hence even) term. The Fibonacci sequence is generated as normal.

3 Largest Prime Factor

Problem: The prime factors of 13195 are 5, 7, 13 and 29. What is the largest prime factor of the number 600851475143?

Answer: 6857
Method: Take two long variables, x and y, with x initialised as 600851475143. Let y equal x with it's smallest divisor removed. Stop when x is prime.

4 Largest Palindrome Product

Problem: A palindromic number reads the same both ways. The largest palindrome made from the product of two 2-digit numbers is $9009 = 91 \times 99$. Find the largest palindrome made from the product of two 3-digit

numbers.

Answer: 906609
Method: let i range from 1 to 999, and for each of these let j range from i to 999. We are now checking every possibility, without repeats. Check each for a palindrome, save the maximum.

5 Smallest multiple

Problem: 2520 is the smallest number that can be divided by each of the numbers from 1 to 10 without any remainder. What is the smallest positive number that is evenly divisible by all of the numbers from 1 to 20?

Answer: 23279560
Method: I check each number until the first number that divides everything (none optimal).

6 Sum square difference

Problem: The sum of the squares of the first ten natural numbers is,

$$1^2 + 2^2 + ... + 10^2 = 385$$

The square of the sum of the first ten natural numbers is,

$$(1 + 2 + ... + 10)^2 = 55^2 = 3025$$

Hence the difference between the sum of the squares of the first ten natural numbers and the square of the sum is $3025385 = 2640$. Find the difference between the sum of the squares of the first one hundred natural numbers and the square of the sum.

Answer: 25164150
Method: Take two variables in a loop from 1 to 100. The first added each number together, the sound adds the squares.

7 10001st prime

Problem: By listing the first six prime numbers: 2, 3, 5, 7, 11, and 13, we can see that the 6^{th} prime is 13. What is the 10001^{st} prime number?

Answer: 104009
Method: Simply check for primes, this is not a very big task for a computer. (Still, a faster solution would be nice).

8 Largest product in a series

Problem: The four adjacent digits in the 1000-digit number that have the greatest product are $9 \times 9 \times 8 \times 9 = 5832$.

$$
\begin{array}{c}
7316717653133062491922511967442657474235534919493 4 \\
9698352031277450632623957831801698480186947885184 3 \\
8586156078911294949545950173795833195285320880551 1 \\
1254069874715852386305071569329096329522744304355 7 \\
6689664895044524452316173185640309871112172238311 3 \\
6222989342338030813533627661428280644448664523874 9 \\
3035890729629049156044077239071381051585930796086 6 \\
7017242712188399879790879227492190169972088809377 6 \\
6572733300105336788122023542180975125454059475224 3 \\
5258490771167055601360483958644670632441572215539 7 \\
5369781797784617406495514929086256932197846862248 2 \\
8397224137565705605749026140797296865241453510047 4 \\
8216637048440319989000889524345065854122758866688 1 \\
1642717147992444292823086346567481391912316282458 6 \\
1786645835912456652947654568284891288314260769004 2 \\
2421902267105562632111110937054421750694165896040 8 \\
0719840385096245544436298123098787992724428490918 8 \\
8458015616609791913387549920052406368991256071760 6 \\
0588611646710940507754100225698315520005593572972 5 \\
7163626956188267042825248360082325753042075296345 0
\end{array}
$$

Find the thirteen adjacent digits in the 1000-digit number that have the greatest product. What is the value of this product?

Answer: 2091059712
Method: Multiplies every 13 digits until the greatest is found.

9 Pythagorean Triplets

Problem:A Pythagorean triplet is a set of three natural numbers, $a < b < c$, for which,

$$a^2 + b^2 = c^2$$

For example, $32 + 42 = 9 + 16 = 25 = 52$. There exists exactly one Pythagorean triplet for which $a + b + c = 1000$. Find the product abc.

Answer: 31875000
Method: Starting with $a = b = 1$ and increasing, letting $c = 1000 - a - b$ and checking for triplets.

10 Summation of primes

Problem: The sum of the primes below 10 is $2 + 3 + 5 + 7 = 17$. Find the sum of all the primes below two million.

Answer: 1308111348
Method: Checks for primes and adds them, re-uses the code from Problem 7.