Let G be some graph on the vertices $\{1, 2, \ldots, n\}$. We denote a proper colouring of G as $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ where vertex i is coloured x_i and each colour comes from $\{0, 1, \ldots, n\}$. A *habitat* is a proper colouring of G and the *suitability* of a habitat is the number of colours used in the colouring. We denote the set of proper colourings of G as $\mathcal{C}(G)$.

Let $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ and $\mathbf{y} = (y_1, y_2, \ldots, y_n)$ be two distinct proper colourings of G. We define $dist(\mathbf{x}, \mathbf{y})$ to be the number of vertices that are coloured differently in the two colourings. Our notion of distance yields that $(\mathcal{C}(G), dist)$ is a metric space.

We can compute a canonical 'path' from $\mathbf{x}$ to $\mathbf{y}$ as follows. Let $I = \{i : x_i \neq y_i\}$ and let i be some vertex from I. Start with the proper colouring $\mathbf{x}$. We build a new colouring $\mathbf{x}'$ by recolouring vertex i to be y_i, although $\mathbf{x}'$ might not be a proper colouring; however, the set of vertices on which $\mathbf{x}'$ is different from $\mathbf{y}$ is $I \setminus \{i\}$. Note that there are no colour-conflicts in $\mathbf{x}'$ between i and any neighbour of i from $\{1, 2, \ldots, i-1\}$ (as $\mathbf{y}$ is a proper colouring) but there might be colour-conflicts between i and some neighbours from $\{i+1, i+2, \ldots, n\}$. Suppose that $J \subseteq \{i+1, i+2, \ldots, n\}$ consists of those vertices j such that (i, j) is an edge of G and $y_i = x_j$. In particular, for any $j \in J$, $x_j \neq y_j$ as in the proper colouring $\mathbf{y}$, vertex i is coloured y_i and vertex j is coloured y_j; hence, $j \in I \setminus \{i\}$. Iteratively recolour each vertex $j \in J$ with the smallest legal colour z_j different to y_j and denote this new proper colouring $\mathbf{x}_1$. The vertices on which $\mathbf{x}_1$ and $\mathbf{y}$ differ is $I \setminus \{i\}$. Iteratively repeat for each vertex of $I \setminus \{i\}$ and we obtain a 'path' of colourings $\mathbf{x}, \mathbf{x}_1, \ldots, \mathbf{x}_{d-1}, \mathbf{y}$, where $|I| = d$.

In the cuckoo optimization algorithm, a cuckoo lays eggs in nest at a distance of not more that ELR away from its current location. In order to choose such a nest, if $ELR = d$ then we might randomly choose at most d vertices of G and iteratively recolour each chosen vertex with the smallest legal colour that is different to the current colour.

We can cluster in metric spaces (see attached paper).

When a cuckoo needs to fly part of the way along a path towards a habitat, we can work with the canonical path as described above.

When we want to vary the vector to a specific habitat, we might choose some vertex that is at distance 1 from the habitat and aim for there.