EE-238
Homework 5

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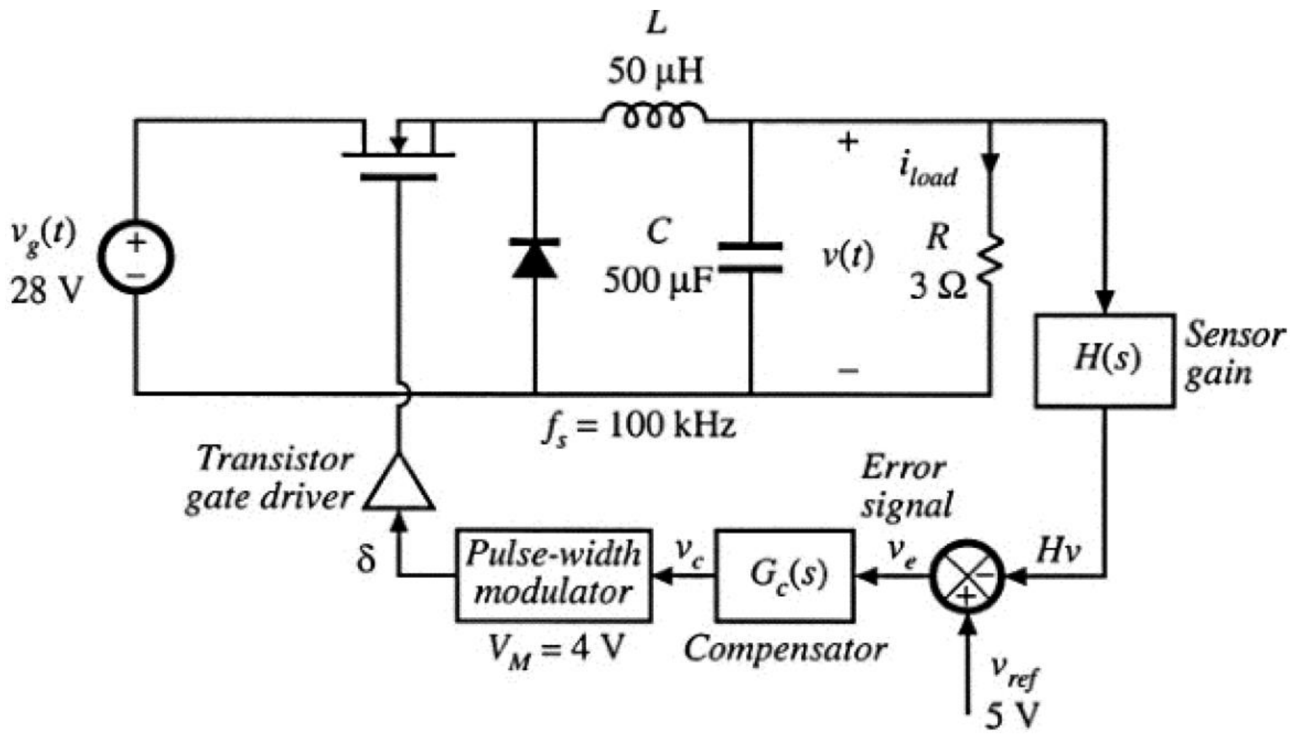
Due Date : 05/23/2017

1- For the Buck converter shown below, it is desired to control the output voltage to be $15\ v$

- a) What should $H(s)$ be equal to and why?
- b) What is the expected system duty ratio assuming ideal characteristics?
- c) Find $G_v(s)$ and $G_d(s)$ of the given system.
- d) Find the natural frequency (f), the quality factor (Q) and the DC gain for both of the transfer functions.
- e) Plot the Bode plots of both of the transfer functions.
- f) Assuming that the only disturbance in the system is the input voltage variations, draw the control block diagram of the system.
- g) Find the loop gain transfer function $T(s)$ with $G_c(s) = 1$.
- h) Draw the Bode Plot of the $T(s)$.
- i) What is the difference between $G_d(s)$ and $T(s)$ bode plots.
- j) Find the system crossover frequency and phase margin.
- k) Simulate the converter in a circuit simulation software using the same parameters given and shown (including the reference voltage, the unity compensator, the Vm value, etc..). Show the system output voltage and inductor current from your circuit simulation file (while applying the feedback control).
- l) Show the system response for a small change in the reference voltage and a small change in the input voltage from both the simulated circuit and your T(s) model with $G_c(s) = 1$ (while applying the feedback control).
- m) Design a PD compensator to attain a crossover frequency of 5 kHz and a phase margin of 52 degrees.
- n) Write the PD compensator transfer function and draw its bode plot.
- o) Draw the bode plot of the $T(s)$ where $G_c(s)$ is the PD compensator and show if you were able to meet the desired control requirements (crossover frequency and phase margin).
- p) Show the system response for a small change in the reference voltage and a small change in the input voltage from both the simulated circuit and your T(s) model with $G_c(s)$ being
the PD compensator transfer function (while applying the feedback control).
- q) Enhance the performance of your PD controller through the addition of an inverted zero (PI controller) so that the transfer function become a PID controller in order to increase the system low frequency gain. Choose the inverted zero frequency to be one tenth of the

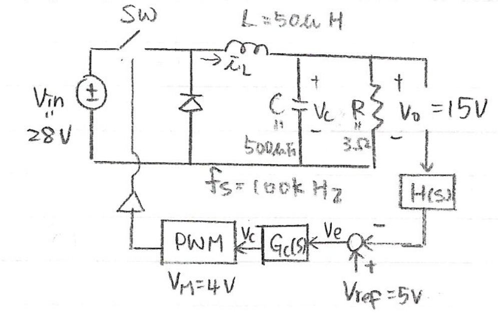
crossover frequency used in m).

- r) Write the PID compensator transfer function and draw its bode plot.
- s) Draw the bode plot of the $T(s)$ where $G_C(s)$ is the PID compensator and show if you are still able to meet the desired control requirements (crossover frequency, phase margin and DC gain).
- t) Show the system response for a small change in the reference voltage and a small change
in the input voltage from both the simulated circuit and your T(s) model with $G_C(s)$ being the PID compensator transfer function (while applying the feedback control).
- u) Comment on the results in l), p) & t).



Q1

Buck



$$G_{vd}(s) = \frac{G_{do}\left(1 - s/\omega_z\right)}{\left(1 + \frac{s}{Q\omega_o} + \left(\frac{s}{\omega_o}\right)^2\right)}$$

$$G_{vg}(s) = \frac{G_{go}}{\left(1 + \frac{s}{Q\omega_o} + \left(\frac{s}{\omega_o}\right)^2\right)}$$

$G_{go} = D$

$G_{do} = V_o/D$

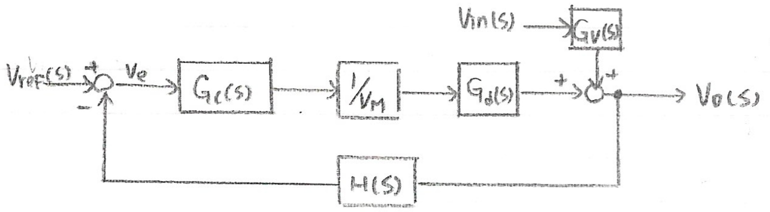
$\omega_o = 1/\sqrt{LC}$

$Q = R\sqrt{C/L}$

$\omega_z = \infty$

$$V_o(s) = V_{ref}(s) * \frac{1}{H(s)} * \frac{T(s)}{1+T(s)} + V_{in}(s) * \frac{G_v(s)}{1+T(s)}$$

$$T(s) = H(s)\,G_c(s)\,G_d(s) * \frac{1}{V_M}$$



a)

if $\|T\| >> 1 \Rightarrow \frac{\hat{V}_o}{\hat{V}_{ref}} = \frac{1}{H(s)} \Rightarrow H(s) = \frac{\hat{V}_{ref}}{\hat{V}_o} = \underline{\frac{1}{3}}$

this is the value we want in steady state.

(b)

$\frac{V_o}{V_{in}} = D = \frac{15}{28} = \underline{0.536}$

(c)

$G_{go} = 0.536 \qquad G_{do} = 28 \qquad \omega_o = \frac{1}{1.58\times10^{-4}} = 6324.56 \qquad Q = 3\sqrt{10} = 9.49$

$$G_{vg} = \frac{0.536}{1 + \frac{s}{60020.07} + \left(\frac{s}{6324}\right)^2} = \frac{0.536}{1 + 1.67\times10^{-5}s + 2.5\times10^{-8}s^2}$$

$$G_{vd} = \frac{28}{1 + 1.67\times10^{-5}s + 2.5\times10^{-8}s^2}$$

(d)

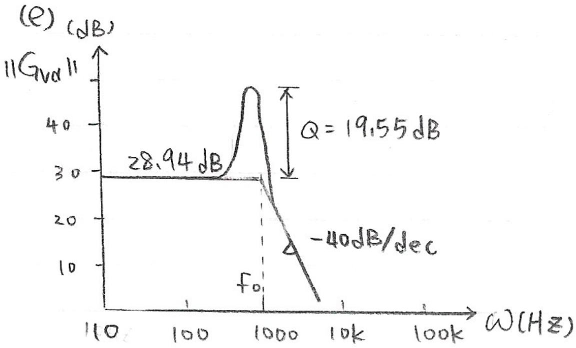
$f_o = \frac{\omega_o}{2\pi} = \underline{1007\ Hz}$

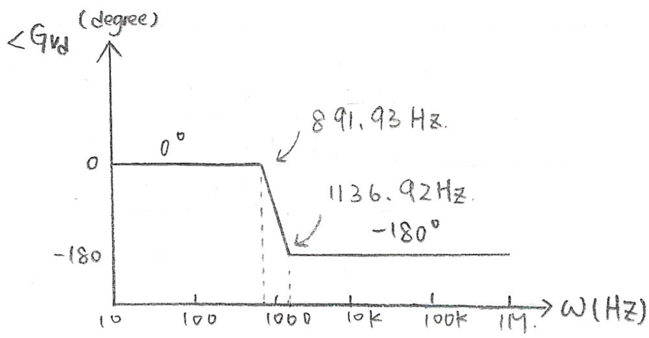
$Q_o = R\sqrt{C/L} = \underline{9.49}$

$\underline{G_{go} = 0.536}$

$\underline{G_{do} = 28}$

(e)





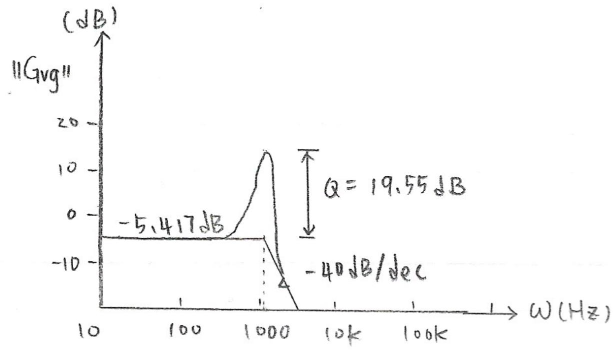
$G_{d0} = 28 \rightarrow 28.94\,dB$

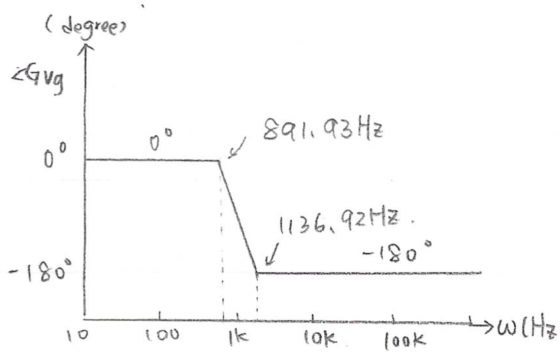
$Q = 9.49 \rightarrow 19.55\,dB$

$f_0 = 1007Hz$

$10^{1/2Q} f_0 = 10^{0.0527} f_0 = 1136.92$

$10^{-1/2Q} f_0 = 891.93\,Hz$

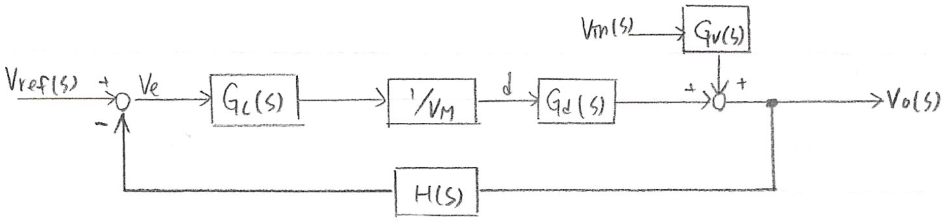




$G_{g0} = 0.536 = -5.417\,dB$

$Q = 19.55\,dB$

(f).

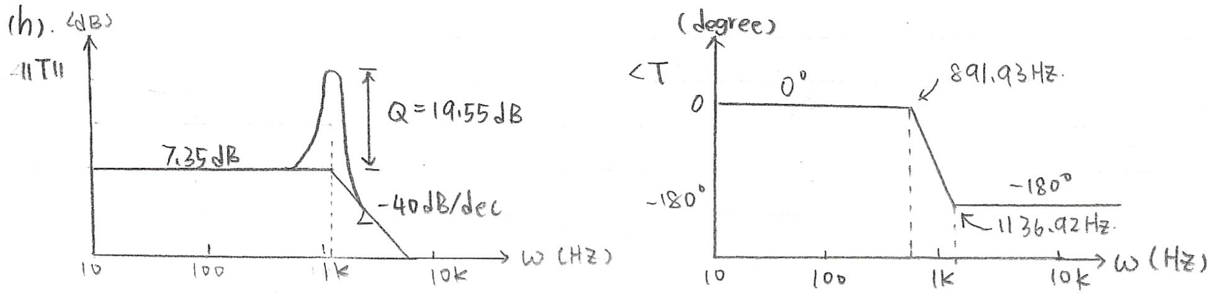


(g)

$$T(s) = H(s)\,G_c(s)\,G_d(s)\,\frac{1}{V_M} = \frac{1}{3} \times 1 \times \frac{28}{1 + 1.67\times10^{-5}s + 2.5\times10^{-8}s^2} \times \frac{1}{4}$$

$$= \underline{\underline{\frac{2.3}{1 + 1.67\times10^{-5}s + 2.5\times10^{-8}s^2}}}$$

(h).



$T_0 = 2.33 \rightarrow 7.35\,dB$

(i)

The Bode plots of $G_d(s)$ and $T(s)$ are really familliar. The difference between them is on the magnitude part. The DC gain is different.

(j)

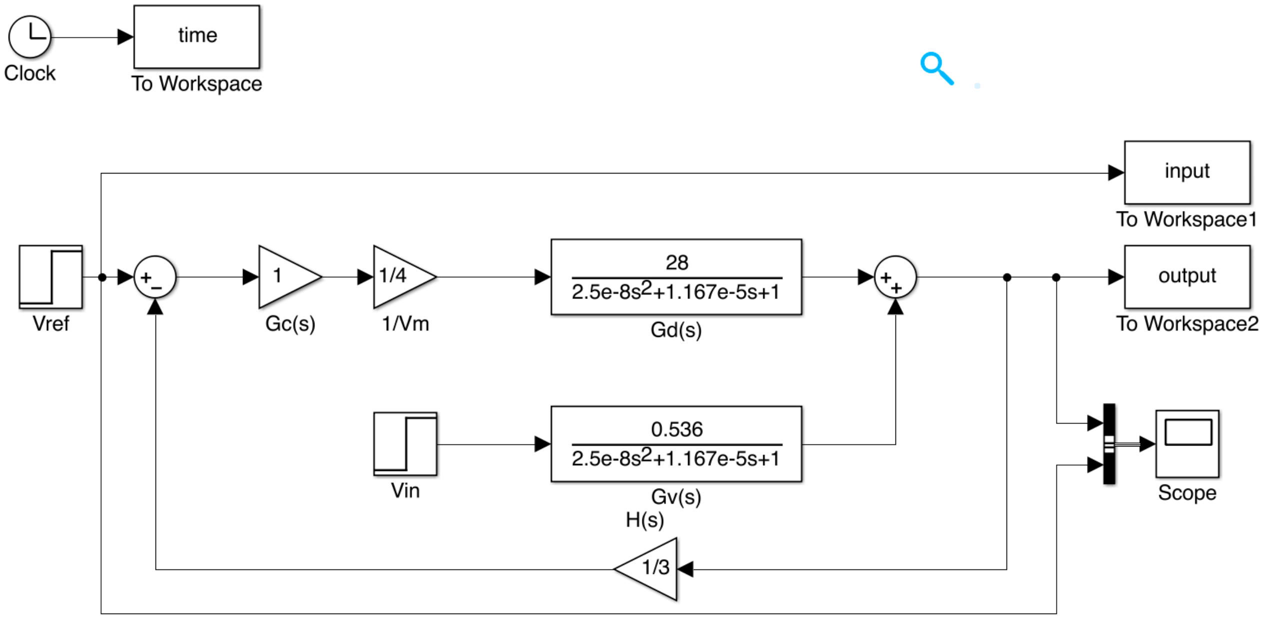
By using MATLAB function [Gm, Pm, Wgm, Wpm] we can get G_d's crossover frequency = 3.19×10^4 = 31900 Hz.
phase margin = 1.09 dB

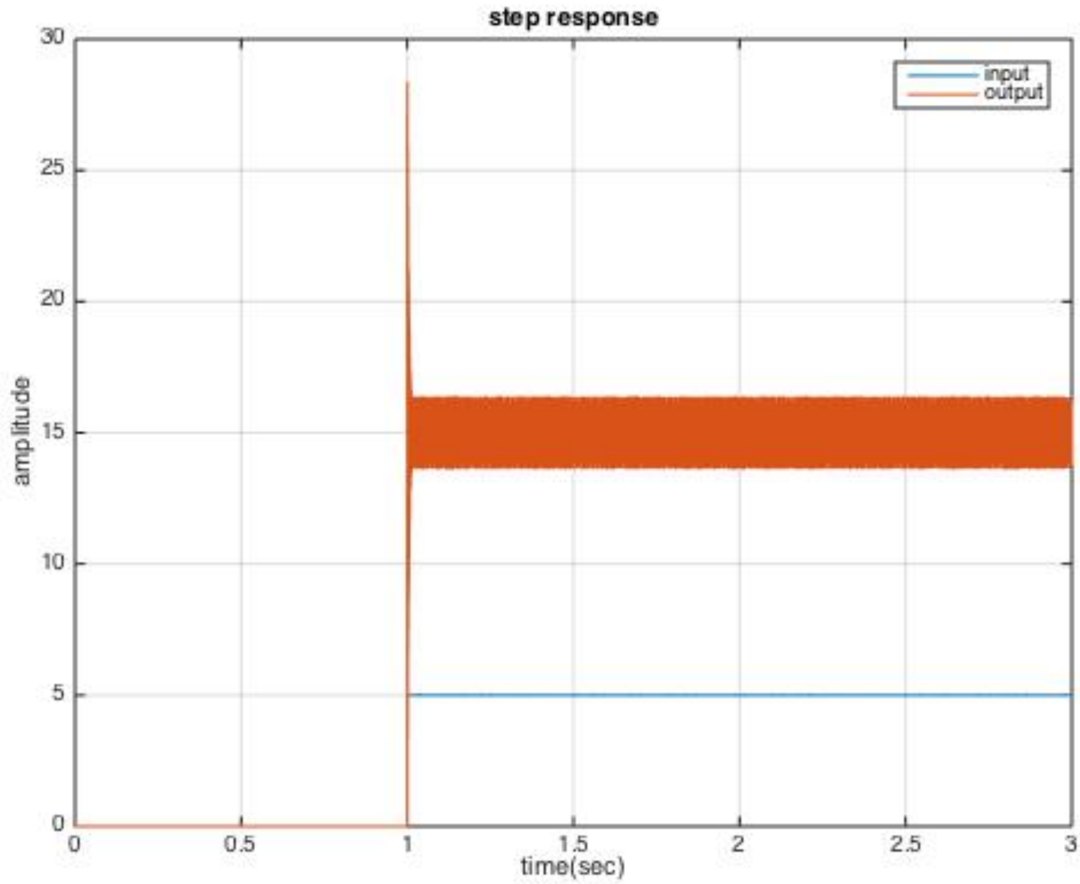
T's crossover frequency = 1.0803×10^4 = 10803 Hz.
phase margin = 4.4346 dB

```
>> Gd = tf ([28] , [2.85*10^-8 1.67*10^-5 1])

Gd =

                28
  -----------------------------
  2.85e-08 s^2 + 1.67e-05 s + 1

Continuous-time transfer function.

>> T = tf ([28/12] , [2.85*10^-8 1.67*10^-5 1])

T =

              2.333
  -----------------------------
  2.85e-08 s^2 + 1.67e-05 s + 1

Continuous-time transfer function.
```

```
>> [Gm,Pm,Wgm,Wpm] = margin(Gd)

Gm =

   Inf


Pm =

    1.0900


Wgm =

   Inf


Wpm =

   3.1898e+04
```

(k)

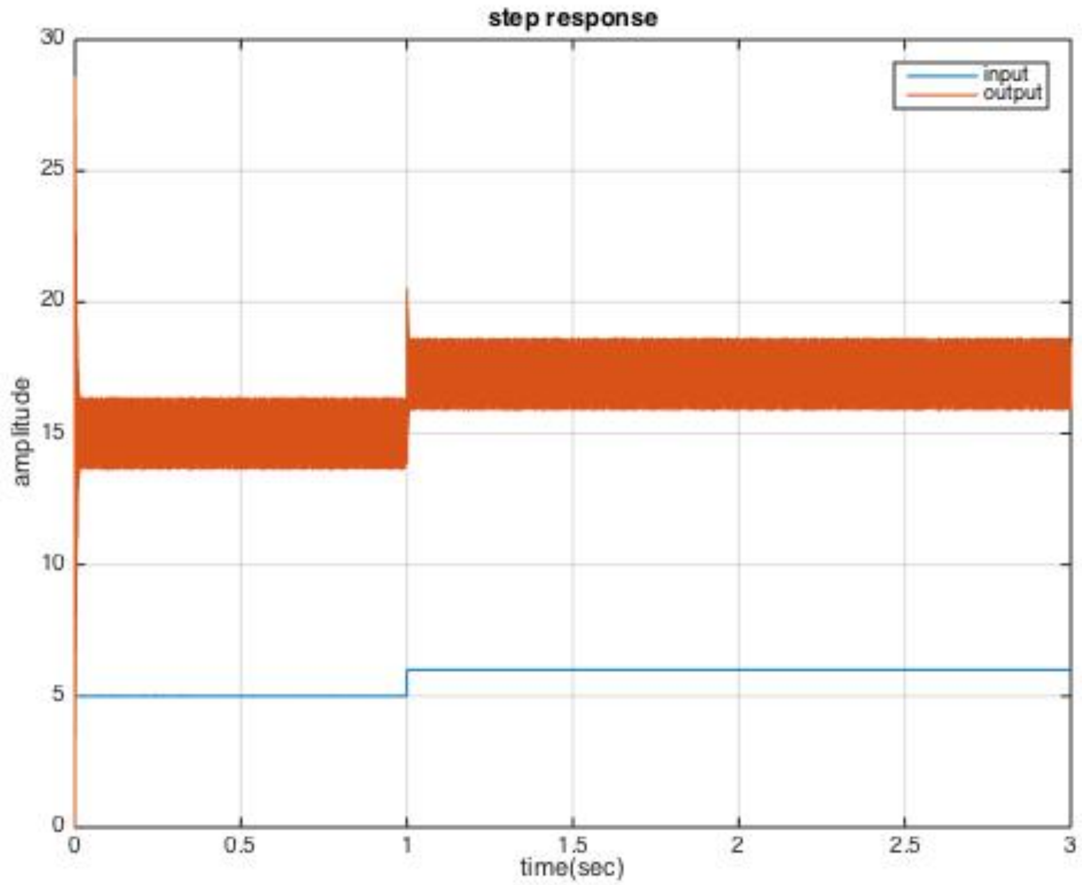




the blue line is V_{ref} not V_{in}

(l)

the blue line is V_{ref} not V_{in}



(m)

PD compensator $\Rightarrow f_c = 5kHz,\ \Phi_m = 52°$

$$G_c(s) = G_{co}\frac{(1+ s/\omega_z)}{(1+ s/\omega_p)}$$

$$f_z = f_c\sqrt{\frac{1-\sin(\theta)}{1+\sin(\theta)}} = 5k\sqrt{\frac{0.212}{1.788}} = 3739.25\ Hz. \Rightarrow \omega_z = 23494.4$$

$$f_p = f_c\sqrt{\frac{1+\sin(\theta)}{1-\sin(\theta)}} = 5k\sqrt{\frac{1.788}{0.212}} = 14521.05\ Hz \Rightarrow \omega_p = 91238.45$$

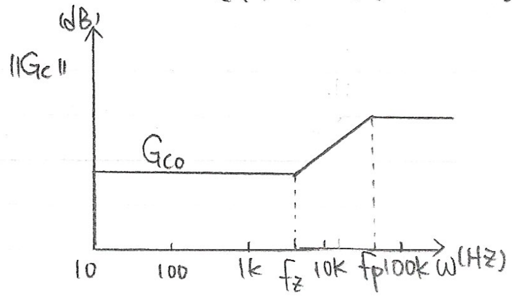
Assume that I want the magnitude of the compensator gain at f_c is unity.

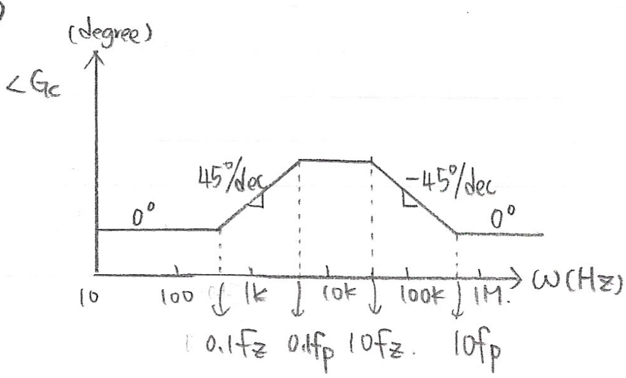
$$\Rightarrow G_{co} = \sqrt{\frac{f_z}{f_p}} = 0.507$$

$$\Rightarrow G_c(s) = \frac{0.507(1+4.256\times10^{-5}s)}{(1+1.096\times10^{-5}s)}$$

(n.)

$$G_c(S) = \frac{0.507\,(1+4.256\times10^{-5}S)}{(1+1.0965\times10^{-5}S)}$$



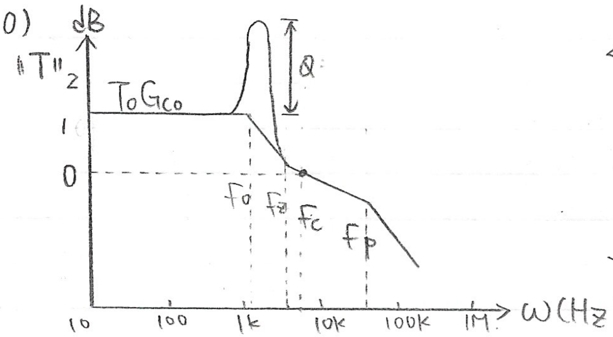


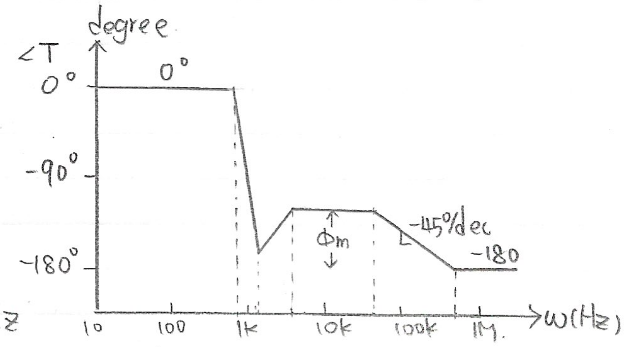
$f_z = 3.74\ kHz$

$f_p = 14.5\ kHz$

$G_{co} = 0.507 \Rightarrow -5.9\,dB$

0)





$T_oG_{co} = 1.181 \Rightarrow 1.445\,dB$

$f_o = 1007\ Hz$

$f_z = 3739\ Hz$

$f_p = 14521\ Hz.$

$f_c = 5\ kHz.$

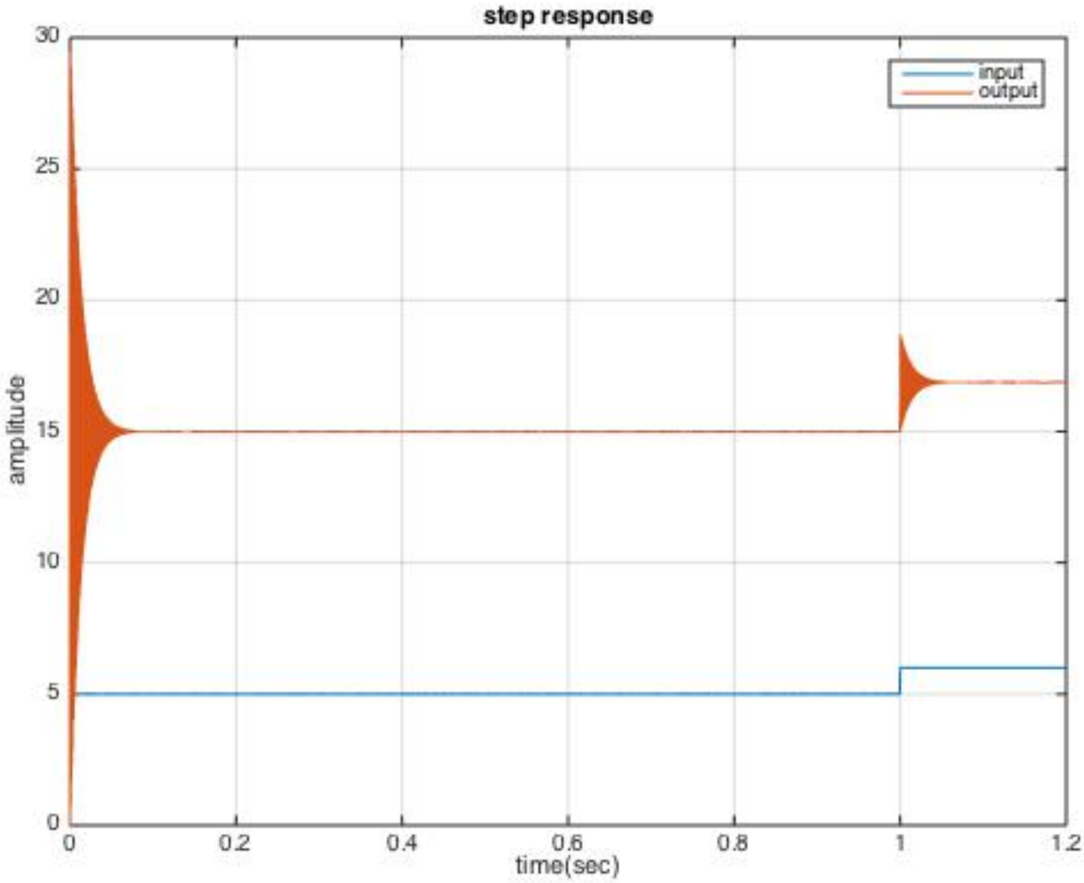
$Q = 19.55\,dB$

$10^{-3} f_o = 892\,Hz$

$10^{3} f_o = 1137\,Hz.$

$S + \omega_L.$

(p) the blue line is V_{ref} not V_{in}



(q)

PI compensator: $G_{c\infty}\left(1+\frac{\omega_L}{S}\right) = (1+3141.6 * S^{-1})$

$f_L = 0.1 f_c = 5k \times 0.1 = 500\,Hz$

$\Rightarrow \omega_L = 3141.6\ Hz$

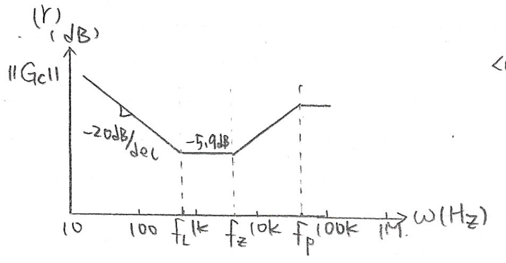
Assume $G_{c\infty} = 1$

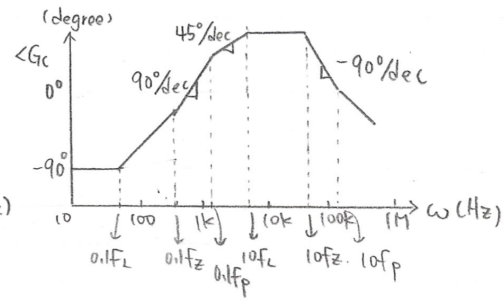
(r)

PID compensator: $G_{co}\, G_{c\infty} \dfrac{(1+\omega_L/S)(1+S/\omega_z)}{(1+S/\omega_p)}$

$$= \frac{0.507\,(1+3141.6/S)(1+4.256\times10^{-5}S)}{(1+1.0965\times10^{-5}S)}$$

(r)





$G_{co}\, G_{c\infty} = 0.507 \rightarrow -5.9\,dB$

$f_L = 500\,Hz$

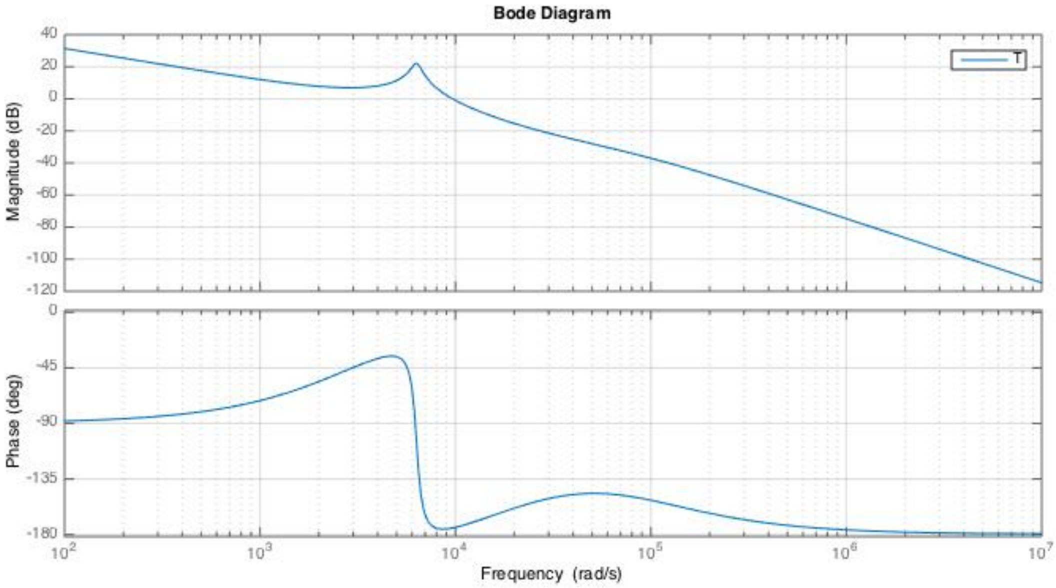
$f_z = 3739\,Hz$

$f_p = 14521\,Hz$

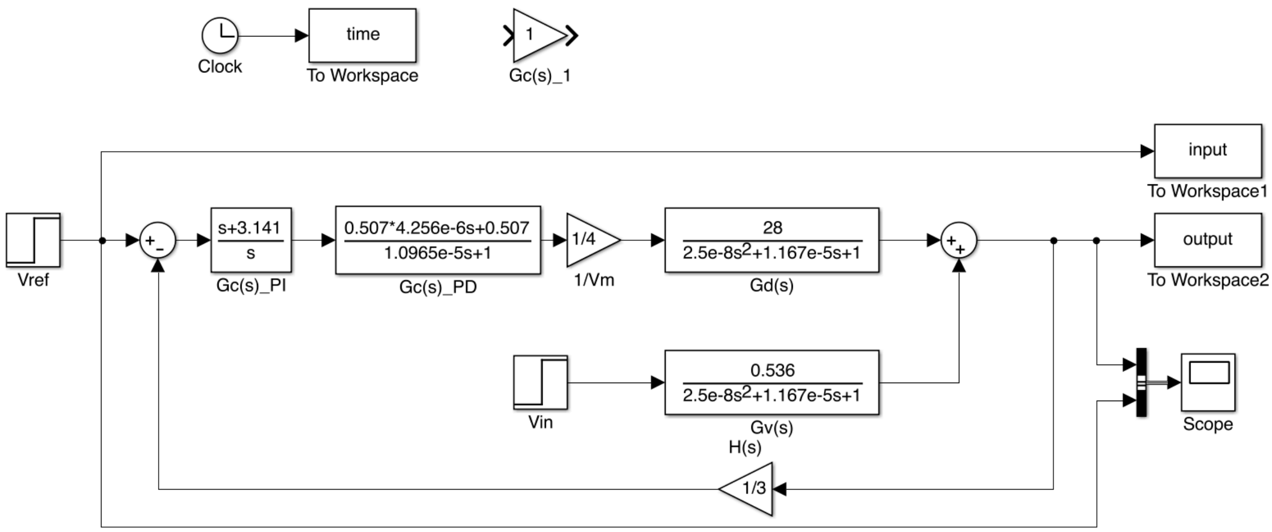
$f_c = 5k\,Hz$

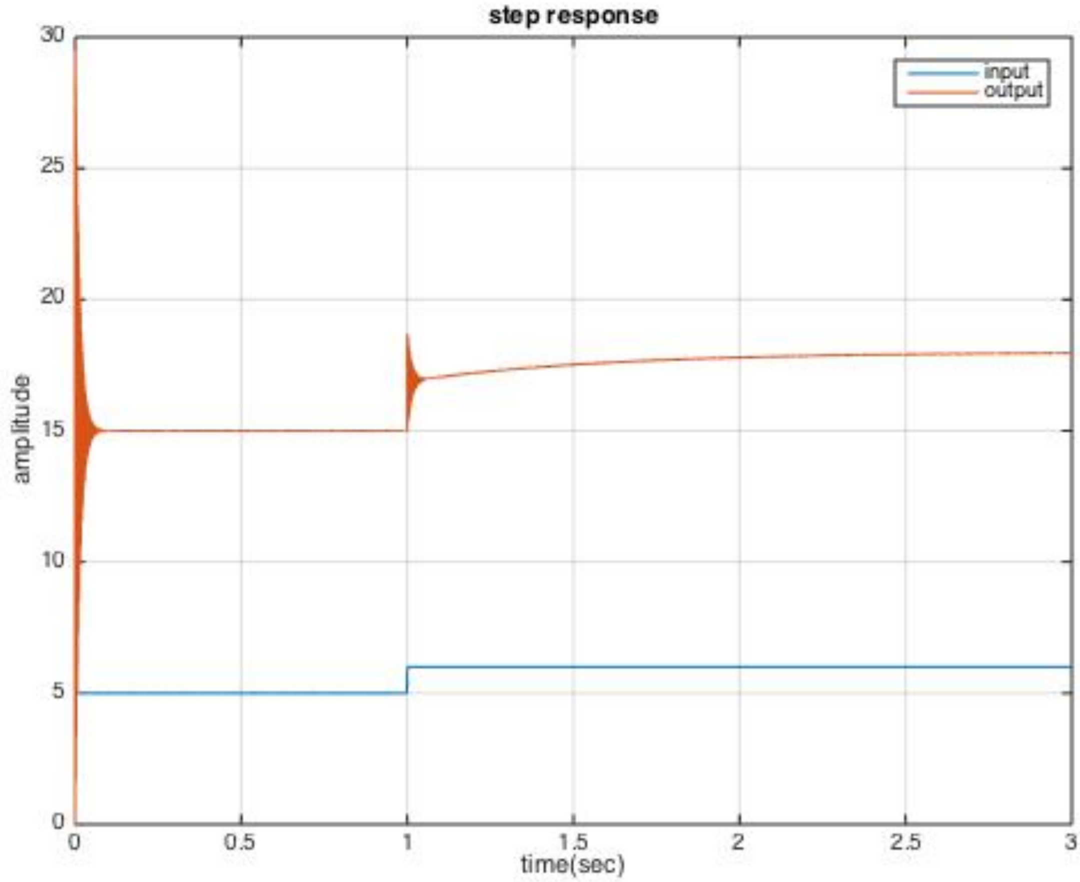
(s)

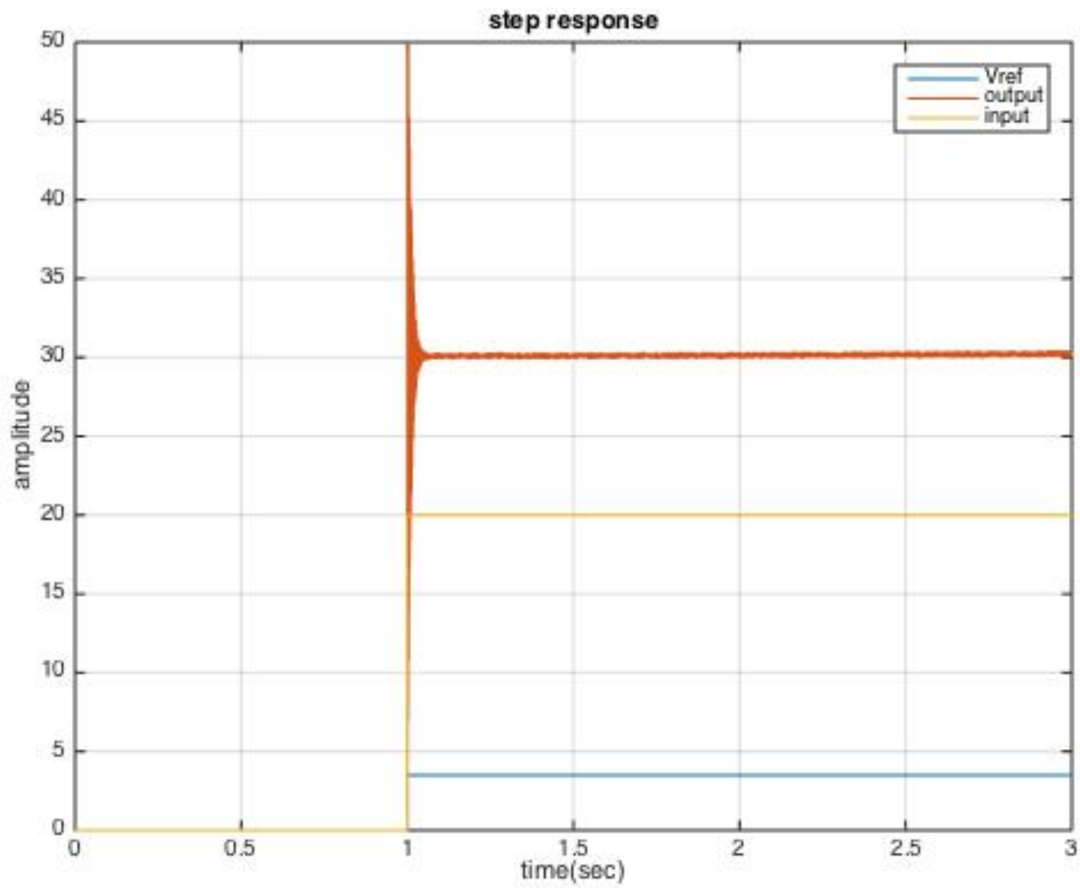
```
>> Gd=tf([28], [2.5e-8 1.67e-5 1]);
>> Gc=tf([0.507*4.26e-5 0.507*(1+0.134) 0.507*3141.6],[1.096e-5 1 0]);
>> H=1/3;
>> Vm=4;
>> T=H*Gc*Gd/Vm;
>> bode(T)
```

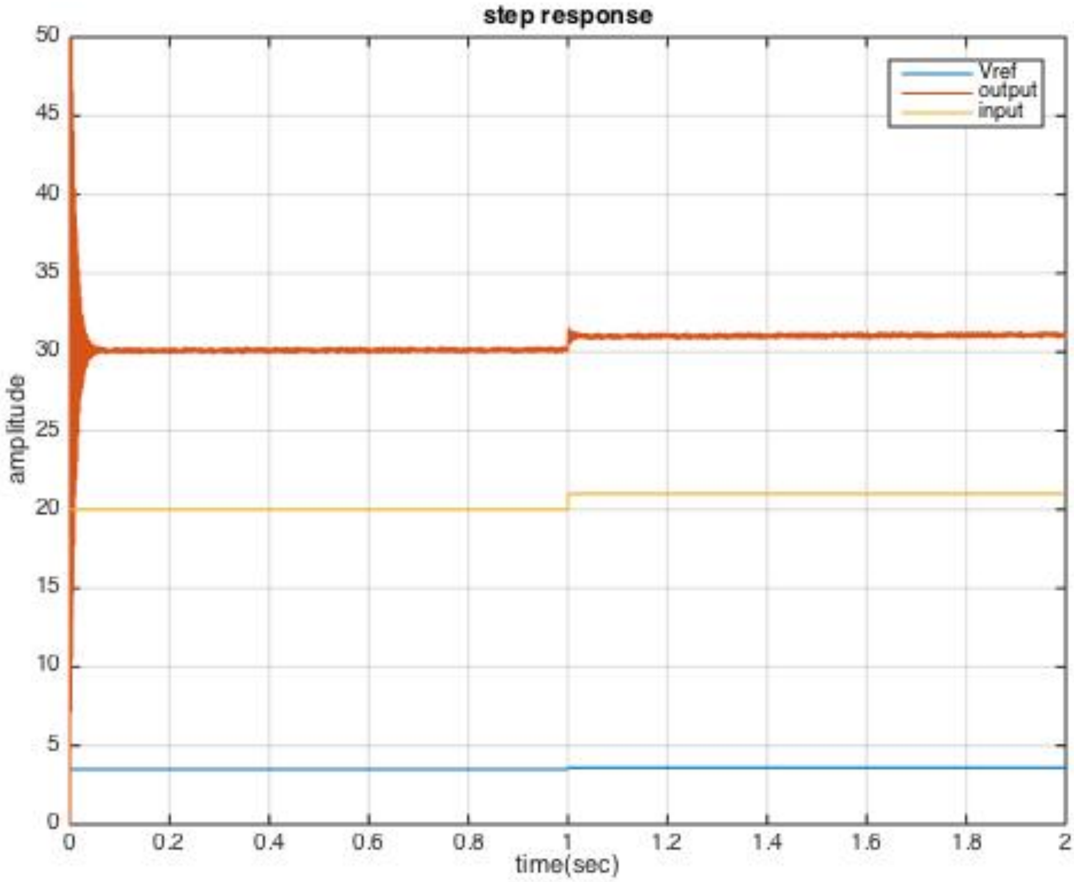


(t)




step response
input
output
amplitude
time(sec)
30
25
20
15
10
5
0
0
0.5
1
1.5
2
2.5
3


step response
Vref
output
input
amplitude
time(sec)
50
45
40
35
30
25
20
15
10
5
0
0
0.5
1
1.5
2
2.5
3



(u)

when $G_c(s)=1$, the damping is huge.
when using PD controller as $G_c(s)$, the damping decrease.
when using PID as $G_c(s)$, the settling time is quicker then using PD. Both of PD and PID reach the steady state.

2- A boost converter is used with the following parameters:

V =20v in

L = 0.5 mH

C = 500 µF

f = 100 kHz s

R = 10 Ω

The boost converter will be regulated using a feedback control similarly to questions 1.
The output voltage is desired to be $35\ v$. The available voltage on the board to be used as a reference voltage is $3.5\ v$, $V = 1$, $a$ssuming ideal components, m

- i) Repeat parts a)-l) similarly to question 1
- ii) Based on the system response and the bode plots, design a compensator to be used as $G_C(s)$
- iii) Show the system response for a small change in the reference voltage and a small change in the input voltage from both the simulated circuit and your T(s) model with$G_C(s)$ being the designed compensator transfer function (while applying the feedback control).
- iv) Comment on the feedback control results.

Q2

i)

a).

$$H(S) = \frac{V_{ref}}{V_o} = \frac{3.5}{35} = \underline{\underline{0.1}}$$

b).

$$\frac{V_o}{V_i} = \frac{1}{1-D} = \frac{35}{20} \Rightarrow \underline{\underline{D = 0.429}}$$

c)

$$G_v(S) = \frac{G_{vo}}{1 + S/Q\omega_o + (S/\omega_o)^2} \qquad G_d(S) = \frac{G_{do}(1 - S/\omega_z)}{1 + S/Q\omega_o + (S/\omega_o)^2}$$

$$G_{vo} = \frac{1}{D'} = 1.75 \qquad G_{do} = V_o/D' = 61.4$$

$$\omega_o = D'/\sqrt{LC} = 1140 \qquad Q = D'R\sqrt{C/L} = 5.7 \qquad \omega_z = D'^2 R/L = 6498$$

$$\Rightarrow \underline{\underline{G_v(S) = \frac{1.75}{1 + 1.54\times10^{-4}S + 7.69\times10^{-7}S^2}}}$$

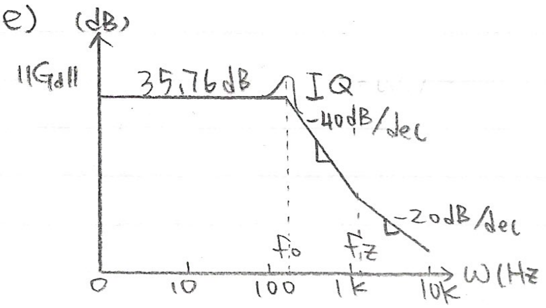
$$\underline{\underline{G_d(S) = \frac{61.4 * (1 - 1.54\times10^{-4}S)}{1 + 1.54\times10^{-4}S + 7.69\times10^{-7}S^2}}}$$

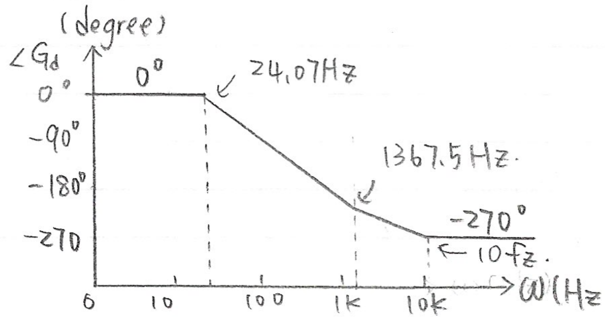
d)

IV)

after applying the compensator, the system became a stable system.

e)





$G_{do} = 61.4 \rightarrow 35.76dB$

$f_o = 181.44$ Hz

$f_z = 1034.19$ Hz

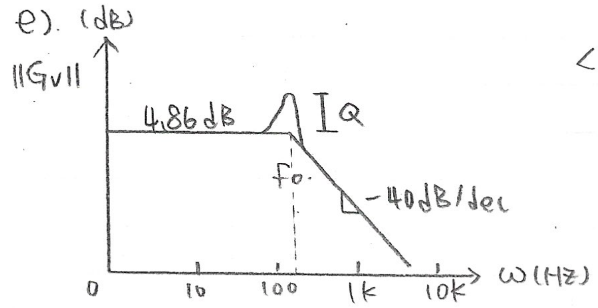
$Q = 5.7 \Rightarrow 15.12dB$

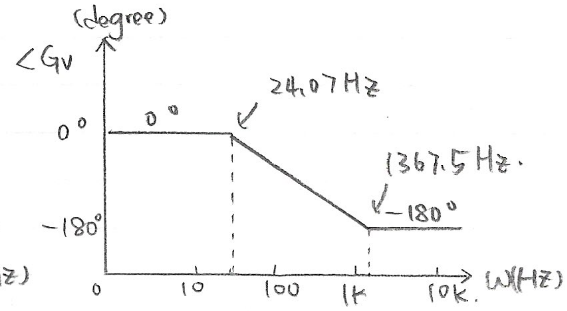
$10^{-3} f_o = 24.07$ Hz

$10^{3} f_o = 1367.5$ Hz

$10 f_z = 10341.9$ Hz

e).





$G_{go} = 1.75 \rightarrow 4.86\ \text{dB}$

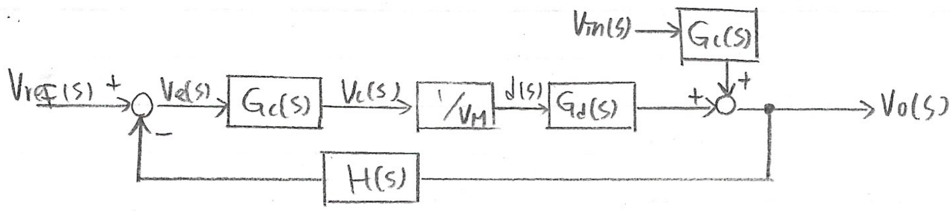
$f_o = 181.44\ \text{Hz}$

$Q = 5.7 \rightarrow 15.12\ \text{dB}$

$10^{-3} f_o = 24.07\ \text{Hz}$

$10^{3} f_o = 1367.5\ \text{Hz}$

f).



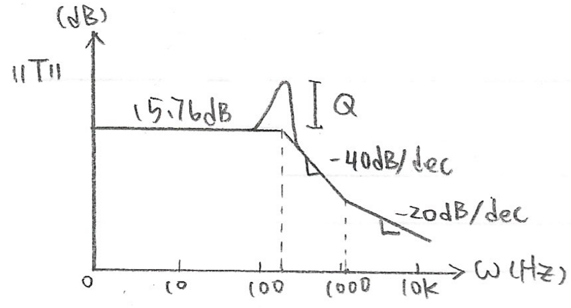
g)

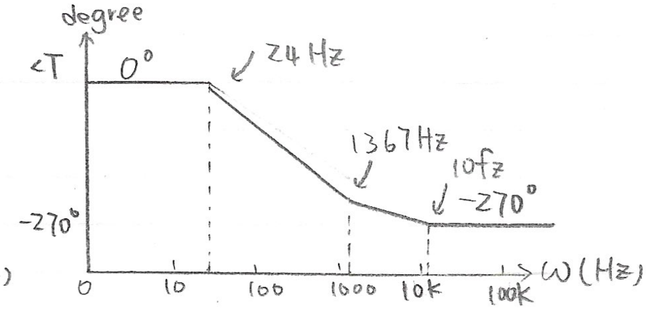
$$T(s) = H(s)\, G_c(s)\, G_d(s)\, {}^{1}\!/\!_{V_M}$$

$$= 0.1 \times 1 \times \frac{61.4\,(1 - 1.54\times10^{-4}s)}{1 + 1.54\times10^{-4}s + 7.69\times10^{-7}s^2} \times \frac{1}{1}$$

$$= \frac{6.14\,(1 - 1.54\times10^{-4}s)}{1 + 1.54\times10^{-4}s + 7.69\times10^{-7}s^2}$$

h)





$T_o = 6.14 \rightarrow 15.76\ \text{dB}$

$f_o = 181.44\ \text{Hz}$

$f_z = 1034\ \text{Hz}$

$Q = 5.7 \rightarrow 15.12\ \text{dB}$

$10^{-3} f_o = 24.07\ \text{Hz}$

$10^{3} f_o = 1367.5\ \text{Hz}$

$10 f_z = 10340\ \text{Hz}$

(i)

T(s) 's Bode plot looks simillar with Gd(s)
But they have different DC gain.

(j).

By using MATLAB function, we can get the crossover frequency = $\underline{3.19\times10^3\ Hz}$, $\Phi_m = \underline{-22.0518°}$

(k)

when Gd(s) has zero at ω_z that makes the system an unstable system ⇒ cannot get the output response and Inductor current.

(l)

Same result as (k)

```
>> H=0.1;
>> Gc=1;
>> Gd=tf([61.4*(-1.54e-4) 61.4],[7.69e-7 1.54e-4 1]);
>> Vm=1;
>> T=H*Gc*Gd/Vm;
>> bode(T)
>> [Gm,Pm,Wgm,Wpm] = margin(T)
Warning: The closed-loop system is unstable.
> In warning at 25
  In DynamicSystem.margin at 65

Gm =

    0.1629


Pm =

  -22.0518


Wgm =

   1.6127e+03


Wpm =

   3.1906e+03
```