Stat 400. Problem Set 8. Due 11/05/24

Linear Combinations or Random Variables

Problem 1. Let $\mathcal{S} = \{(a,b) : a,b \in \{1,2,3,4,5,6\}\}$ be the space used to model two consecutive rolls of a fair die. Consider three random variables defined on S:

$$X(a,b) = a-2,\ Y(a,b) = |a-b|,\ \text{and } Z(a,b) = a,$$

where a is the result of the first die roll and b is the result of the second.

(a) Compute $E(Z)$.
(b) Compute $E(Y)$.
(c) Compute $E(X)$.
(d) Compute $E(XZ)$. Is $E(XZ) = E(X)E(Z)$?
(e) Compute $E(100Z - \pi Y)$.

Problem 2. Consider a game where we toss an unfair coin with $P(H) = .4$ three times. We win \$$n$ if the first H is on the n^{th} toss, or we lose \$1 if we don't get any H. Let X be the number of Heads, and Y be the net winnings.

(a) Calculate the following values/ recall them from Homework 7:
 (i) $E(X)$
 (ii) $E(Y)$
 (iii) σ_X
 (iv) σ_Y
(b) Calculate $E(2X - Y)$.
(c) Calculate σ_{2X-Y}.
(d) Consider a higher stakes game where we win \$$100n$ if the first H is on the n^{th} toss, or we lose \$100 if we don't get any H. What is the expected net winnings of this new game?

Problem 3. Suppose X is a discrete random variable with expected value $E(X)$. Show using the definition of expected value for a discrete random variable that if $Y = 5X-2$, then $E(Y) = 5 \cdot E(X) - 2$. *Hint:* How do the possible values $\mathcal{Y}$ of Y compare to the possible values $\mathcal{X}$ of X?

Problem 4. Suppose the lifetimes of two light bulbs are measured (in 1000s of hours) by random variable X and Y , each having exponential distibution with parameter $\lambda_X = 2$ and $\lambda_Y = 4$.

(a) Compute the expected value of X, $E(X)$.
(b) Compute the expected value of Y, $E(Y)$.
(c) Compute the variance of X, σ_X^2.
(d) Compute the variance of Y, σ_Y^2.
(e) Compute the expected value of $Z = 4X - 3Y$.
(f) Compute the variance of $Z = 4X - 3Y$.

Problem 5. Suppose X is a continuous random variable with expected value $E(X)$. Let $Y = 3X$.

(a) What is the pdf of Y (in terms of the pdf of X)?
(b) Show using the definition of expected value for a continuous random variable that $E(Y) = 3 \cdot E(X)$.

Problem 6. Suppose X is a continuous random variable with expected value $E(X)$. Let $Y = 5X - 2$.

(a) What is the pdf of Y (in terms of the pdf of X)?
(b) Show using the definition of expected value for a continuous random variable that $E(Y) = 5 \cdot E(X) - 2$.