

## Homework 3

● Graded

1 Hour, 21 Minutes Late

Student

Jacob Hauptman

Total Points

23.5 / 24 pts

Question 1

Four coin tosses

6 / 6 pts

✓ - 0 pts Correct

- 1 pt Subsections "dc" and "dd" are wrong
- 1 pt Didn't use  $P(H) = 0.8$  / treat it as a fair coin
- 2 pts Didn't consider permutation in part b,c,d.
- 2 pts Part d is completely wrong.
- 0.5 pts Inaccurate expression of cumulative distribution
- 1 pt Wrong cdf on parts b and c
- 1 pt Part d. (c) (d) are wrong
- 0.5 pts small mistakes such as computation error
- 2 pts Missing a part
- 6 pts No answer loaded
- 0.5 pts missing a subquestion of a part

Question 2

Two die rolls

6 / 6 pts

✓ - 0 pts Correct

- 2 pts Part c is wrong / missing.
- 2 pts Part d is wrong / missing.
- 1 pt inaccurate expression of cdc
- 6 pts No answer loaded

### Question 3

#### Defective Ovens

6 / 6 pts

✓ - 0 pts Correct

- 1 pt Small mistake
- 2 pts Assuming order
- 2 pts Treat it as the case with replacement
- 6 pts No attempt.
- 5 pts Wrong answer
- 2 pts Missing necessary derivation
- 2 pts consider order but miss the order across defective and good ones

### Question 4

#### Indefinite coin tosses

5.5 / 6 pts

- 0 pts Correct
- 1 pt 1(a) wrong formula
- 1 pt 1(b) didn't verify the pmf
- 0.5 pts 1(b) insufficient verification
- 1 pt 1(c) missing formula of expectation
- 1 pt 1(c) missing formula of variance
- 0.5 pts 1(c) wrong formula of expectation

✓ - 0.5 pts 1(c) wrong formula of variance

- 1 pt 2(a) wrong formula
- 1 pt 2(b) wrong formula of expectation
- 0.5 pts small mistake
- 1 pt Missing 2(a)
- 1 pt Missing 2(b)
- 6 pts No answer loaded / presented

Question assigned to the following page: [1](#)

### Homework 3

1a)  $S = \{HHHH, HHH T, HH T H, H T H H, T H H H, H H T T, H T T H, T T H H, \\ T H H T, T H T H, H T H T, H T T T, T T T H, T H T T, T T H T, T T T T\}$

1b)

a)  $X = \{0, 1, 2, 3, 4\}$

b)  $x=0: \bar{X} = \{TTTT\}$

$x=1: \bar{X} = \{HTTT, THTT, TTHT, TTT H\}$

$x=2: \bar{X} = \{HHTT, THHT, TT HH, HTTH, THTH, HTHT\}$

$x=3: \bar{X} = \{HHHT, HH TH, HT HH, TH HH\}$

$x=4: \bar{X} = \{HHHH\}$

c)  $F_{\bar{X}}(0) = 0.2^4 = 0.0016$

$F_{\bar{X}}(1) = F_{\bar{X}}(0) + P(\bar{X}=1) = 0.0016 + 4(0.8)(0.2)^3 = 0.0272$

$F_{\bar{X}}(2) = F_{\bar{X}}(1) + P(\bar{X}=2) = 0.0272 + 6(0.8)^2(0.2)^2 = 0.1808$

$F_{\bar{X}}(3) = F_{\bar{X}}(2) + P(\bar{X}=3) = 0.1808 + 4(0.8)^3(0.2) = 0.5904$

$F_{\bar{X}}(4) = F_{\bar{X}}(3) + P(\bar{X}=4) = 0.5904 + 0.8^4 = 1$

Question assigned to the following page: [1](#)

$$\begin{aligned}
 \text{d)} \quad P_X(0) &= P(X=0) = (0.2)^4 = 0.0016 \\
 P_X(1) &= P(X=1) = 4(0.8)(0.2)^3 = 0.0256 \\
 P_X(2) &= P(X=2) = 6(0.8)^2(0.2)^2 = 0.1536 \\
 P_X(3) &= P(X=3) = 4(0.8)^3(0.2) = 0.4096 \\
 P_X(4) &= P(X=4) = 0.8^4 = 0.4096
 \end{aligned}$$

1c)

$$\text{a)} \quad X = \{0, 1, 2, 3, 4\}$$

$$\text{b)} \quad x=0: X = \{HHHH\}$$

$$x=1: X = \{THHH, HTHH, HHHT, HHHH\}$$

$$x=2: X = \{TTHH, HTTH, HHTT, THHT, THTH, HTHT\}$$

$$x=3: X = \{TTTH, TTHT, THTT, HTTT\}$$

$$x=4: X = \{TTTT\}$$

c)

$$F_X(0) = (0.8)^4 = 0.4096$$

$$F_X(1) = F_X(0) + P(X=1) = 0.4096 + 4(0.2)(0.8)^3 = 0.8192$$

$$F_X(2) = F_X(1) + P(X=2) = 0.8192 + 6(0.2)^2(0.8)^2 = 0.9728$$

$$F_X(3) = F_X(2) + P(X=3) = 0.9728 + 4(0.2)^3(0.8) = 0.9984$$

$$F_X(4) = F_X(3) + P(X=4) = 0.9984 + (0.2)^4 = 1$$

Question assigned to the following page: [1](#)

$$d) P_X(0) = P(X=0) = (0.8)^4 = 0.4096$$

$$P_X(1) = P(X=1) = 4(0.2)(0.8)^3 = 0.4096$$

$$P_X(2) = P(X=2) = 6(0.2)^2(0.8)^2 = 0.1536$$

$$P_X(3) = P(X=3) = 4(0.2)^3(0.8) = 0.0256$$

$$P_X(4) = P(X=4) = (0.2)^4 = 0.0016$$

1d)

$$a) X = \begin{matrix} \text{NO} & \text{YES} \\ \{0, 1, 2, 3, 4\} \end{matrix}$$

$$b) x = \text{NO} : X = \{TTTT, TTTH, TTHT, THTT, HTTT, TTHH, THTH, HHTT, HTTH, HTHT, THTH\}$$

$$x = \text{YES} : X = \{HHHT, HHTH, HTHH, THHH, HHHH\}$$

c)

$$F_X(\text{NO}) = F_X(2) \text{ from part a} = 0.1808$$

$$F_X(\text{YES}) = F_X(4) = F_X(\text{NO}) + \underbrace{P(X=3) + P(X=4)}_{\text{part a}}$$

$$= 0.1808 + 4(0.8)^3(0.2) + (0.8)^4 = 1$$



Questions assigned to the following page: [1](#) and [2](#)

$$d) P_X(\text{No}) = P_X(\{0, 1, 2\}) = 0.0016 + 0.0256 + 0.1536 = 0.1808$$

$$P_X(\text{YES}) = P_X(\{3, 4\}) = 0.4096 + 0.4096 = 0.8192$$

$$2a) S = \{ \overset{1}{(1,1)}, \overset{2}{(1,2)}, \overset{3}{(2,1)}, \overset{4}{(1,3)}, \overset{5}{(3,1)}, \overset{6}{(1,4)}, \overset{7}{(4,1)}, \overset{8}{(1,5)}, \overset{9}{(5,1)}, \overset{10}{(1,6)}, \overset{11}{(6,1)}, \\ \overset{12}{(2,2)}, \overset{13}{(2,3)}, \overset{14}{(3,2)}, \overset{15}{(2,4)}, \overset{16}{(4,2)}, \overset{17}{(2,5)}, \overset{18}{(5,2)}, \overset{19}{(2,6)}, \overset{20}{(6,2)}, \overset{21}{(3,3)}, \overset{22}{(3,4)}, \\ \overset{23}{(4,3)}, \overset{24}{(3,5)}, \overset{25}{(5,3)}, \overset{26}{(3,6)}, \overset{27}{(6,3)}, \overset{28}{(4,4)}, \overset{29}{(4,5)}, \overset{30}{(5,4)}, \overset{31}{(4,6)}, \overset{32}{(6,4)}, \overset{33}{(5,5)}, \\ \overset{34}{(5,6)}, \overset{35}{(6,5)}, \overset{36}{(6,6)} \}$$

2b)

$$a) X = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$$

$$b) x=2: X = \{ \overset{1}{(1,1)} \}$$

$$x=3: X = \{ \overset{2}{(1,2)}, \overset{3}{(2,1)} \}$$

$$x=4: X = \{ \overset{4}{(1,3)}, \overset{5}{(3,1)}, \overset{6}{(2,2)} \}$$

$$x=5: X = \{ \overset{7}{(1,4)}, \overset{8}{(4,1)}, \overset{9}{(2,3)}, \overset{10}{(3,2)} \}$$

$$x=6: X = \{ \overset{11}{(1,5)}, \overset{12}{(5,1)}, \overset{13}{(2,4)}, \overset{14}{(4,2)}, \overset{15}{(3,3)} \}$$

$$x=7: X = \{ \overset{16}{(1,6)}, \overset{17}{(6,1)}, \overset{18}{(2,5)}, \overset{19}{(5,2)}, \overset{20}{(3,4)}, \overset{21}{(4,3)} \}$$

$$x=8: X = \{ \overset{22}{(2,6)}, \overset{23}{(6,2)}, \overset{24}{(3,5)}, \overset{25}{(5,3)}, \overset{26}{(4,4)} \}$$

$$x=9: X = \{ \overset{27}{(3,6)}, \overset{28}{(6,3)}, \overset{29}{(4,5)}, \overset{30}{(5,4)} \}$$

$$x=10: X = \{ \overset{31}{(4,6)}, \overset{32}{(6,4)}, \overset{33}{(5,5)} \}$$

$$x=11: X = \{ \overset{34}{(5,6)}, \overset{35}{(6,5)} \}$$

$$x=12: X = \{ (6,6) \}$$

Question assigned to the following page: [2](#)

c

$$F_X(2) = \frac{1}{36}$$

$$F_X(3) = F_X(2) + P(X=3) = \frac{1}{36} + \frac{2}{36} = \frac{3}{36}$$

$$F_X(4) = F_X(3) + P(X=4) = \frac{3}{36} + \frac{3}{36} = \frac{6}{36}$$

$$F_X(5) = F_X(4) + P(X=5) = \frac{6}{36} + \frac{4}{36} = \frac{10}{36}$$

$$F_X(6) = F_X(5) + P(X=6) = \frac{10}{36} + \frac{5}{36} = \frac{15}{36}$$

$$F_X(7) = F_X(6) + P(X=7) = \frac{15}{36} + \frac{6}{36} = \frac{21}{36}$$

$$F_X(8) = F_X(7) + P(X=8) = \frac{21}{36} + \frac{5}{36} = \frac{26}{36}$$

$$F_X(9) = F_X(8) + P(X=9) = \frac{26}{36} + \frac{4}{36} = \frac{30}{36}$$

$$F_X(10) = F_X(9) + P(X=10) = \frac{30}{36} + \frac{3}{36} = \frac{33}{36}$$

$$F_X(11) = F_X(10) + P(X=11) = \frac{33}{36} + \frac{2}{36} = \frac{35}{36}$$

$$F_X(12) = F_X(11) + P(X=12) = \frac{35}{36} + \frac{1}{36} = 1$$

d

$$P_X(2) = \frac{1}{36}$$

$$P_X(3) = \frac{2}{36}$$

$$P_X(4) = \frac{3}{36}$$

$$P_X(5) = \frac{4}{36}$$

$$P_X(6) = \frac{5}{36}$$

$$P_X(7) = \frac{6}{36}$$

$$P_X(8) = \frac{5}{36}$$

$$P_X(9) = \frac{4}{36}$$

$$P_X(10) = \frac{3}{36}$$

$$P_X(11) = \frac{2}{36}$$

$$P_X(12) = \frac{1}{36}$$

Question assigned to the following page: [2](#)

2c)

a)  $X = \{1, 2, 3, 4, 5, 6\}$

b)

$$X=1: \underline{X} = \{(1,1)\}$$

$$X=2: \underline{X} = \{(1,2), (2,1), (2,2)\}$$

$$X=3: \underline{X} = \{(1,3), (3,1), (2,3), (3,2), (3,3)\}$$

$$X=4: \underline{X} = \{(1,4), (4,1), (2,4), (4,2), (3,4), (4,3), (4,4)\}$$

$$X=5: \underline{X} = \{(1,5), (5,1), (2,5), (5,2), (3,5), (5,3), (4,5), (5,4), (5,5)\}$$

$$X=6: \underline{X} = \{(1,6), (6,1), (2,6), (6,2), (3,6), (6,3), (4,6), (6,4), (5,6), (6,5), (6,6)\}$$

c)

$$F_{\underline{X}}(1) = P(\underline{X} = 1) = \frac{1}{36}$$

$$F_{\underline{X}}(2) = F_{\underline{X}}(1) + P(\underline{X} = 2) = \frac{1}{36} + \frac{3}{36} = \frac{4}{36}$$

$$F_{\underline{X}}(3) = F_{\underline{X}}(2) + P(\underline{X} = 3) = \frac{4}{36} + \frac{5}{36} = \frac{9}{36}$$

$$F_{\underline{X}}(4) = F_{\underline{X}}(3) + P(\underline{X} = 4) = \frac{9}{36} + \frac{7}{36} = \frac{16}{36}$$

$$F_{\underline{X}}(5) = F_{\underline{X}}(4) + P(\underline{X} = 5) = \frac{16}{36} + \frac{9}{36} = \frac{25}{36}$$

$$F_{\underline{X}}(6) = F_{\underline{X}}(5) + P(\underline{X} = 6) = \frac{25}{36} + \frac{11}{36} = 1$$

Question assigned to the following page: [2](#)

d)

$$P_X(1) = \frac{1}{36} \quad P_X(4) = \frac{7}{36}$$

$$P_X(2) = \frac{3}{36} \quad P_X(5) = \frac{9}{36}$$

$$P_X(3) = \frac{5}{36} \quad P_X(6) = \frac{11}{36}$$

2d)

$$a) X = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$$

b)

$$x = -5 : X = \{(1, 6)\}$$

$$x = -4 : X = \{(2, 6), (1, 5)\}$$

$$x = -3 : X = \{(3, 6), (2, 5), (1, 4)\}$$

$$x = -2 : X = \{(4, 6), (3, 5), (2, 4), (1, 3)\}$$

$$x = -1 : X = \{(5, 6), (4, 5), (3, 4), (2, 3), (1, 2)\}$$

$$x = 0 : X = \{(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)\}$$

$$x = 1 : X = \{(6, 5), (5, 4), (4, 3), (3, 2), (2, 1)\}$$

$$x = 2 : X = \{(6, 4), (5, 3), (4, 2), (3, 1)\}$$

$$x = 3 : X = \{(6, 3), (5, 2), (4, 1)\}$$

$$x = 4 : X = \{(6, 2), (5, 1)\}$$

$$x = 5 : X = \{(6, 1)\}$$



Question assigned to the following page: [2](#)

c

$$F_X(-5) = P(X \leq -5) = \frac{1}{36}$$

$$F_X(-4) = F_X(-5) + P(X = -4) = \frac{1}{36} + \frac{2}{36} = \frac{3}{36}$$

$$F_X(-3) = F_X(-4) + P(X = -3) = \frac{3}{36} + \frac{3}{36} = \frac{6}{36}$$

$$F_X(-2) = F_X(-3) + P(X = -2) = \frac{6}{36} + \frac{4}{36} = \frac{10}{36}$$

$$F_X(-1) = F_X(-2) + P(X = -1) = \frac{10}{36} + \frac{5}{36} = \frac{15}{36}$$

$$F_X(0) = F_X(-1) + P(X = 0) = \frac{15}{36} + \frac{6}{36} = \frac{21}{36}$$

$$F_X(1) = F_X(0) + P(X = 1) = \frac{21}{36} + \frac{5}{36} = \frac{26}{36}$$

$$F_X(2) = F_X(1) + P(X = 2) = \frac{26}{36} + \frac{4}{36} = \frac{30}{36}$$

$$F_X(3) = F_X(2) + P(X = 3) = \frac{30}{36} + \frac{3}{36} = \frac{33}{36}$$

$$F_X(4) = F_X(3) + P(X = 4) = \frac{33}{36} + \frac{2}{36} = \frac{35}{36}$$

$$F_X(5) = F_X(4) + P(X = 5) = \frac{35}{36} + \frac{1}{36} = 1$$

d

$$P_X(-5) = \frac{1}{36}$$

$$P_X(1) = \frac{5}{36}$$

$$P_X(-4) = \frac{2}{36}$$

$$P_X(2) = \frac{4}{36}$$

$$P_X(-3) = \frac{3}{36}$$

$$P_X(3) = \frac{3}{36}$$

$$P_X(-2) = \frac{4}{36}$$

$$P_X(4) = \frac{2}{36}$$

$$P_X(-1) = \frac{5}{36}$$

$$P_X(5) = \frac{1}{36}$$

$$P_X(0) = \frac{6}{36}$$

Question assigned to the following page: [3](#)

$$3) X = \{0, 1, 2, 3, 4\}$$

$$\binom{30}{4} = \frac{30!}{26!4!} = 27405$$

$$\binom{5}{k} = \text{choose defect}$$

$$\binom{25}{4-k} = \text{choose not defect}$$

$$P = \frac{\binom{5}{k} \binom{25}{4-k}}{27405}$$

$$P_X(0) = \frac{\binom{5}{0} \binom{25}{4}}{27405} \approx 0.4616$$

$$P_X(1) = \frac{\binom{5}{1} \binom{25}{3}}{27405} \approx 0.4196$$

$$P_X(2) = \frac{\binom{5}{2} \binom{25}{2}}{27405} \approx 0.1095$$

$$P_X(3) = \frac{\binom{5}{3} \binom{25}{1}}{27405} \approx 0.0091$$

$$P_X(4) = \frac{\binom{5}{4} \binom{25}{0}}{27405} \approx 0.0002$$

Question assigned to the following page: [4](#)

4)

$$1a) P_X(n) = (0.2)^{n-1} (0.8) \quad n \in \mathbb{N}$$

$$1b) 1 = \sum_{n=1}^{\infty} P_X(n) = \sum_{n=1}^{\infty} (0.2)^{n-1} (0.8) = \frac{1}{1-0.2} (0.8) = 1$$

$$1c) E(X) = 1(0.8) + 2(0.2)(0.8) + 3(0.2)^2(0.8) + \dots$$

$$= \sum_{n=1}^{\infty} n P_X(n) = \sum_{n=1}^{\infty} n (0.2)^{n-1} (0.8)$$

$$= 0.8 \sum_{n=1}^{\infty} n (0.2)^{n-1} = 0.8 \left( \frac{1}{(1-0.2)^2} \right)$$

$$= 0.8 \left( \frac{1}{0.8^2} \right) = \frac{1}{0.8} = 1.25 = \mu_X$$

$$V(X) = \sum_{n=1}^{\infty} n^2 (0.8) (0.2)^{n-1}$$

$$= 0.8 \sum_{n=1}^{\infty} n^2 (0.2)^{n-1} = \frac{1+0.2}{(1-0.2)^3} = 2.343$$

idk

Question assigned to the following page: [4](#)

2)

a)  $p_X(n) = 0.2^n \cdot 0.8$

b)  $E(X) = \sum_{n=1}^{\infty} 3^n (0.2)^{n-1} (0.8)$

$$= 0.8 \sum_{n=1}^{\infty} 3^n (0.2)^{n-1} = 0.8(7.5) = 6$$