Stat 400. Problem Set 6. Due 10/15/24

Examples of Continuous Random Variables

Problem 1. Let X denote the random variable that calculates the time until a bulb fails (measured in years). As per the manufacturer, X has the exponential distribution with lambda parameter, $\lambda = \frac{1}{3}$.

(a) Find $p = P(X \geq 5)$.

(b) Suppose we randomly pick (with replacement) 10 bulbs from the manufacturing facility, what is the probability that at least 7 of them will last for at least 5 years?

(c) Suppose we randomly pick (with replacement) 10 bulbs from the manufacturing facility. Each of these bulbs will be classified as acceptable/unacceptable, with $P(\text{acceptable}) = p \in (0,1)$. Let N count the number of acceptable bulbs in a sample of 10 bulbs. Calculate $P(N \geq 7)$ as a function of p, say $A(p)$.

(d) Find $p_0 \in (0,1)$ such that $A(p_0) = .95$. How would you interpret this number?

(e) Given the p_0 you found in (d), find x_0 such that $P(X \geq x_0) = p_0$. How would you interpret this number?

Problem 2. Suppose $X \sim \text{Exp}(\lambda)$. Show that

$$P(X > t + s) = P(X > t)P(X > s).$$

Problem 3. There are three types of 4-sided die, with the following probability distributions

Die 1

x	1	2	3	4
$P(x)$	.7	.1	.1	.1

Die 2

x	1	2	3	4
$P(x)$	.2	.3	.3	.2

Die 3

x	1	2	3	4
$P(x)$	.1	.1	.2	.6

A bag contains 3 identical Die 1, 4 identical Die 2, and 3 identical Die 3.

Suppose an experiment consists of randomly picking a die from this bag, and then rolling it. Let X be the random variable taking values 1, 2 or 3 depending on which of Die 1, Die 2, or Die 3 is picked from the bag in Step 1. Let Y be the random variable that keeps track of the outcome of the die roll.

(a) Draw a labelled tree diagram for this experiment.

(b) Calculate the joint distribution table for X and Y.

Problem 4. Let X denote the number Cars sold during a particular month by certain dealership. The pmf of X is given by

x	0	1	2	3	4	5
$P(x)$	.1	.2	.3	.2	.1	.1

70% of all car buyers at the dealership also purchase an extended warranty. Let Y denote the number of car buyers that also buy an extended warranty.

(a) Calculate the joint distribution table for X and Y.
(*Hint*: To calculate $P(X = x, Y = y)$, use the fact that $P(Y = y|X = x)$ will have a binonmial distribution with $p = 0.7$ and $n = x$.)

(b) Calculate the probability of the event where every car buyer also buys extended warranty. That is, calculate $P(X = Y)$.