Homework 6

Graded

1 Day, 19 Minutes Late

Student

Jacob Hauptman

Total Points

21 / 24 pts

Question 1

Exponential probabilities — **5** / 6 pts

1.1 (no title) — **1** / 1 pt

- ✔ **– 0 pts** Correct
- **– 0.5 pts** small mistake

1.2 (no title) — **2** / 2 pts

- ✔ **– 0 pts** Correct
- **– 0.5 pts** small mistake
- **– 1 pt** wrong formula

1.3 (no title) — **1** / 1 pt

- ✔ **– 0 pts** Correct
- **– 0.5 pts** small mistake
- **– 1 pt** wrong formula

1.4 (no title) — **1** / 1 pt

- ✔ **– 0 pts** Correct
- **– 0.5 pts** small mistake / correct formula but no / wrong final result of p_0
- **– 1 pt** Incorrect

1.5 (no title) — **0** / 1 pt

- **– 0 pts** Correct, or wrong but consistent with 1.4
- **– 0.5 pts** small mistake / incomplete
- ✔ **– 1 pt** Incorrect / no math

1 For part e, you should solve $e^{-x_0\lambda} = p_0$

Question 2

Exponential property

6 / 6 pts

✔ **– 0 pts** Correct

– 2 pts Incomplete / no necessary proof

Question 3

Joint distribution dice

5 / 6 pts

3.1 **(no title)**

3 / 3 pts

✔ **– 0 pts** Correct

– 1 pt small mistake

– 1 pt No tree

3.2 **(no title)**

2 / 3 pts

– 0 pts Correct, or wrong numbers but consistent with 3.1

✔ **– 1 pt** No table.

– 1 pt wrong table format

– 1 pt didn't compute joint probabilities

– 0.5 pts computation error

Question 4

Joint distribution cars

5 / 6 pts

4.1 **(no title)**

2 / 3 pts

– 0 pts Correct

– 0.5 pts small mistake / computation error

– 1 pt didn't compute joint probability in table

✔ **– 1 pt** Probabilities w.o/ table or incomplete table

– 2 pts Wrong table / no table

4.2 **(no title)**

3 / 3 pts

✔ **– 0 pts** Correct

– 0.5 pts small mistake

– 1 pt correct idea and formula but wrong numbers plugged in

– 1.5 pts wrong formula

– 2 pts Missing an important part

Questions assigned to the following page: 1.1 and 1.2

STAT400 Homework 6

Jacob Hauptman

1)

a $F_X(x;\lambda) = \begin{cases} 1-e^{-\lambda x} & x \geq 0 \\ 0 & \text{o.w.} \end{cases}$

$$P(X \geq t) = \int_t^\infty \lambda e^{-\lambda x}\,dx = \lambda\left(\frac{e^{-\lambda x}}{-\lambda}\right)\Big|_t^\infty$$

$$= -e^{-\lambda x}\Big|_t^\infty = 0 - (-e^{-\lambda t}) = e^{-\lambda t}$$

$$P(X \geq t=5) = e^{-\frac{5}{3}} \approx 0.189$$

b We already have $P(X \geq 5) \approx 0.189$.

Now $P(Y \geq 7) = \sum_{k=7}^{10} \binom{10}{k} p^k (1-p)^{10-k}$

$$\approx \sum_{k=7}^{10} \binom{10}{k} (0.189)^k (0.811)^{10-k}$$

$$= \binom{10}{7}(0.189)^7(0.811)^3 + \binom{10}{8}(0.189)^8(0.811)^2$$

$$+ \binom{10}{9}(0.189)^9(0.811)^1 + \binom{10}{10}(0.189)^{10}(1)$$

$$= 0.0055 + 0.000048 + 0.00000025 + 0.000000058$$

$$\approx 0.0006$$

Questions assigned to the following page: 1.3, 1.4, and 1.5

c Same as b. Bernoulli is a single trial.

$$A(p)=\sum_{k=7}^{10}\binom{10}{k}p^k(1-p)^{10-k}$$

$$P(N\geq 7)=0.0006$$

d Graphing

$$120P_a^7(1-P_a)^3+15P_a^8(1-P_a)^2+10P_a^9(1-P_a)+P_a^{10}=0.95$$

$$P_a=0.84997$$

95% or more of getting at least 7 acceptable light bulbs.

e

$$P(X\geq X_a)=0.84997$$

$$1-P(X<X_a)=0.84997$$

$$P(X\geq X_a)=1-P(X<X_a)=\sum_{k=0}^{X_a-1}\binom{10}{k}P_a^k(1-P_a)^{10-k}$$

Using calculator $X_a=8$.

Question assigned to the following page: 2

2

$$P(X>t+s) = \int_{t+s}^{\infty} \lambda e^{-\lambda x}\,dx = \lambda\left(\frac{e^{-\lambda x}}{-\lambda}\right)\Big|_{t+s}^{\infty}$$

$$= -e^{\lambda x}\Big|_{t+s}^{\infty} = 0 - (-e^{(t+s)x}) = e^{tx+sx}$$

$$= e^{tx}e^{sx}.$$

$$P(X>t)P(X>s) = \int_{t}^{\infty} \lambda e^{-\lambda x}\,dx \int_{s}^{\infty} \lambda e^{-\lambda x}\,dx$$

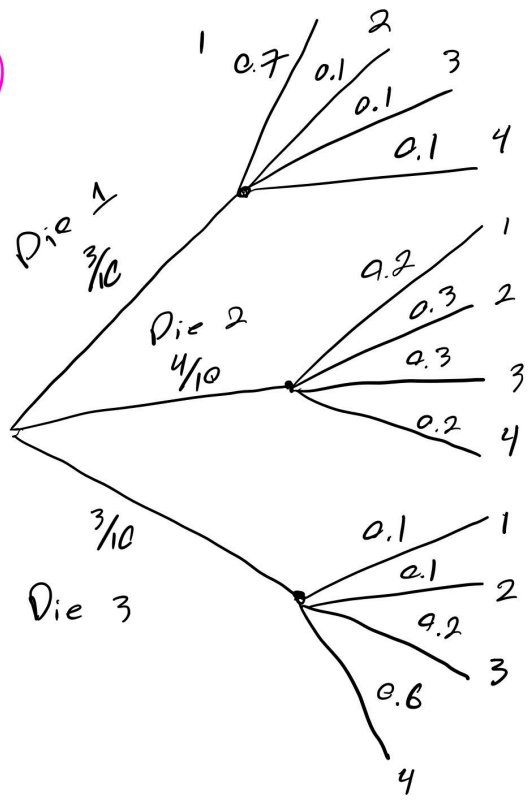
$$= -e^{\lambda x}\Big|_{t}^{\infty} - e^{\lambda x}\Big|_{s}^{\infty} = e^{tx}e^{sx}.$$

So $P(X>t+s) = P(X>s)P(X>t)$

Questions assigned to the following page: 3.1 and 3.2

3)

a



b

Die 1 $(X=1)$

$Y=1$: $P(X,Y)=P(X)P(Y|X)=\frac{3}{10}(0.7)=0.21$

$Y=2$: $P(X,Y)=P(X)P(Y|X)=\frac{3}{10}(0.1)=0.03$

$Y=3$: $P(X,Y)=P(X)P(Y|X)=\frac{3}{10}(0.1)=0.03$

$Y=4$: $P(X,Y)=P(X)P(Y|X)=\frac{3}{10}(0.1)=0.03$

Question assigned to the following page: 3.2

Die 2 ($X=2$)

$y=1: P(X,y)=P(X)P(y|X)=\frac{4}{10}(0.2)=0.08$

$y=2: P(X,y)=P(X)P(y|X)=\frac{4}{10}(0.3)=0.12$

$y=3: P(X,y)=P(X)P(y|X)=\frac{4}{10}(0.3)=0.12$

$y=4: P(X,y)=P(X)P(y|X)=\frac{4}{10}(0.2)=0.08$

Die 3 ($X=3$)

$y=1: P(X,y)=P(X)P(y|X)=\frac{3}{10}(0.1)=0.03$

$y=1: P(X,y)=P(X)P(y|X)=\frac{3}{10}(0.1)=0.03$

$y=1: P(X,y)=P(X)P(y|X)=\frac{3}{10}(0.2)=0.06$

$y=1: P(X,y)=P(X)P(y|X)=\frac{3}{10}(0.6)=0.18$

Question assigned to the following page: 4.1

4

a $P(Y=y|X=x) = \binom{x}{y}(0.7)^{y}(0.3)^{x-y}$

$x=0$

$y=0 : P(x|y) = P(x)P(Y|x) = 0.1(1) = 0.1$

$x=1$

$y=0 : P(x|y) = P(x)P(Y|x) = 0.2(0.3) = 0.06$

$y=1 : P(x|y) = P(x)P(y|x) = 0.2(0.7) = 0.14$

$x=2$

$y=0 : P(x|y) = P(x)P(y|x) = 0.3(0.09) = 0.027$

$y=1 : P(x|y) = P(x)P(y|x) = 0.3(0.42) = 0.126$

$y=2 : P(x|y) = P(x)P(y|x) = 0.3(0.49) = 0.147$

Question assigned to the following page: 4.1

$x=3$

$y=0: P(x|y)=P(x)P(y|x)=0.4(0.027)=0.0054$

$y=1: P(x|y)=P(x)P(y|x)=0.4(0.189)=0.0378$

$y=2: P(x|y)=P(x)P(y|x)=0.4(0.441)=0.0882$

$y=3: P(x|y)=P(x)P(y|x)=0.4(0.343)=0.0686$

$x=4$

$y=0: p(x|y)=P(x)P(y|x)=0.1(0.0081)=0.0008$

$y=1: p(x|y)=P(x)P(y|x)=0.1(0.1089)=0.0109$

$y=2: p(x|y)=P(x)P(y|x)=0.1(0.2646)=0.0265$

$y=3: p(x|y)=P(x)P(y|x)=0.1(0.2646)=0.0265$

$y=4: p(x|y)=P(x)P(y|x)=0.1(0.2401)=0.024$

Questions assigned to the following page: 4.2 and 4.1

$x = 5$

$y=0 : P(x|y) = p(x)p(y|x) = 0.1(0.0024) = 0.0002$

$y=1 : P(x|y) = p(x)p(y|x) = 0.1(0.0328) = 0.0033$

$y=2 : P(x|y) = p(x)p(y|x) = 0.1(0.1323) = 0.0132$

$y=3 : P(x|y) = p(x)p(y|x) = 0.1(0.2263) = 0.0226$

$y=4 : P(x|y) = p(x)p(y|x) = 0.1(0.2637) = 0.0264$

$y=5 : P(x|y) = p(x)p(y|x) = 0.1(0.1681) = 0.0168$

b Add up all $P(x=y)$

$0.1+0.14+0.147+0.0686+0.024+0.0168 = 0.496$

No questions assigned to the following page.