Homework 5

Graded

Student

Jacob Hauptman

Total Points

21.5 / 24 pts

Question 1

(no title) **4.5** / 5 pts

1.1 **(no title)** **2** / 2 pts

✔ **– 0 pts** Correct

1.2 **(no title)** **0.5** / 1 pt

– 0 pts Correct

✔ **– 0.5 pts** missing something

– 1 pt wrong plot

1 what about negative x's?

1.3 **(no title)** **2** / 2 pts

✔ **– 0 pts** attempted

– 2 pts not attempted

Question 2

(no title) **7.5** / 9 pts

2.1 **(no title)** **2** / 2 pts

✔ **– 0 pts** Correct

– 0.5 pts small mistake

– 1.5 pts No plot/wrong plot

– 2 pts Incorrect

2.2 **(no title)** **2** / 2 pts

✔ **– 0 pts** Correct

– 0.5 pts small mistake

– 2 pts incorrect

2.3 **(no title)** **1.5** / 2 pts

– 0 pts Correct

– 0.5 pts small mistake in finding G

✔ **– 0.5 pts** the wrong / no answer for whether X is a continuous random variable or not.

– 1 pt wrong G(x)

– 1.5 pts No work for G(x)

– 2 pts Incorrect

2.4 **(no title)** **1** / 1 pt

✔ **– 0 pts** Correct

– 1 pt Incorrect

2.5 **(no title)** **1** / 2 pts

– 0 pts Correct

✔ **– 1 pt** wrong $n_{0.2}$

– 2 pts both wrong

Question 3

(no title) **4** / 4 pts

3.1 **(no title)** **1** / 1 pt

✔ **– 0 pts** Correct

– 0.5 pts not checking the integral condition

– 0.5 pts not checking the nonnegativity condition

– 0.5 pts small mistake

3.2 **(no title)** **1** / 1 pt

✔ **– 0 pts** Correct

– 0.5 pts small mistake s.t. wrong / no expectation or no / wrong beta's range

– 1 pt Missing / Incorrect

2 $\frac{\alpha\beta}{\beta-1}$

3.3 **(no title)** **1** / 1 pt

✔ **– 0 pts** attempted

– 1 pt not attempted

3.4 **(no title)** **1** / 1 pt

✔ **– 0 pts** attempted

– 1 pt not attempted

Question 4

(no title) **2.5** / 3 pts

4.1 **(no title)** **1** / 1 pt

✔ **– 0 pts** Correct

– 1 pt wrong

4.2 **(no title)** **1** / 1 pt

✔ **– 0 pts** Correct

– 0.5 pts lower / upper bound is wrong

– 1 pt both bounds are wrong

4.3 **(no title)** **0.5** / 1 pt

– 0 pts Correct

✔ **– 0.5 pts** $P(100 \leq X \leq 999)$ instead of $P(100 \leq X < 1000)$

– 1 pt Incorrect

– 0.5 pts small mistake

Question 5

(no title) **3** / 3 pts

5.1 **(no title)** **1** / 1 pt

✔ **– 0 pts** Correct

5.2 **(no title)** **1** / 1 pt

✔ **– 0 pts** Correct

– 0.5 pts small mistake

– 1 pt Incorrect

5.3 **(no title)** **1** / 1 pt

✔ **– 0 pts** Correct

Questions assigned to the following page: 1.2, 1.3, and 1.1

STAT 400 - Homework 5

Jacob Hauptman

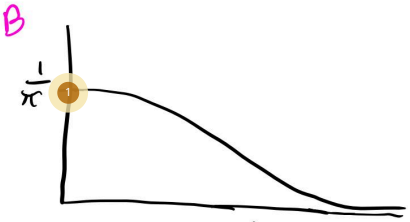
1)

A $\lim_{t\to\infty}\int_{-t}^{t}\frac{1}{1+x^2}dx = \lim_{t\to\infty}\arctan(x)\Big|_{-t}^{t} = \lim_{t\to\infty}\arctan(t)-\arctan(-t)$

$= \frac{\pi}{2}-(-\frac{\pi}{2}) = \pi$. The integral must equal 1 so $c=\frac{1}{\pi}$

and $g(x)=\frac{1}{\pi}f(x)$.

B



C $E(X) = \lim_{t\to\infty}\int_{-t}^{t}\frac{x}{1+x^2}dx = \lim_{t\to\infty}\frac{1}{2}\ln|1+x^2|\Big|_{-t}^{t} = DNE$

Questions assigned to the following page: 2.5, 2.4, 2.1, 2.2, and 2.3

2)

A

No.

B $x \in [0,1]$ $\int_0^1 4x\,dx = 2x^2\Big|_0^1 = 2$

$x \in (1,3]$ $\lim_{t\to 1^+} \int_t^3 4+2x\,dx = \lim_{t\to 1^+} 4x+x^2\Big|_t^3 = 21-5 = 16$

So $\int_0^3 f(x)\,dx = 18$ therefore $c = \frac{1}{18}$

$g(x) = \frac{1}{18} f(x)$

C

$$G(x) = \begin{cases} 0 & x<0 \\ \frac{x^2}{9} & 0 \le x \le 1 \\ \frac{4x+x^2-5}{18} + \frac{1}{9} & 1 < x \le 3 \\ 1 & x > 3 \end{cases}$$

D False, just has to be equal to 1.

E $G(n_p) = p \Rightarrow \frac{n_p^2}{9} = \frac{1}{5} \Rightarrow n_p = \sqrt{\frac{9}{5}}$ $\left(G(0) = 0 \le p \le G(1) = \frac{1}{9}\right)$

$G(n_p) = p \Rightarrow \frac{4n_p + n_p^2 - 5}{18} + \frac{1}{9} = \frac{1}{2} \Rightarrow 4n_p + n_p^2 = 12 \Rightarrow n_p^2 + 4n_p - 12 = 0$

so $n_p = -2 \pm 4 = 2$ $(-6 < 0)$ $(G(1) < p \le G(3))$

Questions assigned to the following page: 3.4, 3.1, 3.2, and 3.3

3)

A $F(x) = \int_{\alpha}^{\alpha} \frac{\beta \alpha^{\beta}}{x^{\beta+1}} dx = \beta \alpha^{\beta} \int_{\alpha}^{\infty} x^{-\beta-1} dx = \beta \alpha^{\beta} \frac{x^{-\beta}}{-\beta} \Big|_{\alpha}^{\infty}$

$= -\alpha^{\beta}[0 - \alpha^{-\beta}] = -\alpha^{\beta}(-\alpha^{-\beta}) = \alpha^{0} = 1$

B $E(x) = \beta \alpha^{\beta} \int_{\alpha}^{\infty} \frac{x}{x^{\beta+1}} dx = \beta \alpha^{\beta} \int_{\alpha}^{\infty} x^{-\beta} dx = \beta \alpha^{\beta} \left[\frac{x^{-\beta+1}}{-\beta+1} \right] \Big|_{\alpha}^{\infty}$

β has to be greater than 1 to be finite.

$= \frac{\beta \alpha^{\beta}}{1-\beta} [0 - \alpha^{1-\beta}] = \frac{\beta \alpha}{1-\beta}$

C $V(x) = E(x^2) - E(x)^2$

$E(x)^2 = \frac{\beta^2 \alpha^2}{(1-\beta)^2}$, $\beta > 1$

$E(x^2) = \beta \alpha^{\beta} \int_{\alpha}^{\infty} \frac{x^2}{x^{\beta+1}} dx = \beta \alpha^{\beta} \int_{\alpha}^{\infty} x^{-\beta+1} dx = \beta \alpha^{\beta} \left[\frac{x^{2-\beta}}{2-\beta} \right] \Big|_{\alpha}^{\infty}$

$= \frac{\beta \alpha^{\beta}}{2-\beta} [0 - \alpha^{2-\beta}] = \frac{-\beta \alpha^2}{2-\beta}$, $\beta > 2$ for finite.

$V(x) = -\frac{\beta \alpha^2}{2-\beta} - \frac{\beta^2 \alpha^2}{(1-\beta)^2}$, $\beta > 2$

D $X = \sqrt[3]{Y}$

So $F(\sqrt[3]{Y}) = \frac{\beta \alpha^{\beta}}{y^{\frac{\beta+1}{3}}} \left(\frac{1}{3} y^{-\frac{2}{3}} \right)$

Questions assigned to the following page: 5.1, 4.1, 4.2, 5.2, 4.3, and 5.3

4)

A $f(x) = \begin{cases} \frac{1}{1000} & 0 \le x \le 1000 \\ 0 & \text{otherwise} \end{cases}$

I'm assuming 0 and 100 inclusive?

B $P(2 \le x \le 90) = \frac{1}{1000}\int_2^{90} dx = \frac{x}{1000}\Big|_2^{90} = \frac{88}{1000}$

C $P(100 \le x \le 999) = \frac{x}{1000}\Big|_{100}^{999} = \frac{899}{1000}$

5)

A $P(a \le x \le b) = P\left(\frac{a-\mu}{\sigma} \le Z \le \frac{b-\mu}{\sigma}\right)$

$= \Phi(1.67) - \Phi(-1) = 0.9525 - 0.1587 = 0.7938$

Used table for the values.

B Looking at table z-score is 1.96 and -1.96

$1.96 = \frac{70+c-70}{3} = \frac{c}{3} \Rightarrow c = 5.88$

C $P(67 \le x \le 75) = P(-1 \le z \le 1.67)$

$= \Phi(1.67) - \Phi(-1) = 0.9525 - 0.1587 = 0.7938$

Binomial Distribution $E(x) = np = 10(0.7938) = 7.938$