Homework 8

● Graded

6 Days, 17 Hours Late

Student

Jacob Hauptman

Total Points

24 / 24 pts

Question 1

(no title) **4** / 4 pts

✔ **– 0 pts** Correct

– 0.5 pts small mistake (no duplicate penalty when repeated mistake occurs)

– 0.5 pts d. didn't draw the conclusion / wrong conclusion

– 1 pt e. missing

Question 2

(no title) **4** / 4 pts

✔ **– 0 pts** Correct

– 0.5 pts small mistakes (no duplicate penalization for repeated mistakes)

– 0.5 pts c. Partially incorrect formula such as missing covariance or other mistakes

– 1 pt c. incorrect

– 1 pt d. incorrect

Question 3

(no title) **4** / 4 pts

✔ **– 0 pts** Correct

Question 4

(no title) **4** / 4 pts

✔ **– 0 pts** Correct

– 0.5 pts small mistakes

– 0.5 pts negative variance

Question 5

(no title) **4** / 4 pts

✔ **– 0 pts** Complete

Question 6

(no title) **4** / 4 pts

✔ **– 0 pts** Correct

– 0.5 pts didn't use chain rule / wrong derivation

– 0.5 pts wrong transformation

– 1.5 pts a. incorrect

Question assigned to the following page: 1

1)

·a

$Z = \{1,2,3,4,5,6\}$

$E(Z) = \frac{1}{6} + \frac{2}{6} + \frac{3}{6} + \frac{4}{6} + \frac{5}{6} + 1 = 1 + \frac{15}{6} = 3.5$

·b

$Y = \{0,1,2,3,4,5\}$

0: Doesn't matter

1: (1,2), (2,1), (2,3), (3,2), (3,4), (4,3), (4,5), (5,4), (5,6), (6,5)

2: (1,3), (3,1), (2,4), (4,2), (3,5), (5,3), (4,6), (6,4)

3: (1,4), (4,1), (2,5), (5,2), (3,6), (6,3)

4: (1,5), (5,1), (2,6), (6,2)

5: (1,6), (6,1)

$E(Y) = \frac{10}{36} + 2\left(\frac{8}{36}\right) + 3\left(\frac{6}{36}\right) + 4\left(\frac{4}{36}\right) + 5\left(\frac{2}{36}\right)$

$= \frac{10+16+18+16+10}{36} = \frac{70}{36} = 1.9\overline{4}$

·c

$X = \{-1,0,1,2,3,4\}$

$E(X) = -\frac{1}{6} + 0 + \frac{1}{6} + \frac{2}{6} + \frac{3}{6} + \frac{4}{6} = \frac{-1+1+2+3+4}{6} = \frac{9}{6} = 1.5$

Questions assigned to the following page: 1 and 2

•d

$XZ(a,b) = a(a-2)$

$XZ = \{-1, 0, 3, 8, 15, 24\}$

$E(XZ) = -\frac{1}{6} + 0 + \frac{3}{6} + \frac{8}{6} + \frac{15}{6} + \frac{24}{6} = \frac{-1+3+8+15+24}{6} = \frac{49}{6} = 8.1\overline{6}$

$E(X)E(Z) = 1.5(3.5) = 5.25$

So $E(XZ) \neq E(X)E(Z)$

•e

$E(100Z - xY) = 100E(Z) - xE(Y) = 350 - 1.9\bar{4}x$

2)

•a Homework 7 is not graded so I will assume its values are right.

(i) $E(X) = 1.008$

(ii) $E(Y) = 1$

(iii) $\sigma_x = 0.8717$

(iv) $\sigma_y = 1.249$

•b

$E(2X - Y) = 2E(X) - E(Y) = 2(1.008) - 1 = 1.016$

•c

$Z = 2X - Y$ where $a = 2$ and $y = -1$.

$V(Z) = a^2V(X) + b^2V(Y) + 2ab\,Cov(X,Y)$

$= 4(0.8717)^2 + (1.249)^2 - 4(0.624)$ Cov from homework 7

$= 2.103$

So $\sigma_z = \sqrt{2.103} = 1.45$

Questions assigned to the following page: 2, 3, and 4

·d

$E(Y)$ was our expected earnings, so $E(100Y) = 100E(Y) = 100$.

3)

$$E(Y) = \sum_{y \in Y} y P(Y = y)$$

So...

$$E(Y) = \sum_{x \in X} (5x-2) P(5X-2 = 5x-2) = \sum_{x \in X} (5x-2) P(X = x)$$

$$= \sum_{x \in X} 5x P(X=x) - \sum_{x \in X} 2 P(X=x) = 5\sum_{x \in X} x P(X=x) - 2\sum_{x \in X} P(X=x)$$

Since the sum of P over all its elements is 1, so $\sum_{x \in X} P(X=x) = 1$.

$$= 5E(X) - 2(1) = 5E(X) - 2$$

4)

·a

$$E(X) = \frac{1}{\lambda_X} = \frac{1}{2}$$

·b

$$E(Y) = \frac{1}{\lambda_Y} = \frac{1}{4}$$

·c

$$V(X) = \frac{1}{\lambda_X^2} = \frac{1}{4}$$

·d

$$V(Y) = \frac{1}{\lambda_Y^2} = \frac{1}{16}$$

·e

$$E(4X-3Y) = 4E(X) - 3E(Y) = 4\left(\tfrac{1}{2}\right) - 3\left(\tfrac{1}{4}\right) = \frac{5}{4} = 1.25$$

·f

$$V(Z) = a^2 V(X) + b^2 V(Y) + 2ab\,Cov(X,Y) = 16\left(\tfrac{1}{4}\right) + 9\left(\tfrac{1}{16}\right) + 0 \quad (\text{Independent})$$

$$= 4.5625$$

Questions assigned to the following page: 5 and 6

5)

•a

$$F_Y(Y) = P(Y \le y) = P(3X \le y) = P(X \le \tfrac{y}{3}) = F_x(\tfrac{y}{3})$$

$$f_y(y) = \frac{d}{dy} F_x(\tfrac{y}{3}) = \tfrac{1}{3} f_x(\tfrac{y}{3})$$

•b

$$E(Y) = \int_{-\infty}^{\infty} y \tfrac{1}{3} f_x(\tfrac{y}{3})\, dy \quad \text{Let } u = \tfrac{y}{3}, \text{ then } dx = 3du.$$

$$= \int_{-\infty}^{\infty} 3u f_x(u)\, du = 3\int_{-\infty}^{\infty} u f_x(u)\, du$$

$3E(X) = 3\int_{-\infty}^{\infty} x f_x(x)\, dx$. Can just rename u to x.

6)

•a

$$F_Y(Y) = P(Y \le y) = P(5X - 2 \le y) = P(X \le \tfrac{y+2}{5}) = F_x(\tfrac{y+2}{5})$$

$$f_y(Y) = \frac{d}{dy} F_x(\tfrac{y+2}{5}) = \tfrac{1}{5} f_x(\tfrac{y+2}{5})$$

•b

$$E(Y) = \int_{-\infty}^{\infty} \tfrac{1}{5} y f_x(\tfrac{y+2}{5})\, dy \quad \text{Let } u = \tfrac{y+2}{5}, \text{ then } dx = 5du$$

$$= \int_{-\infty}^{\infty} (u - \tfrac{2}{5}) f_x(u)(5)\, du = \int_{-\infty}^{\infty} (5u - 2) f_x(u)\, du = \int_{-\infty}^{\infty} 5u f_x(u) - 2 f_x(u)\, du$$

$5E(X) - 2 = \int_{-\infty}^{\infty} 5x f_x(x)\, dx - 2 \quad E(2) = 2$ and $\int_{-\infty}^{\infty} f_x(x)\, dx = 1$, so...

$$= \int_{-\infty}^{\infty} 5x f_x(x)\, dx - \int_{-\infty}^{\infty} 2 f_x(x)\, dx = \int_{-\infty}^{\infty} 5x f_x(x) - 2 f_x(x)\, dx$$

Rename u to x and then it's equal.