

Stat 400. Problem Set 9. Due 11/12/24
Random Samples and the Central Limit Theorem

Problem 1. Suppose the random variable X has the following distribution:

x	2	4	6
$P(x)$.4	.2	.4

- (a) Calculate $E(X)$ and $V(X)$.
- (b) Suppose we are sampling a random sample of size three. Calculate the sampling distribution, expected value, and variance of each of the following statistics:
 - (i) sample total
 - (ii) sample mean
 - (iii) sample max

Problem 2. Assume SAT scores are normally distributed with mean 1520 and standard deviation 320.

- (a) If one SAT score is randomly selected, find the probability that it is between 1200 and 1360.
- (b) If 16 SAT scores are randomly selected, find the probability that they have a mean between 1200 and 1360.
- (c) Why can the central limit theorem be applied?

Problem 3. Suppose we have a random sample of size 64 coming from the distribution:

x	1	2	3	4	5	6
$P(x)$.1	.1	.1	.1	.1	.5

Let \bar{X} denote the sample mean.

- (a) Use the central limit theorem to approximate the following probability:

$$P(|\bar{X} - 2|) > 1.$$

- (b) Why can the central limit theorem be applied?

Problem 4. A survey found that the American family generates an average of 17.2 pounds of glass garbage each year. Assume the standard deviation of the distribution is 2.5 pounds.

- (a) Find the probability that the mean of a sample of 55 families will be between 17 and 18 pounds.
- (b) Why can the central limit theorem be applied?

Problem 5. Suppose that the amount of change statistics students carry is exponentially distributed with a mean of \$0.84.

- (a) If one statistics student is randomly selected, what is the probability that they have between \$0.70 and \$.84?
- (b) If we randomly select 36 statistics students, what is the probability that the average amount they have is between \$0.70 and \$.84?
- (c) If we randomly select 100 statistics students, what is the probability that the average amount they have is between \$0.70 and \$.84?

Problem 6. Let X_i ($i \leq n$) denote whether or not a randomly (with replacement) selected individual approves of the job the President is doing ($X_i = 1$ for approval, $X_i = 0$ for disapproval). Suppose $P(X_i = 1) = .3$. Let $Y = X_1 + X_2 + \dots + X_n$.

- (a) What distribution does Y have? What are the mean and standard deviation?
- (b) How big of a sample do we need to take for the central limit theorem to apply?
- (c) Suppose we randomly sample 100 people. Using the central limit theorem, what is the probability that more than 35 people approve of the job the President is doing?