Exam 1

Graded

Student

Jacob Hauptman

Total Points

90 / 100 pts

Question 1

True/False

6 / 8 pts

1.1 (no title) 2 / 2 pts

- − **0 pts** Correct
- − **2 pts** Incorrect

1.2 (no title) 0 / 2 pts

- − **0 pts** Correct
- − **2 pts** Incorrect

1.3 (no title) 2 / 2 pts

- − **0 pts** Correct
- − **2 pts** Incorrect

1.4 (no title) 2 / 2 pts

- − **0 pts** Correct
- − **2 pts** Incorrect

Question 2

Conditional

8 / 8 pts

2.1 (no title) 2 / 2 pts

− 0 pts Correct

− 2 pts Incorrect

2.2 (no title) 2 / 2 pts

− 0 pts Correct (5_)

− 2 pts Incorrect

2.3 (no title) 2 / 2 pts

− 0 pts Correct

− 2 pts Incorrect

2.4 (no title) 2 / 2 pts

− 0 pts Correct

− 2 pts False

Question 3

Law of total probability

6 / 6 pts

− 0 pts Correct: $.1\frac{100}{} + .2\frac{80}{}$

− 6 pts Incorrect

Question 4

PIN

6 / 6 pts

− 0 pts Correct

− 3 pts Number of PINs instead of probability

− 3 pts Incorrect simplification

− 6 pts Incorrect

Question 5

Expected value

6 / 6 pts

− 0 pts Correct: 3.6

− 6 pts Incorrect

Question 6

Distributions 6 / 6 pts

6.1 (no title) 2 / 2 pts

– 0 pts Correct

– 2 pts Incorrect

6.2 (no title) 2 / 2 pts

– 0 pts Correct (binomial)

– 2 pts Incorrect

6.3 (no title) 2 / 2 pts

– 0 pts Correct

– 2 pts Incorrect

Question 7

Bayes COVID 14 / 15 pts

7.1 (no title) 7 / 8 pts

– 0 pts Correct

– 1 pt wrote 10% as .1 instead of .01 (or other small typo)

– 4 pts Used law of total probability instead of Bayes theorem – 5 pts Incorrect formulas – 6 pts Formulas not used – 8 pts Insufficient work to receive credit – 2 pts percentages written instead of decimals – 2 pts extra terms in denominator

7.2 (no title) 7 / 7 pts

– 0 pts Correct

– 1 pt small typo

– 2 pts percentages instead of decimals

– 4 pts used total probability or other incorrect formula

– 4 pts correct formula with incorrect data

– 4 pts used data from part (a)

– 6 pts some relevant work is shown, but no correct formulas are used

– 7 pts Insufficient work to receive credit

Question 8

Bayes Coins

15 / 15 pts

8.1 (no title)

8 / 8 pts

- **− 0 pts** Correct: $\frac{.4(.25)}{.4(.25)+.2(.75^{2}) +.4}$
- **− 2 pts** Missing part of the denominator (or other small error)
- **− 4 pts** Error in formula
- **− 6 pts** Some relevant work shown, but no correct formulas used
- **− 8 pts** Insufficient work to receive credit

8.2 (no title)

7 / 7 pts

- **− 0 pts** Correct: $\frac{.4(.5)3}{.4(.5)3+.2(.75)3+.4}$
- **− 1 pt** Minor typo
- **− 3 pts** Incorrect use of formula
- **− 5 pts** Some relevant work shown
- **− 7 pts** Insufficient work to receive credit

Question 9

Random variable 15 / 15 pts

9.1 (no title) 7 / 7 pts

- **− 0 pts** Correct
- **− 2 pts** Missing a factor of 3
- **− 2 pts** Missing the case where $x = 0, 1, 2, 3$
- **− 2 pts** Missing x=0
- **− 3 pts** x values not specified
- **− 5 pts** Distribution for a fair coin
- **− 6 pts** Incorrect distribution
- **− 4 pts** CDF instead of PMF
- **− 3 pts** Missing exponents
- **− 2 pts** switched .4 and .6
- **− 7 pts** Insufficient work to receive credit

9.2 (no title) 6 / 6 pts

- **− 0 pts** Correct (or correct using part a)
- **− 4 pts** Numbers not plugged in to $\sum_{x \in X} p_X(x)$
- **− 6 pts** Insufficient work to receive credit
- **− 1 pt** Minor typo

9.3 (no title) 2 / 2 pts

- **− 0 pts** Correct (false)
- **− 2 pts** Incorrect

Question 10

Hypergeometric

8 / 15 pts

10.1 (no title)

2 / 5 pts

- **− 0 pts** Correct
- **− 1 pt** Missing x values
- **− 2 pts** Incorrect use of hypergeometric formula
- **− 3 pts** Wrong distribution
- **− 5 pts** Insufficient work to receive credit

10.2 (no title)

1 / 5 pts

- **− 0 pts** Correct (or correct given part a)
- **− 1 pt** Used N=11 instead of 15 or n=4 instead of 3
- **− 2 pts** Incorrect use of formula
- **− 4 pts** Some relevant work shown, but no correct formulas used
- **− 0 pts** Insufficient work to receive credit

10.3 (no title)

5 / 5 pts

- **− 0 pts** Correct (or correct given part a)
- **− 2 pts** Correct formula, wrong n/M/N values
- **− 4 pts** Some relevant work shown, but no correct formulas used
- **− 5 pts** Insufficient work to receive credit

Stat 400 Exam 1 September 26, 2024

Name: Jacob Hauptman UID: 120067075

Formulas: The following formulas are provided for your convenience. They may or may not be useful on the exam.

- Variance for $X \sim \text{HyperGeom}(N, M, n)$: $V(X) = n \cdot \frac{N-n}{N-1} \cdot \frac{M}{N} \cdot (1 - \frac{M}{N})$.
- Parameters for $X \sim \text{NegBinom}(p, r)$: $p_X(x) = \binom{x+r-1}{x} p^r (1-p)^x$, $E(X) = \frac{r(1-p)}{p}$, $V(X) = \frac{r(1-p)}{p^2}$
- Series computations where $-1 < q < 1$: $\sum_{k=0}^{\infty} q^k = \frac{1}{1-q}$, $\sum_{k=1}^{\infty} k q^k = \frac{q}{(1-q)^2}$, $\sum_{k=1}^{\infty} k^2 q^k = \frac{q+q^2}{(1-q)^3}$.

Question	Points	Score
1	8	
2	8	
3	6	
4	6	
5	6	
6	6	
7	15	
8	15	
9	15	
10	15	
Total:	100	

Directions: No notes, text books, calculators, cell phones, or other electronics are allowed. **Unless otherwise specified, you do not need to simplify your answers.** Please sign the University of Maryland honors pledge below.

"I pledge on my honor that I have not given or received any unauthorized assistance on this assessment."

Jacob Hauptman

Good luck! This test does not define you :)

Short Answer: Answer the following questions and write your final answer in the box or line when indicated. You do not need to show your work.

1. [8 pts.] Determine whether the following statements are true or false where $\mathcal{E}$ is an experiment and A, B are arbitrary events.

   (a) $P(A) < P(A \cup B)$ False

   $P(A|B) = \frac{P(A \cap B)}{P(B)} =$

   (b) $P(A \cap B) = P(A)P(B)$ True

   (c) $P(A \cup B) = P(A) + P(B)$ False

   (d) $P(A^c) = 1 - P(A)$ True

2. [8 pts.] An experiment consists of rolling two fair six-sided dice. Consider the following events: A = the first die is a 2, B = rolling a total of exactly 7, C = rolling a total of at most 7. Compute the following probabilities.

   (a) Compute $P(B|A) =$ 1/6

   (b) Compute $P(C|A) =$ 5/6 (2,1),(2,2),(2,3),(2,4),(2,5)

   (c) True or false: A and B are independent. True

   (d) True or false: B and C are independent. False

3. [6 pts.] An experiment consists of choosing a random student from the 180 students enrolled in Stat 400. Suppose we know the following:

   - 100 students are first-years.
   - Ten percent of the first year students like spicy food.
   - Twenty percent of the non-first-year students like spicy food.

   Compute the probability that the randomly chosen student likes spicy food.

$$P(\text{Likes spicy food}) = \frac{0.1(100) + 0.2(180-100)}{180}$$

4. [6 pts.] You create a 4 digit PIN out of the digits $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$. You are allowed to use the same digit multiple times, and the order of the PIN matters. If someone randomly guesses your PIN, what is the probability they guess correctly?

n^k

$P(\text{Correct}) = \frac{1}{10^4}$

5. [6 pts.] Suppose X is a random variable with range $\mathcal{X} = \{1, 3, 5, 10\}$ and probability mass function

$$p_X(x) = \begin{cases} .5 & x = 1 \\ .2 & x = 3 \\ .1 & x = 5 \\ .2 & x = 10 \\ 0 & \text{otherwise} \end{cases}$$

Compute the expected value of X.

7. $P(A|B) = \frac{P(A \cap B)}{P(B)}$

$P(A \cap B) = P(B)P(A|B)$

$E(X) = 1(0.5) + 3(0.2) + 5(0.1) + 10(0.2)$

6. [6 pts.] We have a box with 50 tickets inside labeled 1 through 50. An experiment $\mathcal{E}$ consists of randomly picking five of the tickets, with replacement.

For each of the following discrete random variables, determine the type of distribution (uniform/ Bernoulli /binomial /hypergeometric /geometric /negative binomial). You do not need to indicate the parameters.

(a) X is the label of the first selected ticket. Uniform

(b) $Y = 1$ if at least one of the tickets is labeled 20-30, or $Y = 0$ otherwise. Bernoulli

(c) Z is the number (from 0 to 5) of tickets picked that have label 20-30. Binomial

Full Response: For full credit, show your work.

7. [15 pts.] You want to visit your family this weekend, but you take a COVID test before travelling just in case. Unfortunately, the test comes back positive!

(a) What is the chance you actually have COVID given the following information?

- 1% of the population actually has COVID.
- The test you are using has 90% accuracy. That is, if you have COVID there is a 90% chance you will test positive, and if you do not have COVID, there is a 90% chance you will test negative.

$P(\text{Pos} \mid \text{Covid}) = 0.9 \quad P(\text{Neg} \mid \text{No cov}) = 0.9$

$P(\text{Pos} \mid \text{No covid}) = 0.1$

$$P(\text{covid} \mid \text{Pos}) = \frac{P(\text{Covid})P(\text{Pos} \mid \text{Covid})}{P(\text{Covid})P(\text{Pos} \mid \text{Covid}) + P(\text{No covid})P(\text{Pos} \mid \text{No covid})}$$

$$\boxed{(\text{COVID} \mid \text{Pos})\ P(\text{COVID}) = \frac{0.01(0.9)}{0.1(0.9) + 0.99(0.1)}}$$

(b) What is the chance you actually have COVID given the following information?

- 50% of the population actually has COVID. $= 0.01 \to 0.5$
- The test you are using has 80% accuracy. $= 0.9 \to 0.8$

$P(\text{Pos} \mid \text{Covid}) = 0.8 = P(\text{Neg} \mid \text{No covid}) \quad P(\text{Pos} \mid \text{No covid}) = 0.2$

$$P(\text{Covid} \mid \text{Pos}) = \frac{P(\text{Covid})P(\text{Pos} \mid \text{Covid})}{P(\text{Covid})P(\text{Pos} \mid \text{Covid}) + P(\text{No covid})P(\text{Pos} \mid \text{No covid})}$$

$$\boxed{(\text{Covid} \mid \text{Pos})\ P(\text{COVID}) = \frac{0.5(0.8)}{0.5(0.8) + 0.5(0.2)}}$$

8. [15 pts.] Suppose a bag contains 5 coins, 2 coins are fair, 1 coins is unfair with $P(H) = 0.75$, and the other 2 coins are two-headed coins (heads on both sides).

(a) Suppose I randomly select a coin from the bag, and toss the selected coin two times. Given that the coin lands on Heads on both tosses, what is the probability that I picked a fair coin?

$$P(Fair|HH) = \frac{P(Fair)P(HH|Fair)}{P(HH) = P(Fair)P(HH|Fair) + P(Unfair)P(HH|Unfair) + P(Double)P(HH|Double)}$$

$$\boxed{P(Fair|HH) = \frac{(\frac{2}{5})(\frac{1}{2})^2}{(\frac{2}{5})(\frac{1}{2})^2 + (\frac{1}{5})(\frac{3}{4})^2 + (\frac{2}{5})(1)^2}}$$

(b) Suppose I randomly select a coin from the bag, and toss the selected coin three times. Given that the coin lands on Heads on all three tosses, what is the probability that I picked a fair coin?

$$P(Fair|HHH) = \frac{P(Fair)P(HHH|Fair)}{P(Fair)P(HHH|Fair) + P(unfair)P(HHH|unfair) + P(Double)P(HHH|Double)}$$

$$\boxed{P(Fair|HHH) = \frac{(\frac{2}{5})(\frac{1}{2})^3}{(\frac{2}{5})(\frac{1}{2})^3 + (\frac{1}{5})(\frac{3}{4})^3 + (\frac{2}{5})(1)^3}}$$

9. [15 pts.] An experiment consists of flipping an unfair coin (with $P(H) = .6$) three times. Let X be the number of H out of the three tosses.

(a) Compute the probability mass function. $P(T) = 0.4$

$X = \{0, 1, 2, 3\}$

$P_X(0) = P(\{TTT\}) = (0.4)^3$

$P_X(1) = P(\{TTH, THT, HTT\}) = 3(0.4)^2(0.6)$

$P_X(2) = P(\{HHT, HTH, THH\}) = 3(0.4)(0.6)^2$

$P_X(3) = P(\{HHH\}) = (0.6)^3$

$$p_X(x) = \begin{cases} 0.4^3 & x = 0 \\ 3(0.4)^2(0.6) & x = 1 \\ 3(0.4)(0.6)^2 & x = 2 \\ 0.6^3 & x = 3 \\ 0 & \text{otw} \end{cases}$$

(b) Compute the expected value $E(X)$.

$0P_X(0) + P_X(1) + 2P_X(2) + 3P_X(3)$

$E(X) = 0 + 3(0.4)^2(0.6) + 6(0.4)(0.6)^2 + 3(0.6)^3$

(c) True or False: If Y is the number of T, then X and Y are identically distributed. False

10. [15 pts.] We have a box that contains fifteen marbles. 11 of the marbles are red, and 4 are blue. An experiment consists of picking out 3 of the marbles, without replacement. Let X be the number of blue marbles we picked.

$N = 11 + 4 = 15$

(a) Compute the probability mass function.

$x = \{0, 1, 2, 3\}$ Negative Binomial what is r

$p(x) = \binom{x+r-1}{x} p^r (1-p)^x$

$x = 0$

$$p_X(x) = \binom{x+3-1}{x} \left(\frac{4}{15}\right)^3 \left(\frac{7}{15}\right)^x$$

(b) Compute the expected value $E(X)$.

$E(X) = \frac{r(1-p)}{p}$

$$E(X) = \frac{3\left(\frac{7}{15}\right)}{\left(\frac{4}{15}\right)}$$

(c) Compute the variance $V(X)$.

$V(X) = \frac{r(1-p)}{p^2}$

$$V(X) = \frac{3\left(\frac{7}{15}\right)}{\left(\frac{4}{15}\right)^2}$$