Homework 9

● Graded

5 Days, 16 Hours Late

Student

Jacob Hauptman

Total Points

23 / 24 pts

Question 1

(no title) **3.5** / 4 pts

- **– 0 pts** Correct
- **– 0.5 pts** small mistake (no duplicate penalization for repeated mistakes)
- **– 0.5 pts** b. i missing / wrong exp and var
- **– 0.5 pts** b. ii missing / wrong exp and var
- ✔ **– 0.5 pts** b. iii missing / wrong exp and var
- **– 0.5 pts** b. i, ii missing / wrong var
- **– 1 pt** a. no attempt / wrong answer without explanation
- **– 1 pt** b. i no attempt / wrong answer without explanation
- **– 1 pt** b. ii no attempt / wrong answer without explanation
- **– 1 pt** b. iii no attempt / wrong answer without explanation

Question 2

(no title) **4** / 4 pts

- ✔ **– 0 pts** Correct
- **– 0.5 pts** small mistake
- **– 0.5 pts** b. incomplete

Question 3

(no title) **3.5** / 4 pts

- **– 0 pts** Correct
- ✔ **– 0.5 pts** P(|X-2|<1) instead of P(|X-2|>1) or wrong direction of inequality: $\Phi(X)$ or $1-\Phi(X)$
- **– 1 pt** wrong sample variance
- **– 0.5 pts** small mistake

Question 4

(no title) **4** / 4 pts

✔ **– 0 pts** Correct

– 0.5 pts small mistake

– 1 pt wrong variance sample

Question 5

(no title) **4** / 4 pts

✔ **– 0 pts** Correct

– 1 pt missing part b

– 1 pt missing part c

– 1 pt wrong variance

– 0.5 pts small mistake

Question 6

(no title) **4** / 4 pts

✔ **– 0 pts** Correct

– 0.5 pts small mistake at part a

– 0.5 pts not checking the binomial conditions or making wrong conclusion in part b.

– 0.5 pts small mistake at part c

– 4 pts missing

Question assigned to the following page: 1

1)

.a $E(X) = 2(0.4) + 4(0.2) + 6(0.4) = 4$

$V(X) = (2-4)^2(0.4) + (4-4)^2(0.2) + (6-4)^2(0.4) = 3.2$

.b

c.i) $T_0 = X_1 + X_2 + X_3$

$T_0(2,2,2) = 6 \quad T_0(2,2,4) = 8 \quad T_0(2,2,6) = 10$

$T_0(2,4,2) = 8 \quad T_0(2,4,4) = 10 \quad T_0(2,4,6) = 12$

$T_0(2,6,2) = 10 \quad T_0(2,6,4) = 12 \quad T_0(2,6,6) = 14$

$T_0(4,2,2) = 8 \quad T_0(4,2,4) = 10 \quad T_0(4,2,6) = 12$

$T_0(4,4,2) = 10 \quad T_0(4,4,4) = 12 \quad T_0(4,4,6) = 14$

$T_0(4,6,2) = 12 \quad T_0(4,6,4) = 14 \quad T_0(4,6,6) = 16$

$T_0(6,2,2) = 10 \quad T_0(6,2,4) = 12 \quad T_0(6,2,6) = 14$

$T_0(6,4,2) = 12 \quad T_0(6,4,4) = 14 \quad T_0(6,4,6) = 16$

$T_0(6,6,2) = 14 \quad T_0(6,6,4) = 16 \quad T_0(6,6,6) = 18$

$P(6) = (0.4)^3 = 0.064$

$P(8) = 3(0.4)^2(0.2) = 0.096$

$P(10) = 3(0.4)^2(0.4) + 3(0.4)(0.2)^2 = 0.24$

$P(12) = 6(0.4)(0.2)(0.4) + (0.2)^3 = 0.2$

$P(14) = 3(0.4)(0.4)^2 + 3(0.2)^2(0.4) = 0.24$

$P(16) = 3(0.2)(0.4)^2 = 0.096$

$P(18) = (0.4)^3 = 0.064$

$P_{T_0}(6)$	$P_{T_0}(8)$	$P_{T_0}(10)$	$P_{T_0}(12)$	$P_{T_0}(14)$	$P_{T_0}(16)$	$P_{T_0}(18)$
0.064	0.096	0.24	0.2	0.24	0.096	0.064

$E(T_0) = 3E(X) = 3(4) = 12$

$V(T_0) = 3V(X) = 3(3.2) = 9.6$

Questions assigned to the following page: 2 and 1

(ii) $\bar{X} = \frac{T_o}{n} = \frac{T_o}{3}$

So we have

$P_{T_o}(2)$	$P_{T_o}(\frac{8}{3})$	$P_{T_o}(\frac{10}{3})$	$P_{T_o}(4)$	$P_{T_o}(\frac{14}{3})$	$P_{T_o}(\frac{16}{3})$	$P_{T_o}(6)$
0.064	0.096	0.24	0.2	0.24	0.096	0.064

$E(\bar{X}) = \frac{1}{n} E(T_o) = \frac{12}{3} = 4$

$V(\bar{X}) = \frac{1}{n^2} V(T_o) = \frac{1}{9}(9.6) = 1.0\overline{6}$

(iii)

$\max\{X_1, X_2, X_3\} = 2 : (2,2,2)$

$\max\{X_1, X_2, X_3\} = 4 : (2,2,4), (2,4,2), (2,4,4), (4,2,2), (4,4,2), (4,2,4), (4,4,4)$

$\max\{X_1, X_2, X_3\} = 6 : (2,6,2), (2,6,4), (4,6,2), (6,2,2), (6,4,2), (6,6,2), (4,6,4), (6,2,4), (6,4,4), (6,6,4), (2,2,6), (2,4,6), (2,6,6), (4,2,6), (4,4,6), (4,6,6), (6,2,6), (6,4,6), (6,6,6)$

$P_{max}(2) = (0.4)^3 = 0.064$

$P_{max}(4) = 3(0.4)^2(0.2) + 3(0.4)(0.2)^2 + (0.2)^3 = 0.152$

$P_{max}(6) = 3(0.4)^2(0.4) + 6(0.4)(0.2)(0.4) + 3(0.2)^2(0.4) + 3(0.4)(0.4)^2 + 3(0.2)(0.4)^2 + (0.4)^3 = 0.784$

$P_{max}(2)$	$P_{max}(4)$	$P_{max}(6)$
0.064	0.152	0.784

2)

- a $P(1200 < X < 1360) = P(\frac{1200-1520}{320} < Z < \frac{1360-1520}{320}) = \Phi(-0.5) - \Phi(-1) = 0.1498$
- b $\sigma_{\bar{X}} = \frac{\sigma_X}{\sqrt{n}} = \frac{320}{\sqrt{16}} = 80$

  $P(\frac{1200-1520}{80} < Z < \frac{1360-1520}{80}) = \Phi(-2) - \Phi(-4) = 0.0228$
- c It is already normally distributed.

Questions assigned to the following page: 3, 4, and 5

3)

·a $\mu_x = E(x) = 0.1 + 2(0.1) + 3(0.1) + 4(0.1) + 5(0.1) + 6(0.5) = 4.5$

$E(x^2) = 0.1 + 4(0.1) + 9(0.1) + 16(0.1) + 25(0.1) + 36(0.5) = 23.5$

$V(x) = E(x^2) - E(x)^2 = 23.5 - 4.5^2 = 3.25 \quad \sigma_x = \sqrt{3.25} = 1.803$

I am assuming it's supposed to be $P(|x-2|>1)$ not $P(|x-2))>1$.

$|x-2|>1 \Rightarrow -1<x-2<1$

$-1<x-2 \qquad x-2<1$

$1<x \qquad\quad x<3$

So $\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}} = \frac{1.803}{8} = 0.225$, then $P(1<X<3) = P\left(\frac{1-4.5}{0.225} < Z < \frac{3-4.5}{0.225}\right)$

$= \Phi(-6.\bar{6}) - \Phi(-15.\bar{5})$

·b n is large, $n \geq 30 \Rightarrow 64 > 30$

4)

·a $\mu_x = 17.2 \quad \sigma_x = 2.5$

$\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}} = \frac{2.5}{\sqrt{55}} \approx 0.337$

$P(17<\bar{X}<18) = P\left(\frac{17-17.2}{0.337} < Z < \frac{18-17.2}{0.337}\right) \approx \Phi(2.37) - \Phi(-0.59) \approx 0.7142$

·b n is large, $n \geq 30 \Rightarrow 55 > 30$

5)

·a $\mu_x = 0.84 \quad \mu_x = \frac{1}{\lambda} \Rightarrow \lambda = \frac{1}{\mu_x} = \frac{1}{0.84} = 1.19$

$F(x) = 1 - e^{-\lambda x}$

So $F(0.84) - F(0.7) = 1 - e^{-0.84(1.19)} - 1 + e^{-0.7(1.19)} = 0.0667$

·b $\sigma_x = \mu_x = 0.84$ so $\sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{n}} = \frac{0.84}{6} = 0.14$

$P(0.7<\bar{X}<0.84) = P\left(\frac{0.7-0.84}{0.14} < Z < \frac{0.84-0.84}{0.14}\right) = \Phi(0) - \Phi(-1) = 0.3413$

Questions assigned to the following page: 5 and 6

·c $\sigma_{\bar{x}} = \frac{0.84}{10} = 0.084$

$P(0.7 < \bar{X} < 0.84) = P\left(\frac{0.7-0.84}{0.084} < Z < \frac{0.84-0.84}{0.084}\right) = \Phi(0) - \Phi(-1.67) = 0.4525$

6)

·a Binomial Distribution. $\mu_x = E(x) = np = 0.3n$

$\sigma_x = \sqrt{V(x)} = \sqrt{np(1-p)} = \sqrt{0.3n(0.7)} = \sqrt{0.21n}$

·b $np > 5$ and $n(1-p) > 5$

$0.3n > 5$ $\quad$ $0.7n > 5$

$n > 16.\overline{6}$ $\quad$ $n > 7.14$

$16.\overline{6} < 17.14$ so n must be greater than or equal to 17.14, $n \geq 17.14$.

·c $\mu_x = 0.3(100) = 30$ $\quad \sigma_x = \sqrt{0.21(100)} = 4.583$

$P(Y > 35) = P\left(\frac{35-30}{4.583} < Z\right) \approx 1 - \Phi(1.09) = 0.1379$