Homework 4

Graded

22 Hours, 14 Minutes Late

Student

Jacob Hauptman

Total Points

20 / 24 pts

Question 1

Uniform

4 / 4 pts

- ✔ **− 0 pts** Correct

Question 2

Bernoulli

4 / 4 pts

- ✔ **− 0 pts** Correct
- **− 1 pt** Wrong variance formula.
- **− 1 pt** wrong probability of success

Question 3

Bernoilli parameters

1 / 4 pts

- **− 0 pts** Correct
- **− 1 pt** Part a is wrong.
- ✔ **− 1 pt** Part b is wrong.
- ✔ **− 1 pt** Part c is wrong
- ✔ **− 1 pt** Part d is wrong.

1. total of 19 outcomes have max=3
2. There 11 outcomes with two consecutive 6s
3. Total 8 outcomes have max <=2

Question 4

Binomial

3.5 / 4 pts

- **– 0 pts** Correct
- ✔ **– 0.5 pts** Calculation error in part b
- **– 0.5 pts** No final answer in part b (Hence no comparing in part c)
- **– 0.5 pts** Not using pmf to compute Exp or Var in part b.
- **– 1 pt** Not using pmf to compute Exp and Var in part b.
- **– 1 pt** ignoring X=5 and X=6

Question 5

Hypergeometric

3.5 / 4 pts

- **– 0 pts** Correct
- ✔ **– 0.5 pts** calculation error in part b
- **– 0.5 pts** No final answer in part b.
- **– 1 pt** Not using pmf to compute Exp / Var in part b
- **– 1 pt** No attempt / irrelevant answer for part d
- **– 0.5 pts** Didn't compute variance

Question 6

Binomial/Hypergeometric with bigger parameters

4 / 4 pts

- ✔ **– 0 pts** Attempted
- **– 4 pts** Not attempted

Questions assigned to the following page: 1, 2, and 3

9.24.2024

STAT400 Homework 4

1a) Binomial Distribution.

1b) $\frac{N+1}{2} = \frac{1001}{2} = 500.5$

1c) $\frac{(N+1)(N-1)}{12} = \frac{1001(999)}{12} = 83333.25$

2a) $\mathcal{X} = \{win, lose\}$

$P_X(win) = \frac{1}{1000}$

$P_X(lose) = \frac{999}{1000}$

$P_X(x) = p^x(1-p)^{x-1} = \left(\frac{1}{1000}\right)^x\left(\frac{999}{1000}\right)^{x-1}$

2b) $E(X) = p = \frac{1}{1000}$

2c) $V(X) = p(1-p) = \frac{1}{1000}\left(\frac{999}{1000}\right) = \frac{999}{1000000}$

3a) $P(Sum \geq 5) = 1 - P(Sum < 5) = 1 - \frac{3+1+0+0}{216} = 1 - \frac{1}{54} = \frac{53}{54} \approx 0.981$

3b) $P(max \leq 2) = \frac{1+3}{216} = \frac{1}{54} \approx 0.0185$

3c) $P(max = 3) = \frac{3+3}{216} = \frac{6}{216} = \frac{1}{36} \approx 0.02\overline{7}$

113 (x3)
223 (x3)

Questions assigned to the following page: 3, 4, and 5

3d) $P(2 \text{ consecutive } 6\text{'s}) = \frac{2}{216} = \frac{1}{108} \approx 0.0093$

$x\,6\,6$

$6\,6\,x$

4a) $P_X(x) = \binom{n}{x} p^x (1-p)^{n-x} = \binom{6}{x}\left(\frac{1}{3}\right)^x\left(\frac{2}{3}\right)^{6-x}$

4b) $E(X) = \sum_{x=1}^{6} x\,P_X(x) = 1\binom{6}{1}\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^5 + 2\binom{6}{2}\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^4$

$+3\binom{6}{3}\left(\frac{1}{3}\right)^3\left(\frac{2}{3}\right)^3 + 4\binom{6}{4}\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^2$

$+5\binom{6}{5}\left(\frac{1}{3}\right)^5\left(\frac{2}{3}\right)^1 + 6\binom{6}{6}\left(\frac{1}{3}\right)^6$

$\binom{6}{1} = 6 \quad \binom{6}{2} = \frac{6!}{4!2!} = 15 \quad \binom{6}{3} = \frac{6!}{3!3!} = 20$

$\binom{6}{4} = \frac{6!}{2!4!} = 15 \quad \binom{6}{5} = \frac{6!}{5!} = 6 \quad \binom{6}{6} = 1$

$E(X) = 2 = \mu_x$

$V(X) = \sum_{x=1}^{6} (x - \mu_x)^2 P_X(x) \approx 0.98$

4c) $E(X) = np = 6\left(\frac{1}{3}\right) = 2$

$V(X) = np(1-p) = 6\left(\frac{1}{3}\right)\left(\frac{2}{3}\right) = \frac{12}{9} = \frac{4}{3}$

$2 = 2 \qquad 0.98 \neq \frac{4}{3}$

The expected value equal but variance is off.

5a) $P_X(x) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}} = \frac{\binom{4}{x}\binom{8}{6-x}}{\binom{12}{6}} \qquad \binom{12}{6} = 924$

Question assigned to the following page: 5

5b) $E(X) = \sum_{x=1}^{4} x P_X(x) = P(1) + 2P(2) + 3P(3) + 4P(4)$

$= 2$

$P(1) = 0.\overline{24}$

$P(2) = 0.\overline{45}$

$P(3) = 0.\overline{24}$

$P(4) = 0.\overline{03}$

$V(X) = \sum_{x=1}^{4} (x-\mu_X)^2 P_X(x) = (1-2)^2 P(1) + (2-2)^2 P(2) + (3-2)^2 P(3)$

$+ (4-2)^2 P(4) = 0.\overline{606}$

5c) $E(X) = n\left(\frac{M}{N}\right) = 6\left(\frac{4}{12}\right) = 2$

$V(X) = \left(\frac{N-n}{N-1}\right) n \left(\frac{M}{N}\right)\left(1-\frac{M}{N}\right) = \left(\frac{12-6}{12-1}\right)(6)\left(\frac{4}{12}\right)\left(1-\frac{4}{12}\right)$

$= \left(\frac{6}{11}\right)(6)\left(\frac{4}{12}\right)\left(\frac{8}{12}\right) = 0.727$

Expected is the same but variance is sort of close

5d) The expected value using the formulas is the same but the variance is different.

$E(X)$: $\overset{\text{geo}}{2} = \overset{\text{hyp geo}}{2}$

$V(X)$: $\frac{4}{3} \neq 0.727$

Question assigned to the following page: 6

6a) $P_X(x) = \binom{n}{x} p^x (1-p)^{n-x}$

$P_X(x) = \binom{6}{x} \left(\frac{1}{3}\right)^x \left(\frac{2}{3}\right)^{6-x}$

6b) $P_X(x) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$

$P_X(x) = \frac{\binom{40}{x}\binom{80}{6-x}}{\binom{120}{6}}$

6c) Hyper does replacement while not hyper does without replacement. Hyper geometric has to account for more variables too.