Stat 400. Problem Set 3. Due 09/17/24

Random Variables

Problem 1. Suppose we toss a biased coin with $P(H) = 0.8$ four times.

(a) Write out the sample space $\mathcal{S}$ for this experiment.

(b) Suppose $X(\omega)$ =the number of heads in ω.
   (a) Calculate χ.
   (b) For each $x \in \chi$, write out the set $\{X = x\}$.
   (c) Calculate the cumulative distribution function of X.
   (d) Calculate the probability mass function of X.

(c) Suppose $X(\omega)$ =the number of tails in ω.
   (a) Calculate χ.
   (b) For each $x \in \chi$, write out the set $\{X = x\}$.
   (c) Calculate the cumulative distribution function of X.
   (d) Calculate the probability mass function of X.

(d) Suppose $X(\omega)$ = are there more heads than tails in ω?
   (a) Calculate χ.
   (b) For each $x \in \chi$, write out the set $\{X = x\}$.
   (c) Calculate the cumulative distribution function of X.
   (d) Calculate the probability mass function of X.

Problem 2. Suppose we roll a fair die twice.

(a) Write out the sample space $\mathcal{S}$ for this experiment.

(b) Suppose $X(\omega)$ = the sum of the numbers in ω.
   (a) Calculate χ.
   (b) For each $x \in \chi$, write out the set $\{X = x\}$.
   (c) Calculate the cumulative distribution function of X.
   (d) Calculate the probability mass function of X.

(c) Suppose $X(\omega)$ = the maximum of the numbers in ω.
   (a) Calculate χ.
   (b) For each $x \in \chi$, write out the set $\{X = x\}$.
   (c) Calculate the cumulative distribution function of X.
   (d) Calculate the probability mass function of X.

(d) Suppose $X(\omega)$ = the first number minus the second number in ω.
   (a) Calculate χ.
   (b) For each $x \in \chi$, write out the set $\{X = x\}$.
   (c) Calculate the cumulative distribution function of X.
   (d) Calculate the probability mass function of X.

Problem 3. An appliance store receives a shipment of 30 ovens, 5 of which are defective. The store manager selects 4 ovens at random, without replacement, and tests to see if they are defective. Let X be the number of defectives found. Calculate the probability mass function of X.

Problem 4. Suppose we toss a biased coin with $P(H) = 0.8$ indefinitely (infinitely many times).

(1) Let X be the random variable that counts the number of tosses until the first H.
   (a) Calculate the probability mass function of X.
   (b) Verify that the function you calculated is indeed a pmf (check each condition).
   (c) Calculate the expected value and variance of X.

(2) Let's say you win 3^n dollars if the first H shows up on the n^{th} toss. Let $X(\omega)$ be your winnings for the run of the game ω.
   (a) Calculate the probability mass function of X.
   (b) Calculate $E(X)$ (your expected earnings).