Stat 400. Problem Set 2. Due 09/10/24
Conditional Probability, Bayes' Theorem, and Counting

Directions: You must show all work to receive full credit. Turn in your completed assignment to gradescope by 11:59pm on 9/10.

Problem 1. An experiment consists of rolling two fair six-sided dice. Consider the following events: $E =$ rolling a total of 7, $F =$ the first die is a 6, $G =$ the first die is a 4, $H =$ rolling at total of at least 10. Compute the following probabilities.

   (a) $P(E|F)$
   (b) $P(E|H)$
   (c) $P(H|F)$

Problem 2. Consider the same experiment and events from Problem 1. For each of the following, show your work by using the definition of independent events.

   (a) Are F and G independent?
   (b) Are F and H independent?
   (c) Are E and F independent?

Problem 3. You go to a tea tasting party with your coworkers where there will be white and green tea. You want to impress your coworkers with your tea-tasting ability. Here is what you know going into the party:

   • 90% of the teas at the party will be green teas.
   • When you taste a green/white tea, you can guess the correct color of tea 60% of the time.

You taste the first tea at the party, and you *think* it tastes white. What is the probability that it actually is white?

Problem 4. Suppose a bag contains 7 coins, 2 coins are fair, 2 coins are unfair with $P(H) = 0.2$, and the other 3 coins are two-headed coins (heads on both sides).

   (1) Suppose Amy randomly selects a coin from the bag, and tosses the selected coin two times. Given that the coin lands on Heads on both tosses, what is the probability that Amy tossed a fair coin?
   (2) Suppose Amy randomly selects a coin from the bag, and tosses the selected coin three times. Given that the coin lands on Heads on all three tosses, what is the probability that Amy tossed a fair coin?

Problem 5. You are creating a 4 digit password out of the letters $\{a, A, b, B, c, C, d, D, e, E, f, F\}$. How many possible passwords are there if:

   (a) You can use a letter more than once, and the password *is not* case-sensitive (a is treated the same as A).
   (b) You can use a letter more than once, and the password *is* case-sensitive.
   (c) Each letter can only be used once, and the password is not case-sensitive.
   (d) Each letter can only be used once, and the password *is* case-sensitive.
   (e) You can use a letter more than once and the password is not case-sensitive, but one (*and only one*) of the 4 digits needs to be a special character: !, ?, or & .

Problem 6. Suppose a restaurant menu has 8 different appetizers, 6 different entrees, and 6 different dessert options. A meal consists of two appetizers, two entrees, and three desserts. Calculate the total number of meals possible if:

   (a) If we are allowed to repeat any of the items on the list, and the order in which we consume the food matters.
   (b) If we are not allowed to repeat any of the items on the list, and the order in which we consume the food does not matter.