Stat 400. Problem Set 10. Due 11/19/24
Point Estimation

Problem 1. Consider the negative binomial distribution with parameters r and p. Show that $\hat{p} = \dfrac{r-1}{X+r-1}$ is an unbiased estimator for p.
Hint: Write an infinite sum for $E(\hat{p})$, and cancel out $x+r-1$ inside the sum.

Problem 2. Suppose we get a random sample $\{X_1, ..., X_m\}$ from a population with mean μ and variance σ^2, and a random sample $\{Y_1, ..., Y_n\}$ from a population with mean μ and variance $9\sigma^2$.

(a) Show that for any fixed number $\delta \in (0,1)$, the estimator
$$\hat{\mu} = \delta\overline{X} + (1-\delta)\overline{Y}$$
is an unbiased estimator for μ.

(b) Compute the variance of $\hat{\mu}$ (your answer will involve m, n, σ, δ).

(c) Find the value of δ that minimizes $V(\hat{\mu})$. (Hint: calculate the derivative with respect to δ)

Problem 3. Let $\{X_1, ..., X_n\}$ be a random sample from a population with the normal distribution with mean μ and standard deviation σ. Consider the statistic
$$S_{xx} = \sum_{i=1}^{n}(X_i - \overline{X})^2.$$

(a) Calculate the expected value $E(S_{xx})$.

(b) Find an unbiased estimator for σ^2. This estimator is called the sample variance, and is denoted S^2.

Problem 4. Using a rod of (unknown) length ℓ, you lay out a square plot of side length ℓ. So the area is ℓ^2. Suppose you take n independent measurements X_i, each with mean ℓ and variance σ^2.

(a) Is $\overline{X}$ an unbiased estimator for ℓ? Justify your answer.

(b) Calculate the bias of $\overline{X}^2$. Is $\overline{X}^2$ an unbiased estimator of ℓ^2? If not, is $\overline{X}^2$ an overestimate or an underestimate?

(c) For what value of K is $\overline{X}^2 - KS^2$ an unbiased estimator of ℓ^2?