

Stat 400. Problem Set 4. Due 09/24/24
Discrete Random Variables

Problem 1. Suppose you have a lottery where every ticket has an equal chance of winning, and there are 1000 tickets labeled from 1 to 1000. Let X be the number of the selected ticket.

- (a) Which type of distribution is this?
- (b) Calculate the expected value of the selected ticket.
- (c) Calculate the variance.

Problem 2. Suppose you enter a lottery where every ticket has an equal chance of winning, and there are 1000 tickets labeled from 1 to 1000. Let X be the Bernoulli random variable where Success is winning the lottery.

- (a) Calculate the pmf.
- (b) Calculate the expected value.
- (c) Calculate the variance.

Problem 3. Suppose a trial consists of rolling a six-sided fair die three times. For each of the descriptions of trials, identify the relevant Bernoulli parameter, i.e. $P(\text{Success}) = p$.

- (a) the trial is a success if the sum of the three die rolls is greater than or equal to 5.
- (b) the trial is a success if the max of the three die rolls is less than or equal to 2.
- (c) the trial is a success if the max of the three die rolls is exactly equal to 3.
- (d) the trial is a success if two consecutive 6's show up in three rolls.

Problem 4. Suppose $N = 12$ (population size), $M = 4$ (number of successes), and $n = 6$ (sample size).

- (a) Write down the probability mass function for the Binomial distribution with parameters n and $p = \frac{M}{N}$.
- (b) Calculate the expected value and the variance using the pmf and the definitions of expected value and variance of a discrete random variable.
- (c) Compare your answer of expected values and variances to those that you get using the formulas presented in class for the Binomial distribution.

Problem 5. Suppose $N = 12$ (population size), $M = 4$ (number of successes), and $n = 6$ (sample size).

- (a) Write down the probability mass function for the Hypergeometric distribution with parameters N, M , and n .
- (b) Calculate the expected value and the variance using the pmf and the definitions of expected value and variance of a discrete random variable.
- (c) Compare your answer of expected values and variances to those that you get using the formulas presented in class for the Hypergeometric distribution.
- (d) Compare the values of the probability mass functions for the Hypergeometric and Binomial random variables that you have calculated in this problem and the one before.

Problem 6. Now suppose $N = 120, M = 40$, and $n = 6$.

- (a) Calculate the probability mass function for the Binomial distribution with parameters $n = 6$ and $p = \frac{40}{120}$.
- (b) Calculate the probability mass function for Hypergeometric random variable with parameters $N = 120, M = 40$, and $n = 6$.
- (c) Compare the probability mass functions for the Hypergeometric and Binomial random variables that you have calculated above. Compared with the situation in the previous problem, what do you notice here?