Matrix Completion

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Initiative

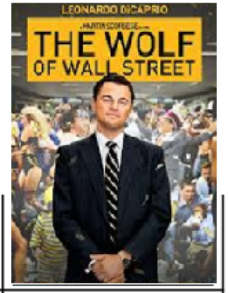
Matrix completion is the task of filling in the missing entries of a partially observed matrix. Some questions:

- Is it generally feasible for $\mathrm{M} \in \mathbb{R}^{m \times n}$?
- mn measurement are required
- what if the matrix is low rank
- number of degrees of freedom is $r(m+n-r) \ll mn$.
- what if the matrix is sparse

Application

- Movie Recommender System
- Image Inpainting

Movies

Users				
Bob	4	?	?	4
Alice	?	5	4	?
Joe	?	5	?	?
Sam	5	?	?	?

Algorithms

$$\min \quad \operatorname{rank}(\mathrm{X}) \quad \text{s.t.} \quad X_{ij} = M_{ij},\ (i,j) \in \Omega$$

- Convex relaxation of the rank

$$\min \quad \|\mathrm{X}\|_* \quad \text{s.t.} \quad X_{ij} = M_{ij},\ (i,j) \in \Omega$$

- Singular Value Thresholding

$$\min \quad \tau\|\mathrm{X}\|_* + \frac{1}{2}\|\mathrm{X}\|_F^2 \quad \text{s.t.} \quad X_{ij} = M_{ij},\ (i,j) \in \Omega$$

- Robust PCA

$$\min_{\mathrm{L} \in \mathrm{R}^{m \times n}} \|\mathrm{L}\|_* + \lambda\|\mathrm{S}\|_1 \quad \text{s.t.} \quad \mathrm{M} = \mathrm{L} + \mathrm{S} \tag{1}$$

Image Inpainting

Movie Recommender System