

Matrix Completion

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Initiative

Matrix completion is the task of filling in the missing entries of a partially observed matrix. Some questions:

- ▶ Is it generally feasible for $M \in \mathbb{R}^{m \times n}$?
- ▶ mn measurement are required
- ▶ what if the matrix is low rank?
- ▶ number of degrees of freedom is $r(m + n - r) \ll mn$.
- ▶ what if the matrix is sparse?

Application

- ▶ Movie Recommender System
- ▶ Image Inpainting

	Movies			
				
Users	Bob	4	?	?
Alice	?	5	4	?
Joe	?	5	?	?
Sam	5	?	?	?



Algorithms

$$\min \text{rank}(X) \quad \text{s.t.} \quad X_{ij} = M_{ij}, \quad (i,j) \in \Omega$$

- Convex relaxation of the rank

$$\min \|X\|_* \quad \text{s.t.} \quad X_{ij} = M_{ij}, \quad (i,j) \in \Omega$$

- Singular Value Thresholding

$$\min \tau \|X\|_* + \frac{1}{2} \|X\|_F^2 \quad \text{s.t.} \quad X_{ij} = M_{ij}, \quad (i,j) \in \Omega$$

- Robust PCA

$$\min_{L \in \mathbb{R}^{m \times n}} \|L\|_* + \lambda \|S\|_1 \quad \text{s.t.} \quad M = L + S \quad (1)$$

Image Inpainting

We can recover the missing pixels of an image

Original Image



Image Inpainting

First we add different levels of noise [0.1, 0.2, 0.3, 0.4, 0.5, 0.6] to the original image to generate damaged images:

10



20



30



40



50



60



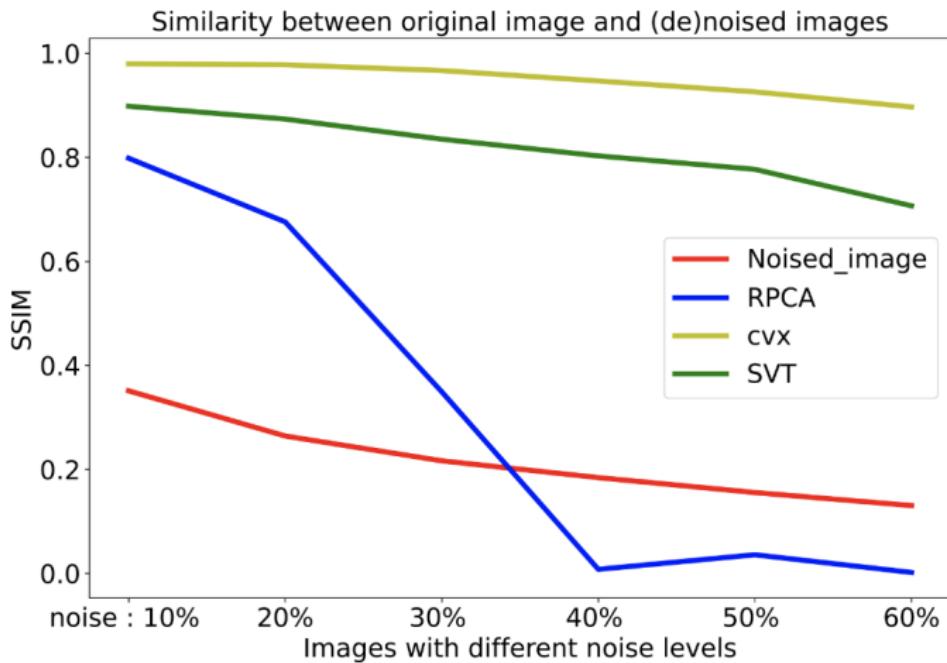
Image Inpainting

Running the algorithms on the distorted image we get:

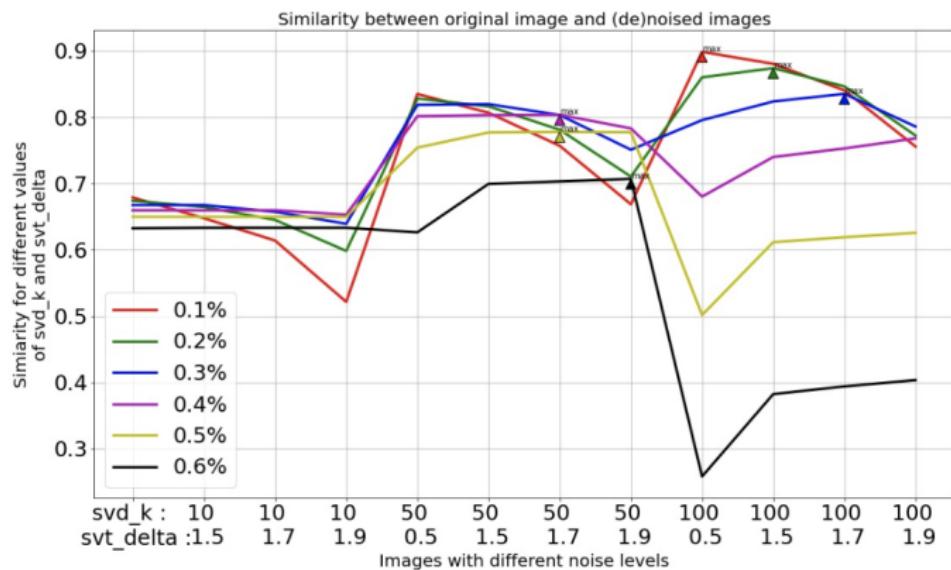


Performance of Algorithms

We use the Structural Similarity index to compare quantitatively the algorithms used for matrix completion.

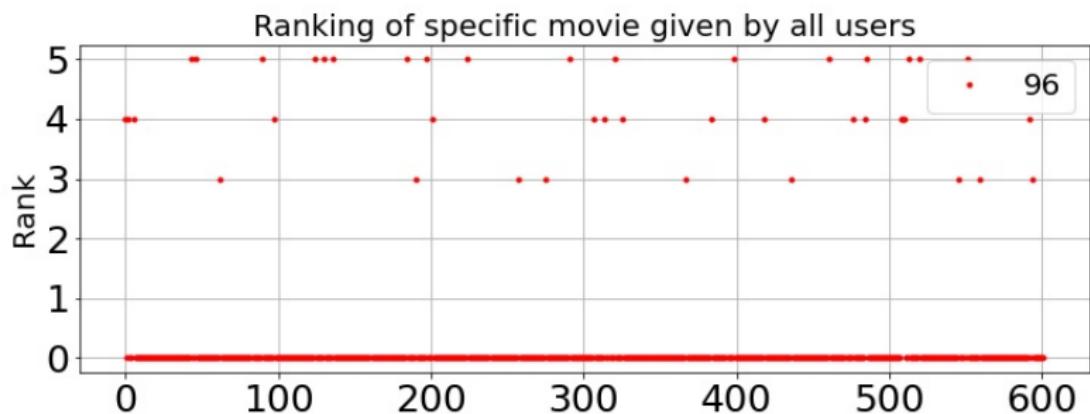


Performance of SVT for different parameters.



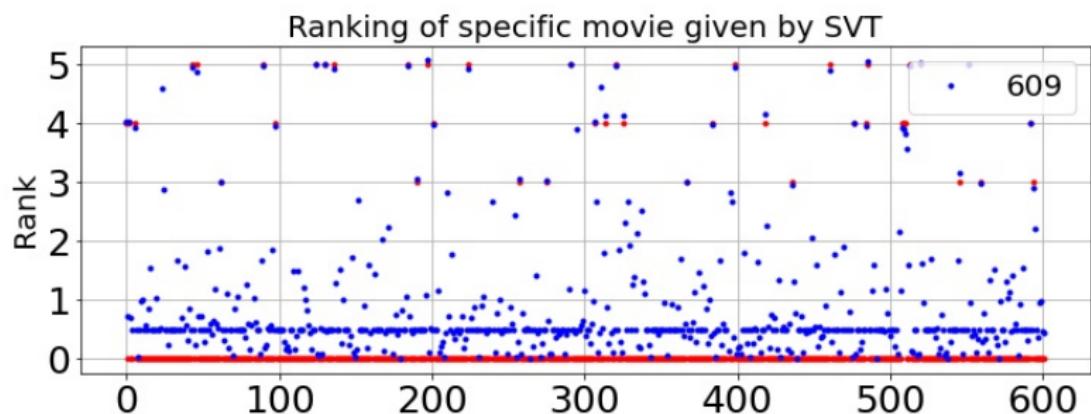
Movie Recommender System

- ▶ SVT and Movie Recommender



Movie Recommender System

► SVT and Movie Recommender



Movie Recommender System

► SVT and Movie Recommender

184 : actual rate -- 4.0	prediction: -- 4.0
185 : actual rate -- 4.0	prediction: -- 4.0
190 : actual rate -- 4.0	prediction: -- 4.0
192 : actual rate -- 2.0	prediction: -- 2.0
199 : actual rate -- 3.5	prediction: -- 3.0
200 : actual rate -- 5.0	prediction: -- 4.0
201 : actual rate -- 4.0	prediction: -- 4.0
205 : actual rate -- 5.0	prediction: -- 5.0
212 : actual rate -- 3.5	prediction: -- 4.0
213 : actual rate -- 3.0	prediction: -- 2.0
215 : actual rate -- 3.0	prediction: -- 3.0
216 : actual rate -- 4.0	prediction: -- 4.0
218 : actual rate -- 3.5	prediction: -- 4.0
219 : actual rate -- 5.0	prediction: -- 5.0
222 : actual rate -- 3.5	prediction: -- 4.0

Movie Recommender System

► SVT and Movie Recommender

Table 1. Top 15 movies recommended for user 1

User 1 Recommendation	
Movie Name	Rating
The Yards (2000)	5.2810
Up in Smoke (1978)	5.2301
Lawrence of Arabia (1962)	5.2236
The Princess Bride (1987)	5.1427
Arsenic and Old Lace (1944)	5.1302
Some Kind of Wonderful (1987)	5.1139
A Walk on the Moon(1999)	5.1086
The Wizard of Oz (1939)	5.1084
Dumb and Dumber (1994)	5.0728
Tombstone (1993)	5.0620
Home Alone 2 (1992)	5.0614
Dirty Dancing (1987)	5.0567
The French Connection (1971)	5.0545
A Fish Called Wanda (1988)	5.0497
Fargo (1996)	5.0439