CSCI 104
Binary Search Trees

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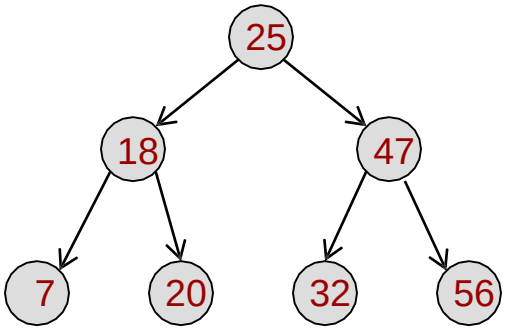
Properties, Insertion and Removal

BINARY SEARCH TREES

Binary Search Tree

- Binary search tree = binary tree where all nodes meet the property that:
  - All values of nodes in left subtree are less than or equal to the parent's value
  - All values of nodes in right subtree are greater than the parent's value



If we wanted to print the values in sorted order would you use an pre-order, in-order, or post-order traversal?

BST Insertion

- Important: To be efficient (useful) we need to keep the binary search tree balanced
- Practice: Build a BST from the data values below
  - To insert an item walk the tree (go left if value is less than node, right if greater than node) until you find an empty location, at which point you insert the new value

Insertion Order: 25, 18, 47, 7, 20, 32, 56

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- https://www.cs.usfca.edu/~galles/visualization/BST.html

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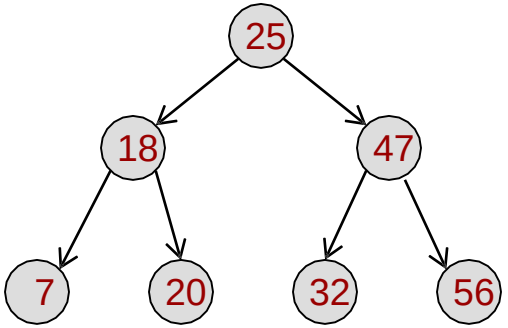

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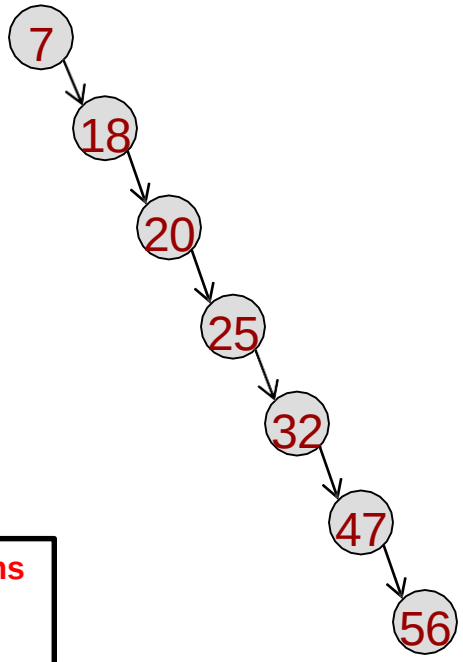
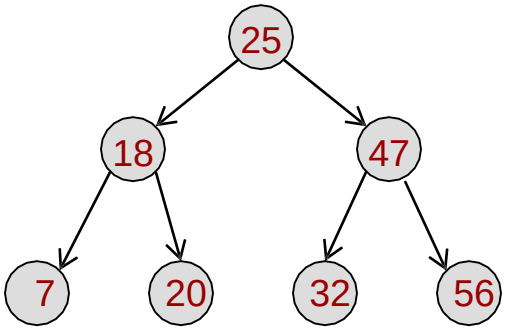


A major topic we will talk about is algorithms to keep a BST balanced as we do insertions/removals

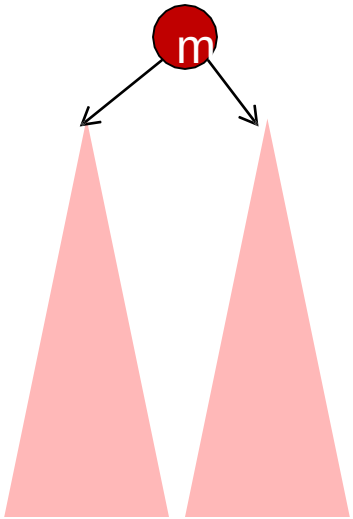
BST Find

- Exercise: Given a node to a root of a tree and a value, write a recursive function **find** that returns a pointer to the node containing the value or nullptr if not found:
- **shared_ptr<Node> find(shared_ptr<Node> root, int value);**



Successors & Predecessors

- Let's take a quick tangent that will help us understand how to do **BST Removal**
- Given a node in a BST
  - Its predecessor is defined as the next smallest value in the tree
  - Its successor is defined as the next biggest value in the tree
- Where would you expect to find a node's successor?
- Where would find a node's predecessor?



Predecessors

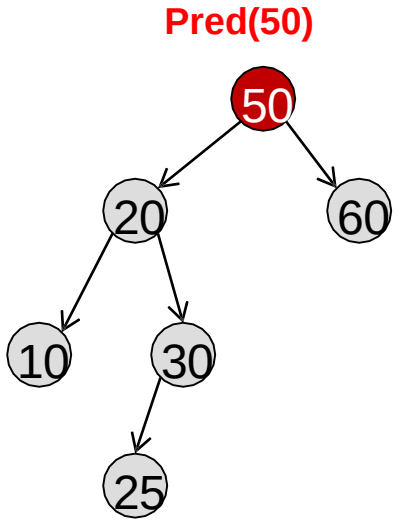
- If left child exists, predecessor is the right most node of the left subtree (case 1)
- Else walk up the ancestor chain

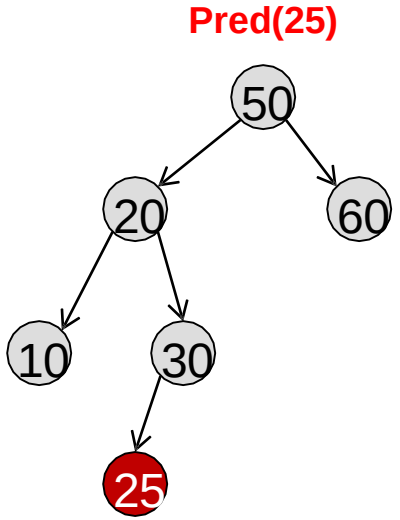
  if traverse the first right child pointer

  - Parent of right child is predecessor (case 2)

  else

  - reach root, then there is no predecessor (case 3)





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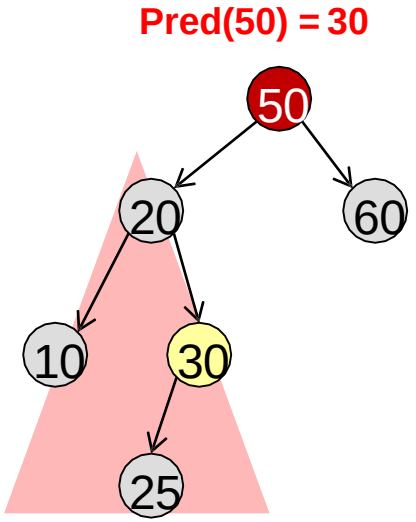

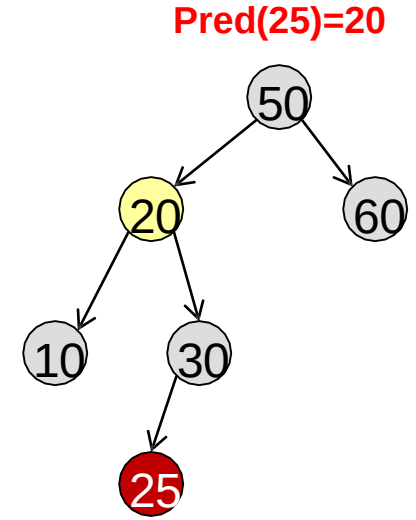

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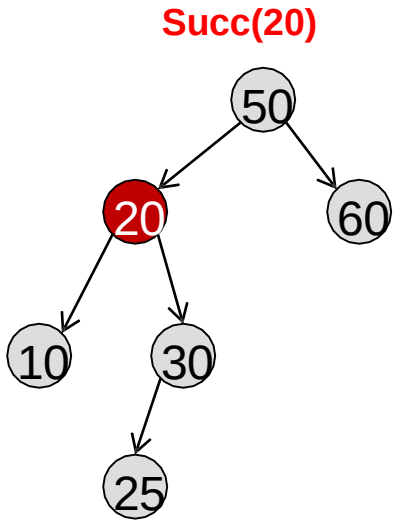

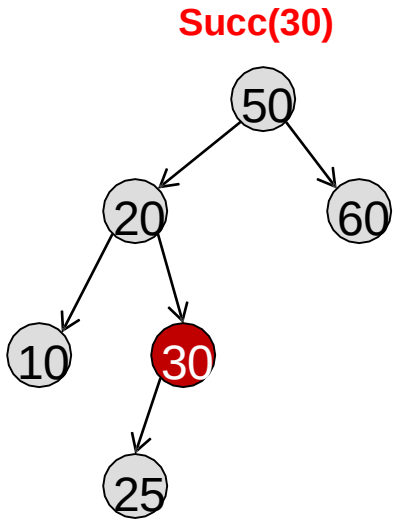

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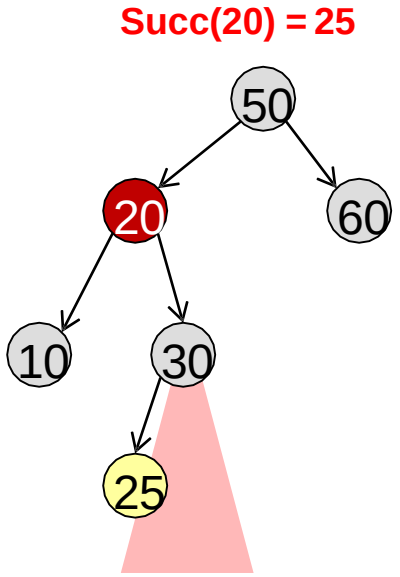
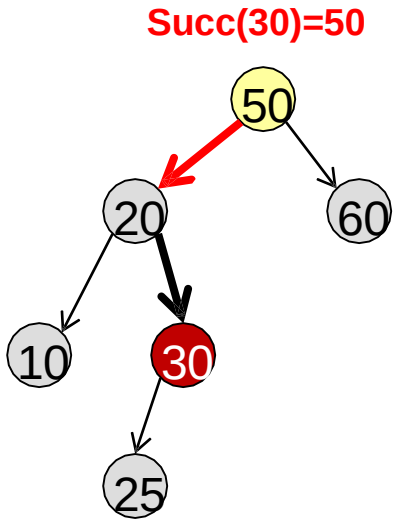
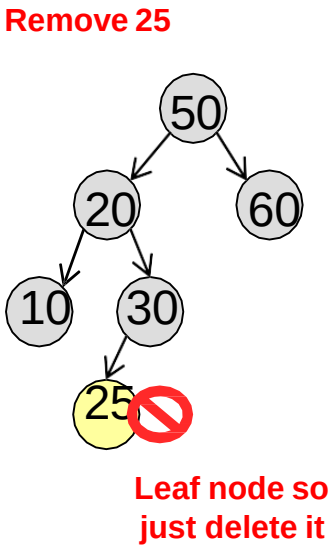




BST Removal

- To remove a value from a BST…
  - First find the node containing value using search
  - If the node is in a leaf node, simply remove that leaf node



BST Removal

- To remove a value from a BST…
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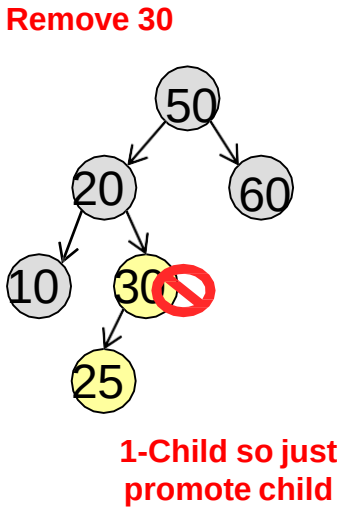
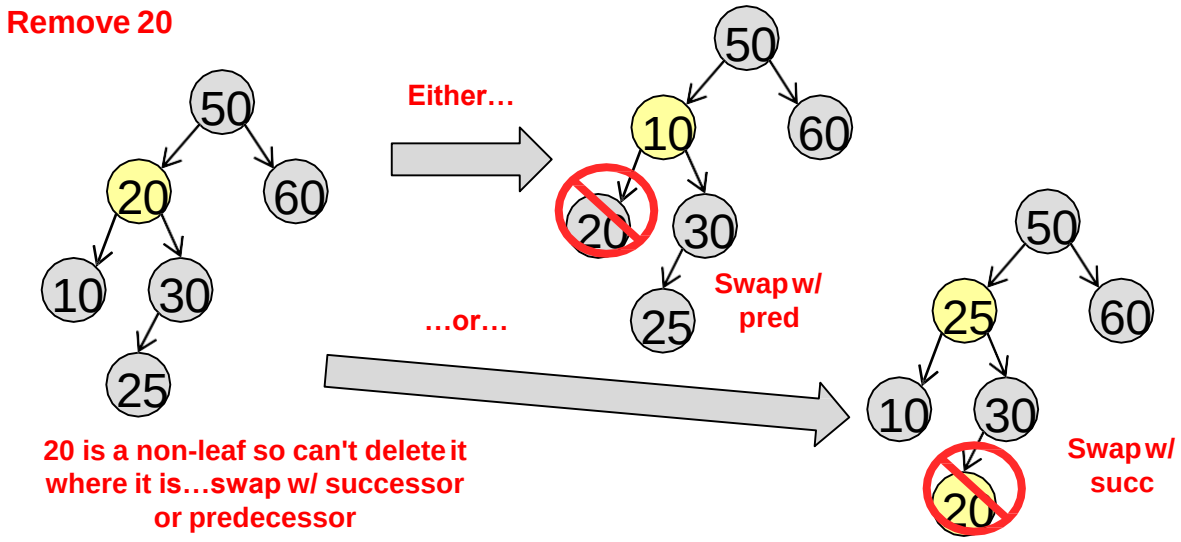
  - If the value is in a non-leaf node with only one child, promote the child



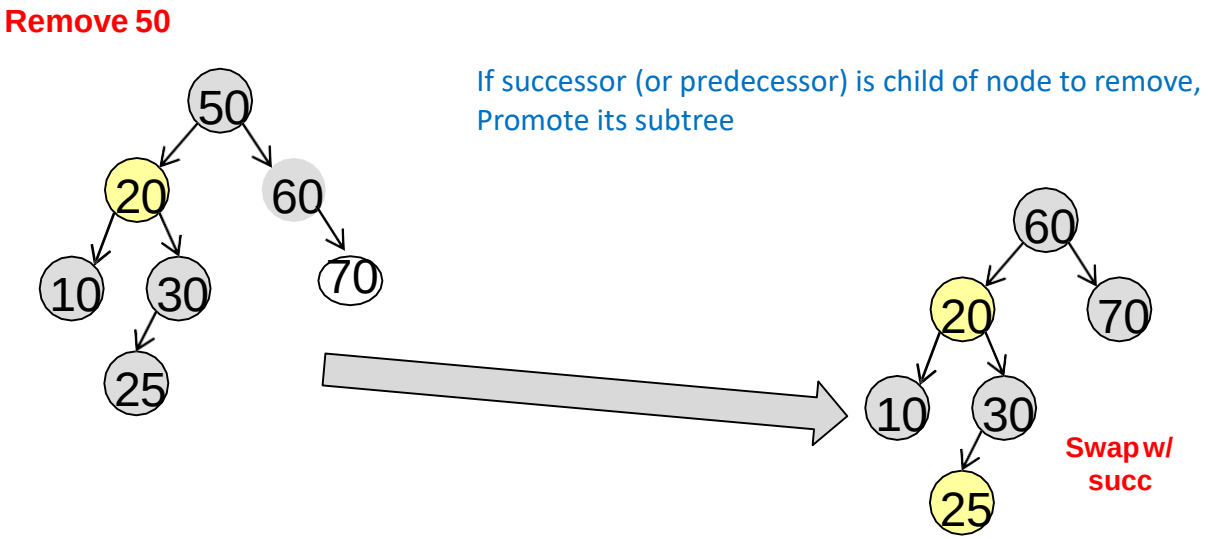
BST Removal

- To remove a value from a BST…
  - First find the node containing value using search
  - If non-leaf node with two children, swap the value with its in-order successor or predecessor and then remove the value
  - What if successor and predecessor are not leaf nodes?



BST Removal

- To remove a value from a BST…
  - First find the node
  - If the node non-leaf node with two children, swap the value with its in-order successor or predecessor and then remove the value
  - A predecessor is either a leaf or has no right child
  - A successor is either leaf or has no left child



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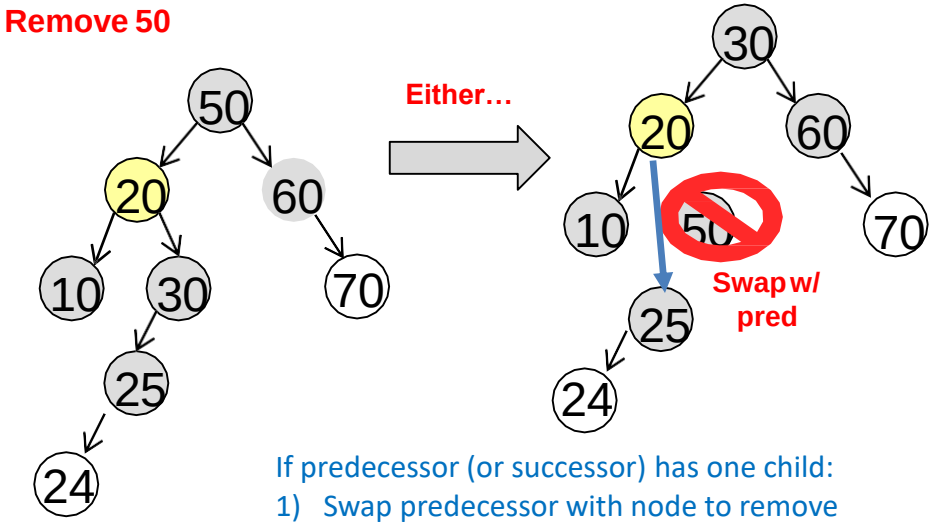

If predecessor (or successor) has one child:

1) Swap predecessor with node to remove
2) Remove the predecessor node and promote its only child (and its subtree)

BST Efficiency

- Insertion
  - Balanced: O(log n)
  - Unbalanced: O(n)
- Removal
  - Balanced : O(log n)
  - Unbalanced: O(n)
- Find/Search
  - Balanced : O(log n)
  - Unbalanced: O(n)

```
#include<iostream>
using namespace std;

// Bin. Search Tree
template <typename T>
class BST
{
 public:
 BTree();
 ~BTree();
 virtual bool empty() = 0;
 virtual void insert(const T& v) = 0;
 virtual void remove(const T& v) = 0;
 virtual T* find(const T& v) = 0;
};
```

Trees & Maps/Sets

- C++ STL "maps" and "sets" use binary search trees internally to store their keys (and values) that can grow or contract as needed
- This allows O(log n) time to find/check membership
  - BUT ONLY if we keep the tree balanced!