In [29]:
```
using SymPy
@syms k1 k2 c1 c2 L₁ L₂ t m g J z₁ z₂ z₃ z₄ u h₀ l1₀ l2₀ θ₀ y₀
θ = SymFunction("θ")(t)
y = SymFunction("y")(t)
θshift = SymFunction("θshift")(t)
yshift = SymFunction("yshift")(t)
```

Out[29]: $\mathrm{yshift}(t)$

$l1_0$ and $l2_0$ are the undeformed lengths of the two springs
h_0 is the hight of the reference line
θ_0 and y_0 are the equilibrium points

In [30]:
```
y₁ = -L₁*sin(θ) + y
ẏ₁ = diff(y₁,t)
y₂ = L₂*sin(θ) + y
ẏ₂ = diff(y₂,t)
```

Out[30]:
$$L_2\cos(\theta(t))\frac{d}{dt}\theta(t)+\frac{d}{dt}y(t)$$

In [31]:
```
F_eqn = k1*(y₁+h₀-l1₀) + c1*ẏ₁ + k2*(y₂+h₀-l2₀) + c2*ẏ₂ + m*diff(y,t,t)
```

Out[31]:
$$c_1\left(-L_1\cos(\theta(t))\frac{d}{dt}\theta(t)+\frac{d}{dt}y(t)\right)+c_2\left(L_2\cos(\theta(t))\frac{d}{dt}\theta(t)+\frac{d}{dt}y(t)\right)+gm$$
$$+k_1\left(-L_1\sin(\theta(t))+h_0-l1_0+y(t)\right)+k_2\left(L_2\sin(\theta(t))+h_0-l2_0+y(t)\right)+m\frac{d^2}{dt^2}y(t)$$

In [32]:
```
M_eqn = L₁*cos(θ)*(k1*(y₁+h₀-l1₀) + c1*ẏ₁) - L₂*cos(θ)*(k2*(y₂+h₀-l2₀) +
```

Out[32]:
$$-J\frac{d^2}{dt^2}\theta(t)+L_1\Bigg(c_1\left(-L_1\cos(\theta(t))\frac{d}{dt}\theta(t)+\frac{d}{dt}y(t)\right)+k_1\big(-L_1\sin(\theta(t))+h_0-l1_0+y(t)$$
$${}_2\left(c_2\left(L_2\cos(\theta(t))\frac{d}{dt}\theta(t)+\frac{d}{dt}y(t)\right)+k_2\left(L_2\sin(\theta(t))+h_0-l2_0+y(t)\right)\right)\cos$$

In [33]:
```
# Kinetic Energy
T = 1//2*m*diff(y,t)^2 + 1//2*J*diff(θ,t)^2
```

Out[33]:
$$\frac{J\left(\frac{d}{dt}\theta(t)\right)^2}{2}+\frac{m\left(\frac{d}{dt}y(t)\right)^2}{2}$$

In [34]:
```
# Potential Energy
V = m*g*y + 1//2*k1*(y₁+h₀-l1₀)^2 + 1//2*k2*(y₂+h₀-l2₀)^2
```

Out[34]:
$$gmy(t)+\frac{k_1\left(-L_1\sin(\theta(t))+h_0-l1_0+y(t)\right)^2}{2}+\frac{k_2\left(L_2\sin(\theta(t))+h_0-l2_0+y(t)\right)^2}{2}$$

In [35]:
```
# Dissipation Term
D = 1//2*c1*(diff(y₁,t))^2 + 1//2*c2*(diff(y₂,t))^2
```

Out[35]:

$$\frac{c_1\left(-L_1\cos\left(\theta(t)\right)\frac{d}{dt}\theta(t)+\frac{d}{dt}y(t)\right)^2}{2}+\frac{c_2\left(L_2\cos\left(\theta(t)\right)\frac{d}{dt}\theta(t)+\frac{d}{dt}y(t)\right)^2}{2}$$

In [36]:
```
L = T - V
L_eqn1 = diff(diff(L,diff(θ,t)),t) - diff(L,θ) + diff(D,diff(θ,t))
```

Out[36]:

$$J\frac{d^2}{dt^2}\theta(t)-L_1c_1\left(-L_1\cos\left(\theta(t)\right)\frac{d}{dt}\theta(t)+\frac{d}{dt}y(t)\right)\cos\left(\theta(t)\right)-L_1k_1\left(-L_1\sin\left(\theta(t)\right)+h_0-l1\right.$$
$$+L_2c_2\left(L_2\cos\left(\theta(t)\right)\frac{d}{dt}\theta(t)+\frac{d}{dt}y(t)\right)\cos\left(\theta(t)\right)+L_2k_2\left(L_2\sin\left(\theta(t)\right)+h_0-l2_0+y(t\right.$$

In [37]:
```
L_eqn2 = diff(diff(L,diff(y,t)),t) - diff(L,y) + diff(D,diff(y,t)) |> s
```

Out[37]:

$$-c_1\left(L_1\cos\left(\theta(t)\right)\frac{d}{dt}\theta(t)-\frac{d}{dt}y(t)\right)+c_2\left(L_2\cos\left(\theta(t)\right)\frac{d}{dt}\theta(t)+\frac{d}{dt}y(t)\right)+gm$$
$$-k_1\left(L_1\sin\left(\theta(t)\right)-h_0+l1_0-y(t)\right)+k_2\left(L_2\sin\left(\theta(t)\right)+h_0-l2_0+y(t)\right)+m\frac{d^2}{dt^2}y(t)$$

In [38]:
```
M_eqn + L_eqn1 |> simplify
```

Out[38]: 0

cordinate shift!

In [ ]:
```

```

In [39]:
```
AllDotsGoToZero = Dict(diff(θ,t,t)=>0,diff(θ,t)=>0,diff(y,t,t)=>0,diff(
Equil_F = F_eqn |> subs(AllDotsGoToZero) |> subs(y=>y₀,θ=>θ₀)
```

Out[39]:

$$gm+k_1\left(-L_1\sin\left(\theta_0\right)+h_0-l1_0+y_0\right)+k_2\left(L_2\sin\left(\theta_0\right)+h_0-l2_0+y_0\right)$$

In [40]:
```
Equil_M = M_eqn |> subs(AllDotsGoToZero) |> subs(y=>y₀,θ=>θ₀)
```

Out[40]:

$$L_1k_1\left(-L_1\sin\left(\theta_0\right)+h_0-l1_0+y_0\right)\cos\left(\theta_0\right)-L_2k_2\left(L_2\sin\left(\theta_0\right)+h_0-l2_0+y_0\right)\cos\left(\theta_0\right)$$

In [41]:
```
horiz_Equil = [(Equil_F |> subs.(θ=>0)), (Equil_M|> subs.(θ=>0))]
```

Out[41]:

$$\left[\begin{array}{r} gm+k_1\left(-L_1\sin\left(\theta_0\right)+h_0-l1_0+y_0\right)+k_2\left(L_2\sin\left(\theta_0\right)+h_0-l2_0\right. \\ L_1k_1\left(-L_1\sin\left(\theta_0\right)+h_0-l1_0+y_0\right)\cos\left(\theta_0\right)-L_2k_2\left(L_2\sin\left(\theta_0\right)+h_0-l2_0+y_0\right)\mathrm{cc} \end{array}\right.$$

In [42]:
```
Linear_Rule = Dict(z₁=>θ , z₂=>y , z₃=>diff(θ,t), z₄=>diff(y,t))
```

Out[42]:

$$\begin{cases} z_3 & => & \frac{d}{dt}\theta(t) \\ z_1 & => & \theta(t) \\ z_4 & => & \frac{d}{dt}y(t) \\ z_2 & => & y(t) \end{cases}$$

In [43]:
```
z = [z₁; z₂; z₃; z₄].subs(Linear_Rule)
EOM = solve([F_eqn,M_eqn],[diff(y,t,t),diff(θ,t,t)])
ż = diff.(z)
Reverse_Linear_Rule = Dict(θ=>z₁ , y=>z₂ , diff(θ,t)=>z₃, diff(y,t)=>z₄
ż = ż.subs(EOM).subs(Reverse_Linear_Rule)
```

Out[43]:

$$\left[\begin{array}{c} \\ \frac{\left(-L_1^2 c_1 z_3 \cos\left(z_1\right) - L_1^2 k_1 \sin\left(z_1\right) + L_1 c_1 z_4 + L_1 h_0 k_1 - L_1 k_1 l1_0 + L_1 k_1 z_2 - L_2^2 c_2 z_3 \cos\left(z_1\right) - L_2^2 k_2 \sin\left(z_1\right) - L_2 c_2 z_4 - L_2 h_0 k_2 + L_2 k_2}{J} \\ \frac{L_1 c_1 z_3 \cos\left(z_1\right) + L_1 k_1 \sin\left(z_1\right) - L_2 c_2 z_3 \cos\left(z_1\right) - L_2 k_2 \sin\left(z_1\right) - c_1 z_4 - c_2 z_4 - gm - h_0 k_1 - h_0 k_2 +}{m} \end{array} \right.$$

In [44]:
```
A = fill(k1,(4,4))
    for i=1:4
        A[i,1] = diff(ż[i],z₁)
        A[i,2] = diff(ż[i],z₂)
        A[i,3] = diff(ż[i],z₃)
        A[i,4] = diff(ż[i],z₄)
    end
A
```

Out[44]:

$$\left[\begin{array}{c} \\ \frac{\left(L_1^2 c_1 z_3 \sin\left(z_1\right) - L_1^2 k_1 \cos\left(z_1\right) + L_2^2 c_2 z_3 \sin\left(z_1\right) - L_2^2 k_2 \cos\left(z_1\right)\right) \cos\left(z_1\right)}{J} \\ - \frac{\left(-L_1^2 c_1 z_3 \cos\left(z_1\right) - L_1^2 k_1 \sin\left(z_1\right) + L_1 c_1 z_4 + L_1 h_0 k_1 - L_1 k_1 l1_0 + L_1 k_1 z_2 - L_2^2 c_2 z_3 \cos\left(z_1\right) - L_2^2 k_2 \sin\left(z_1\right) - L_2 c_2 z_4 - L_2 h_0 k_2 + L}{J} \\ \frac{-L_1 c_1 z_3 \sin\left(z_1\right) + L_1 k_1 \cos\left(z_1\right) -}{} \end{array} \right.$$

In [45]:
```
B = fill(k1,(4,1))
    for i = 1:4
        B[i] = diff(ż[i],u)
    end
B
```

Out[45]:

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

In [ ]:
```

```

Computational Dynamics Homework4 Part2 4.5 Airfoil

March 5, 2020

1 The Airfoil

1.1 Lagrange's Equations

First, define the Kinetic Energy

```
using SymPy
@vars kt k J t m p_1 p_2
theta = SymFunction("theta")(t)
y = SymFunction("y")(t)
T = 1//2*m*diff(y,t)^2 + 1//2*J*diff(theta,t)^2
```

$$\frac{J\left(\frac{d}{dt}\theta(t)\right)^2}{2} + \frac{m\left(\frac{d}{dt}y(t)\right)^2}{2}$$

The potential energy

```
V = 1//2*kt*theta^2 + 1//2*k*y^2
```

$$\frac{ky^2(t)}{2} + \frac{kt\theta^2(t)}{2}$$

Define the Langrangian

```
L = T-V
```

$$\frac{J\left(\frac{d}{dt}\theta(t)\right)^2}{2} - \frac{ky^2(t)}{2} - \frac{kt\theta^2(t)}{2} + \frac{m\left(\frac{d}{dt}y(t)\right)^2}{2}$$

Get equations of motion

```
eqn1 = diff(diff(L,diff(theta,t)),t) -diff(L,theta)
```

$$J\frac{d^2}{dt^2}\theta(t) + kt\theta(t)$$

```
eqn2 = diff(diff(L,diff(y,t)),t) -diff(L,y)
```

$$ky(t) + m\frac{d^2}{dt^2}y(t)$$

1.2 Hamilton's Equations

Get generalized momenta

```
zero1 = diff(L,diff(theta,t)) - p_1
zero2 = diff(L,diff(y,t)) - p_2
rule_1 = solve([zero1,zero2],[diff(theta,t),diff(y,t)])
```

```
Dict{Any,Any} with 2 entries:
  Derivative(y(t), t)     => p_2/m
  Derivative(theta(t), t) => p_1/J
```

1.3 But the equations of motion do not have any momentum dependence.

Build the Hamiltonian and get equations of motion

```
H = T + V
H = H.subs(rule_1)
H_eqn_1 = diff(diff(L,diff(theta,t)),t) + diff(H,theta)
```

$$J\frac{d^2}{dt^2}\theta(t) + kt\theta(t)$$

```
H_eqn_2 = diff(diff(L,diff(y,t)),t) + diff(H,y)
```

$$ky(t) + m\frac{d^2}{dt^2}y(t)$$

4.7 Rotating pendulum of doom

March 5, 2020

I think this problem is different from the furuta pendulum because:

1. There is no mass spoken of for the horizontal arm.
2. There is no moment of inertia for the second arm, and instead there is a spring.
3. I don't think I should be using Rotational matrices

1 Define Energy

1.1 Kinetic Energy of Arm 1

```
using SymPy
@vars J m L0 t g k d
theta = SymFunction("theta")(t)
Omega = SymFunction("Omega")(t)
L = SymFunction("L")(t)
Torque = SymFunction("Torque")(t)
T1 = 1//2*J*Omega^2
```

$$\frac{J\Omega^2(t)}{2}$$

1.2 Potential Engergy of Arm 1

```
V1 = 0;
```

1.3 Kinetic Energy of Arm 2

The Kinetic Energy of Arm 2 is tough. The first term is rotational energy, the second term is the translational energy in the direction of the spring. The last term I am not sure about. I think there needs to be translational energy in the y-z plane (This needs to account for the fact that the coordinates themselves are rotating).

```
v = Omega*d*sin(theta) + Omega*d*cos(theta) + diff(theta,t)*L - Omega*L*sin(theta)
I_theta = 1//2*m*L^2
T2 = 1//2*I_theta*diff(theta,t)^2 + 1//2*m*diff(L,t)^2 + 1//2*m*v
```

$$\frac{m\left(d\Omega(t)\sin\left(\theta(t)\right)+d\Omega(t)\cos\left(\theta(t)\right)-L(t)\Omega(t)\sin\left(\theta(t)\right)+L(t)\frac{d}{dt}\theta(t)\right)}{2}+\frac{mL^2(t)\left(\frac{d}{dt}\theta(t)\right)^2}{4}+\frac{m\left(\frac{d}{dt}L(t)\right)^2}{2}$$

1.4 Potential Energy of Arm 2

```
V2 = m*g*L*(1-cos(theta)) + 1//2*k*(L-L0)^2
```

$$gm\left(1-\cos\left(\theta(t)\right)\right)L(t)+\frac{k\left(-L_0+L(t)\right)^2}{2}$$

---

2 Get Equations of Motion via Lagrange

```
T = T1 + T2
V = V1 + V2
Langr = T-V
L_eqn1 = diff(diff(Langr,diff(theta)),t) - diff(Langr,theta)
```

$$gmL(t)\sin\left(\theta(t)\right)-\frac{m\left(-d\Omega(t)\sin\left(\theta(t)\right)+d\Omega(t)\cos\left(\theta(t)\right)-L(t)\Omega(t)\cos\left(\theta(t)\right)\right)}{2}+\frac{mL^2(t)\frac{d^2}{dt^2}\theta(t)}{2}+mL(t)\frac{d}{dt}$$

```
L_eqn2 = Torque - (diff(diff(Langr,diff(Omega)),t) - diff(Langr,Omega))
```

$$J\Omega(t)+\frac{m\left(d\sin\left(\theta(t)\right)+d\cos\left(\theta(t)\right)-L(t)\sin\left(\theta(t)\right)\right)}{2}+\mathrm{Torque}\left(t\right)$$

```
L_eqn1 = diff(diff(Langr,diff(L)),t) - diff(Langr,L)
```

$$gm\left(1-\cos\left(\theta(t)\right)\right)+\frac{k\left(-2L_0+2L(t)\right)}{2}-\frac{m\left(-\Omega(t)\sin\left(\theta(t)\right)+\frac{d}{dt}\theta(t)\right)}{2}-\frac{mL(t)\left(\frac{d}{dt}\theta(t)\right)^2}{2}+m\frac{d^2}{dt^2}L(t)$$