In [29]:
```
using SymPy
@syms k1 k2 c1 c2 L₁ L₂ t m g J z₁ z₂ z₃ z₄ u h₀ l1₀ l2₀ θ₀ y₀
θ = SymFunction("θ")(t)
y = SymFunction("y")(t)
θshift = SymFunction("θshift")(t)
yshift = SymFunction("yshift")(t)
```

Out[29]: $\mathrm{yshift}(t)$

$l1_0$ and $l2_0$ are the undeformed lengths of the two springs
h_0 is the hight of the reference line
θ_0 and y_0 are the equilibrium points

In [30]:
```
y₁ = -L₁*sin(θ) + y
ẏ₁ = diff(y₁,t)
y₂ = L₂*sin(θ) + y
ẏ₂ = diff(y₂,t)
```

Out[30]:

$$L_2 \cos(\theta(t)) \frac{d}{dt}\theta(t) + \frac{d}{dt}y(t)$$

In [31]:
```
F_eqn = k1*(y₁+h₀-l1₀) + c1*ẏ₁ + k2*(y₂+h₀-l2₀) + c2*ẏ₂ + m*diff(y,t,t)
```

Out[31]:

$$c_1\left(-L_1\cos(\theta(t))\frac{d}{dt}\theta(t) + \frac{d}{dt}y(t)\right) + c_2\left(L_2\cos(\theta(t))\frac{d}{dt}\theta(t) + \frac{d}{dt}y(t)\right) + gm + k_1\left(-L_1\sin(\theta(t)) + h_0 - l1_0 + y(t)\right) + k_2\left(L_2\sin(\theta(t)) + h_0 - l2_0 + y(t)\right) + m\frac{d^2}{dt^2}y(t)$$

In [32]:
```
M_eqn = L₁*cos(θ)*(k1*(y₁+h₀-l1₀) + c1*ẏ₁) - L₂*cos(θ)*(k2*(y₂+h₀-l2₀) +
```

Out[32]:

$$-J\frac{d^2}{dt^2}\theta(t) + L_1\left(c_1\left(-L_1\cos(\theta(t))\frac{d}{dt}\theta(t) + \frac{d}{dt}y(t)\right) + k_1\left(-L_1\sin(\theta(t)) + h_0 - l1_0 + y(t)\right.\right.$$

$$_2\left(c_2\left(L_2\cos(\theta(t))\frac{d}{dt}\theta(t) + \frac{d}{dt}y(t)\right) + k_2\left(L_2\sin(\theta(t)) + h_0 - l2_0 + y(t)\right)\right)\cos$$

In [33]:
```
# Kinetic Energy
T = 1//2*m*diff(y,t)^2 + 1//2*J*diff(θ,t)^2
```

Out[33]:

$$\frac{J\left(\frac{d}{dt}\theta(t)\right)^2}{2} + \frac{m\left(\frac{d}{dt}y(t)\right)^2}{2}$$

In [34]:
```
# Potential Energy
V = m*g*y + 1//2*k1*(y₁+h₀-l1₀)^2 + 1//2*k2*(y₂+h₀-l2₀)^2
```

Out[34]:

$$gmy(t) + \frac{k_1\left(-L_1\sin(\theta(t)) + h_0 - l1_0 + y(t)\right)^2}{2} + \frac{k_2\left(L_2\sin(\theta(t)) + h_0 - l2_0 + y(t)\right)^2}{2}$$

In [35]:
```
# Dissipation Term
D = 1//2*c1*(diff(y₁,t))^2 + 1//2*c2*(diff(y₂,t))^2
```

Out[35]:
$$\frac{c_1\left(-L_1\cos\left(\theta(t)\right)\frac{d}{dt}\theta(t)+\frac{d}{dt}y(t)\right)^2}{2}+\frac{c_2\left(L_2\cos\left(\theta(t)\right)\frac{d}{dt}\theta(t)+\frac{d}{dt}y(t)\right)^2}{2}$$

In [36]:
```
L = T - V
L_eqn1 = diff(diff(L,diff(θ,t)),t) - diff(L,θ) + diff(D,diff(θ,t))
```

Out[36]:
$$J\frac{d^2}{dt^2}\theta(t) - L_1c_1\left(-L_1\cos\left(\theta(t)\right)\frac{d}{dt}\theta(t)+\frac{d}{dt}y(t)\right)\cos\left(\theta(t)\right) - L_1k_1\left(-L_1\sin\left(\theta(t)\right)+h_0-l1$$
$$+ L_2c_2\left(L_2\cos\left(\theta(t)\right)\frac{d}{dt}\theta(t)+\frac{d}{dt}y(t)\right)\cos\left(\theta(t)\right) + L_2k_2\left(L_2\sin\left(\theta(t)\right)+h_0-l2_0+y(t$$

In [37]:
```
L_eqn2 = diff(diff(L,diff(y,t)),t) - diff(L,y) + diff(D,diff(y,t)) |> s
```

Out[37]:
$$-c_1\left(L_1\cos\left(\theta(t)\right)\frac{d}{dt}\theta(t)-\frac{d}{dt}y(t)\right)+c_2\left(L_2\cos\left(\theta(t)\right)\frac{d}{dt}\theta(t)+\frac{d}{dt}y(t)\right)+gm$$
$$-k_1\left(L_1\sin\left(\theta(t)\right)-h_0+l1_0-y(t)\right)+k_2\left(L_2\sin\left(\theta(t)\right)+h_0-l2_0+y(t)\right)+m\frac{d^2}{dt^2}y(t)$$

In [38]:
```
M_eqn + L_eqn1 |> simplify
```

Out[38]: 0

cordinate shift!

In [ ]:
```

```

In [39]:
```
AllDotsGoToZero = Dict(diff(θ,t,t)=>0,diff(θ,t)=>0,diff(y,t,t)=>0,diff(
Equil_F = F_eqn |> subs(AllDotsGoToZero) |> subs(y=>y₀,θ=>θ₀)
```

Out[39]:
$$gm+k_1\left(-L_1\sin\left(\theta_0\right)+h_0-l1_0+y_0\right)+k_2\left(L_2\sin\left(\theta_0\right)+h_0-l2_0+y_0\right)$$

In [40]:
```
Equil_M = M_eqn |> subs(AllDotsGoToZero) |> subs(y=>y₀,θ=>θ₀)
```

Out[40]:
$$L_1k_1\left(-L_1\sin\left(\theta_0\right)+h_0-l1_0+y_0\right)\cos\left(\theta_0\right)-L_2k_2\left(L_2\sin\left(\theta_0\right)+h_0-l2_0+y_0\right)\cos\left(\theta_0\right)$$

In [41]:
```
horiz_Equil = [(Equil_F |> subs.(θ=>0)), (Equil_M|> subs.(θ=>0))]
```

Out[41]:
$$\left[\begin{array}{r} gm+k_1\left(-L_1\sin\left(\theta_0\right)+h_0-l1_0+y_0\right)+k_2\left(L_2\sin\left(\theta_0\right)+h_0-l2_0 \\ L_1k_1\left(-L_1\sin\left(\theta_0\right)+h_0-l1_0+y_0\right)\cos\left(\theta_0\right)-L_2k_2\left(L_2\sin\left(\theta_0\right)+h_0-l2_0+y_0\right)\mathrm{cc} \end{array}\right.$$

In [42]:

```
Linear_Rule = Dict(z₁=>θ , z₂=>y , z₃=>diff(θ,t), z₄=>diff(y,t))
```

Out[42]:

$$\begin{cases} z_3 & => & \frac{d}{dt}\theta(t) \\ z_1 & => & \theta(t) \\ z_4 & => & \frac{d}{dt}y(t) \\ z_2 & => & y(t) \end{cases}$$

In [43]:

```
z = [z₁; z₂; z₃; z₄].subs(Linear_Rule)
EOM = solve([F_eqn,M_eqn],[diff(y,t,t),diff(θ,t,t)])
ż = diff.(z)
Reverse_Linear_Rule = Dict(θ=>z₁ , y=>z₂ , diff(θ,t)=>z₃, diff(y,t)=>z₄
ż = ż.subs(EOM).subs(Reverse_Linear_Rule)
```

Out[43]:

$$\left[\begin{array}{c} \\ \\ \frac{\left(-L_1^2 c_1 z_3 \cos\left(z_1\right) - L_1^2 k_1 \sin\left(z_1\right) + L_1 c_1 z_4 + L_1 h_0 k_1 - L_1 k_1 l1_0 + L_1 k_1 z_2 - L_2^2 c_2 z_3 \cos\left(z_1\right) - L_2^2 k_2 \sin\left(z_1\right) - L_2 c_2 z_4 - L_2 h_0 k_2 + L_2 k_2}{J} \\ \frac{L_1 c_1 z_3 \cos\left(z_1\right) + L_1 k_1 \sin\left(z_1\right) - L_2 c_2 z_3 \cos\left(z_1\right) - L_2 k_2 \sin\left(z_1\right) - c_1 z_4 - c_2 z_4 - gm - h_0 k_1 - h_0 k_2 + }{m} \end{array}\right.$$

In [44]:

```
A = fill(k1,(4,4))
    for i=1:4
        A[i,1] = diff(ż[i],z₁)
        A[i,2] = diff(ż[i],z₂)
        A[i,3] = diff(ż[i],z₃)
        A[i,4] = diff(ż[i],z₄)
    end
A
```

Out[44]:

$$\left[\begin{array}{c} \\ \frac{\left(L_1^2 c_1 z_3 \sin\left(z_1\right) - L_1^2 k_1 \cos\left(z_1\right) + L_2^2 c_2 z_3 \sin\left(z_1\right) - L_2^2 k_2 \cos\left(z_1\right)\right) \cos\left(z_1\right)}{J} \\ -\frac{\left(-L_1^2 c_1 z_3 \cos\left(z_1\right) - L_1^2 k_1 \sin\left(z_1\right) + L_1 c_1 z_4 + L_1 h_0 k_1 - L_1 k_1 l1_0 + L_1 k_1 z_2 - L_2^2 c_2 z_3 \cos\left(z_1\right) - L_2^2 k_2 \sin\left(z_1\right) - L_2 c_2 z_4 - L_2 h_0 k_2 + L}{J} \\ -L_1 c_1 z_3 \sin\left(z_1\right) + L_1 k_1 \cos\left(z_1\right) - \end{array}\right.$$

In [45]:

```
B = fill(k1,(4,1))
    for i = 1:4
        B[i] = diff(ż[i],u)
    end
B
```

Out[45]:

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

In [ ]: