4.7 Rotating pendulum of doom

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I think this problem is different from the furuta pendulum because:

1. There is no mass spoken of for the horizontal arm.
2. There is no moment of inertia for the second arm, and instead there is a spring.
3. I don't think I should be using Rotational matrices

1 Define Energy

1.1 Kinetic Energy of Arm 1

```
using SymPy
@vars J m L0 t g k d
theta = SymFunction("theta")(t)
Omega = SymFunction("Omega")(t)
L = SymFunction("L")(t)
Torque = SymFunction("Torque")(t)
T1 = 1//2*J*Omega^2
```

$$\frac{J\Omega^2(t)}{2}$$

1.2 Potential Engergy of Arm 1

```
V1 = 0;
```

1.3 Kinetic Energy of Arm 2

The Kinetic Energy of Arm 2 is tough. The first term is rotational energy, the second term is the translational energy in the direction of the spring. The last term I am not sure about. I think there needs to be translational energy in the y-z plane (This needs to account for the fact that the coordinates themselves are rotating).

```
v = Omega*d*sin(theta) + Omega*d*cos(theta) + diff(theta,t)*L - Omega*L*sin(theta)
I_theta = 1//2*m*L^2
T2 = 1//2*I_theta*diff(theta,t)^2 + 1//2*m*diff(L,t)^2 + 1//2*m*v
```

$$\frac{m\left(d\Omega(t)\sin\left(\theta(t)\right)+d\Omega(t)\cos\left(\theta(t)\right)-L(t)\Omega(t)\sin\left(\theta(t)\right)+L(t)\frac{d}{dt}\theta(t)\right)}{2}+\frac{mL^2(t)\left(\frac{d}{dt}\theta(t)\right)^2}{4}+\frac{m\left(\frac{d}{dt}L(t)\right)^2}{2}$$

1.4 Potential Energy of Arm 2

```
V2 = m*g*L*(1-cos(theta)) + 1//2*k*(L-L0)^2
```

$$gm\left(1-\cos\left(\theta(t)\right)\right)L(t)+\frac{k\left(-L_0+L(t)\right)^2}{2}$$

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2 Get Equations of Motion via Lagrange

```
T = T1 + T2
V = V1 + V2
Langr = T-V
L_eqn1 = diff(diff(Langr,diff(theta)),t) - diff(Langr,theta)
```

$$gmL(t)\sin\left(\theta(t)\right)-\frac{m\left(-d\Omega(t)\sin\left(\theta(t)\right)+d\Omega(t)\cos\left(\theta(t)\right)-L(t)\Omega(t)\cos\left(\theta(t)\right)\right)}{2}+\frac{mL^2(t)\frac{d^2}{dt^2}\theta(t)}{2}+mL(t)\frac{d}{dt}$$

```
L_eqn2 = Torque - (diff(diff(Langr,diff(Omega)),t) - diff(Langr,Omega))
```

$$J\Omega(t)+\frac{m\left(d\sin\left(\theta(t)\right)+d\cos\left(\theta(t)\right)-L(t)\sin\left(\theta(t)\right)\right)}{2}+\mathrm{Torque}\left(t\right)$$

```
L_eqn1 = diff(diff(Langr,diff(L)),t) - diff(Langr,L)
```

$$gm\left(1-\cos\left(\theta(t)\right)\right)+\frac{k\left(-2L_0+2L(t)\right)}{2}-\frac{m\left(-\Omega(t)\sin\left(\theta(t)\right)+\frac{d}{dt}\theta(t)\right)}{2}-\frac{mL(t)\left(\frac{d}{dt}\theta(t)\right)^2}{2}+m\frac{d^2}{dt^2}L(t)$$