Computational Dynamics Homework4 Part2 4.5 Airfoil

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1 The Airfoil

1.1 Lagrange's Equations

First, define the Kinetic Energy

```
using SymPy
@vars kt k J t m p_1 p_2
theta = SymFunction("theta")(t)
y = SymFunction("y")(t)
T = 1//2*m*diff(y,t)^2 + 1//2*J*diff(theta,t)^2
```

$$\frac{J\left(\frac{d}{dt}\theta(t)\right)^2}{2} + \frac{m\left(\frac{d}{dt}y(t)\right)^2}{2}$$

The potential energy

```
V = 1//2*kt*theta^2 + 1//2*k*y^2
```

$$\frac{ky^2(t)}{2} + \frac{kt\theta^2(t)}{2}$$

Define the Langrangian

```
L = T-V
```

$$\frac{J\left(\frac{d}{dt}\theta(t)\right)^2}{2} - \frac{ky^2(t)}{2} - \frac{kt\theta^2(t)}{2} + \frac{m\left(\frac{d}{dt}y(t)\right)^2}{2}$$

Get equations of motion

```
eqn1 = diff(diff(L,diff(theta,t)),t) -diff(L,theta)
```

$$J\frac{d^2}{dt^2}\theta(t) + kt\theta(t)$$

```
eqn2 = diff(diff(L,diff(y,t)),t) -diff(L,y)
```

$$ky(t) + m\frac{d^2}{dt^2}y(t)$$

1.2 Hamilton's Equations

Get generalized momenta

```
zero1 = diff(L,diff(theta,t)) - p_1
zero2 = diff(L,diff(y,t)) - p_2
rule_1 = solve([zero1,zero2],[diff(theta,t),diff(y,t)])
```

```
Dict{Any,Any} with 2 entries:
  Derivative(y(t), t)     => p_2/m
  Derivative(theta(t), t) => p_1/J
```

1.3 But the equations of motion do not have any momentum dependence.

Build the Hamiltonian and get equations of motion

```
H = T + V
H = H.subs(rule_1)
H_eqn_1 = diff(diff(L,diff(theta,t)),t) + diff(H,theta)
```

$$J\frac{d^2}{dt^2}\theta(t) + kt\theta(t)$$

```
H_eqn_2 = diff(diff(L,diff(y,t)),t) + diff(H,y)
```

$$ky(t) + m\frac{d^2}{dt^2}y(t)$$