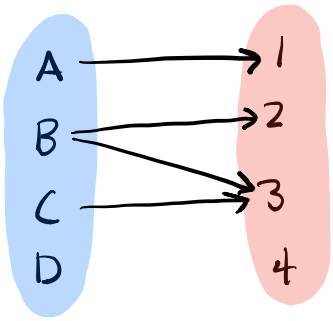
§2.1: Functions

Def: A relation is a rule that assigns elements in one set to elements in another set.

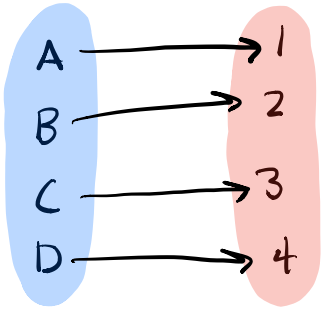
Def: A function is a rule that assigns each element in one set to a unique element in a second set.

Ex Relation or Function?

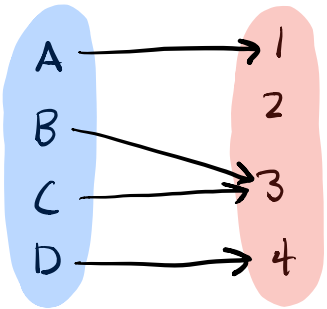
(i)



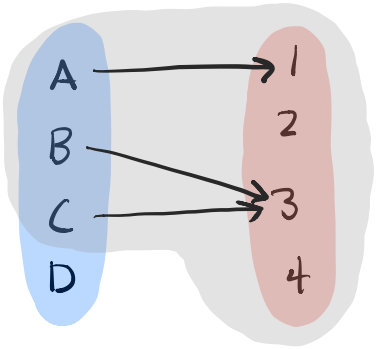
(ii)



(iii)



(iv)



Def: A relation is a set of ordered pairs.

Def: A function is a set of ordered pairs in which no two pairs have the same first coordinate and different second coordinates.

(i) $\{(A,1),(B,2),(B,3),(C,3)\}$ relation

(ii) $\{(A,1),(B,2),(C,3),(D,4)\}$ function

(iii) $\{(A,1),(B,3),(C,3),(D,4)\}$ function

(iv) $\{(A,1),(B,3),(C,3)\}$ function

* all functions are relations, but not all relations are functions

Ex) Determine whether each relation is a function

#25) $\{(-1,-1),(2,2),(3,3)\}$

#26) $\{(0.5,7),(0,7),(1,7),(9,7)\}$

#27) $\{(25,5),(25,-5),(0,0)\}$

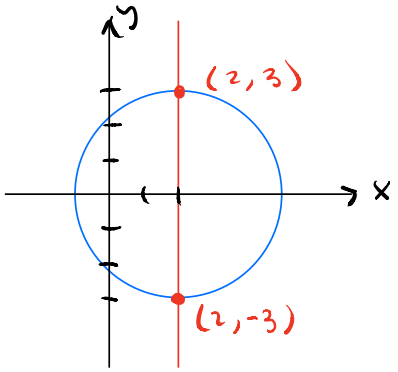
#29)

x	y
3	6
4	9
3	12

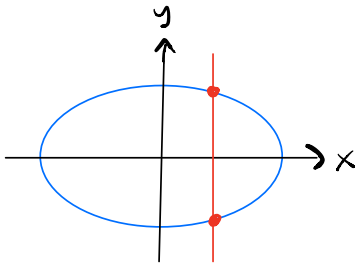
#31)

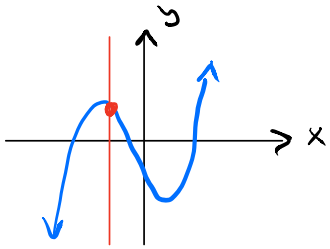
x	y
-1	1
1	1
-5	1
5	1

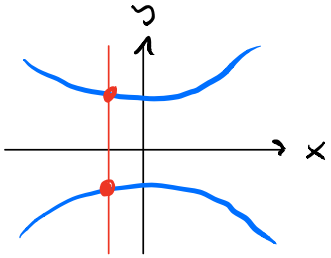
Identifying a function from a graph



Thm: A graph is the graph of a function if and only if there is no vertical line that crosses the graph more than once.







Ex) Determine whether each equation defines y as a function of x.

#33) $y = 3x - 8$ ✓

ex) $y = x^2$ ✓

ex) $x = y^2 \rightarrow y = \pm\sqrt{x}$ ✗

ex) $x = y^3 \rightarrow y = x^{1/3}$ ✓

Domain and Range

Def: The <u>domain</u> of a relation is the set of all first coordinates and the <u>range</u> is the set of all second coordinates

Ex) Determine the domain and range consider only real #s

#46) $\{(1,2),(2,4),(3,8),(4,16)\}$

D: $\{1,2,3,4\}$

R: $\{2,4,8,16\}$

#47) $\{(x,y) \mid y = 4\}$

D: $(-\infty, \infty)$

R: $\{4\}$

#50) $y = x^2 + 8$

* what is the smallest value y can obtain

D: $(-\infty, \infty)$

R: $[8, \infty)$

#52) $x + 2 = \sqrt{y}$ $\longrightarrow$ $x = \sqrt{y} - 2$

D: $[-2, \infty)$

R: $[0, \infty)$

$y \geq 0$ and $x = \sqrt{y} - 2$

#54) $y = \sqrt{5-x}$

D: $(-\infty, 5]$

R: $[0, \infty)$

$5 - x \geq 0$

$-x \geq -5$

$x \leq 5$

#56) $x = -|y|$

D: $(-\infty, 0]$

R: $(-\infty, \infty)$

Function Notation.

Ex) $h = \{(1,4), (6,0), (7,9)\}$, $f(x) = \sqrt{x-3}$

a) $h(6) = \boxed{0}$

b) $f(7) = \sqrt{7-3}$

$= \sqrt{4}$

$= \boxed{2}$

c) x if $f(x) = 5$

$5 = \sqrt{x-3}$

$25 = x - 3$

$x = \boxed{28}$

Ex) $f(x) = 3x^2 - x$

ex) $f(\☺) = 3\☺^2 - \☺$

#68) $f(w) = 3w^2 - w$

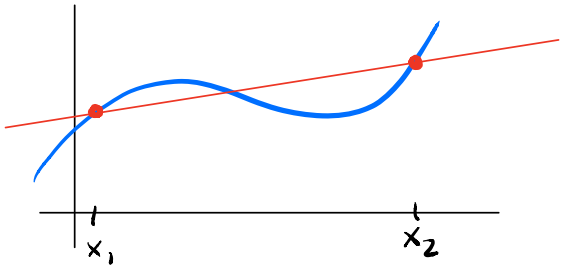
#74) $f(x+h) = 3(x+h)^2 - (x+h)$

$= 3(x^2 + 2hx + h^2) - (x+h)$

$= 3x^2 + 6hx - x + h^2 - h$

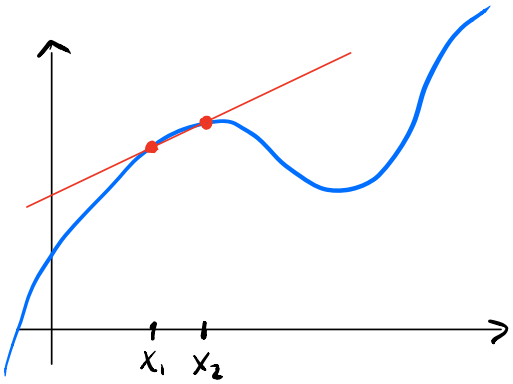
$= 3x^2 + (6h-1)x + h^2 - h$

Average Rate of Change



Slope of the lines passing through the two points.

$$\frac{\Delta f}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_1 - x_2}$$



What if those two points get close to one another?

$x_1 = x$

$x_2 = x + h$ where h is small.

$$\frac{\Delta f}{\Delta x} = \frac{f(x+h) - f(x)}{x + h - x}$$

$$= \frac{f(x+h) - f(x)}{h}$$

ex) $f(x) = 3x^2 - x$

$f(x+h) = 3x^2 + (6h-1)x + h^2 - h$

$f(x) = 3x^2 - x$

$$\frac{f(x+h) - f(x)}{h} = \frac{3x^2 + (6h-1)x + h^2 - h - (3x^2 - x)}{h}$$

$$= \frac{(3x^2 - 3x^2) + 6hx + (-x + x) + h^2 - h}{h}$$

$$= \frac{6hx - h + h^2}{h}$$

$$= \frac{h(6x-1+h)}{h}$$

$$= 6x - 1 + h$$

Now suppose $h \to 0$ $\quad \frac{f(x+h)-f(x)}{h} \to 6x-1$

↑ this is a derivative!