§1.6: Miscellaneous Equations.

Factoring Higher-Degree Equations (Special Cases)

#2) $x^3 - x^2 - 5x + 5 = 0$

$= (x^3 - x^2) - (5x - 5)$

$= x^2(x-1) - 5(x-1)$

$= (x-1)(x^2 - 5)$

$\begin{array}{ll} x - 1 = 0 & x^2 - 5 = 0 \\ \quad x = 1 & \quad x^2 = 5 \\ & \quad x = \pm\sqrt{5} \end{array}$

$\boxed{\{1, \pm\sqrt{5}\}}$

SOAP

#10) $w^4 + 8w = 0$

$= w(w^3 + 8)$

$= w(w^3 + 2^3)$

$= w(w+2)(w^2 - 2w + 4)$

$a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$

$w = 0$ or $w + 2 = 0$ or $w^2 - 2w + 4 = 0$

$\qquad\qquad\quad w = -2$

$$w = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(4)}}{2(1)}$$

$$= \frac{2 \pm \sqrt{4 - 16}}{2}$$

$$= \frac{2}{2} \pm \frac{1}{2}\sqrt{-12}$$

$$= 1 \pm \frac{1}{2}\sqrt{(-1)(4)(3)}$$

$$= 1 \pm \frac{1}{2}\sqrt{-1}\sqrt{4}\sqrt{3}$$

$$= 1 \pm i\sqrt{3}$$

$\boxed{\{1 \pm i\sqrt{3}\}}$

Equations Involving Square Roots.

#12) $\sqrt{x-1} = x - 7$

$\left(\sqrt{x-1}\right)^2 = (x-7)^2$

$x - 1 = x^2 - 14x + 49$

$-x + 1 \qquad\qquad -x + 1$

both negative
same sign

$0 = x^2 \ominus 15x \oplus 50$

$= (x-10)(x-5)$

$\{10, 5\}$ possible.

$x - 10 = 0 \quad OR \quad x - 5 = 0$

$x = 10 \qquad\qquad x = 5$

no imaginary
$x - 1 \geq 0$
$x \geq 1$

square root is not negative.
$x - 7 \geq 0$
$x \geq 7$

Check: $\sqrt{10-1} \overset{?}{=} 10 - 7$ $\qquad\qquad$ $\sqrt{5-1} \overset{?}{=} 5 - 7$

$\sqrt{9} \overset{?}{=} 3$ ✓ $\qquad\qquad$ $\sqrt{4} \overset{?}{=} -2$ ✗

$\boxed{\{10\}}$

#26) $\sqrt{3x-2} - \sqrt{x-2} = 2$

$\left(\sqrt{3x-2}\right)^2 = \left(2 + \sqrt{x-2}\right)^2$

$3x - 2 = \left(2 + \sqrt{x-2}\right)\left(2 + \sqrt{x-2}\right)$

$= 4 + 2\sqrt{x-2} + 2\sqrt{x-2} + \left(\sqrt{x-2}\right)^2$

$3x - 2 = 4 + 4\sqrt{x-2} + x - 2$

$3x - 2 = x + 2 + 4\sqrt{x-2}$

$-x - 2 \qquad -x - 2$

$(2x - 4)^2 = \left(4\sqrt{x-2}\right)^2$

$(2x - 4)^2 = 4^2\left(\sqrt{x-2}\right)^2$

$= 16(x-2)$

$$(2x-4)(2x-4) = 16x - 32$$

$$4x^2 - 8x - 8x + 16 = 16x - 32$$

$$4x^2 - 16x + 16 = 16x - 32$$
$$\underline{\qquad -16x + 32 \qquad -16x + 32}$$

$$4x^2 - 32x + 48 = 0$$

$$x^2 - 8x + 12 = 0$$

$$(x - 6)(x - 2) = 0 \qquad\qquad \longrightarrow \{6, 2\} \text{ possible}$$

$$x - 6 = 0 \quad \text{OR} \quad x - 2 = 0$$

$$x = 6 \qquad\qquad x = 2$$

Check: $\sqrt{3x-2} - \sqrt{x-2} = 2$

$x = 6:$ $\sqrt{3(6)-2} - \sqrt{6-2}$

$\quad = \sqrt{18-2} - \sqrt{4}$

$\quad = \sqrt{16} - \sqrt{4}$

$\quad = 4 - 2$

$\quad = 2 \checkmark$

$x = 2:$ $\sqrt{3(2)-2} - \sqrt{2-2}$

$\quad = \sqrt{6-2} - \sqrt{0}$

$\quad = \sqrt{4} - 0$

$\quad = 2 \checkmark$

$$\boxed{\{6, 2\}}$$

Equations with Rational Exponents $\qquad a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m$

#28) $x^{2/3} = \dfrac{1}{2}$

$\left(x^{\frac{1}{3}}\right)^2 = \dfrac{1}{2}$

$x^{1/3} = \pm\sqrt{\dfrac{1}{2}}$

$\left(x^{1/3}\right)^3 = \left(\pm\dfrac{\sqrt{2}}{2}\right)^3$

$x = \pm\dfrac{(\sqrt{2})^3}{2^3}$

$\quad = \pm\dfrac{(\sqrt{2})^2 \cdot \sqrt{2}}{2^3}$

$\quad = \pm\dfrac{2 \cdot \sqrt{2}}{8}$

$\quad = \pm\dfrac{\sqrt{2}}{4}$

$$\boxed{\left\{\pm\dfrac{\sqrt{2}}{4}\right\}}$$

#34) $(s-2)^{-1/2} = \frac{1}{3}$

$$\left((s-2)^{1/2}\right)^{-1} = \frac{1}{3}$$

$$\left((s-2)^{1/2}\right)^{2} = (3)^{2}$$

$$s-2 = 9$$

$$s = 11$$

$\boxed{\{11\}}$

Equations of Quadratic Type

$$au^2 + bu + c = 0$$

#38) $x^4 - x^2 - 12 = 0$ $u = x^2$

$$(x^2)^2 - x^2 - 12 = 0$$

$$u^2 - u \ominus 12 = 0$$

$$(u-4)(u+3) = 0$$

$$u = 4 \quad or \quad u = -3$$

$x^2 = 4 \quad or \quad x^2 = -3$

$x = \pm\sqrt{4} \quad or \quad x = \pm\sqrt{-3}$

$x = \pm 2 \qquad\qquad x = \pm i\sqrt{3}$

$\boxed{\{\pm 2, \pm i\sqrt{3}\}}$

#44) $\frac{1}{(x-3)^2} + \frac{2}{x-3} - 24 = 0$ $u = \frac{1}{x-3}$

$$\left(\frac{1}{x-3}\right)^2 + 2\left(\frac{1}{x-3}\right) - 24$$

$$u^2 + 2u \ominus 24 = 0$$

$$(u+6)(u-4) = 0$$

$$u = -6 \quad or \quad u = 4$$

$\frac{1}{x-3} = -6 \quad or \quad \frac{1}{x-3} = 4$

$x-3 = -\frac{1}{6} \quad or \quad x-3 = \frac{1}{4}$

$x = 3 - \frac{1}{6} \quad or \quad x = 3 + \frac{1}{4}$

$\quad = \frac{18}{6} - \frac{1}{6} \qquad x = \frac{12}{4} + \frac{1}{4}$

$\quad = \frac{17}{6} \qquad\qquad = \frac{13}{4}$

$\boxed{\left\{\frac{17}{6}, \frac{13}{4}\right\}}$

*use $w = u^2 + 2u$

#46) $(u^2 + 2u)^2 - 2(u^2 + 2u) - 3 = 0$ $\{\pm 1, -3\}$

$w^2 - 2w - 3 = 0$

$(w + 1)(w - 3) = 0$

$\boxed{\{\pm 1, -3\}}$

$w + 1 = 0$ or $w - 3 = 0$

$u^2 + 2u \oplus 1 = 0$ or $u^2 + 2u \ominus 3 = 0$

$(u + 1)(u + 1) = 0$ or $(u + 3)(u - 1) = 0$

$u + 1 = 0$ or $u + 3 = 0$ or $u - 1 = 0$

$u = -1$ or $u = -3$ or $u = 1$

see section 3.2 for synthetic division.

Absolute Value Equations

#54) $|a^2 - 1| = 1$

$a^2 - 1 = 1$ or $a^2 - 1 = -1$
$+1 \quad +1$ $+1 \quad +1$

$a^2 = 2$ or $a^2 = 0$

$\boxed{\{0, \pm\sqrt{2}\}}$

$a = \pm\sqrt{2}$ $a = 0$

#56) $|z^2 - 12| = \boxed{z}$

$z^2 - 12 = z$ or $z^2 - 12 = -z$

$z^2 - z \ominus 12 = 0$ OR $z^2 + z - 12 = 0$

$(z + 3)(z - 4) = 0$ or $(z + 4)(z - 3) = 0$

$z + 3 = 0$ $z - 4 = 0$ $z + 4 = 0$ $z - 3 = 0$

$\{\pm 3, \pm 4\}$ possible

Check: $|z^2 - 12| \geq 0$ so right hand side cannot be < 0.

$\Rightarrow$ $\boxed{\{3, 4\}}$

#64) $|2x - 3| = |2x + 7|$

By original equation the are equal or opposite

$2x - 3 = 2x + 7$ or $2x - 3 = -(2x + 7)$

$\underline{-2x + 3 \quad -2x + 3}$ $2x - 3 = -2x - 7$

$\quad\quad 0 = 10$ ✗ $\underline{+2x + 3 \quad +2x + 3}$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad 4x = -4$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad x = -1$

Check: $|2(-1) - 3| \overset{?}{=} |2(-1) + 7|$ $\boxed{\{-1\}}$

$\quad\quad\quad\quad\quad |-5| \overset{?}{=} |5|$ ✓

Desk Work:

#65) $\sqrt{16x + 1} - \sqrt{6x + 13} = -1$

$\quad\quad \{\frac{1}{2}\}$

#73) $\left(\frac{x-2}{3}\right)^2 - 2\left(\frac{x-2}{3}\right) + 10 = 0$

$\quad\quad \{5 \pm 9i\}$

#82) $(1 - 2m)^{-5/3} = -\frac{1}{32}$

$\quad\quad \{\frac{9}{2}\}$

#84) $|x^2 + 7x| = x^2 - 4$

$\quad\quad \{-4\}$