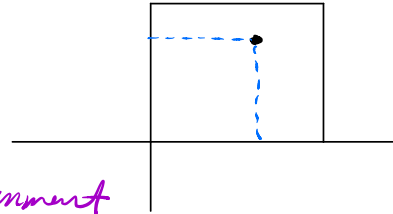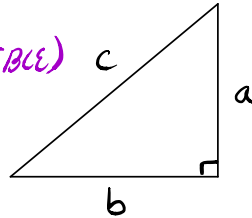# §1.3: Equations and Graphs in Two Variables

Pythagorean Theorem and Distance Formula
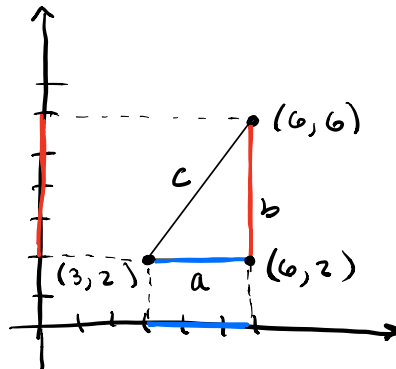
Pitagoris

Samis, Greece (570 - 495 BCE)

$$a^2 + b^2 = c^2$$

René Descartes (1596 - 1650 CE)
- Philosopher and mathematician enlightenment
- liked to stay in bed late.
- once saw a fly walking on the ceiling
- describe the fly trajectory with #s
  if using one of the corners of his room as a reference point
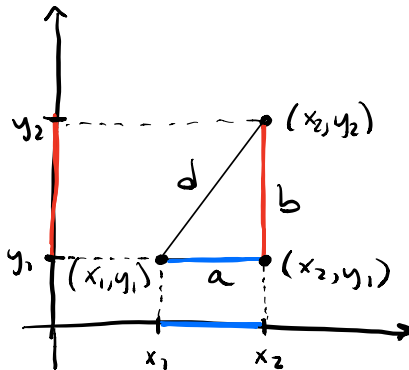
Geometry to algebra

$a = 6 - 3 = 3$
$b = 6 - 2 = 4$

$$c = \sqrt{a^2 + b^2}$$
$$= \sqrt{3^2 + 4^2}$$
$$= \sqrt{9 + 16}$$
$$= \sqrt{25}$$
$$= 5$$

(6,6)

c

b

(3,2) a (6,2)

(x_2, y_2)

d

b

(x_1, y_1) a (x_2, y_1)
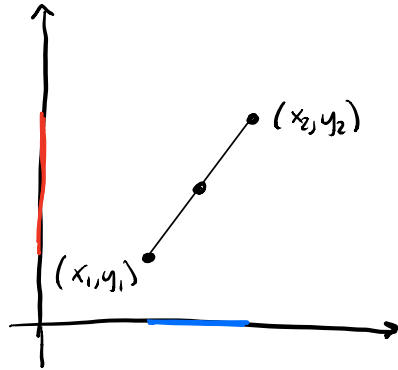
y_2 y_1 x_1 x_2

$a = x_2 - x_1$
$b = y_2 - y_1$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Doesn't matter which
point is (x_1, y_1) and (x_2, y_2)

Ex. Ask for numbers

Midpoint Formula

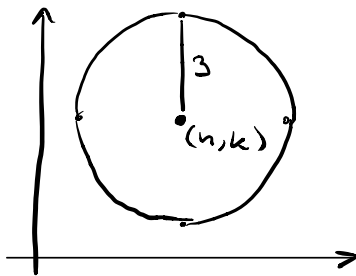$$MP = \left(\frac{x_1 + x_2}{2} , \frac{y_1 + y_2}{2} \right)$$

$(7,2), (5,-3)$
$(-1,4), (-7,3)$

The equation of a circle (standard form)

$$(x-h)^2 + (y-k)^2 = r^2$$

where the center is (h,k) and r is the radius.

#36) Determine the center and radius of the circle and sketch the graph

$$x^2 + (y-2)^2 = 16$$

center: $(0,2)$
radius: 4

Ex) Center at $(-7, 2)$ passing through $(5, 7)$.

$$(x - (-7))^2 + (y - 2)^2 = r^2$$

$$r^2 = (-7 - 5)^2 + (2 - 7)^2$$
$$= (-12)^2 + (-5)^2$$
$$= 144 + 25$$
$$= 169$$

$$(x + 7)^2 + (y - 2)^2 = 169$$
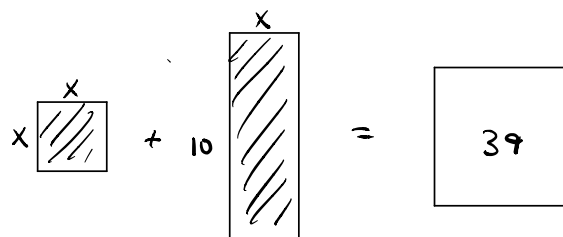
Ex) $x^2 + 6x + y^2 - 8y = 0$

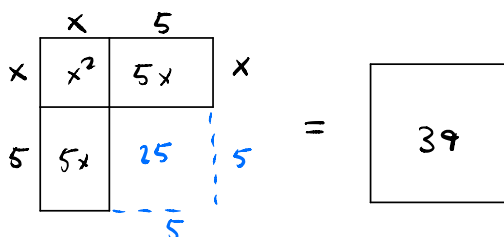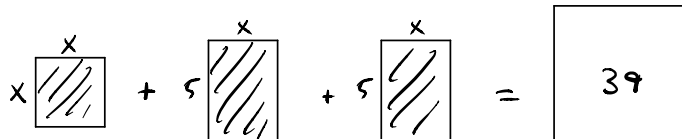propose this is an equation of a circle

aside, completing the square

Completing the square.

$x^2 + \underline{10}x = 39$
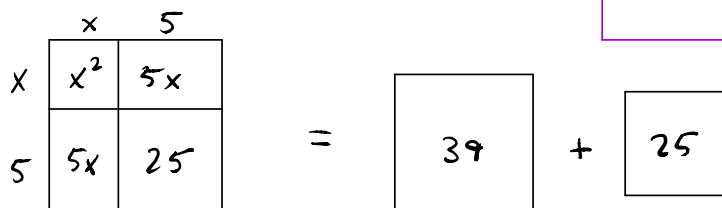
Write as $(x + \_\_)^2 = \_\_$

$x^2 + 5x + 5x = 39$

$= \boxed{39} + \boxed{25}$ $x^2 + 10x + 25 = 39 + 25$
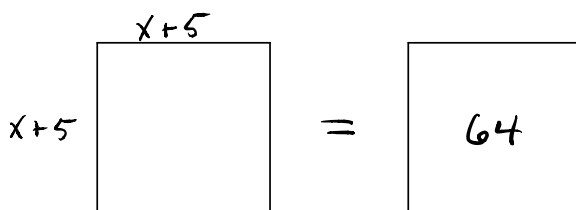
$(x+5)^2 = 64$

Recap $x^2 + 10x + \left(\frac{10}{2}\right)^2 = 39 + \left(\frac{10}{2}\right)^2$

$x^2 + bx = d$

$x^2 + bx + \left(\frac{b}{2}\right)^2 = d + \frac{b}{2}$

$\left(x + \frac{b}{2}\right)^2 = d + \frac{b}{2}$

Ex) $x^2 - 14x = 51$
$(x^2 - 14x + (-7)^2) = 51 + (-7)^2$
$x^2 - 14x + 7^2 = 100$
$(x - 7)^2 = 100$

* we will see this
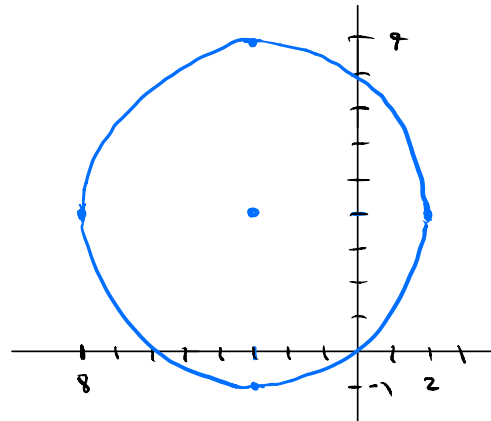again in chap 2.

Ex) Sketh the graph of the circle

$$x^2 + 6x + y^2 - 8y = 0$$

$$(x^2 + 6x + 9) + (y^2 - 8y + 16) = 0 + 9 + 16$$

$$(x + 3)^2 + (y - 4)^2 = 25 = 5^2$$

center = $(-3, 4)$

radius = 5

The Line.

Standard Form $\quad Ax + By = C \quad\quad$ A, B, C are real

ex. $\quad 2x + 3y = 5 \quad\quad x = 4 \quad\quad y = 9$

<u>x-intercept</u> — where line crosses x-axis
y-intercept — where line crosses y-axis

How to find intercepts?

Ex) Find the x and y-intercepts of the lines
$\quad$ a) $\quad 2x - 3y = 12 \quad\quad$ what is the x-axis? $\quad y = 0$
$\quad\quad\quad\quad\quad\quad\quad\quad\quad$ what is the y-axis? $\quad x = 0$

x-intercept : what is x when $y = 0$?

$$2x \quad 3 \cdot 0 = 12 \quad\quad\quad (6, 0)$$
$$2x = 12$$
$$x = 6$$

y-intercept : what is y when $x = 0$?

$$2 \cdot 0 - 3y = 12$$
$$-3y = 12$$
$$y = -4 \quad\quad\quad (0, -4)$$
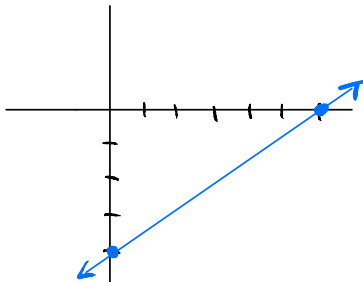
b) $4x + 6y = 12$

x-intercept:

y-intercept:

c)   ask for 3 #s. $-\_x + \_y = \_$