

§ 2.4: Operations with Functions.

Basic Operations with Functions

Def: For two functions f and g , the sum, difference, product, and quotient functions $f+g$, $f-g$, $f \cdot g$, and f/g , respectively are defined as follows

1. $(f+g)(x) = f(x) + g(x)$
2. $(f-g)(x) = f(x) - g(x)$
3. $(f \cdot g)(x) = f(x) \cdot g(x)$
4. $(f/g)(x) = f(x)/g(x)$ provided $g(x) \neq 0$.

Ex1) $f(x) = 3\sqrt{x} - 2$, $g(x) = x^2 + 5$. Find and simplify each expression

$$\begin{aligned} a) (f+g)(4) &= f(4) + g(4) \\ &= (3\sqrt{4} - 2) + (4^2 + 5) \\ &= (3 \cdot 2 - 2) + (16 + 5) \\ &= 4 + 21 = 25 \end{aligned}$$

$$\begin{aligned} b) (f-g)(x) &= f(x) - g(x) \\ &= (3\sqrt{x} - 2) - (x^2 + 5) \\ &= 3\sqrt{x} - x^2 - 2 - 5 \\ &= 3\sqrt{x} - x^2 - 7 \end{aligned}$$

$$\begin{aligned}
 c) (f \cdot g)(0) &= f(0) \cdot g(0) \\
 &= (3\sqrt{0} - 2)(0^2 + 5) \\
 &= (0 - 2)(0 + 5) \\
 &= (-2)(5) = -10
 \end{aligned}$$

$$d) \left(\frac{f}{g}\right)(9) = \frac{f(9)}{g(9)} = \frac{3\sqrt{9} - 2}{9^2 + 5} = \frac{3 \cdot 3 - 2}{81 + 5} = \frac{7}{86}$$

Ex 2) $f = \{(1, 3), (2, 8), (3, 6), (5, 9), (6, 0)\}$
 $g = \{(1, 6), (2, 11), (3, 0), (4, 1), (6, 1)\}$

Find each expression and state the domain.

a) $f + g = \{(1, 9), (2, 19), (3, 6), (6, 1)\}$ * leave the first coordinate alone
 $D: \{1, 2, 3\}$ * both functions must be defined

b) $f \cdot g = \{(1, 18), (2, 88), (3, 0), (6, 0)\}$
 $D: \{1, 2, 3\}$

c) $\frac{f}{g} = \{(1, \frac{1}{2}), (2, \frac{8}{11}), (6, 0)\}$

Ex 3) $f(x) = \sqrt{x}$, $g(x) = 3x + 1$, $h(x) = x - 1$

Find each function and state the domain

$$D(f) = [0, \infty) \quad D(h) = (-\infty, \infty)$$

$$D(g) = (-\infty, \infty)$$

$$a) f+g \Rightarrow (f+g)(x) = f(x) + g(x) = \sqrt{x} + 3x + 1$$

$$D(f+g) = D(f) \cap D(g) = [0, \infty) \cap (-\infty, \infty) = [0, \infty)$$

$$b) \frac{g}{f} \quad (g/f)(x) = \frac{g(x)}{f(x)} = \frac{3x+1}{\sqrt{x}} \text{ without}$$

$$\begin{aligned} D(g/f) &= (D(g) \cap D(f)) \setminus \{ \text{where } f(x) = 0 \} \\ &= ((-\infty, \infty) \cap [0, \infty)) \setminus \{ 0 \} \\ &= [0, \infty) \setminus \{ 0 \} \\ &= (0, \infty) \end{aligned}$$

$$c) g \cdot h$$

$$\begin{aligned} (g \cdot h)(x) &= g(x) \cdot h(x) \\ &= (3x+1)(x-1) \\ &= 3x^2 - 3x + x - 1 \\ &= 3x^2 - 2x - 1 \end{aligned}$$

$$\begin{aligned} D(g \cdot h) &= D(g) \cap D(h) \\ &= (-\infty, \infty) \cap (-\infty, \infty) \\ &= (-\infty, \infty) \end{aligned}$$

$$d) g-h$$

$$\begin{aligned} (g-h)(x) &= g(x) - h(x) \\ &= 3x+1 - (x-1) \\ &= 3x+1 - x + 1 \\ &= 2x+2 \end{aligned}$$

$$\begin{aligned} D(g-h) &= D(g) \cap D(h) = (-\infty, \infty) \cap (-\infty, \infty) \\ &= (-\infty, \infty) \end{aligned}$$

Composition of Functions.

Def: If f and g are two functions, the composition of f and g , written $f \circ g$, is defined by the equation

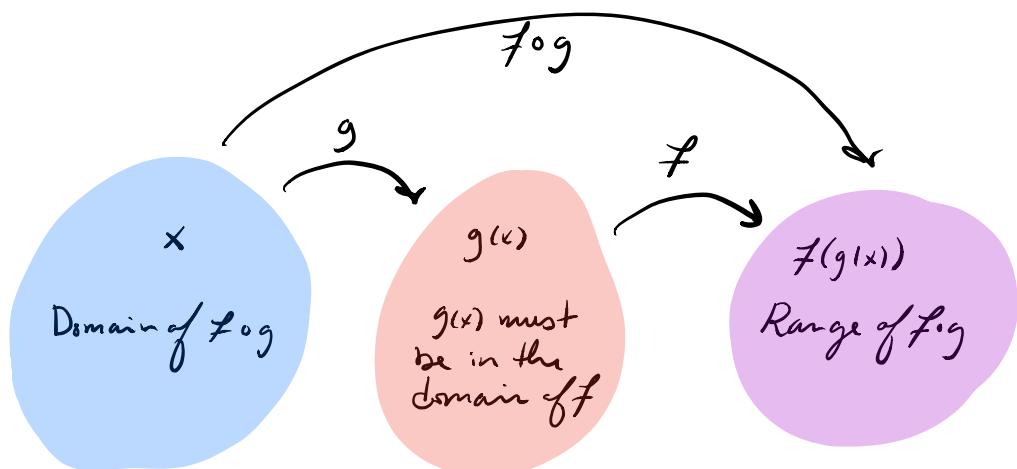
$$(f \circ g)(x) = f(g(x))$$

provided that $g(x)$ is in the domain of f .

The composition of g and f , written $g \circ f$, is defined by

$$(g \circ f)(x) = g(f(x))$$

provided that f is in the domain of g .



$$\text{Ex 4) } f = \{(3, 8), (4, 9), (5, 10)\}$$

$$g = \{(1, 4), (2, 5), (3, 6)\}$$

Find $f \circ g$

$$f \circ g = \{(1, 9), (2, 10), (3, \cancel{3})\}$$

$$= \{(1, 9), (2, 10)\}$$

$$\text{Ex 5) } f(x) = \sqrt{x}, g(x) = 2x - 1, h(x) = x^2$$

Find the value of each expression

$$\begin{aligned} a) (f \circ g)(5) &= f(g(5)) \\ &= f(2 \cdot 5 - 1) \\ &= f(9) \\ &= \sqrt{9} \\ &= 3 \end{aligned}$$

$$b) g \circ f(5) = g(f(5)) = g(\sqrt{5}) = 2 \cdot \sqrt{5} - 1$$

$$\begin{aligned} c) (h \circ g \circ f)(9) &= h(g(f(9))) \\ &= h(g(3)) \\ &= h(5) \\ &= 25 \end{aligned}$$

$$Ex(6) \quad f(x) = \sqrt{x^1}, \quad g(x) = 2x - 1, \quad h(x) = x^2$$

Find each composition function and state its domain

$$D(f) = [0, \infty), \quad D(g) = (-\infty, \infty), \quad D(h) = (-\infty, \infty)$$

- a) $f \circ g$ b) $g \circ f$ c) $f \circ h$ d) $h \circ f$

$$a) (f \circ g)(x) = f(g(x)) = f(2x - 1) = \sqrt{2x - 1}$$

$$\text{Find domain } 2x - 1 \geq 0 \Rightarrow 2x \geq 1 \Rightarrow x \geq \frac{1}{2}$$

$$D(f \circ g) = [\frac{1}{2}, \infty)$$

$$b) (g \circ f)(x) = g(f(x)) = g(\sqrt{x}) = 2\sqrt{x} - 1$$

$$\text{Find domain } x \geq 0$$

$$D(g \circ f) = [0, \infty)$$

$$c) (f \circ h)(x) = f(h(x)) = f(x^2) = \sqrt{x^2} = |x|$$

$$D(f \circ h)(x) = (-\infty, \infty)$$

$$d) (h \circ f) = h(f(x)) = h(\sqrt{x}) = (\sqrt{x})^2 = x$$

$$D(h \circ f) = [0, \infty)$$

Ex 7) Writing a function as a composition

$$f(x) = \sqrt{x}, g(x) = x - 3, h(x) = 2x$$

Write each function as a composition of appropriate functions chosen from f, g, h

a) $F(x) = \sqrt{x-3}$ b) $G(x) = x-6$ c) $H(x) = 2\sqrt{x}-3$

a) $F(x) = f(x-3) = f(g(x)) = (f \circ g)(x)$
 $\Rightarrow F = f \circ g$

b) $G(x) = x-6 = (x-3)-3$
 $= g(x-3) = g(g(x)) = (g \circ g)(x)$

$$G = g \circ g$$

c) $H(x) = 2\sqrt{x}-3$
 $= 2f(x)-3$
 $= h(f(x))-3$
 $= g(h(f(x)))$
 $= (g \circ h \circ f)(x)$

$$\Rightarrow H = g \circ h \circ f$$