§1.4: Linear Equations in Two Variables

Recall, the standard form of a line.

$$Ax + By = C$$

Slope

Def: The slope of a line which passes through the points (x_1, y_1) and (x_2, y_2) with $x_1 \neq x_2$ is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

rise/run →

Ex) Find the slope of the line passing through the two points.

#12) $(2, -1), (5, -3)$ — x_1, y_1 and x_2, y_2

$$m = \frac{-3 - (-1)}{5 - 2} = \frac{-3 + 1}{5 - 2} = \boxed{\frac{-2}{3}}$$

#13) $(5, 2), (-3, 2)$

$$m = \frac{2 - 2}{-3 - 5} = \frac{0}{-8} = \boxed{0}$$

#18) $(-7, 2), (-7, 6)$

$$m = \frac{6 - 2}{-7 - (-7)} = \frac{4}{0}$$

← slope is undefined

no slope

Ex) Determine the slope of $-2x + 5y = 10$.

Hint: Find any two points on the line. x-int, y-int.

x-int: $(-5, 0)$
y-int: $(0, 2)$

$$m = \frac{2-0}{0-(-5)} = \frac{2}{5}$$

Does it matter which two points you choose?

$$\frac{y-2}{x-0} = \frac{2}{5}$$

$$y - 2 = \frac{2}{5}x \quad \Rightarrow \quad y = \frac{2}{5}x + 2$$

Point-Slope Form.

If we know the slope and one point on the line (x_1, y_1) then, if $x \neq x_1$, then the other point is irrelevant.

$$\frac{y-y_1}{x-x_1} = m \longrightarrow y - y_1 = m(x - x_1)$$

given the slope and one point we can find the equation of the line.

Ex. Find the equation (in standard form) of the line passing through the points

#20) $(\overset{x_1}{-2}, \overset{y_1}{1}), (\overset{x_2}{3}, \overset{y_2}{5})$

$$m = \frac{5-1}{3-(-2)} = \frac{4}{5}$$

$$y - 1 = \frac{4}{5}(x - (-2))$$

$$y - 1 = \frac{4}{5}(x+2)$$

$$3(y-1) = 4(x+2)$$

$$3y - 3 = 4x + 8$$

$$\boxed{-4x + 3y = 12}$$

Slope-Intercept form

Point Slope $y - y_1 = m(x - x_1)$

$y = mx - mx_1 + y_1$

$y = mx + \underbrace{(y_1 - mx_1)}_{b}$

$y = mx + b$

what is b? When $x = 0$, $y = b$ $\quad (0, b)$ ← y-intercept.

Ex) Write the equation in slope intercept form and identify the slope

#35) $3x - 5y = 10$

$\frac{-5y}{-5} = \frac{-3x + 10}{-5}$

$y = \frac{-3x + 10}{-5}$

$= \frac{-3x}{-5} + \frac{10}{-5}$

$y = \frac{3}{5}x - 2$

$m = 3/5$, $b = -2$

#38) $y + 5 = -3(x - (-1))$

$y + 5 = -3(x + 1)$

$\frac{y + 5}{-5} = \frac{-3x - 3}{-5}$

$y = -3x - 8$

$m = -3$, $b = -8$

#44) line through (6,9) with slope $-1/3$

$y - 9 = -\frac{1}{3}(x - 6)$

$y - 9 = -\frac{1}{3}x + 2$

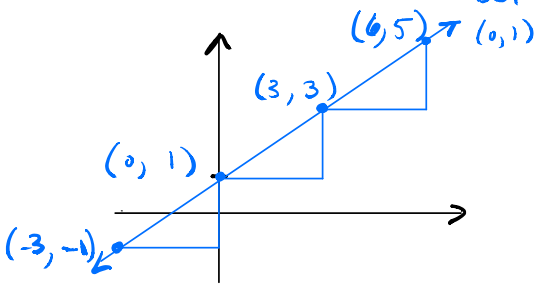
$+9 \quad\quad +9$

$y = -\frac{1}{3}x + 11$

$m = -\frac{1}{3}, \; b = 11$

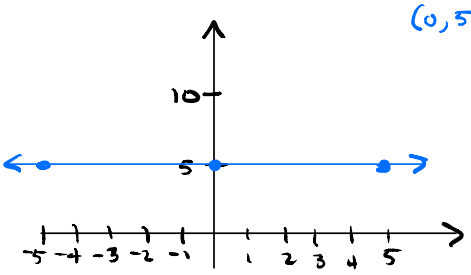
Use slope intercept point to graph

#48) $y = \frac{2}{3}x + 1$ $m = \frac{2}{3}$ $b = 1$



#54) $y - 5 = 0$

$y = 5$ $m = 0$ $b = 5$ $(0,5)$



Def: Two non-vertical lines in the coordinate plane are parallel if and only if their slopes are equal.

Def: Two lines with slopes m_1 and m_2 are perpendicular if and only if $m_1 m_2 = -1$

$m_2 = \frac{-1}{m_1}$

Ex) Find the standard form of

#78) A line parallel to $4x + 9y = 5$ containing $(-4, 2)$

$4x + 9y = 5$

$-4x \quad -4x$

$\frac{9y}{9} = \frac{-4x + 5}{9}$

$y = -\frac{4}{9}x + \frac{5}{9}$

$m = -\frac{4}{9}$, $(-4, 2)$

$y - 2 = -\frac{4}{9}(x - (-4))$

$y - 2 = -\frac{4}{9}(x + 4)$

$9y - 18 = -4(x + 4)$

$9y - 18 = -4x - 16$

$+4x + 18 \quad +4x + 18$

$4x + 9y = 2$

#80) A line perpendicular to $y = 9x + 5$ containing $(5, 4)$

$m_1 = 9 \Rightarrow m_2 = \frac{-1}{m_1} = -\frac{1}{9}$

$-9x + y = 5$

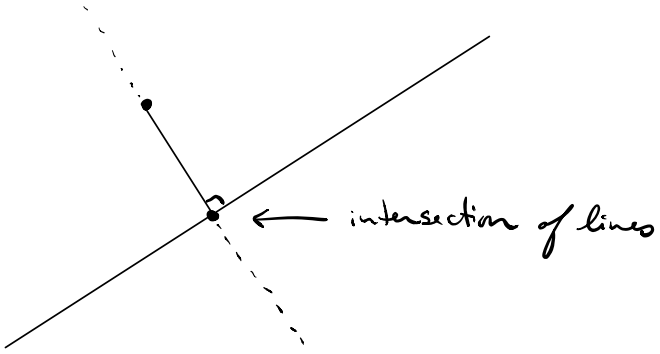
$9(y - 4) = -\frac{1}{9}(x - 5) \cdot 9$

$9y - 36 = -(x - 5)$

$9y - 36 = -x + 5$

$x + 9y = 41$

#112) Find the exact distance from the point $(-4, 8)$ to the line $3x + 4y = 9$



$3x + 4y = 9$
$-3x \quad -3x$
$4y = -3x + 9$
$y = -\frac{3}{4}x + \frac{9}{4}$

$m_1 = -\frac{3}{4} \Rightarrow m_2 = \frac{4}{3}$

$y - 8 = \frac{4}{3}(x - (-4))$
$y - 8 = \frac{4}{3}x + \frac{16}{3}$
$y = \frac{4}{3}x + \frac{16}{3} + \frac{24}{3}$
$= \frac{4}{3}x + \frac{40}{3}$

$y = -\frac{3}{4}x + \frac{9}{4}$

$y = \frac{4}{3}x + \frac{40}{3}$

$-\frac{3}{4}x + \frac{9}{4} = \frac{4}{3}x + \frac{40}{3}$
$+\frac{3}{4}x \quad -\frac{40}{3} \quad +\frac{3}{4}x \quad -\frac{40}{3}$

$\left(\frac{9}{4} - \frac{40}{3}\right) = \left(\frac{4}{3}x + \frac{3}{4}x\right)$

$-\frac{133}{12} = \frac{25x}{12}$

$x = -\frac{133}{25}$

$y = \frac{156}{25}$

$(-4, 8), \left(-\frac{133}{25}, \frac{156}{25}\right)$

$$d = \sqrt{\left(-4 + \frac{133}{125}\right)^2 + \left(8 - \frac{156}{25}\right)^2}$$

$$= \sqrt{\frac{1089}{625} + \frac{1936}{25}}$$

$$= \sqrt{\frac{3025}{625}}$$

$$= \sqrt{\frac{121}{25}}$$

$$= \frac{11}{5}$$