§ P.6: Rational Expressions

Factoring Procedure

Review section P.5

To factor $ax^2 + bx + c$

1. Find two numbers whose sum is b and whose product is ac.
2. Replace b by the sum of these two numbers
3. Factor the result by grouping.

Ex) $x^2 + 10x + 16$ $\quad$ $ac = 16$ $\quad$ $b = 10$

$(x+2)(x+8)$

$$= x^2 + 2x + 8x + 16$$
$$= x(x+2) + 8(x+2)$$
$$= (x+8)(x+2)$$

2, 8

4, 4

Ex) $x^2 - 12x + 20$ $\quad$ $ac = 20$ $\quad$ $b = -12$

$(x-2)(x-10)$

$$= x^2 - 2x - 10x + 20$$
$$= x(x-2) - 10(x-2)$$
$$= (x-10)(x-2)$$

$-2, -10$ $\quad$ -12

since $+7x$ only need to consider ~~small~~ negative factors

Ex) $6y^2 + 7x - 5$ $\quad$ $ac = -30$ $\quad$ $b = 7$

$$= 6y^2 - 3y + 10y - 5$$
$$= (6y^2 - 3y) + (10y - 5)$$
$$= 3y(2y-1) + 5(2y-1)$$
$$= (3y+5)(2y-1)$$

$-2, 15$ $\quad$ 13

$-3, 10$ $\quad$ 7

Note for x^2+bx+c

if $b>0$ and $c>0$, then the factors are $(x+\_)(x+\_)$

if $b<0$ and $c>0$, the the factors are $(x-\_)(x-\_)$

if $c<0$, then the factors are $(x+\_)(x-\_)$

Factoring Special Products

$$a^2+2ab+b^2 = (a+b)^2$$

$$a^2-2ab+b^2 = (a-b)^2$$

$$a^2-b^2 = (a+b)(a-b)$$

$$\left.\begin{aligned} a^3-b^3 &= (a-b)(a^2+ab+b^2) \\ a^3+b^3 &= (a+b)(a^2-ab+b^2) \end{aligned}\right\} \text{SOAP}$$

Rational Expression

Def: A <u>rational expression</u> is a ratio of two polynomials in which the denominator is not the zero polynomial.

* the denominator can be zero for some values, but not all values

Def: The <u>domain</u> of a rational expression is the set of all real numbers that can be used in place of the variable.

Find the domain of each rational expression.

#8) $\dfrac{x^2-1}{x-5}$

$x-5 \neq 0$

$\{x \mid x \neq 5\}$

#10) $\dfrac{2x-3}{(x+1)(x-3)}$

$x+1 \neq 0 \qquad x-3 \neq 0$

$\{x \mid x \neq -1,\ x \neq 3\}$

Ex) $\dfrac{x}{x^2+1}$

all real

Reducing to Lowest Terms

#18) $\dfrac{a^2-b^2}{b-a}$

$$= \frac{(a+b)(a-b)}{(b-a)} \cdot \frac{(-1)}{(-1)}$$

$$= \frac{-(a+b)(a-b)}{(-1)(-a+b)}$$

$$= \frac{-(a+b)(a-b)}{(a-b)}$$

$$= -(a+b) \cdot \cancel{\frac{a-b}{a-b}}^{\,1}$$ divide out to one

$$= -a-b$$

#22) $\dfrac{t^3u^7}{-t^8u^5}$

$$= -\frac{t^3}{t^8} \cdot \frac{u^7}{u^5}$$

$$= -\frac{1}{t^{8-3}} \cdot u^{7-5}$$

$$= -\frac{u^2}{t^5}$$

Multiplying and Dividing Rational Expressions

#28) $\dfrac{14w}{51y} \cdot \dfrac{3w}{7y}$

$$= \frac{\cancel{3} \cdot \cancel{14}^{2}}{{}_{17}\cancel{51} \cdot \cancel{7}} \frac{w^2}{y^2}$$

$$= \frac{2}{17}\frac{w^2}{y^2}$$

$$= \frac{2w^2}{17y^2}$$

#34) $\dfrac{a^3-b^3}{a^2-2ab+b^2} \div \dfrac{2a^2+2ab+2b^2}{9a^2-9b^2}$

$$= \frac{a^3-b^3}{a^2-2ab+b^2} \cdot \frac{9a^2-9b^2}{2a^2+2ab+2b^2}$$

$$= \frac{\cancel{(a-b)}\cancel{(a^2+ab+b^2)}}{\cancel{(a-b)^2}} \cdot \frac{9(a+b)\cancel{(a-b)}}{2\cancel{(a^2+ab+b^2)}}$$

$$= \frac{9}{2}(a+b)$$

Adding and Subtracting Rational Expressions

Ex) $\frac{7}{5}\cdot\frac{3}{3}+\frac{2}{3}\cdot\frac{5}{5}$ LCD

$= \frac{7\cdot 3+2\cdot 5}{15}$

$= \frac{21+10}{15} = \frac{31}{15}$

#52) $\frac{7}{2x-4} - \frac{3}{2x-4}$

$= \frac{7-3}{2x-4}$

$= \frac{4}{2x-4}$

$= \frac{4}{2(x-2)}$

$= \frac{2}{x-2}$

#54) $\frac{-7}{3a^2b}\cdot\frac{2b}{2b} + \frac{4}{6ab^2}\cdot\frac{a}{a}$

$= \frac{-14b+4a}{6a^2b^2}$

$= \frac{2(2a-7b)}{6a^2b^2}$

$= \frac{2a-7b}{3a^2b^2}$

#62) $\frac{x-1}{x^2+x-6} - \frac{x-2}{x^2+4x+3}$

$= \frac{(x-1)}{(x+3)(x-2)}\cdot\frac{(x+1)}{(x+1)} - \frac{(x-2)}{(x+1)(x+3)}\cdot\frac{(x-2)}{(x-2)}$

$= \frac{(x-1)(x+1)-(x-2)^2}{(x+3)(x-2)(x+1)} = \frac{x^2-1-(x^2-4x-4)}{(x+3)(x-2)(x+1)}$

$= \frac{4x-5}{(x+3)(x-2)(x+1)}$

Complex Fractions

$6 = 2 \cdot 3$
$4 = 2 \cdot 2$
$LCM(6,4) = 2 \cdot 2 \cdot 3$
$= 12$

#71)

$$\frac{\frac{25}{36a}}{\frac{10a}{27}}$$

$$= \frac{25}{36a} \cdot \frac{27}{10a}$$

$$= \frac{5^{\cancel{2}} \cdot 3^3}{6^2 \cdot 2 \cdot \cancel{5}\, a^2}$$

$$= \frac{5 \cdot 3^{\cancel{3}\,1}}{2^2 \cdot \cancel{3^2} \cdot 2\, a^2}$$

$$= \frac{5 \cdot 3}{2^3\, a^2}$$

$$= \frac{15}{8a^2}$$

#74)

$$= \frac{\frac{2}{6xy} - \frac{1}{4x}}{\frac{1}{3y^2} + \frac{1}{2x}} \cdot \frac{12xy^2}{12xy^2}$$

$LCM(3y^2, 2x) = 3y^2 \cdot 2x = 6xy^2$
$LCM(6xy, 4x) = 6xy \cdot 2 = 12xy$
$LCM(6xy^2, 12xy) = 12xy^2$

$$= \frac{\frac{{}^{4}\cancel{24}\,\cancel{x}\,y^{\cancel{2}\,1}}{\cancel{6}\cancel{x}\cancel{y}} - \frac{{}^{3}\cancel{12}\,\cancel{x}\,y^2}{\cancel{4}\cancel{x}}}{\frac{{}^{4}\cancel{12}\,x\,\cancel{y^2}\,1}{\cancel{3}\cancel{y^2}} + \frac{{}^{6}\cancel{12}\,\cancel{x}\,y}{\cancel{2}\cancel{x}}}$$

$$= \frac{4y - 3y^2}{4x + 6y}$$