§1.5: Quadratic Equations

Def: A <u>quadratic equation</u> is an equation of the form

$$ax^2 + bx + c = 0,$$

where $a, b,$ and c are real with $a \neq 0$.

Zero Factor Property:

If A and B are algebraic expressions, then $AB = 0$ is equivalent to the compound statement $A = 0$ OR $B = 0$.

<u>Factoring</u>

Factoring Procedure — Review section P.5

To factor $ax^2 + bx + c$

1. Find two numbers whose sum is b and whose product is ac.
2. Replace b by the sum of these two numbers
3. Factor the result by grouping.

#6) $x^2+2x-8=0$

$(x+4)(x-2)=0$

$x+4=0$ or $x-2=0$

$x=-4$ or $x=2$

$\{-4,2\}$

#12) $12x^2-17x+6=0$

m	n	$m+n=-17$?
-1	-72	✗
-2	-36	✗
-3	-24	✗
-4	-18	✗
-6	-12	✗
-8	-9	✓

$12x^2-8x-9x+6=0$

$(12x^2-8x)-(9x-6)=0$

$4x(3x-2)-3(3x-2)=0$

$(4x-3)(3x-2)=0$

$4x-3=0$ or $3x-2=0$

$x=\frac{3}{4}$ or $x=\frac{2}{3}$

$\{\frac{3}{4},\frac{2}{3}\}$

#14) $(w-1)(w-2)=6$

$\Rightarrow w-1=6$ or $w-2=6$?

$w^2-2w-w+2=6$

$w^2-3w-4=0$

$(w+1)(w-4)=0$

$w+1=0$ or $w-4=0$

$w=-1$ or $w=4$

$\{-1,4\}$

Square Root Property

For any real number k $x^2=k$ is equivalent to $x=\pm k$.

#16) $x^2-8=0$

$x^2=8$

$x=\pm\sqrt{8}$

$x=\pm 2\sqrt{2}$

$\{\pm 2\sqrt{2}\}$

#26) $(x-3)^2=-20$

$x-3=\pm\sqrt{-20}$

$x-3=\pm i\sqrt{4\cdot 5}$

$x-3=\pm i\cdot 2\sqrt{5}$

$x=3\pm 2i\sqrt{5}$

$\{3\pm 2i\sqrt{5}\}$

Solving by Completing the Square

#36) $x^2 - 10x + 5 = 0$

$x^2 - 10x + \underline{25} = -5 + 25$

$(x-5)^2 = 20$

$x - 5 = \pm\sqrt{20}$

$x - 5 = \pm 2\sqrt{5}$

$x = 5 \pm 2\sqrt{5}$

$\boxed{\{5 \pm 2\sqrt{5}\}}$

#~~44~~) $5x^2 + 4x + 3 = 0$

$x^2 + \frac{4}{5}x + \frac{3}{5} = 0$

$x^2 + \frac{4}{5}x + \underline{\left(\frac{2}{5}\right)^2} = -\frac{3}{5} + \left(\frac{2}{5}\right)^2$

$\left(x + \frac{2}{5}\right)^2 = -\frac{3}{5} + \frac{4}{25}$

$\left(x + \frac{2}{5}\right)^2 = -\frac{11}{25}$

$x + \frac{2}{5} = \pm\sqrt{-\frac{11}{25}}$

$x + \frac{2}{5} = \pm i\frac{\sqrt{11}}{5}$

$x = -\frac{2}{5} \pm i\frac{\sqrt{11}}{5}$

$\boxed{-\frac{2}{5} \pm i\frac{\sqrt{11}}{5}}$

The Quadratic Formula

This will be a 10pt question on the exam.

$$ax^2 + bx + c = 0$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}\cdot\frac{4a}{4a}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{\sqrt{4a^2}}$$

$$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$$

no abs ?

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

let $a \to -a$, $b \to -b$, $c \to -c$

$$x = \frac{-(-b) \pm \sqrt{b^2 - 4(-a)(-c)}}{2(-a)}$$

$$= -\frac{-(-b) \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-b \mp \sqrt{b^2 - 4ac}}{2a}$$

#12) $12x^2 - 17x + 6 = 0$ $\quad 17^2 = 289$

$$x = \frac{17 \pm \sqrt{17^2 - 4(12)(6)}}{2(12)}$$

$$= \frac{17 \pm \sqrt{289 - 288}}{24}$$

$$= \frac{17 \pm 1}{24}$$

$\frac{18}{24}$ OR $\frac{16}{24}$

$\left\{\frac{3}{4}, \frac{2}{3}\right\}$

Strategy for solving $ax^2 + bx + c = 0$

1. If $b = 0$, solve $ax^2 + c = 0$ for x^2 and use the square root property
2. If $ax^2 + bx + c$ is easily factored, solve by factoring. If $a < 0$, multiply each side of eq by -1 for easier factoring.
3. If all else fails, use the quadratic formula.