§P.4: Polynomials

Def: If n is a nonnegative integer and $a_0, a_1, a_2, \ldots, a_n$ are real numbers, then

$$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

is a polynomial in one variable x.

degree: highest power

leading coefficient: coefficient of term with highest power

Evaluating Polynomials

$P(x) = x^2 - 3x + 2$ $\qquad$ $M(x) = -x^3 + 5x^2 - x + 2$

#15) $P(-2) = (-2)^2 - 3(-2) + 2$

$= 4 + 6 + 2$

$= 12$

#17) $M(-3) = (-3)^3 + 5(-3)^2 - (-3) + 2$

$= (27) + 5(9) + 3 + 2$

$= 27 + 45 + 3 + 2$

$= (27+3) + (45+2)$

$= 30 + 47$

$= 77$

Adding and Subtracting Polynomials

#19) $(3x^2 - 4x) + (5x^2 + 7x - 1)$

$= (3x^2 + 5x^2) + (-4x + 7x) - 1$

$= 8x^2 + 3x - 1$

#22) $(x^2 + 2x + 4) - (x^2 + 4x + 4)$

$= x^2 + 2x + 4 - x^2 - 4x - 4$

$= (x^2 - x^2) + (2x - 4x) + (4 - 4)$

$= 0 + (-2x) + 0$

$= -2x$

Multiplying polynomials

Use distributive property

#30) $-2m(m^2 - 3m + 9)$

$= -2m(m^2) - 2m(-3m) - 2m(9)$

$= -2m^3 + 6m^2 - 18m$

#32) $(-w^2 - 5w + 6)(w + 5)$

$= -w^2(w+5) - 5w(w+5) + 6(w+5)$

$= -w^2 \cdot w - w^2 \cdot 5 - 5w(w) - 5w(5) + 6(w) + 6(5)$

$= -w^3 - 5w^2 - 5w^2 - 25w + 6w + 30$

$= -w^3 - 10w^2 - 19w + 30$

Multiplying Binomials using FOIL

First Outer Inner Last

#40) $(z-3)(z-4)$

$= z^2 - 4z - 3z + 12$

$= z^2 - 7z + 12$

Ex.) $(a+b)(a-b)$

$= a^2 (-ab + ba) - b^2$

$= a^2 - b^2$

#46) $(5x-3)^2 \neq 25x^2 - 9$

$= (5x-3)(5x-3)$

$999 \cdot 75$

$(1000-1)(100-25)$

$= 100000 - 100 - 25000 + 25$

$= 75000 - 75$

$= 74925$

Multiplying Radicals using FOIL

#50) $(5+\sqrt{6})(2+\sqrt{6})$

$= 10 + 5\sqrt{6} + 2\sqrt{6} + \sqrt{6}\sqrt{6}$

$= 10 + 7\sqrt{6} + 6$

$= 16 + 7\sqrt{6}$

Special Products

$(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

$(a+b)(a-b) = a^2 - b^2$

#60) $(2z^b + 1)(2z^b - 1)$

$= (2z^b)^2 - (1)^2$

$= 4z^{2b} - 1$

Using Conjugates to Rationalize a denominator.

#68) $\frac{2}{3+\sqrt{5}} \cdot 1$

$= \frac{2}{3+\sqrt{5}} \cdot \frac{3-\sqrt{5}}{3-\sqrt{5}}$

$= \frac{2(3-\sqrt{5})}{(3+\sqrt{5})(3-\sqrt{5})}$ from special product

$= \frac{6-2\sqrt{5}}{3^2-5}$

$= \frac{6-2\sqrt{5}}{4}$

$= \frac{3-\sqrt{5}}{2}$

Long Division

#82) $(x^2 - 3x - 54) \div (x - 9) = x + 6$

$$
\begin{array}{r}
x + 6 \\
x - 9 \overline{)\, x^2 - 3x - 54} \\
-(x^2 - 9x) \quad \downarrow \\
\hline
6x - 54 \\
-(6x - 54) \\
\hline
0
\end{array}
$$

#86) Find quotient and remainder when $3x^2 - x + 4$ is divided by $x + 2$

$$
\begin{array}{r}
3x - 7 \quad r\ 18 \\
x + 2 \overline{)\, 3x^2 - x + 4} \\
-(3x^2 + 6x) \\
\hline
-7x + 4 \\
-(-7x - 14) \\
\hline
18
\end{array}
$$

$$\Rightarrow \quad \frac{3x^2 - x + 4}{x + 2} = 3x - 7 + \frac{18}{x+2}$$

#92) $\dfrac{x^2 + 4x + 5}{x + 1}$