§ 1.1: Linear, Rational, & Absolute Value Eq.

Def: A linear equation in one variable is an equation of the form $ax + b = 0$ where $a, b \in \mathbb{R}$ and $a \neq 0$.

Properties of Equality:

If A and B are algebraic equations and $C \in \mathbb{R}$, then the following are equivalent to $A = B$:

1. $A + C = B + C$
2. $A - C = B - C$
3. $CA = CB \qquad (C \neq 0)$
4. $\dfrac{A}{C} = \dfrac{B}{C} \qquad (C \neq 0)$

#14) $\quad -2x + 3 = 0$

$\qquad \qquad \quad -3 \qquad \quad -3$

$\qquad \qquad \dfrac{-2x}{-2} = \dfrac{-3}{-2}$

$\qquad \qquad \quad x = \frac{3}{2}$

Solution set *

$\boxed{\left\{ \frac{3}{2} \right\}}$

#18) $\quad 4x - 3 = 6x - 1$

$\{-1\}$

Classifying an Equation

Class	Number of Solutions
Conditional	Finitely many (you can list them)
Identity	Infinitely many (but not necessarily all)
Inconsistent	No Solutions.

Ex. Classify and find the solution set

#27) $3(x-6) = 3x - 18$ → identity $\mathbb{R}$

#32) $2(x+1) = 3x + 2$ → conditional $\{0\}$

#33) $3(x-6) = 3x + 18$ → inconsistent $\emptyset$

Equations w/ Rational Expressions

Ex) $30\left(\dfrac{x}{2} + \dfrac{x}{3}\right) = \left(\dfrac{1}{5}\right) \cdot 30$

$$15x + 10x = 6$$
$$25x = 6$$
$$x = \frac{6}{25}$$

$\boxed{\text{conditional} \quad \left\{\frac{6}{25}\right\}}$

#36) $\dfrac{x(x+2)}{(x+2)} = x$ → $\boxed{\text{identity}, \; \{x \in \mathbb{R} \mid x = -2\}}$

Ex $\dfrac{x}{x+3} - \dfrac{x}{x-3} = \dfrac{1}{x^2-9}$

$\left(\dfrac{x}{x+3} - \dfrac{x}{x-3} = \dfrac{1}{(x+3)(x-3)} \right) \cdot (x+3)(x-3)$ $\qquad (x \neq \pm 3)$

$x(x-3) - x(x+3) = 1$

$\cancel{x^2} - 3x - \cancel{x^2} - 3x = 1$

$\qquad -6x = 1$

$\qquad x = -\frac{1}{6}$

$\boxed{\text{conditional}, \ \{-\frac{1}{6}\}}$

#43)

$\left(\dfrac{1}{x-3} - \dfrac{1}{x+3} = \dfrac{6}{x^2-9} \right) \cdot (x+3)(x-3) \qquad x \neq \pm 3$

$x+3 - (x-3) = 6$

$x+3 - x+3 = 6$

$6 = 6$

identity, $\{x \in \mathbb{R} \mid x \neq \pm 3\}$

Equations involving Absolute Value

Ex) $|x| = 4$

$x = 4$ OR $x = -4$ $\{-4, 4\}$

#68) $|x - 7| = 0$

$\begin{aligned} x - 7 &= 0 \\ +7 & +7 \\ x &= 7 \end{aligned}$ OR $\begin{aligned} x - 7 &= -0 \\ +7 & +7 \\ x &= 7 \end{aligned}$ $\{7\}$

#71) $|2x - 3| = 7$

$\begin{aligned} 2x - 3 &= 7 \\ +3 & +3 \\ \frac{2x}{2} &= \frac{10}{2} \\ x &= 5 \end{aligned}$ OR $\begin{aligned} 2x - 3 &= -7 \\ +3 & +3 \\ \frac{2x}{2} &= \frac{-4}{2} \\ x &= -2 \end{aligned}$ $\{-2, 5\}$

#74) $\frac{3}{2} \cdot \frac{2}{3} |x + 4| = 8 \frac{3}{2}$

$|x + 4| = 12$

$\begin{aligned} x + 4 &= 12 \\ -4 & -4 \\ x &= 8 \end{aligned}$ OR $\begin{aligned} x + 4 &= -12 \\ -4 & -4 \\ x &= -16 \end{aligned}$ $\{-16, 8\}$

#70) $|x + 9| = -6$

$\begin{aligned} x + 9 &= -6 \\ -9 & -9 \\ x &= -15 \end{aligned}$ OR $\begin{aligned} x + 9 &= 6 \\ -9 & -9 \\ x &= -3 \end{aligned}$ $\{-15, -3\}$?

$\emptyset$

#75) $2|x+5| - 10 = 0$ $\{-10, 0\}$

#79) $2|x| + 7 = 6$ $\emptyset$

Ex) $|x| = |x-1|$ Homework

#86) $(x-3)^2 = x^2 - 9$ $\{3\}$

#89) $\frac{x}{2} + 1 = \frac{1}{4}(x-6)$ $\{-10\}$

#90) $-\frac{1}{6}(x+3) = \frac{1}{4}(3-x)$ $\{15\}$

#92) $\frac{x-3}{5} - \frac{y}{2}' = 5$ $\{-12\}$

#98) $5|7-3x| + 2 = 4|7-3x| - 1$ $\emptyset$

#101) $\frac{4}{x+3} - \frac{3}{2-x} = \frac{7x+1}{x^2+x-6}$ $\{x \in \mathbb{R} \mid x \neq -3, x \neq 2\}$

#103) $\frac{x-2}{x-3} = \frac{x-3}{x-4}$ $\emptyset$

#104) $\frac{y-1}{y+4} = \frac{y+1}{y-2}$ $\{-\frac{1}{4}\}$