Unit 6: Introduction to linear regression

2. Outliers and inference for regression

Sta 101 - Spring 2015

Duke University, Department of Statistical Science

April 1, 2015

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Slides posted at http://bitly.com/sta101sp15

1. Housekeeping

2. Main ideas
    1. R^2 assesses model fit -- higher the better
    2. Inference for regression uses the T distribution
    3. Conditions for regression
    4. Type of outlier determines how it should be handled

3. Summary

- Project 2: *https://stat.duke.edu/courses/Spring15/sta101.001/projects/project2.html*

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Response: annual_murders_per_mil
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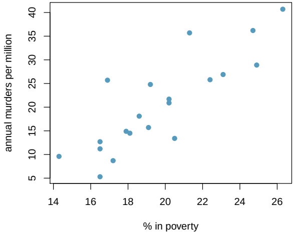
Clicker question

R^2 for the regression model for predicting annual murders per million based on percentage living in poverty is roughly 71%. Which of the following is the correct interpretation of this value?



(a) 71% of the variability in percentage living in poverty is explained by the model.

(b) 84% of the variability in the murder rates is explained by the model, i.e. percentage living in poverty.

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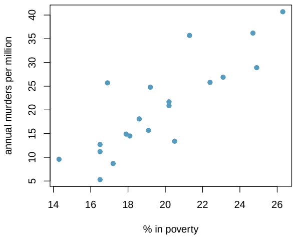

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- Confidence intervals for a slope:
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- Independence of residuals (and hence observations) $\rightarrow$ depends on data collection method, often violated for time-series data -- important for inference

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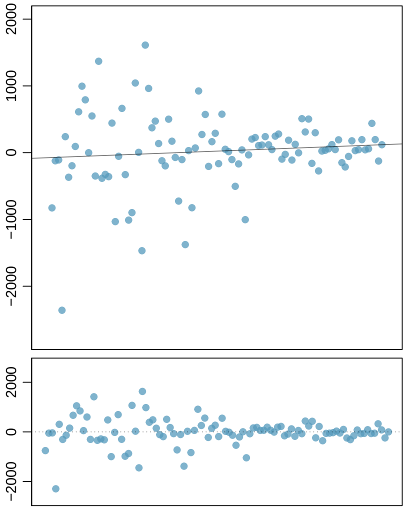
What condition is this linear model obviously and definitely violating?

(a) Linear relationship
(b) Non-normal residuals
(c) Constant variability
(d) Independence of observations



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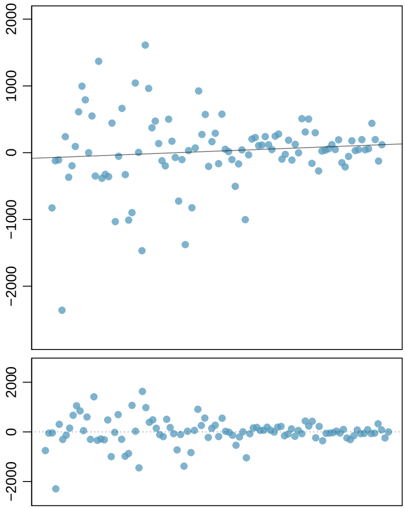

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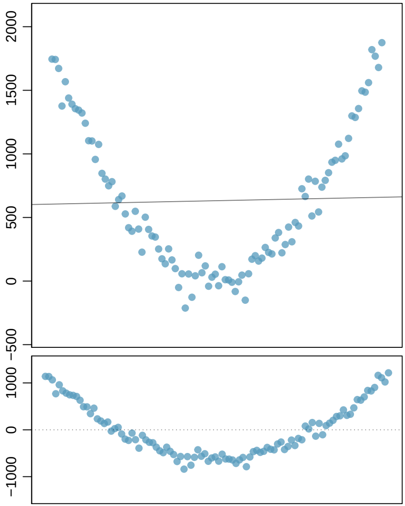

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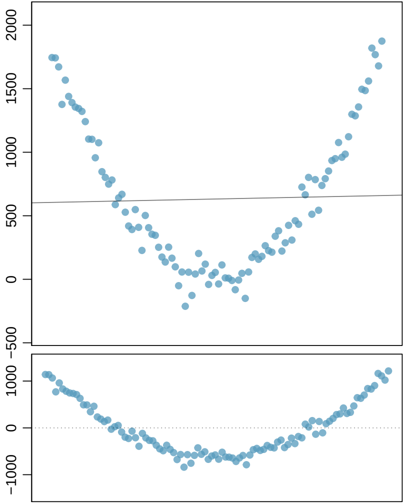

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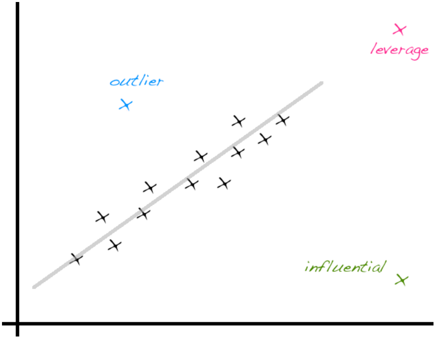

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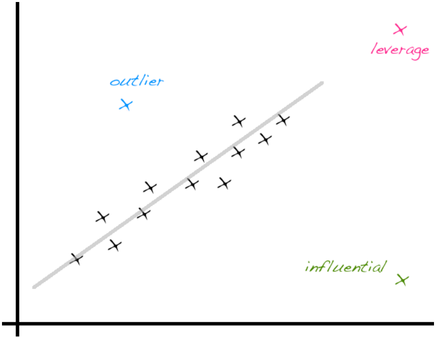

- *Outlier* is an unusual point without these special characteristics (this one likely affects the intercept only)
- If clusters (groups of points) are apparent in the data, it might be worthwhile to model the groups separately.

Application exercise: 6.2 Linear regression

See course website for details

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