ETM540 Final Exam

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1 Production Planning

The Portland company, Weatherman, makes a single, very specialized multipurpose tool. Demand for the next four quarters are known to be 700, 1200, 550, and 1000 units respectively. The production cost is $1250 per unit. The per unit cost to hold one unit in inventory is 25% of the production cost per quarter. An inventory buffer of 25 units must be maintained at the end of every quarter (including at the end of the planning horizon) for warranty issues and other purposes. Due to production capacity limitations, no more than 1000 units can be produced in any quarter. At the beginning of the planning, there are 30 units in inventory.

1.1 Formulate an explicit mathematical optimization model for this problem using LaTeX in RMarkdown.

Data:

- C_t^P = Production cost of each unit
- C_t^H = Hold Cost
- L_t^H = Minimum amount of units for inventory buffer.
- L_t^P = Maximum amounts of units that can be produced in a given Quarter.
- B^I = Inventory on hand at start of Q1.
- D_t = Demand for Quarter t.

Decision variables:

- x_t is the number of units to produce it Quarter t.
- z_t is the inventory of units at the end of Quarter t.

$$
\begin{aligned}
\text{min} \quad & \sum_{t=1}^{4} (C_t^P \cdot x_t + C_t^H \cdot x_t) \\
\text{s.t.:} \quad & z_1 = B^I + x_1 - D_1 \\
& z_t = z_{t-1} + x_t - D_t, \; \forall t > 1 \\
& x_t \leq L^P, \; \forall t \\
& z_t \geq L^H, \; \forall t \\
& x_t, \; z_t \geq 0, \; \forall \; t
\end{aligned}
$$

1.2 Implement and solve your optimization model using ompr.

```
promod <- MIPModel()
promod <- add_variable (promod, Vx[tt], tt=1:nTime,
                        lb=0, type="continuous") # Production volume.
promod <- add_variable (promod, Vz[tt], tt=1:nTime,
                        lb=0, type="continuous") # Inventory at end of quarter.
promod <- set_objective(promod, sum_expr(prodcost * Vx[tt] + invcost * prodcost
                                         * Vz[tt], tt=1:nTime), "min")
promod <- add_constraint (promod, beg_inv + Vx[1]
                          - dem [1] == Vz[1] ) #first week backlog
promod <- add_constraint (promod, Vz[tt-1] + Vx[tt] - dem [tt] == Vz[tt],
                          tt = 2:nTime ) #following weeks backlogs
promod <- add_constraint (promod, Vx[tt] <= lmaxpro, tt = 1:nTime)
#Maximum amount that can be produced per period.

promod <- add_constraint (promod, Vz[tt] >= lmininv, tt = 1:nTime)
#Min amount that must be held in inventory

prores <- solve_model(promod, with_ROI(solver = "glpk"))
prores$status
```

```
## [1] "success"
```

```
prores$objective_value
```

```
## [1] 4400000
```

```
Sol <- rbind (
  t(as.matrix(as.numeric(get_solution (prores, Vx[tt])[,3]))),
  t(as.matrix(as.numeric(get_solution (prores, Vz[tt])[,3]))))

var_list <- c("$x_{t}$", "$z_{t}$")
Sol <- cbind(var_list, Sol)
colnames(Sol) <- c("Variable", "$Q_{1}$", "$Q_{2}$", "$Q_{3}$", "$Q_{4}$")
rownames(Sol) <- c("Tool Production", "Tool Inventory (EOQ)")


kbl (Sol, booktabs=T, escape=F,
     caption="Production Planning Over Time") |>
  kable_styling(latex_options = "hold_position")
```

Table 1: Production Planning Over Time

	Variable	Q_1	Q_2	Q_3	Q_4
Tool Production	x_t	895	1000	550	1000
Tool Inventory (EOQ)	z_t	225	25	25	25

1.3 Interpret and discuss the solution to the problem.

In the above production model, the goal is to reduce the production costs. The costs are not only the production cost, but also the holding costs per quarter.

Running the model indicates a successful optimization with the minimum production cost of $\$4,400,400$, while being able to meet the demand, for each quarter. This also fulfills the constraints of maximum production per quarter (1000 units), and the inventory buffer of a minimum of 25 units to be held at all times.

1.4 Modified Example Formulation

> Weatherman is looking at investing in a manufacturing process that will decrease their manufacturing cost per item to \$1100 per unit. Production capacity would increase to 2000 units per quarter but there is a setup cost of \$5000 each quarter in which units are produced.
>
> Create a new optimization formulation for this modified situation

To do this version of the model we need to add the below variables.

Data:

- C_t^s = New Setup costs

Decision variables:

- y_t if a setup cost for production for each quarter t in which units are produced; 0 otherwise.

$$
\begin{aligned}
\min \quad & \sum_{t=1}^{4}(C_t^P \cdot x_t + C_t^H \cdot x_t + C_t^S \cdot y_t) \\
\text{s.t.:} \quad & z_1 = B^I + x_1 - D_1 \\
& z_t = z_{t-1} + x_t - D_t, \ \forall t > 1 \\
& x_t \leq L^P, \ \forall t \\
& z_t \geq L^H, \ \forall t \\
& x_t, \ z_t \geq 0, \ \forall \ t \\
& y_t \ \in \{0,1\} \ \forall \ t
\end{aligned}
$$

1.5 Implement and Solve the modified model. Discuss the results

```
prores2$status
```

```
## [1] "success"
```

```
prores2$objective_value
```

```
## [1] 3817000
```

```
Sol2 <- rbind (
  t(as.matrix(as.numeric(get_solution (prores2, Vx[tt])[,3]))),
  t(as.matrix(as.numeric(get_solution (prores2, Vz[tt])[,3]))))

var_list <- c("$x_{t}$", "$z_{t}$")
Sol2 <- cbind(var_list, Sol2)
colnames(Sol2) <- c("Variable", "$Q_{1}$", "$Q_{2}$", "$Q_{3}$", "$Q_{4}$")
rownames(Sol2) <- c("Tool Production", "Tool Inventory (EOQ)")


kbl (Sol2, booktabs=T, escape=F,
     caption="Production Planning Over Time Adjusted") |>
  kable_styling(latex_options = "hold_position")
```

Table 2: Production Planning Over Time Adjusted

	Variable	Q_1	Q_2	Q_3	Q_4
Tool Production	x_t	695	1200	550	1000
Tool Inventory (EOQ)	z_t	25	25	25	25

1.6 Compare the results between the models in light of managerial insights

Below (Table 3) we see the comparison between the 2 models. With the new addition of the \$5000 setup costs, and the increase in production capacity the initial thought would be that we should try to focus production in the first quarter or so, then try to save money by not producing units in the later quarters. In the comparison under the new conditions we do see a reduction in total production costs of $\$583,000$ ($\$4,400,400$ to $\$3,817,000$). The models shows us that the high carrying costs are our main focus and since the carrying cost are a function of production costs, the model focused on reducing the amount of units being held each quarter to the minimum 25 units.

The company should pursue investing in the new manufacturing process.

Table 3: Production Planning Over Time Comparison

	Variable	Q_1	Q_2	Q_3	Q_4
Tool Production	x_t	895	1000	550	1000
Tool Inventory (EOQ)	z_t	225	25	25	25
Tool Production Adusted	x_2t	695	1200	550	1000
Tool Inventory (EOQ) Adjusted	z_2t	25	25	25	25

2 DEA

> Is DEA a tool for measuring relative or absolute efficiency? Explain and discuss your reason(s) for your answer. (Aim for one or two paragraphs.)

A DEA takes empirical data to measure productivity efficiency. It measures the efficiency of decision making units (DMU's) in the data set.

The efficiency that you are measuring is relative efficiency. This means you are measuring the efficiency relative to another entity. For example you have a business that has several branches each with different number of employees and varying amounts of transactions. Using a DEA you can find out the most efficient branch, and see how far off the other branches might be. This can help management make staffing decisions or other business

3 Computational Complexity

> Does changing a set of decision variables from continuous to general integers tend to increase, decrease, or leave unchanged the computational complexity of optimization? Explain and discuss your reason(s) for your answer. (Aim for one or two paragraphs.)

Changing variable from continuous to integers increases computational complexity a great deal. Forcing a variable to an integer vs floating point, forces the solver to lay the solutions on an integer versus what could be the optimum floating point solution. Ways around this is a procedure called "branch and bound", or a time constraint on the solver that gives you the "best solution up to a time limit".

4 Sensitivity Analysis

> Describe a situation or application where the results of sensitivity analysis are of greater importance than the specific optimal decision variable values. (Aim for one or two paragraphs.)

Sensitivity Analysis can help a business decide where extra resources could best be realized. For example if they have funds to build a new facility, sensitivity analysis could help them to decide what function that faculty should focus on.

Where are they constrained? Maybe the storage fees are high because they pay a 3rd party to hold excess inventory. Should they focus on a warehouse. Maybe if they had more production capacity they would be more profitable. Should they focus on factory space. A sensitivity analysis could help them decide the best business descion.