Using the KS-test to determine confidence intervals for the period spacings of g-mode pulsations in KIC 11558725

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ABSTRACT

After analyzing three years of *Kepler* data of the subdwarf B star KIC 11558725, asymptotic period spacings of 244 ± 0.32 and 144 ± 0.46 seconds were found for the *ell* = 1 and 2 gravity (g-) mode period sequences, respectively. Using the Kolmogorov-Smirnov (KS-) test, confidence intervals were calculated to determine the statistical significance of the period spacings. Confidence intervals of 99.9%, 99%, 95%, and 90% were calculated using randomized period lists. Each list contained 100 integers ranging from 3000 to 10000 seconds (ie: the period range for g-mode pulsations). The KS-test was performed on a list of KIC 11558725's g-mode pulsation periods. By plotting the confidence intervals from the randomization test on the KS-test for KIC 11558725, it is clear that the aforementioned period spacings are intrinsic to the star.

INTRODUCTION

Subdwarf B (sdB) stars have been known to exhibit asymptotic period spacing in their g-mode pulsations since 2011 (Reed M.D., Baran A., Quint A.C., Kawaler S. J., et. al. 2011, MNRAS, 414, 2885). In asteroseismology, deviations from asymptotic period spacings can be used to constrain convective depths (Montgomery M.H., 2005, ApJ, 633, 1142) and infer interior chemical composition gradients (Kawaler S.D., Bradley P.A., 1994, ApJ, 427, 415; Degroote P. et al., 2010, Nat, 464, 259). Deviations from asymptotic period spacings occur in all of the sdB stars observed by *Kepler* during its K1 mission.

Non-radial pulsations are characterized using three quantum numbers *n*, *ell*, and *m;* the number of radial nodes, surface nodes, and azimuthal nodes, respectively. For integer values of *n* where *n* >> *ell*, the period of the m = 0 components of g-mode pulsations can be written in the following expression (Unno W., Osaki Y., Ando H., Shibahashi H., 1979, Nonradial Oscillations of Stars. Univ. of Tokyo Press, Tokyo, and others).

$$\Pi_{\ell,n} = \frac{\Pi_o}{\sqrt{\ell\,(\ell+1)}}n + \epsilon$$

Therefore, since Π_o and ϵ are constants, the change in Π between consecutive radial overtones can be written as follows.

$$\Delta\Pi_\ell = \frac{\Pi_o}{\sqrt{\ell(\ell+1)}}$$

From this equation we see that the *ell* value for each pulsation can be observationally determined using period spacings, if the pulsations can be designated to a period sequence with a particular spacing. It has been shown by M. D. Reed et al. in 2011 that the KS-test is a sufficient test to uncover these period spacings. After extrapolating period sequences with these spacings, observational mode identifications can be determined for the g-mode pulsations of sdB stars (Reed M.D., Baran A., Quint A.C., Kawaler S. J., et. al. 2011, MNRAS, 414, 2885).

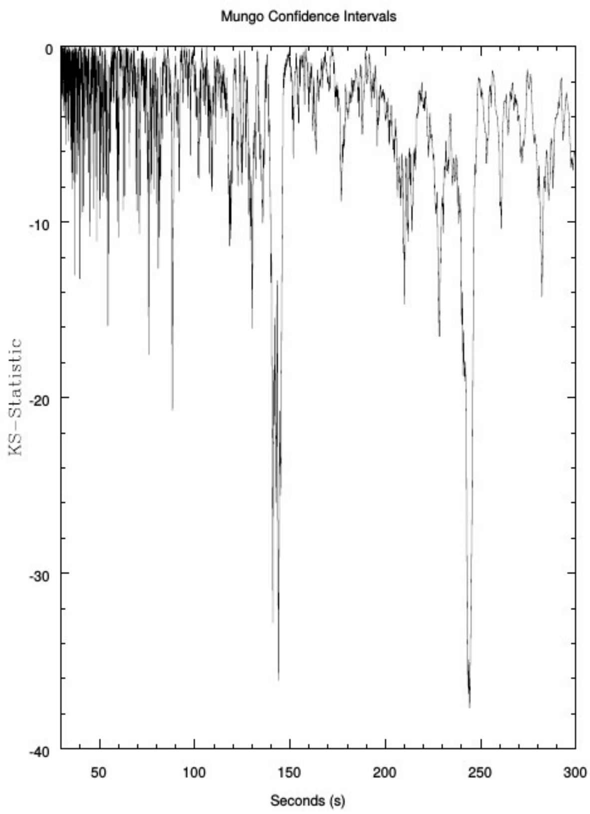
2 ASYMPTOTIC PERIOD SPACINGS AND CONFIDENCE INTERVALS

In the previous analysis of KIC 11558725 done by Reed et al. 2011 and Telting et al. 2012, the period spacings for *ell* = 1 and 2 were identified as $\Delta\Pi_1$ = 246.8 seconds and $\Delta\Pi_2$ = 142.6 seconds, and $\Delta\Pi_1$ = 248.68 seconds and $\Delta\Pi_2$ = 143.37, respectively (Reed M.D., Baran A., Quint A.C., Kawaler S.J., et. al. 2011, MNRAS, 414, 2885; Telting J.H., et al. 2012, A&A, 544, A1). The current analysis of KIC 11558725 is to aimed to confirm the period spacings of Reed et al. and Telting et al. while using the higher resolution, 3 years of short cadence *Kepler* data.

We began our search for the asymptotic period spacings by making a list of the highest amplitude g-mode peaks, and using the KS-test to determine the potential period spacings. The results of our KS-test are plotted in Figure 1.

Noticeably, there are large negative values at approximately 244 and 143 seconds which correspond to the *ell* = 1 and *ell* = 2 period spacings, respectively. We searched for these spacings in our period list to determine, specifically, the periods that fall on each sequence. Using the highest amplitude modes and looking at their period differences, we found a candidate list of 5 radially consecutive overtones for both *ell* = 1 and 2. We fitted these candidates using a linear regression fit. Once a

Figure 1: KS-test for KIC 11558725



line of best fit was determined, we extrapolated the sequences through all radial overtones. Periods that fell within 10% of our sequences were considered as part of the sequence. Once new sequence members were identified, the linear regression fit was updated, extrapolated, and again used to find other sequence members. In total, we identified 60 *ell* = 1 and 66 *ell* = 2 pulsations using asymptotic period spacings. The spacings of these sequences were determined to be 244 ± 0.32 seconds for *ell* = 1 and 144 ± 0.46 seconds for *ell* = 2

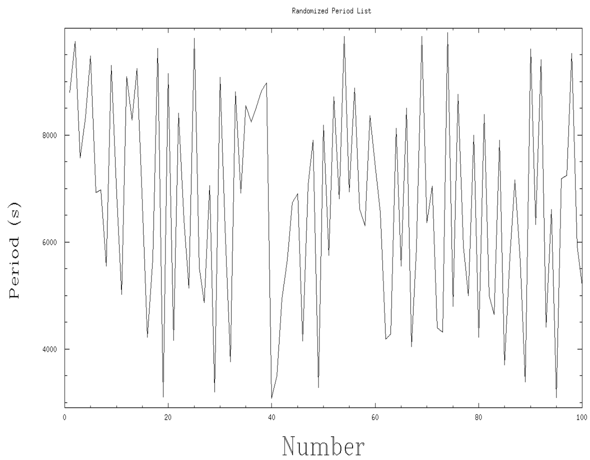
In order to determine whether these sequences are intrinsic to the star, a randomization test was performed and confidence levels were calculated.

3 RANDOMIZATION AND ITERATIVE KS-TESTING

In order to determine whether the period spacings uncovered by the KS-test of KIC 11558725 are statistically significant, the probability of randomized period lists must be explored. More specifically, the lowest KS-statistic given by any random period list (with similar list length and range as compared to our known list) should not exceed the depth of the peaks in the KS-test for KIC 11558725.

For the purpose of this project a confidence interval of 99.9% will be the highest computed. Also, considering the list of periods used to compute the KS-test for KIC 11558725 consisted of ~100 to ~200 values, a total of 1000 randomized period lists ranging from 3000 to 10000 seconds and containing 100 values were generated. Each period list was generated using a python script called rand_numb.py. It utilizes the *random* package in python and calls the function random.randrange(). An example of the randomization of these period lists is shown in Figure 2.

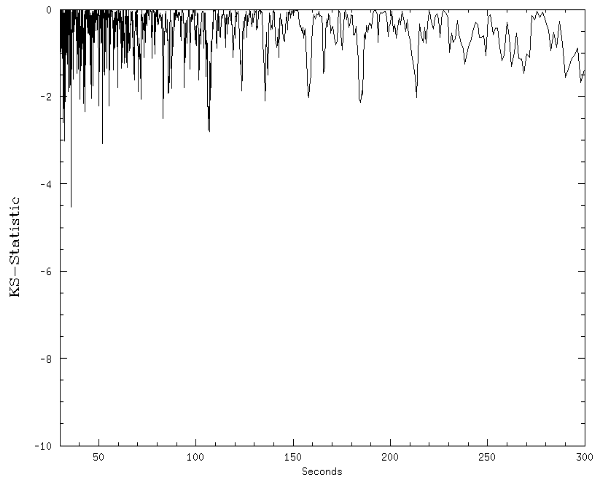
Figure 2: Randomized Period List



The smallest time step of each of my randomized period lists is one second. Translated to frequency-space, this implies that the largest frequency-splitting of my randomized lists is ~0.01 μHz. Also, the resolution of the *Kepler* data for KIC 11558725 is a 1.5/T value of ~0.018 μHz. This means that the randomized period lists are in a state that can be accurately compared with the data

In order to calculate the confidence intervals of the KS-test, the test was computed for each of period lists. The program that performs the KS-test requires parameters such as input file, integer precision, maximum period spacing to compute, and minimum period spacing to compute. A shell script was developed to automate this function for each of the 1000 randomized period lists. An example of the KS-test for a random period list is shown below in Figure 3.

Figure 3: KS-test of randomized period list



4 CONFIDENCE INTERVALS, STATISTICAL SIGNIFICANCE, AND RESULTS

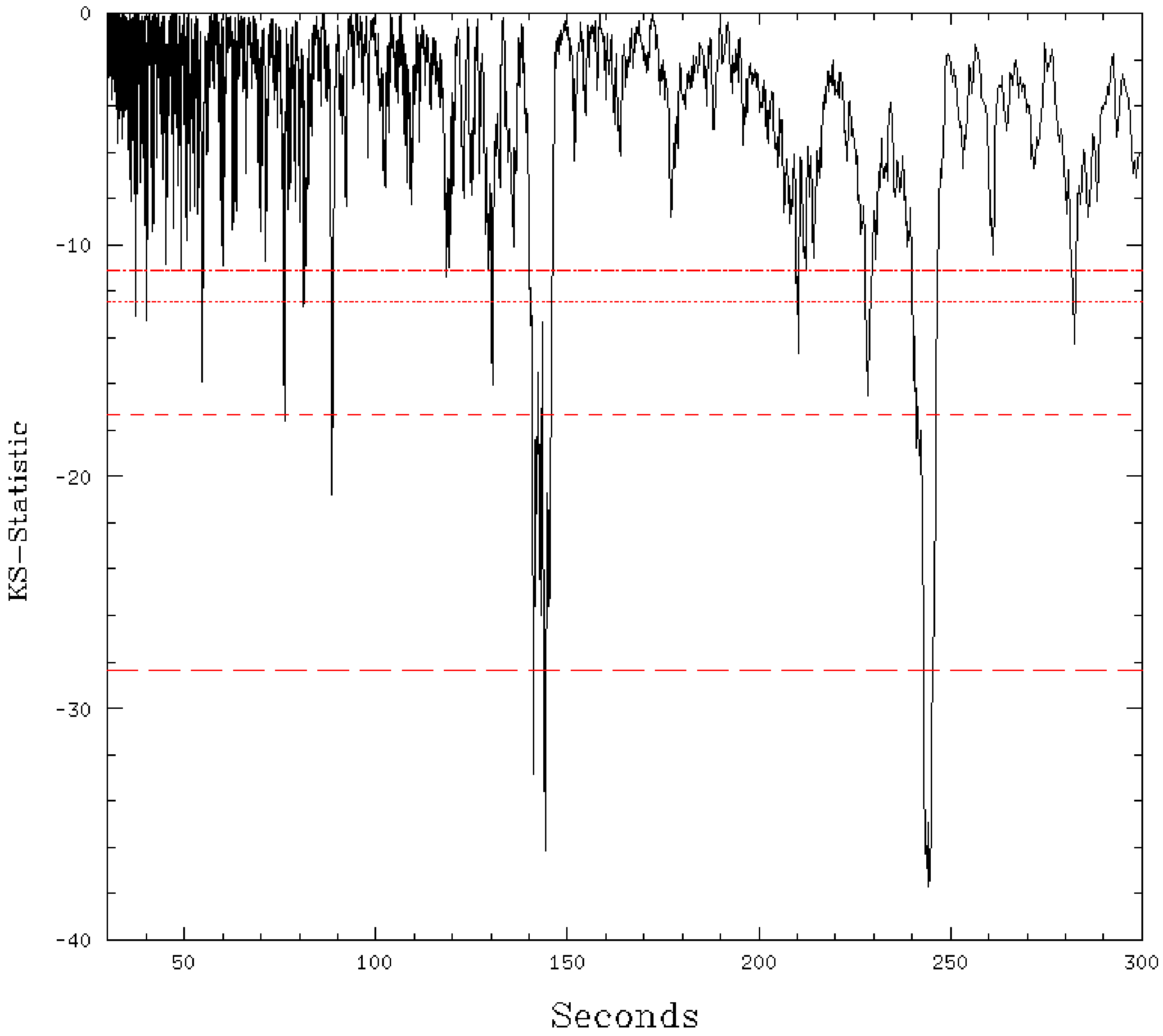
As you can see, the lowest peak in Figure 3 is ~-4.5. The minimum value in each of the 1000 KS-test's was recorded in a list. From this list confidence intervals were calculated. The depth of the single deepest peak is ~-28. This demonstrates that only 1 out of 1000 KS-test's (when computed using randomized period lists containing 100 values in a range from 3000 to 10000) will reach this value. Meaning that 99.9% of the remaining minimum KS-statistic's lie above this value.

To put it another way, if the peaks in a KS-test computed using an observed period list are lower than this value, we are 99.9% confident that it is not due to random chance. Thus the period spacing implied by these peaks is statistically significant and therefore intrinsic to the star. Finally, the confidence intervals calculated from our randomization test are given in terms of the KS-statistic as follows:

99.9% ≡ -28.43
99.0% ≡ -17.33
95.0% ≡ -12.49
90.0% ≡ -11.13

The confidence intervals were plotted on the KS-test for KIC 11558725. The results are shown in Figure 4 where the stated confidence intervals are designated by the red lines. It is clear that the peaks in the KS-test for KIC 11558725 are considerably lower than the 99.9% confidence threshold. Thus, with a 99.9% confidence, the randomization test has shown that the period spacings in KIC 11558725 are intrinsic to the star.

Figure 4: KS-test of KIC 11558725 with confidence intervals



References

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