1 SU(N) generators

The SU(N) generators are generally represented as traceless hermitian matrices,

$$\mathrm{Tr}\,(T_a) = 0 \text{ and } T_a = T_a^\dagger \tag{1}$$

where $a = 1, 2, \cdots, N^2 - 1$.

1.1 Fundamental representation

The generators in the fundamental representation are written by $N \times N$ matrices T_a which satisfy

$$T_a T_b = \frac{1}{N}\,\delta_{ab}\mathbf{I}_N + \frac{1}{\sqrt{2}} \sum_{c=1}^{N^2-1} (if_{abc} + d_{abc})T_c, \tag{2}$$

where the f are, so called structure constants, antisymmetric in all indices while d are symmetric in all indices. As a consquence, we also have

$$\begin{aligned} [T_a, T_b] &= i\sqrt{2} \sum_{c=1}^{N^2-1} f_{abc} T_c, && (3) \\ \{T_a, T_b\} &= \frac{2}{N}\,\delta_{ab}\mathbf{I}_N + \sqrt{2} \sum_{c=1}^{N^2-1} d_{abc} T_c. && (4) \end{aligned}$$

For $N = 2$, the generators are nothing but Pauli matrices with appropriate normalizations:

$$u_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad u_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad u_3 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{5}$$

By generalizing the generators of $SU(2)$ to $SU(N)$, we can obtain the generators for arbitary N.

1. Off-diagonal matrix (u_1-type): for $i < j$, take $1/\sqrt{2}$ to ij and ji elements, and 0 to all others. The total number of these elements is $N(N-1)/2$.
2. Off-diagonal matrix (u_2-type): for $i < j$, take $-i/\sqrt{2}$ to ij elements, $i/\sqrt{2}$ to ji, and 0 to all others. The total number of these matrices is $N(N-1)/2$.
3. Diagonal matrix: take $1/\sqrt{k(k-1)}$ to ii elements for $i < k$, $-(k-1)/\sqrt{k(k-1)}$ to kk for $i = k$, and 0 to all others, where $k = 2, 3, \cdots, N$. The total number of these matrices is $N - 1$.

1.2 Adjoint representation

In the adjoint representation the generators are represented by $(N^2-1)\times(N^2-1)$ matrices whose matrix elements are defined by the structure constants:

$$(T_a)_{bc} = -if_{abc}. \tag{6}$$

The structure constants can be easily evaluated by using the generators in the fundamental representation. By taking trace after multiplying Eq. (3) by T_d and using the normalization condition of $\mathrm{Tr}\,(T_aT_b) = \delta_{ab}$, one may obtain

$$f_{abd} = -\frac{i}{\sqrt{2}}\,\mathrm{Tr}\,([T_a, T_b]T_d). \tag{7}$$