This technique has the distinct advantage that it can be applied to the problem of interpolation of observations to one or more irregularly-spaced points; of course, it also can be applied to the problem of interpolating atmospheric variable values to a regular grid although it does require much more computer time than the previously discussed successive approximation technique.

One very important application of this technique at NSSFC is in the area of determination of erroneous data. That is, the method can be applied to estimate a variable at the site of the observation; if the resulting value differs greatly from the reported value, one can be sure that a data error has been found and necessary steps can be taken to insure that the bad data is eliminated or properly corrected.

A subroutine which determines the analysis of a variable at points of a rectangular grid has been written for the NSSFC CDC 3100 computer and is included in Appendix B (MOBAN).

APPENDIX A

The Polar Stereographic Map Projection

The nearly spherical surface of the earth may be mapped by means of projections which transpose points on the earth's surface into points of an image surface. The two most common types of maps employed in meteorology are the Lambert conformal and the polar stereographic. Saucier (1955) gives some of the details of these projections as well as others which are useful for special applications. Here we discuss the polar stereographic projection.

For our purposes the image surface is a plane which passes through the earth at 60 deg. N. latitude. Thus, we are concerned with a secant projection. The latitude at which the plane intersects the earth is normally referred to as the standard latitude or parallel of the projection and will be denoted as φ_o. Along the standard parallel, distance on the image surface is equal exactly to distance on a spherical earth.

The image scale, σ, is defined as the ratio of image surface distance to earth distance. Thus, σ has a value of one along the standard parallel and is greater or less than one at other latitudes as shown below.

The polar stereographic projection is a conformal map. That is, there is a one-to-one correspondence between angles on the map and angles on the earth's surface. Also, on a conformal map the image scale must be the same in all directions in the vicinity of a point.

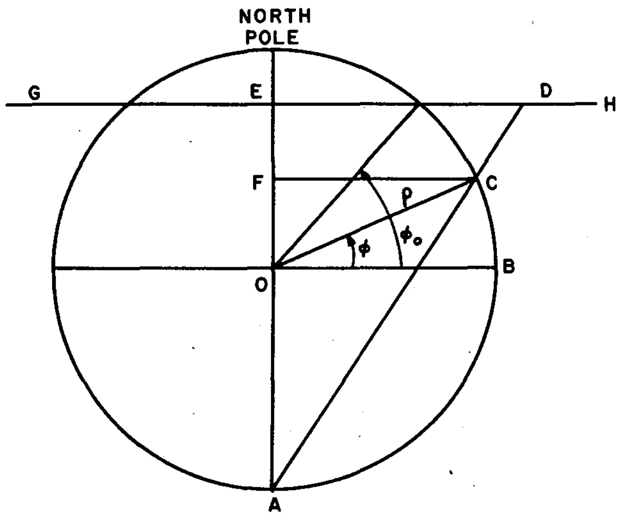

Figure A1. Scheme for polar stereographic map projection with standard parallel at latitude φ_o.

Figure A1 shows the scheme for the polar stereographic map projection. This is the simplest and most convenient of the projections in use in meteorology; it is a perspective projection for which the point of projection is the south pole (for a northern hemisphere map).

The image scale factor, σ, is given by the ratio of the radius of a latitude circle on the image surface to its corresponding radius on the earth. Point C on the earth at latitude φ is projected upon the secant plane GH at point D. Thus, the image scale σ is equal to the ratio of the lengths of the two line segments ED and FC. That is,

$$\sigma = \frac{ED}{FC} \ . \qquad \text{(A.1)}$$

The line segment FC is the radius of the latitude circle passing through C and is equal to $\rho \cos \varphi$, where ρ is the mean radius of the earth. It easily can be shown that the length of the line segment ED, which is the radius of the latitude circle on the image plane, is equal to $\rho(1 + \cos \Psi_o) \tan \frac{\Psi}{2}$, where Ψ is the co-latitude. Thus,

$$\sigma = \frac{\rho(1 + \cos \Psi_o) \tan (\frac{\Psi}{2})}{\rho \cos \varphi} = \frac{1 + \sin \varphi_o}{1 + \sin \varphi} \ . \qquad \text{(A.2)}$$

The variation of the image scale factor with latitude is shown in figure A2. The radius of any latitude circle on the image plane also may be expressed as

$$r = \rho \ \sigma \cos \varphi \ . \qquad \text{(A.3)}$$

A polar stereographic map can be constructed by introducing the map scale, m. This reduction scale is constant for the entire map and is usually printed on the chart (e.g., 1:10,000,000). It is defined by the ratio

$$m = \frac{\text{Distance on map}}{\text{Distance on image plane}} \ .$$

Thus the radius of a latitude circle on the map is given by

$$R = m \rho \sigma \cos \varphi \; . \qquad (A.4)$$

On a polar stereographic map, lines of constant latitude are concentric circles about the pole, of radius R. Lines of constant longitude are straight lines radiating from the pole and are spaced at the same angular increment on the map as on the earth.

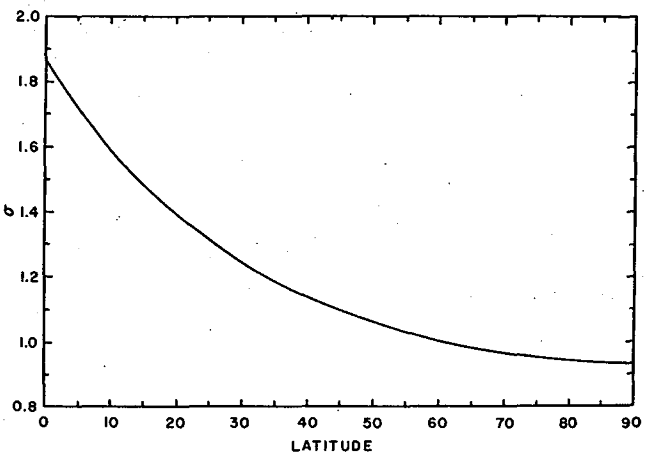

Figure A2. Image scale variation with latitude on the polar stereographic map projection with standard parallel at 60 deg.

a. Space coordinates and finite-difference space coordinates

The simplest rectangular coordinate system on the image plane that one could choose is described below. It is convenient to put the origin of the coordinate system (x,y) at the north pole. Then, on the image plane,

$$\begin{aligned} x &= r \cos \lambda = \rho \sigma \cos \varphi \cos \lambda \; , \\ y &= r \sin \lambda = \rho \sigma \cos \varphi \sin \lambda \; , \end{aligned} \qquad (A.5)$$

where λ is the deviation (λ_o - longitude of point) of longitude from the standard longitude, λ_o. The x-axis lies along the chosen standard longitude line onthe image plane and the y-axis is along the east-west direction at the point of intersection of the y-axis with the standard longitude line. The coordinate system is illustrated in figure A3.

The latitude and deviation of longitude of a point (x,y) can be calculated according to

$$\begin{aligned} \varphi &= \frac{\pi}{2} - 2 \tan^{-1} \left[\frac{(x^2 + y^2)^{\frac{1}{2}}}{\rho \, (1 + \sin \varphi_o)} \right] \\ \lambda &= \tan^{-1} [y/x] \; . \end{aligned} \qquad (A.6)$$

The above expressions for the space coordinates can be evaluated at equally spaced points on a rectangular grid. A constant space increment Δ (i.e., constant on the image plane) is chosen so that the finite difference formulae for the cartesian coordinates may be written as

$$x = (i - 1)\,\Delta + x_o,\ i = 1, 2, \text{----} I\ ,$$
$$y = (j - 1)\,\Delta + y_o,\ j = 1, 2, \text{----} J\ . \tag{A.7}$$

Here we have allowed for the arbitrary translation of the origin of the coordinate system by including the numbers x_o and y_o in the formulas.

b. Velocity components on the image plane

The velocity components, U and V, are related to the observed wind velocity by

$$U = -\,C \sin \alpha\ ,$$
$$V = -\,C \cos \alpha\ , \tag{A.8}$$

where C is the magnitude of the horizontal wind velocity and α is the observed wind direction. This is illustrated in figure A4.

The velocity components along the x- and y-axes of the previously described rectangular coordinate system are u and v, respectively. Figure A5 reveals that these components are related to U and V by

$$u = -U \sin \lambda - V \cos \lambda\ ,$$
$$v = U \cos \lambda - V \sin \lambda\ . \tag{A.9}$$

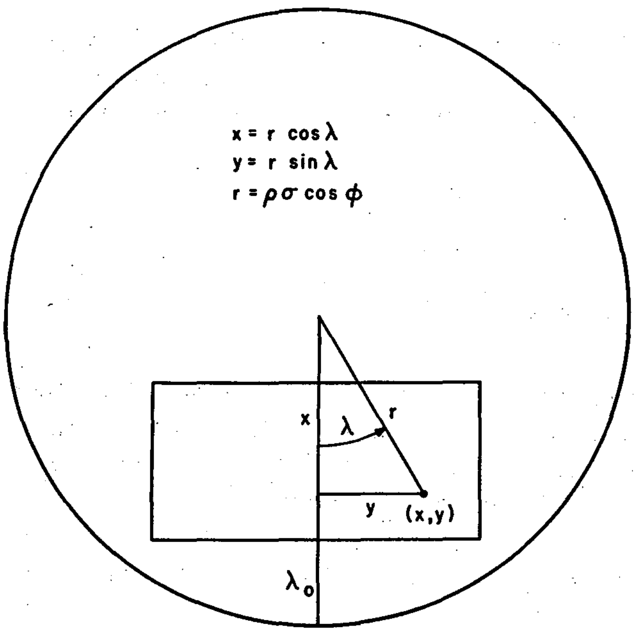

Figure A3. The image plane for the polar stereographic map projection. The computational grid is oriented so that the y-axis is perpendicular to the standard longitude, λ_o.

Substitution for U and V in (A.9) from (A.8) yields

$$u = C \cos (\lambda - \alpha) = \sigma^{-1}\, \dot{x}\ ,$$
$$v = C \sin (\lambda - \alpha) = \sigma^{-1}\, \dot{y}\ , \tag{A.10}$$

where $\dot{x}$ and $\dot{y}$ are velocity components along the x- and y-axes on the image plane, i.e., distance on the image plane per unit time.

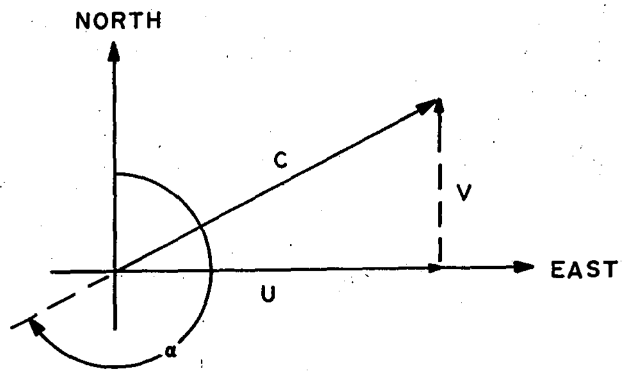

Figure A4. East-west and north-south components of the horizontal wind velocity.

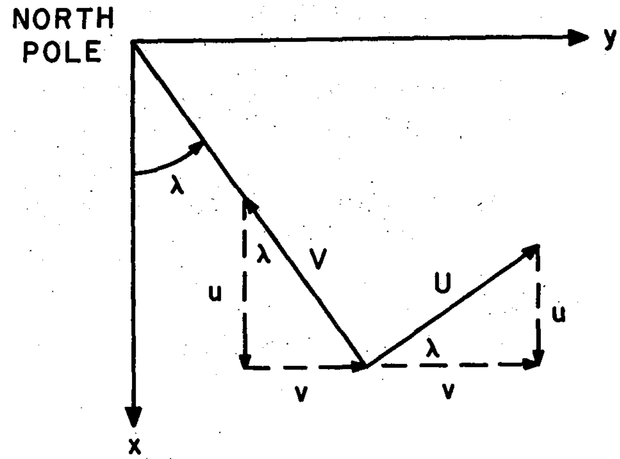

Figure A5. Relation of velocity components (u,v) to the east-west and north-south components of the horizontal wind.

APPENDIX B

Use of Objective Analysis Routines and Program Lists

This appendix contains listings of several FORTRAN subroutines utilized for various objective analyses on the CDC 3100 computer at NSSFC, Kansas City, Missouri. PROGRAM SAMPLE which utilizes some of the routines, is also included. All analysis routines do not have the same features; the user should add those he desires or delete those not needed. Subroutines included in this summary are:

OBAN - This is a general purpose routine to be used when the analysis of a single scalar field is desired.

TOBAN - This routine is similar to OBAN; it provides for economical analysis of two fields, simultaneously.

HOBAN - This routine should be used for constant pressure height analyses; observed heights and winds are used in the analysis.

BOBAN - This routine is similar to OBAN except that the data must be sorted; this routine should be used when surface data is to be analyzed because it is much more efficient than OBAN. A listing of BOBAN is given on pages 62 through 65 of this report.

SORTOB - This subroutine sorts data in the desired manner for use in BOBAN. A listing of this subroutine is given on pages 61 and 62 of this report.