Project 1 - Helicopters!

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Introduction

The aim of this project is to optimize certain design parameters for the construction of a series of helicopter toys. The ideal result is a helicopter design that maximizes hang-time with minimal variance in the times across drops. However, as the budget is limited for the testing of various designs, we cannot perform more than 100 drops across all candidate designs. It is important that we perform these test drops in as economical a fashion as possible–that is, we would like to extract the most information possible about the effects of specific design decisions from each trial run as we can.

A factorial or fractional factorial experimental design is well-suited to a situation like this. Through this procedure, we may define factors with levels that represent different values of the design parameters of interest. Then, having collected data replicates for each specified combination of factor levels, we may perform an analysis to ascertain the effect of each factor along with the effects of possible interactions between factors. Knowing these effects will allow us to determine the optimal design having maximal hang-time and minimal variance.

Design

A few test runs were performed to better understand how to make the helicopters and determined what changes to the design might impact the performance. Given the results from these tests, the following factors will considered in the construction of each candidate helicopter design:

Factor	**Description**	**Low (-)**	**High (+)**
A	Total size	6.5”	11.0”
B	Blade length	4.75”	5.75”
C	Blade width	2.1”	3.0”
D	Stabilizer length	2.75”	4.75”
E	Stabilizer width	0.5”.	1.25”

For convenience we are referring to the bottom section of the helicopter that extends downward beneath the blades as the “stabilizer”. We also noted during our initial tests that taping the bottom of the stabilizer is universally beneficial to the hang-time and stability of the helicopter. We will therefore being applying tape in all designs.

We have elected to conduct a 2_V^{5-1} fractional factorial experiment to analyze the effects of these factors on the hang-time. The generator for this experiment is $I = ABCDE$. This resulted in 16 distinct factor combinations with 5 replicates for each combination, totaling 80 drops. The combinations and data are given in the following plot:

A	B	C	D	E	1	2	3	4	5
-	-	-	-	+	5.05	3.61	4.20	4.20	4.72
-	-	-	+	-	2.84	3.94	3.30	4.45	4.39
-	-	+	-	-	3.29	3.35	3.68	3.68	3.93
-	-	+	+	+	4.12	3.67	4.66	3.36	4.08
-	+	-	-	-	3.35	3.74	3.55	3.93	4.21
-	+	-	+	+	4.13	3.95	3.54	4.72	4.06
-	+	+	-	+	3.21	3.54	3.22	3.29	4.52
-	+	+	+	-	4.39	5.31	4.32	4.85	6.16
+	-	-	-	-	4.13	4.27	4.13	3.35	3.74
+	-	-	+	+	3.80	3.68	3.15	3.34	3.54
+	-	+	-	+	4.08	3.87	3.55	4.32	4.07
+	-	+	+	-	3.69	3.36	4.21	3.42	3.88
+	+	-	-	+	5.83	5.58	5.96	6.16	6.53
+	+	-	+	-	3.84	3.62	4.72	4.66	4.26
+	+	+	-	-	3.66	3.42	4.27	3.47	3.36
+	+	+	+	+	3.61	3.68	4.14	4.19	3.74

Analysis

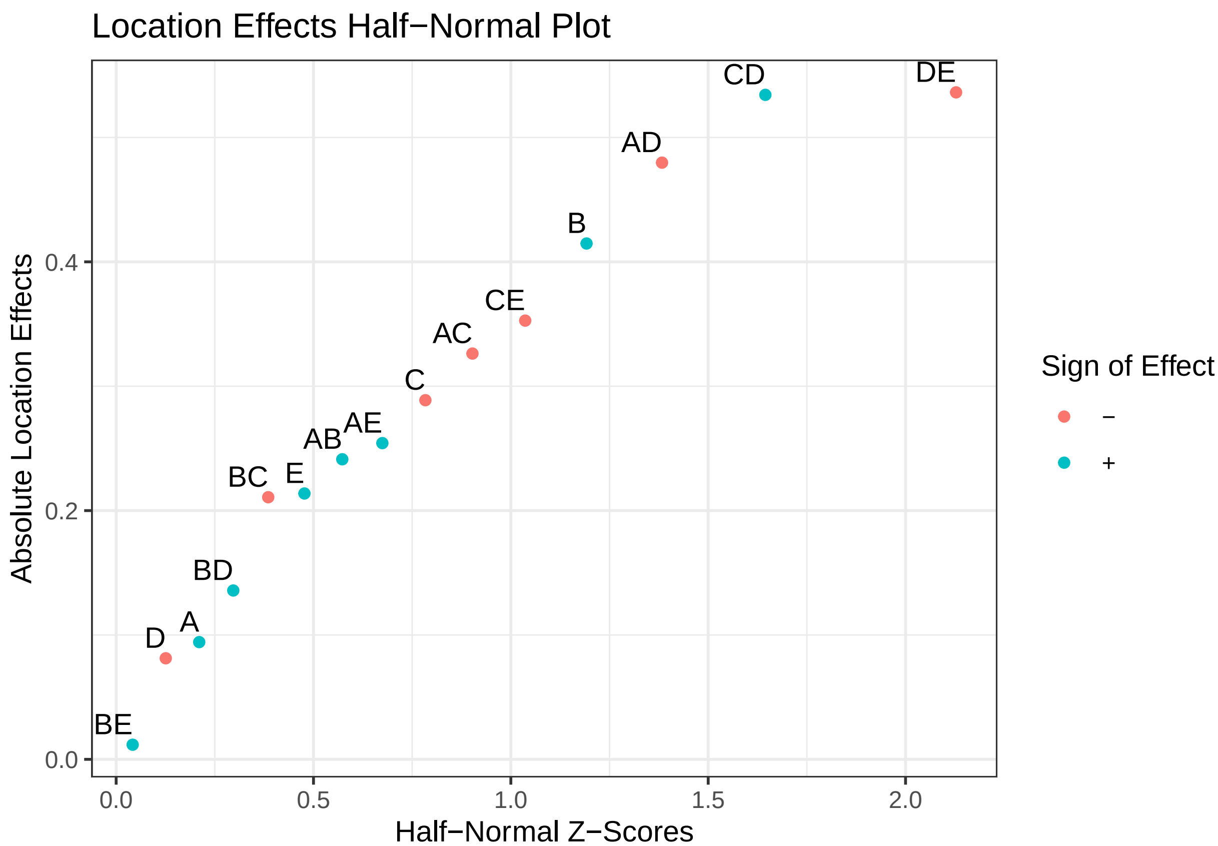
```r
data <- read_excel("flight_data.xlsx") %>%
  pivot_longer(r1:r5, names_to="rep", values_to="time") %>%
  mutate_at(.funs=funs(ifelse(.=='+', 1, -1)),
            .vars=vars(A:E)) %>%
  select(-rep)
```

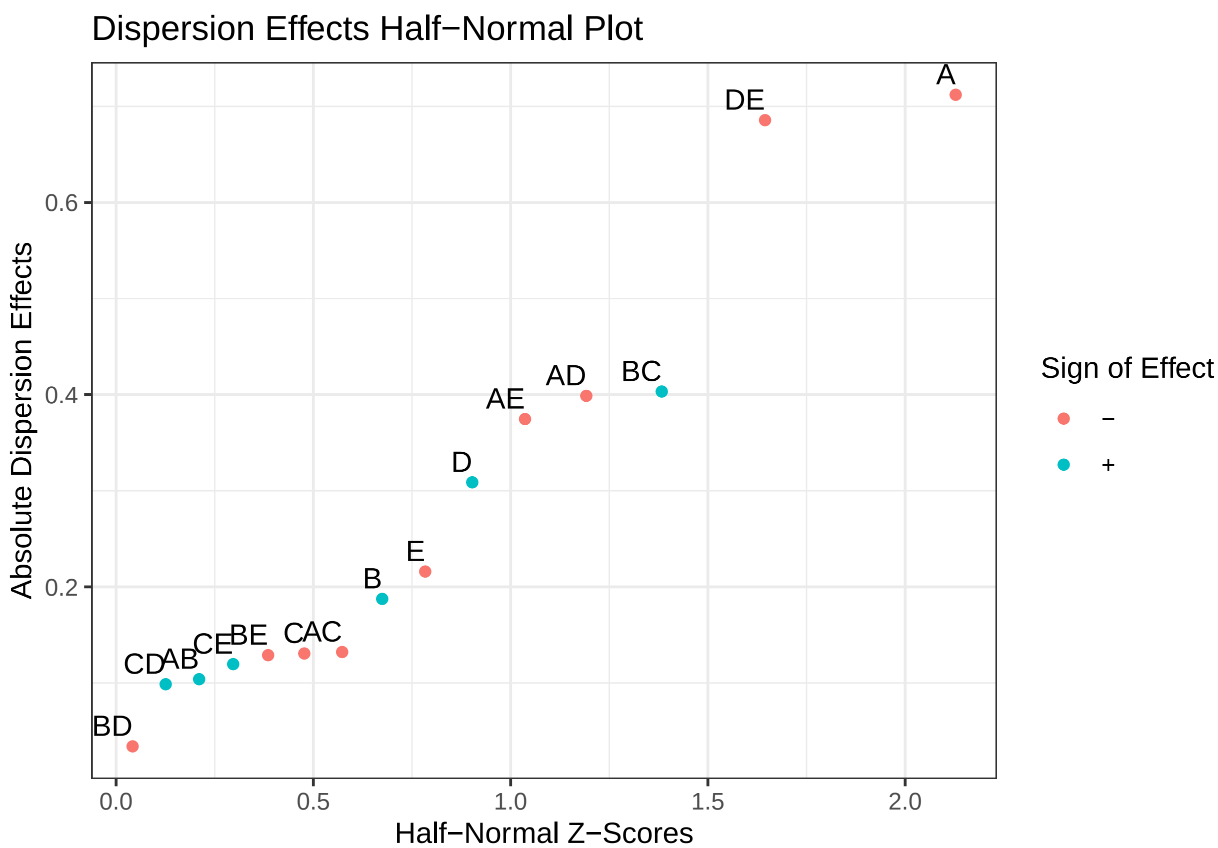
```r
# calculate main location effect
main.loc <- function(df, fac, resp) {
  # for column selection
  fac  <- sym(fac)
  resp <- enquo(resp)

  # calculate main effect
  df %>%
    group_by_at(1:5) %>%
    summarise(y.bar=mean(!!resp)) %>%
    group_by(!!fac) %>%
    summarise(mean.y.bar=mean(y.bar)) %>%
    mutate(to.add=!!fac*mean.y.bar) %>%
    pull(to.add) %>% sum()
}

main.disp <- function(df, fac, resp) {
  # for column selection
  fac  <- sym(fac)
  resp <- enquo(resp)

  # calculate main effect
  df %>%
    group_by_at(1:5) %>%
    summarise(lns.2=log(var(!!resp))) %>%
```

```
    group_by(!!fac) %>%
    summarise(mean.lns.2=mean(lns.2)) %>%
    mutate(to.add=!!fac*mean.lns.2) %>%
    pull(to.add) %>% sum()
}

two.fac.loc <- function(df, fac, resp) {
  facs <- strsplit(fac, "")[[1]]
  fac1 <- sym(facs[1])
  fac2 <- sym(facs[2])
  resp <- enquo(resp)

  # calculate two-factor location
  eff.1 <- df %>%
    filter(!!fac2>0) %>%
    main.loc(., facs[1], !!resp)
  eff.2 <- df %>%
    filter(!!fac2<0) %>%
    main.loc(., facs[1], !!resp)
  0.5 * (eff.1 - eff.2)
}

two.fac.disp <- function(df, fac, resp) {
  facs <- strsplit(fac, "")[[1]]
  fac1 <- sym(facs[1])
  fac2 <- sym(facs[2])
  resp <- enquo(resp)

  # calculate two-factor dispersion
  eff.1 <- df %>%
    filter(!!fac2>0) %>%
    main.disp(., facs[1], !!resp)
  eff.2 <- df %>%
    filter(!!fac2<0) %>%
    main.disp(., facs[1], !!resp)
  0.5 * (eff.1 - eff.2)
}
```

```
# function wrappers
flight.main.loc     <- function(fac) { main.loc(data, fac, time) }
flight.main.disp    <- function(fac) { main.disp(data, fac, time) }
flight.two.fac.loc  <- function(fac) { two.fac.loc(data, fac, time) }
flight.two.fac.disp <- function(fac) { two.fac.disp(data, fac, time) }

# effect labels
labels=c("A",  "B",  "C",  "D",  "E",
         "AB", "AC", "AD", "AE", "BC",
         "BD", "BE", "CD", "CE", "DE")

# effect table
eff.tbl <- tibble(
  label    = labels,
  loc.eff  = c(sapply(labels[1:5],   flight.main.loc),
```

```
                 sapply(labels[6:10],  flight.two.fac.loc),
                 sapply(labels[11:15], flight.two.fac.loc)),
  disp.eff = c(sapply(labels[1:5],   flight.main.disp),
                 sapply(labels[6:10],  flight.two.fac.disp),
                 sapply(labels[11:15], flight.two.fac.disp))
)
```






```
loc.all <- abs(eff.tbl$loc.eff)
cutoff <- 3.75 * median(loc.all)
int.sub <- loc.all[loc.all <= cutoff]
pse <- 1.5 * median(int.sub)

lenth.med <- eff.tbl %>%
  mutate(loc.abs=abs(loc.eff),
         disp.abs=abs(disp.eff)) %>%
  mutate(loc.cut=3.75*median(loc.abs),
         disp.cut=3.75*median(disp.abs)) %>%
  mutate(loc.inc=ifelse(loc.abs<=loc.cut, T, F),
         disp.inc=ifelse(disp.abs<=disp.cut, T, F)) %>%
  filter(loc.inc, disp.inc) %>%
  summarise(loc.pse=1.5*median(loc.abs),
            disp.pse=1.5*median(disp.abs)) %>%
  mutate(ier=2.16)

lenth.tbl <- bind_cols(eff.tbl, lenth.med) %>%
  mutate(loc.sig=abs(loc.eff)/loc.pse > ier,
         disp.sig=abs(disp.eff)/disp.pse > ier) %>%
  mutate(label=labels) %>%
  select(label, loc.sig, disp.sig)

lenth.tbl %>% kable(col.names=c("Factors",
                                "Loc. Significant?",
                                "Disp. Significant?"),
```

Factors	Loc. Significant?	Disp. Significant?
A	FALSE	TRUE
B	FALSE	FALSE
C	FALSE	FALSE
D	FALSE	FALSE
E	FALSE	FALSE
AB	FALSE	FALSE
AC	FALSE	FALSE
AD	FALSE	FALSE
AE	FALSE	FALSE
BC	FALSE	FALSE
BD	FALSE	FALSE
BE	FALSE	FALSE
CD	FALSE	FALSE
CE	FALSE	FALSE
DE	FALSE	TRUE

```
                      align=c('l', 'c', 'c')) %>%
  kable_styling(position="center") %>%
  row_spec(0, bold=T)
```

Optimization

An optimal design is one that stays in the air as long as possible with consistency between drops. In more rigorous terms, we want a design that maximizes the expected hang-time while minimizing dispersion.