POISSON PROCESSES AND 911 CALLS

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INTRODUCTION

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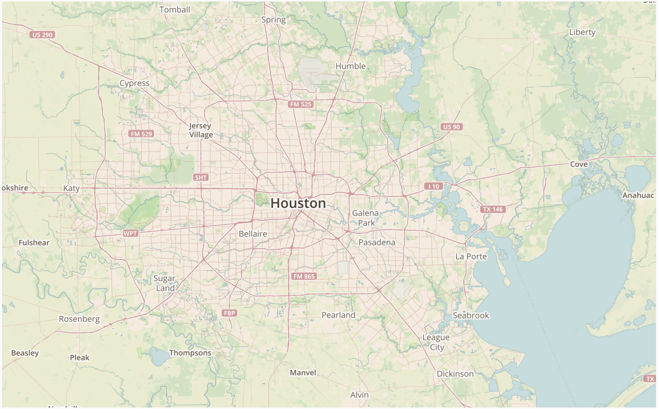

Data: 1389 heat-related 911 calls, 2006-2010

How many 911 calls will come in and where they will come from?

Spatial data is:

- nonlinear
- highly correlated

so how do we model it?

MODEL

We use a Poisson point process model

- $N \sim Pois(\int_{\mathcal{S}} \Lambda(s))$
- $f(\mathrm{s}_1, \mathrm{s}_2, \ldots, \mathrm{s}_N | N) = \prod_{i=1}^{N} \frac{\Lambda(\mathrm{s}_i)}{\int_{\mathcal{S}} \Lambda(\mathrm{s})}$
- $\Lambda(\mathrm{s})$ is the **intensity function**
- $\mathrm{s}_1, \mathrm{s}_2, \ldots, \mathrm{s}_N$ where N = 1389 are the latitude-longitude coordinates of the observed 911 calls
- $\mathcal{S}$ is the set of all latitude-longitude coordinates in Houston

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$\Lambda(\mathrm{s})$ is the key!

It can be shown the likelihood is:

$$L(\Lambda(\mathbf{s})) = \exp\left(-\int_{\mathcal{S}} \Lambda(\mathbf{s})\right) \prod_{i=1}^{N} \Lambda(\mathbf{s}_i)$$

- To simplify we are going to divide $\mathcal{S}$ into a grid
- $\mathbf{s_1}^*, \mathbf{s_2}^*, \ldots, \mathbf{s_K}^*$ where K = 1428 are the latitude-longitude coordinates of the prediction locations
- This allows us to discretize $\Lambda(\mathbf{s})$ to make modeling easier:

$$\Lambda(\mathbf{s}) = \delta \sum_{k=1}^{K} \frac{\lambda_k}{|\mathcal{G}_k|} \mathbb{1}\left\{\mathbf{s} \in \mathcal{G}_k\right\}$$

where $\lambda_k \in (0, 1)$ and $\sum_{k=1}^{K} \lambda_k = 1$

MODEL

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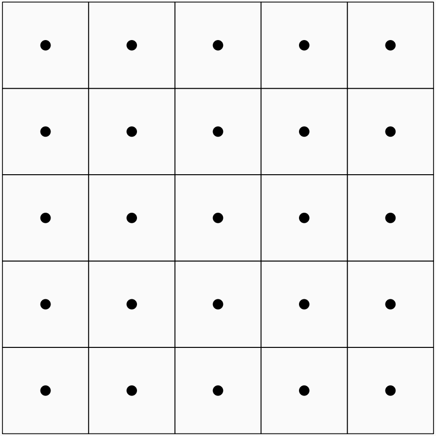

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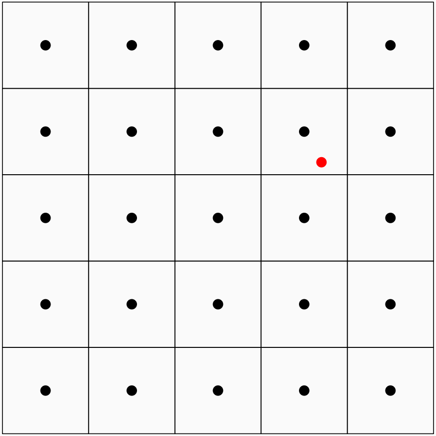

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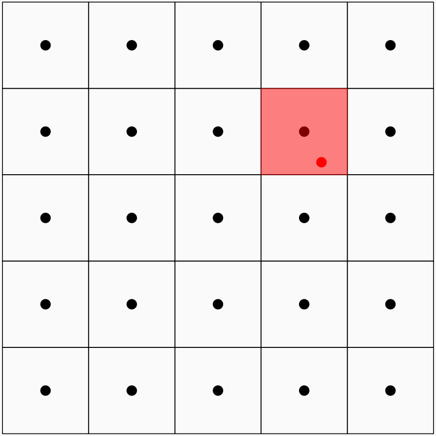

With

$$\Lambda(\mathsf{s}) = \delta \sum_{k=1}^{K} \frac{\lambda_k}{|\mathcal{G}_k|} \mathbb{1}\left\{\mathsf{s} \in \mathcal{G}_k\right\}$$

the likelihood can be rewritten as

$$\begin{aligned} L(\Lambda(\mathsf{s})) &= \exp\left(-\delta\right) \delta^N \prod_{i=1}^{N} \frac{\lambda_{\{k:\mathsf{s}_i \in \mathcal{G}_k\}}}{|\mathcal{G}_k|} \\ &= \exp\left(-\delta\right) \delta^N \prod_{i=1}^{K} \frac{\lambda_k^{N_k}}{|\mathcal{G}_k|} \end{aligned}$$

FITTING THE MODEL

$\delta \sim \text{gamma}(0.001, 0.001)$, conjugate

How can we use the spatial smoothness in $\Lambda(\mathbf{s})$ to estimate $\lambda_1, \ldots, \lambda_k$?

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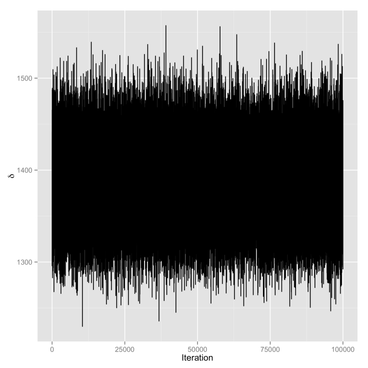
We use a Gaussian process:

- Let $\lambda_k \propto \exp(\lambda_k^*)$
- Let $\boldsymbol{\lambda}^* = \begin{pmatrix} \lambda_1^* \\ \vdots \\ \lambda_k^* \end{pmatrix} \sim \mathcal{N}(\mathbf{0}, \sigma^2 \mathsf{M}_\nu(D|\phi))$
- Then it follows that $\lambda_k = \frac{\exp(\lambda_k^*)}{\sum_{j=1}^K \exp(\lambda_j^*)}$

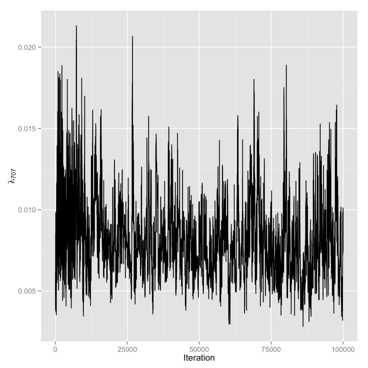
With $\boldsymbol{\lambda}^* \sim \mathcal{N}(0, \sigma^2 \mathsf{M})$,

$$\pi(\boldsymbol{\lambda}^*) L(\boldsymbol{\lambda}^* | N) \propto \exp\left(-\frac{1}{2\sigma^2} (\boldsymbol{\lambda}^*)' \mathsf{M}^{-1} (\boldsymbol{\lambda}^*)\right) \prod_{k=1}^{K} \left(\frac{\exp(\lambda_k^*)}{\sum_{j=1}^{K} \exp(\lambda_j^*)}\right)^{N_k}$$

Using a Metropolis within Gibbs sampler, we generate 100,000 draws for δ and $\boldsymbol{\lambda}^*$



Trace for δ

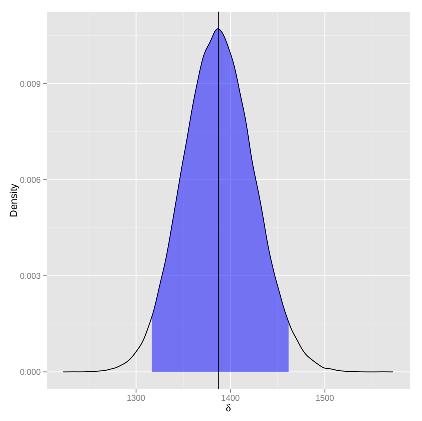


Trace for λ_{707}

RESULTS

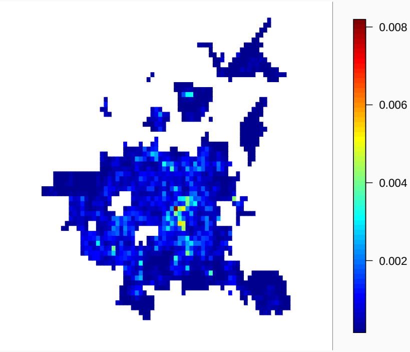
RESULTS

Posterior Density of δ



Posterior Mean: 1387.65

Posterior Density of $\boldsymbol{\lambda}$



CONCLUSIONS

Successfully answered the questions:

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So what?

Next steps:

- Add in a lagged effect for temperature
- Analyze demographics (e.g. age, race, etc.)
- Merge 911 call map with a similar map for heat-related mortalities

QUESTIONS?