POISSON PROCESSES AND 911 CALLS

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INTRODUCTION

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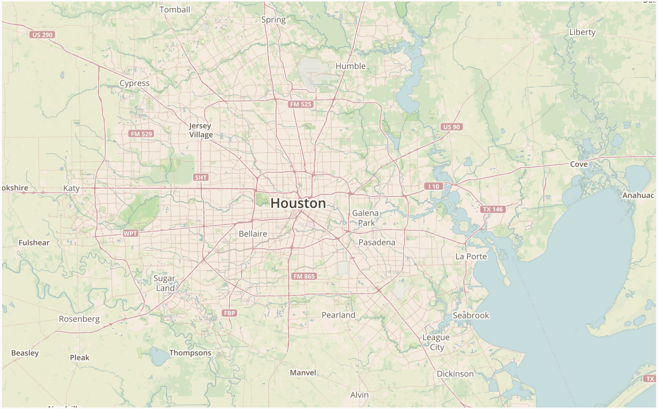
- Extreme heat scenarios pose a significant threat to public health
- It is of interest for researchers to understand how the effect of heat varies by location


Tomball
Spring
Liberty
US 290
Humble
Cypress
FM 525
US 90
Jersey Village
FM 529
Cove
SHT
I 10
Anahuac
Katy
Houston
TX 146
Galena Park
WPT
Bellaire
Fulshear
Pasadena
La Porte
Sugar Land
FM 865
FBP
Pearland
Seabrook
Rosenberg
League City
Beasley
Pleak
Thompsons
Manvel
Dickinson
TX
Alvin

Our data includes information about 1389 heat-related 911 calls

How many 911 calls will come in and where they will come from?

Spatial data is:

- nonlinear
- highly correlated

so how do we model it?

We use a point process model

- Models both frequency and location of events
- Relies on an intensity function, $\Lambda(x)$
- $\Lambda(x)$ integrates to the expected number of events
- If we normalize $\Lambda(x)$ it becomes the density of event location

$\Lambda(x)$ is the key!

MODEL

- $\mathsf{s_1, s_2, \ldots, s_N}$ where N = 1389 are the latitude-longitude coordinates of the observed 911 calls
- $\mathcal{S}$ is the set of all latitude-longitude coordinates in Houston
- $N \sim Pois(\int_{\mathcal{S}} \Lambda(s))$
- $f(\mathsf{s_1, s_2, \ldots, s_N} | N) = \prod_{i=1}^{N} \frac{\Lambda(\mathsf{s_i})}{\int_{\mathcal{S}} \Lambda(\mathsf{s})}$

Our likelihood is:

$$L(\Lambda(\mathbf{s})) = \frac{\exp(-\int_{\mathcal{S}} \Lambda(\mathbf{s}))\left(\int_{\mathcal{S}} \Lambda(\mathbf{s})\right)^{N}}{N!} \prod_{i=1}^{N} \frac{\Lambda(\mathbf{s}_{\mathrm{i}})}{\int_{\mathcal{S}} \Lambda(\mathbf{s})}$$

which is equivalent to:

$$L(\Lambda(\mathbf{s})) = \exp\left(-\int_{\mathcal{S}} \Lambda(\mathbf{s})\right) \prod_{i=1}^{N} \Lambda(\mathbf{s}_i)$$

- To simplify we are going to divide $\mathcal{S}$ into a grid
- $\mathbf{s}_1{}^*, \mathbf{s}_2{}^*, \ldots, \mathbf{s}_K{}^*$ where K = 1428 are the latitude-longitude coordinates of the prediction locations
- This allows us to discretize $\Lambda(\mathbf{s})$ to make modeling easier:

$$\Lambda(\mathbf{s}) = \delta \sum_{k=1}^{K} \frac{\lambda_k}{|\mathcal{G}_k|} \mathbb{1}\left\{\mathbf{s} \in \mathcal{G}_k\right\}$$

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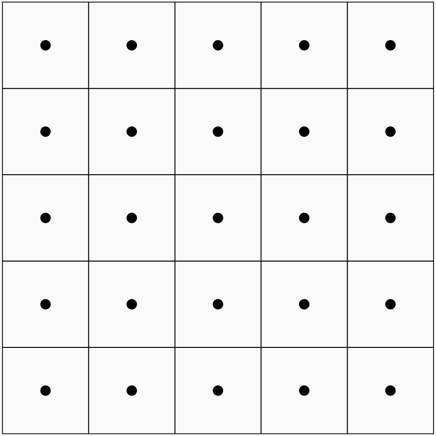
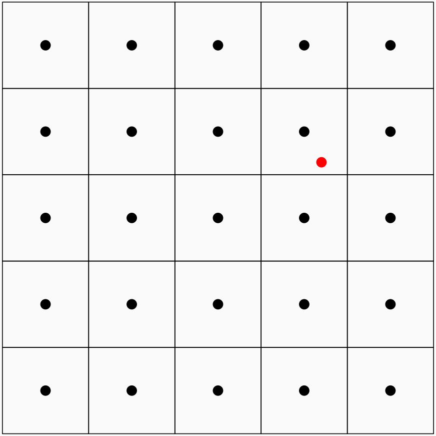
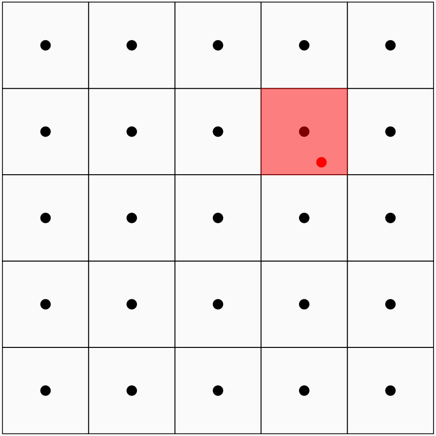
- δ is the expected number of 911 calls
- λ_k is the probability that a 911 call occurs in grid cell $\mathcal{G}_k$
- $\mathcal{G}_k$ is the set of spatial coordinates that are closest to grid point $\mathbf{s}_k^*$
- $|\mathcal{G}_k| = \int_{\mathcal{G}_k} d\mathbf{s}$ is the the “area” of grid cell $\mathcal{G}_k$
- $\lambda_k \in (0, 1)$ and $\sum_{k=1}^{K} \lambda_k = 1$



MODEL



MODEL



With $\Lambda(\mathsf{s}) = \delta \sum_{k=1}^{K} \frac{\lambda_k}{|\mathcal{G}_k|} \mathbb{1}\left\{\mathsf{s} \in \mathcal{G}_k\right\}$ our likelihood can be written

$$L(\Lambda(\mathsf{s})) = \exp\left(-\delta\right) \delta^N \prod_{i=1}^{N} \frac{\lambda_{\{k:\mathsf{s}_i \in \mathcal{G}_k\}}}{|\mathcal{G}_k|}$$

FITTING THE MODEL

If we use a gamma$(0.001, 0.001)$ distribution as the prior for δ then the posterior distribution for δ is

$$\begin{aligned}\pi(\delta)L(\delta|N) &\propto \exp(-\delta)\delta^N \exp(-0.001\delta)\delta^{0.001-1} \\ &= \exp(-1.001\delta)\delta^{N+0.001-1}\end{aligned}$$

which we recognize as the kernel of a gamma$(N + 0.001, 1.001)$ distribution

How can we use the spatial smoothness in $\Lambda(\mathbf{s})$ to estimate $\lambda_1, \ldots, \lambda_k$?

We use a Gaussian process:

- Let $\lambda_k \propto \exp(\lambda_k^*)$
- Let $\boldsymbol{\lambda}^* = \begin{pmatrix} \lambda_1^* \\ \vdots \\ \lambda_k^* \end{pmatrix} \sim \mathcal{N}(\mathbf{0}, \sigma^2 \mathbf{M})$
- $\mathbf{M}$ is a $K \times K$ correlation matrix with ij^{th} element $M_\nu(\|\mathbf{s}_i^* - \mathbf{s}_j^*\| \,|\phi)$ where $M_\nu(d|\phi)$ is the Matérn correlation function with smoothness ν and decay ϕ
- Then it follows that $\lambda_k = \frac{\exp(\lambda_k^*)}{\sum_{j=1}^{K} \exp(\lambda_j^*)}$

If we use an inverse-gamma(0.01, 0.01) distribution as the prior for σ^2 then the posterior distribution is:

$$\pi(\sigma^2)L(\sigma^2|\boldsymbol{\lambda}^*) \propto (\sigma^2)^{-(0.01+\frac{N}{2})} \exp\left\{-\frac{1}{2\sigma^2}\left[0.01+\frac{1}{2}(\boldsymbol{\lambda}^*)'\mathsf{M}^{-1}(\boldsymbol{\lambda}^*)\right]\right\}$$

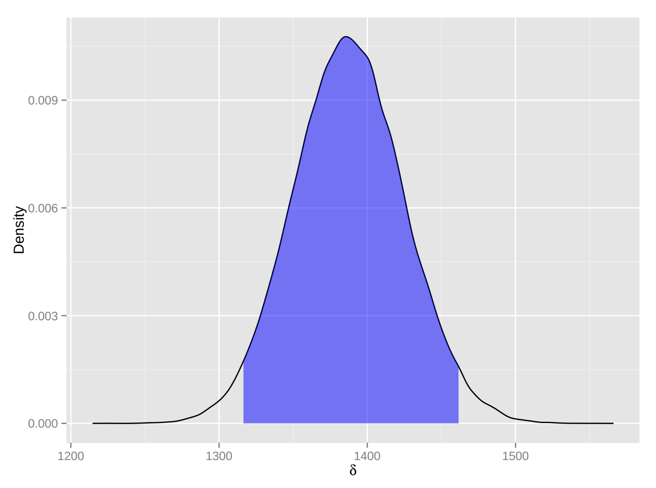
which we recognize as the kernel of an inverse-gamma$(0.01+\frac{N}{2}, 0.01+\frac{1}{2}(\boldsymbol{\lambda}^*)'\mathsf{M}^{-1}(\boldsymbol{\lambda}^*))$ distribution

$$L(\boldsymbol{\lambda}) \propto \prod_{i=1}^{K} \lambda_k^{N_k}$$

RESULTS

RESULTS

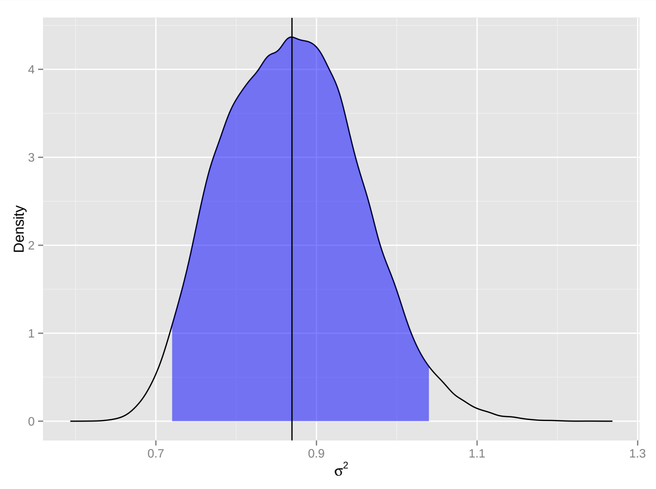
Posterior Density of δ



Posterior Mean: 1387.65

RESULTS

Posterior Density of σ^2



Posterior Mean: 0.87

Posterior Density of $\boldsymbol{\lambda}$