The Plasma Blackout Problem Equation Cheat Sheet

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Intro to the Blackout Problem

Plasma Electron Frequency

$$\omega_e = \sqrt{\frac{n_e e^2}{\epsilon_0 m_e}}\ rad/sec \tag{1}$$

This can be approximated as

$$f_e \approx 8.79\sqrt{n_e}\ Hz \tag{2}$$

Lorentz Windows

These are the equations used by Kim et. al. in the computer modeling of a plasma sheath in the vicinity of an $\mathbf{E} \times \mathbf{B}$ field [1].

[1] Minkwan Kim, Michael Keidar, and Iain D. Boyd. Analysis of an Electromagnetic Mitigation Scheme for Reentry Telemetry Through Plasma. *Journal of Spacecraft and Rockets*, 45(6):1223–1229, 2008

Ion Transport Equations

$$\nabla \cdot (\mathbf{V_i} n) = 0 \tag{3}$$

$$m_i n(\mathbf{V_i} \cdot \nabla \mathbf{V_i}) = en(\mathbf{E} + \mathbf{V_i} \times \mathbf{B}) - m_i n \nu_c \mathbf{V_i} \tag{4}$$

Current Density of Plasma Sheath The current density of the plasma is derived from the generalized Ohm's Law, and includes the Hall effect:

$$\mathbf{j} = \sigma\left(\mathbf{E} + \frac{kT_e}{e}\nabla \ln n - \frac{\mathbf{j} \times \mathbf{B}}{en} + (\mathbf{V_i} \times \mathbf{B})\right) \tag{5}$$

The x and z components of the current density used in the 2D simulations simplify to:

$$j_x = \sigma\left(E_x + \frac{kT_e}{e}\frac{\partial \ln n}{\partial x} - V_y B_z\right) \tag{6}$$

$$j_z = \sigma\left(E_z + \frac{kT_e}{e}\frac{\partial \ln n}{\partial z}\right) \tag{7}$$

Current Density Conservation

$$\nabla \cdot \mathbf{j} = 0 \tag{8}$$

$$e = mc2 \tag{9}$$

References

[1] Minkwan Kim, Michael Keidar, and Iain D. Boyd. Analysis of an Electromagnetic Mitigation Scheme for Reentry Telemetry Through Plasma. *Journal of Spacecraft and Rockets*, 45(6):1223–1229, 2008.